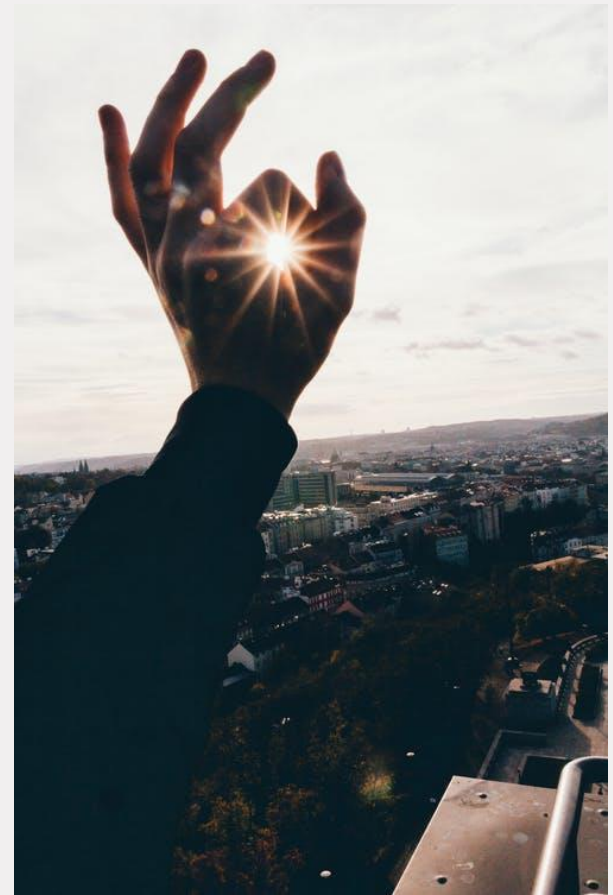


# Sun position



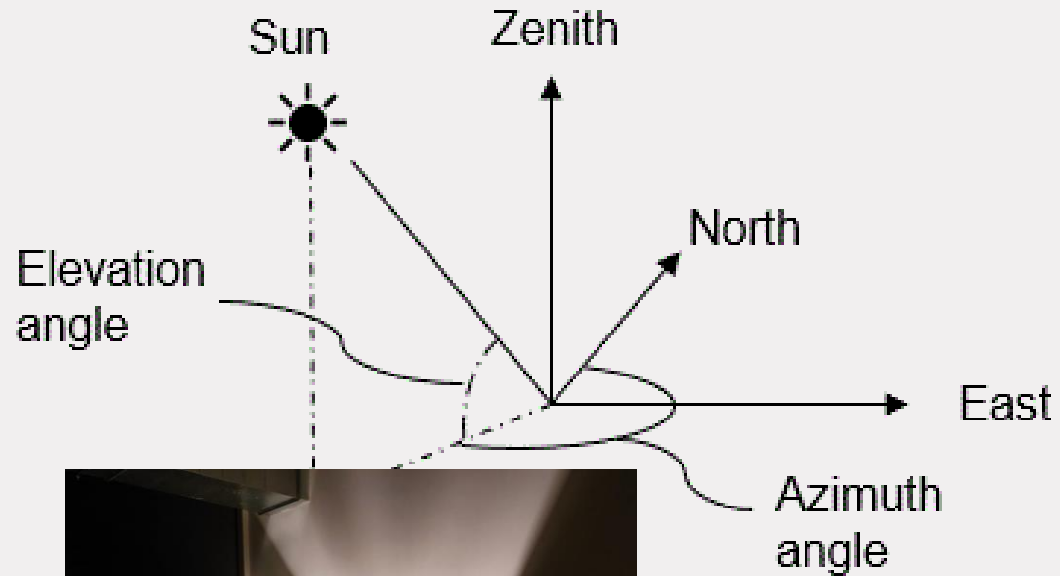
# Sun position

Sun position:

- Sun elevation angle  $\gamma_S$
- Sun azimuth angle  $\alpha_S$

Depends on:

- Geographical latitude  $\varphi_B$
- Sun declination  $\delta_S$
- Hour angle  $\tau_S$



