

Axiom

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Contents

1	Preface	1
I	The Foundation	3
2	Introduction	5
3	Logic	7
3.1	Introduction	7
3.2	Defining an Axiom	8
3.3	Formal Logic	9
3.3.1	Classical logic	9
3.3.1.1	Propositional logic	9
3.3.1.2	Truth Tables	10
4	Philosophy of Mathematics	13
5	Metaphysics	15
6	*Epistemology	17
7	Philosophy of Science	19

8 Philosophy of Physics	21
9 Foundations of Physics	23
10 Foundations of Mathematics	25
11 Foundation of Computations	27
12 Foundations of Mathematics	29
13 Theology	31
14 Ethics	33
15 Social Philosophy	35
16 Personal Philosophy	37
 II Notes	 39
17 Introduction	41
18 Textbooks	43
18.1 Introduction	43
18.2 Theoretical Physics by Georg Joos	43
18.3 On Sympathetic Reduction in Classical Me- chanics	46
18.4 Magnetic Fields and Magnetic Diagnos- tics for Tokamak Plasmas by Alan Wooton	46

18.5 Advanced MHD with Applications to Laboratory and Astrophysical Plasmas by Cambridge	46
18.6 Hamiltonian description of the ideal Fluid by P.J.Morrison	47
18.7 Classical Dynamics a Modern Perspective	47
18.7.1 Chapter 6	47
18.7.2 Chapter 7	48
19 Video classes	49
19.1 Differential Geometry - Robert Davie . . .	49
19.1.1 1-forms	49
19.1.2 2-forms	49
19.1.3 3-forms	50
19.1.4 k-form	50
19.1.5 Introduction to Exterior calculus .	50
19.1.6 Exterior calculus-2	51
19.1.7 Wedge Product-2	51
19.1.8 Introduction to the Hodge Star Operation	52
19.1.9 Hodge star 2	53
19.1.10 The push forward of vectors on manifolds	53
19.1.11 Pull-Back in k-forms	54
19.1.12 Pull-Back of volume forms	56
19.1.13 Integration on Manifolds Using the Pullback of Volume Forms	57
19.1.14 Line Integrals on Manifolds	58
19.1.15 Classical Stokes Theorem in Manifolds	59

19.2 Symplectic geometry classical mechanics	59
19.2.1 Lecture 1	59
19.2.2 Lecture 2	59
20 Coding	61
20.1 *Linux Terminal	61
20.2 *Github	61
20.3 *FORTRAN	61
20.4 *Python	61
III Research	63
21 Introduction	65
22 Previous Research	67
22.1 3D Modeling of Non-Equilibrium Dynamics in Compressed Plasma Using Lattice Boltzmann Method	67
22.1.1 3D Modeling of Non-Equilibrium Dynamics in Compressed Plasma Using Lattice Boltzmann Method Introduction	68
22.1.2 Physical Model	69
22.1.3 Density Distribution Function	70
22.1.4 Potential Energy Distribution	70
22.1.5 Hyperbolic Maxwell Equations	71
22.1.6 Kinetic Non-Equilibrium Method	71
22.1.7 Discussion	72

22.2A Hamiltonian Framework on ICF Implosions Rocket Equation Based on Rayleigh–Taylor Instabilities	72
22.2.1 Background	72
22.2.2 Problem Statement	73
22.2.3 Assumptions	74
22.2.4 Framework	74
22.2.5 Shell Kinematics	75
22.2.6 Shell Areal Density	75
22.2.7 Compressed-Fuel Pressure	76
22.2.8 Variational Principles	76
22.2.8.1 Phase-Space Lagrangian	76
22.2.8.2 Conservation Laws	77
22.2.9 Conclusion	77
22.3 Basic MHD	78
22.3.1 MHD Module	78
22.3.2 Hamiltonian MHD	79
22.4A Metriplectic Formulation of Reduced Magnetohydrodynamics with Resistivity	79
22.4.1 Introduction	79
22.4.2 Defining the Hamiltonian	80
22.4.3 Entropy and Dissipation	80
22.4.4 Metric Bracket and Metriplectic Structure	80
22.4.5 Combined Dynamics	81
22.5 Death Star	81
22.6 Interstellar travel	82
22.6.1 Speeds and forces	82
22.6.2 Magnetic Fusion Plasma Drive	85

22.6.2.1 Theoretical Basis	85
22.6.3 *Optimization	86
22.6.4 *Special Circumstances	86
22.6.4.1 *Strong Magnetic Fields	86
22.6.4.2 *Gravitational Fields	86
22.7 Kerr Black hole stuff	86
22.7.1 Energy (finish)	86
22.7.2 *Multiversal Travel	88
22.7.3 *Other effects	88
22.8 *Extreme Theoretical Work	88
22.9 Astroid Game	88
22.9.1 Introduction	88
22.9.2 Classical	88
22.9.3 Newtonian Gravity	88
22.9.4 Dark Matter	88
22.9.5 Relativistic	88
23 The Structure of Motion: A Philosophical Journey Through Hamiltonian Mechanics	89
23.1 Introduction	89
23.1.1 Physical Reality and Frameworks	90
23.1.2 Philosophical Context*	92
23.1.3 Notion of Time, States, and Trajectories	92
23.1.4 Introduction to Phase Space	93
23.1.5 Teleology vs Causality	94
23.1.6 Role of Variational Principles in Mechanics	95
23.2 Axiomatic Mechanics	95

23.2.1	Axiomatic Derivation of ...	95
23.2.1.1	Introduction	95
23.2.1.2	Axiom of Real numbers	95
23.2.1.3	Axiom of Euclidean Geometry	96
23.2.1.4	Axiom of Set Theory (Basic)	96
23.2.1.5	Axiom of Calculus (Differential and Integral Calculus)	97
23.2.1.6	Axioms of Variational Calculus	98
23.2.1.7	Axioms of Linear Algebra (Basic)	98
23.2.1.8	Axioms of Variational Calculus	99
23.2.1.9	Physical Assumptions (Not Strictly Mathematical Axioms)	99
23.2.2	Axiomatic Derivation of Hamiltonian Mechanics	100
23.2.3*	Hamiltonian Mechanics ad a Geometric Theory	102
23.2.4*	Poisson Brackets and the Algebra of Dynamics	102
23.2.5*	Canonical Transformations: Symmetry, Simplicity, an Structure	102
23.3	Philosophical Implications*	102
23.3.1*	Time in Essence	102
23.3.2*	Hamiltonian Constraints and the Problem with Time	102
23.4	Modern Hamiltonian	102

23.4.1 Modern Research	102
23.4.1.1*Hamiltonian Formulation in Plasma Physics	102
23.4.1.2*Hamiltonian and Metriplectic Mechanics	102
23.4.1.3*Symplectic Integrators: Pre- serving the Structure of Na- ture	102
23.4.1.4*HNNN(Hamiltonian Neural Networks)	102
23.4.2 Fun Mess Around	102
23.4.2.1 Non-Canonical Poisson Brack- ets	102

24 Omega-X 107

24.1 Pre-planing(part 1)	107
24.1.1 Ideas & Overarching Plan	107
24.1.2 Metriplectic 4-bracket algorithm for constructing thermodynamically con- sistent dynamical systems	108
24.1.2.1 Math	108
24.1.3 A thermodynamically consistent dis- cretizations of 1D thermal-fluid mod- els using their metriplectic 4-bracket structure	109
24.1.3.1 Math	109
24.1.3.2 Coding	112
24.1.3.3 Future Work	113
24.1.4 Metriplectic Framework for Dissi- pative Magneto-Hydrodynamics . .	113

24.1.4.1	My Introduction	113
24.1.4.2	Notes on introduction . . .	114
24.1.5	METRIPECTIC FORMULATION OF VISCO-RESISTIVE MHD	115
24.1.6A	least Action principle for visco- resistive Hall Magnetohydrodynamic with metriplectic reformulation . .	117
24.1.6.1	Abstract	117
24.1.6.2	Metriplectic bracket, Hamil- tonian and Entropy for Vis- coresistive Hall MHD	117
24.1.7	Variational discretizations of viscous and resistive magnetohydrodynam- ics using structure-preserving finite elements	118
24.1.7.1	Abstract	118
24.2	part 2	118
24.2.1	Coding(1D fluid)	118
24.2.1.0.1	Introduction	118
24.2.1.0.2	mesh_1d	119
24.2.1.0.3	states_1d	121
24.2.1.0.4	projection_matrix_1d	123
24.2.1.0.5	mass_1d	124
24.2.1.0.6	momentum_1d . .	125
24.2.1.0.7	entropy_1d	127
24.2.1.0.8	velocity_1d	129
24.2.1.0.9	EOS_1d.thermal .	130
24.2.1.0.10	time_integrator_1d	131
24.2.1.0.11	b_1d	133

24.2.1.0.12D.Thermal.driver	134
24.2.1.1Plans	135
24.3part 3	136
24.3.1Notes	136
24.3.1.1Notes on module FORTRAN	136
24.3.1.2Diminsionales programming	137
24.3.1.3Generalize	137
24.3.2Plan	137
24.3.3New Structure	138
25 Axiom	139
25.1 *Structure	139
25.2 LaTeX	139
25.2.1 Division of Structure	139
25.2.2 Commands	140
IV Essays	143
26 Introduction	145
V Journaling	147
27 *Introduction	149
28 Plans	151
28.1 Now	151
28.1.1 Self-Education	151
28.1.2 Research	156

28.1.3 Other activities	158
28.2 Pre-University	158
28.2.1 Independent Learning	158
28.3 Undergrad	159
28.3.1 Classes	159
28.3.1.1 Requirments	159
28.3.1.1.1 General	159
28.3.1.1.2 Academic Physics Major .	160
28.3.1.1.3 Applied Maths Major . .	161
28.3.1.1.4 Computer Science Minor	161
28.3.1.1.5 Electives	162
28.3.1.1.6 Total	163
28.3.1.2 Schedule	165
28.3.1.3 First Year	165
28.3.1.4 Second Year	166
28.3.1.5 Third year	167
28.3.1.6 Fourth year	167
28.3.1.7 Others	168
28.3.1.8 Possible summer	169
28.4 *Grad	169
28.5 *Early career	169
28.6 *Late Career	169

29 Personal-Analysis 171

29.1 Introduction	171
29.2 Disgrace and Pride	171
29.3 On Ayn Rand	175
29.4 On the Pursuit of Thought	178

29.5 Nietzsche's Sovereign Man and Morality .	179
29.6 Half-Growth and Half-Death	182
29.7 Lack of Aesthetic	186
29.8 Boredom	187
29.9 Why Physics	188
29.10 Lack of Resistance	189
29.11 Am I Misanthropic?*	190
29.12 Depth of Feeling	191
29.13 Language Usage*	192
29.14 Euphesus*	194

Chapter 1

Preface

"If you start with the right axioms, the rest will follow."— David Hilbert

This book "Axiom" is a continuation of my previous project *Lógos*. Which was mostly a collection of essays on several topics; physics, philosophy, religion, civilization, psychology, personal plans, etc.

It was previously meant to work through my ideas, using the fact I was writing it out. It allowed me to keep track of the dozens of ideas and how they all interacted with each other. Developing on the complexity without forgetting and creating a full framework with defined axioms.

Creating this new project is for multiple reasons.

1. To create a more organized book

- What I mean is have an order, like create a fully formalized epistemology and basic metaphysics and then using that to create a fully organized Ontology. Rather than a chaotic

coll

2. To mark the end of my high school career and beginning of University
3. Lógos had become too big and had too many errors
 - Too big to run on Overleaf, too many errors to run directly in the terminal.
 - This will start smaller, and once it is bigger will be made without errors so it can be run on the terminal once it gets bigger. Also more modular
4. Be able to rewrite with my new knowledge to update it.
5. Write it with a better understanding of the packages, LaTeX, and even create my own custom packages

In the end, it serves the same purpose; to help me better understand things through writing, develop things iteratively over time using writing, and write out notes to remember things for the future.

As of writing this section, the text will be broken into three parts. Part one being "The Foundation." It would be a axiomatic approach to creating the foundations needed for future systems. Two, being "Notes" on things that I have learned. Three, being writings on my "Projects"; mostly physics research projects but also projects in further developing philosophy or other similar things. Finally, "Life in Practice" being more practical things; plans for the future, self-analysis, ideals on society, etc.

Part I

The Foundation

Chapter 2

Introduction

As mentioned in the preface, this section is made to axiomatically develop the foundations needed.

To illustrate, in order to develop physical theories, mathematics must be developed, a philosophy of science must be made, metaphysics, epistemology, and formal logic. Thus, a complete theory of all of these things and more should be created. One that has fundamental, expressible axioms.

Also, most of these theories rely on one another, so a logical chronology must be defined. At the moment of writing this it is defined to be logic → metaphysics → Epistemology → philosophy of science → philosophy of physics → mathematics → physics → theology → ethics → social philosophy → personal philosophy.

One additional thing to add, you may notice the separation of metaphysics and the philosophy of physics. These are both meant to signify a difference where metaphysics represents extremely fundamental questions like does existence exist. Things that must be answered before epistemology can be fully introduced. The next

section, philosophy of physics, are slightly less fundamental, like the ontology of entropy. Questions that require a theory of epistemology to be answered.

Chapter 3

Logic

3.1 Introduction

Logic is an important tool for the creation of all formal systems. It is the foundation of mathematics, which is for the most part the foundation of everything else. It is also simply important to anything else by clarify concepts, giving them formal definitions, and insures consistency. Allowing for a bedrock behind all analytical thinking.

Though, before I can get into it too much, I should define what the study of logic is. It is the branch of philosophy that critically examines the fundamental nature, scope, and principles of logic itself.

