Solar crown observation mission study

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1 introduction

2 observation zone

2.1 computing the position of the observation zone

At first for simplify things we will first consider that the sun and the occulting body are aligned along the X axis.

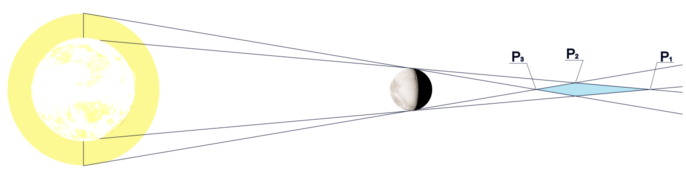

Figure 1: major points of the observation zone

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A point is in the observation zone if the sun is totally hidden and the crown is visible around the occulting body. This zone has the shape of a double cone that we can represent by the rotation of a triangle.

The triangle could be defined by its three points P_1,P_2 et P_3, with as coordinates $(P_{1x}, 0, 0)$, $(P_{2x}, P_{2y}, 0)$,$(P_{3x}, 0, 0)$

The value of P_{1x} and P_{3x} can be easily computed using the Thales theorem.

$$P_{1x} = \frac{\bar{D}R_l}{R_s - R_l}$$

$$P_{3x} = \frac{\bar{D}R_l}{R_s(\alpha+1)-R_l}$$

Where $\bar{D}$ is the Solar-Moon distance, R_s the Solar Radius ,R_l the Lunar radius and α is the solar crown's added size in solar radius (the more *alpha* is near of zero the more the solar crown's radius is near of the solar radius meaning that we will observe part of the crown near of the solar surface)

We can then compute the position of P_2 by computing the intersection of the lines linking the surface of the Moon and the points P_1 and P_3

$$P_{2x} = \frac{P_{1x}tan(\theta_1) + P_{3x}tan(\theta_3)}{tan(\theta_1)+tan(\theta_3)}$$

$$P_{2y} = tan(\theta_1)(P_{1x} - P_{2x})$$

with

$$\theta_1 = \sin^{-1}\left(\frac{R_l}{P_{1x}}\right)$$

$$\theta_3 = \sin^{-1}\left(\frac{R_l}{P_{3x}}\right)$$

with $\alpha = 0.05$

2.2 approximation of the zone

We could use directly the result obtained earlier in the simulation but as checking if the device is in the observation zone will be one of the most reapeated computing of the code i think it is usefull to look for approximation that could reduce computing.

The Position of the points P_1,P_2 and P_3 relative to the moon will change depending of the moon position around the earth, hovewer the shape of the the zone in itself will not change much (with a dimension error smaller than 0.5%)

We can obtain a good approximation of the observation zone at any position of the moon around the Earth by computing the shape of the observation zone with the moon at the origin and then move it and rotate it to place it at the right position.

We name $\hat{P}_1$, $\hat{P}_2$ and $\hat{P}_3$ the points of the observation zone when the moon is at the origin (which is fixed at the average distance between the Earth and Sun D). the points P_1 and P_3 are collinear with the vector R_{ls} witch are the position of the Moon relative to the Sun.

As the value of P_1 and P_3 are proportional to the distance between the Moon and Sun

Also we take the points $\hat{P}_1$ $\hat{P}_3$ so they are colinear with the X axis. We can conclude of the following formula of the points P_3 and P_1.

$$P_3 = R_{lt} + \hat{P_{3x}}D^{-1}R_{ls} \tag{1}$$

$$P_1 = R_{lt} + \hat{P_{1x}} D^{-1} R_{ls} \tag{2}$$

A point S is considered in the observation zone if the following inequality are verified :

$$\begin{aligned} &||a|| < ||b|| p_1 \\ &||a|| < O - ||b|| p_2 \\ &||S|| > ||P_3|| \end{aligned} \tag{3}$$

with b being the projection of $S - P_3$ on the axis Sun-Moon, $a = S - P_3 - b$. p_1, p_2 and O are real value: here this parameter will be approximated by their value when the Moon is at the origin :

$$\begin{aligned} p_1 &= \frac{\hat{P_2 y}}{\hat{P_2 x} - \hat{P_3 x}} \\ p_2 &= \frac{\hat{P_2 y}}{\hat{P_1 x} - \hat{P_2 x}} \\ O &= \hat{P_2 y} + (\hat{P_2 x} - \hat{P_3 x}) p_2 \end{aligned}$$

Note that the last condition was added just to avoid a symetric virtual zone that will appear before the P_3 point because of the symetry of the norm function.

