Computational Methods for the Interaction of Light and Matter

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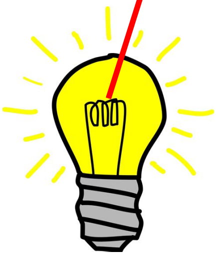

Where does radiation interact with matter?

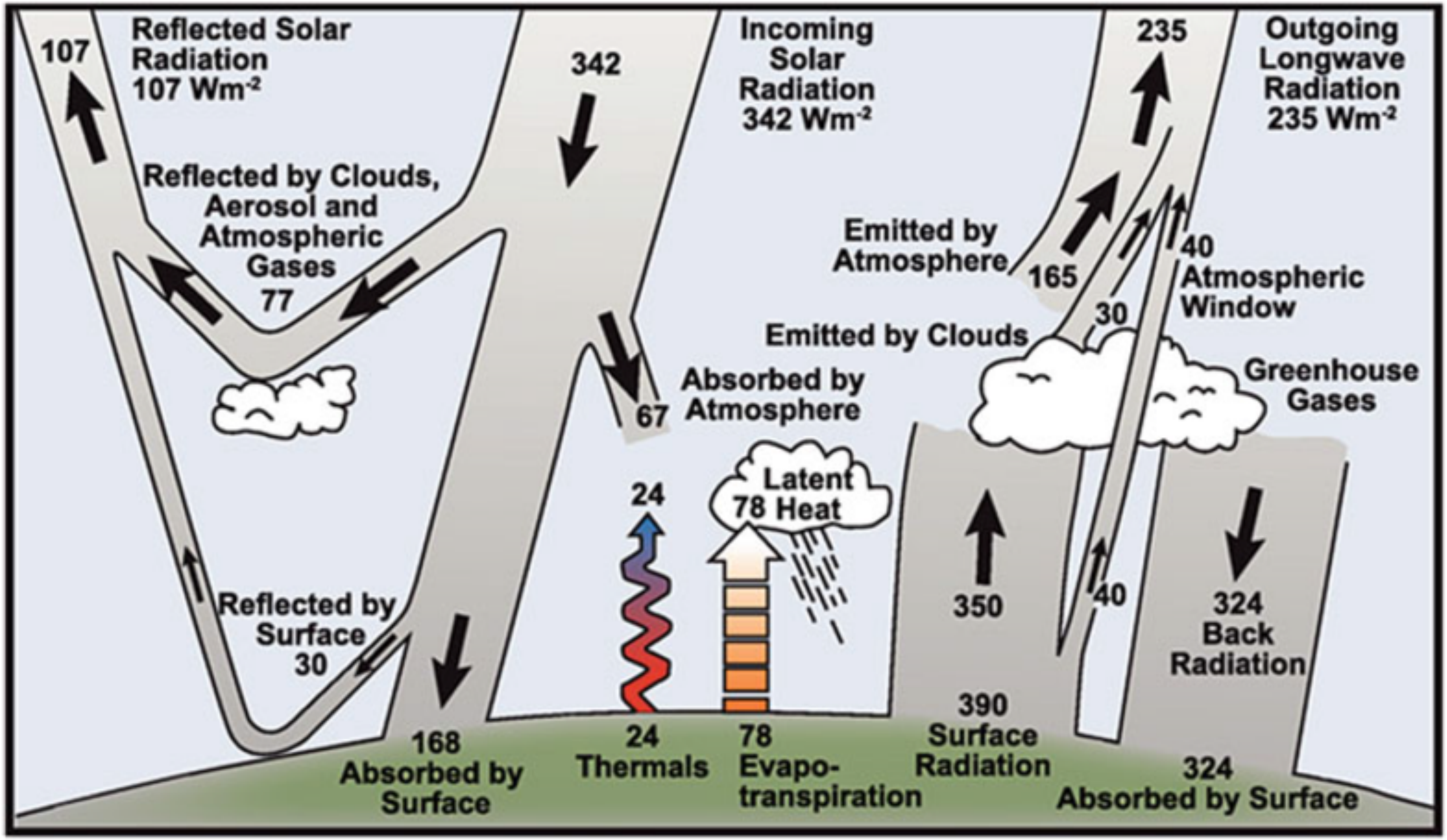

Fig. 4.3 Estimate of the Earth's annual and global average energy balance (from IPCC AR4 2007)

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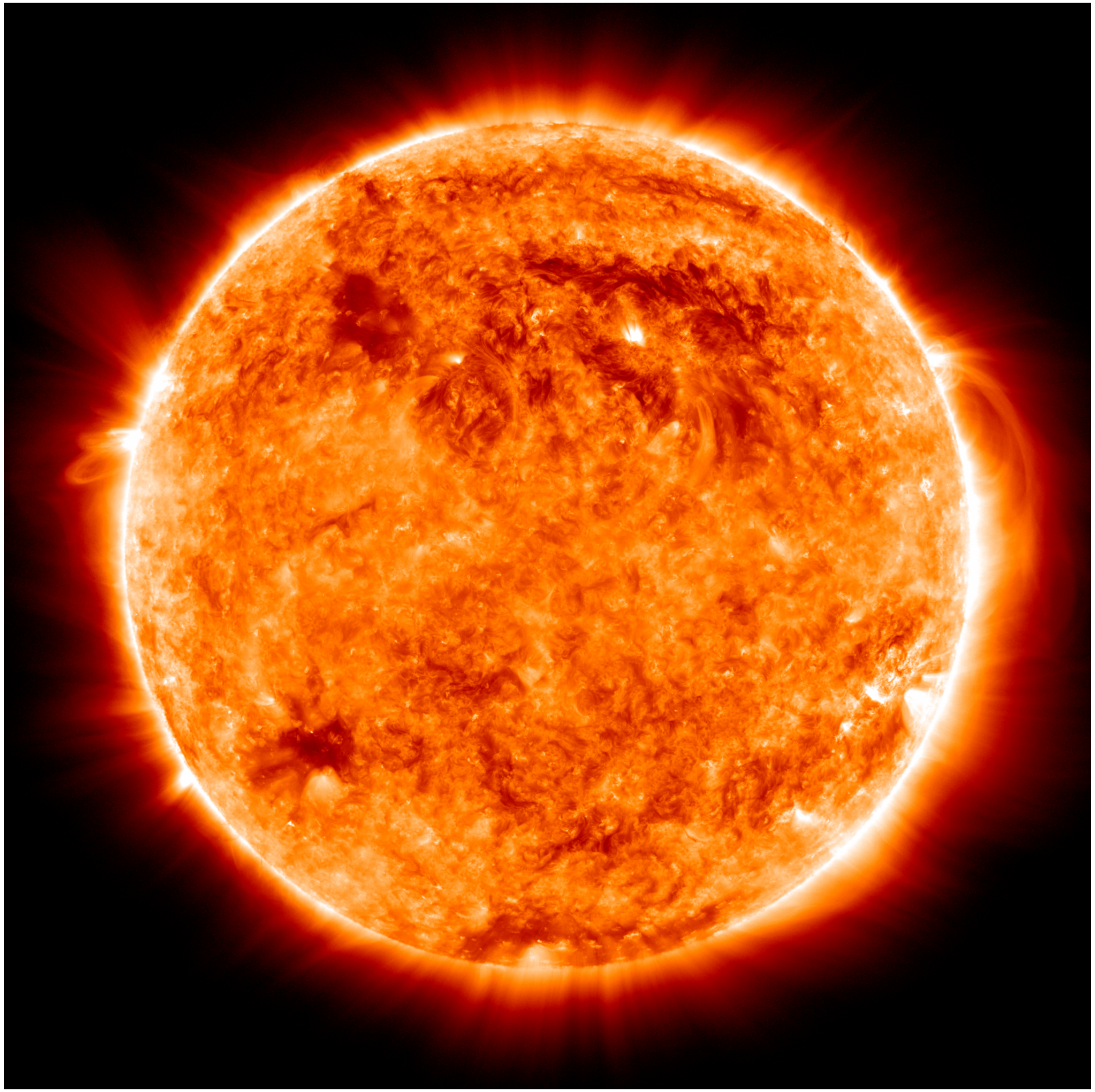

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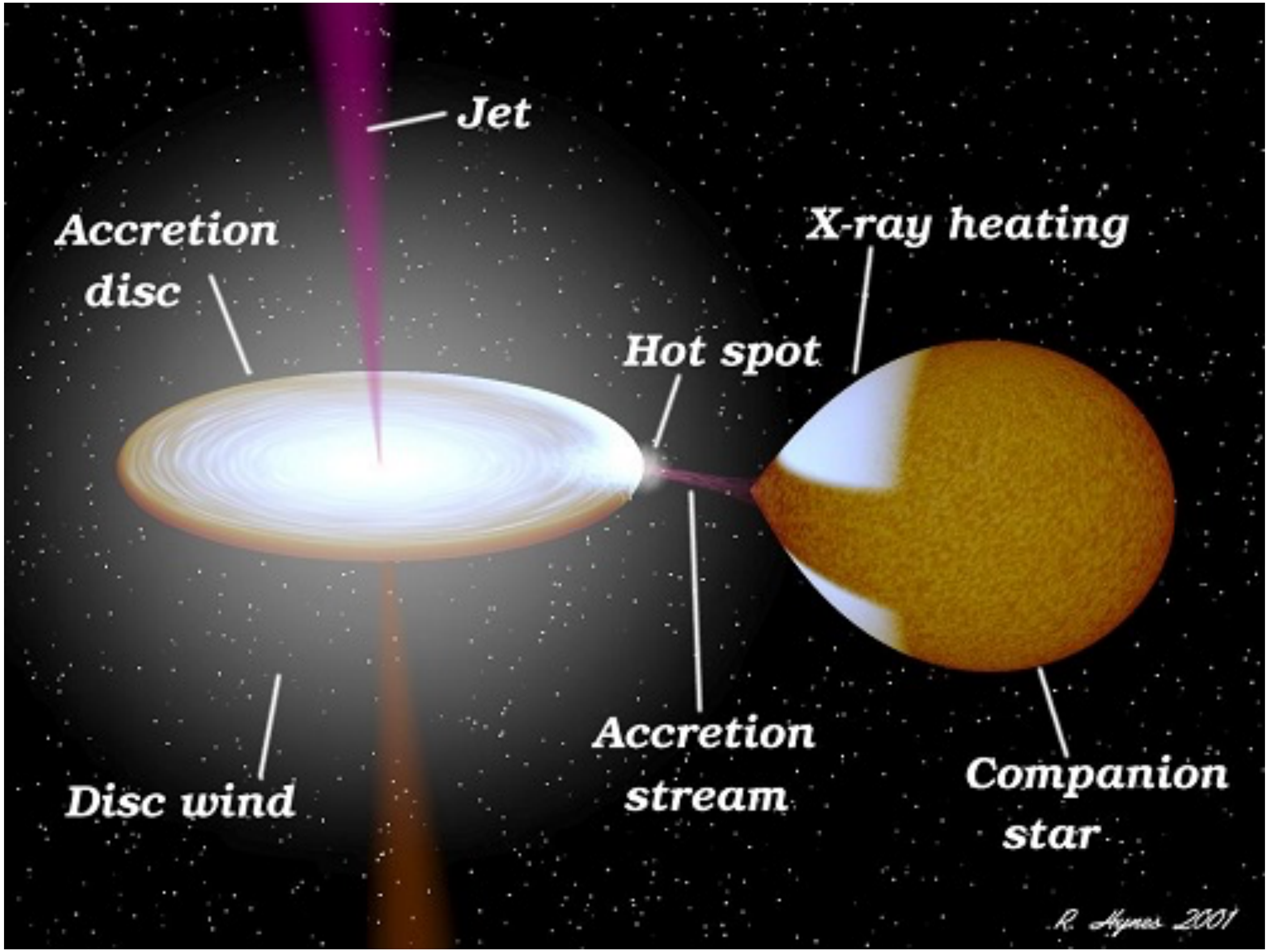

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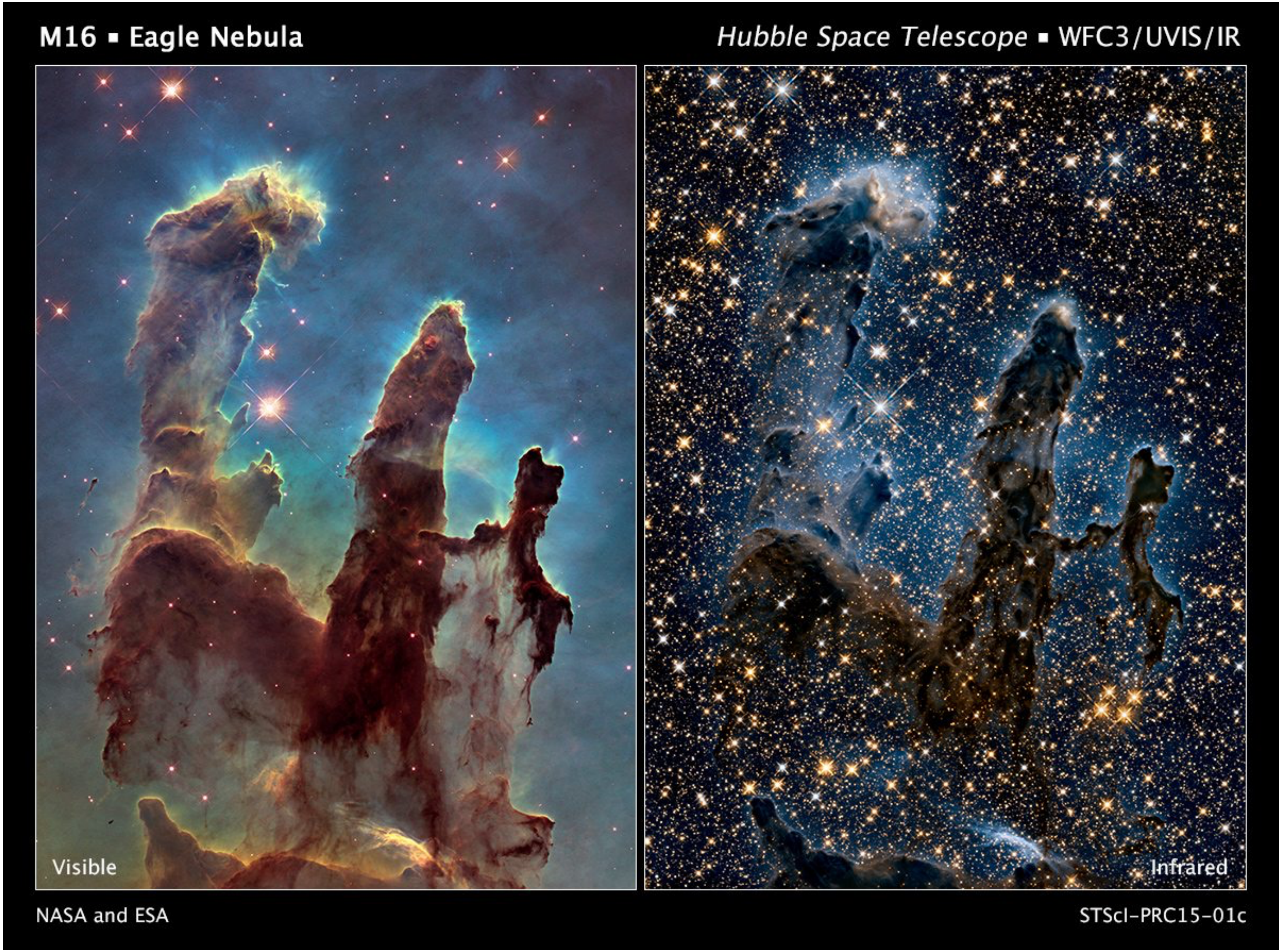