It has a couple fo subfields as well.

- Philosophy of Logic
 - The study of logic at a high level, analyzing its nature and scope.

- Formal Logic
 - A creation and usage of symbolic logic symbols to create systems; mainly mathematics.
- Metalogic
 - The technical, mathematical study of the properties of formal logical systems. The philosophy of logic often uses the results of metalogic (like Gödel's incompleteness theorems) to address its conceptual questions, such as the limits of a formal axiomatic system

3.2 Defining an Axiom

This entire text is named Axiom, for it is the foundation. To define systems axiomatically and then build them out into their true complexity is not only one of my favorite things to do, but as an extension of that it is what this book is trying to do. (Obviously, I wouldn't do something like this if I didn't enjoy it.)

So let's define it. As I mention extensively in my last book, *Lógos*, we must define things extremely rigorously. By testing against edge cases and keep it with consistent.

A good way to start is with the dictionary definition. "a statement accepted as true as the basis for argument or inference." For instance, our understanding of scientific realism is the understanding of the fact that our observations of reality are accurate.

A more defined definition can then be defined to say that an axiom is a statement that either cannot be proved or cannot be proved currently that is used

within a greater statement that can be proved upon its edifice. These axioms, while don't require 'proof' generally desire some type of reasoning even if it is not objective or purely analytical in nature. For instance, the above arguments for the objective nature of reality aren't true objective proof, they still exist as semi-arguments. While we cannot prove the objectivity of reality with true and objective reasoning, some reasoning can be applied to get some kind of 'proof' of the statement so that further studies can be applied based upon that said axiom.

An astute observer can find that these mean the same thing, my is more of an explanation so, the dictionary definition does work:

Definition 3.1. *Axiom: a statement accepted as true as the basis for argument or inference.*

3.3 Formal Logic

Let us start with formal logic, a system of symbols to be able to represent logic. Then build from there.

3.3.1 Classical logic

The most basic form of logic is classical logic, the examination of simple singular objects.

3.3.1.1 Propositional logic

The most basic form of classical logic, but instead of telling you now, I will develop it.

3.3.1.2 Truth Tables

Negation is the first and simplest form. The symbol is \neg , so $\neg A$ is the negation of A.

so the table can be constructed

A	$\neg A$
-----	----------

T	F
-----	-----

F	T
-----	-----

Where T, represents true, and F represents false.

A further complication is the conjunction operator \wedge or "and." It combines two functions into a greater set. (a union of function)

A	B	$A \wedge B$
-----	-----	--------------

T	T	T
-----	-----	-----

T	F	F
-----	-----	-----

F	T	F
-----	-----	-----

F	F	F
-----	-----	-----

$A \wedge B$ is only true when both "A" and "B" are true.

\vee is the inverse, only one of the functions must be true. It is called a disjunction.

A	B	$A \vee B$
-----	-----	------------

T	T	T
-----	-----	-----

T	F	T
-----	-----	-----

F	T	T
-----	-----	-----

F	F	F
-----	-----	-----

$A \implies B$ means "if A, then B" meaning that A needs

to happen for B or that A causes B.

A	B	$A \implies B$
T	T	T
T	F	F
F	T	T
F	F	T

Chapter 4

Philosophy of Mathematics

Chapter 5

Metaphysics

Chapter 6

***Epistemology**

Chapter 7

Philosophy of Science

Chapter 8

Philosophy of Physics

Chapter 9

Foundations of Physics

Chapter 10

Foundations of Mathematics

Chapter 11

Foundation of Computations

Chapter 12

Foundations of Mathematics

Chapter 13

Theology

Chapter 14

Ethics

Chapter 15

Social Philosophy

Chapter 16

Personal Philosophy

Part II

Notes

Chapter 17

Introduction

This part is for the purpose of writing my notes on other people's work, rather than my own. Whether it be notes from a class, a textbook, a regular book, or anything of that nature. Pretty simple.

Chapter 18

Textbooks

18.1 Introduction

Here I will write my notes from books I have read or am reading.

18.2 Theoretical Physics by Georg Joos

The vector analysis portion is interesting due to the fact that it takes something as simple as vector analysis and brings rigorous ideas to it like that $\oint ds = 0$ for close surfaces.

I also never really thought about using vector analysis instead of tensors.

The rest of curl, gauss, and such is very simple. Though, now it is getting into tensors though vectors.

$$dv_x = ds \nabla v_x$$

$$dv_y = ds \nabla v_y$$

$$dv_z = ds \nabla v_z$$

So therefore $dv = ds \nabla v$

To calculate v , three vectors or nine scalar must be known.

Another interesting addition is the fact that in physics, symmetric tensors can be represented as a surface to the second degree.

One thing I have noticed is I need a more intuitive grasp into the relationships between curl, div, laplace, and grad. They use it a lot to simplify the calculations.

$$\nabla^2 f = \nabla \cdot (\nabla f)$$

The Laplace operator is equal to the divergence of the gradient.

$$\nabla \times (\nabla f) = 0$$

For smooth scalar fields. The curl of the gradient is zero.

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0$$

The same is true for the divergence of the curl. $\nabla^2 \mathbf{F} = \nabla(\nabla \cdot \mathbf{F}) - \nabla \times (\nabla \times \mathbf{F})$ For vector fields, the Laplace operator has different relations.

Next, onto calculus of variations. I will derive Euler-Lagrange differential equation.

Let: \tilde{y} be a neighboring function to y . Where ϵ be a small quantity and $\eta(x)$ be a arbitrary function of x . so if $\tilde{y} = y + \epsilon\eta$ then $\tilde{y}' = y' + \epsilon\eta'$. Here we stipulate that the two functions \tilde{y} and y converge at the beginning and end. Thus, η must vanish at the ends. So if we substitute an integral I , we find that it becomes a function

of ϵ . Then we require that $I(\epsilon)$ must have an extreme value of $\epsilon = 0$. Here it is in mathematical terms:

$$I(\epsilon) = \int_{x_0}^{x_1} F(x, y + \epsilon\eta, y' + \epsilon\eta') dx = \text{extremum for } \epsilon = 0$$

This gives us a simple way of determining the extreme value for a given integral. The condition is:

$$\left(\frac{dI}{d\epsilon}\right)_{\epsilon=0} = 0$$

We can then expand the integrand function F in Taylor's series, according to the powers of ϵ .

The differentiate with respect to ϵ .

This expression then vanishes for $\epsilon = 0$. Thus then simply remains the condition for the extremum.

Integrate this to get the Euler-Lagrange differential equation:

$$\frac{d}{dx} \frac{\partial F(x, y, y')}{\partial y'} - \frac{\partial F(x, y, y')}{\partial y} = 0$$

For writing constraining forces, we find it to be

$$Z = \lambda \text{ grad } G$$

Where $G(x, y, z)$ is the equation of the surface

18.3 On Sympathetic Reduction in Classical Mechanics

18.4 Magnetic Fields and Magnetic Diagnostics for Tokamak Plasmas by Alan Wooton

18.5 Advanced MHD with Applications to Laboratory and Astrophysical Plasmas by Cambridge

For wide variety of MHD instabilities operating in tokamaks, represented by normal modes of the form (assuming that cylindrical approximation and the toroidal representation may be ignored):

$$f(\psi, \vartheta, \varphi, t) = \sum_m \tilde{f}(\psi) e^{i(m\vartheta + \eta\varphi - \omega t)}$$

Is only unstable for perpendicular wave vectors.

The reason is the enormous field line bending energy of the Alfvén waves

18.6 Hamiltonian description of the ideal Fluid by P.J.Morrison

18.7 Classical Dynamics a Modern Perspective

18.7.1 Chapter 6

A lot more importation stuff I forgot to write down, well I will start in chapter 6: If multiple canonical transformations, directly transforming would be canonical. $w \rightarrow w' \rightarrow w''$ then $w \rightarrow w''$. This is the group Composition Law.

Next is the Associativity of Composition Law: if $w' = \phi(w)$, $w'' = \Phi(w')$, and $w''' = \xi(w'')$ so no matter what order of transformation they will be the same, whether $(T_3(T_2T_2))$ or $(T_3T_2)T_1$ or anything of the such.

Identity: The identity transformation $w' = w$ obviously obeys the conditions for being canonical.

Inverses: If $w \rightarrow w'$ is canonical, then so is the transformation $w' \rightarrow w$.

"Thus all the properties needed to define a group are trivially obeyed. (In Chapter 12 we discuss group structure in a little more detail). "

The canonical group is characterized by the dimension, $2k$, of the phase space.

The subgroups of such a phasespace is very large, though three can be easily defined.

Another subgroup is the contact transformations.(when a poisson bracket can be formed)

Note: changing coordinate systems can satisfy phase space rules, but isn't generally considered to be a transformation

"Intuitively speaking, a finite canonical transformation that can be connected continuously to the identity should be built up as a succession of infinitesimal transformations."

To express the relation between Poisson Brackets and canonical transformations through differential equations:

$$\frac{d\omega}{d\theta} = \{\omega^\mu, \phi(\omega)\}_\omega = \epsilon^{\mu\nu} \frac{\partial \phi(\omega)}{\partial \omega^\nu}$$

Let us first clarify the meaning of this equation. We look upon the w^μ as functions of θ ; ϕ is a function with a fixed functional form, and w^μ are the unknowns in the differential equation. Thus on the right-hand side of this equation, the arguments of ϕ are just the quantities we are trying to solve for. From the theory of differential equations, we know that there is a unique solution for w^μ if we are given the values of w^μ at $\theta = 0$. Calling these boundary values of w^μ as w_0^μ , the solution of the equation can be written as:

$$\omega^\mu = \varphi^\mu(\omega_0; \theta), \varphi^\mu(\omega_0; 0) = \omega_0^\mu$$

18.7.2 Chapter 7

Chapter 19

Video classes

19.1 Differential Geometry - Robert Davie

19.1.1 1-forms

These are linear functionals that map tangent vectors at a point on a manifold to real numbers. In local coordinates on a manifold M , a 1-form can be written as $\omega = \omega_i dx^i$, where ω_i are smooth functions and dx^i are basis 1-forms (dual to coordinate basis vectors). For example, the differential of a function $df = \frac{\partial f}{\partial x^i} dx^i$ is a 1-form. They are used to measure vectors, like gradients or line integrals.

19.1.2 2-forms

These are antisymmetric bilinear maps that take two tangent vectors and produce a real number. In coordinates, a 2-form looks like $\omega = \omega_{ij} dx^i \wedge dx^j$, where \wedge

denotes the wedge product (ensuring antisymmetry: $dx^i \wedge dx^j = -dx^j \wedge dx^i$). 2-forms are used to measure oriented areas, such as in surface integrals or the electromagnetic field tensor in physics.

19.1.3 3-forms

These are antisymmetric trilinear maps, taking three tangent vectors to a real number. In coordinates, a 3-form is $\omega = \omega_{ijk} dx^i \wedge dx^j \wedge dx^k$, with antisymmetry in all indices. They measure oriented volumes and are used in integrals over 3-dimensional submanifolds, like in fluid dynamics or general relativity.

19.1.4 k-form

This can then be made general as

$$\omega = \sum_{i_1 \dots i_k} \omega_{i_1 \dots i_k} dx^{i_1} \dots \wedge dx^{i_k}$$

19.1.5 Introduction to Exterior calculus

An exterior derivative maps k-forms into k+1-forms.

$$d : \wedge^k(M) \rightarrow \wedge^{k+1}(M)$$

and must follow these rules
 Linearity: $d(\alpha + \beta) = d\alpha + d\beta$
 Leibniz(product rule): $\wedge(\alpha + \beta) = d\alpha \wedge \beta + (-1)^k \alpha \wedge d\beta$
 Nilpotency: $d(d\alpha) = 0$

$$\therefore d\omega = \sum_{i < j} \left(\frac{\partial \omega_i}{\partial x_j} - \frac{\partial \omega_j}{\partial x_i} \right) dx^i \wedge dx^j$$

In 3D, the exterior derivative of the 2-form corresponds to the divergence in vector field.

Allows for integration of manifolds. Shows that the integral of the exterior derivative of a differential form over a manifold is equal to the integral of the form itself over the boundary of the manifold:

$$\int_M d\omega = \int_{\partial M} \omega$$

This unifies vector calculus in areas like divergence or green's theorem.

It also shows vector flow

Lie derivatives measure the change of forms

19.1.6 Exterior calculus-2

The exterior for a k-form is

$$d\omega = \sum_{n=1}^k \left(\sum \frac{\partial x_{i_1 \dots i_n}}{\partial x^{i_n}} dx^{i_n} \right) \wedge dx^{i_1} \wedge \dots \wedge dx^{i_{n-1}}$$

4-form is the highest form

Back to integration of manifolds with Stokes theorem. While $\int_M d\omega = \int_{\partial M} \omega$ is true, $\int dx \wedge dy$ and $\int dx dy$ represent slightly different mathematical concepts. In differential forms, it is the integral of an oriented area. In Multi-variable calculus it is the integral over a region in the xy-plane

19.1.7 Wedge Product-2

Properties of the wedge product: Anti-symmetry: $\omega \wedge \eta = (-1)^{pq} \eta \wedge \omega$. Where p & q are the dimensionality of

ω, η

Symmetric group: S_{p+q} is the group of all possible permutations.

Permutation and sign of permutations: σ is essentially a specific way of ordering a set of elements. The sign of permutation $sign(\sigma)$ tells us if the permutation can be achieved in an even or odd number of swaps. Where odd is negative and even is positive

Role of these two things: The sign tells us the sign of a wedge product given a permutation. $dx^1 \wedge dx^2 \rightarrow -dx^2 \wedge dx^1$. The symmetric groups tells us all possible permutations.

$$\therefore (\omega \wedge \eta)(X_1, \dots, X_{p+q}) = \frac{1}{p!q!} \sum_{\sigma \in S_{p+q}} sign(\sigma) \omega(X_{\sigma(1)}, \dots, X_{\sigma(p)}), \\ \eta(X_{\sigma(p+1)}, \dots, X_{\sigma(p+q)})$$

This $\frac{1}{p!q!}$ ensures correct normalization.

