Mission d'occulatation lunaire

Vincent Callegari

August 6, 2024

1 introduction

2 observation zone

2.1 computing the position of the observation zone

during the study of these problems, we will take the earth as the orgin in cartesian coordinate. The axis Earth-Sun at time t0 will be defined as the X axis and the axis X and Y define the earth ecliptic plane.

for now we fix the time at (0,0)

a point is in the observation zone if the sun is totally hidden and the crown is visible around the occulting body. This zone has the shape of a double cone that we can represent by the rotation of a triangle.

the triangle could be defined by its three points P_1,P_2 et P_3, with as coordinates $(P_{1x}, 0, 0)$, $(P_{2x}, P_{2y}, 0)$,$(P_{3x}, 0, 0)$

the value of P_{1x} and P_{3x} can be easily computed usingt the Thales theorem.

$$P_{1x} = \frac{\bar{D}R_l}{R_s - R_l}$$

$$P_{3x} = \frac{\bar{D}R_l}{R_s(\alpha + 1) - R_l}$$

where $\bar{D}$ is the Solar-Moon distance, R_s the Solar Radius ,R_l the Lunar radius and α is the solar crown's added size in solar radius (the more *alpha* is near of zero the more the solar crown's radius is near of the solar raidus meaning that we will observe part of the crown near of the solar surface)

we can then compute the position of P_2 by computing the intersection of the lines linking the surface of the Moon and the points P_1 and P_3

$$P_{2x} = \frac{P_{1x}tan(\theta_1) + P_{3x}tan(\theta_3)}{tan(\theta_1) + tan(\theta_3)}$$

$$P_{2y} = tan(\theta_1)(P_{1x} - P_{2x})$$

with

$$\theta_1 = \sin^{-1}\left(\frac{R_l}{P_{1x}}\right)$$

$$\theta_3 = \sin^{-1}\left(\frac{R_l}{P_{3x}}\right)$$

with $\alpha = 0.05$

2.2 approximation of the zone

the Position of the points P_1,P_2 and P_3 relative to the moon will change depending of the moon position around the earth, hovewer the shape of the the zone in itself will not change much (with a dimension error smaller than 0.5%)

we can obtain a good approximation of the observation zone at any position of the moon around the Earth by computing the shape of the observation zone with the moon at the origin and then move it and rotate it to place it at the right position.

We name $\hat{P}_1$, $\hat{P}_2$ and $\hat{P}_3$ the points of the observation zone when the moon is at the origin. the points P_1 and P_3 are collinear with the vector R_{ls} witch are the position of the Moon relative to the Sun.

Also the points $\hat{P}_1$ $\hat{P}_3$ are colinear with the X axis. We can conclude of the following formula of the points P_3 and P_1

$$P_3 = R_{lt} + \hat{P_{3x}} D^{-1} R_{ls} \tag{1}$$

$$P_1 = R_{lt} + \hat{P_{1x}} D^{-1} R_{ls} \tag{2}$$

a point S is consider in the observation zone if the following inequality are verified :

$$\begin{aligned} ||a|| &< ||b|| p_1 \\ ||a|| &< O - ||b|| p_2 \\ ||S|| &< ||P_3|| \end{aligned} \tag{3}$$

note that the last condition was added just to avoid a symetric virtual zone that will appear before the P_3 point because of the symetry of the norm function.

with b being the projection of $S - P_3$ on the axis Sun-Moon, $a = S - P_3 - b$. p_1, p_2 and O are real value: here this parameter will be approximated by their value when the Moon is at the origin :

$$\begin{aligned} p_1 &= \frac{\hat{P_2}y}{\hat{P_2}x - \hat{P_3}x} \\ p_2 &= \frac{\hat{P_2}y}{\hat{P_1}x - \hat{P_2}x} \\ O &= \hat{P_2}y + (\hat{P_2}x - \hat{P_3}x) p_2 \end{aligned}$$

if we want to increase the precision of the observation zone we can take in account the variation in length of the observation zone by scaling the parameter b by
$\frac{||\hat{P_1}-\hat{P_3}||}{||P_1-P_3||}$

