



# Comparison of FRF measurements and mode shapes determined using optically image based, laser, and accelerometer measurements

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## ABSTRACT

Today, accelerometers and laser Doppler vibrometers are widely accepted as valid measurement tools for structural dynamic measurements. However, limitations of these transducers prevent the accurate measurement of some phenomena. For example, accelerometers typically measure motion at a limited number of discrete points and can mass load a structure. Scanning laser vibrometers have a very wide frequency range and can measure many points without mass-loading, but are sensitive to large displacements and can have lengthy acquisition times due to sequential measurements. Image-based stereo-photogrammetry techniques provide additional measurement capabilities that complement the current array of measurement systems by providing an alternative that favors high-displacement and low-frequency vibrations typically difficult to measure with accelerometers and laser vibrometers. Within this paper, digital image correlation, three-dimensional (3D) point-tracking, 3D laser vibrometry, and accelerometer measurements are all used to measure the dynamics of a structure to compare each of the techniques. Each approach has its benefits and drawbacks, so comparative measurements are made using these approaches to show some of the strengths and weaknesses of each technique. Additionally, the displacements determined using 3D point-tracking are used to calculate frequency response functions, from which mode shapes are extracted. The image-based frequency response functions (FRFs) are compared to those obtained by collocated accelerometers. Extracted mode shapes are then compared to those of a previously validated finite element model (FEM) of the test structure and are shown to have excellent agreement between the FEM and the conventional measurement approaches when compared using the Modal Assurance Criterion (MAC) and Pseudo-Orthogonality Check (POC).

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## 1. Introduction and motivation

Modal testing can be performed using a variety of different experimental techniques. Accelerometer, laser vibrometer, and stereo-photogrammetry measurement systems all have advantages and drawbacks, so each must be implemented where they will be most effectively employed. Accelerometers are by far the most traditional and widely used sensors employed in modal testing. Their ease of use allows for quick, broadband measurements to be made, however the effects of mass-loading (especially at higher frequency ranges or for lightweight structures) can corrupt a measurement and a large channel count can be challenging due to cost and bookkeeping. Laser Doppler vibrometers provide a non-contacting,

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broadband alternative to accelerometers, but large displacements and rigid body motion can dramatically contaminate the data and measurements over an area need to be scanned sequentially. Conversely, three-dimensional (3D) digital image correlation (DIC) and 3D point-tracking (3DPT) (or dynamic photogrammetry) are both displacement based approaches that analyze stereo image pairs to measure the 3D motion of surface patterns or specific points, respectively. Stereophotogrammetry has been used for many years in the field of solid mechanics to measure full field displacement and strain, but only very recently has the technique been exploited for dynamic applications to measure vibration [1–6]. As with laser vibrometry, a line of sight for both cameras must be maintained with the measurement points of interest. Furthermore, surface preparation is required for both techniques; a speckle pattern is applied to a test object prior to imaging for DIC while 3D point-tracking tracks high-contrast, circular targets. Three-dimensional point-tracking monitors the response at these discrete targets, while DIC is capable of providing a relatively continuous measurement on the order of tens of thousands of points, throughout a continuously patterned surface. More details and an overview of DIC measurement can be found in Ref. [6].

The primary motivation for exploring DIC as a measurement approach for structural dynamics is the fact that this non-contacting full-field technique can take shape measurements at thousands of points on the surface of an object in a single snapshot. With this abundant wealth of information now available from test, the ability to perform meaningful correlation with large scale finite element models is greatly improved as well as to monitor the full-field transient response of a structure in a single test.

Likewise, with the advent of digital cameras, optical point-tracking is becoming a more common method to track the motion of optical targets that are attached to a rigid or flexible body. To date, 3DPT has not been validated within the field of structural dynamics as a non-contacting vibration measurement tool. When evaluating the performance of any new system or technique, it is critical to compare the new approach to existing measurement methods or to analytical solutions. To accomplish this end, a well documented and understood test article were chosen to compare the image-based approaches to established measurement techniques. A structure known as the “Base-Upright” (BU) was chosen for its well-known dynamic characteristics to compare these four measurement approaches (DIC, 3DPT, laser vibrometry, and accelerometers). Test setups and measurement considerations for each case are addressed. Each test is correlated to a well known and highly accurate finite element model and to the other test cases using the Modal Assurance Criterion (MAC) or Pseudo-Orthogonality Check (POC).

To obtain accurate mode shapes from accelerometer and vibrometer FRF measurements, modal parameter estimation must be performed. With both optical based systems studied in this paper, the mode shapes are measured directly using low-speed cameras and forced normal mode testing (FNMT). A summary of past work using accelerometers, laser Doppler vibrometers, and low-speed cameras is presented, followed by a thorough discussion of the setup and results of tests run with high-speed cameras. Finally, conclusions are drawn to highlight the strengths and weaknesses of 3D point-tracking using high-speed cameras relative to the other measurement techniques.

The use of digital image correlation for vibration measurement is currently in its infancy. To the authors' knowledge, this paper represents the first attempt to obtain frequency response function measurements for a vibrating structure using 3DPT while simultaneously validating the shape of the two optically based measurements to the two other traditional vibration measurements as well as to a finite element model. Some of the advantages of the technique are discussed along with limitations that must be considered.