If we want to increase the precision of the observation zone we can take in account the variation in length of the observation zone by scaling the parameter b by

$\frac{||\hat{P_1} - \hat{P_3}||}{||P_1 - P_3||}$

This approximation is quite precise because the real equation of the point P_1,P_{2x} and P_3 are linea, meaning there is an error only on the position of P_{2y}.

2.3 numeric application

We have the following distances :

$$\begin{aligned} R_l &= 1.7374 \times 10^6 m \\ R_s &= 6.955 \times 10^8 m \\ D_{sl} &= 1.496 \times 10^{11} m \\ \Delta_D = 2D_{tl} &= 7.69496 \times 10^8 m \end{aligned}$$

We get the following value for the point of the Zone for $\alpha = 0.05$

$$\begin{aligned} P_{3x} &= 3.577 \times 10^8 m \\ P_{1x} &= 3.758 \times 10^8 m \\ P_{1x} - P_{3x} &= 1.7928 \times 10^7 m \\ P_{2x} &= 3.655 \times 10^8 m \\ P_{2y} &= 4.248 \times 10^4 m \end{aligned}$$

with

$$\begin{aligned}
\Delta_{P_{3x}} &= 1.835 \times 10^6 m \\
\Delta_{P_{1x}} &= 1.927 \times 10^6 m \\
\Delta_{P_{1x}-P_{3x}} &= 9.198 \times 10^4 m \\
\Delta_{P_{2x}-P_{3x}} &= 4.487 \times 10^4 m \\
\Delta_{P_{2x}} &= 1.880 \times 10^6 m \\
\Delta_{P_{2y}} &= 4.935 \times 10^{-3} m
\end{aligned}$$

We can see that by using the approximation defined earlier, we are neglecting variation of order lower than thousand of killometer ($10^6 m$).If we add the optimisation on the value of b we are now neglecting variation of order lower than ten killometerss ($10^4 m$) leaving only error of a couple killometers on the posisiton of P_{2x} which is not a lot against the size of the observation zone : (around $20000 km$ by $100 km$).

About the error on P_{2y} the error is of the order of a millimeter and can clearly be neglected.

3 Observation analysis

3.1 definition of the problem

the problem is describe as follow :
The Earth Moon Sun system is simulated with the Sun being fixed at the origin. The Coordinate system is then centered without rotating it at the Earth Moon barycenter. The motion of the device is computed by taking in accoun the gravitional pull of the Earth an Moon but we don't apply the force of gravity of the sun, this mean that the device will experience the gravity of the sun as if it wa at the altitude of the Earth-Moon barycenter.This won't make a huge difference considering that the device will stay between 2 Moon-Earth distance from Earth. the trajectory of Earth and Moon will be precompute before running any optimisation algorithm.

the initial position of the Earth and Moon that will be use in this article are their position in January 1st midnight, 2025. their position are retrieved on the horyzon system of Nasa : https://ssd.jpl.nasa.gov/horizons/app.html

we consider the position and shape of the observation using the formula and approximation defined earlier, with the initial value of the observation zone being computed if the moon was at the origin at time t0. this give the following equation :

$$P_3(R_l) = R_l + \hat{P_{3x}} D^{-1} R_{ls}$$

with $\hat{P_3}$ being the position of the point P_3 when the moon is at the origin ,R_l being the Moon position. and R_{ls} is the position of the Moon relatively to the sun.

the objectif is to find trajectory that allow repeated and long observations.

after testing some possible orbit configuration for the device, it look like most of the observation last only a few minutes, but some precise configuration that we will discuss later can last for multiples hour. This space of solutions suggest that it is better to optimize only one observations that will last longer instead of trying to find trajectory that will make multiples observations but very short ones.