19.1.8 Introduction to the Hodge Star Operation

This operator allows us to relate manifolds. Crucial in formulating coordinate free theories. It transforms k -forms into $(n-k)$ -forms. Specifically it is an isomorphism between the spaces of k -forms and $(n-k)$ -forms on an n -dimensional Riemannian (M, g) . Where g is the metric tensor. It is denoted as:

$$\star : \wedge^k(T^*M) \rightarrow \wedge^{n-k}(T^*M)$$

The way it works is hard to write out, so just re-

member it. To construct a hodge star

$$\omega \wedge \star \eta = \langle \omega, \eta \rangle \text{vol}_M$$

Where $\langle \cdot, \cdot \rangle$ is the inner product and Vol is the volume form of M . Defined as $\text{vol}_M = \sqrt{|g|} dx^1 \wedge \dots \wedge dx^n$ Where $|g|$ is the determinate of the Metric Tensor.

The equation is long, so just look it up, it is simple but long.

19.1.9 Hodge star 2

Hodge star establishes duality between geometric theories. Involution: Doing it twice returns it to the original Linear transformations: You know what this means Interacts like multiplication with exterior derivative: $\star(d\omega) = d(\star\omega)$ The co-differential is defined using the Hodge Star.

19.1.10 The push forward of vectors on manifolds

A pushforward is a way to map a vector on a tangent space onto another manifold's tangent space.

Properties:

Linearity: The pushforward $\phi_* : T_p M \rightarrow T_{\phi(p)} N$ is a linear map. For vectors $v, w \in T_p M$ and scalars a, b ,

$$\phi_*(av + bw) = a\phi_*v + b\phi_*w$$

Chain rule: If you have another smooth map $\psi : N \rightarrow P$, the pushforward satisfies the composition rule:

$$(\psi \circ \phi)_* = \psi_* \circ \phi_*$$

This follows from the chain rule for derivatives.

Action on curves: If you think of a vector $v \in T_p M$ as the tangent to a curve $\gamma : (-\epsilon, \epsilon) \rightarrow M$ with $\gamma(0) = p$, then $\phi_* v$ is the tangent vector to the curve $\phi \circ \gamma$ at $\phi(p)$.

The pushforward $\phi_* v \in T_{\phi(p)} N$ is given by:

$$\phi_* v = v^i \frac{\partial \phi^j}{\partial x^i} \frac{\partial}{\partial y^j}$$

Here, v^i are the components of v , and $\frac{\partial \phi^j}{\partial x^i}$ are the Jacobian entries. The result is a vector in $T_{\phi(p)} N$, expressed in the basis $\left\{ \frac{\partial}{\partial y^1}, \dots, \frac{\partial}{\partial y^n} \right\}$.

19.1.11 Pull-Back in k-forms

Given a smooth map $\phi : M \rightarrow N$ between manifolds M (dimension m) and N (dimension n), and a k -form $\omega \in \Omega^k(N)$ on N , the pullback $\phi^* \omega \in \Omega^k(M)$ is a k -form on M . The pullback essentially "transfers" ω from N to M by composing it with the map ϕ . The pullback is defined pointwise. For a point $p \in M$, and k tangent vectors $v_1, \dots, v_k \in T_p M$, the pullback $\phi^* \omega$ at p is:

$$(\phi^* \omega)_p(v_1, \dots, v_k) = \omega_{\phi(p)}(\phi_* v_1, \dots, \phi_* v_k)$$

Here:

$\phi_* : T_p M \rightarrow T_{\phi(p)} N$ is the pushforward (differential) of ϕ , which maps each tangent vector v_i to $\phi_* v_i$. $\omega_{\phi(p)} \in \wedge^k T_{\phi(p)}^* N$ is the k -form ω evaluated at $\phi(p)$, acting on the pushed-forward vectors.

This definition ensures that $\phi^*\omega$ is a k -form on M , as it takes k vectors in T_pM and produces a number.

Key Properties of the Pullback

Linearity: The pullback $\phi^* : \Omega^k(N) \rightarrow \Omega^k(M)$ is a linear map.

Preserves degree: The pullback of a k -form is a k -form.

Functoriality: For maps $\phi : M \rightarrow N$ and $\psi : N \rightarrow P$,

$$(\psi \circ \phi)^* = \phi^* \circ \psi^*$$

Commutates with exterior derivative: For any k -form ω ,

$$d(\phi^*\omega) = \phi^*(d\omega)$$

This is a powerful property, making pullbacks compatible with the exterior calculus.

Wedge product: The pullback respects the wedge product:

$$\phi^*(\omega \wedge \eta) = (\phi^*\omega) \wedge (\phi^*\eta)$$

Computing the Pullback in Coordinates To compute $\phi^*\omega$ in practice, we typically use local coordinates. Here's the step-by-step process:

Choose coordinates:

On M , use coordinates (x^1, \dots, x^m) around p .

On N , use coordinates (y^1, \dots, y^n) around $\phi(p)$.

The map ϕ is expressed as $y^i = \phi^i(x^1, \dots, x^m)$.

Express the k -form:

Let $\omega \in \Omega^k(N)$ be written in coordinates as:

$$\omega = \sum_I a_I(y) dy^{i_1} \wedge \dots \wedge dy^{i_k}$$

where $I = (i_1, \dots, i_k)$ is a multi-index with $1 \leq i_1 < \dots < i_k \leq n$, and $a_I(y)$ are smooth functions on N .

Pull back the form:

The pullback is:

$$\phi^* \omega = \sum_I (a_I \circ \phi) \phi^*(dy^{i_1} \wedge \dots \wedge dy^{i_k})$$

Compute $a_I \circ \phi$: Replace y with $\phi(x)$, so $a_I(y) = a_I(\phi(x))$.

Compute the pullback of the basis k -form:

$$\phi^*(dy^{i_1} \wedge \dots \wedge dy^{i_k}) = \phi^*(dy^{i_1}) \wedge \dots \wedge \phi^*(dy^{i_k})$$

For a 1-form dy^j , the pullback is:

$$\phi^*(dy^j) = d(y^j \circ \phi) = d(\phi^j(x)) = \frac{\partial \phi^j}{\partial x^\ell} dx^\ell$$

(using the chain rule, summing over ℓ). Thus:

$$\phi^*(dy^{i_1} \wedge \dots \wedge dy^{i_k}) = \left(\frac{\partial \phi^{i_1}}{\partial x^{\ell_1}} dx^{\ell_1} \right) \wedge \dots \wedge \left(\frac{\partial \phi^{i_k}}{\partial x^{\ell_k}} dx^{\ell_k} \right)$$

This is a k -form on M , expressed in the dx^ℓ basis.

Simplify:

Expand the wedge products and collect terms. The result is a sum of terms involving $dx^{\ell_1} \wedge \dots \wedge dx^{\ell_k}$, with coefficients that depend on the Jacobian $\frac{\partial \phi^j}{\partial x^\ell}$ and $a_I(\phi(x))$.

19.1.12 Pull-Back of volume forms

The pullback of a volume form $\omega = f(y) dy^1 \wedge \dots \wedge dy^n$ on N under a map $\phi : M \rightarrow N$ is an n -form on M , computed

as:

$$\phi^*\omega = (f \circ \phi) \cdot \det \left(\frac{\partial \phi^i}{\partial x^{j_k}} \right) dx^{j_1} \wedge \cdots \wedge dx^{j_n}$$

If $\dim M = \dim N$, and ϕ is a local diffeomorphism, $\phi^*\omega$ is a volume form on M . If $\dim M < \dim N$, the pullback is zero. The Jacobian determinant captures the volume scaling, making this operation central to integration and geometric transformations.

19.1.13 Integration on Manifolds Using the Pullback of Volume Forms

The pullback of volume forms allows for integration of manifolds. Which is useful in the integration of curved coordinate systems.

volumes generalize "dx" or "dxdy."

The steps in integration of manifolds using volume forms is:

Step 1: M : The domain of integration (often \mathbb{R}^n or a simpler manifold). N : The target manifold with a volume form ω . $\phi : M \rightarrow N$: A smooth map, typically a parametrization or diffeomorphism. $f : N \rightarrow \mathbb{R}$: The function to integrate (if $f = 1$, we're computing the volume).

Express the volume form:

Write $\omega = f_\omega(y) dy^1 \wedge \cdots \wedge dy^n$ in coordinates on N .

Compute the pullback:

Compute $\phi^*\omega$, which involves:

Substituting $y = \phi(x)$ into $f_\omega(y)$. Computing the Jacobian determinant of ϕ . Forming $\phi^*\omega = (f_\omega \circ \phi) \cdot \det \left(\frac{\partial \phi^i}{\partial x^j} \right) dx^1 \wedge \cdots \wedge dx^n$ (if $m = n$).

Set up the integral:

The integral becomes:

$$\int_{\phi(M)} f \omega = \int_M (f \circ \phi) \cdot \phi^* \omega$$

In coordinates, if $\phi^* \omega = g(x) dx^1 \wedge \cdots \wedge dx^n$, and M corresponds to a region $U \subset \mathbb{R}^n$:

$$\int_U (f \circ \phi)(x) \cdot g(x) dx^1 \cdots dx^n$$

Evaluate the integral:

Perform the integral over U using standard multi-variable calculus techniques, accounting for the Jacobian determinant and any orientation changes.

19.1.14 Line Integrals on Manifolds

Parameterize the curve: Define the curve $\gamma(t) = (x(t), y(t), \dots)$ for $t \in [a, b]$.

Define the one-form: Let the one-form be $\omega = f(x, y, \dots)dx + g(x, y, \dots)dy + \dots$

Calculate the pullback: Substitute the coordinate functions $x(t), y(t), \dots$ and their differentials into the one-form. This gives the pullback one-form $\gamma^* \omega$ on the interval $[a, b]$. For example, if $\omega = f(x, y)dx + g(x, y)dy$ and $\gamma(t) = (x(t), y(t))$, the pullback is $\gamma^* \omega = (f(x(t), y(t)) \frac{dx}{dt} + g(x(t), y(t)) \frac{dy}{dt}) dt$.

Integrate the pullback: Integrate the resulting expression with respect to t from a to b .

This gives the value of the line integral:

$$\int_{\gamma} \omega = \int_a^b \left(f(x(t), y(t)) \frac{dx}{dt} + g(x(t), y(t)) \frac{dy}{dt} \right) dt$$

19.1.15 Classical Stokes Theorem in Manifolds

19.2 Symplectic geometry classical mechanics

19.2.1 Lecture 1

Symplectic doesn't work well in dissipative structures.

It also goes over the definition of local euclidian space, which is pretty basic.

They also go over how you can make an object unique by maxionable defining coordinate systems.

This is given by an atlas of coordinate charts

To get a differntiable structure, an atlas is described through coordinate charts $(U_{\alpha}, \phi_{\alpha})$ such that:

Each $\psi_{\alpha} : U_{\alpha} \rightarrow R^n$ is a homeimorphism Also, on overlaps between $U_{\alpha} \cap U_{\beta}$ it is continously differentiable.

19.2.2 Lecture 2

They go over catogories in a brief overview, and that this lecture will be going over a map of differential geometry.

So, let us start with a manifold: $f(p) \in N$

Now there isn't much we can say about this basic manifold, though let's add a bit more context. Say that manifolds seem like flat-space(euclidian) locally.

So we can take the charts in the manifold to project euclidian space upon it to differentiate.

When k is ∞ f is smooth.

Chapter 20

Coding

20.1 *Linux Terminal

20.2 *Github

20.3 *FORTRAN

20.4 *Python

Part III

Research

Chapter 21

Introduction

Chapter 22

Previous Research

These are brief/lazy copies/explanations of things I have written previously made projects. Copied from my last book Logos

22.1 3D Modeling of Non-Equilibrium Dynamics in Compressed Plasma Using Lattice Boltzmann Method

The construction of a theoretical model, magnetohydrodynamic lattice boltzmann method(MHD-LBM) model for 3D compressed plasma, using a finite volume scheme is constructed. The hyperbolic Maxwell equations, which satisfy the elliptic constraints of Maxwell's equations and the constraint of charge conservation, are used to simulate the electromagnetic field. The flow field and electromagnetic field are coupled to simulate a compressible plasma through the electromagnetic force and magnetic induction equations. This model can further be applied to create a quantitative simulation to model

complex nonequilibrium effects of compressed plasma to provide mesoscopic physical insights into the flow mechanism of a shock wave in a supersonic plasma.

Theoretical model, Finite volume scheme, hyperbolic maxwell equations, and nonequilibrium effects

22.1.1 3D Modeling of Non-Equilibrium Dynamics in Compressed Plasma Using Lattice Boltzmann Method Introduction

The study of nonequilibrium dynamics in compressible plasma is a critical area of research in plasma physics, with significant implications for both theoretical understanding and practical applications. Compressible plasmas are found in a variety of contexts, such as astrophysical phenomena(1), industrial processes(2,3), and fusion. Understanding the complex interactions and behaviors of plasma under nonequilibrium conditions is essential for advancing these fields.

Previous research has primarily focused on two-dimensional models(4) While these models have provided valuable insights, they often fall short in capturing the full complexity of three-dimensional plasma dynamics. The limitations of these models highlight the need for more advanced simulation techniques that can accurately represent the intricate behaviors of compressible plasma in three dimensions.