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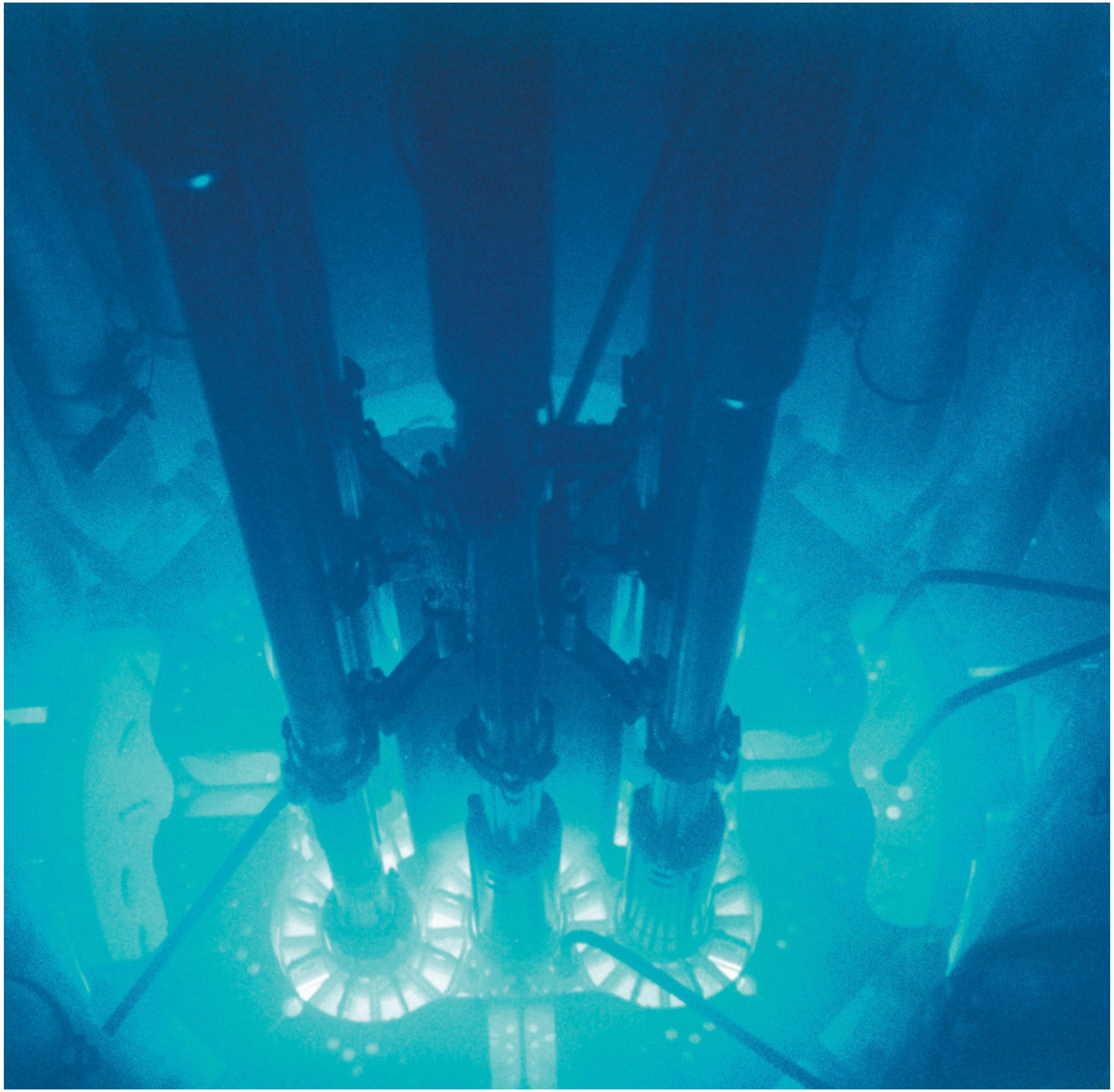


Concept

- Theory of radiative transfer and the interaction of radiation and matter
- Numerical methods for solving this problem under various conditions
- Applications

Concept

- Theory of radiative transfer and the interaction of radiation and matter
  - Formal Transfer Equation
  - Scattering and absorption/emission by solid particles ("continuum transfer")
  - Quantum transitions in gaseous media ("line transfer")
  - The difficulty of Radiative Transfer: Non-local interaction
  - Mathematical formalisms

Concept

- Numerical methods for solving this problem under various conditions:
  - Monte Carlo
  - Moment methods
  - Lambda Iteration
  - Short characteristics
  - Finite Volume, Discontinuous Galerkin schemes
- Here is where YOU come in:
  - Bring your own laptop, with Python (with standard libs: numpy, matplotlib), C compiler, Fortran compiler
  - You will program your own mini-versions of these algorithms.
  - And you will learn to use the big codes, too.

Concept

- Applications:
  - Astrophysical objects
  - Atmospheres of planets

Requirements:

- Have some experience with programming and plotting of data

- Basic mathematics, basic physics, though we will recap most of it.

Weekly schedule

- Wednesdays: 2 hours from 11:15-13:00
  – INF 205 / SR C

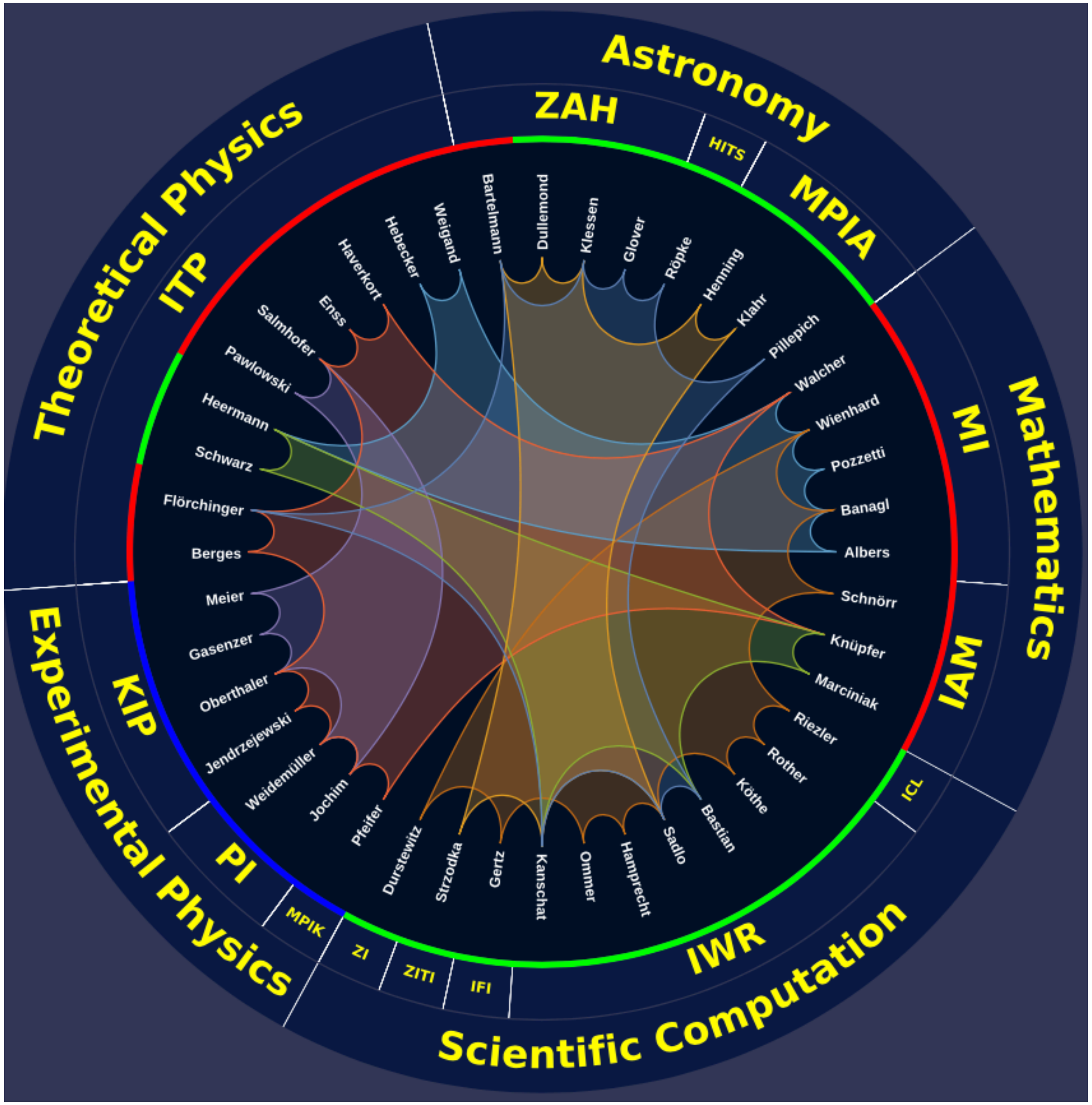
- Fridays: 1 hour from 12:15-13:00
  – INF 205 / SR B (different room!) Check...

  – (Discussion on time)

Examination

- Oral exam or project
- Credit points (if exercises done): 6

Excellence Cluster STRUCTURES



Questions?

How to describe light

Measuring radiation

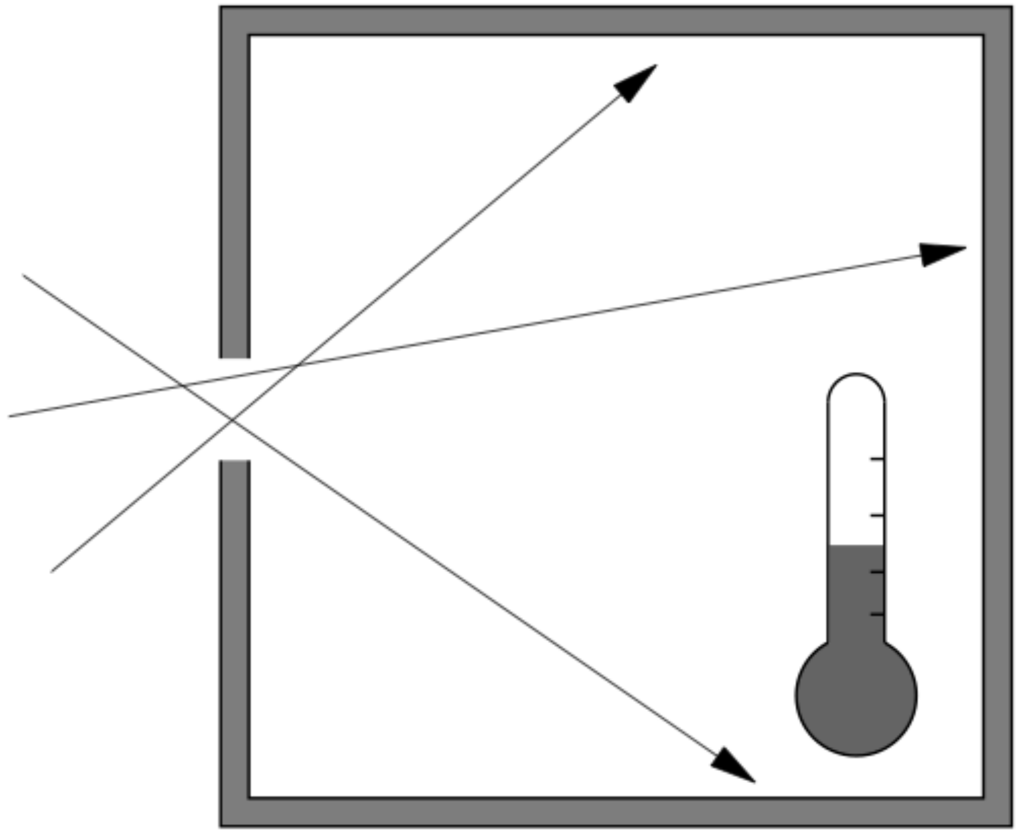
CAREFUL: Just a thought experiment

Radiation flux:

Energy increase per unit surface and time

$$[F] = \frac{\text{erg}}{\text{cm}^2 \cdot \text{s}}$$

Important: Must start with very cold cavity.

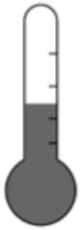


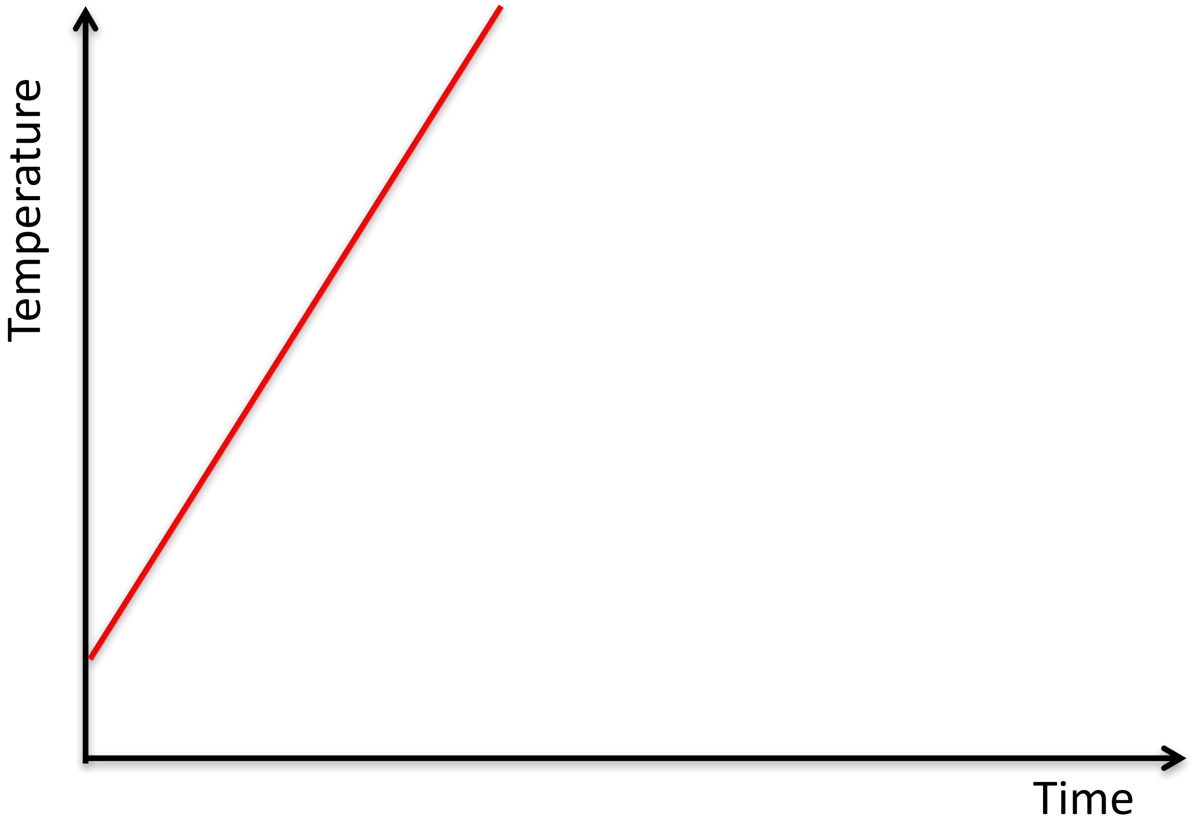
Measuring radiation

The increase of temperature per unit of time is an indication of how much radiation passes through the pupil.

The conversion factor between dT/dt into Flux depends on the details of the instrument.

CAREFUL: Just a thought experiment



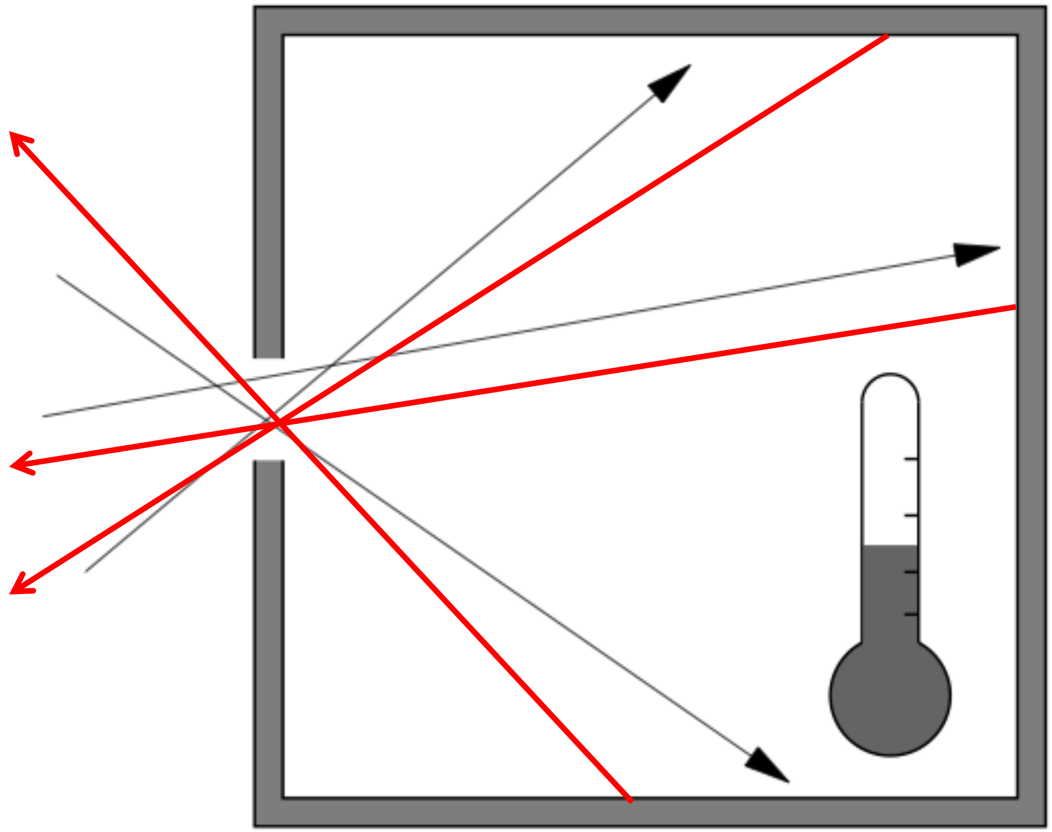


Measuring radiation

You have to start with a very cold cavity, otherwise the temperature starts to saturate.

The cavity will start to radiate itself, and eventually as much radiation escapes through the pupil as enters through it. We then reach saturation.

This is a fundamental physical effect! Cannot be avoided.