in order to make multiple observation, it may be better to take orbital period that are multiple of the moon's, this way the Moon and thus the observation zone would be at the same place when the device get to the observation zone again.

hovewer considering that the Sun is moving, it may be better to make the device orbital period equal to the Moon's synodic period but i haven't make enough comparative data to know which one is the best.

from this we can conclude the following formula for the semi major axis of the device.

$$a_s = a_l k^{\frac{2}{3}} \left(\frac{\mu_t}{\mu_t + \mu_l} \right)^{\frac{1}{3}}$$

with

$$P_s = kP_l$$

considering that the objective is to make observation, it seem logical that the device will be in the observation zone at one time or another. This is why i've decided to define the device initial position to be inside the observation zone. In addition the fact that the observation zone is almost flat (around 100 times longer than thick), we can consider the starting position of the device to be on the segment $[P_3, P_1]$. We will also need a initial time t_0 to know when the device cross this segment.

this mean that we can defined the initial position of the device with the following function

$$r_0(t, \lambda) = P_3(t)\lambda + (1 - \lambda)P_1(t)$$

where P_3 and P_1 are computed using the Moon position and the formula that we show earlier.

as we have fixed the semi major axis of the device wa can conclude its speed using this formula :

$$||\dot{r}(0)|| = \sqrt{\frac{2\mu}{||S_0||} - \frac{\mu}{a_s}}$$

we then just have to choose the direction of the speed vector $\dot{r}_0$ using two angle θ_s and ϕ_s.

we must compile the dynamics of the device in order to get the observation time. Here's the dynamics :

$$\begin{bmatrix} \dot{\overrightarrow{R_s}} \\ \dot{\overrightarrow{V_s}} \end{bmatrix} = \begin{bmatrix} \overrightarrow{V_s} \\ \frac{-\mu_l}{||R_s - R_l||^3}(\overrightarrow{R_s} - \overrightarrow{R_l}) + \frac{-\mu_t}{||R_s - R_t||^3}(\overrightarrow{R_s} - \overrightarrow{R_t}) \end{bmatrix}$$

with as initial condition

$$\begin{bmatrix} \overrightarrow{R_0} \\ \overrightarrow{V_0} \end{bmatrix} = \begin{bmatrix} \lambda\left(\widehat{P_1} - \widehat{P_3}\right) + R_{lt} + \frac{\widehat{P_{3x}}}{D}R_{ls} \\ \sqrt{\mu\left(\frac{2}{||R_0||} - \frac{1}{a_s}\right)}\widehat{v_0}(\theta_s, \phi_s) \end{bmatrix}$$

the differential equation must be solve before and after the initial time t_0 to get the time when the device exit and enter the observation zone.

the payoff function that will be used to evaluate the interest of one observation is defined as follow

$$\int_{-\tau_d}^{\tau_u} dx = \tau_d + \tau_u$$

where τ_d and τ_u are the time when the device enter and exit the observation zone.

we will use the equation (2) to evaluate if the device is in the observation zone at a time t.

to conclude we have the following variables :

- the initial time

on a donc un espace de dimension 4 : $(\nu, \lambda, \theta_s, \phi_s) = [0, 2\pi] \times [0, 1] \times [0, 2\pi] \times [0, \pi]$

l'espace est contraint mais étant donné que la plupart des dimension sont des angles et qu'il suffit de donner un score de 0 si λ est en dehors du domaine on peut considéré que l'espace est égal à $\mathbb{R}^4$ pour avoir un problème sans contrainte.

4 Result

When we are working with a simple 2D problem with circular Moon orbit , there are an optimum near the value $(2\pi/3, 0.5, 38/45\pi, 0)$ that give observation time of almost 20h.

We managed to get observation time this long because the speed of the object and the observation Zone are equivalent. Meaning that the device can stay in the observation zone for a long time. As the Observation Zone velocity are nearly the same as the Moon, we can simplify for now by saying that the device must have the same speed as the Moon.