The Lattice Boltzmann Method (LBM) offers a powerful tool for modeling fluid dynamics at the microscopic level.(4,5) Unlike traditional computational fluid dynamics methods, such as the Navier-Stokes equation, the LBM is particularly well-suited for simulating

22.1 3D Modeling of Non-Equilibrium Dynamics in Compressed Plasma Using Lattice Boltzmann

Method 69
~~nonequilibrium effects and complex boundary conditions.~~ Its ability to handle multiphase flows and incorporate microscopic interactions makes it an ideal choice for studying compressible plasma dynamics.(5)

This paper aims to develop a comprehensive 3D model using the LBM to simulate nonequilibrium effects in compressible plasma. By leveraging the strengths of the LBM, we seek to overcome the limitations of previous models and provide a more accurate and detailed representation of plasma behavior. Our research focuses on the coupling of flow and electromagnetic fields, exploring the interactions and dynamics that arise under nonequilibrium conditions.

22.1.2 Physical Model

Kinetic Equation

Li and Zhong(10) introduced the potential energy distribution function as well as a compressed DDF Lattice Boltzmann equation. A potential energy distribution function can be added so the Boltzmann BGK can obtain an adjustable specific heat ratio or Prandtl number(4)

This new Boltzmann Kinetic equation can be written as followed:

$$\frac{\partial f_k}{\partial t} + (\mathbf{e}_k \cdot \nabla) f_k + \mathbf{a} \cdot \nabla_e f_k = -\frac{1}{\tau_f} (f_k - f_k^{eq})$$

$$\frac{\partial h_k}{\partial t} + (\mathbf{e}_k \cdot \nabla) h_k + \mathbf{a} \cdot \nabla_e h_k = -\frac{1}{\tau_h} (h_k - h_k^{eq}) + \frac{z_k}{\tau_{hf}} (f_k - f_k^{eq})$$

Where k is the direction of discrete velocity. f_k is the density distribution function. h_k is the potential energy distribution function, while f_k^{eq} is the equilibrium distribution function. e_k is the discrete velocity component, and τ_f is the relaxation time of the density distribution function. τ_h is the relaxation time of the potential energy distribution function.

This can also be defined as:

$$\tau_{fh} = \frac{\tau_f}{\tau_h}$$

The force term can be approximated as:

$$\mathbf{a} \cdot \nabla_e f \approx \mathbf{a} \cdot \nabla_e f^{eq} = -\frac{\mathbf{a} \cdot (\mathbf{e}_k - \mathbf{u})^2}{RT} f^{eq}$$

22.1.3 Density Distribution Function

$$\begin{aligned} f_k^{n+1}(x_i, y_j, z_l) = f_k^n(x_i, y_j, z_l) &- \Delta t \left(\frac{F_{k,i+1/2,j,l} - F_{k,i-1/2,j,l}}{\Delta x} + \frac{F_{k,i,j+1/2,l} - F_{k,i,j-1/2,l}}{\Delta y} \right. \\ &\left. - \Delta t \cdot \frac{1}{\tau} (f_k - f_k^{eq}) \right) \end{aligned}$$

22.1.4 Potential Energy Distribution

$$\begin{aligned} h_k^{n+1}(x_i, y_j, z_l) = h_k^n(x_i, y_j, z_l) &- \Delta t \left(\frac{G_{k,i+1/2,j,l} - G_{k,i-1/2,j,l}}{\Delta x} + \frac{G_{k,i,j+1/2,l} - G_{k,i,j-1/2,l}}{\Delta y} \right. \\ &\left. - \Delta t \cdot \frac{1}{\tau_h} (h_k - h_k^{eq}) + \frac{z_k}{\tau_{hf}} (f_k - f_k^{eq}) \right) \end{aligned}$$

22.1 3D Modeling of Non-Equilibrium Dynamics in Compressed Plasma Using Lattice Boltzmann

Method 22.1.5 Hyperbolic Maxwell Equations 71

The previously used Maxwell equations have been enhanced with Lagrange multipliers, Ψ and Φ , to combine with the evolution equation. These are the new Maxwell equations in derivative form:

$$\begin{aligned}\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} + \gamma \nabla \psi &= 0 \\ \frac{\partial \mathbf{E}}{\partial t} - c^2 \nabla \times \mathbf{B} + \chi c^2 \nabla \phi &= -\frac{\mathbf{J}}{\epsilon_0} \\ \frac{\partial \Psi}{\partial t} + \gamma c^2 \nabla \cdot \mathbf{B} &= 0 \\ \frac{\partial \Phi}{\partial t} + \chi \nabla \cdot \mathbf{E} &= 0\end{aligned}$$

Where \mathbf{E} is the electric field, \mathbf{B} is the magnetic field, Ψ and Φ are introduced divergence variables, γ and χ are divergence error propagation speeds, c is the speed of light, and ϵ_0 is the vacuum permittivity.

22.1.6 Kinetic Non-Equilibrium Method

The nonequilibrium effects on compressed plasma are observed through differences in molecular speed. Each non-equilibrium kinetic moment can be represented as the difference between the corresponding kinetic moment and the local equilibrium kinetic moment. The equations of kinetic moments may be used to find the nonequilibrium quantities.

$$\begin{aligned}M_{f,m}^{neq} &= M_{f,m} - M_{f,m}^{eq} \\ M_{h,m}^{neq} &= M_{h,m} - M_{h,m}^{eq}\end{aligned}$$

22.1.7 Discussion

In this paper, a 3D MHD-LBM analytical model was constructed for compressed kinetic plasma. Future research must be done in developing a computer model and validating said computer model. A third-order finite volume MUSCL could be applied to this analytical model combined with a D3Qx density equilibrium distribution function to create an easily solvable numerical solution. While electromagnetic fluxes were evaluated through Steger-Warming flux vector splitting. MHD-LBM models have shown to have greater accuracy when it comes to plasma shockwaves, therefore MHD-LBM models have good reason for continual development.

22.2 A Hamiltonian Framework on ICF Implosions Rocket Equation Based on Rayleigh–Taylor Instabilities

22.2.1 Background

Inertial Confinement Fusion (ICF) Implosions: One of the primary methods of heating fusion environments is through lasers. In this approach, high-energy lasers heat spherical fuel capsules, causing them to implode. This implosion leads to an increase in pressure and heat on the fuel, creating the necessary environments for fusion.

Hamiltonian: A Hamiltonian is a function in classical mechanics that describes the total amount of energy in the system. From here, equations of motion are

Rayleigh–Taylor Instabilities: Rayleigh–Taylor instabilities are specific types of instabilities caused by forceful interactions between higher and lower density fluids. In the context of ICF Implosions, when the higher density outer shell interacts with the lower-density inside, it creates perturbations through finger-like structures and bubbles that interfere with the efficiency of the implosion.

Mass Ablation (Rocket Effect): During the process of ICF Implosion, the interaction between the outer shell and the inner fuel causes mass to be sprayed off, giving a rocket-like effect.

22.2.2 Problem Statement

Achieving commercial fusion energy through inertial-confinement-fusion (ICF) remains one of the most significant scientific and engineering challenges of our day. One of the key obstacles is optimizing the interactions between high-energy lasers and fuel capsules, which are prone to Rayleigh–Taylor instabilities during implosions. These instabilities can lead to inefficient energy confinement and hinder the overall success of the fusion process. Traditional numerical simulations are computationally expensive and often lack the accuracy needed to address this issue. Therefore, there is a need for a robust analytical framework to model these instabilities and provide new insights for optimizing the interactions. This research aims to develop a Hamiltonian framework for ICF implosions, specifically focusing on Rayleigh–Taylor instability and mass ablation. By leveraging this theoretical approach, we seek to provide deeper insights and practical solutions

for optimizing laser-capsule interactions in fusion experiments.

22.2.3 Assumptions

Thin-Shell Approximation: The thickness of the shell is assumed to be insignificant compared to the size of the imploding object. This approximation simplifies the creation of the model and the subsequent numerical analysis. It is valid given that the perturbations caused by the instabilities are much larger than the actual thickness of the shell.

Deceleration: This model is specifically for an imploding spherical shell that decelerates as it converges onto the compressed fluid within its interior.

Acceleration: This model does not consider the acceleration phase of an ICF capsule implosion at the beginning of the capsule-laser interaction.

22.2.4 Framework

In order to analyze the dynamics of the imploding shell, it must first be parameterized through two Lagrangian coordinates ϑ and ϕ . Here, ϑ corresponds to the polar angle θ , and ϕ corresponds to the azimuthal angle. The position vector is $\mathbf{X} = \mathbf{X}(t, \phi, \vartheta)$. Thus, the parameterization can be expressed as:

$$\mathbf{X} = \mathbf{X}(t, \phi, \vartheta)$$

The derivative of the surface is given by:

$$\frac{d\mathbf{X}}{dt}$$

22.2 A Hamiltonian Framework on ICF Implosions Rocket Equation Based on Rayleigh–Taylor Instabilities

75

~~Reference: D. E. Ruiz; Degradation of performance~~
in ICF implosions due to Rayleigh–Taylor instabilities:
A Hamiltonian perspective. Phys. Plasmas 1 December
2024; 31 (12): 122701.

22.2.5 Shell Kinematics

The force differential can be described by:

$$dF = p(t) \cdot d\mathbf{A} \times \mathbf{n}$$

where $p(t)$ is the function of pressure with respect to time, and the cross product involves the vectors following the surface of the shell. This leads to the derivation of several important quantities, such as the velocity vector field, position vector, and centrifugal forces, which are relevant to the shape and dynamics of the shell.

An example of the centrifugal force is:

$$F_{\text{centrifugal}} = m \cdot \omega^2 \cdot r$$

22.2.6 Shell Areal Density

The change in density of the shell is critical to its dynamics. Given that total mass is not conserved in this equation, deriving it becomes more complex.

$$\rho_{\text{areal}} = \frac{M(t)}{A}$$

where $M(t)$ is the mass at time t , and A is the area of the shell's surface.

22.2.7 Compressed-Fuel Pressure

The pressure that decelerates the shell can be described in several ways depending on the known information and boundary conditions. One example is:

$$P = P_0 \left(\frac{V_0}{V} \right)^\gamma$$

where P_0 is the initial pressure, V_0 is the initial volume, γ is the adiabatic index, and V is the current volume.

22.2.8 Variational Principles

To mathematically solve the Hamiltonian framework, given that the equation is asymptotic, variation principles are applied.

22.2.8.1 Phase-Space Lagrangian

This formulation describes the dynamics of a system by combining its configuration space (position variables) and momentum space into a single framework, called phase space.

Euler-Lagrange Equations

Through the principle of least action, a set of differential equations are derived to provide an analytical solution to complex nonlinear functions. For example, for radial velocity:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{r}} \right) - \frac{\partial L}{\partial r} = 0$$

22.2 A Hamiltonian Framework on ICF Implosions Rocket Equation Based on Rayleigh–Taylor Instabilities

77

where L is the Lagrangian of the system.

22.2.8.2 Conservation Laws

Generally, conservation laws are employed to show that certain parts of the Euler-Lagrange or phase-space Lagrangian are conserved. However, due to mass ablation, parts of the fuel and capsule are lost, which complicates these conservation laws.

22.2.9 Conclusion

Summary: A Hamiltonian framework was developed to include both Rayleigh–Taylor Instabilities and mass ablation in an inertial confinement fusion implosion, leading to a set of variational principles for future work on optimizing ICF implosions in practical applications.

Significance: Analytical analysis can provide future insights into optimizing ICF implosions in fusion reactors. Additionally, nonlinear numerical models can be developed based on these principles, leading to less computationally expensive models.

Limitations:

1. This model does not account for more complex laser-plasma interaction phenomena (e.g., bremsstrahlung, x-ray losses, alpha heating).
2. This model does not consider the initial acceleration phase of the interaction.
3. This model uses the thin-shell approximation, which may not be accurate when perturbations caused by instabilities are smaller than the shell thickness.

Future Works:

1. Quasilinear Models To study additional insights from this model, a quasilinear model must be developed for analytical study.
2. Acceleration Phase Incorporating the initial acceleration phase into the model will increase its accuracy.
3. Nonlinear Growth Calculations Developing nonlinear numerical simulations will be useful in generating further insights and advancing current models.

22.3 Basic MHD

22.3.1 MHD Module

The first section of it is initializing the values of things like magnetic field, pressure, and velocity. Future work will be done to have it so such values can be edited in a config file.

Next is computing the current density. The analytical equation can be written as

$$J = \frac{1}{\mu} \nabla \times B$$

Which in finite difference is:

$$J_z(i, j) = \frac{1}{\mu_0} \left[\frac{B_y(i+1, j) - B_y(i-1, j)}{2\Delta x} - \frac{B_x(i+1, j) - B_x(i-1, j)}{2\Delta y} \right]$$

Next we have

Update: I have advanced the MHD simulations time steps, magnetic field calculations, current, pressure calculations, and density calculations. Beyond that I have created animations based on magnetic field and made graphical outputs for heat, velocity, temperature, and magnetic field.

I also created another one specifically made for Tokamak geometry.

22.3.2 Hamiltonian MHD

My next project was creating a Hamiltonian based MHD simulation also in FORTRAN.

I explicitly created simulation of plasma thrusters with it.

22.4 A Metriplectic Formulation of Reduced Magnetohydrodynamics with Resistivity

22.4.1 Introduction

Here I will use the Strauss formulation of RMHD. Then I will incorporate resistivity. This resistivity typically breaks Hamiltonian structure, but I will use metriplectic to preserve the geometric identities through adding a separate antisymmetric bracket.