2.3 numeric application

we have the following distances :

$$
\begin{aligned}
R_l &= 1.7374 \times 10^6 m \\
R_s &= 6.955 \times 10^8 m \\
D_{sl} &= 1.496 \times 10^{11} m \\
\Delta_D = 2D_{tl} &= 7.69496 \times 10^8 m
\end{aligned}
$$

we get the following value for the point of the Zone for $\alpha = 0.05$

$$
\begin{aligned}
P_{3x} &= 3.577 \times 10^8 m \\
P_{1x} &= 3.758 \times 10^8 m \\
P_{1x} - P_{3x} &= 1.7928 \times 10^7 m \\
P_{2x} &= 3.655 \times 10^8 m \\
P_{2y} &= 4.248 \times 10^4 m
\end{aligned}
$$

with

$$
\begin{aligned}
\Delta_{P_{3x}} &= 1.835 \times 10^6 m \\
\Delta_{P_{1x}} &= 1.927 \times 10^6 m \\
\Delta_{P_{1x}-P_{3x}} &= 9.198 \times 10^4 m \\
\Delta_{P_{2x}-P_{3x}} &= 4.487 \times 10^4 m \\
\Delta_{P_{2x}} &= 1.880 \times 10^6 m \\
\Delta_{P_{2y}} &= 4.935 \times 10^{-3} m
\end{aligned}
$$

we can see that by using the approximation defined earlier, we are neglecting variation of order lower than thousand of killometer ($10^6 m$).If we add the optimisation on the value of b we are now neglecting variation of order lower than ten killometerss ($10^4 m$) leaving only error of a couple killometers on the posisiton of P_{2x} which is not a lot against the size of the observation zone : (around $20000 km$ by $100 km$).

about the error on P_{2y} the error is of the order of a millimeter and can clearly be neglected.

3 problème

on va considérer le problème suivant:

la lune suit une orbite circulaire autour de la Terre de rayon $a = 384000km$.

la forme de la zone d'observation de la Lune est considérée comme étant égale à la zone d'observation de la lune si elle se trouvait à l'origine (la position de la terre). La position du point P_3 est déterminé par la formule suivante: avec

$$P_3(R_l) = R_{lt} + \hat{P_{3x}} D^{-1} R_{ls}$$

avec $\hat{P}_3$ étant la position du point P_3 quand la lune est à l'origine ,R_{lt} est la position de la lune relativement à la Terre. et R_{ls} est la position de la Lune relativement au Soleil.

(on a $R_{ls} = R_{lt} + D\hat{x}$);

le but est de trouver des orbites Kepleriennes qui effectue des observations répété et les plus longues possibles.

Les temps d'observation peuvent beaucoup varier allant d'une durée de quelques minutes à plusieurs heures. dans la suite on va donc se concentrer sur une seule observation.

in order to make multiple observation, it may be better to take orbital period that are multiple of the moon's.

Hovewer choosing period between the moon orbital period or its synodic period could up to debate.

from this we can conclude the following formula for the semi major axis.

$$a_s = a_l k^{\frac{2}{3}}$$

avec

$$P_s = kP_l$$

étant donné que l'objectif est de faire une observation, on peut faire partir le satellite directement de la zone d'observation.

De plus la dimension de la zone étant très étirée (environ $10000km \times 100km \times 100km$) on peut considérer que le satellite coupera forcement le segment $[P_3, P_1]$, on peut donc décrire la position initiale du satellite à l'aide de l'anomalie vraie de la lune ν et un scalaire λ entre 0 et 1. la position initiale du satellite devient :

$$S_0 = \lambda(P_1 - P_3) + P_3(R_l(\nu))$$

avec

$$R_l(\nu) = \begin{bmatrix} r(\cos\Omega\cos\theta - \sin\Omega\sin\theta\cos\ i) \\ r(\sin\Omega\cos\theta - \cos\Omega\sin\theta\cos\ i) \\ r\sin\theta\sin\ i \end{bmatrix}$$

avec $\theta = \nu + \omega$

pour l'instant on est en 2D donc l'équation se simplifie par:

$$X_l(\nu) = \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{bmatrix}$$

Maintenant que l'on connait la position du satellite on peut déterminer sa vitesse à l'aide de la formule suivante:

$$||\dot{S}(0)|| = \sqrt{\frac{2\mu}{||S_0||} - \frac{\mu}{a_s}}$$

on peut ensuite determiner l'orientation de la vitesse initiale avec deux angles θ_s et ϕ_s.