The satellite describe a loop inside the observation zone, meaning that it is possible to get two fairly long observation very near to each other if the tip of the loop is outside of the observation zone. ascending if there is a good chance that these solutions are less efficient that a solution with the entire loop inside the observation zone, it could be usefull to improve the objective function to detect when the object reenter the observation zone after a short time.

For the model implemented, i've intentionally kept high value of α to ensure that the loop stayed in the observation zone.

we can easily determine that there are two point in the lunar orbit where we can obtain very long observation time:

The satellite can have the same speed vector as the moon only if the following equation is verified :

$$\sqrt{\frac{2\mu}{R_s} - \frac{\mu}{a_s}} = \sqrt{\frac{2\mu}{R_l} - \frac{\mu}{a_l}}$$

Considering that the moon's speed is constant (because of its small eccentricity) and that the period of the device is the same as the moon (ie $a_s = a_l$), we get the relation :

$$R_s = R_l$$

as the satellite is in the obsevation zone, the region in which the satelite can cross the observation zone and at the same speed as the moon is the intersection between the possible observation zone space (which is in this simplified case simplified by an ellipse) and a sphere of radius R with $R = R_l$ in this case.

There are two point in the orbit that satisfy these condition.

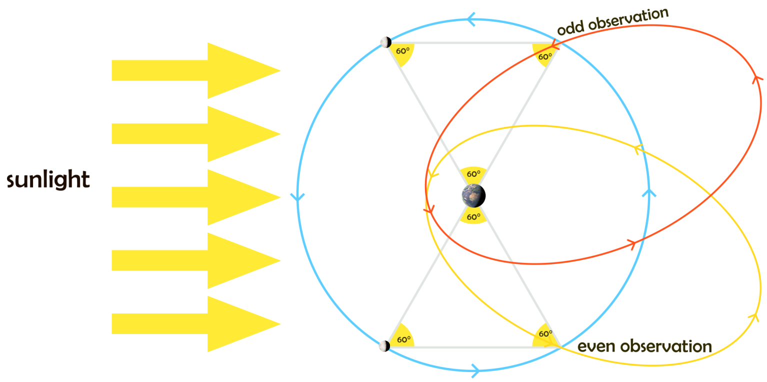


Figure 2: keplerian orbit allowing to make long observation in a simplified model (fixed sun, circular orbit of the moon and observation zone distance equal to moon's orbit radius)

Using the solution that we found in 2D we can try to use it as an initial condition for 3D problems, the solution found with varying Ω value and considering the eccentricity of the Moon tend to show that there are always a solution near this point.

A question that could arise from these observation can be about the importance of the device period over its speed in the observation zone. In other word, maybe it is better to force the device to match the speed of the observations zone instead of making it matcht the speed needed to get a fixed period.

To answer this question i've made a quick test where for a set of position of the observation zone i've lanched the device with the same velocity vector as the observation zone, the result gave pretty high observation time of multiple hours, with the lowest possible time occuring when the observation time is near the Earth. This is not a problem as this kind of trajectory where not possible anyway because it require speed of around $1km \cdot s^2$ at low altitudes.

We can also see Three main peak, the first two are the one we find earlier with observations time of around 18 hours, and there a third one that is a little bit higher with observation time of around 24 hours.

The fact that we make longer observation when the speed of of the observation zone match the speed of a circular orbit at its current position could be linked to the fact that the radial speed of the device start decreasing after crossing this altitude which could allow the device to make an oscilation around the observation point.

about the last peak, the proximity of the observation from the Earth umbra and the fact that the corresponding device will have around the energy to escape the Earth sphere of influence (the speed of the Moon two time further than the Moon) make it not the better choice for reapeated observation.

It could be an improvement of the method to drop the constraint on the speed norm and replace the two angle argument by a simple vector. Hovewer tests show worse result in most case when using only this algotithm to find solution while using this algorithm with the solution of the previous ones as initial condition barelly improve the solution.

4.1 finding initial guess

Now that we know where the observation roughly are we can create an algorithm that could easily find initial guess to find observation.

as the speed of the device is constrained by its semi major axis (by the equation $\sqrt{\frac{2\mu}{R_s} - \frac{\mu}{a_l}}$), we just have to find the point where the observation zone speed matches the speed of the device.