## 2. Description of the test article and finite element model

The BU was designed to be a time-invariant mechanical structure with well-spaced, directional modes that could be identified easily. Fig. 1a shows the BU with the primary dimensions labeled. The base plate is  $24 \times 24$  in ( $0.6096 \times 0.6096$  m<sup>2</sup>) in dimension and rigidly bolted to the concrete laboratory floor at four locations, while the upright

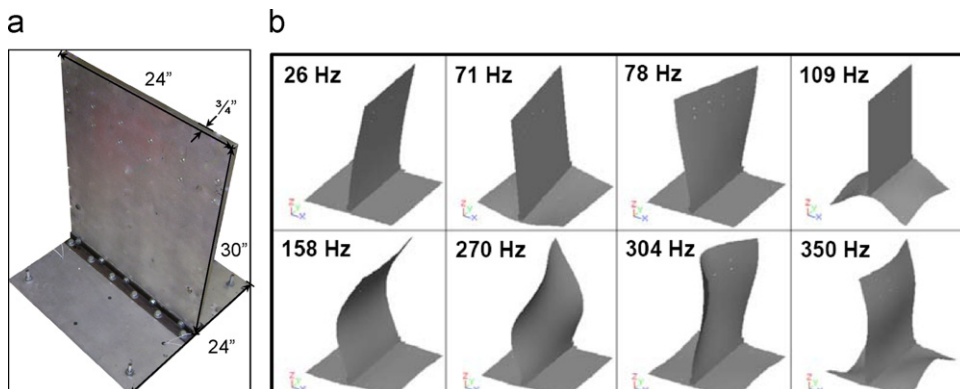


Fig. 1. (a) Photo of the base-upright (BU) with dimensions and (b) first 8 analytical frequencies and mode shapes of the BU.

is  $24 \times 30 \text{ in}^2$  ( $0.6096 \times 0.762 \text{ m}^2$ ) in dimension. Both plates are made from  $3/4''$  (1.905 cm) thick aluminum and are connected by two steel angle brackets and set of bolts. A finite element model (FEM) is available and has been shown to be very well correlated to other measured test data from previous studies [7–10]. The FEM was assembled with solid elements and has approximately 58,000 degrees of freedom (DOF). For reference, the analytical frequencies and mode shapes for the first 8 modes are shown in Fig. 1b. In previous work, excellent correlation between both laser and accelerometer measurements and the BU finite element model was presented [10]. Comparisons were made in terms of the Modal Assurance Criterion (MAC) and Pseudo-Orthogonality Check (POC). The average resonant frequency difference is less than 2.5% and the minimum MAC is greater than 0.97 for the first 7 modes, confirming that the model is a very good representation of the true structure. Depending on the DAQ system being used, necessary changes in the test setup were made when transitioning from the more traditional modal approaches to DIC and 3D point-tracking testing. The two configurations are described in detail below.

### 2.1. Accelerometer and laser Doppler vibrometer test setup

When the accelerometer and laser data was acquired, shaker excitation was provided at an angle  $45^\circ$  relative to all three principle axes so that all modes would be excited. Although white noise random excitation could have been used, it was not considered in this work. The force excitation was a pseudo-random signal having a bandwidth from 0 to 400 Hz. Thirty averages were taken with 1600 lines of spectral resolution to obtain adequate coherence over the frequency range of interest. The shaker mounted to the BU with the numbered laser and accelerometer measurement points indicated by red dots is shown in Fig. 2a. An overlay of FRFs in the z-direction measured by an accelerometer and the laser Doppler vibrometer at point 3 is shown in Fig. 2b. A frequency domain, polynomial curve fitter was then used to extract the modal parameters and mode shapes.

### 2.2. DIC & 3D point-tracking test setup (low-speed cameras)

The two optically based measurement techniques (DIC and 3DPT) monitored the response of the BU at the same eight locations as performed for the accelerometer and laser tests. Comparisons were made only at eight points on the structure such that all of the approaches could be compared at several locations simultaneously while sufficiently representing the dynamics of the structure. Previously performed and more detailed laser scans of the structure (using 85 points) indicate that these eight measurement locations are sufficient to capture the mode shapes of the structure over the frequency range of interest [10]. The prior results showed that that the 8 point measurement produces essentially the same correlation results as the 85 point test. Therefore, the use of only 8 points does not seriously degrade the overall results obtained when compared to the larger set of 85 points. For higher order mode comparison, more measurement locations would have to be made.

While DIC can be used over the whole surface [2], an alternative approach with discrete patches of patterns was employed. This was done to illustrate that equivalent data could be extracted without having to pattern the entire structure. The surface treatment applied to the BU needed for the DIC test can be seen in Fig. 3a. To create the patches, flat white spray paint coated the areas of interest. A black permanent marker was then used to create the speckle pattern. Circular optical targets with adhesive backs were subsequently applied over the speckle pattern at the eight locations collocated with the accelerometers and laser measurement points prior to performing the 3D point-tracking test. At these