la dynamique du satellite et de la lune doivent être calculé pour calculer le temps de l'observation.

on à la dynamique suivante :

$$\begin{bmatrix} \dot{\overrightarrow{R_s}} \\ \dot{\overrightarrow{V_s}} \\ \dot{\overrightarrow{R_l}} \\ \dot{\overrightarrow{V_l}} \end{bmatrix} = \begin{bmatrix} \overrightarrow{V_s} \\ \frac{-\mu}{||R_s||^3}\overrightarrow{R_s} \\ \overrightarrow{V_l} \\ \frac{-\mu}{||R_l||^3}\overrightarrow{R_l} \end{bmatrix}$$

avec comme condition initiale :

$$\begin{bmatrix} \overrightarrow{R_{s0}} \\ \overrightarrow{V_{s0}} \\ \overrightarrow{R_{l0}} \\ \overrightarrow{V_{l0}} \end{bmatrix} = \begin{bmatrix} \lambda\left(\widehat{P_1} - \widehat{P_3}\right) + R_{lt} + \frac{\widehat{P_{3x}}}{D} R_{ls} \\ \sqrt{\mu\left(\frac{2}{||R_{s0}||} - 2\right)}\widehat{v_0}(\theta_s, \phi_s) \\ r(\nu)\begin{bmatrix} (\cos\Omega\cos\theta - \sin\Omega\sin\theta\cos\ i) \\ (\sin\Omega\cos\theta + \cos\Omega\sin\theta\cos\ i) \\ \sin\theta\sin\ i \end{bmatrix} \\ \sqrt{\frac{\mu}{p}}\begin{bmatrix} -\cos\Omega(\sin\theta + e\ \sin\omega) - \sin\Omega(\cos\theta + e\ \cos\omega)\cos\ i \\ -\sin\Omega(\sin\theta + e\ \sin\omega) + \cos\Omega(\cos\theta + e\ \cos\omega)\cos\ i \\ (\cos\theta + e\ \cos\omega)\sin\ i \end{bmatrix} \end{bmatrix}$$

la dynamique devra être simulée après et avant l'état initial pour trouver l'instant où l'objet entre et sort de la zone.

la fonctions d'objectif est définit comme suit :

$$\int_{-\tau_d}^{\tau_u} dx = \tau_d + \tau_u$$

où τ_d est l'instant où l'objet rentre dans la zone d'observation et τ_u l'instant où l'objet en sort.

l'équation utilisé pour vérifier si l'objet est dans la zone est l'équation (2).

les paramètre de contrôle sont :

- la position de la lune ν qui définit la position de la zone d'observation.
- la position initiale λ de l'objet dans la zone d'observation qui est simplifier par un segment allant de P_3 à P_1.
- les angles θ_s et ϕ_s qui définissent l'orientation de la vitesse de l'objet.

on a donc un espace de dimension 4 : $(\nu, \lambda, \theta_s, \phi_s) = [0, 2\pi] \times [0, 1] \times [0, 2\pi] \times [0, \pi]$

l'espace est contraint mais étant donné que la plupart des dimension sont des angles et qu'il suffit de donner un score de 0 si λ est en dehors du domaine on peut considéré que l'espace est égal à $\mathbb{R}^4$ pour avoir un problème sans contrainte.

4 Result

When we are working with a simple 2D problem with circular Moon orbit , there are an optimum near the value $(2\pi/3, 0.5, 38/45\pi, 0)$ that give observation time of almost 20h.

We managed to get observation time this long because the speed of the object and the observation Zone are equivalent. Meaning that the device can stay in the observation zone for a long time. As the Observation Zone velocity are nearly the same as the Moon, we can simplify for now by saying that the device must have the same speed as the Moon.

The satellite describe a loop inside the observation zone, meaning that it is possible to get two fairly long observation very near to each other if the tip of the loop is outside of the observation zone. Even if there is a good chance that these solutions are less efficient that a solution with the entire loop inside the observation zone, it could be usefull to improve the objective function to detect when the object reenter the observation zone after a short time.