If we consider that the observation zone center is at the position of the point P_{2x} we get :

$$P_{2x} = R_l + \frac{D_{p2}}{D_{ts}}(R_l - R_s)$$

where D_{p2} is the distance between the moon and the point P_{2x} where the moon is at the origin. the velocity of P_{2x} is then straightforward to compute

$$\dot{P_{2x}} = \dot{R_l} + \frac{D_{p2}}{D_{ts}}(\dot{R_l} - \dot{R_s})$$

we can then get from this the following payoff formula :

$$D_{obs} = \left(||\dot{P_{2x}}|| - \sqrt{\frac{2\mu}{P_{2x}} - \frac{\mu}{a_l}}\right)^2$$

which is a one dimension equation depending of time only, which make it way easier to solve.

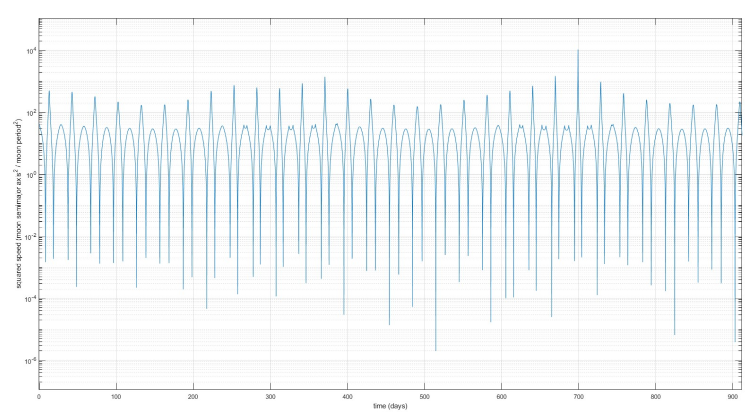


Figure 3: relative speed between the device and the observation zone

we can see a periodic signal with a period equal to the synodic period of the device. A period is made of two local maximum corresponding to the time when the observation zone is the closest to Earth (the highest sharp peak)and when the observation zone is the farthest from Earth(the soft peak). There also two local minimum which correspond to the two points of interest. Theses minimum are around 5 days before and after the highest peak ($\frac{1}{6}$ time the synodic period exactly).

This configuration allow find estimate of time where the observation happen :

The first step consist of finding the high peak, the sharp peak would always be higher than 100 because of the high speed of the device near Earth and the soft peak would be always under 100 (because the speed difference can't be above 2π making the value of D_{obs} smaller than $4\pi^2 \approx 39$).

this give us the following constrained problem:

$$t^0 = \arg\max D_{obs}(t)_{0 \leq t; D_{obs}(t) \geq 100}$$

by taking an initial guess at $t_0^0 = 0$ or finding the an estimation by hand should give us the first peak.

we can then find every peak by solving the same problem with an initial guess of $t_0^n = t^{n-1} + P_{sy}$ where t^{n-1} is the time of the last peak and P_{sy} is the synodic period of the moon.

we can then find the points of interests by finding the minimum of the function D_{obs} by taking the the following initial guess :

$$\mathrm{O}_0^{2n} = t^n - \frac{P_{sy}}{6}$$
$$\mathrm{O}_0^{2n+1} = t^n + \frac{P_{sy}}{6}$$

A good thing about this method is that each problems (except the first one if we take $t_0^0 = 0$) can be easily sove with a simple gradient descent method. take also note that the first observation may be before 0 if the first peak happen at a time smaller than a sixth of the Synodic period.

At last we can compute the vector X_0 for each of these time and then apply the real method to compute the best set of value for each observation.

the problem of this simplified model is that we fixed the period of the device at the period of the Moon which was an hypothesis to reduce thrust needed between transfert but not realy an argument for maximizing the observation time.

in fact I've noticed that observation time were higher when the period of the device was slightly reduced (around 0.92 Lunar period). I didn't manage to find the exact cause of this pheonomenon, i think a better model than this one could be around...

For the rest of this article the value of the device period was fixed at 0.92 as it give longer observation time.