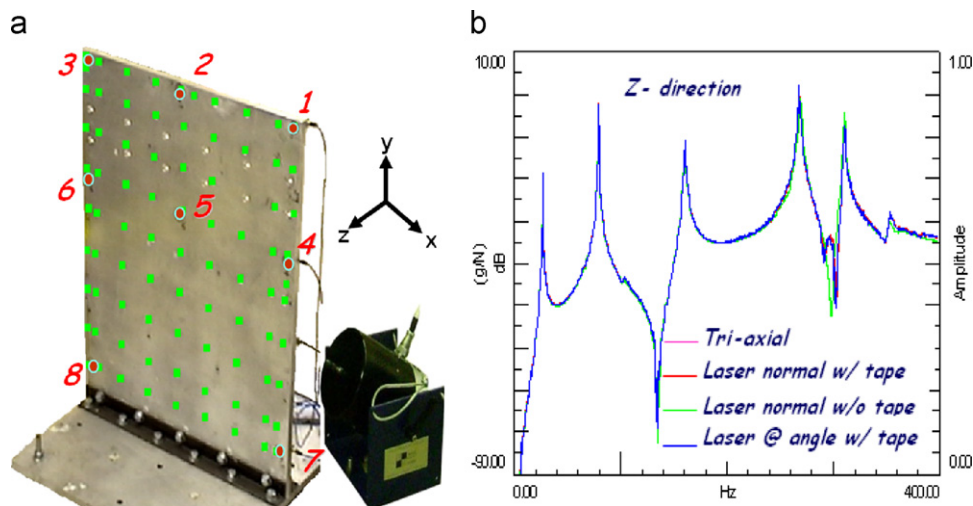
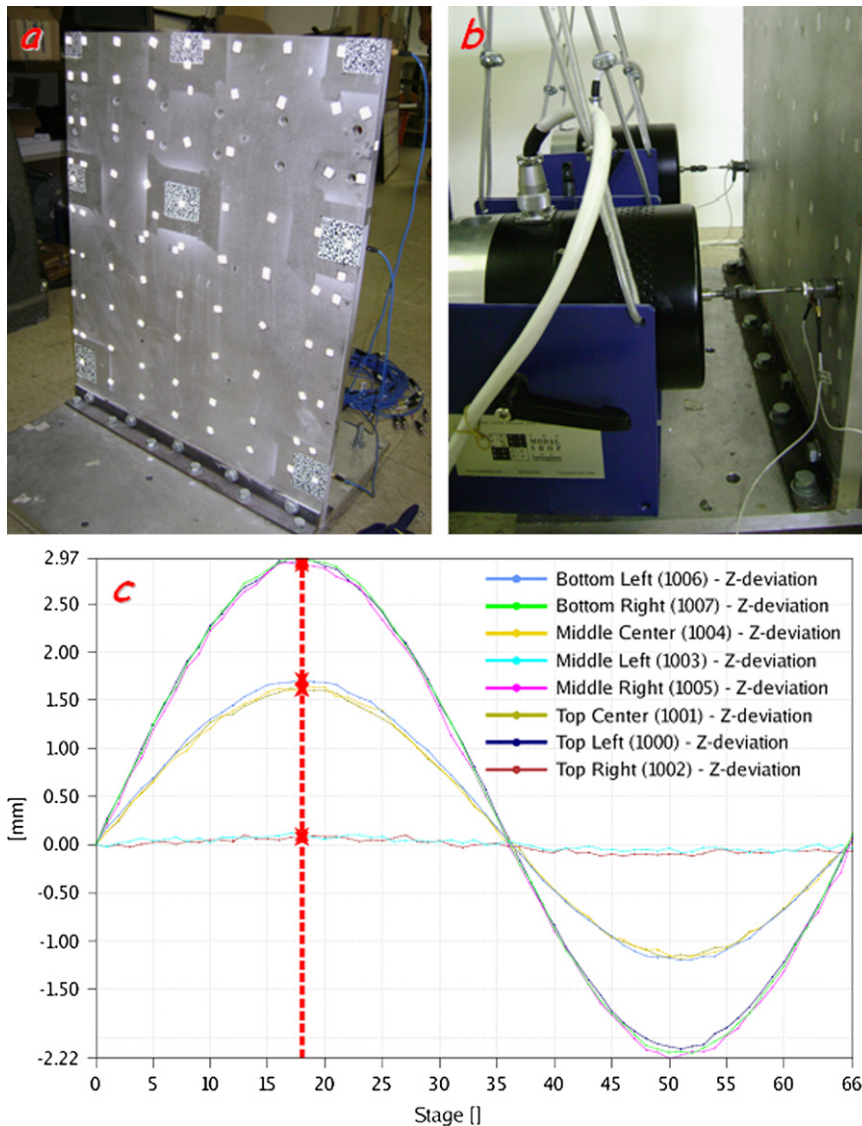


Fig. 2. (a) BU structure with shaker and measurement points and (b) sample FRF from a previous study [10].



**Fig. 3.** (a) BU structure with patterned surface; (b) shaker orientation for optical testing; and (c) sample output from a dynamic photogrammetry test.

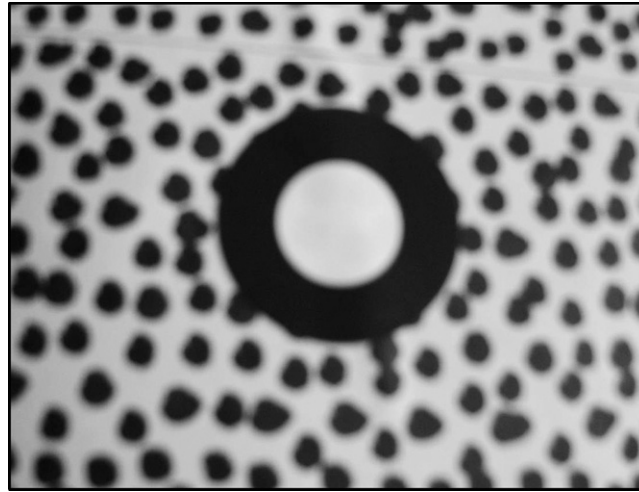
measurement locations optical targets were tracked between frames by the 3D point-tracking system Pontos<sup>TM</sup> manufactured by GOM. An example of each can be seen in Fig. 4.