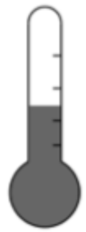
Measuring radiation

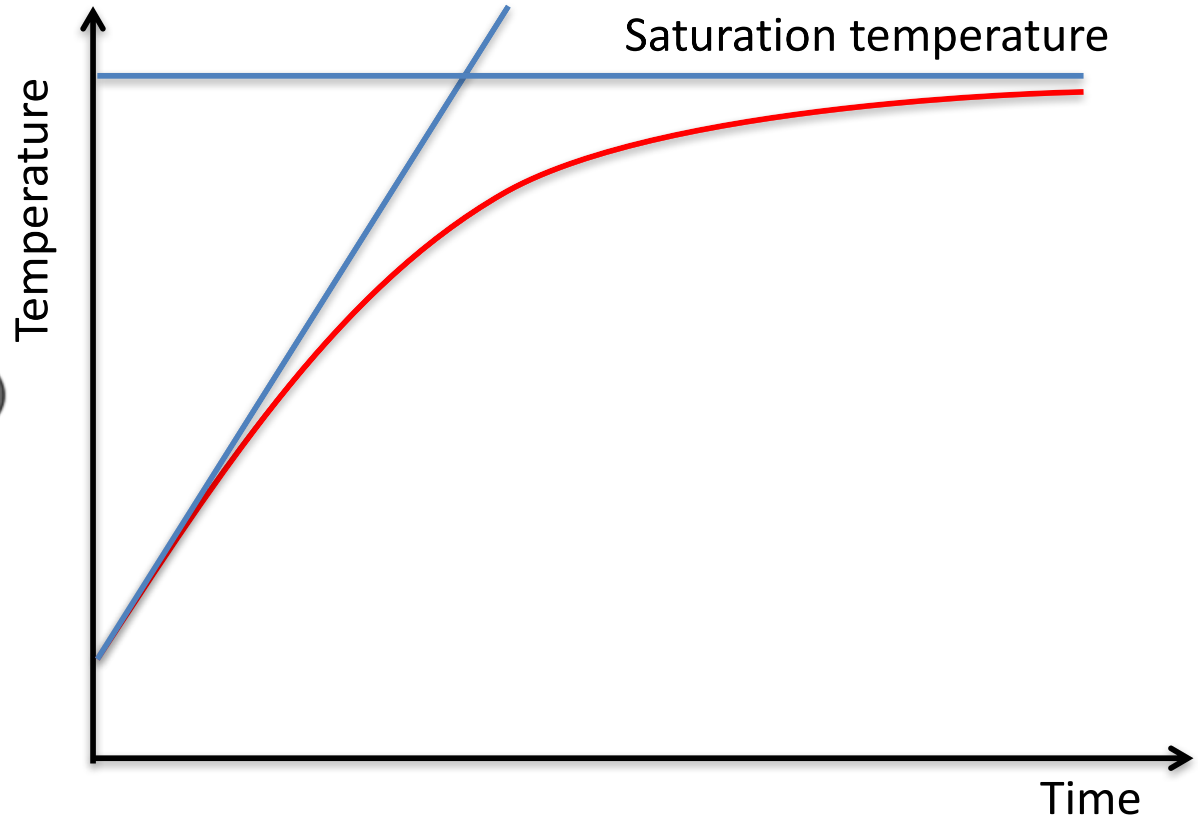
The saturation temperature is exactly defined by physics:

$$T_{\text{sättigung}} = \left(\frac{F}{\sigma_{SB}}\right)^{1/4}$$

$$\sigma_{SB} = 5.67 \times 10^{-5} \frac{\text{erg}}{\text{cm}^2 \text{K}^4 \text{s}}$$

(Stefan-Boltzmann Konstante)





Blackbody radiation

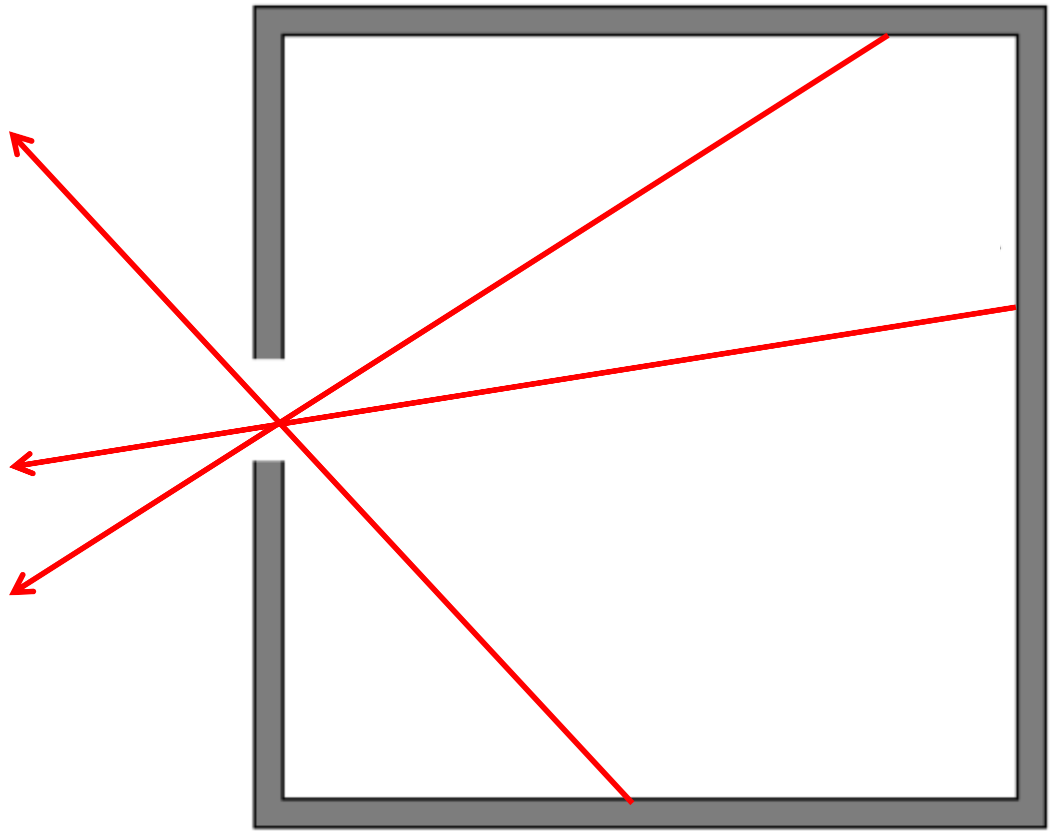
Apparently the walls of the cavity produce a well-defined radiation field:

When you look through the pupil you see exactly the following flux:

$$F = \sigma_{SB} T^4$$

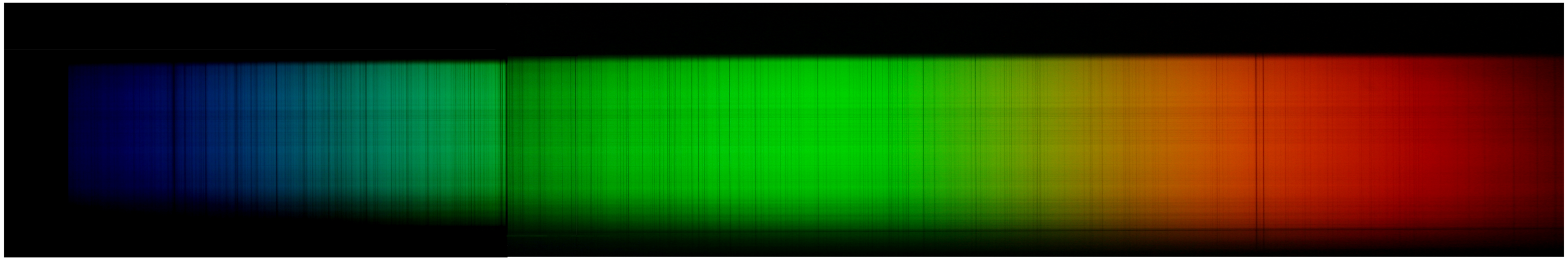
This follows direction from quantum thermodynamics

The perfect cavity is called a "blackbody", and the radiation field emerging from it "blackbody radiation".



Wavelength, frequency

Frequeny ν [Hz]

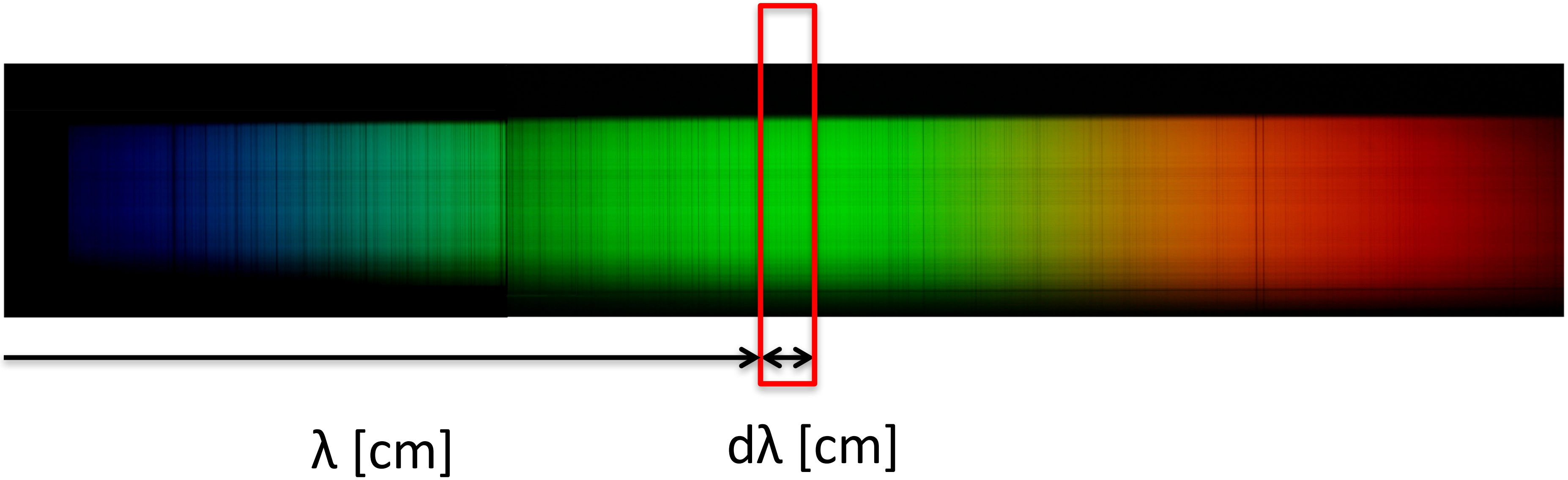


Wavelength λ [cm]

$$\lambda = \frac{c}{\nu}$$

$$c = 2.9979 \times 10^{10}\ \mathrm{cm/s}$$

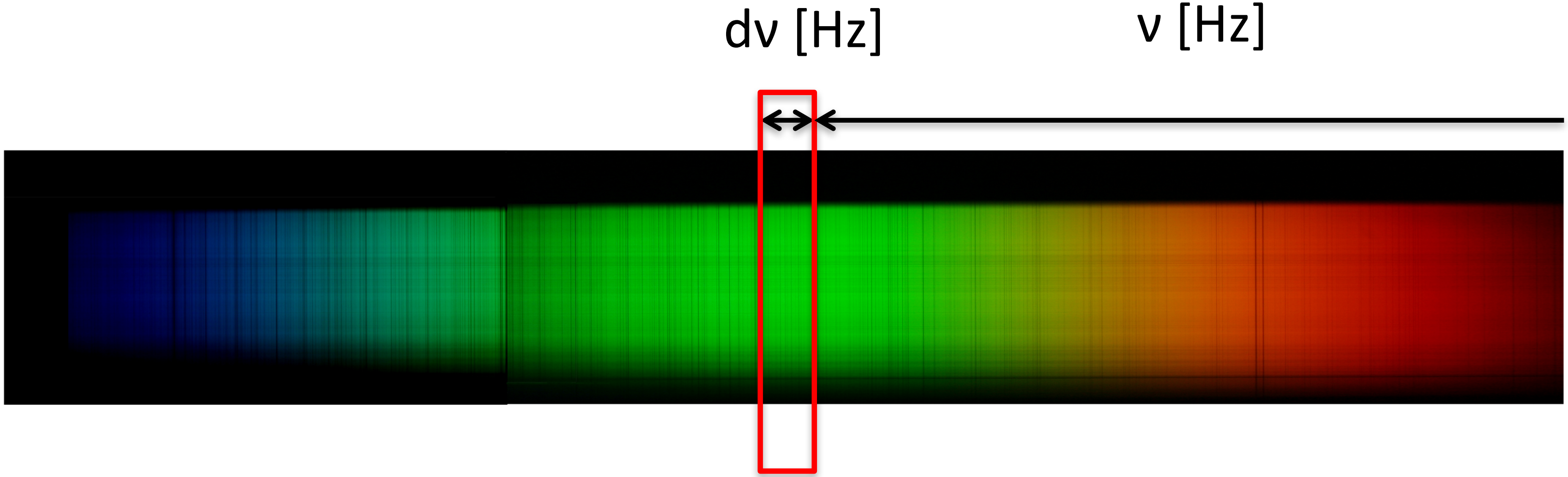
Spectrum: Flux per unit wavelength



$$F_\lambda \equiv \frac{dF}{d\lambda}$$

$$F = \int_0^\infty F_\lambda d\lambda$$

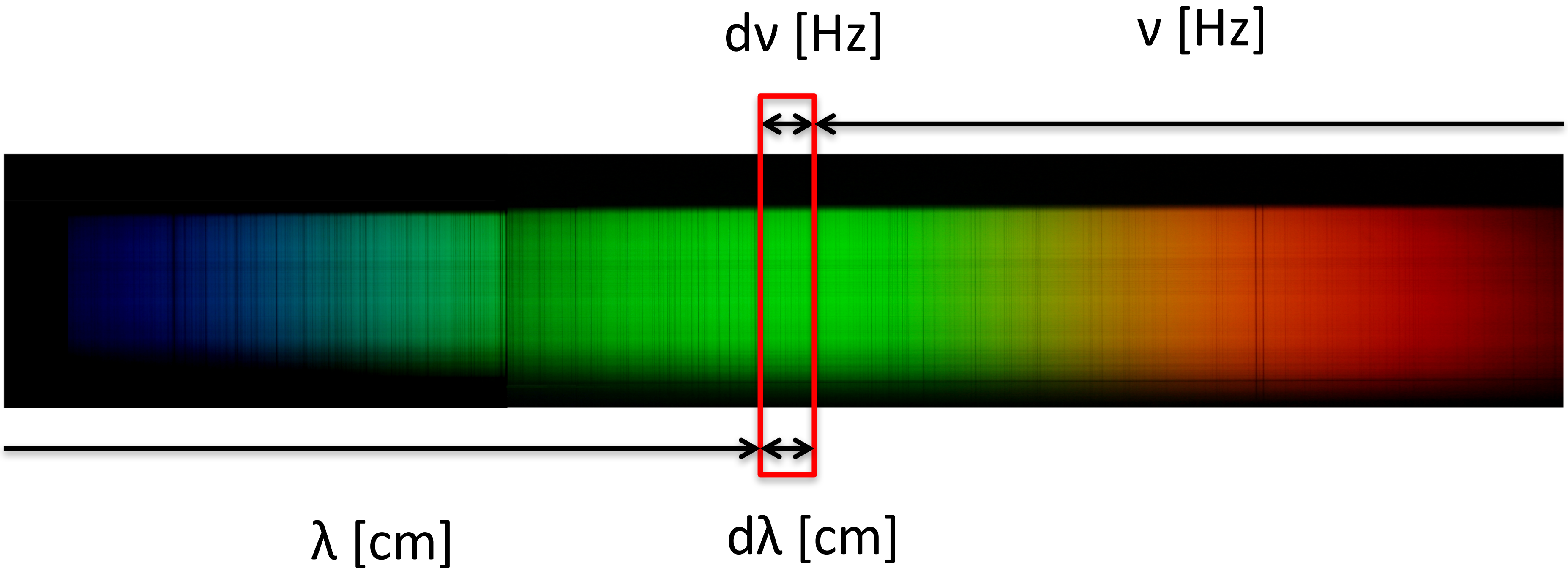
Spectrum: Flux per unit wavelength



$$F_\nu \equiv \frac{dF}{d\nu}$$

$$F = \int_0^\infty F_\nu d\nu$$

One can use both, but be careful...