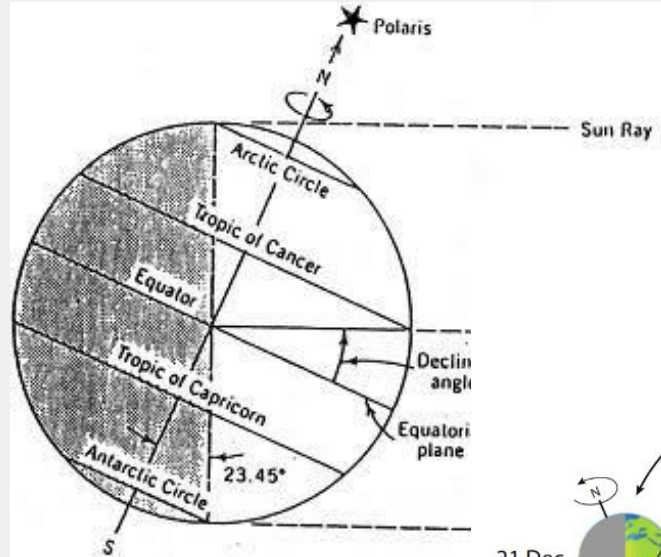
## Sun position

## Sun position:

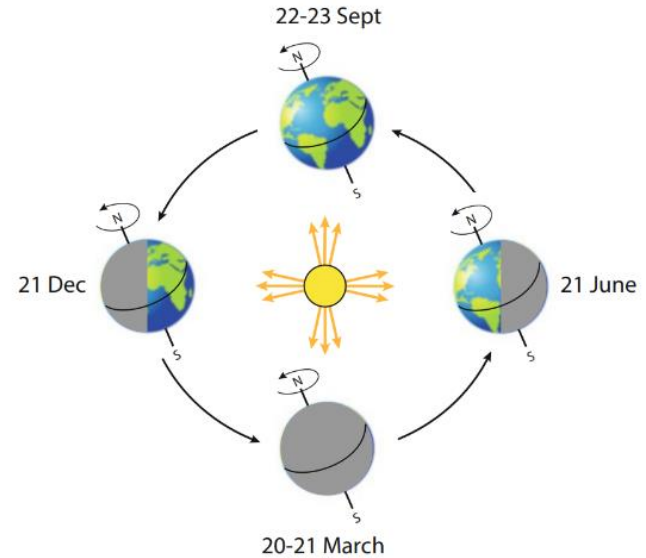
- Sun elevation angle  $\gamma_S$
- Sun azimuth angle  $\alpha_S$

Depends on:

- Geographical latitude  $\varphi_B$
- **Sun declination  $\delta_S$**
- Hour angle  $\tau_S$



The **earth's equator is tilted** 23.45 degrees with respect to the plane of the earth's orbit around the sun, so at various times during the year, as the earth orbits the sun, declination **varies from 23.45 degrees north to 23.45 degrees south.**



Van Bommel. Interior Lighting (2019).

# Sun position

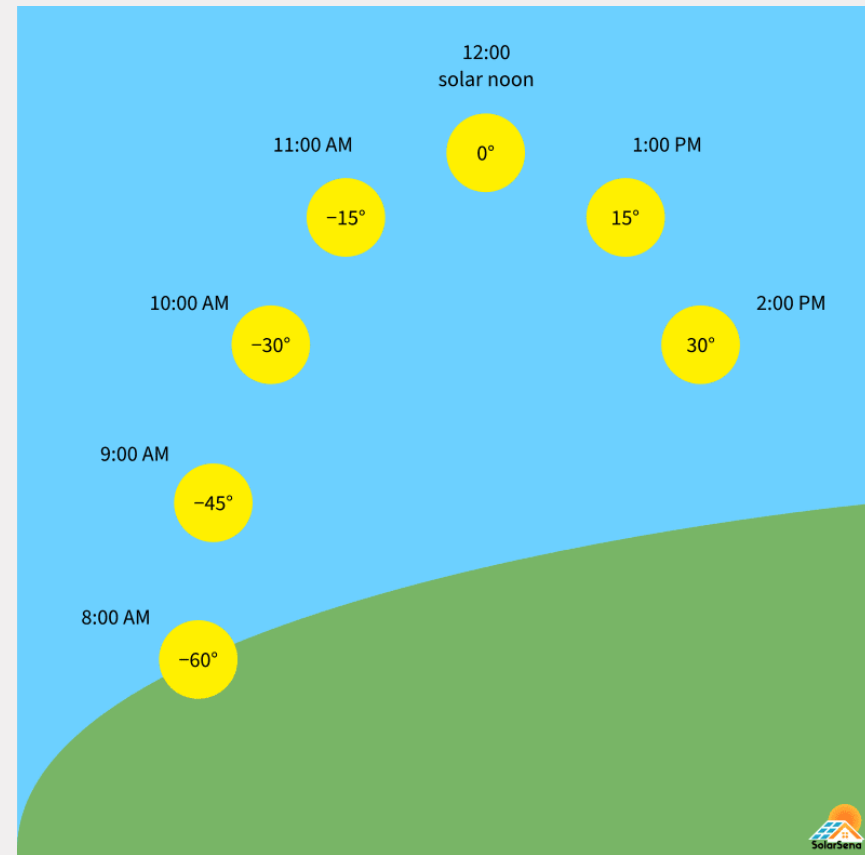
Sun position:

- Sun elevation angle  $\gamma_S$
- Sun azimuth angle  $\alpha_S$

Depends on:

- Geographical latitude  $\varphi_B$
- Sun declination  $\delta_S$
- **Hour angle  $\tau_S$**

The hour angle is the angular displacement of the sun east or west of the local meridian due to **rotation of the earth on its axis** at  $15^\circ$  per hour with **morning being negative and afternoon being positive**.



# Sun position | Calculation

$$\gamma_S = \arcsin(\cos \varphi_B \cdot \cos \delta_S \cdot \cos \tau_S + \sin \varphi_B \cdot \sin \delta_S)$$

$$\alpha_S = \begin{cases} 180^\circ - \arccos\left(\frac{\sin \gamma_S \cdot \sin \varphi_B - \sin \delta_S}{\cos \gamma_S \cdot \cos \varphi_B}\right) & \text{for } TST \leq 12.00 \text{ h} \\ 180^\circ + \arccos\left(\frac{\sin \gamma_S \cdot \sin \varphi_B - \sin \delta_S}{\cos \gamma_S \cdot \cos \varphi_B}\right) & \text{for } TST > 12.00 \text{ h} \end{cases}$$

$\varphi_B$ : Geographical latitude

North  $\alpha_S=0^\circ$

$\delta_S$ : Sun declination

East  $\alpha_S=90^\circ$

$\tau_S$ : Hour angle

South  $\alpha_S=180^\circ$

TST: True Solar Time

West  $\alpha_S=270^\circ$

# Sun position | Calculation

Geographical latitude  $\varphi_B$  of Eindhoven:  $51.4^\circ$



<https://www.fonq.nl/product/city-shapes-skyline-eindhoven-klein/354910/>

# Sun position | Calculation

The sun declination  $\delta_S$  can be approximated by Fourier series:

$$\begin{aligned}\delta_S(J) &= 0.3948 - 23.2559 \times \cos(J' + 9.1^\circ) - 0.3915 \\ &\times \cos(2 \times J' + 5.4^\circ) - 0.1764 \times \cos(3 \times J' + 26^\circ)\end{aligned}$$