Because the optical approaches measure shapes directly, forced normal mode testing was conducted to drive the structure at resonance. Two shakers were mounted near the base of the upright, as shown in Fig. 3b. The camera pair then captured a series of images throughout 2–3 cycles using a phase-stepping approach. The amplitude and phase of vibration at the 8 accelerometer points were monitored and used as feedback to tune the 2 shaker inputs such that the BU exhibited single mode behavior. Fig. 3c shows an example of the motion at the 8 measurement points for the 26 Hz mode for one of the 3D point-tracking measurements. Note that each signal is in-phase, which is to be expected when measuring what is essentially the first bending mode of the BU. The three points along the top of the upright are moving at roughly the same amplitude while those along the midline are moving in phase with approximately half the amplitude as the top points. Likewise, the bottom measurement points display very little motion since they are located near the root of the upright portion of the structure.

The maximum values of displacement during a test were used when correlating the optical results to the finite element model, laser, and acceleration mode shapes because they naturally have the highest signal-to-noise ratio. In this example, the shape used was measured in Stage 18 of the image series, as indicated by the red cursor in Fig. 3c.

When any forced normal mode test is run, several shakers are typically needed at multiple locations to excite different mode shapes. Because only two shakers were used, not all modes of the BU could be excited and measured with the dual shaker configuration. Only bending or torsion modes of the upright were targeted for this study.





**Fig. 4.** Prepared measurement surface: speckled area for DIC and a circular target for dynamic photogrammetry.

Also, a drawback to any displacement-base measurement technique is that relatively high frequency phenomena cannot be measured due to the low displacements associated with their vibrations and the inherent noise in the optical measurement. For these tests, the working distance between the 2 Mega-pixel cameras and BU was approximately 2 m so the entire upright could be seen in the field of view, resulting in an out-of-plane noise floor of approximately 40  $\mu\text{m}$ . The noise floor can be determined theoretically based on the experimental parameters of the experiment. However an alternate and more robust metric for estimating the noise floor is performed by imaging a stationary object and observing maximum displacement calculated by the stereophotogrammetry software. This approach takes into account natural ground vibrations and natural variations in the structure or environment that are always present during a test. For this set of experiments approximately 800 stages were recorded of the BU at rest and the maximum displacement was calculated to be 40  $\mu\text{m}$ . These physical limitations of the FNM test setup and the measurement noise only allowed the first and third modes of the structure to be captured using DIC and 3D point-tracking and only these two modes were used in this study.

Prior to testing, the photogrammetric principles of triangulation and bundle adjustment [11] are used extensively in the determination of the cameras' positions relative to each other. DIC packages typically include calibration panels or crosses with a series of dots on a rigid flat surface. The calibration panel is sized so that it fills the desired field-of-view for the measurements to follow. The distance between the dots is input into the software. This allows the position of the cameras relative to each other and the internal distortion parameters of each lens to be accounted for after a series of snapshots of the calibration panel are analyzed by the DIC software. During the tests, the exposure time for the cameras was always less than the inverse of the frame rate and was dependent on the lighting for each specific test conducted.

Studies of vibrations often require the measurement of small displacements. Therefore, the sensitivity of DIC measurement must be understood, especially for measurements in the out-of-plane direction. The relation of the pixel size to the object sample size must be known. This is typically done by dividing the width of the field-of-view by the number of active pixels across the width of the pixel array to obtain a distance per pixel. The sensitivity will decrease as this number gets larger. For the system tested (Aramis<sup>TM</sup>), the facet pattern matching is iterated until a stability of 0.001 pixels is mathematically reached. The in-plane physical accuracy is approximately 1/100 pixel while the out-of-plane accuracy is approximately 1/30 pixel [12–14]. These sensitivity values vary depending on a number of factors including lighting and speckle pattern effectiveness. Once measurements are made, the data is post-processed, which usually takes a few minutes, depending on the number of images recorded.

### 3. Experimental testing and results

#### 3.1. Comparison of the four measurement methods using FNMT

To evaluate the performance of the four measurement techniques, the modal assurance criterion (MAC) values were calculated using the FEMtools<sup>TM</sup> software package [15]. The mode shapes resulting from the four experimental approaches (accelerometer, laser Doppler vibrometer (LDV), DIC, and 3DPT) are overlaid with the mode shape predicted by the finite element model and is shown in Fig. 5. Tables 1 and 2 summarize the correlation results for modes 1 and 3, respectively, comparing all four sensing methods to the finite element model and to each other.

Overall, the results are very good. The accelerometer and laser MAC values are all above 99.5 when compared to the FEM for the two modes evaluated. When 3D point-tracking (3DPT) was used, MACs of at least 99.7 were obtained. DIC yielded slightly lower results for both modes 1 and 3 with values of 99.3 and 97.8, respectively. Originally, the value