$F_\nu \neq F_\lambda$ aber $\boxed{\nu F_\nu = \lambda F_\lambda}$

Spectrum of a blackbody

$$F_\nu = \pi B_\nu$$

$$\boxed{B_\nu \equiv \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(h\nu / k_B T\right) - 1}}$$

Planck-Function

$h = 6.6262 \times 10^{-27}\ \mathrm{erg \cdot s}$ Planck constant

$k_B = 1.3807 \times 10^{-16}\ \mathrm{erg/K}$ Boltzmann constant

Spectrum of a blackbody

$$B_\nu \equiv \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(h\nu / k_B T\right)-1}$$

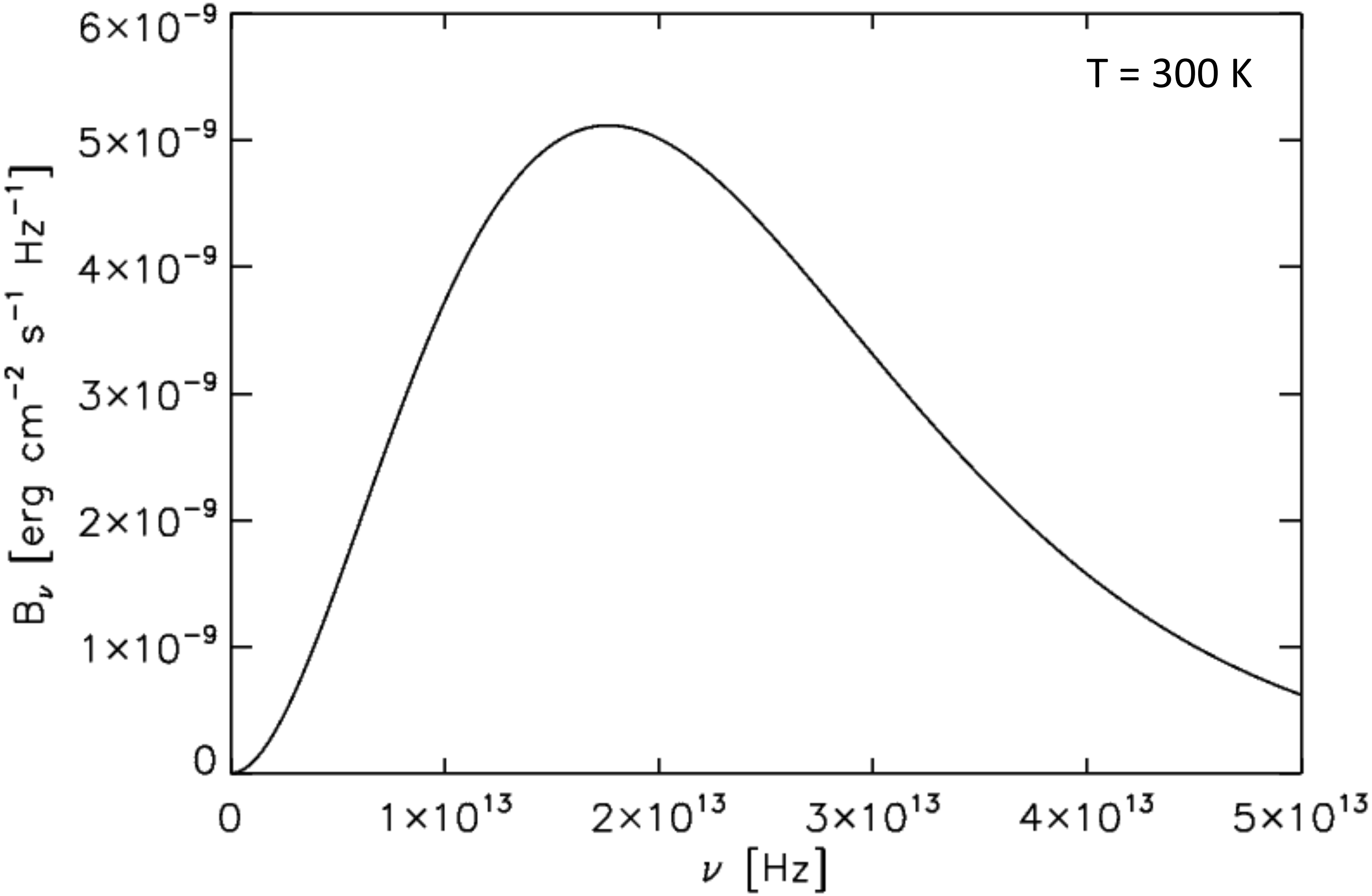
Spectrum of a blackbody

$$B_\nu \equiv \underbrace{2}_{\text{One photon has 2 degrees of freedom}} \underbrace{\frac{\nu^2}{c^2}}_{\text{Density of quantum states}} \underbrace{h\nu}_{\text{Photon energy}} \underbrace{\frac{1}{\exp\left(h\nu / k_B T\right) - 1}}_{\text{Occupation number (Quantum-Statistics)}}$$

This only works if one considers radiation as consisting of quantized energy units. Max Planck discovered this in 1900 when he (as an act of desperation) tried to describe blackbody radiation with Boltzmann statistics, even though initially he did not seriously believe in light quanta! Without realizing it, he started the revolution of quantum mechanics.

Spectrum of a blackbody

$$B_\nu \equiv \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(h\nu / k_B T\right) - 1}$$



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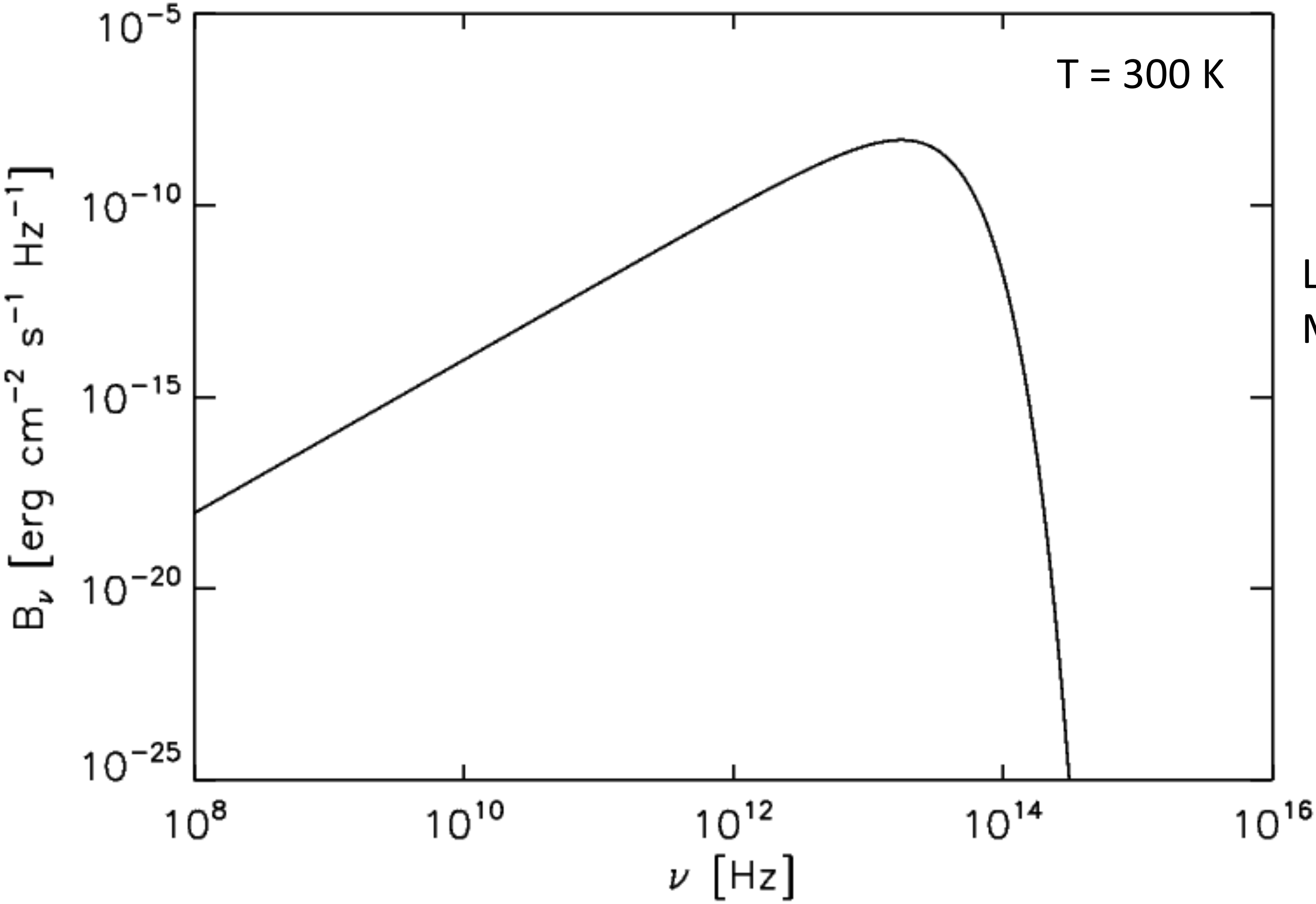


Log-Log Plot:
Much better!

Spectrum of a blackbody

$$B_\nu \equiv \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(h\nu / k_B T\right)-1} \overset{(h\nu<<k_B T)}{\approx} \frac{2\nu^2}{c^2}k_B T$$

Rayleigh-Jeans Limit

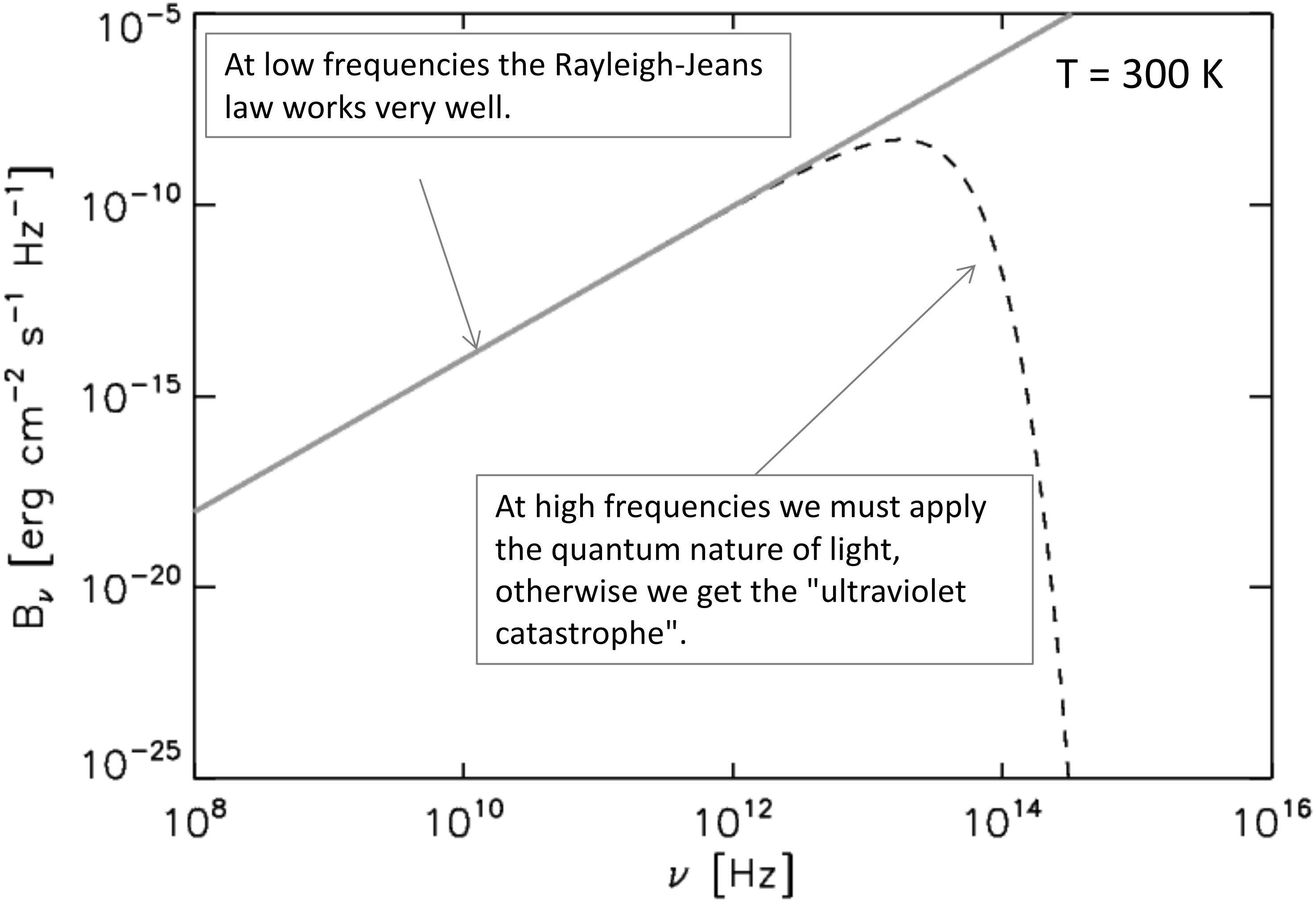
Proof:

$$\exp\left(\frac{h\nu}{k_B T}\right)-1 \overset{\text{Taylor}}{=} 1+\left(\frac{h\nu}{k_B T}\right)+\frac{1}{2}\left(\frac{h\nu}{k_B T}\right)^2+\ldots-1 \overset{(h\nu<<k_B T)}{\approx} \frac{h\nu}{k_B T}$$

Spectrum of a blackbody

$$B_\nu \equiv \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/k_BT)-1} \overset{(h\nu<<k_BT)}{\approx} \frac{2\nu^2}{c^2}k_BT$$