22.4.2 Defining the Hamiltonian

For Strauss MHD, a reduced MHD model that works for strongly magnetized Plasmas, we will formulate the Hamiltonian system.

$$\mathcal{H}[\omega, \psi] = \frac{1}{2} \int (\phi\omega + \psi j) dx dy$$

Here the $\phi\omega$ represents the kinetic energy with ψj represents the magnetic energy.

Now onto the Noncanonical Poisson Structure for this ideal Hamiltonian.

$$\{F, G\} = \int \omega \left[\frac{\delta F}{\delta \omega}, \frac{\delta G}{\delta \omega} \right] dx dy + \int \psi \left(\left[\frac{\delta F}{\delta \omega}, \frac{\delta G}{\delta \psi} \right] - \left[\frac{\delta G}{\delta \omega}, \frac{\delta F}{\delta \psi} \right] \right) dx dy$$

22.4.3 Entropy and Dissipation

Now to add dissipation. While generally adding this will break Hamiltonian structure, we will use a couple of tools.

Though first we must define our function we are using, or more accurately functional. This is our functional for entropy:

$$S[\psi] = \frac{1}{2} \int \psi^2 dx dy$$

22.4.4 Metric Bracket and Metriplectic Structure

To incorporate we define our symmetric bracket. We make sure to construct it so that it only affects the

magnetic flux variable ψ , since resistivity only affects the magnetic field lines and not vorticity directly.

$$(F, G) = \int \frac{\delta F}{\delta \psi} \eta \nabla^2 \frac{\delta G}{\delta \psi} dx dy$$

This has

- Symmetry $((F, G)) = ((G, F))$
- Positive semi-definiteness: $((S, S)) \leq 0$
- Energy Conservation: $((\mathcal{H}, F)) = 0$ for any F

22.4.5 Combined Dynamics

22.5 Death Star

Not to long ago(from when I wrote this section) I tried to simulate the death star using FLASH-X(which is really a testament to who I am as a person, getting my hands on government codes and the fist thing I do is play around with pop-culture based planetary destruction) anyways. It failed. The laser dynamics were based upon old code that was outdated and I couldn't figure it out. Though I did learn a lot about Flash-X as a system, which is good.

So for it... what am I doing, this doesn't make sense if I am trying to teach myself through practical action of writing but I already did the most practical action of all, doing it. This is kinda useless.

I later went back and ended up actually making the simulation.

22.6 Interstellar travel

22.6.1 Speeds and forces

Before we can get to all of the more complex stuff about design and the effects of extreme environments of specific interstellar space travel we must first look at how forces effect speeds at relativistic speeds. Here is the math stuff in simplified form:

We all know that force is the derivative of the momentum over time:

$$F = \frac{dp}{dt}$$

Where momentum can be expand for relativistic momentum:

$$p = \gamma mv$$

You can then expand force to be (this is a constant mass, we will look at that in a second)

$$F = m \frac{d(\gamma v)}{dt}$$

One thing to keep in mind for future maths is that in simplifying $\dot{\gamma}$ is equal to $\frac{\gamma v}{c^2} \frac{dv}{dt}$ With the force you can calculate acceleration with

$$a = \frac{F}{m\gamma} \frac{1}{1 + \frac{\gamma^2 v^2}{c^2}}$$

Due to the fact most rockets use propulsion which lowers the mass, we must include mass variations:

$$F = \frac{\gamma mv}{dt}$$

Where you must use the product rule to expand

$$F = \dot{\gamma}mv + \gamma\dot{m}v + \gamma m\dot{v}$$

Which can be further expanded to

$$F = mv\frac{\gamma v}{c^2}\frac{dv}{dt} + \gamma v\frac{dm}{dt} + \gamma m\frac{dv}{dt}$$

With \dot{m} being the infinitesimal change in mass over time.

To calculate acceleration you must find $\frac{dv}{dt}$. First subtract the middle equation without acceleration within it.

$$F - \gamma v\frac{dm}{dt} = mv\frac{\gamma v}{c^2}\frac{dv}{dt} + \gamma m\frac{dv}{dt}$$

Then factor for acceleration

$$F - \gamma v\frac{dm}{dt} = \frac{dv}{dt}(mv\frac{\gamma v}{c^2} + \gamma m)$$

Then divide the parentheses:

$$(F - \gamma v\frac{dm}{dt})\frac{1}{mv\frac{\gamma v}{c^2} + \gamma m} = a$$

Now, this is all well and good, now let us find the position. To take position from acceleration you must take a double integral, but this integral can only be solved numerically(maybe it could be approximated)

Another possible way to solve this is through Hamiltonians/Lagrangians (I will choose Hamiltonians)

Let us first start with the Hamiltonian expressed for relativistic speeds in covariant form

First the Lagrangian must be calculated: The four-

momentum is

$$P^\mu = m\gamma(c, v)$$

Energy:

$$E = \gamma mc^2$$

Therefore the Lagrandian is:

$$L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}}$$

Now to find generalized momentum:

$$p = \frac{\partial L}{\partial v}$$

$$p = \gamma mv$$

Now the Hamiltonian

$$H = p \cdot v - L$$

Therefore a relativistic Hamiltonain can be represented as

$$H = \sqrt{(pc)^2 + (mc^2)^2}$$

Now to include acceleration

$$H(r, p) = \sqrt{(pc)^2 + (mc^2)^2} + V(r)$$

Now to make mass variable for the rockets:

$$H(r, p, t) = \sqrt{(p(t)c)^2 + (m(t)c^2)^2} + H_{thrust}$$

Now to incorporate with equations of motion:

$$\frac{dr}{dt} = \frac{pc^2}{\sqrt{(p(t)c)^2 + (m(t)c^2)^2}}$$

Now, this works for the time for most people, now let us look at proper time

$$H(r, p, \tau) = \sqrt{(p(\tau)c)^2 + (m(\tau)c^2)^2} + H_{thrust}$$

For specific cases, these equations will be evolved to include them, but this is the general theoretical framework.

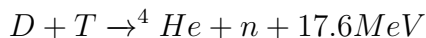
One thing I will add, is the force will likely be variable, a couple of days of 2-3 g, most of the time in g then reversing after the half-way point.

22.6.2 Magnetic Fusion Plasma Drive

22.6.2.1 Theoretical Basis

Magnetic Fusion Plasma Drive is a type of propulsion system specifically for interstellar travel.

The basis of this concept is that there is a fusion reactor undergoing Thermal Nuclear fusion:



The spent fuel then being shot out the nozzle as propulsion.

22.6.3 *Optimization

22.6.4 *Special Circumstances

22.6.4.1 *Strong Magnetic Fields

22.6.4.2 *Gravitational Fields

22.7 Kerr Black hole stuff

22.7.1 Energy (finish)

Kerr Black holes, or spinning black holes are a great source of energy. They have extreme amounts of radial kinetic energy that we are actually capable of 'bleeding' off through several methods.

First, we must ask why we are able to do this. Well, because a rotating object with a strong gravitational field 'drags' spacetime, that spacetime can affect the direction, speed, and even waves in a way that can amplify them.

First, Penrose(really like that dude) found that if you break apart an object rotating a Kerr black hole, the object that escapes gains energy. This process is challenging given that the object that escapes must have a velocity of $v > \frac{c}{2}$. There is a whole lot of math involved in proving this, but I will skip ahead.

The next that is found is that electromagnetic waves can be amplified. That if they are reflected around the black hole, they will gain energy and be amplified(this process is different from the Doppler shift and does not change the frequency.

After a lot, and I mean a lot of very complected math-

ematics you can find the amplification factor as

$$Z = 8r_+^2 T_{H(2l+1)} (r_+ - r_-)^{2l} \left[\frac{\Gamma(1+l-s)\Gamma(1+l+s)}{2l+1)!!\Gamma(l+1)\Gamma(2l+1)} \right]^2 \times, \\ \sinh\left(\frac{m\Omega_H}{r_+ T_H}\right) \Gamma\left(l - \frac{im\Omega_H}{\pi r_+ T_H} + 1\right) \Gamma\left(l + \frac{im\Omega_H}{\pi r_+ T_H} + 1\right)$$

For more information visit [here](#)

Now the point of this is to take strange theoretical and apply it, so I will apply it for actual application. For this I will assume the average mass of $10 M_\odot$ or ten solar masses. A spin of 0.98(if 1 refers to an extremal Kerr black hole)

First we must find out what a spin 0.98 really means for a $10M_\odot$ black hole. We can find this with a simple equation with a being the spin parameter, r being the radius, and M being the mass

$$\Omega = \frac{ac^3}{2GM r_h}$$

Though, for a kerr black hole the is slightly different to the swarzschild radius.

$$r_H = \frac{GM}{c^2} \left(1 + \sqrt{1 - \frac{ac}{GM}} \right)$$

You know, I wanted to optimize the black hole engine, but I don't want to do that anymore, so I choice to move on.

22.7.2 *Multiversal Travel

22.7.3 *Other effects

22.8 *Extreme Theoretical Work

22.9 Astroid Game

22.9.1 Introduction

I have recently created a Python game based on the Atari game Asteroid.

22.9.2 Classical

Just like the regular game, nothing special.

22.9.3 Newtonian Gravity

This adds that the asteroids and UFO's mass, so they attract each other and add more challenges.

22.9.4 Dark Matter

Add the challenge of invisible mass attractors, though when you are close by they light up.

22.9.5 Relativistic

This adds time dilatation(time skips or slow downs), length contractions(changing shapes of objects), black holes, and even doopler effect.

Chapter 23

The Structure of Motion: A Philosophical Journey Through Hamiltonian Mechanics

“The Hamiltonian formalism of mechanics, especially in its canonical form, has been an inexhaustible source of inspiration for modern theoretical physics.” — Albert Einstein

23.1 Introduction

This chapter is planned to be an extreme examination of Hamiltonian mechanics, especially from a philosophical, logic, and abstract way. Eventually, it will go from the basic logic and axioms underlying it, to the metaphysics and epistemology that are derived, and finally to advanced and modern usages. The primary purpose of this exercise is for me to find the gaps in my

The Structure of Motion: A Philosophical Journey 90 Through Hamiltonian Mechanics

knowledge and gain a more intuitive and deep understanding of these concepts. (It was also written in 10th grade before the rest of the book so may not have the same standards.)

The reasoning for this is due to the extremely complex and abstract nature of such mechanics. To see the world through 'flows' instead of forces.

23.1.1 Physical Reality and Frameworks

Now, there are many theories, models, descriptions, framework, and so much more. Though, what really differentiates them? What makes one thing one and the other the other. While I won't truly go into extreme linguistic detail for every single one, I will go into moderate detail within the art to get the general idea down.

Let's focus on frameworks, because that is what a Hamiltonian is (others define it as schemes which is another great word and arguably more accurate, but I will refer to it as a framework because it has a more intuitive grasp). It is an entirely different framework than Newtonian. It comes from an entirely different perspective, looking into geometric identities rather than the causal relation of forces and such interactions. Also, other theories and identities are derived through it, independent of the actual reality of it. For instance, the flow of electromagnetic interactions, relativistic, or even gravitational are all derived from this concept of the flow of energies through sympathetic geometry.

Even beyond that, Hamiltonian mechanics can represent any function that changes; statistics, viruses, and many others. This is due, to that Hamiltonian's is a pure mathematic relational concept rather than

based upon physical axioms. For instance, Newtonian mechanics is based upon the axiom of inertia. Though, I will add one interesting involvement is the idea of privileging Hamiltonian over Lagrangian and stating that Lagrangian is a subset of Hamiltonian rather than its own framework.

Then this begins to ask our questions of scientific realism vs anti-realism (I found out the more neutral language is instrumentalism). Basically, what makes such theories more fundamentalist, predictive modeling or metaphysical truth. Further, this now engages the complexities of mathematical axiomatic reduction or physical.

Let's start with realism vs instrumentalism. I have already made my stance clear, having metaphysical truth in important but not in the way that non-metaphysical schemes should be thrown away. Though, this then brings about the question of what frameworks have greater metaphysical truth, but I will later look into this in later sections.

This now leads to the question of mathematical abstraction vs physical realism. Though, one thing I will add is that the physical axioms of Newton's laws are based upon falsified ideas like absolute space and time (but you can get around such semantics by employing Whewell's axioms of Mechanics [1st: "Every change is produced by a cause." 2nd says that: "Causes are measured by their effects." Finally, the third remain unchanged from Newton's formulation.] Now back to the main event, I feel that the structuralism of Hamiltonian is better due to the fact that when you break down physics to the quantum the classical forces, inertia, and such break down and you can see that the structure is what remains. That while forces still interact as abstract

The Structure of Motion: A Philosophical Journey 92 Through Hamiltonian Mechanics

'causes of change" through bosons, their classical and 'physical' realism fails to encapsulate their extreme complexity.

23.1.2 Philosophical Context*

The Kantian influence on Sir Hamilton is clear....

23.1.3 Notion of Time, States, and Trajectories

Let's start with time. I have already explained my thoughts on time, but here I will deepen my explanation. Time, is the factor on which states evolve. It is then evolution of entropy. This axiom is very simple and intuitive. While objective empirical proving the nature of time is extremely challenging due to its abstract nature. For a more in-depth analysis of the nature of time visit Chapter 1, section 3&7.

One thing I will discuss is time reversibility. Now, Hamiltonian mechanics works best for time reversal symmetries(it can still work in some use cases but rarely.). In fact, time reversal symmetry is a corner stone of much of modern physics, especially quantum mechanics, but this interferes with our intuitive grasp of reality, the second law of thermal dynamics, and even dark energy; in essence the emergent properties of reality seem to interfere with our 'fundamental mathematical' derivations. So I will examine these questions; both for their own state and because it continues our question of the fundamental nature of reality. I will also attempt to discuss imaginary time with regards to quantum mechanics.