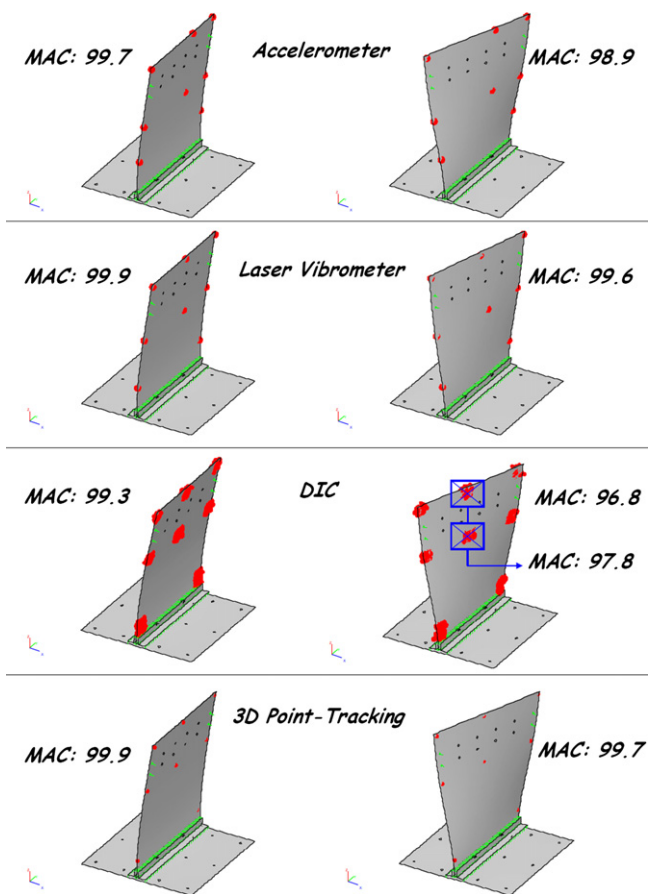


Fig. 5. Correlation of the mode shapes predicted by the FEM to the various sensing methods, for modes 1 (26 Hz) and 3 (71 Hz).

Table 1  
Comparison of MAC values for mode 1 (26 Hz).

Case	Laser Mode 1	Accel Mode 1	DIC Mode 1	DP Mode 1	FE Model
Laser Mode 1	100	99.5	99.7	99.8	99.9 <sup>+</sup>
Accel Mode 1	99.5	100	99.7	99.6	99.8
DIC Mode 1	99.7	99.7	100	99.4	99.3
DP Mode 1	99.8	99.6	99.4	100	99.9

Table 2  
Comparison of MAC values for mode 3 (78 Hz).

Case	Laser Mode 3	Accel Mode 3	DIC Mode 3	DP Mode 3	FE Model
Laser Mode 3	100	98.4	99.6	99.5	99.6
Accel Mode 3	98.4	100	97.3	98.5	99.5
DIC Mode 3	99.6	97.3	100	95.6	97.8
DP Mode 3	99.5	98.5	95.6	100	99.7

between the FEM and the DIC measurement for mode 3 was 96.8, but removing the center two patches improved the MAC by a full point as indicated in Fig. 5. These patches lie along what is essentially the nodal line for the torsional mode of the BU. The structure exhibits little to no displacement response in this area, and therefore the measurement has the lowest signal to noise ratio along the centerline. Removing these points from the correlation reduced the variance between the measurement and the model.

When comparing two experimental sets, the purity of the finite element model's mode shapes is not considered and variance on the measurements can compound to provide biased results. However, for the comparisons made, good

correlation was obtained for both modes (1 and 3); all MAC values were 95.6 and higher when the empirical results from any two different measurement approaches were correlated.

The greatest advantage of the FNMT phase-stepping approach is that the mode shapes are measured directly, so no post-processing is necessary (beyond the stereophotogrammetry calculations required to determine displacements of course). Unfortunately, time and frequency domain results cannot be obtained since motion of the structure is essentially harmonic and observed only at the peak amplitude in the oscillation cycle. To address this deficiency, tests were also conducted using high-speed cameras which satisfy the Nyquist–Shannon sampling criteria.

Typically for most optical testing, cameras are generally defined by their spatial resolution and frame rate (temporal resolution). The following section describes the use of high- and low-speed cameras in conjunction with stereophotogrammetry. “High” and “low” are relative terms, so a distinction needs to be made for this paper. In general, a “high-speed” camera is one whose frame rate exceeds 1000 frames per second (FPS). Within the context of this work, however, a “high-speed” camera is classified as one which satisfies the Nyquist–Shannon sampling theorem, i.e. the frame rate of the camera is greater than twice highest frequency of interest. Conversely, a “low-speed” camera does not satisfy this theorem. Typically, this shortcoming would render an acquisition system ineffective. However, even if budget and hardware limitations restrict one to low-speed cameras when presented with high speed phenomena, measurements can be made in conjunction with forced normal mode testing (FNMT) or by using a phase-stepping technique, as long as these vibration measured is periodic [6].

### 3.2. High speed camera testing experimental setup and considerations

The two impact tests performed on the BU were designed to target different modes of the structure. A 3 lb modal hammer was used to impact the structure. Time data for the hammer was captured using separate, synchronized data acquisition systems. Digital signal processing was carried out in MATLAB and curvefitting was performed using PolyMAX [16,17].

In the first test, a pair of 1.3 Mega-pixel cameras measured the response of the BU due to a perpendicular 1500 lbf impact near Point 1 (see Fig. 2a), at one of the top corners of the BU. In this case, only the out-of-plane modes of the upright were excited. The frame rate was set to 500 fps, corresponding to a Nyquist frequency of 250 Hz. Three-dimensional point-tracking data was calculated at 30 points on the BU, including the 8 measurement points common to the other tests. The sampled time response data were used to calculate frequency spectra (linear, auto, cross, etc.). Multiple spectrums are averaged together and in this case two averages were taken.

For the second impact modal test, the BU was impacted with approximately 4000 lbf at the same point as the shaker input as was conducted for the accelerometer and LDV tests, to excite both the in-plane and out-of-plane modes. Images were taken at a rate of 250 fps with a pair of 3.6 Mega-pixel cameras. The goal was to acquire the first two in-plane modes of the BU which were not measurable from the previous impact test. For this test the Nyquist frequency was 125 Hz and therefore sufficient to capture the modes of interest. A total of 4 averages were taken and the response was tracked with 40 photogrammetric targets distributed throughout the surface of the structure.