$\delta_S$ : Sun declination

J: Day of the year (Jan 1=1, Dec 31=365)

$$J' = \frac{360^\circ \times J}{365}$$

# Sun position | Calculation

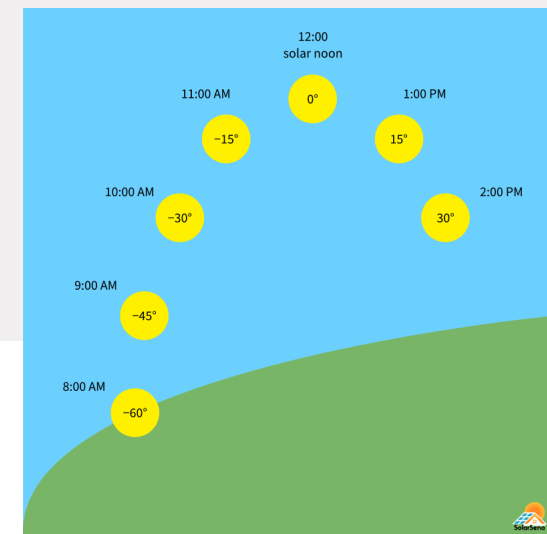
The hour angle  $\tau_s$  can be calculated:

For calculating the sun position, it is necessary to specify the hour angle  $\omega_\eta$

$$\omega_\eta = (12,00 \text{ h} - TST) \times 15^\circ$$

The hour angle  $\omega_\eta$  is counted from the meridian as positive towards the afternoon and negative towards the morning. The following then applies for solar altitude

EN 17037: 2018





# Sun position | Calculation

The hour angle  $\tau_S$  can be calculated:

$$\tau_S = \begin{cases} -15^\circ \cdot |(12:00h - TST)| & \text{for } TST < 12.00 \\ 15^\circ \cdot |(12:00h - TST)| & \text{for } TST > 12.00 \end{cases}$$

$$TST = LT + \frac{\lambda_B - \lambda_S}{15} + ET$$

$$\begin{aligned} ET(J) &= 0.0066 + 7.3525 \times \cos(J' + 85.9^\circ) + 9.9359 \\ &\times \cos(2 \times J' + 108.9^\circ) + 0.3387 \times \cos(3 \times J' + 105.2^\circ) \end{aligned}$$

“correction” in minutes

$\tau_S$ : Hour angle

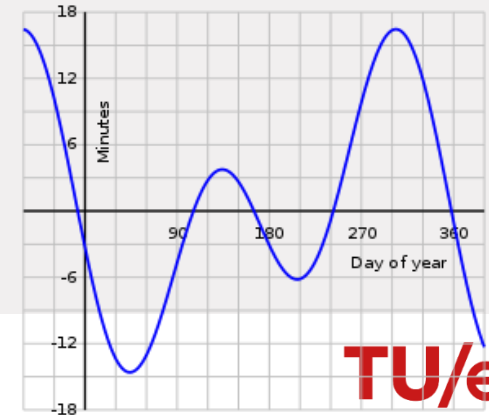
TST: True Solar Time

LT: Local clock Time

$\lambda_B$ : Geographical longitude

$\lambda_S$ : Longitude of the standard time meridian (15° in the NL)

ET: Equation of Time



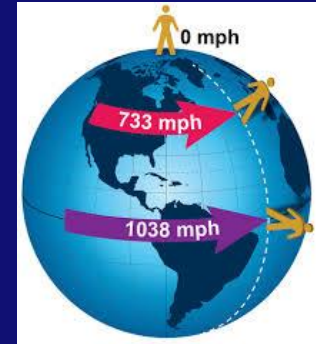
24 hours

# Sun position | Calculation



What is the width in degrees between two time zones?