Actually, upon further thought the nature of time will be further developed through after the derivation of analytical mechanics.

Next will be on trajectories; now like many other ideas presented within this book, that may seem simple but the derivation of the ideas will be thorough and show its true complexity. Within Hamiltonian mechanics, the trajectories hold special geometric identities due to the nature of motion, but what can we derive from this simple observations

23.1.4 Introduction to Phase Space

Another question that arises from a Hamiltonian view of metaphysics is Phase Space. The fact that, according to our theories, phase space almost seems real, but yet it is so very different than what we observe. Or, we think so.

Obviously, there is the idea of the conflict between the teleology that comes as a consequence and our view of causality. I will address this in a second.

A simpler idea is that, from a perspective view, change becomes fundamental and any identity becomes simply emergent. This isn't truly new, simply more emergent in phase space, so I will move on.

One new thing, is that Phase Space allows for entropy irreversibility. Traditional theories find time reversibility to be a corner stone, this going against it.

This is due to the fact that entropy increases

$$S = -k_B \sum_i p_i \log p_i$$

The Structure of Motion: A Philosophical Journey 94 Through Hamiltonian Mechanics

Where the volume of the phase space stays the same

$$Vol(\phi_t(\Omega_M)) = Vol(\Omega_M)$$

Lastly, the 6D phase space seems very different to ours. Having generalized position and momentum coordinates is odd. An instrumentalist view makes this simple, but a scientific realism makes this difficult. Sadly, I don't have the knowledge to come up with.

23.1.5 Teleology vs Causality

My privileging of causal relations has already become clear, but now I will justify my thoughts through a Hamiltonian view point. While many claim that the nature of Hamiltonian mechanics necessitates a teleology, and my stated view on structuralism seems to contradict my views on causality, but I will address these claims categorically.

First, the nature of Hamiltonian mechanics necessitates a teleological view of the natural world. This statement will seemingly logical, lacks the tautological strength required of such an extreme statement. For one, Hamilton himself had a clear belief in causality(while this obviously isn't enough, I will explain why he and I see it that way.) First, the concepts of causal forces(forces are by definition causation principles) can be derived through the inverse gradient of potential energy. Second, the connection of the 'principle of least' action and teleology is flimsy at best. This is because their main argument is based upon intuition of mathematic alone not physical realism(I know this goes against stated structuralism but I will come back to that), the reason I state this against due to the fact that, math-

emational intuition in the face of metaphysical truth. For instance, as stated earlier the teleological 'energy' is found as a derivation of causal force. Finally, returning to our arguments placed in the first chapter.

23.1.6 Role of Variational Principles in Mechanics

23.2 Axiomatic Mechanics

23.2.1 Axiomatic Derivation of ...

23.2.1.1 Introduction

For any mathematical physical axiomatic derivation, a mathematical derivation is first required. I won't go super in depth but I will still go over it for rigors sake. I may go back and do a deeper dive but for now I will simply put these things.

23.2.1.2 Axiom of Real numbers

Why needed: Physical quantities like position, velocity, time, energy, and action are represented by real numbers. The calculus used in mechanics relies on the properties of real numbers.

Key axioms:

- Commutative, associative, and distributive properties for addition and multiplication.
- Existence of additive and multiplicative identities (0 and 1).

The Structure of Motion: A Philosophical Journey 96 Through Hamiltonian Mechanics

- Existence of additive and multiplicative inverses (for non-zero numbers).
- Completeness axiom: Every non-empty set of real numbers with an upper bound has a least upper bound (ensures continuity, critical for calculus).

Role: These axioms enable arithmetic operations, inequalities, and the construction of functions like the Lagrangian $L(q, \dot{q}, t)$.

23.2.1.3 Axiom of Euclidean Geometry

Why needed: If the system involves spatial coordinates (e.g., particles moving in 3D space), Euclidean geometry provides the framework for defining positions and distances.

Key axioms(informal):

- Points, lines, and planes exist.
- A straight line can be drawn between any two points.
- Distance between points is defined (e.g., via the Pythagorean theorem).

Role: Defines generalized coordinates q (e.g., Cartesian or polar coordinates) and kinetic energy terms like $\frac{1}{2}m\dot{x}^2$. Note: For abstract systems (e.g., in generalized coordinates), geometry may be less critical, but it's foundational for physical intuition.

23.2.1.4 Axiom of Set Theory (Basic)

Why needed: Sets are used to define the domain of variables (e.g., time $t \in \mathbb{R}$, coordinates $q \in \mathbb{R}^n$) and func-

tions.

Key axioms(informal):

- Existence of sets: Sets can be formed to represent collections of objects (e.g., possible paths of a system).
- Union, intersection, and Cartesian product: Allow combining and manipulating sets.
- Axiom of choice (implicitly): Ensures a choice of path exists in variational problems.

Role: Provides the language for defining functions, spaces, and the configuration space of a system.

23.2.1.5 Axiom of Calculus (Differential and Integral Calculus)

Why needed: The principle of stationary action involves integrals (action $S = \int L dt$) and derivatives (in Euler-Lagrange equations).

Key concepts (built on axioms of real numbers):

- Limits: Define continuity and differentiability of functions like $L(q, \dot{q}, t)$.
- Derivatives: Partial derivatives (e.g., $\frac{\partial L}{\partial q}$) are used to describe rates of change.
- Integrals: The Riemann integral defines the action S .
- Fundamental theorem of calculus: Links derivatives and integrals, essential for variational calculus.

Role: Enables the formulation of the action and the variation $\delta S = 0$.

23.2.1.6 Axioms of Variational Calculus

Why needed: The principle of stationary action requires finding the path that makes the action stationary, which is a problem in variational calculus.

Key axioms(informal):

- **Functional:** The action S is a functional (a function of functions, e.g., paths $q(t)$).
- **Variation:** Small changes in the path $\delta q(t)$ are used to compute δS .
- **Stationary condition:** The path satisfies $\delta S = 0$, leading to the Euler-Lagrange equations.

Role: Provides the mathematical machinery to derive the equations of motion from the action.

23.2.1.7 Axioms of Linear Algebra (Basic)

Why needed: For systems with multiple coordinates or degrees of freedom, vectors and matrices describe generalized coordinates q_i and momenta p_i .

Key axioms:

- **Vector space axioms:** Addition and scalar multiplication of vectors (e.g., coordinates in \mathbb{R}^n).
- **Linearity:** Operations like partial derivatives in Hamilton's equations are linear.

Role: Supports the formulation of the Hamiltonian $H(q, p, t)$ and phase space (the space of (q, p)).

23.2.1.8 Axioms of Variational Calculus

Why needed: The principle of stationary action requires finding the path that makes the action stationary, which is a problem in variational calculus.

Key axioms(informal):

- **Functional:** The action S is a functional (a function of functions, e.g., paths $q(t)$).
- **Variation:** Small changes in the path $\delta q(t)$ are used to compute δS .
- **Stationary condition:** The path satisfies $\delta S = 0$, leading to the Euler-Lagrange equations.

Role: Provides the mathematical machinery to derive the equations of motion from the action.

23.2.1.9 Physical Assumptions (Not Strictly Mathematical Axioms)

While not mathematical axioms, certain physical assumptions are mathematically formalized:

- Time is continuous and one-dimensional ($t \in \mathbb{R}$). (Though Hamiltonians can work in other forms)
- Energy is well-defined: Kinetic energy T and potential energy V are functions of coordinates and velocities.
- Differentiability: The Lagrangian L is sufficiently smooth (at least twice differentiable) to allow partial derivatives
-

The Structure of Motion: A Philosophical Journey 100 Through Hamiltonian Mechanics

Role: These ensure the mathematical framework applies to physical systems.

23.2.2 Axiomatic Derivation of Hamiltonian Mechanics

Two hundred years ago, William Rowan Hamilton reformulated classical mechanics using the **principle of stationary action**, a single axiom that unifies the dynamics of physical systems. This principle states that the path taken by a system between two times makes the action S stationary.

The **action** is defined as:

$$S = \int_{t_1}^{t_2} L(q, \dot{q}, t) dt,$$

where $L = T - V$ is the **Lagrangian**, with T as kinetic energy, V as potential energy, q as generalized coordinates, and \dot{q} as their time derivatives.

The system follows the path where $\delta S = 0$. Varying the action:

$$\delta S = \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \delta \dot{q} \right) dt = 0.$$

Integrating by parts on the second term:

$$\int_{t_1}^{t_2} \frac{\partial L}{\partial \dot{q}} \delta \dot{q} dt = \left. \frac{\partial L}{\partial \dot{q}} \delta q \right|_{t_1}^{t_2} - \int_{t_1}^{t_2} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) \delta q dt.$$

Since $\delta q = 0$ at t_1, t_2 , we get the **Euler-Lagrange equations**:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0.$$

Hamilton defined the **Hamiltonian** as:

$$H(q, p, t) = \sum_i \dot{q}_i p_i - L(q, \dot{q}, t),$$

where $p_i = \frac{\partial L}{\partial \dot{q}_i}$ are generalized momenta. Typically, $H = T + V$.

Using the Legendre transform, the dynamics are governed by **Hamilton's equations**:

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}.$$

Example: For a particle in gravity, $L = \frac{1}{2}m\dot{h}^2 - mgh$. The momentum is $p = \frac{\partial L}{\partial \dot{h}} = m\dot{h}$. The Hamiltonian is:

$$H = \frac{p^2}{2m} + mgh.$$

Hamilton's equations yield:

$$\dot{h} = \frac{p}{m}, \quad \dot{p} = -mg,$$

reproducing the equation of motion $m\ddot{h} = -mg$.

The principle of stationary action simplifies dynamics, applies to diverse systems, and underpins modern physics, including quantum mechanics and relativity. Hamilton's work remains a cornerstone of theoretical physics.

23.2.3 *Hamiltonian Mechanics ad a Geometric Theory

23.2.4 *Poisson Brackets and the Algebra of Dynamics

23.2.5 *Canonical Transformations: Symmetry, Simplicity, an Structure

23.3 Philosophical Implications*

23.3.1 *Time in Essence

23.3.2 *Hamiltonian Constraints and the Problem with Time

23.4 Modern Hamiltonian

23.4.1 Modern Research

23.4.1.1 *Hamiltonian Formulation in Plasma Physics

23.4.1.2 *Hamiltonian and Metriplectic Mechanics

23.4.1.3 *Symplectic Integrators: Preserving the Structure of Nature

23.4.1.4 *HNNN(Hamiltonian Neural Networks)

23.4.2 Fun Mess Around

23.4.2.1 Non-Canonical Poisson Brackets

A Poisson bracket is a bilinear, antisymmetric operation $[F, G]$ that defines the time evolution of a functional

F via $\dot{F} = [F, H]$, where H is the Hamiltonian. Non-canonical brackets arise when the phase space variables (e.g., density, velocity, magnetic field in MHD) do not follow the standard canonical structure $\{q_i, p_j\} = \delta_{ij}$.

Properties: The bracket must satisfy:

- Antisymmetry: $[F, G] = -[G, F]$
- Leibniz Rule: $[F, GH] = [F, G]H + G[F, H]$
- Jacobi Identity: $[F, [G, H]] + [G, [H, F]] + [H, [F, G]] = 0$

For this I will be defining my own Poisson bracket specifically for plasma thrusters.

For this I will obviously need to define the variables. Things like:

- Mass density: $\rho(\mathbf{r}, t)$
- Velocity field: $\mathbf{v}(\mathbf{r}, t)$
- Magnetic field: $\mathbf{B}(\mathbf{r}, t)$
- Entropy or internal energy: $s(\mathbf{r}, t)$ or $\epsilon(\rho, s)$

Now define the Hamiltonian, for this I will use the pre-established ideal-MHD, but in all reality for plasma thrusters I should use a more expanded model.

$$H[\rho, \mathbf{v}, \mathbf{B}, s] = \int \left(\frac{1}{2} \rho v^2 + \rho \epsilon(\rho, s) + \frac{B^2}{2\mu_0} \right) d^3x$$

Next, I must define the phase spaces and constraints. Incompressibility or magnetic field divergence.

The Structure of Motion: A Philosophical Journey 104 Through Hamiltonian Mechanics

Next I propose a bracket system, with many of them being

$$[F, G] = \int \sum_{i,j} \frac{\delta F}{\delta \xi_i} J_{ij} \frac{\delta G}{\delta \xi_j} d^3x$$

Such that Ideal MHD is:

$$[F, G] = - \int \left\{ \rho \left[\frac{\delta F}{\delta \rho}, \frac{\delta G}{\delta \rho} \right] + \left[\frac{\delta F}{\delta \mathbf{v}}, \frac{\delta G}{\delta \mathbf{v}} \right] \cdot \left(\frac{\mathbf{B}}{\rho} \times \nabla \times \frac{\delta G}{\delta \mathbf{B}} \right) + \frac{1}{\rho} \left(\nabla \times \frac{\delta F}{\delta \mathbf{B}} \right) \cdot \left(\nabla \times \frac{\delta G}{\delta \mathbf{B}} \right) \right\} d^3x$$

Another approach is using lie-groups. The Lie-Poisson approach is a method to construct non-canonical Poisson brackets by reducing a canonical Hamiltonian system on a large phase space (e.g., particle coordinates) to a smaller phase space of collective variables (e.g., fluid fields in MHD). It is rooted in the symmetry properties of the system's configuration space, described by a Lie group.

...

The other way is through Casimir invariants. Casimir invariants are functionals C that commute with all functionals F under the Poisson bracket: $[C, F] = 0$. They are conserved quantities that arise from the degeneracy of the non-canonical bracket and provide constraints on the system's dynamics.