For the model implemented, i've intentionally kept high value of α to ensure that the loop stayed in the observation zone.

we can easily determine that there are two point in the lunar orbit where we can obtain very long observation time:

The satellite can have the same speed vector as the moon only if the following equation is verified :

$$\sqrt{\frac{2\mu}{R_s} - \frac{\mu}{a_s}} = \sqrt{\frac{2\mu}{R_l} - \frac{\mu}{a_l}}$$

Considering that the moon's speed is constant (because of its small eccentricity) and that the period of the device is the same as the moon (ie $a_s = a_l$), we get the relation :

$$R_s = R_l$$

as the satellite is in the obsevation zone, the region in which the satelite can cross the observation zone and at the same speed as the moon is the intersection between the possible observation zone space (which is in this simplified case simplified by an ellipse) and a sphere of radius R with $R = R_l$ in this case.

There are two point in the orbit that satisfy these condition.

Using the solution that we found in 2D we can try to use it as an initial condition for 3D problems, the solution found with varying Ω value and considering the eccentricity of the Moon tend to show that there are always a solution near this point.

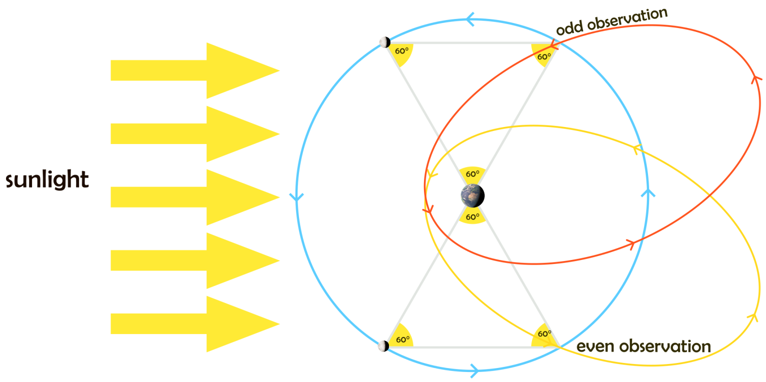

Figure 1: keplerian orbit allowing to make long observation in a simplified model (fixed sun, circular orbit of the moon and observation zone distance equal to moon's orbit radius)

A question that could arise from these observation can be about the importance of the device period over its speed in the observation zone. In other word, maybe it is better to force the device to match the speed of the observations zone instead of making it matcht the speed needed to get a fixed period.

To answer this question i've made a quick test where for a set of position of the observation zone i've lanched the device with the same velocity vector as the observation zone, the result gave pretty high observation time of multiple hours, with the lowest possible time occuring when the observation time is near the Earth. This is not a problem as this kind of trajectory where not possible anyway because it require speed of around $1km \cdot s^2$.

We can also see Three main peak, the first two are the one we find earlier with observations time of around 18 hours, and there a third one that is a little bit higher with observation time of around 24 hours.

The fact that we make longer observation when the speed of of the observation zone match the speed of a circular orbit at its current position could be linked to the fact that the radial speed of the device start decreasing after crossing this altitude which could allow the device to make an oscilation around the observation point.

about the last peak, the proximity of the observation from the Earth umbra and the fact that the corresponding device will have around the energy to escape the Earth sphere of influence (the speed of the Moon two time further than the Moon) make it not the better choice for reapeated observation

4.1 finding initial guess

Now that we know where the observation roughly are we can create an algorithm that could easily find initial guess to find observation.

as the speed of the device is constrained by its semi major axis (by the equation $\sqrt{\frac{2\mu}{R_s} - \frac{\mu}{a_l}}$), we just have to find the point where the observation zone speed matches the speed of the device.

If we consider that the observation zone center is at the position of the point P_{2x} we get :

$$P_{2x} = R_l + \frac{D_{p2}}{D_{ts}}(R_l - R_s)$$

where D_{p2} is the distance between the moon and the point P_{2x} where the moon is at the origin. the velocity of P_{2x} is then straightforward to compute

$$\dot{P_{2x}} = \dot{R_l} + \frac{D_{p2}}{D_{ts}}(\dot{R_l} - \dot{R_s})$$

we can then get from this the following payoff formula :

$$D_{obs} = \left(||\dot{P_{2x}}|| - \sqrt{\frac{2\mu}{P_{2x}} - \frac{\mu}{a_l}} \right)^2$$

which is a one dimension equation depending of time only, which make it way easier to solve.