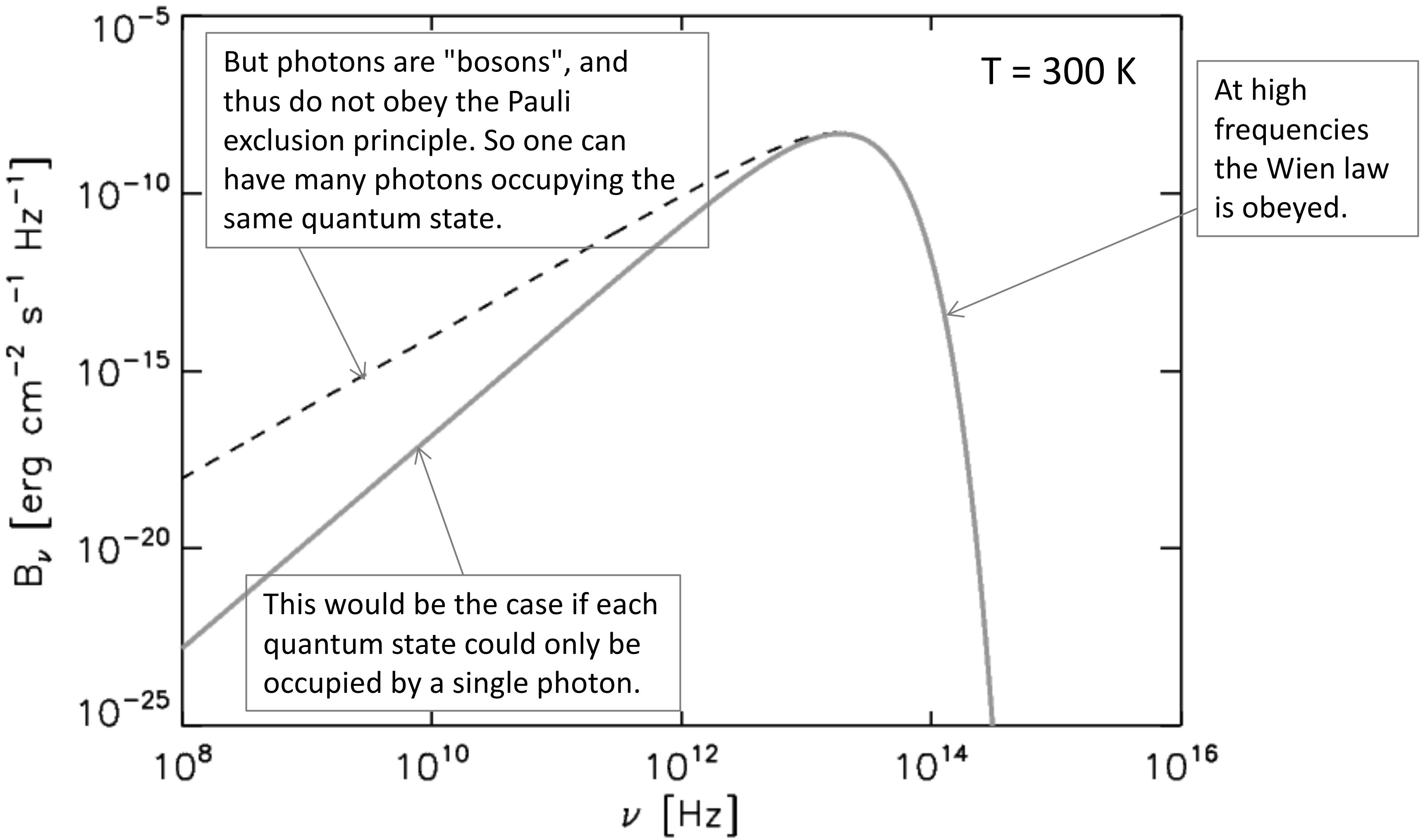
Rayleigh-Jeans Limit



Spectrum of a blackbody

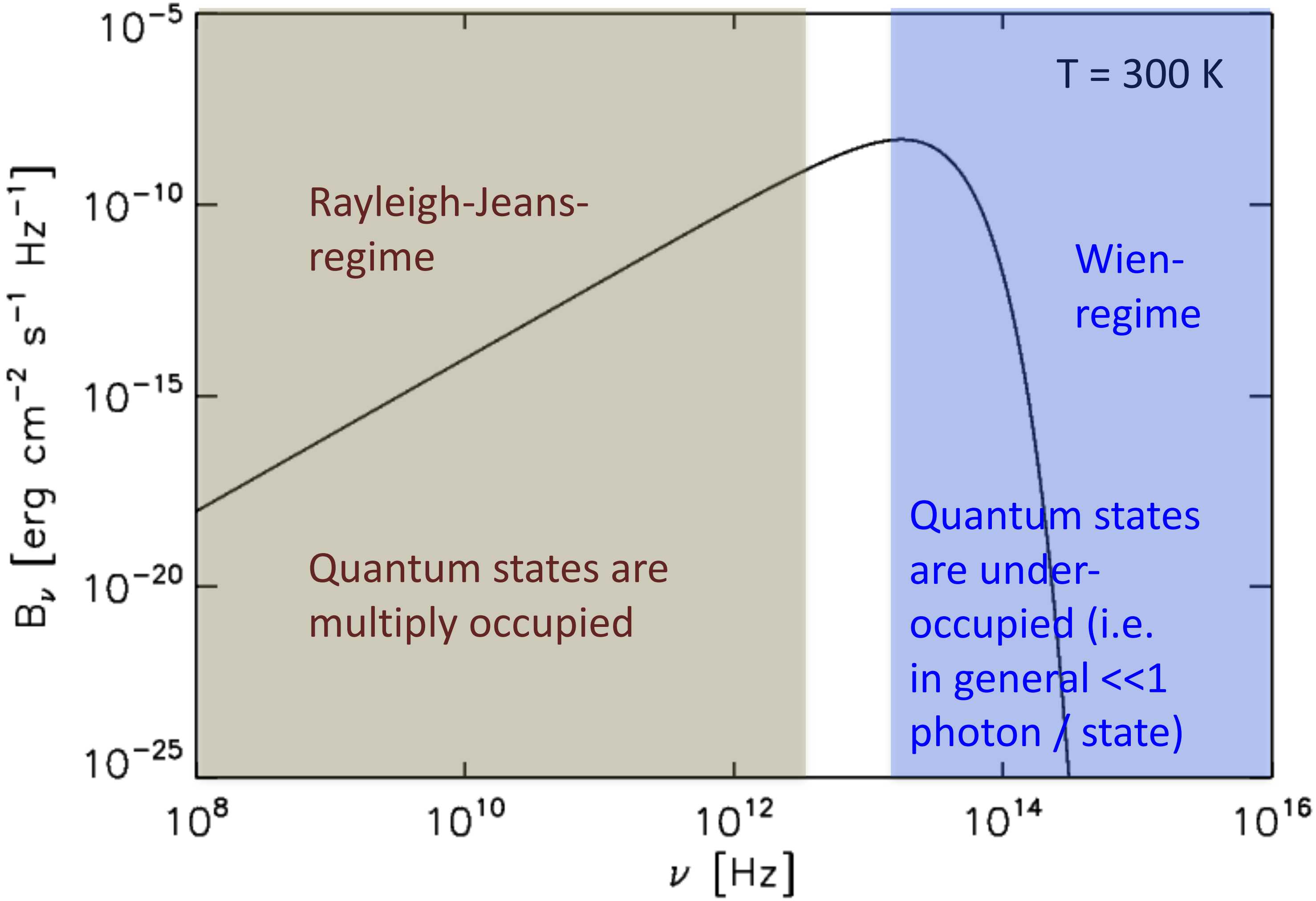
$$B_\nu \equiv \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/k_BT)-1} \overset{(h\nu>>k_BT)}{\approx} \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{k_BT}\right)$$

Wien Limit



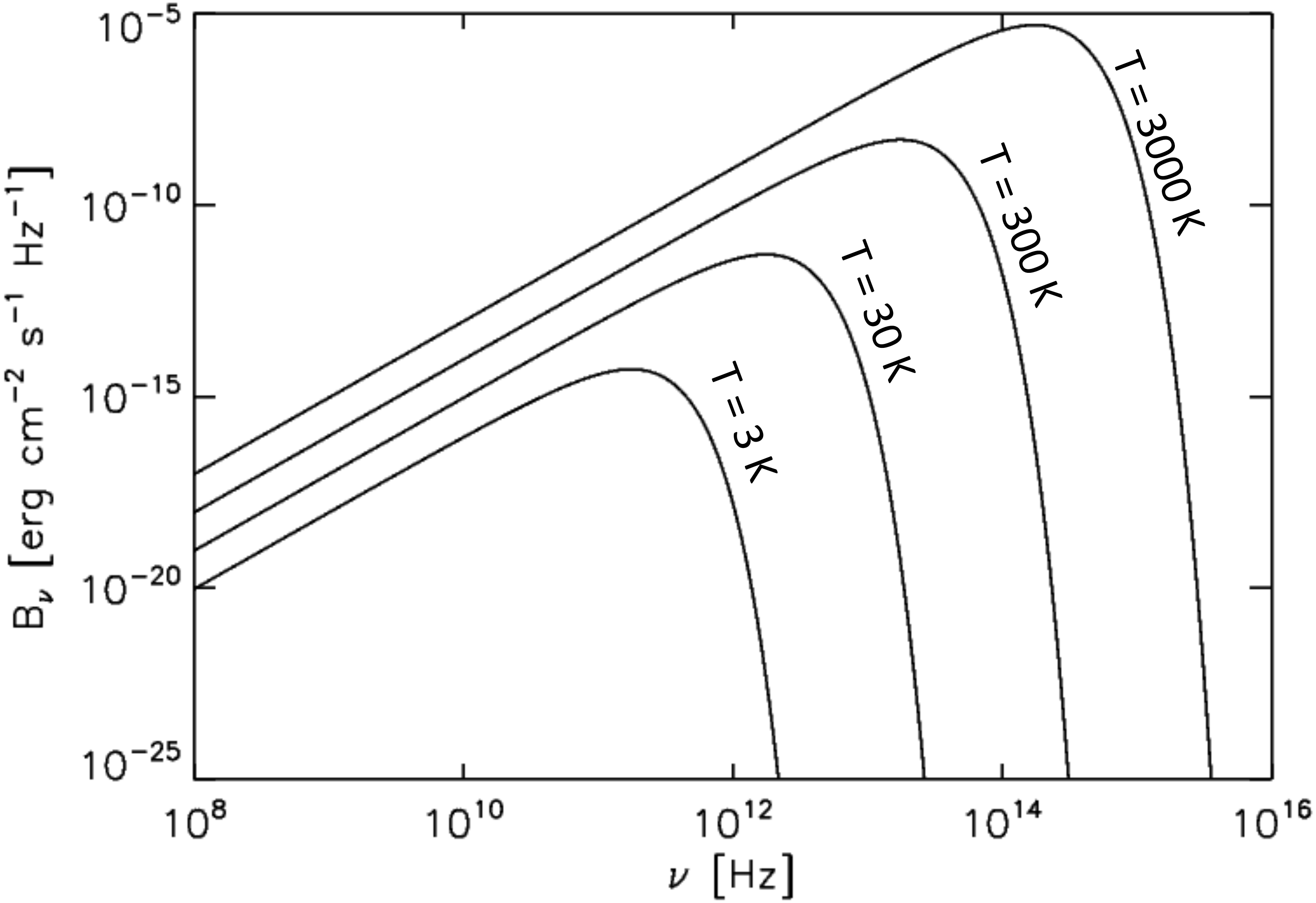
Spectrum of a blackbody

$$B_\nu \equiv \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(h\nu / k_B T\right) - 1}$$



Spectrum of a blackbody

$$\nu_{\max} \approx 3\frac{k_B T}{h}$$ (Wien displacement law)

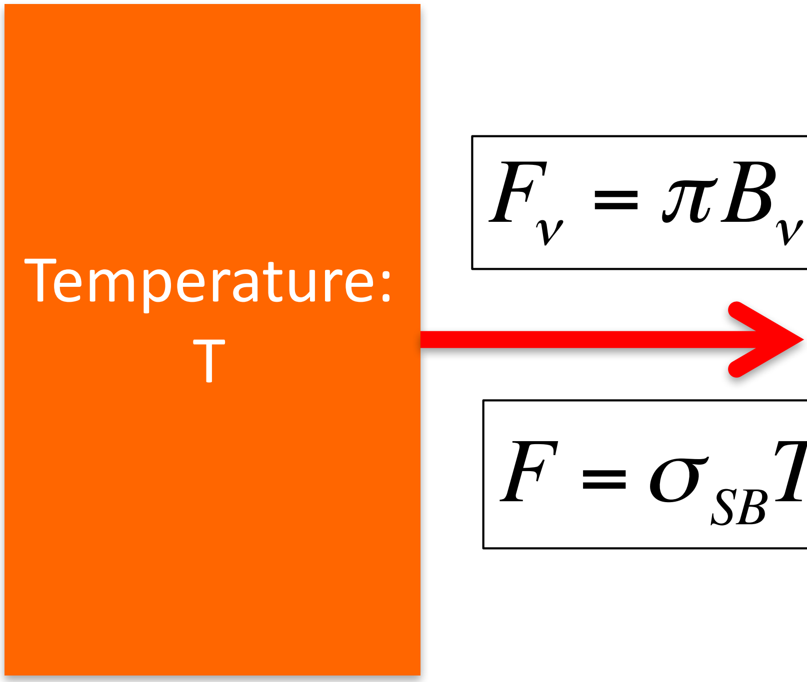


Urheber: Unbekannt. Quelle: http://www.volunteerlocal.com/blog/tag/strike-while-the-iron-is-hot/

Flux from a blackbody

The integral of the flux over all frequencies (wavelengths) gives the total (=bolometric) flux:

$$F = \int_0^\infty F_\nu \, d\nu = \pi \int_0^\infty B_\nu(T) \, d\nu = \sigma_{SB} T^4$$

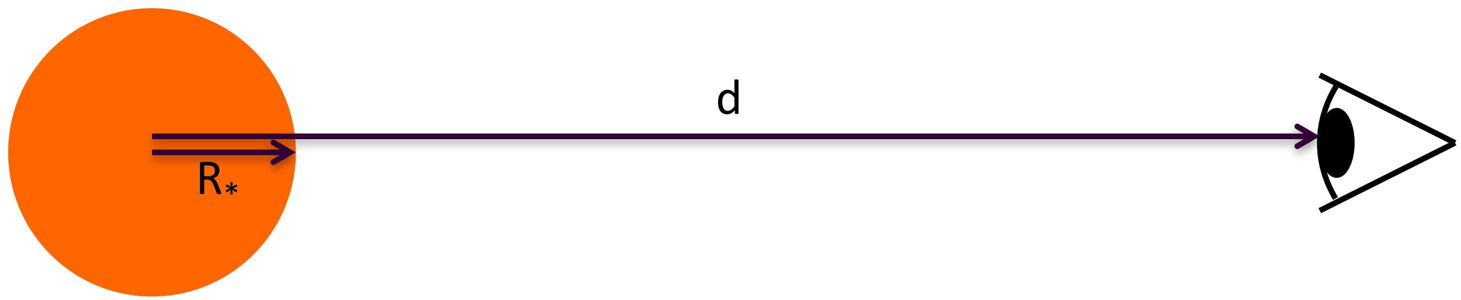


$$\boxed{F_\nu = \pi B_\nu(T)} \qquad \frac{\mathrm{erg}}{\mathrm{s} \cdot \mathrm{cm}^2 \cdot \mathrm{Hz}}$$

$$\boxed{F = \sigma_{SB} T^4} \qquad \frac{\mathrm{erg}}{\mathrm{s} \cdot \mathrm{cm}^2}$$

Star as a blackbody

To first order we can describe a star as a blackbody radiator



Flux at the surface:

$$F = \sigma_{SB} T_*^4$$

Luminosity:

$$L = 4\pi R_*^2 \sigma_{SB} T_*^4$$

Observed flux:

$$F = \left(\frac{R_*}{d}\right)^2 \sigma_{SB} T_*^4$$

Star as a blackbody

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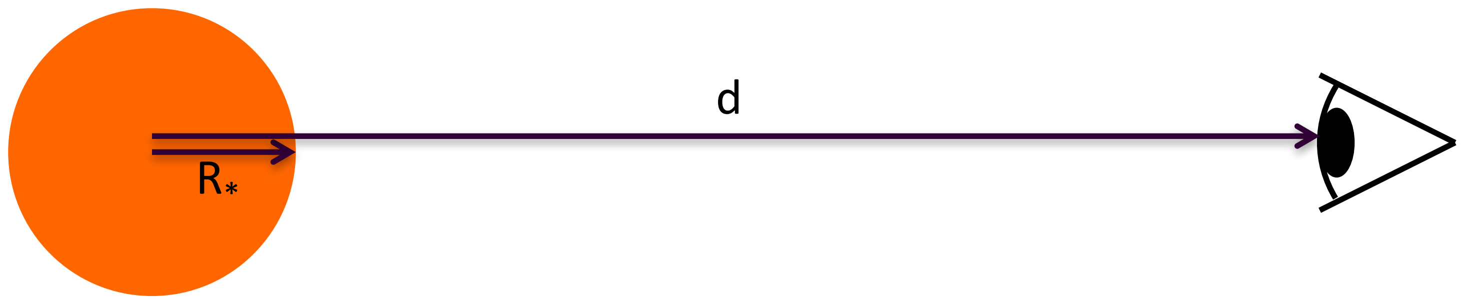

Flux at the surface:

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Luminosity:

$$L_\nu = 4\pi^2 R_*^2 B_\nu(T_*)$$

Observed flux:

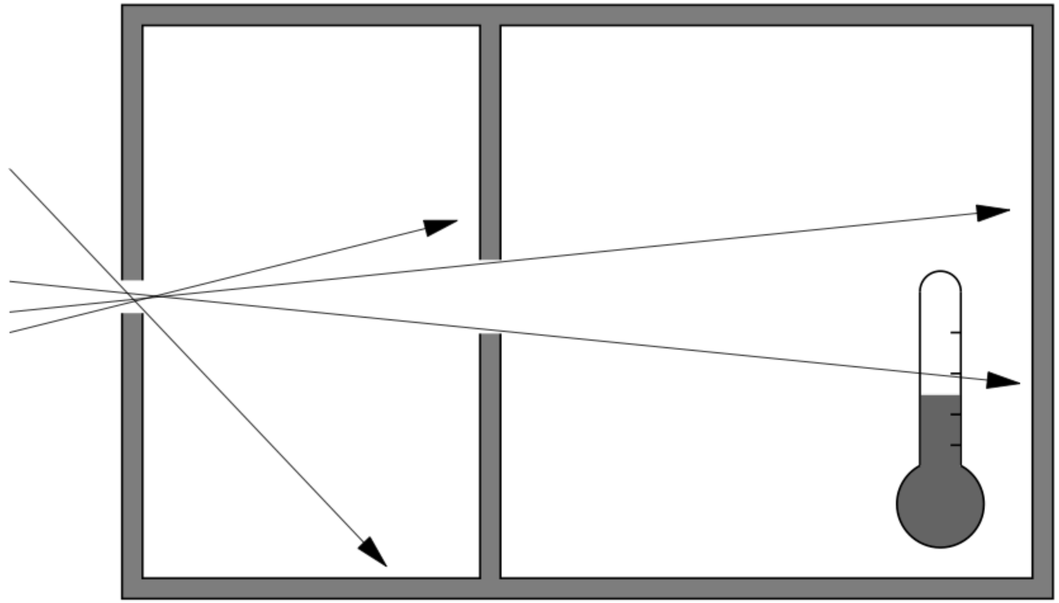
$$F_\nu = \left(\frac{R_*}{d}\right)^2 \pi B_\nu(T_*)$$

Radiation is more than just flux:

Introducing the "intensity" (="radiance")

Directional information in radiation

„Intensity“ I_ν



Warning: just schematic. Real instruments work differently!

Dimension (CGS units):

$$[I_\nu] = \frac{\text{erg}}{\text{cm}^2 \cdot \text{s} \cdot \text{Hz} \cdot \text{ster}}$$

Depends on position, frequency and direction:

$$I(\vec{x}, \nu, \vec{n})$$

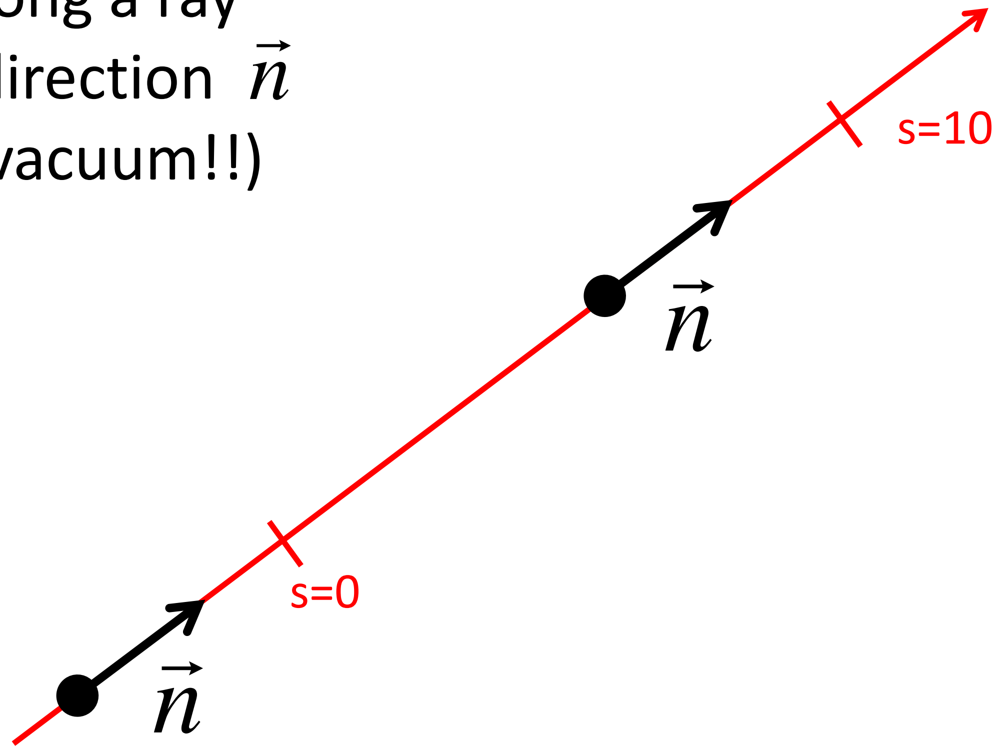
In vacuum, where radiation can move freely, we have:

$$\vec{n} \cdot \nabla I(\vec{x}, \nu, \vec{n}) = 0$$

Directional information in radiation

$$\vec{n} \cdot \nabla I(\vec{x}, \nu, \vec{n}) = 0 \qquad \frac{dI(\vec{x}, \nu, \vec{n})}{ds} = 0$$

Intensity in direction $\vec{n}$
is constant along a ray
that goes in direction $\vec{n}$
(only true in vacuum!!)