Degeneracy: Non-canonical Poisson brackets are degenerate, meaning their Poisson tensor J_{ij} has a non-trivial kernel. Casimirs live in this kernel, satisfying:

$$J_{ij} \frac{\delta C}{\delta \xi_j} = 0$$

Physical Role: Casimirs represent invariants tied to the system's topology or symmetries, such as helicity

in MHD, which constrain the evolution of plasma in thrusters.

Identification: Casimirs are found by solving the functional equation $[C, F] = 0$ for all F , often using the Lie algebra's cohomology.

**The Structure of Motion: A Philosophical Journey
106 Through Hamiltonian Mechanics**

Chapter 24

Omega-X

24.1 Pre-planing(part 1)

24.1.1 Ideas & Overarching Plan

Start with replicating 1D thermal fluid model, then start with more complex models, then experiment with other ways to do it, then dive deeper into the mathematics and hopefully come up with something(maybe metriplectic integrator), then create a code base, compare these systems to other traditional models, and create tokamak simulation(maybe, not sure if possible yet) that is it so far. Hopefully these ideas will evolve.

Basically, the whole idea is to create a Flash-X like project with Metriplectic formulism underlying the the dynamics rather than lagrangian or Newtonian. Obviously much simpler though.

24.1.2 Metriplectic 4-bracket algorithm for constructing thermodynamically consistent dynamical systems

24.1.2.1 Math

1: Identify dynamical variables 2: Propose energy and entropy functionals 3: Find Poisson bracket F, G for which entropy S is a Casimir invariant, $F, S = 0 \forall F$ 4: Construct metriplectic 4-bracket $(F, K; G, N)$ via Kulkarni-Nomizu product by a new method that separates local thermodynamics from phenomenological quantities, giving the EoMs as Poisson bracket + 4-bracket

To find the Σ and M a new method is derived.

$$\partial_t \xi^\alpha = \{\xi^\alpha, H\} + \nabla \cdot J^\alpha$$

$$\alpha = 1, 2 \dots N - 1$$

$$\partial_t \xi^N = \{\xi^N, H\} + \nabla \cdot J^N + Z_\alpha \cdot \tilde{L}^{\alpha\beta} \cdot Z_\beta$$

where $\xi_N = \sigma$, the entropy density. Above splits Hamiltonian and conservative. Which leads to:

$$M(dF, dG) = F_{\xi_N} G_{\xi_N}$$

$$\Sigma(dF, dG) = \nabla F_{\xi_\alpha} \frac{L^{\alpha\beta}}{H_{\xi_N}} \nabla (G_{\xi_\beta})$$

24.1.3 A thermodynamically consistent discretizations of 1D thermal-fluid models using their metriplectic 4-bracket structure

24.1.3.1 Math

Edit: so I don't keep forgetting, ρ is mass density, m is momentum density, and σ is entropy density. Then for the thermodynamic quantities: η is specific entropy, u is velocity, and T is temperature. Let $V_h = v_h \subset H^1(\Omega)$ be the degree- p continuous Galerkin finite element space defined over a uniform grid, τ_h , on Ω : i.e.

$$V_h = v_h \in H^1(\Omega) : v_h|_K \in P_p(K), \forall K \in T_h$$

The discretizations is accomplished using the method of lines by positing that all dynamical fields have spatial dependence modeled in this Galerkin subspace. However, rather than discretizing the equations of motion themselves, we discretize the weak forms implied by the metriplectic formulation.

Let $(\rho_h, m_h, \sigma_h) \in V_h \times V_h \times V_h$. So that the discretized Hamiltonian and entropy can be given as:

$$H^h[\rho_h, m_h, \sigma_h] = \int_{\Omega} \left[\frac{1}{2} \frac{m_h^2}{\rho_h} + \rho_h U\left(\rho_h, \frac{\sigma_h}{\rho_h}\right) \right] dx$$

$$S^h[\sigma_h] = \int_{\Omega} \sigma_h dx$$

Then the Metriplectic 4 Bracket can be expressed through:

$$(F^h, K^h, G^h, N^h)_h = -\frac{1}{\text{Re}} \int_{\Omega} \frac{1}{T_h} \left[(K^h \partial_x F_{m_h}^h - F^h \partial_x K_{m_h}^h) \right],$$

$$\begin{aligned} & (N^h \partial_x G_{m_h}^h - G^h \partial_x N_{m_h}^h) +, \\ & \frac{1}{\text{Pr}\gamma - 1} \frac{1}{T_h} (K^h \partial_x F^h - F^h \partial_x K^h) [(N^h \partial_x G^h - G^h \partial_x N^h)] dx \end{aligned}$$

Then the poission bracket is thus:

$$\begin{aligned} \{F^h, H^h\}_h(u_h) = & -(m_h \partial_x u_h, \phi_m)_{L^2} + (m_h u_h, \partial_x, \phi_m)_{L^2} - \\ & (\rho_h \partial_x \eta_h, \phi_m)_{L^2} + (\rho_h u_h, \partial_x, \phi_\rho)_{L^2} - (\sigma_h \partial_x T_h, \phi_m)_{L^2} + (\sigma_h u_h, \partial_x, \phi_\sigma)_{L^2} \end{aligned}$$

Now, one thing I will say is that this is not technically a Poisson bracket, because it fails to satisfy the Jacobi identity. This is because it is impossible (or at least nobody has figured out a way to discretize fluid poission brackets.

To then find equations of motion, adding them together $F^h = \{F^h, H^h\}_h + (F^h, H^h; S^h, H^h)_h$ where F^h is the observable.

In the case of momentum we must let $\phi_\rho = \phi_\sigma = 0$ so that only ϕ_ρ the momentum test function in the finite element(FE) test space V_h .

$$(\phi_m, \partial_t m_h) + (m_h \partial_x u_h, \phi_m)_{L^2} - (m_h u_h, \partial_x, \phi_m)_{L^2} + (\rho_h \partial_x \eta_h, \phi_m)_{L^2} +$$

Next, I will have $\phi_m = \phi_\sigma = 0$ to get continuity equations:

$$(\phi_\rho, \partial_g \rho_h)_{L^2} - (\rho_h u_h, \partial_x \phi_\rho)_{L^2}$$

Finally the entropy equation is derived in a similar fashion:

$$(\phi_\sigma, \partial_g \sigma_h)_{L^2} - (\sigma_h u_h, \partial_x \phi_\sigma)_{L^2} + \frac{1}{Re} [(\frac{(\partial_x u_h)^2}{T_h}, \phi_\sigma)_{L^2} - \frac{1}{Pr} \frac{\gamma}{\gamma - 1} [(\frac{\partial_x T_h}{T_h}, \phi_\sigma)_{L^2} - (\frac{\partial_x T_h}{T_h}, \phi_\sigma)_{L^2}]]$$

To prove energy conservation

$$(\frac{u_h^{n+1} - u_h^n}{\Delta t}, \delta h_h) = \frac{H(u_h^{n+1}) - H(u_h^n)}{\Delta t} = 0$$

Positive entropy production can be found through:

$$\frac{s_h^{n+1} - s_h^n}{\Delta t} = \frac{1}{Re} [(\frac{(\partial_x u_h^n)^2}{T_h^n}, 1)_{L^2} + \frac{1}{Pr} \frac{\gamma}{\gamma - 1} (\frac{(\partial_x T_h^n)^2}{T_h^n}, 1)_{L^2}] \geq 0$$

Let's figure out all we need to find for each equation, starting with momentum. Start with $M_{ii} = \phi_m$, this should be equal to dx, but I will confirm. Also need the time derivative of the momentum variational, u_h variational, ρ_h variational, T_h , η_h , and $\frac{1}{Re}$, reynolds number, decide later

$$M_{ii} = dx$$

$$u_h = (\rho_h, m_h, \sigma_h)$$

ρ_h , T_h , and η_h are the respective fields, initialized simply and evolved through the equation. Obviously ρ_h, m_h , and σ_h evolve through their metriplectic equations of "motion." T_h , η_h , are evolved through these equations:

$$(\eta_h + \frac{m_h^2}{2\rho_h^2} - U(\rho_h, \frac{\sigma_h}{\rho_h}) - \rho_h \partial_1 U(\rho_h, \frac{\sigma_h}{\rho_h}) + \frac{\rho_h}{\sigma_h} \partial_2 U(\rho_h, \frac{\sigma_h}{\rho_h}, \phi_\eta))_{L^2} = 0$$

$$(u_h - \frac{m_h}{\rho_h}, \phi_u) = 0$$

$$(T_h - \partial_2 U(\rho_h, \frac{\sigma_h}{\rho_h}) \phi_T)_{L^2} = 0$$

Nothing new is added in continuity

Entropy adds: Pr and γ

24.1.3.2 Coding

The first thing is to use a Galerkin Method for projecting PDEs into a finite-dimensional function space. (can base off of Firedrake, FEniCS, Galerkin, or another) M_{ii} in mesh.f90

The second thing to do is to initialize the parameters and states (use mesh to calculate field) in states.f90 (Pr , Re , γ for constants; initialize $(\rho_h, \sigma_h, m_h, \eta_h, T_h, u_h)$)

Then functionals: calculates pointwise functional derivatives of the Total Hamiltonian and Entropy. in functionals.f90 (Update $\rho_h, \sigma_h, m_h, u_h$)

EOS.f90 is used to update the thermodynamic points (η_h, T)

Time step the program, by using Gauss-Legendre implicit Runge-Kutta methods in time_integration.f90

Run the Program (using the driver and makefile)

Input the outputs in io.f90

Some notes: $(f, g)_{L^2} = \int_{\Omega} f(x) \cdot g(x)$

An example would be, for a point in the momentum equation $(\phi_m, \partial_t m_h)_{L^2}$, or the mass matrix times the derivative of momentum functional as a function of time. Thus it makes it so $\phi_m = M_{ii} = dx$ thus, $(\phi_m, \partial_t m_h)_{L^2} = dx \cdot \frac{m_h}{\partial t}$

24.1.3.3 Future Work

The paper presents several ways to improve, or at least look into improving through future work.

Mainly, they mention that many different structure preserving methods exist and may be more suitable.

I also need to figure out how to make this codebase work for more examples

24.1.4 Metriplectic Framework for Dissipative Magneto-Hydrodynamics

24.1.4.1 My Introduction

In my time trying to figure out how to numerically solve metriplectic equations, someone beat me to dissipative magneto-hydrodynamics. Oh well, I can build off of.

Beyond that, the abstract introduces the paper better than I could, " The metriplectic framework, which permits to formulate an algebraic structure for dissipative systems, is applied to visco-resistive Magneto-Hydrodynamics (MHD), adapting what had already been done for non-ideal Hydrodynamics (HD). The result is obtained by extending the HD sym metric bracket and free energy to include magnetic field dynamics and resistive dissipation. The correct equations of motion are obtained once one of the Casimirs of the Poisson bracket for ideal MHD is identified with the total thermodynamical entropy of the plasma. The metriplectic framework of MHD is shown to be invariant under the Galileo Group. The metriplectic structure also permits to obtain the asymptotic equilibria toward which the dynamics of the system evolves. This scheme is finally adapted to the two-dimensional incompressible resis-

tive MHD, that is of major use in many applications.”

24.1.4.2 Notes on introduction

They have an interesting word choice, ”with its noble descendants of path integral representations,” or Algebraization of dynamical systems appears to be the final destination of that virtuous route.

MSTDOF means ”microscopic, statistically treated, degrees of freedom”

In this introduction they derive the metriplectic existence quite simply. Starting with the poisson bracket

$$\dot{f} = \{f, g\}$$

In this case

$$0 = \{S, H\}$$

Thus it is the Casimir functionals of the Poisson bracket

$$\{C, f\} = 0 \forall f$$

From here we can define the Hamiltonian free energy.

$$F = H + \lambda C$$

Where λ is an arbitrary constant left. In MSTDOF, it is the minus of the temperature.

To then expand the system, $\langle\langle f, g \rangle\rangle$ via F . So that $\langle\langle f, g \rangle\rangle = \{f, h\} + (f, g)$ and (f, g) is symmetric, bilinear and semi-definite.

Evolution is then given by

$$\dot{f} = \langle\langle f, F \rangle\rangle$$

(the symmetric bracket (f, g) will be defined so to cancel out the presence of the coefficient λ , removing it from the equations of motion).

(f, g) is defined so that H is conserved.

$$(H, f) = 0 \forall f$$

Thus metriplectic evolution reads:

$$\dot{f} = \{f, H\} + \lambda(f, C)$$

The Casimir to mimic entropy undergoes an evolution described by:

$$\dot{C} = \lambda(C, C)$$

Now, I like this introduction. Clearly showing the basic understanding on Metriplectic dynamics.

24.1.5 METRIPLECTIC FORMULATION OF VISCO-RESISTIVE MHD

The system observed is fully ionized plasma. Where dissipation takes place due to the finite viscosity and resistivity of the fluid. Also, heat conductivity is finite. Here is the SO(3)-covariant form.(expressing physical equations in a way that is invariant under rotations in three-dimensional space)

$$\partial_t \rho + \partial_i(\rho u^i) = 0$$

$$\partial_t B^i + \varepsilon^{ijk} \partial_j E_k = 0, \quad \partial_i B^i = 0$$

$$\partial_t(\rho u^i) + \partial_j(\rho u^i u^j + p \delta^{ij}) = \varepsilon^{ijk} J_j B_k + \partial_j(\eta \partial_j u^i)$$

$$\partial_t s + \partial_i(s u^i) = \frac{\eta}{T} \partial_i u^j \partial_j u^i + \frac{\kappa}{T^2} \partial_i T \partial_i T + \frac{\mu}{T} J^i J_i$$

There is quite a bit to this equation, but I will move on.

