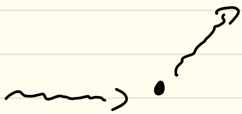
Scattering and Discrete Ordinates

$$n_\Omega \cdot \nabla I_\nu + \alpha I = \varepsilon^2 \alpha B_\nu(T)$$

$$+ \frac{1}{4\pi} \int_\nu \int \underbrace{R}_{\text{redistribution function}} (\Omega' \to \Omega ; \nu' \to \nu)\, I_{\nu'}(x, n_\Omega')\, d\Omega' \, d\nu'$$

Assumption: all interaction of light and matter is accounted for by this equation, which can be justified for the stationary equation in certain cases discussed below

Scattering mechanisms



1) Rayleigh scattering:

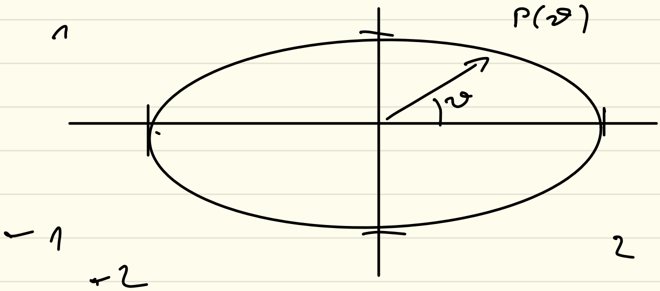
a (low energy) photon hits a small ball and changes its direction, while not changing its energy

$$\Rightarrow \quad R(\Omega' \to \Omega;\ \nu' \to \nu) = \underbrace{P(\Omega' \to \Omega)}_{\text{phase function}} \, \underbrace{\delta(\nu' - \nu)\, e(\nu)}_{\text{some efficiency}}$$

$$P(\Omega' \to \Omega) \sim 1 + \cos^2 \measuredangle(\Omega', \Omega) = 1 + \cos^2 \vartheta$$

$$\text{efficiency} \sim \frac{1}{\lambda^4}$$

Approximation for large wave length



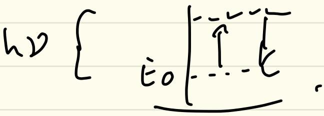
Phase function diagram

[Thompson scattering: scattering at electrons, leads to same phase function

2) Mie scattering: photons scattered by spheres of any size: formulas for conservation of energy and momentum, Maxwell equations for electromagnetic waves allow computation of phase function

Note: Mie scattering is elastic, no change of energy of photon

3) Resonance scattering: a photon hits a molecule and transfers one of the electrons to a higher energy state. Shortly after, the electron drops down to the ground state and releases a photon of the same energy



The new photon does not remember the direction of the old.

$$P(\Omega' \to \Omega) = 1/4\pi$$

Note: While Mie and Rayleigh scattering are instantaneous, resonance scattering involves a delay. Nevertheless, it can be neglected in many cases. Also, in cases of high intensity, the effect may saturate, since the upper energy levels are overpopulated

Fluorescence !!

4) Compton - Scattering

$$\lambda' - \lambda \sim 1 - \cos\vartheta$$

Many different redistribution functions, more to come. General principles:

1) Scattering conserves energy within the radiation field:

$$\int_{\mathbb{R}^+} \int_{S^2} \Big[I_\nu(x, n_\Omega) - \int_{\mathbb{R}^+}\int_{S^2} \overset{R()}{\diagup}\, I_{\nu'}(x, n_{\Omega'})\, d\Omega'\, d\nu' \Big]\, d\Omega\, d\nu = 0$$

$\Rightarrow$ Normalization: $\iint R(\ldots) = 1$

2) Reversibility of light rays

$$R(\Omega' \to \Omega, \nu' \to \nu) = R(-\Omega \to -\Omega', \nu' \to \nu)$$

3) Isotropic materials

Line profiles

A spectral line is sharp ($\delta(\nu)$) if an infinite periodic wave hits.

Lorenz - damping

$$L(\nu) \sim \frac{\gamma}{(2\pi\Delta\nu)^2 + \left(\frac{\gamma}{2}\right)^2}$$

$\Delta\nu = \nu - \nu_0$

resonance frequency

γ : damping parameter

Doppler shift

$$D(\nu) \sim e^{-\left(\frac{\nu-\nu_0}{w}\right)^2}$$

w: Doppler width determined by temperature

$$\varphi(\nu) = \int_{-\infty}^{\infty} L(\nu-\nu')\, D(\nu')\, d\nu'$$

normalized such that $\int_{-\infty}^{\infty} \varphi(\nu)\, d\nu = 1$

$$R(\Omega' \to \Omega; \nu' \to \nu) = \frac{1}{4\pi}\, \varphi(\nu)\, \varphi(\nu')$$