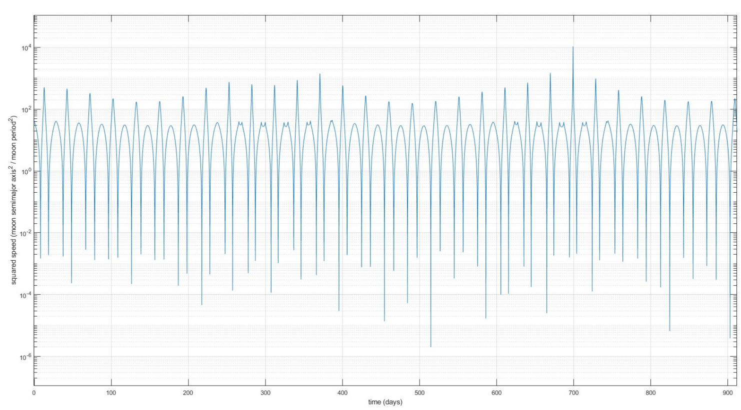

Figure 2: relative speed between the device and the observation zone

we can see a periodic signal with a period equal to the synodic period of the device. A period is made of two local maximum corresponding to the time when the observation zone is the closest to Earth (the highest sharp peak)and when the observation zone is the farthest from Earth(the soft peak). There also two local minimum which correspond to the two points of interest. Theses minimum are around 5 days before and after the highest peak ($\frac{1}{6}$ time the synodic period exactly).

This configuration allow find estimate of time where the observation happen :

The first step consist of finding the high peak, the sharp peak would always be higher than 100 because of the high speed of the device near Earth and the soft peak would be always under 100 (because the speed difference can't be above 2π making the value of D_{obs} smaller than $4\pi^2 \approx 39$).

this give us the following constrained problem:

$$t^0 = \arg\max D_{obs}(t)_{0\leq t; D_{obs}(t)\geq 100}$$

by taking an initial guess at $t_0^0 = 0$ or finding the an estimation by hand should give us the first peak.

we can then find every peak by solving the same problem with an initial guess of $t_0^n = t^{n-1} + P_{sy}$ where t^{n-1} is the time of the last peak and P_{sy} is the synodic period of the moon.

we can then find the points of interests by finding the minimum of the function D_{obs} by taking the the following initial guess :

$$\mathrm{O}_0^{2n} = t^n - \frac{P_{sy}}{6}$$
$$\mathrm{O}_0^{2n+1} = t^n + \frac{P_{sy}}{6}$$

A good thing about this method is that each problems (except the first one if we take $t_0^0 = 0$) are convex allowing them to be solve quite easily. take also note that the first observation may be before 0 if the first peak happen at a time smaller than a sixth of the Synodic period.

5 low thrust transfert

now that we know that the best configuration to make observation are with no relative speed, it mean that we can compare the different Δv needed to transfert from one observation to another.

the goal of this computing is to get an idea of which subset of observations could be done (as it is very likely that we won't be able to make the device attend all the observations).

For now we will consider the simulation over one year, with the earth at the origin, the sun moving around the earth along a perfect circle with a radius of one UA at a constant speed and. The moon moving allong a keplerian orbit with the following component :

$$\Omega = 0$$
$$\omega = 0$$
$$i = 5$$
$$e = 0.054$$
$$\nu = 0$$

The value of ν correspond to the true anomaly of the moon at time $t = 0$. the value of ω and Ω are set at 0 for now but we will test other value later.