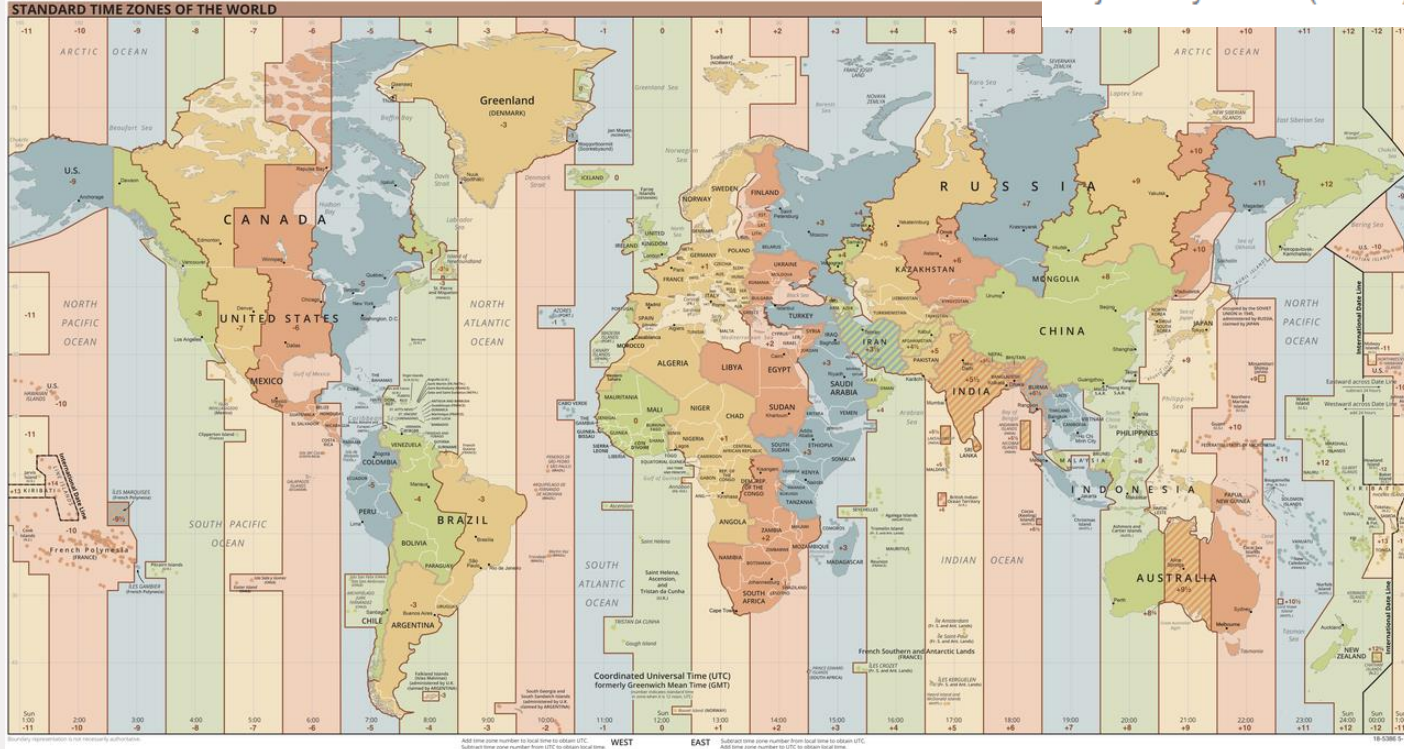
rock climbing  
ice fishing jogging  
hiking running  
weight lifting  
video games  
kayaking  
swimming  
bungee jumping



## Sun position | Calculation

16:38

vrijdag 24 november 2023 (GMT+6:30)  
Tijd in Myanmar (Birma)

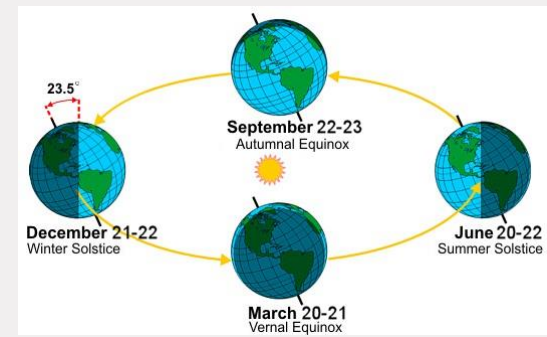


# Sun position | Calculation

## Equation of time

The variation in the length of an apparent solar day is a result of two factors.

1. The Earth's orbit is elliptical—it's a slightly squashed circle, not a perfect circle. When the Earth is closer to the Sun, it travels more quickly along its orbit.
  2. The Earth's spin axis is tilted—around the solstices, one of the Earth's poles is leaning toward the Sun. Around the equinoxes, neither pole is leaning toward the Sun.
- Our clocks and watches ignore these differences in the Sun's movement. To these steadily ticking devices, every day is an average day of exactly 24 hours.
  - This form of steady, average time is called mean solar time.
  - The difference between apparent solar time and mean solar time is called the equation of time.




# Sun position | Calculation

Calculate the sun position ( $\gamma_S$ ,  $\alpha_S$ ) in the NL ( $\lambda_B = 5^\circ$ ,  $\varphi_B = 52^\circ$ ) for today at 3pm.

## 1. Determine the day of the year

Day = 328



A word cloud of outdoor activities, with 'video games' being the largest word in red. Other words include 'kayaking' in blue, 'bungee jumping' in light blue, 'running' in red, 'jogging' in green, 'hiking' in dark blue, 'weight lifting' in light blue, 'rock climbing' in yellow-green, 'swimming' in light blue, and 'ice fishing' in dark blue.

bungee jumping  
running  
jogging  
kayaking  
video games  
hiking weight lifting  
rock climbing  
swimming  
ice fishing

# Sun position | Calculation

Calculate the sun position ( $\gamma_S$ ,  $\alpha_S$ ) in the NL ( $\lambda_B = 5^\circ$ ,  $\varphi_B = 52^\circ$ ) for today at 3pm.

## 2. Determine the TST

$$J' = \frac{360^\circ \times J}{365}$$

$$\begin{aligned} ET(J) &= 0.0066 + 7.3525 \times \cos(J' + 85.9^\circ) \\ &+ 9.9359 \times \cos(2 \times J' + 108.9^\circ) + 0.3387 \\ &\times \cos(3 \times J' + 105.2^\circ) \end{aligned}$$

$$TST = LT + \frac{\lambda_B - \lambda_S}{15} + ET$$

$\lambda_S$ : Longitude of the standard time meridian ( $15^\circ$ )

$$J' = 323.5068$$

$$ET = 13.1756 \text{ min}$$

$$ET = 0.2196 \text{ h}$$

$$\begin{aligned} TST &= 14,5529 \\ &(14:33) \end{aligned}$$

bungee jumping  
running  
jogging  
kayaking  
video games  
hiking weight lifting  
rock climbing  
swimming  
ice fishing