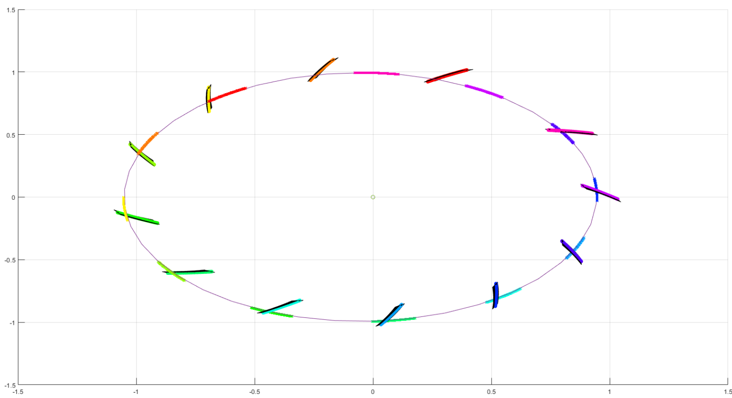


Figure 4: descending observation zone over the course of one year, the position of the moon and the device is also added, the result show in this graph doesn't take in accout the variation of the moon orbit caused by the sun

4.2 impact of alpha on the observation time

i've made a few test of the algorithm with mulitple value of alpha. I've fetch all observations happening between january 4th of 2025 and january 2nd of 2027. here's the observations time we got from theses:

alpha	10%	5%	2%	1%	0.5%
maximum	28h01min	21h26min	15h02min	11h31min	8h46min
minimum	23h30min	18h00min	12h42min	9h42min	6h34min
median	25h26min	19h29min	13h40min	10h30min	8h06min
mean	25h35min	19h35min	13h47min	10h33min	8h04min

the result with the observation zone at 5% will be used in the rest of the article.

5 low thrust transfert

5.1 low thrust model

we want to move from a point R_i at time t_i to a point R_f at time t_f with initial velocity V_i and final velocity V_f.

the device can be control with a low thrust engine that can generate an acceleration of maximum magnitude A_{max} in any direction, as the final objective is to use a solar sail, the variation of mass is neglected.

the passive dynamic of the device is the following :

$$\begin{bmatrix}\dot{r}\\ \dot{v}\end{bmatrix} = \begin{bmatrix} v \\ \sum_{\beta\in B} \frac{\mu_\beta}{\|r_\beta - r\|^3}(r_\beta - r)\end{bmatrix}$$

where B is the set of every bodies that attract the device (for now the two body are The Earth and the Moon). the dynamics of the bodies must be computed before (we suppose that the device does not influence the dynamics of the system).

the acceleration of the thruster can be take in account by adding a vector the equations of speed :

$$f(x,u) = \dot{x} = \begin{bmatrix}\dot{r}\\ \dot{v}\end{bmatrix} = \begin{bmatrix} v \\ u + \sum_{\beta\in B} \frac{\mu_\beta}{\|r_\beta - r\|^3}(r_\beta - r)\end{bmatrix}$$

we want to minimize the Δ_v of the transfert, so the running payoff is :

$$r(x,u) = -\|u\|$$

we also want to be at the right position and velocity at the time t_f so the terminal payoff is :

$$g(x(t_f)) = -(r(t_f) - R_f)^2 - (v(t_f) - V_f)^2$$

we get the following hamiltonian :

$$H(x,p,u) = \begin{bmatrix} v \\ u + \sum_{\beta\in B} \frac{\mu_\beta}{\|r_\beta - r\|^3}(r_\beta - r)\end{bmatrix}^t \cdot p - \|u\|$$

using the Pontryagin maximum principle theorem we can compute the differential of $p = \begin{bmatrix}p_r\\ p_v\end{bmatrix}$

$$\begin{aligned}\dot{p} &= -\nabla_x H \\ &= -\begin{bmatrix} 0 & I_3 \\ \sum_{\beta\in B} \frac{\mu_\beta}{\|r_\beta - r\|^3}\left(\frac{3(r_\beta - r)\cdot(r_\beta - r)^t}{\|r_\beta - r\|^2} - I_3\right) & 0\end{bmatrix}^t \cdot p \\ &= \begin{bmatrix} \sum_{\beta\in B} \frac{\mu_\beta}{\|r_\beta - r\|^3}\left(I_3 - \frac{3(r_\beta - r)\cdot(r_\beta - r)^t}{\|r_\beta - r\|^2}\right)\cdot p_v \\ -p_r\end{bmatrix}\end{aligned}$$