The experimental considerations in the two impact tests were very similar to those of any other impact modal survey, but with a few additional considerations. As with any impact test, the consistency of the input was a concern because of the variations in the force input due to the human excitation. While the level of amplitude varied somewhat, the input spectra were monitored to ensure a fairly uniform amount of energy was distributed across the frequency ranges of interest. Also, force/exponential windows were applied as needed to reduce the effects of leakage. For these impact tests the peak amplitude (for an exponentially decaying response due to an impulse) was on the order of 6–10 mm, while the noise floor was on order of 4–10  $\mu\text{m}$ .

Beyond these standard concerns, the main issue was the synchronous triggering of the cameras with the other data acquisition systems. Typically, a DAQ system is triggered from the force signal generated by the impact hammer with a pre-trigger delay that captures the beginning part of the transient that would have otherwise been lost. Modern high-end, high-speed cameras have pre-trigger capabilities, but these were not available during the first round of testing. Both systems were triggered via an external source prior to impact, so the timing of the impact was not consistent. Due to the timing variation, different windows had to be applied for each average.

In the second impact test, the use of a different timing scheme and trigger synchronization mechanism was investigated. The linear power spectra calculated from the time-domain displacements measured by the imaging systems were used to approximate mode shapes. This procedure provided useful data that was used in the correlation studies and for comparisons with the results from the finite element model and with tri-axial accelerometers.

### 3.3. Experimental results and correlation to the FEM for the impact testing

The initial results obtained from the first impact test (out-of-plane impact, 500 fps) showed a very high level of correlation to the reference finite element model. Tables 3–5 summarize the correlation results for the first impact test. For modes 1 (26 Hz), 3 (78 Hz), and 5 (158 Hz), the diagonal MAC values are 99.9+%, 99.8%, and 98.1%, respectively. The average frequency difference is  $-0.14\%$ . The diagonal POC terms are all within 3.2% of perfect correlation and the off-diagonal terms are all less than 2%.

**Table 3**

Mode shape pairs for the FEM and the first (500 fps) impact test.

Pair	FEA Mode #	FEA Freq. (Hz)	3DPT Mode #	3DPT Freq. (Hz)	Diff. (%)	MAC
1	1	26.03	1	25.79	0.94	99.9 <sup>+</sup>
2	3	77.68	2	78.03	−0.44	99.8
3	5	158.01	3	158.16	−0.09	98.1

**Table 4**

MAC matrix comparing the FEM and the first (500 fps) impact test.

<i>Modal assurance criteria</i>				
FEM (Hz)	3D point-tracking			
	25.79 Hz	78.03 Hz	158.16 Hz	
26.03	<b>99.99<sup>+</sup></b>	0.10	11.30	
70.69	19.00	67.90	19.00	
77.68	0.10	<b>99.80</b>	0.20	
108.8	22.30	32.40	49.80	
158.01	10.30	0.00	<b>98.10</b>	

**Table 5**

POC matrix comparing the FEM and the first (500 fps) impact test.

<i>Pseudo-orthogonality check</i>				
FEM (Hz)	3D point-tracking			
	25.79 Hz	78.03 Hz	158.16 Hz	
26.03	<b>1.0143</b>	−0.0105	0.0045	
77.68	−0.0104	<b>1.0200</b>	−0.0198	
158.01	−0.0089	0.0128	<b>0.9681</b>	

**Table 6**

Mode shape pairs for the FEM and the 250 fps oblique impact test.

Pair	FEA Mode #	FEA Freq. (Hz)	3DPT Mode #	3DPT Freq. (Hz)	Diff. (%)	MAC
1	1	26.03	1	25.94	0.35	99.8
2	2	70.69	2	63.94	9.55	99.6
3	3	77.68	3	78.19	−0.65	98.9
4	4	108.8	4	98.69	9.29	99.6

**Table 7**

MAC matrix for the FEM and the 250 fps oblique impact test.

<i>Modal assurance criteria</i>				
FEM	3D point-tracking			
	25.94	63.94	78.19	
26.03	<b>99.8</b>	0	0.2	0
70.69	0.1	<b>99.6</b>	0	0
77.68	0.2	0	<b>98.9</b>	0
108.8	0.1	0.4	0	<b>99.6</b>

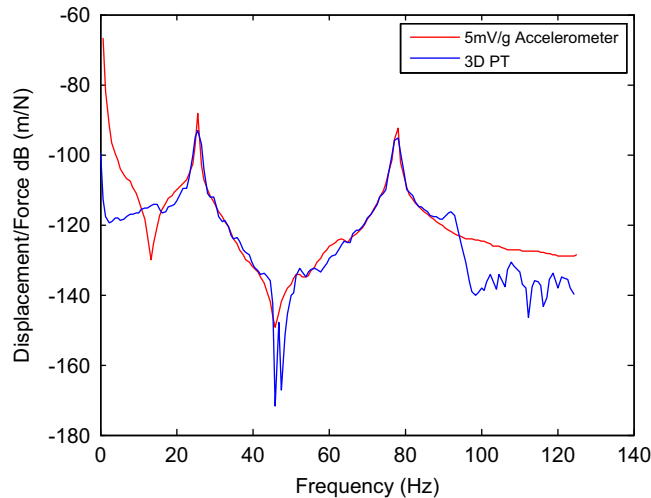
Despite the phase-lag between the input and outputs, the results of the oblique impact modal test are excellent. Tables 6–8 summarize the mode shape pairs and MAC and POC results, respectively. Though the 5th mode was not measured, the 2 additional in-plane modes were measured very clearly. When comparing the 3D point-tracking mode shapes with the FEM, the diagonal MAC values through the first 4 consecutive modes are at least 98.9% with a maximum



**Table 8**

POC Matrix for the FEM and the 250 fps oblique impact test.