we can achieve zero velocity observation if the velocity of the device at the observation zone ($\sqrt{\frac{2\mu}{R_s} - \frac{\mu}{a_l}}$) is the same as the velocity of the observation zone. As the position of the point P_{2x} is given by the following formula :

$$P_{2x} = R_l + \frac{D_{p2}}{D_{ts}}(R_l - R_s)$$

where D_{p2} is the distance between the moon and the point P_{2x} where the moon is at the origin. so we have:

$$\dot{P_{2x}} = \dot{R_l} + \frac{D_{p2}}{D_{ts}}(\dot{R_l} - \dot{R_s})$$

we can get from this the following payoff formula :

$$D_{obs} = \left(||\dot{P_{2x}}|| - \sqrt{\frac{2\mu}{P_{2x}} - \frac{\mu}{a_l}} \right)^2$$

ploting the value of this function over one year give the follwoing graph :

we can see that we have a almost a symetric and a periodic function, this mean that we just need to find the first optimum and then we could compute good initial value to find the other optimum of the function using the following formulas :

$$O_{2n\,i} = nT - O_1 + a$$
$$O_{2n+1\,i} = nT + O_1 + a$$

where T is the synodic period of the Moon, $O_{k\,i}$ is the initial value used to find the k^{th} optimum of the function using a simple gradient descent method O_1 is the value of the first optimum of the function relative to a.

and a is the first time the Sun-Earth-Moon system align (when projected on the ecliptic plane of earth) in this order, for now the value a is equal to zero.

5.1 low thrust model

we want to move from a point R_i at time t_i to a point R_f at time t_f with initial velocity V_i and final velocity V_f.

the device can be control with a low thrust engine that can generate an acceleration of maximum magnitude A_{max} in any direction, as the final objective is to use a solar sail, the variation of mass is neglected.

the passive dynamic of the device is the following :

$$\begin{bmatrix} \dot{r} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ \sum_{\beta \in B} \frac{\mu_\beta}{\|r_\beta - r\|^3}(r_\beta - r) \end{bmatrix}$$

where B is the set of every bodies that attract the device (for now the two body are The Earth and the Moon). the dynamics of the bodies must be computed before (we suppose that the device does not influence the dynamics of the system).

the acceleration of the thruster can be take in account by adding a vector the equations of speed :

$$f(x,u) = \dot{x} = \begin{bmatrix} \dot{r} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ u + \sum\limits_{\beta \in B} \frac{\mu_\beta}{\|r_\beta - r\|^3}(r_\beta - r) \end{bmatrix}$$

we want to minimize the Δ_v of the transfert, so the running payoff is :

$$r(x,u) = -\|u\|$$

we also want to be at the right position and velocity at the time t_f so the terminal payoff is :

$$g(x(t_f)) = -(r(t_f) - R_f)^2 - (v(t_f) - V_f)^2$$

we get the following hamiltonian :

$$H(x,p,u) = \begin{bmatrix} v \\ u + \sum\limits_{\beta \in B} \frac{\mu_\beta}{\|r_\beta - r\|^3}(r_\beta - r) \end{bmatrix}^t \cdot p - \|u\|$$

using the Pontryagin maximum principle theorem we can compute the differential of $p = \begin{bmatrix} p_r \\ p_v \end{bmatrix}$

$$\begin{aligned} \dot{p} =& -\nabla_x H \\ =& -\begin{bmatrix} 0 & I_3 \\ \sum\limits_{\beta \in B} \frac{\mu_\beta}{\|r_\beta - r\|^3} \left(\frac{3(r_\beta - r)\cdot(r_\beta - r)^t}{\|r_\beta - r\|^2} - I_3 \right) & 0 \end{bmatrix}^t \cdot p \\ =& \begin{bmatrix} \sum\limits_{\beta \in B} \frac{\mu_\beta}{\|r_\beta - r\|^3} \left(I_3 - \frac{3(r_\beta - r)\cdot(r_\beta - r)^t}{\|r_\beta - r\|^2} \right) \cdot p_v \\ -p_r \end{bmatrix} \end{aligned}$$

we know that $\forall t$, $u = \arg\max_{u \in B(0,A_{max})} H(x,p,u)$. We can rewrite the hamiltonian as follow :

$$\begin{aligned} H(x,p,u) =& C(x,p) + u \cdot p_v - \|u\| \\ =& C(x,p) + \|u\|(\|p_v\| \cos(\alpha_{\widehat{up_v}}) - 1) \end{aligned}$$

with this form we can easily conclude that

$$\begin{cases} \|u\| = A_{max} & \text{if } \|p_v\| \cos(\alpha_{\widehat{up_v}}) \geq 1 \\ \|u\| = 0 & \text{else} \end{cases}$$