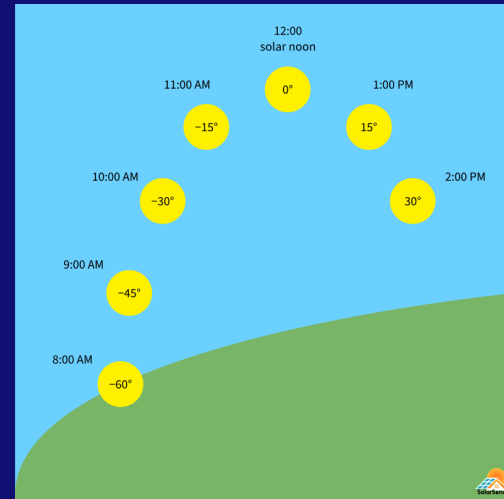
# Sun position | Calculation

Calculate the sun position ( $\gamma_S$ ,  $\alpha_S$ ) in the NL ( $\lambda_B = 5^\circ$ ,  $\varphi_B = 52^\circ$ ) for today at 3pm.

## 3. Determine the hour angle

$$\tau_S = \begin{cases} -15^\circ \cdot |(12:00h - TST)| & \text{for } TST < 12.00 \\ 15^\circ \cdot |(12:00h - TST)| & \text{for } TST > 12.00 \end{cases}$$

$$\tau_S = 38.2939^\circ$$



bungee jumping  
running  
jogging  
kayaking  
video games  
hiking weight lifting  
rock climbing  
swimming  
ice fishing

# Sun position | Calculation

Calculate the sun position ( $\gamma_S$ ,  $\alpha_S$ ) in the NL ( $\lambda_B = 5^\circ$ ,  $\varphi_B = 52^\circ$ ) for today at 3pm.

## 4. Determine the sun declination

$$\begin{aligned}\delta_S(J) &= 0.3948 - 23.2559 \times \cos(J' + 9.1^\circ) - 0.3915 \\ &\times \cos(2 \times J' + 5.4^\circ) - 0.1764 \times \cos(3 \times J' + 26^\circ)\end{aligned}$$

$$\delta_S = -20.4227^\circ$$

### Sun declination:

The earth's equator is tilted 23.45 degrees with respect to the plane of the earth's orbit around the sun, so at various times during the year, as the earth orbits the sun, declination varies from 23.45 degrees north to 23.45 degrees south.

bungee jumping  
running  
jogging  
kayaking  
video games  
hiking weight lifting  
rock climbing  
swimming  
ice fishing



# Sun position | Calculation

$$\tau_S = 38.2939^\circ$$

$$\delta_S = -20.4227^\circ$$

$$(\gamma_S, \alpha_S) =$$

$$(10.2448^\circ, 216.1678^\circ)$$

$$(\gamma_S, \alpha_S) = (10^\circ, 216^\circ)$$

Calculate the sun position  $(\gamma_S, \alpha_S)$  in the NL ( $\lambda_B = 5^\circ$ ,  $\varphi_B = 52^\circ$ ) for today at 3pm.

## 5. Determine the sun position $(\gamma_S, \alpha_S)$

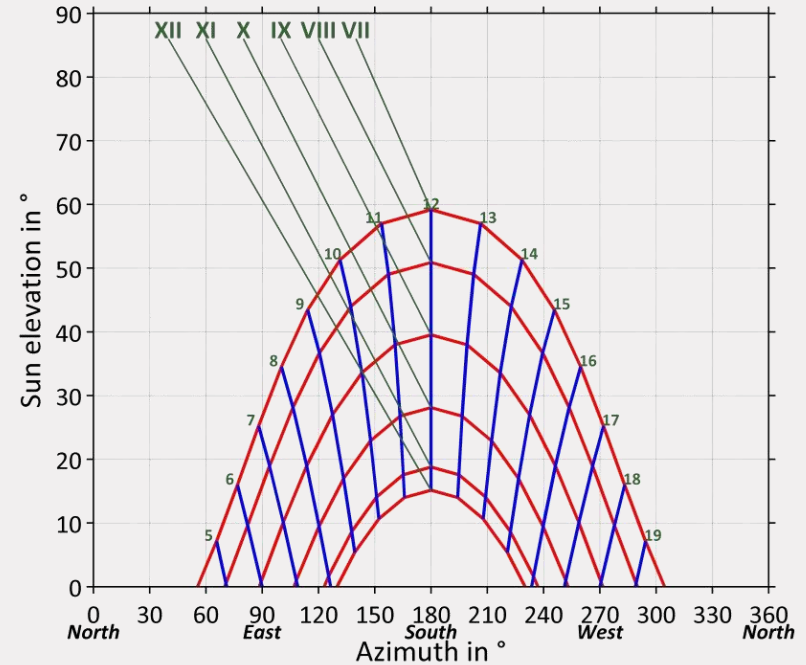
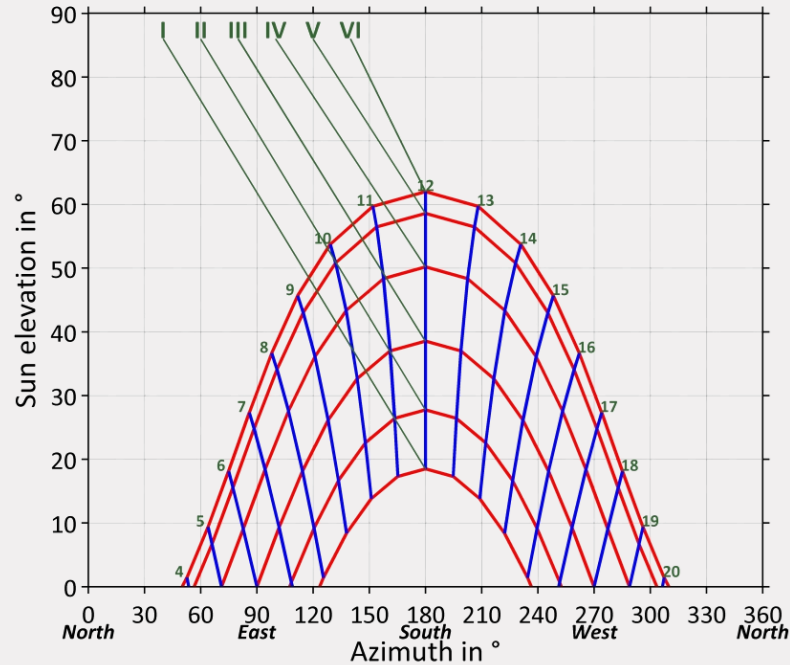
$$\gamma_S = \arcsin(\cos \varphi_B \cdot \cos \delta_S \cdot \cos \tau_S + \sin \varphi_B \cdot \sin \delta_S)$$