<i>Pseudo-orthogonality check</i>				
FEM	3D point-tracking			
	25.94	63.94	78.19	98.69
26.03	<b>0.9999</b>	0.0000	0.0392	0.0000
70.69	0.0240	<b>0.9984</b>	0.0000	0.0010
77.68	0.0221	0.0000	<b>1.0008</b>	0.0000
108.8	0.0339	0.0669	0.0000	<b>1.0000</b>

**Fig. 6.** FRF comparison of 3DPT to a collocated accelerometer—Pt. 1 out-of-plane.

off-diagonal value of 0.4%. The POC values also indicate a very high level of correlation to the FEM. At most, the diagonal terms deviate from 1.00 by 0.16% and the maximum off-diagonal term is 6.69%. In all correlation studies, including accelerometer and laser Doppler vibrometer test results, there is at least a 5% difference between the finite element model and the experimental results for the in-plane modes. The stereophotogrammetry oblique impact data show even more deviation—the differences for modes 2 and 4 are 9.55% and 9.29%, respectively.

### 3.4. Frequency response function estimation using impact testing

To confirm that these natural frequencies are consistent with another traditional sensor, the 3D point-tracking frequency response functions and pole locations were compared to the results obtained from collocated tri-axial accelerometers that measured data simultaneously during the oblique testing. The out-of-plane motion measured at points 1 and 3 are compared in Figs. 6 and 7, respectively. At the peaks, the two measurement types agree very well. The dip at approximately 15 Hz is only observed in the accelerometer data. This is an example in which there is no sufficient displacement in the vibrating structure to be measured by the optical system. The accelerometer has a lower measurement noise floor at this frequency compared to the stereophotogrammetry approach and the dip is likely to be an anti-resonance that the imaging system cannot measure accurately. The curves for the 3DPT FRFs are not as smooth as those from the accelerometers, indicating a lower signal to noise ratio. This is even more apparent for the in-plane measurements shown in Figs. 8 and 9.

In addition to visually inspecting the FRFs, the poles for the 3DPT and accelerometers were compared. The pole locations from the average 3D point-tracking linear input spectra and those from the FRFs from the tri-axial accelerometers at points 1 and 3 (top corners) is compared in Table 9. The maximum difference between any two corresponding poles is 1.45%, so the 3DPT and accelerometer results are consistent. Furthermore, the results obtained for the laser and accelerometer studies were obtained as part of a previous study over the course of 2008 and 2009, while the 3DPT experiments were conducted in March 2010. Within the lab, the BU has been used as a test bed from which to compare mode shapes and natural frequencies. Throughout the period of testing, the BU has not been disassembled. Therefore the typical inconsistencies associated with bolted joints and changing boundary conditions have not been

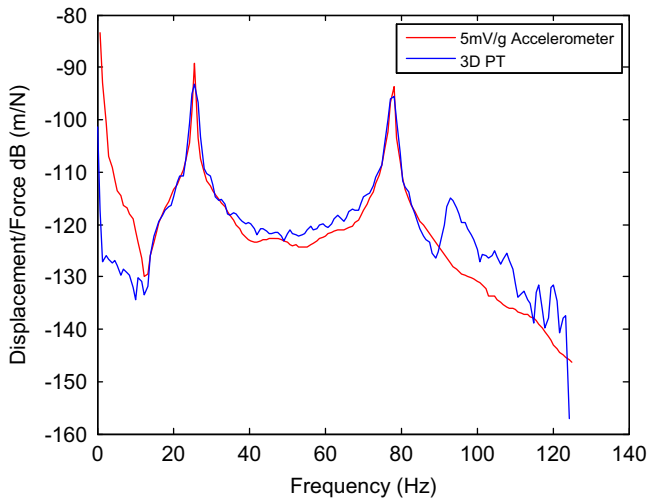


Fig. 7. FRF comparison of 3DPT to a collocated accelerometer—Pt. 3 out-of-plane.

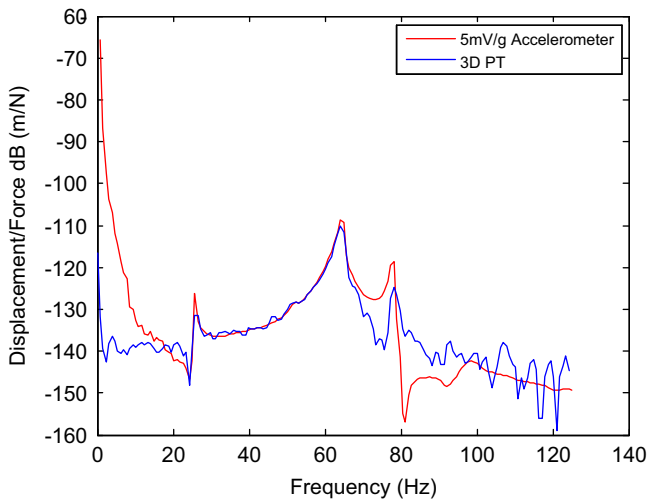


Fig. 8. FRF comparison of 3DPT to a collocated accelerometer—Pt. 1 horizontal.