we can then observe that $\|p_v\| \cos(\alpha_{\widehat{ap_v}})$ cannot be greater than 1 if $\|p_v\| < 1$. Also if $\|p_v\| \geq 1$, it is obvious that taking $\alpha_{\widehat{up_v}} = 0$ (i.e. u and p_v are collinear) maximize the factor $\|p_v\|\cos(\alpha_{\widehat{uap_v}})$. So we can conclude the following formula :

$$\begin{cases} u = A_{max}\widehat{p_v} & \text{if } \|p_v\| \geq 1 \\ u = 0 & \text{else} \end{cases}$$

to solve the problem numerically, we then just have to find the correct value p_i so the terminal value of x correspond to the target value.

6 result low thrust

to compute the solution, i've used the julia library : OptimalControl,

the starting point of the device is the exting point where of starting observation and the ending point is the enter point of the ending observation.

To simplify the next explication i will separate the observations into two category : the even observations and the odd observation. The odd observations are the first observation that happen on the moon orbit and the even one are the second one. another way to see this is to consider that the odd observations are the observation occuring when the device is aproaching earth and the odd observation is when the device is moving away from earth

6.1 observation

The algorithm had a lot of struggle to find an optimum for moving to two adjacent observations, which was excpected as the two orbit of adjacent optimum are very different and the time interval is small meaning that important Δ_v are needed to move from the starting point to the ending point. hovewer the algorithm find solution for transition between even observation and between odd observation. This was also excpected as the starting and objective orbit are very similar with the main difference being only a rotation of $\frac{P_{moon}}{P_{sol}}2\pi$ rad caused by the movement of the sun.

for 1 in 2 transfert, the maximum acceleration found was betweeen $170\mu N \cdot \text{Kg}$ and $200\mu N \cdot \text{Kg}$ which is around the acceleration we can get from electric engine but not from solar sail.

there are also another type of transfert that caught my interest, it is transfert from odd observation to even observation (1 in 3 transfert starting from odd observation) which seem to be very easy to do with maximum acceleration ranging from $50\mu N \cdot \text{Kg}$ to $95\mu N \cdot \text{Kg}$. It truned out that the angle between an odd orbit and a even orbit orbit is around the same length of the angle describe by the movement of the sun in $\frac{4}{3}$ of the moon synodic period, this mean that the starting and ending orbit in such transfert are almost the same with the only main task being to change the true anomaly. This transferts could look like they are a good alternative to reduce maximum acceleration during transfert but the problem is that once we reached the odd observation, we have to make a transfert from even to odd and in this case the situation is reverse with the angles being added and not substracted, causing high maximum acceleration.

In fact making fewer observation doesn't seem to reduce the maximum acceleration needed or even the Δ_V which could look counter intuitive at first glance but is not when we think aboout the fact that orbits become increasingly different from each other with time. This mean that there shold be some kind of minimum power needed for making observations which must be around the amount of power needed to rotate the observation orbit by 360° every year. Though it is possible to jump a lot of observations to the point that we don't need to rotate the orbit all around the Earth. Like with the obvious example of making an observation every year, when the sun is roughly at the same place relative to the Earth-Moon system. But the counter part is that we will be making way less observations ... at most one every 6 mouths.

6.2 scenario with solar sail

Using solar sails with their actual performance level (around $50\mu m \cdot \mathrm{s}^2$) won't allow to make a lot of observations, with the only plausible scenario that far could be to make an odd observation follow by a even observation 40 days later taking advantage of the low maximum acceleration of these transfert. Then spend multiples mounths aligning for another odd observations and repeating the process in a cycle of at least 8 mouths which is still way better than what we could do on Earth. But even this scenario may be infeasible once the additional sail constraint on the control are added.

another possibility could be about the third observation that i had consider to too impractical to be usefull at first but may give better result when making only a few observation per years.

6.3 scenario with electric propulsion

What look like the best scenario at this point would be to put two devices capable of delivering acceleration of the order of $300\mu m \cdot \mathrm{s}^2$, one for the odd observation and one for the even observation. This would allow to attends every observations which mean that we will be able to observe the solar corona around 2.8% of the mission time with even the possibility of observing the sun and the sun corrona at the same time if one device aim at the sun during the observation of the other device