$$\alpha_S = \begin{cases} 180^\circ - \arccos\left(\frac{\sin \gamma_S \cdot \sin \varphi_B - \sin \delta_S}{\cos \gamma_S \cdot \cos \varphi_B}\right) & \text{for } TST \leq 12.00 \text{ h} \\ 180^\circ + \arccos\left(\frac{\sin \gamma_S \cdot \sin \varphi_B - \sin \delta_S}{\cos \gamma_S \cdot \cos \varphi_B}\right) & \text{for } TST > 12.00 \text{ h} \end{cases}$$

bungee jumping  
running  
jogging  
kayaking  
video games  
hiking weight lifting  
rock climbing  
swimming  
ice fishing

Practice exercise  
available!

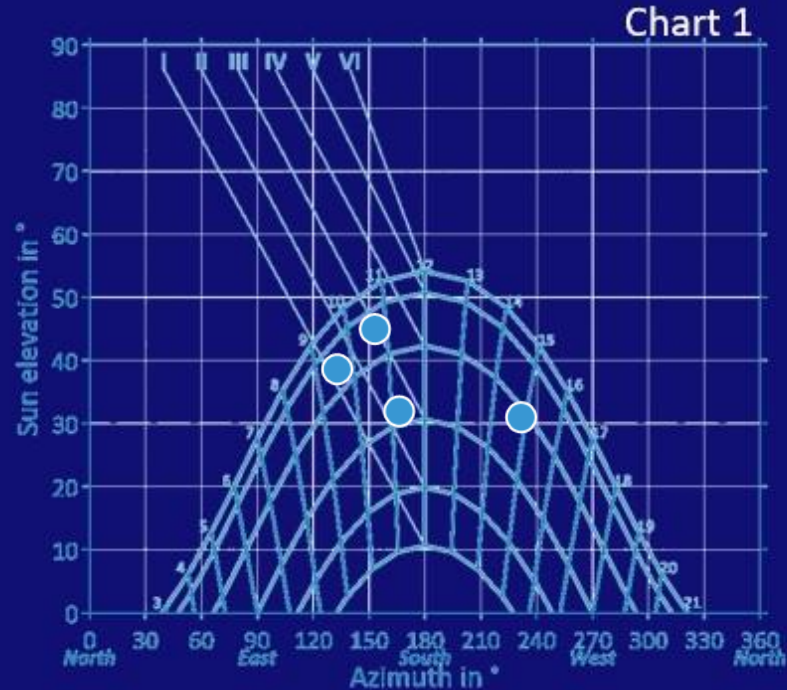
# Sun position | Sun chart



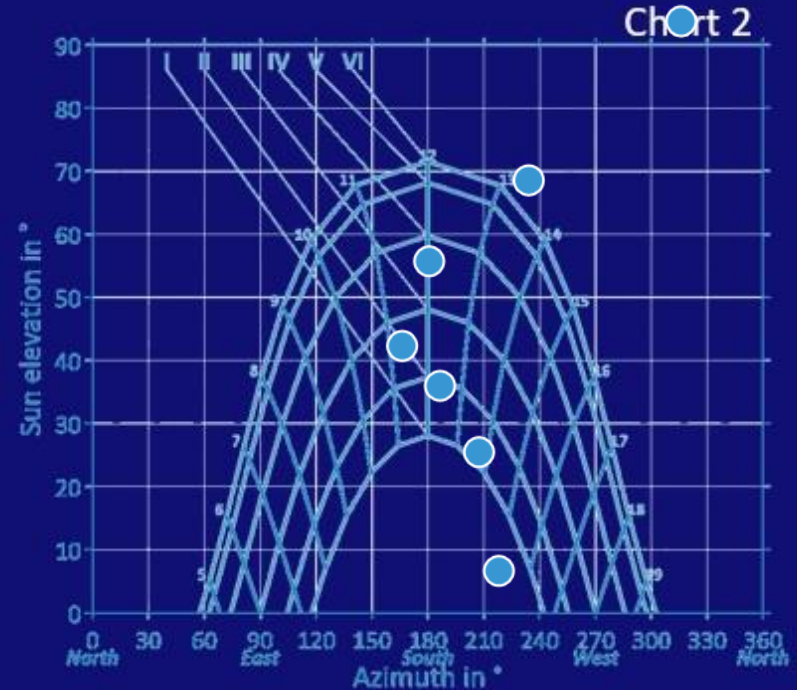
$$\varphi_B(\text{Eindhoven}) = 51.4^\circ$$

# Sun position | Sun chart

One sun chart belongs to Rome and one to Stockholm. Which chart belongs to Rome?