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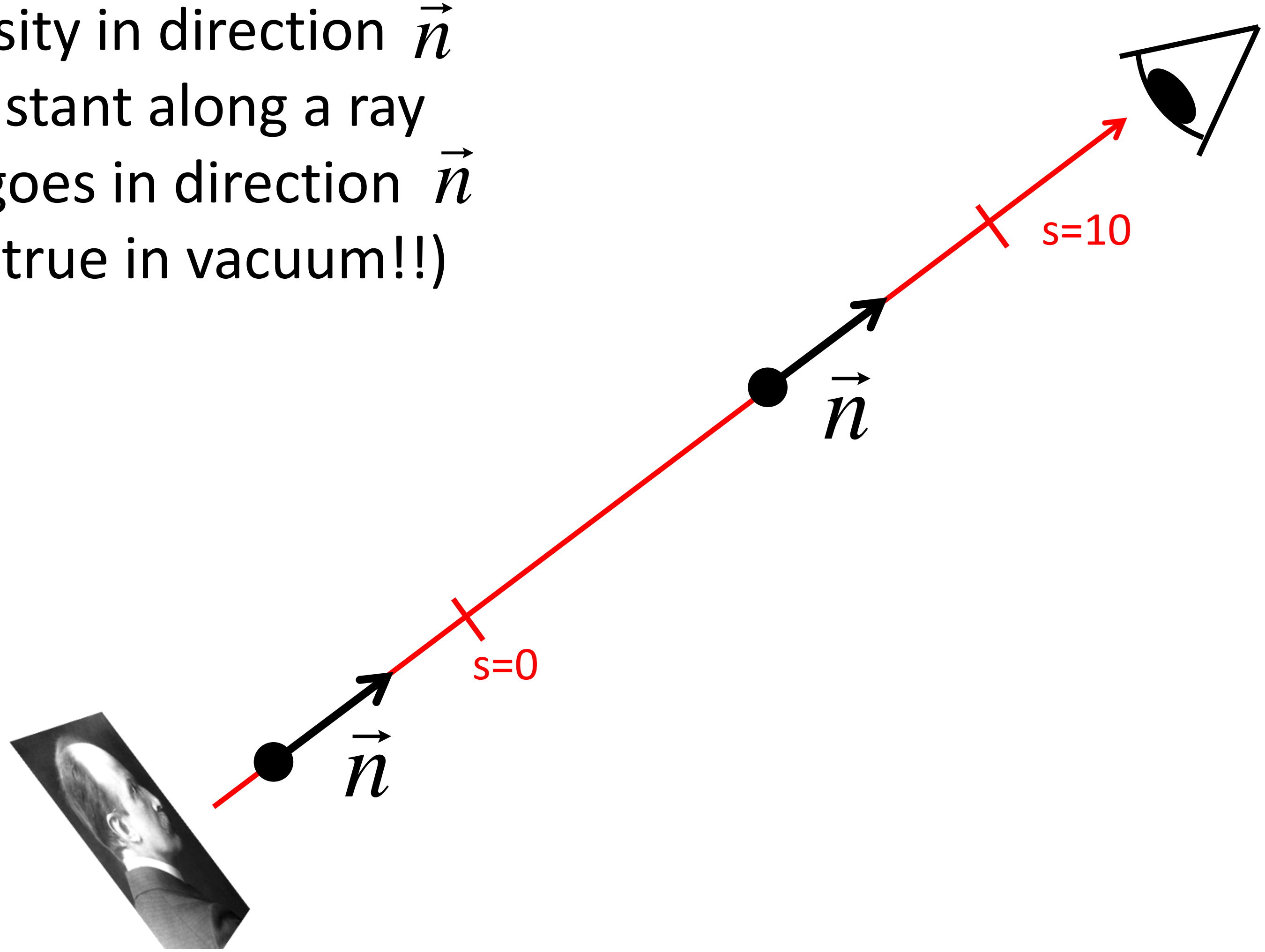

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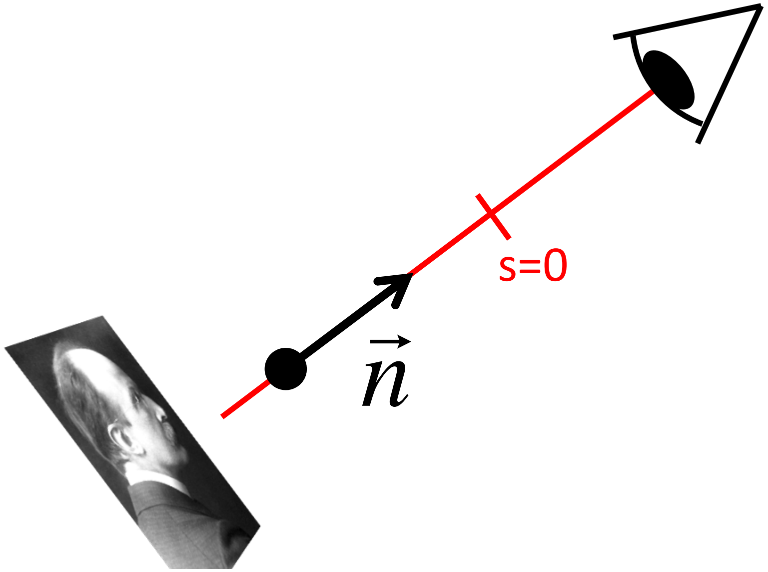


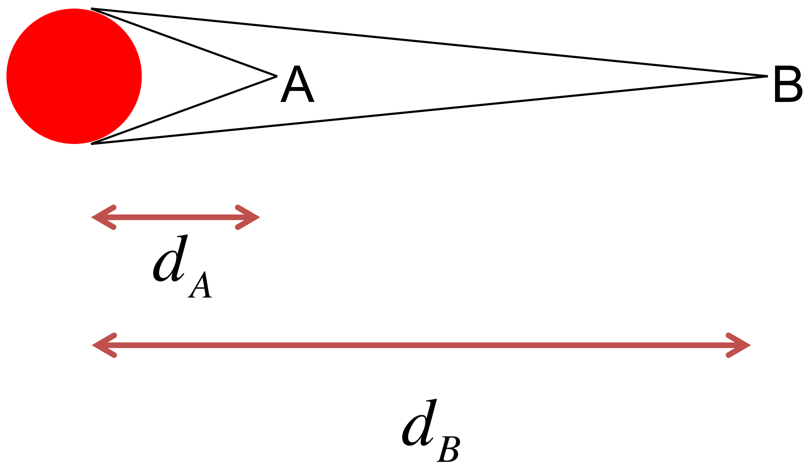
The image gets bigger on the rhetina,
but it does not shine more brightly.
So indeed, intensity is constant!

Flux versus intensity of a star

Why does the measured flux of a star go as d^{-2} while the intensity remains constant with d?

$$F_A = \frac{d_B^2}{d_A^2} F_B$$

and for $d \gg R_*$ we also have: $\Delta\Omega_A = \frac{d_B^2}{d_A^2} \Delta\Omega_B$



$$F = I\,\Delta\Omega$$

$$\downarrow$$

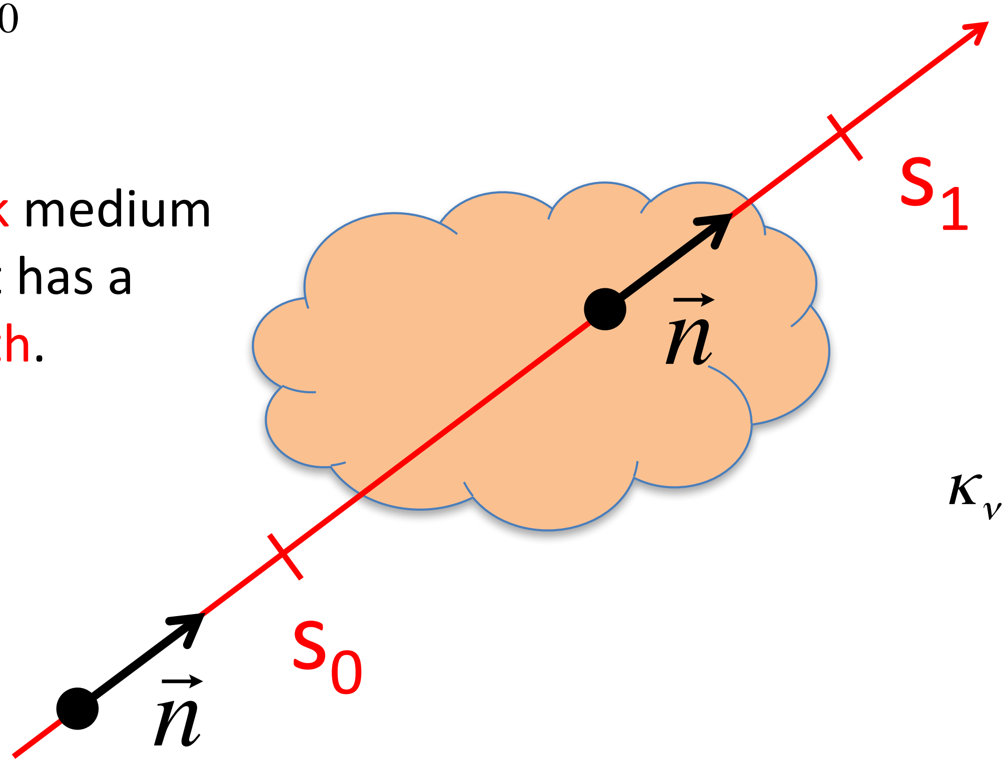
$$I = \text{const}$$

Optical depth (optically thick/thin)

The „optical depth“ of an object along a ray is
„how many mean free paths units are there along the ray?“

$$\tau_\nu = \int_{s_0}^{s_1} \rho \kappa_\nu \, ds$$

An optically thick medium is a medium that has a high optical depth.



κ_ν is the opacity

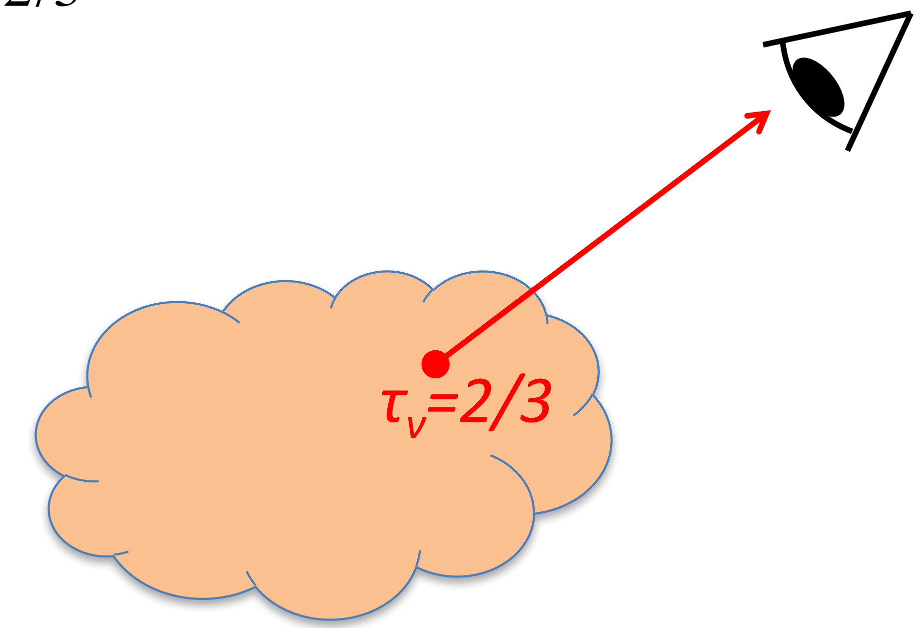
Mean free path:

$$l_\nu = \frac{1}{\rho \kappa_\nu}$$

Eddington-Barbier Rule

A rough estimate of the observed Intensity I_ν of an object/cloud is:

$$I_\nu^{\rm obs} \approx B_\nu(T_{\tau_\nu=2/3})$$



In other words: You see that, which lies around optical depth of 2/3.

Radiative transfer: A short review

Radiative Transfer is a 7-dimensional problem
(that's *one* of the reasons it is so hard and expensive to solve):

$$I(x, y, z, \theta, \phi, \nu, t) \qquad [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}}]$$

Usually: semi-steady-state:

$$I(x, y, z, \theta, \phi, \nu) \qquad [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}}]$$

If the emission and extinction coefficients are known, you can reduce this to the Formal Transfer Equation along a single ray:

$$I(s, \nu) \qquad [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,ster^{-1}}]$$

Radiative transfer: A short review

Formal Transfer Equation along a ray:

$$\frac{dI_\nu}{ds} = \rho\kappa_\nu (S_\nu - I_\nu)$$

Over length scales larger than $1/\rho\kappa_\nu$ intensity I tends to approach source function S.

Photon mean free path:

$$l_{\mathrm{free},\nu} = \frac{1}{\rho\kappa_\nu}$$

Optical depth of a cloud of size L:

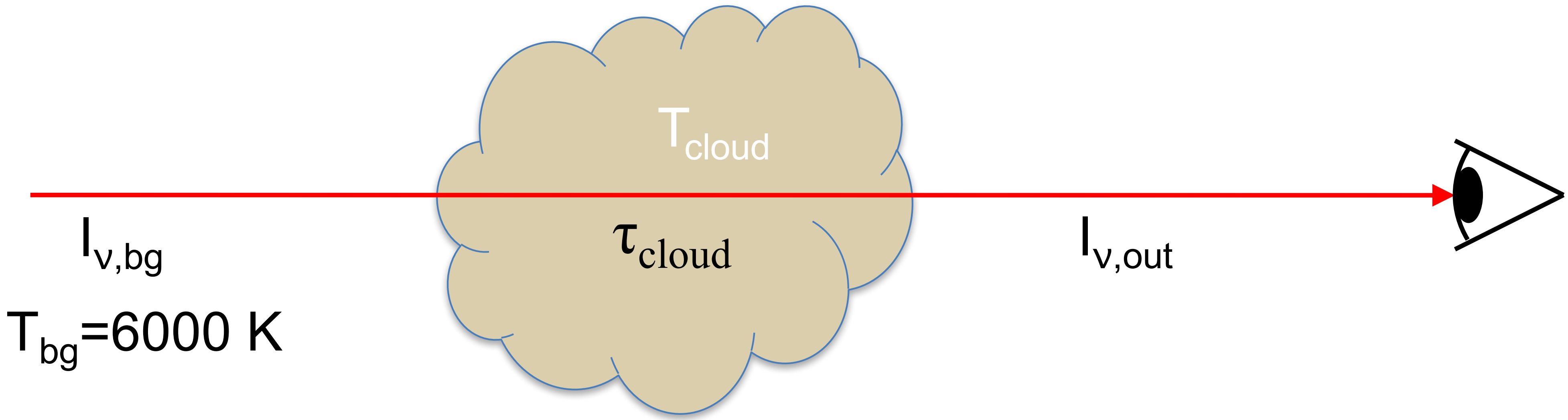
$$\tau_\nu = \frac{L}{l_{\mathrm{free},\nu}} = L\rho\kappa_\nu$$