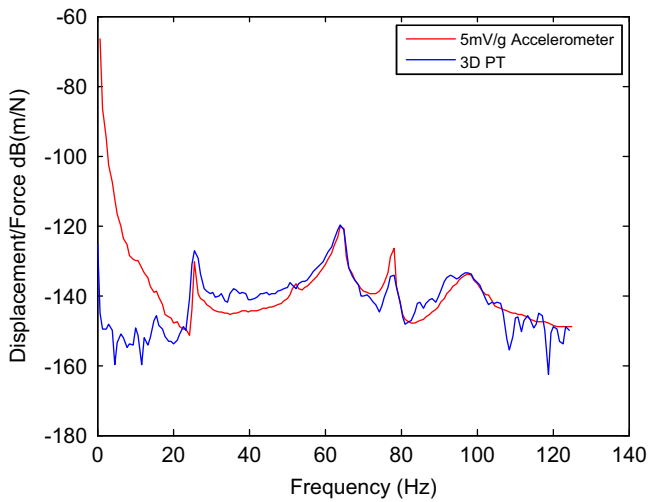


Fig. 9. FRF comparison of 3DPT to a collocated accelerometer—Pt. 1 vertical.

**Table 9**

Comparison of 3DPT and accelerometer poles for the oblique impact test.

3D point-tracking poles (Hz)	Tri-axial accelerometer poles (Hz)	Diff. (%)
25.94	25.65	1.11
63.94	64.44	–0.78
78.19	77.78	0.52
98.69	97.26	1.45

introduced. No appreciable changes have been observed in the modes shapes or natural frequencies of the BU over time when remeasured using the same techniques months or even years later.

#### 4. Observations and conclusions

Each measurement technique investigated in this comparison has its advantages and drawbacks, but in the end all yielded good, consistent measurements. The biggest difference between the traditional and the optical techniques is the approach taken to measure multiple modes. Accelerometers and laser vibrometers measure multiple modes over a broad frequency range point by point. Conversely, the two optical approaches measure all points simultaneously, one mode at a time.

Accelerometers provide an inertial reference frame, so establishing the calibration and orientation procedures (required by the scanning laser vibrometer and optical measurements) can be simpler. However, mass-loading has to be taken into consideration, especially when performing a test that requires numerous or roving accelerometers.

The laser Doppler vibrometer has the widest dynamic capabilities as it can measure velocities from as low as a few Hertz up to 80 kHz or more. Low frequency measurements must be taken with care. Measuring structures that exhibit large displacements or rigid body motion is difficult for most laser Doppler vibrometers, because the specific position on the structure at which the laser is pointing can readily change. Therefore the laser measurement point is susceptible to movement relative to the structure. To obtain both rigid body modes and flexible modes with one type of accelerometer can be challenging due to the difference in measurement amplitudes at very low frequencies where rigid body modes occur along with higher frequencies where flexible modes occur. Another possible use for DIC or 3D point-tracking is the measurement of rigid body motion of a test article suspended in a free-free condition.

The main advantages of the DIC approach is the large number of measurement points provided and the fact that the strain throughout these patterned patches can be measured directly. With the analysis configuration (facet settings) used, each patch – approximately  $3 \times 3$  in<sup>2</sup> (7.62 cm) – had on average 400 effective measurement points. Had the entire face of the BU been patterned while maintaining the same resolution, the number of points measured would be on the order of 30,000–40,000. It should be noted that the laser vibrometer has the ability to sequentially measure a comparable number of points, but the time needed to acquire the data would be quite large. In either case, the limiting factors become processing power and memory.

As opposed to tracking facets of pixels from image to image, 3D point-tracking tracks the 3D motion of circular targets. The number of measurement points is on the order of the more traditional modal tests, so the computation time relative to DIC is greatly reduced. Because discrete targets are being tracked, local strain cannot be calculated as with DIC unless numerous targets are used. A benefit to either optical technique is their ability to measure high amounts of rigid body motion. Cabling and tracking of the targets do not have to be considered as long as the test piece remains within the field of view. Conversely, the primary disadvantage to both optical approaches is that the low amplitude displacements associated with high frequency vibrations can fall below the noise floor of the optical measurement. When mode shapes are to be measured, these optical tests require at least a partial modal test in preparation of FNM testing, as well as feedback transducers (accelerometers or vibrometers).

The results indicate that when low-speed cameras are used in conjunction with forced-normal-mode-testing, both digital image correlation and 3D point-tracking can accurately capture mode shapes as long as measurable displacements are present. Combining DIC and 3DPT with high-speed cameras enables simultaneous measurement of multiple modes over a wide frequency range. The high-speed 3DPT results were the best obtained in these studies through the first 5 modes of the Base-Upright. Nearly all MAC values were better than 99%. Though not presented here, time traces from DIC could theoretically be exported and used to calculate mode shapes just as was done with the 3D point-tracking. A higher point-density may be obtained with DIC, but amount of data recorded may prove unwieldy. For a given measurement, unless local strain data is desired, the use of 3DPT is recommended.

Stereophotogrammetry techniques provide additional measurement capabilities that compliment the current array of traditional measurements by providing an alternative that favors high-displacement and low-frequency vibrations typically difficult to measure with accelerometers and laser vibrometers. The techniques presented within generated results that showed a very high level of correlation to the reference finite element model of the Base-Upright. Therefore, the application of the image-based systems for vibrations and modal analysis is promising. The research presented within this paper reveals that the use of the optically based displacement measurement (DIC and 3DPT) provides an alternative

measurement technique that increases the envelope over which engineers can now make useful and accurate vibration measurements.

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