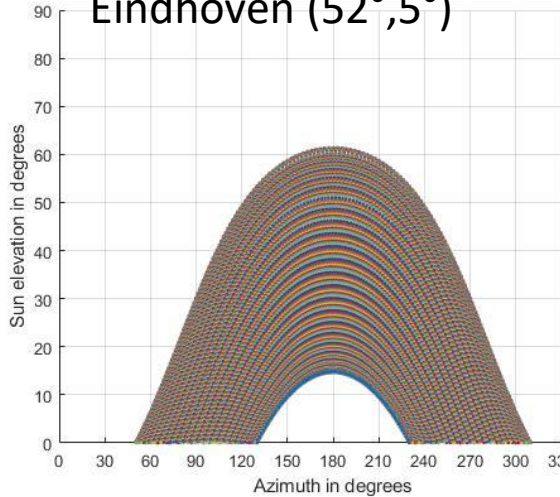
$$\varphi_B(\text{Stockholm}) = 59.3^\circ$$



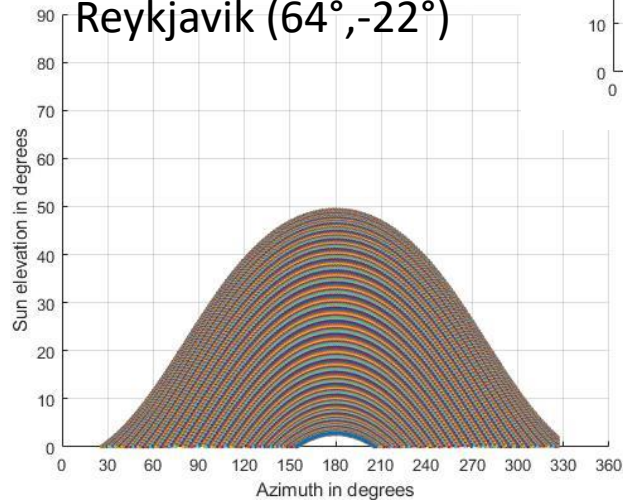
$$\varphi_B(\text{Rome}) = 41.9^\circ$$

# Sun position | Sun chart

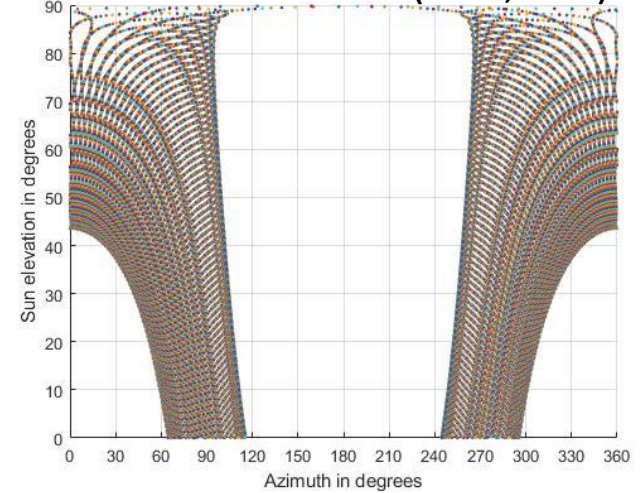
Eindhoven ( $52^\circ, 5^\circ$ )



Reykjavik ( $64^\circ, -22^\circ$ )



Rio de Janeiro ( $-23^\circ, -43^\circ$ )



# Sun position | Sunrise and sunset

$$\gamma_S = \arcsin(\cos \varphi_B \cdot \cos \delta_S \cdot \cos \tau_S + \sin \varphi_B \cdot \sin \delta_S)$$

Set  $\gamma_S = 0$  and solve for time:

$$t_{sr} = 12 - \frac{\arccos(-\tan \varphi_B \cdot \tan \delta_S)}{15}$$

$$t_{ss} = 12 + \frac{\arccos(-\tan \varphi_B \cdot \tan \delta_S)}{15}$$

*In TST!*

$\varphi_B$ : Geographical latitude

$\delta_S$ : Sun declination

$\tau_S$ : Hour angle

# Sun position | Sunrise and sunset

At what time (LT) will the sun set in  
Eindhoven today?

- A. 16:00
- B. 16:06
- C. 16:13
- D. 16:28
- E. 16:33

$$\delta_S = -20.4227^\circ$$

$$t_{ss,TST} = 16.5496 \text{ h} \\ (16:33)$$

$\varphi_B$ : Geographical latitude  
 $\delta_S$ : Sun declination  
 $\tau_S$ : Hour angle

$$t_{ss} = 12 + \frac{\arccos(-\tan \varphi_B \cdot \tan \delta_S)}{15}$$

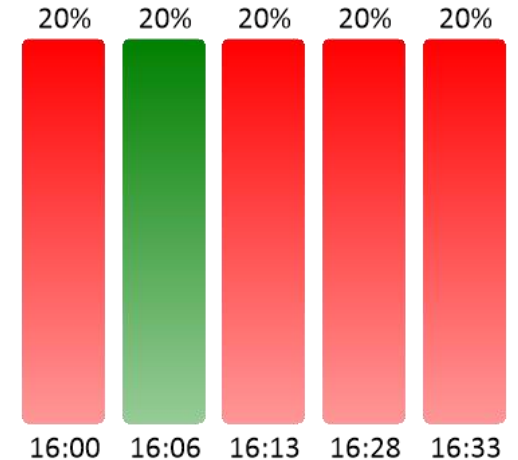
$$TST = LT + \frac{\lambda_B - \lambda_S}{15} + ET$$

$\lambda_S$ : Longitude of the  
standard time meridian (15°)

$$ET = 11.556 \text{ min}$$

$$ET = 0.1926 \text{ h}$$

$$T_{ss,LT} = 16.1025 \text{ h} \\ (16:06)$$



# Sun position | Sunrise and sunset



- Different equations for ET
- Definition of sun rise/sun set

UDC: 526:681.14 Keywords: Computation; Declination; Solar position; Time.