we know that $\forall t$, $u = \arg\max_{u\in B(0,A_{max})} H(x,p,u)$. We can rewrite the hamiltonian as follow :

$$\begin{aligned}H(x,p,u) =& C(x,p) + u\cdot p_v - \|u\| \\ =& C(x,p) + \|u\|(\|p_v\|\cos(\alpha_{\widehat{up_v}}) - 1)\end{aligned}$$

with this form we can easily conclude that

$$\begin{cases} \|u\| = A_{max} & \text{if } \|p_v\|\cos(\alpha_{\widehat{up_v}}) \geq 1 \\ \|u\| = 0 & \text{else} \end{cases}$$

we can then observe that $\|p_v\|\cos(\alpha_{\widehat{ap_v}})$ cannot be greater than 1 if $\|p_v\| < 1$. Also if $\|p_v\| \geq 1$, it is obvious that taking $\alpha_{\widehat{up_v}} = 0$ (i.e. u and p_v are collinear) maximize the factor $\|p_v\|\cos(\alpha_{\widehat{uap_v}})$. So we can conclude the following formula :

$$\begin{cases} u = A_{max}\widehat{p_v} & \text{if } \|p_v\| \geq 1 \\ u = 0 & \text{else} \end{cases}$$

to solve the problem numerically, we then just have to find the correct value p_i so the terminal value of x correspond to the target value.

6 result low thrust

to compute the solution, i've used the julia library : OptimalControl,

the starting point of the device is the exting point where of starting observation and the ending point is the enter point of the ending observation.

To simplify the next explication i will separate the observations into two category : the ascending observations and the descending observation. The descending observations are observations happening when the moon is around it last crescent, in this configuration the device must aproach Earth to make an observation. The ascending observation is the opposite, happening when the moon is around its first crescent and needed the device to moving away from earth.

6.1 observation

The algorithm had a lot of struggle to find an optimum for moving to two adjacent observations, which was excpected as the two orbit of adjacent optimum are very different and the time interval is small meaning that important Δ_v are needed to move from the starting point to the ending point. hovewer the algorithm find solution for transition between ascending observation and between descending observation. This was also excpected as the starting and objective orbit are very similar with the main difference being only a rotation of $\frac{P_{moon}}{P_{sol}}2\pi$ rad caused by the movement of the sun.

for 1 in 2 transfert, the maximum acceleration found was betweeen $170\mu N \cdot \text{Kg}$ and $200\mu N \cdot \text{Kg}$ which is around the acceleration we can get from electric engine but not from solar sail.

there are also another type of transfert that caught my interest, it is transfert from descending observation to ascending observation (1 in 3 transfert starting from descending observation) which seem to be very easy to do with maximum acceleration ranging from $50\mu N \cdot \mathrm{Kg}$ to $95\mu N \cdot \mathrm{Kg}$. It truned out that the angle between an descending orbit and a ascending orbit orbit is around the same length of the angle describe by the movement of the sun in $\frac{4}{3}$ of the moon synodic period, this mean that the starting and ending orbit in such transfert are almost the same with the only main task being to change the true anomaly. This transferts could look like they are a good alternative to reduce maximum acceleration during transfert but the problem is that once we reached the descending observation, we have to make a transfert from ascending to descending and in this case the situation is reverse with the angles being added and not substracted, causing high maximum acceleration.

In fact making fewer observation doesn't seem to reduce the maximum acceleration needed or ascending the Δ_V which could look counter intuitive at first glance but is not when we think aboout the fact that orbits become increasingly different from each other with time. This mean that there shold be some kind of minimum power needed for making observations which must be around the amount of power needed to rotate the observation orbit by 360° every year. considering the result that we have now, the device should be able to deliver a delta V of around $5Km \cdot s^{-1}$ every year, which is not that far of what solar sail could do , but we don't take in account here the constraint given by solar sail that will likely increase this value.