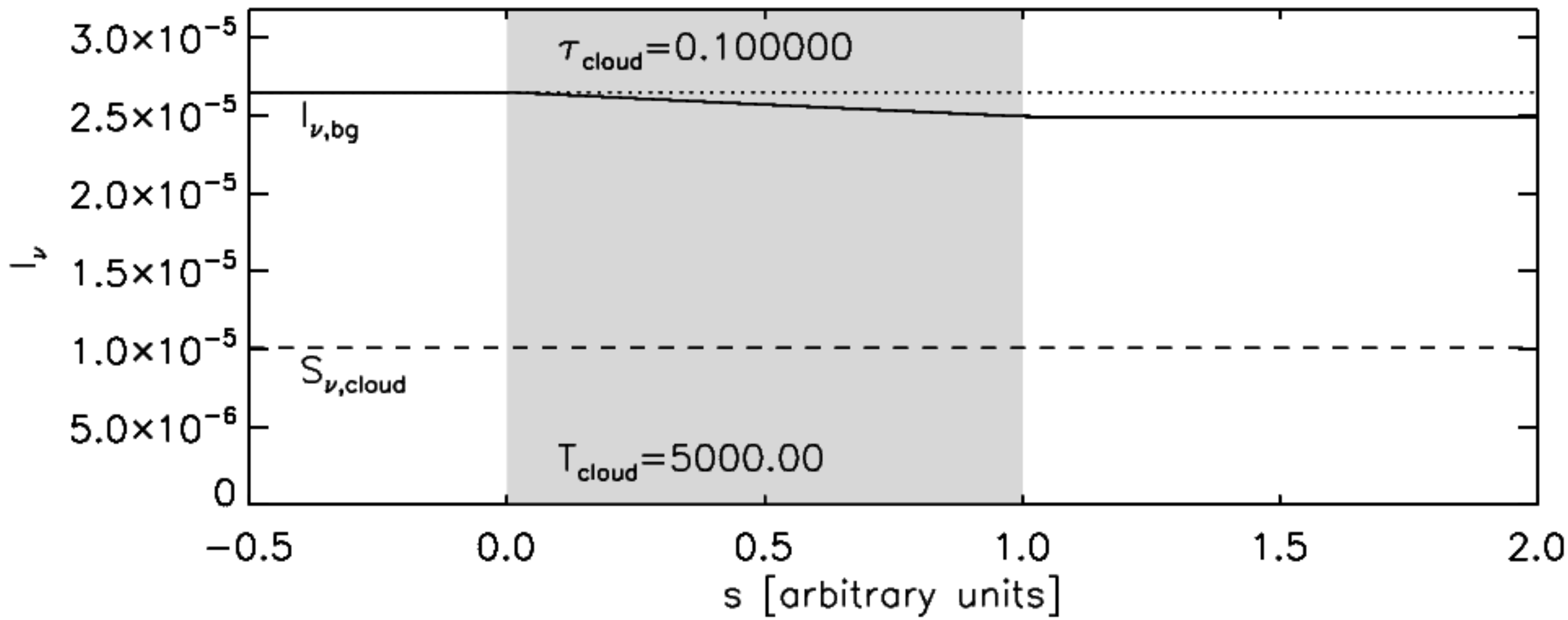
In case of local thermodynamic equilibrium: S is Planck function:

$$S_\nu = B_\nu(T)$$

Kirchhoff's law

Rad. trans. through a cloud of fixed T





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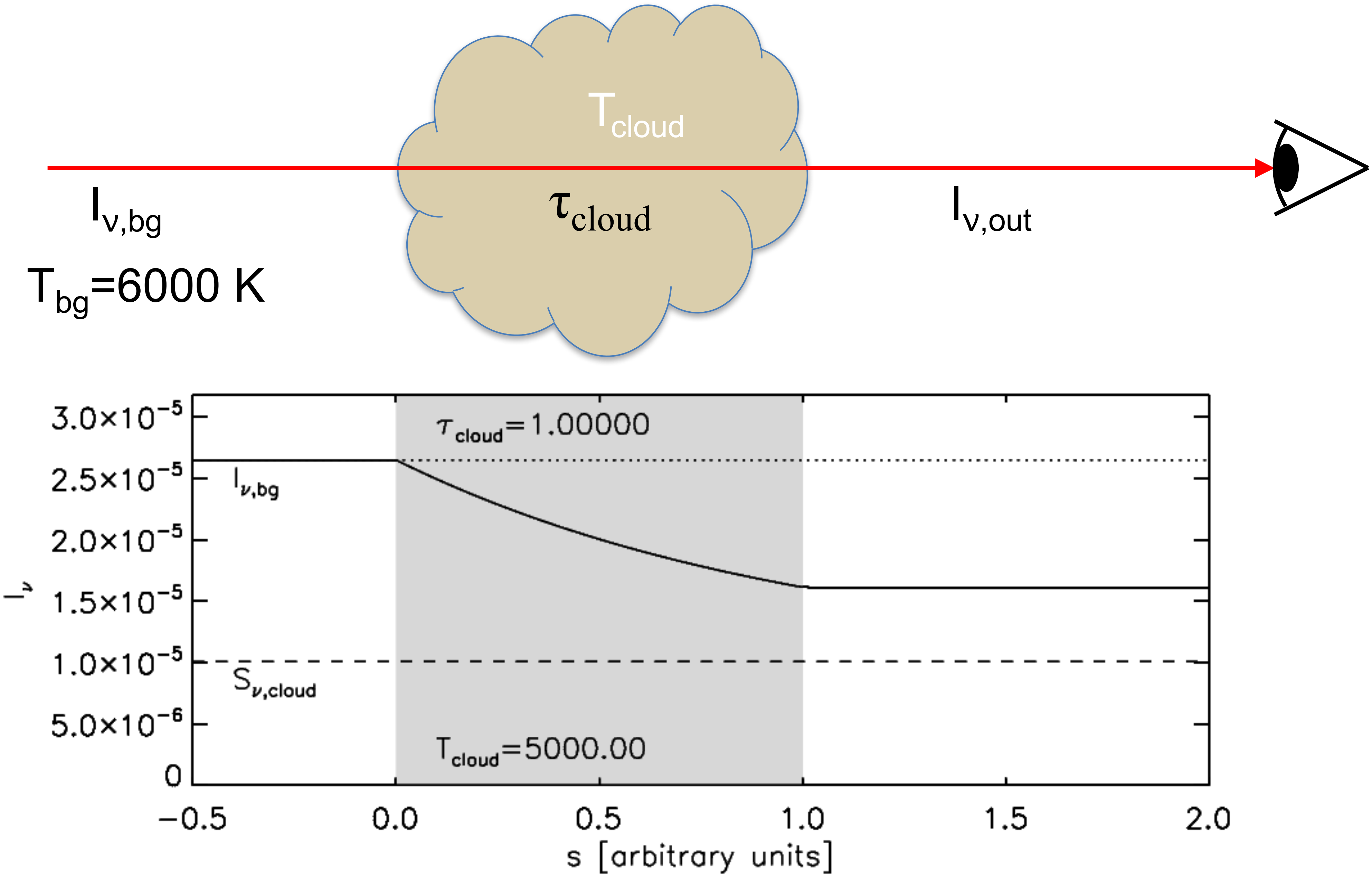

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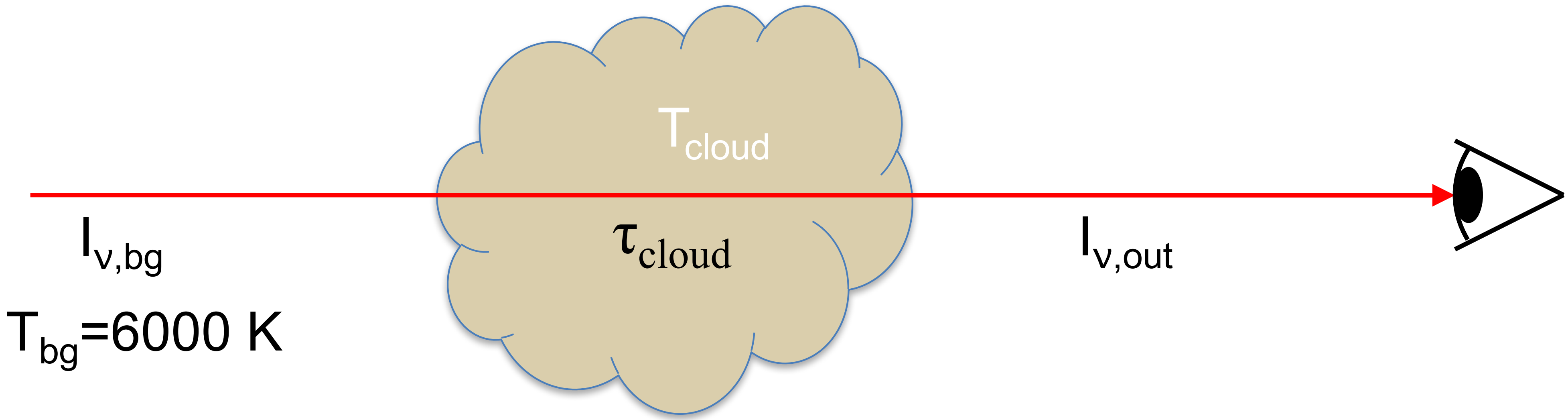

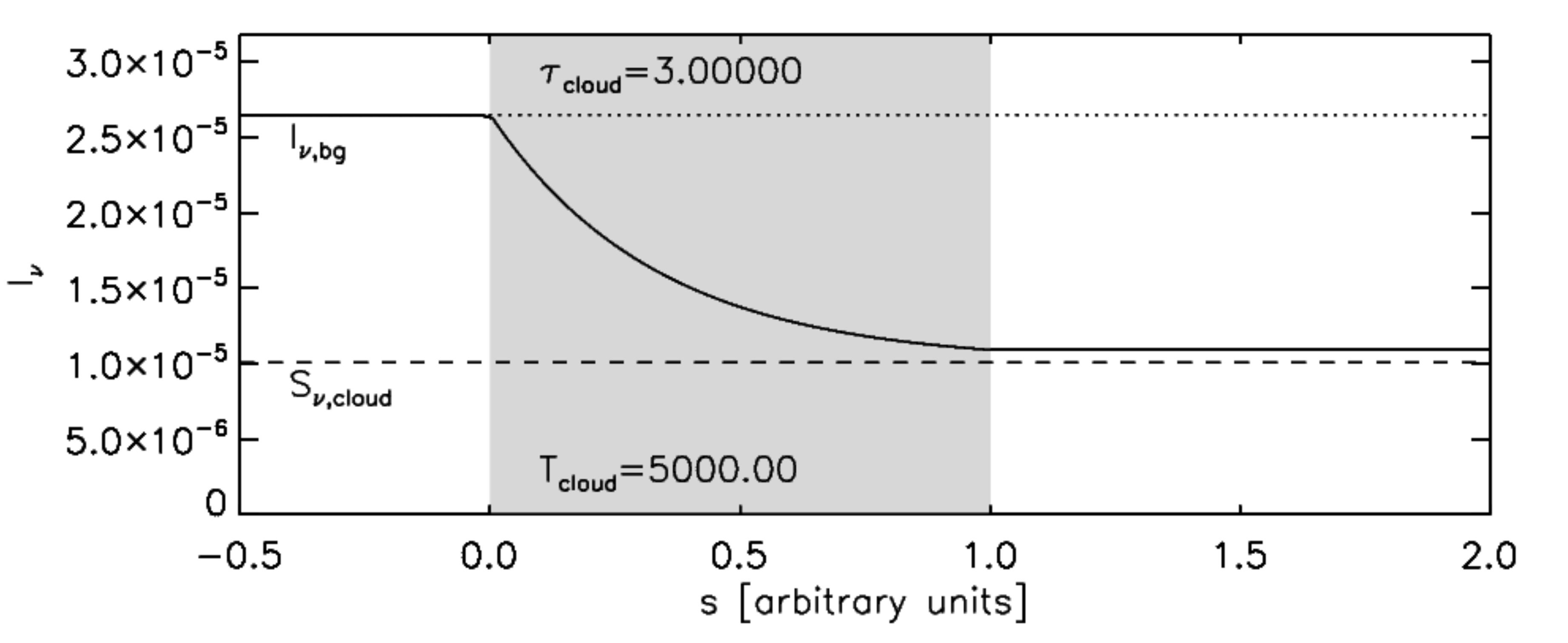

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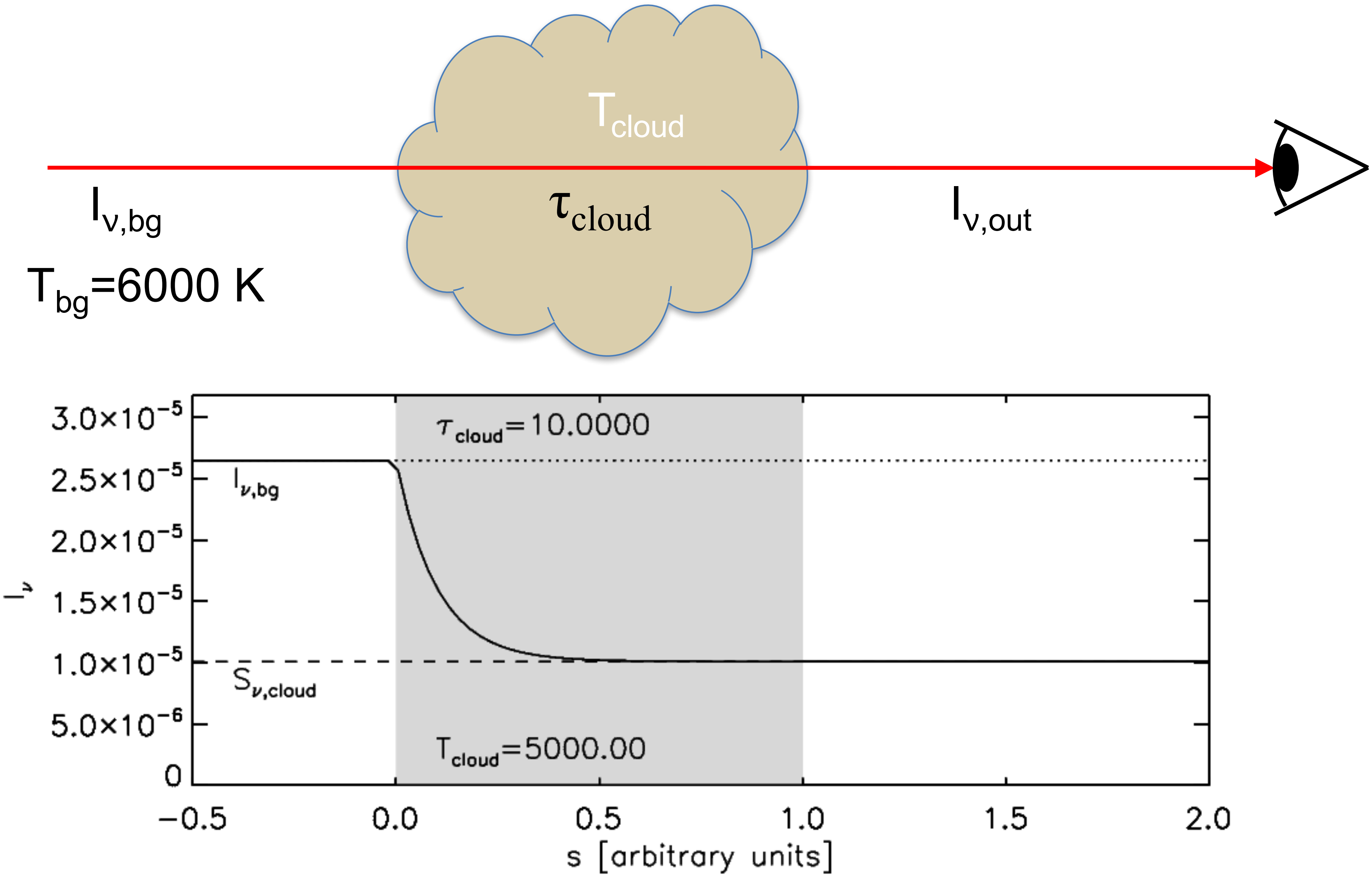