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## A Note on Solar Declination and the Equation of Time

Geoffrey G. Roy

Department of Computer Science

Michael Rodrigo, and Wong Kar King

School of Architecture

University of Western Australia, Nedlands 6009

*Over the last two decades there have been several proposals for empirical relationships for computing the position of this sun. With the rapid development of computer technology, especially micro computers, such formulae are likely to become more widely used. This paper attempts a comparison between several of the readily available expressions, with the view to studying the differences and accuracy of the formulae.*

### Introduction

Accurate knowledge of the position of the sun is important in many types of studies concerning the design of buildings and their immediate environments. Such issues as control of direct sun penetration into windows and the effects of shading onto adjoining buildings are significant issues for the designer. We are concerned with issues of glare, heat loads, impacts on solar energy collectors and even landscaping design.

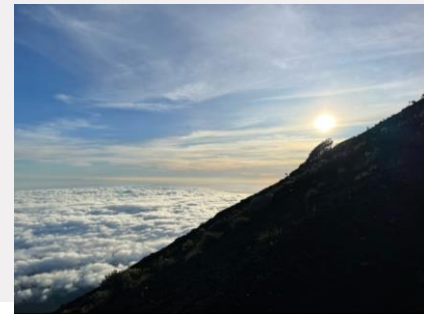
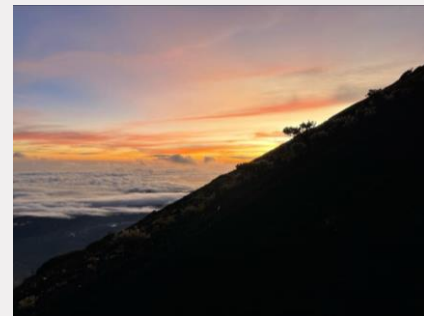
The position of the sun at any time of day, day of year and location on the earth's surface is essential for the analysis of these problems to be undertaken. To enable us to compute the sun's position we require some form of

on the earth's surface. For this observer, the sun is located by two angular measures, the Altitude and the Azimuth. The altitude is the angle between the sun position and a horizontal plane. The Azimuth is the horizontal angle between the sun position and true north. These are expressed in the following usual form:

$$\sin(\text{Altitude}) = \cos(D) \cos(H) \cos(L) + \sin(D) \sin(L) \quad (1)$$

$$\cos(\text{Azimuth}) = \frac{\cos(L) \sin(D) - \sin(L) \cos(D) \cos(H)}{\cos(\text{Altitude})} \quad (2)$$

where, D is the declination, L is the latitude and H is the hour angle. The azimuth angle requires adjustment for pre and post noon times. Of the required terms in Eqs. 1



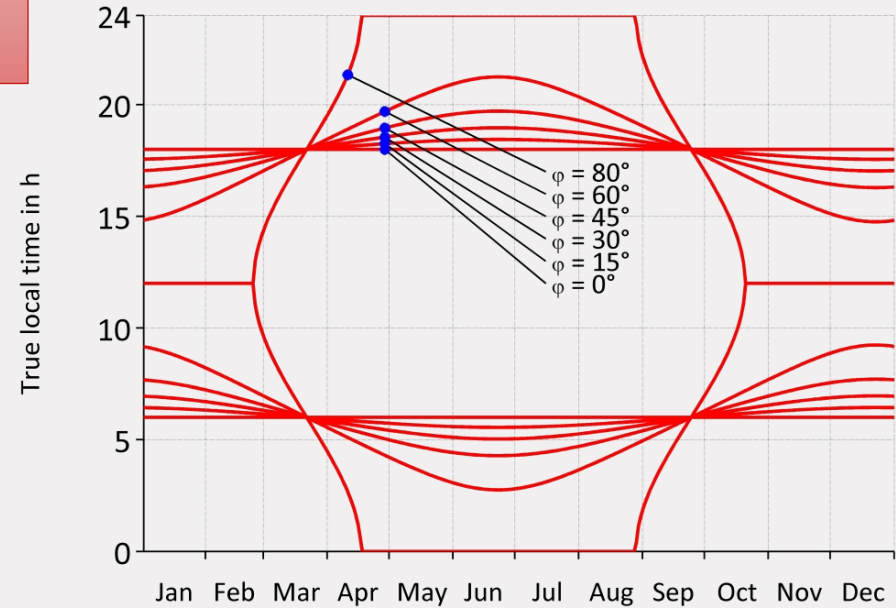


# Sun position | Daylight duration

$$T_D = t_{ss} - t_{sr} = 2 \cdot \frac{\arccos(-\tan \varphi_B \cdot \tan \delta_S)}{15}$$

$$T_a = \sum_{i=1}^{365} T_{D,i}$$

$T_a$  (Eindhoven) = 4406 hours





# Reference material

- EN 17037:2018. Daylight in buildings