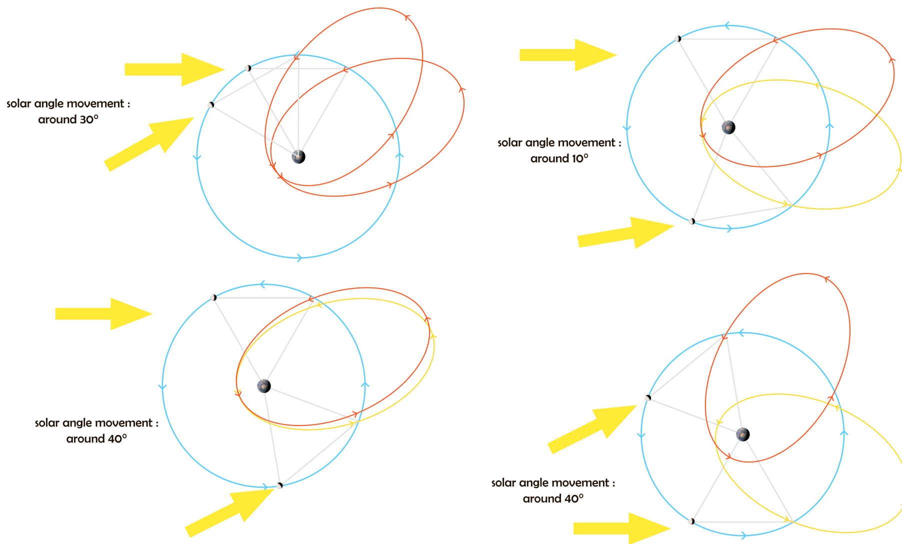
with this said it is possible to jump a lot of observations to the point that we don't need to rotate the orbit all around the Earth. Like with the obvious example of making an observation every year, when the sun is roughly at the same place relative to the Earth-Moon system. But the counter part is that we will be making way less observations ... at most one every 6 mouths.

here some illustration to better understand the different possible transfert (note that theses graphics show approximative position of observations orbits and are not made from real data):

here we can see that the top left transfert (descending observations to descending observation) is relatively easy to do with having to rotate a little bit the orbit in around 29 days. Hovewer, making a transfert from and descending to ascending observations like in the top right would need the device to make a bigger change in orbit in abour 3 times less time. moving from ascending to descending observation will be even harder as the orbit move apart from each other instead of getting closer.

the bottom left illustration show the easiest transfert where the sun magnitude change compensate almost exactly the angle between descending and ascending observation making the transfert only a question of changing the true anomaly.

like for the two other transfert, moving from ascending to descending observation are harder as the sun magnitude change cumulate instead of cancelling the angle between descending and ascending observation.



6.2 scenario with solar sail

Using solar sails with their actual performance level (around $50\mu m \cdot \mathrm{s}^2$) won't allow to make a lot of observations, with the only plausible scenario that far could be to make an descending observation follow by a ascending observation 40 days later taking advantage of the low maximum acceleration of these transfert. Then spend multiples mounths aligning for another descending observations and repeating the process in a cycle of at least 8 mouths which is still way better than what we could do on Earth. But ascending this scenario may be infeasible once the additional sail constraint on the control are added.

another possibility could be about the third observation that i had consider to too impractical to be usefull at first but may give better result when making only a few observation per years.

6.3 scenario with electric propulsion

What look like the best scenario at this point would be to put two devices capable of delivering acceleration of the order of $300\mu m \cdot \mathrm{s}^2$, one for the descending observation and one for the ascending observation. This would allow to attends every observations which mean that we will be able to observe the solar corona around 2.8% of the mission time with ascending the possibility of observing the sun and the sun corrona at the same time if one device aim at the sun during the observation of the other device.

To get more data about these scenario i've added the equation of mass to the dynamics of the system. for the next result i've use the following theoretical spacecraft :

the device has $50Kg$ of payload with $40Kg$ of fuel, the engine used is the

RIT-10 evo from