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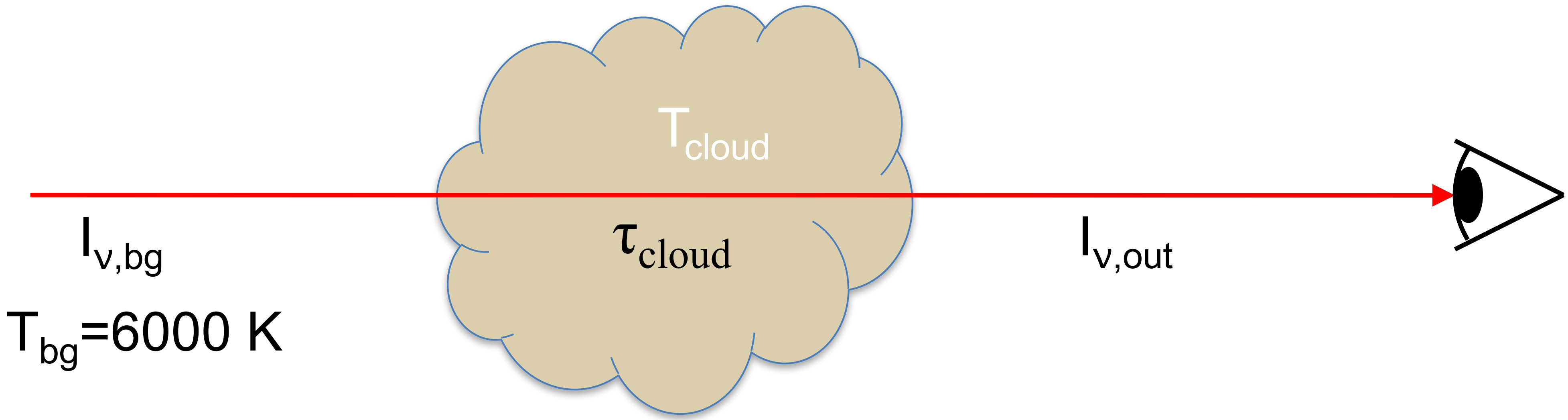
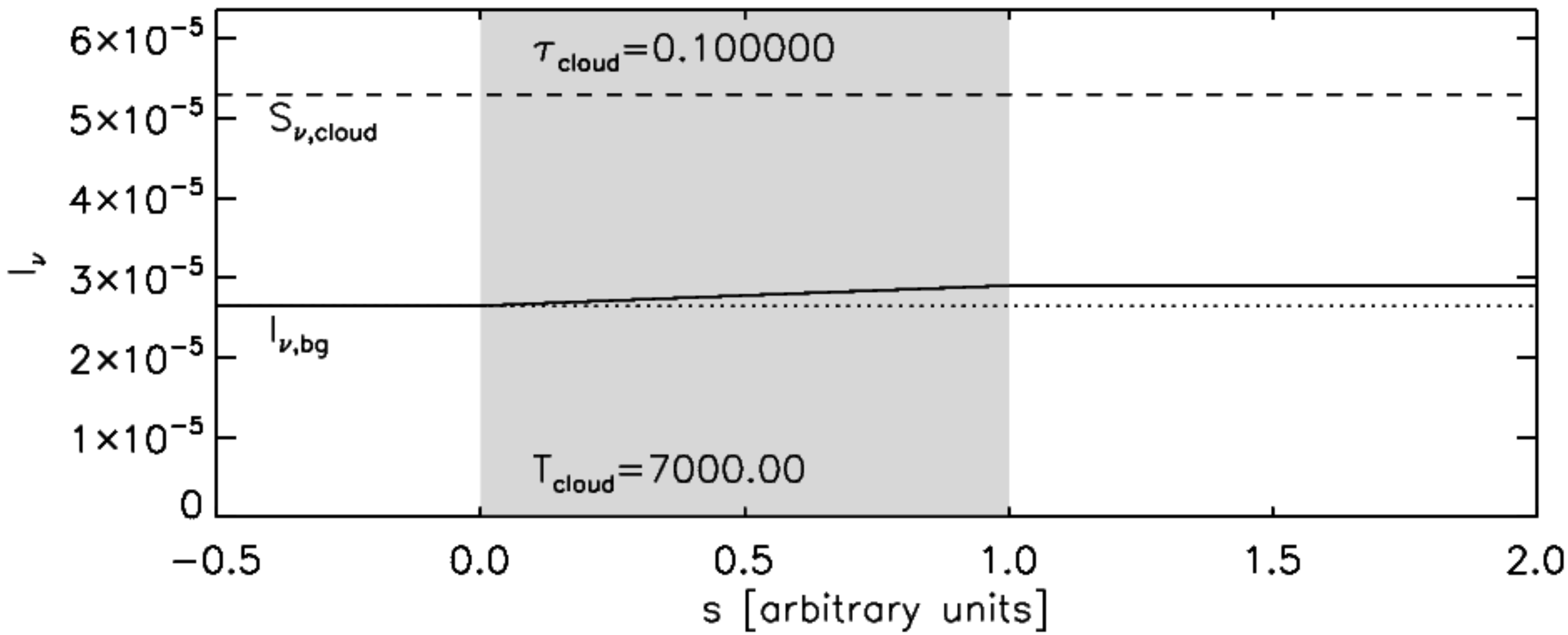
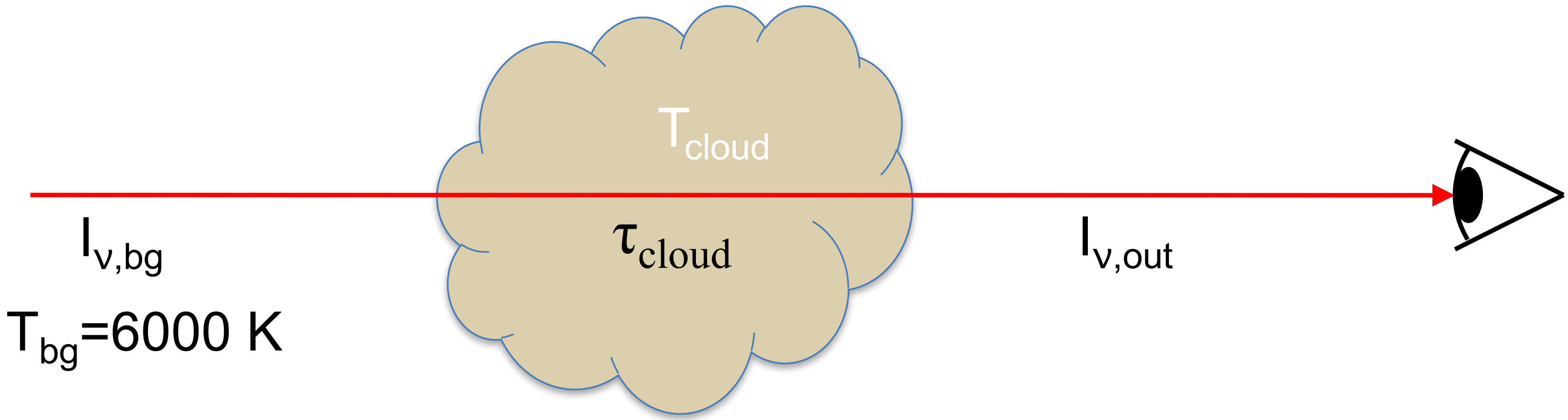
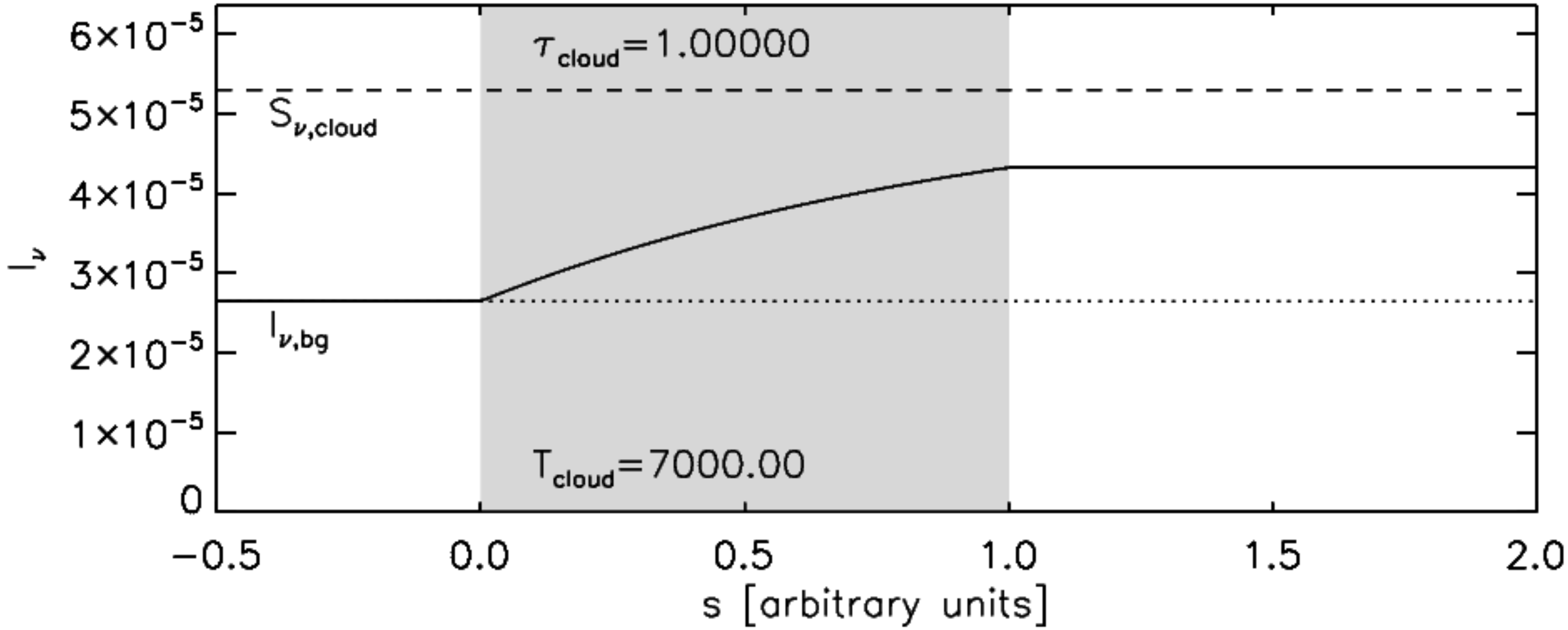


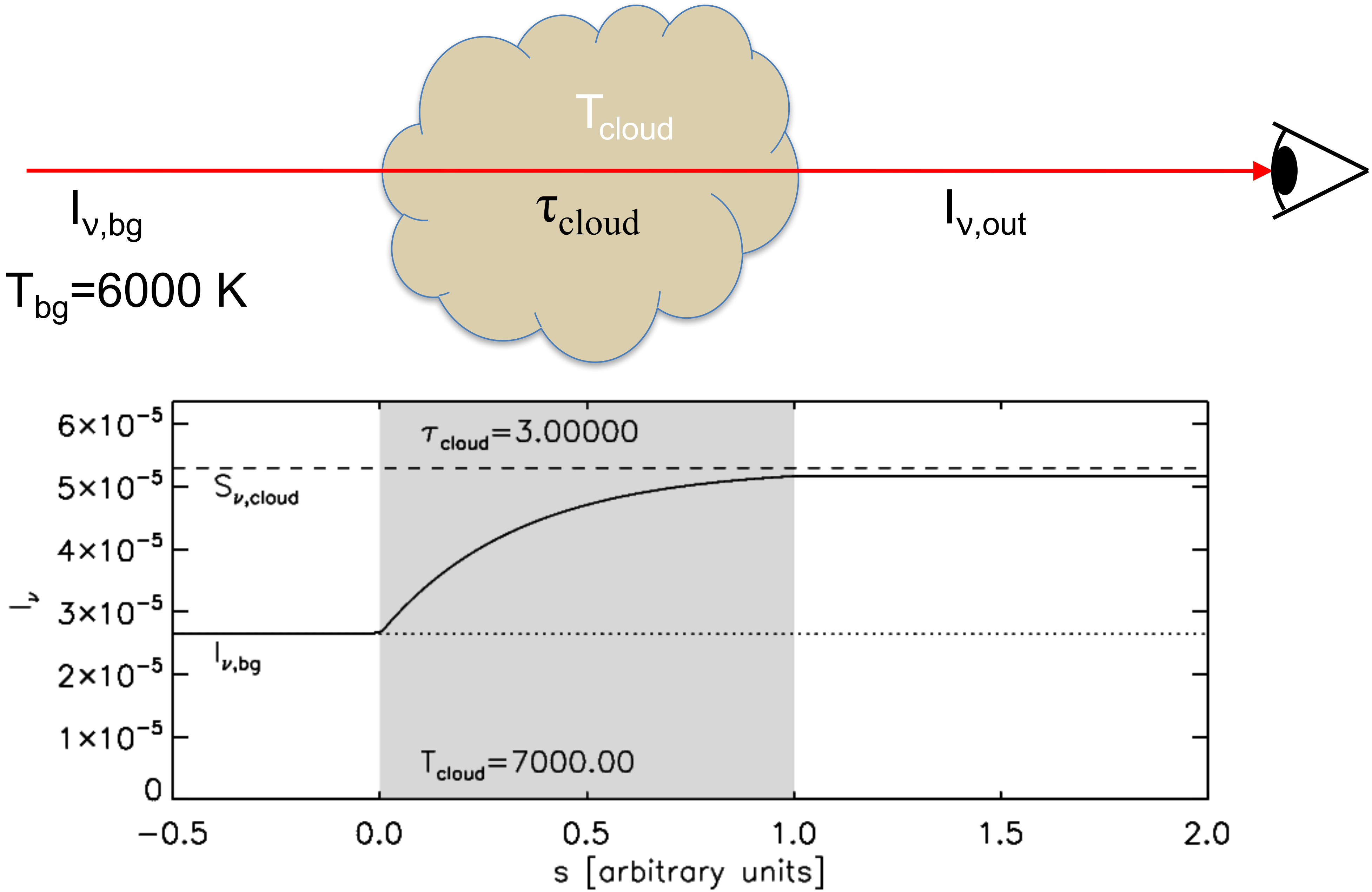


Rad. trans. through a cloud of fixed T

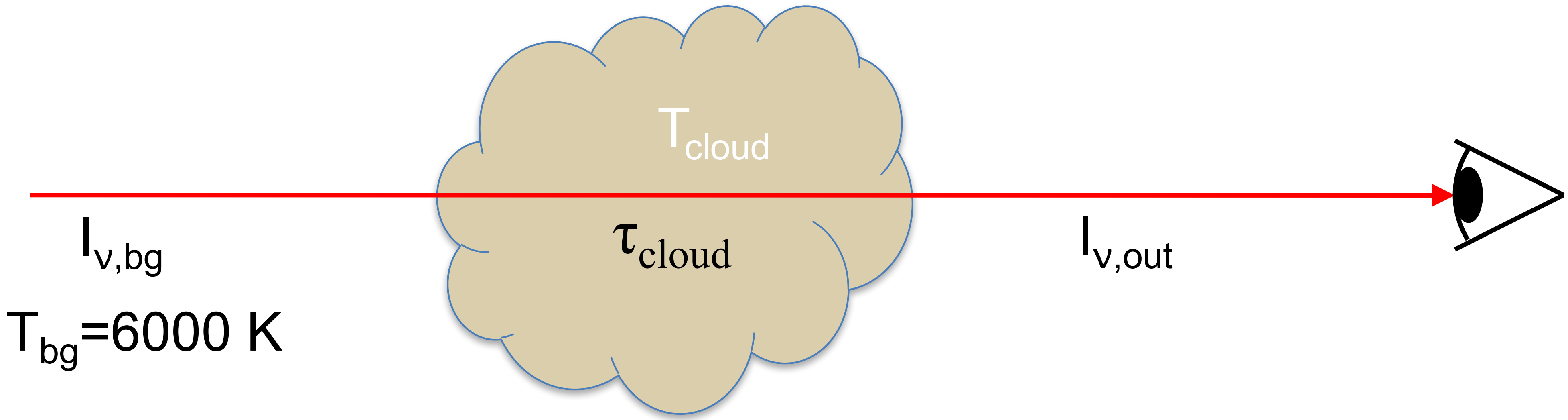


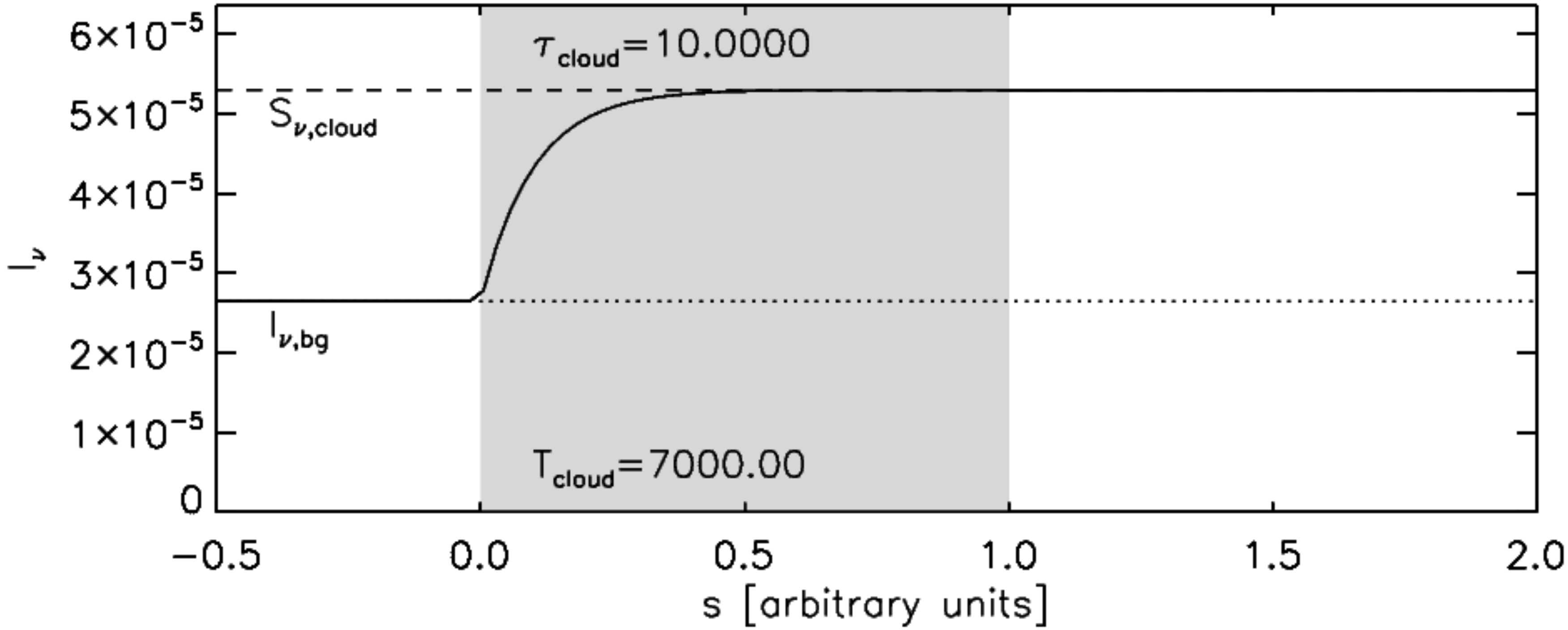


Rad. trans. through a cloud of fixed T



Rad. trans. through a cloud of fixed T





Formal radiative transfer solution

Radiative transfer equation again:

$$\frac{dI_\nu}{ds} = \rho\kappa_\nu (S_\nu - I_\nu)$$

Observed flux from single-temperature slab:

$$I_\nu^{\mathrm{obs}} = I_\nu^0 e^{-\tau_\nu} + (1 - e^{-\tau_\nu}) B_\nu(T) \qquad \tau_\nu = L\rho\kappa_\nu$$

$$\approx \tau_\nu B_\nu(T)$$

for $\tau_\nu << 1$ and $I_\nu^0 = 0$