One thing I will add is a new notation. $\dot{H} \stackrel{\partial}{=} 0$, it simply means that means that \dot{H} and 0 only differ by a boundary term.

They then describe the poission bracket for ideal MHD(when dissipation does not happen, thus traditional poission bracket).

This equation has multiple Casimir observables. The paper mentions:

$$C[\rho, s] = \int_{\mathbb{D}} \rho \varphi(s) d^3x$$

Which within is both the total mass and entropy.

$$M[\rho] = \int_{\mathbb{D}} \rho d^3x$$

$$S[\rho, s] = \int_{\mathbb{D}} \rho s d^3x$$

Thus, mass and entropy are conserved. There are some other, but obviously for this purpose, use the entropy.

Taking the dissipative equations from the to combine $D = (\vec{D}^\rho, \vec{D}^v, \vec{D}^B, \vec{D}^s)$ something called an 8-uple. So we must want to define our metriplectic bracket as $\psi = (\psi, D)$ where ψ is the dynamic variables.

Uhm.. I just realized this wasn't the paper I thought it was. Good read, but I don't think I will continue these notes.

24.1.6 A least Action principle for visco-resistive Hall Magnetohydrodynamic with metriplectic reformulation

24.1.6.1 Abstract

The abstract persented says,

”We present a new variational formulation for Viscous and resistive Hall Magnetohydrodynamic. We first find a variational principle for ideal HMHD by applying the physical assumptions leading to HMHD at the lagrangian level, and then we add the viscous and resistive terms by the means of constrained variations. We also provide a metriplectic reformulation of our formulation, based on two canonical Lie-Poisson brackets for the ideal part and metric 4-brackets for the dissipative part.”

They then go into all of these, I will begin with the Metriplectic for notes alone.

24.1.6.2 Metriplectic bracket, Hamiltonian and Entropy for Viscoresistive Hall MHD

They use the Inclusive curvaturelike tensor found in Morrison’s recent work. Where the dissipation is included using a 4-bracket that have the same symmetries as a curvature tensor.

They take the assumption that the viscous tensor can be found to be

$$\sigma = \Lambda \nabla u_i$$

Where Λ is symmetrize and positive.

24.1.7 Variational discretizations of viscous and resistive magnetohydrodynamics using structure-preserving finite elements

24.1.7.1 Abstract

We propose a novel structure preserving discretization for viscous and resistive magnetohydrodynamics. We follow the recent line of work on discrete least action principle for fluid and plasma equation, incorporating the recent advances to model dissipative phenomena through a generalized Lagrange-d'Alembert constrained variational principle. We prove that our semi discrete scheme is equivalent to a metriplectic system and use this property to propose a Poisson splitting time integration. The resulting approximation preserves mass, energy and the divergence constraint of the magnetic field. We then show some numerical results obtained with our approach. We first test our scheme on simple academic test to compare the results with established methodologies, and then focus specifically on the simulation of plasma instabilities, with some tests on non Cartesian geometries to validate our discretization in the scope of tokamak instabilities.

This paper

24.2 part 2

24.2.1 Coding(1D fluid)

24.2.1.0.1 Introduction My first step to creating my codebase is to create it for the simplest form. This is

specifically made for the purpose of figuring things out, documenting it, and such things. So, I will be writing down each file.

24.2.1.0.2 mesh_1d Obviously, this mesh program creates the mesh for the program. The size N , the nodes x_nodes and the mass matrix M_{ii} . The rest are self explanatory but I will go into what the mass matrix is. It represents the discrete inner product of basis functions.

$$M_{ii} = (\phi_i, \phi_j)_{L^2} = \int_{\Omega} \phi_i(x) \phi_j(x) dx$$

So, M_{ii} becomes a metric tensor for program. It turns continuous inner products into discrete sums over coefficients.

It can be approximated through the "lumped mass matrix" where $M_{ii} = dx$ Though, I use the trapezoidal mass matrix:

$$M_{ii}(1) = 0.5d0 * dx$$

$$M_{ii}(N) = 0.5d0 * dx$$

$$M_{ii}(2 : N - 1) = dx$$

For more information visit, <https://arxiv.org/pdf/2008.03883>

Listing 24.1: mesh_1d.f90

```

1 module mesh_1d
2   implicit none
3   private
4   public :: initialize_mesh, x_nodes, dx, N,
        Mii
5
6   ! Parameters

```

```

7  integer, parameter :: N = 100          !
   Number of spatial grid points (nodes)
8  real(8), parameter :: L = 1.0d0       !
   Length of the domain
9
10 ! Mesh data
11 real(8), dimension(N) :: x_nodes      !
   Coordinates of each mesh node
12 real(8) :: dx                         !
   Uniform cell spacing
13
14 ! Mass matrix (diagonal approximation here,
   for full FEM use full M)
15 real(8), dimension(N) :: Mii          !
   Lumped mass matrix diagonal
16
17 contains
18
19 subroutine initialize_mesh()
20   integer :: i
21
22   dx = L / (N - 1)
23
24   ! Initialize node positions
25   do i = 1, N
26     x_nodes(i) = (i - 1) * dx
27   end do
28
29   ! Lumped mass matrix: each diagonal entry
   is dx
30   Mii = dx
31
32   ! Uncomment below to use trapezoidal mass
   matrix
33   Mii(1) = 0.5d0 * dx
34   Mii(N) = 0.5d0 * dx
35   Mii(2:N-1) = dx

```

```

36
37     end subroutine initialize_mesh
38
39
40 end module mesh_1d

```

24.2.1.0.3 states_1d States then initializes the constants and fields. First the constants(parameters) used throughout the code. Re , Reynolds numbers. γ , compression of the fluid. Pr , Prandtl number, ratio of viscous to thermal diffusion.

Then, it initializes the discretion fields of entropy, momentum, mass, temperature, and specific entropy.

This should be modularized for the future to make this codebase iterative.

Now, what exactly to do. I think for this, have constants in a separate one. Then for the fields, I am not sure at this moment.

Listing 24.2: states_1d.f90

```

1  module states_1d
2      use mesh_1d
3      implicit none
4      private
5      public :: m_h, rho_h, sigma_h, eta_h, T_h
6      public :: initialize_states
7
8      ! Field arrays
9      real(8), allocatable :: m_h(:)           !
10         Momentum density
11     real(8), allocatable :: rho_h(:)          ! Mass
12         density
13     real(8), allocatable :: sigma_h(:)        !
14         Entropy density

```

```

12  real(8), allocatable :: eta_h(:)      !
    Specific entropy (diagnostic)
13  real(8), allocatable :: T_h(:)       !
    Temperature (diagnostic)
14  real(8), parameter, public :: gamma = 1.4
15  real(8), parameter, public :: Pr = 0.71
16  real(8), parameter, public :: Re = 1000.0
17
18  contains
19
20  subroutine initialize_states()
21      implicit none
22      integer :: i
23
24      ! Allocate fields
25      allocate(m_h(N))
26      allocate(rho_h(N))
27      allocate(sigma_h(N))
28      allocate(eta_h(N))
29      allocate(T_h(N))
30
31      ! Set initial conditions (example:
    Gaussian density bump)
32      do i = 1, N
33          rho_h(i) = 1.0d0 + 0.2d0 * exp(-100.0d0
    * (x_nodes(i) - 0.5d0)**2)
34          eta_h(i) = 1.0d0
35          sigma_h(i) = rho_h(i) * eta_h(i)
36          m_h(i) = 0.5d0
37          T_h(i) = (0.4d0) * eta_h(i)  ! Ideal
    gas ! Uses EOS
38      end do
39
40  end subroutine initialize_states
41
42  end module states_1d

```


24.2.1.0.4 projection_matrix_1d Here the creation of two subroutines to simplify future calculations. First, `initialize_projection_matrix()`, that simply creates matrix. Next, `apply_weak_derivative(f, d_proj)`. This does:

$$proj = \sum_{i=1}^n \left(\sum_{j=1}^n D_{i,j} \right) * f$$

Where f is the observable. For future work, I think all I need to do for this one is create multiple for different dimensions.

Listing 24.3: `projection_matrix_1d.f90`

```

1  module projection_matrix_1d
2      use mesh_1d
3      implicit none
4      real(8), allocatable :: D(:, :)
5      public :: initialize_projection_matrix,
6               apply_weak_derivative
7
8  contains
9
10     subroutine initialize_projection_matrix()
11         integer :: i
12         real(8) :: dphi_dx
13         if (allocated(D)) then
14             deallocate(D)
15         end if
16
17         allocate(D(N, N))
18         D = 0.0d0
19
20         dphi_dx = 1.0d0 / dx
21
22         ! Loop over elements and assemble D
23         do i = 2, N-1

```

```

23      ! Local contributions from basis
        functions over each element
24      D(i, i-1) = -dphi_dx / 2.0d0
25      D(i, i ) =  0.0d0
26      D(i, i+1) =  dphi_dx / 2.0d0
27  end do
28
29      ! Optional: Neumann (zero derivative) at
        boundaries
30      D(1,:) = 0.0d0
31      D(N,:) = 0.0d0
32  end subroutine initialize_projection_matrix
33
34      ! Applies D to f to compute (phi_i,
        partial_x f)
35  subroutine apply_weak_derivative(f, d_proj)
36      implicit none
37      real(8), intent(in)  :: f(N)
38      real(8), intent(out) :: d_proj(N)
39      integer :: i
40
41      d_proj = 0.0d0
42      do i = 1, N
43          d_proj(i) = sum(D(i,:) * f(:))
44      end do
45  end subroutine apply_weak_derivative
46
47  end module projection_matrix_1d

```

24.2.1.0.5 Listing 24.4: mass_1d.f90

```

1  module mass_1d
2      use mesh_1d
3      use projection_matrix_1d
4      implicit none
5  contains

```

```

6
7  subroutine compute_mass_flux(N, rho_h, u_h,
   rho_rhs)
8      integer, intent(in) :: N
9      real(8), intent(in) :: rho_h(N), u_h(N)
10     real(8), intent(out) :: rho_rhs(N)
11     real(8) :: flux(N)
12     integer :: i
13
14     ! Compute flux
15     do i = 1, N
16         flux(i) = rho_h(i) * u_h(i)
17     end do
18
19     ! Apply weak derivative
20     call initialize_projection_matrix()
21     call apply_weak_derivative(flux, rho_rhs)
22
23     ! Divide by dx
24     do i = 1, N
25         rho_rhs(i) = -rho_rhs(i) / dx
26     end do
27
28     ! Boundary conditions
29     rho_rhs(1) = 0.0d0
30     rho_rhs(N) = 0.0d0
31
32     end subroutine compute_mass_flux
33
34 end module mass_1d

```

Listing 24.5: momentum_1d.f90

24.2.1.0.6 momentum_1d

```

1  module momentum_1d
2      use states_1d, only: Re
3      use mesh_1d

```

```

4   use projection_matrix_1d
5   implicit none
6   contains
7
8   subroutine compute_momentum_rhs(N, rho_h,
9       m_h, sigma_h, eta_h, T_h, rhs_m)
10      integer, intent(in) :: N
11      real(8), intent(in) :: rho_h(N), m_h(N),
12          sigma_h(N), eta_h(N), T_h(N)
13      real(8), intent(out) :: rhs_m(N)
14      real(8) :: u_h(N), du_proj(N), mu(N),
15          dmu_proj(N), deta_proj(N), dT_proj(N)
16      integer :: i
17
18      ! Compute velocity
19      do i = 1, N
20          if (rho_h(i) > 1.0d-12) then
21              u_h(i) = m_h(i) / rho_h(i)
22          else
23              u_h(i) = 0.0d0
24          end if
25      end do
26
27      ! Apply weak derivatives
28      call initialize_projection_matrix()
29      call apply_weak_derivative(u_h, du_proj)
30      do i = 2, N-1
31          mu(i) = m_h(i) * u_h(i)
32      end do
33      call apply_weak_derivative(mu, dmu_proj)
34      call apply_weak_derivative(eta_h,
35          deta_proj)
36      call apply_weak_derivative(T_h, dT_proj)
37
38      ! Compute RHS
39      do i = 2, N-1
40          rhs_m(i) = 0.0d0

```

```

37     rhs_m(i) = rhs_m(i) - m_h(i) * du_proj(
        i) * dx
38     rhs_m(i) = rhs_m(i) + dmu_proj(i) * dx
39     rhs_m(i) = rhs_m(i) - rho_h(i) *
        deta_proj(i) * dx
40     rhs_m(i) = rhs_m(i) - sigma_h(i) *
        dT_proj(i) * dx
41     rhs_m(i) = rhs_m(i) - (1.0d0 / Re) *
        du_proj(i) * dx
42 end do
43
44 ! Boundary conditions
45 rhs_m(1) = 0.0d0
46 rhs_m(N) = 0.0d0
47
48 end subroutine compute_momentum_rhs
49
50 end module momentum_1d

```

24.2.1.0.7 Listing 24.6: entropy_1d.f90

```

1 module entropy_1d
2     use states_1d
3     use mesh_1d
4     use projection_matrix_1d
5     implicit none
6 contains
7
8     subroutine compute_entropy_rhs(N, sigma_h,
        u_h, T_h, rhs_sigma)
9         implicit none
10        ! Dummy arguments
11        integer, intent(in) :: N
12        real(8), intent(in) :: sigma_h(N), u_h(N)
        , T_h(N)
13        real(8), intent(out) :: rhs_sigma(N)

```

```

14
15      ! Local variables
16      real(8) :: flux(N), dx_u(N), dx_T(N)
17      real(8) :: visc_prod(N), heat_prod(N),
18                heat_grad(N), heat_term(N)
19      integer :: i
20
21      ! Compute flux
22      do i = 1, N
23          flux(i) = sigma_h(i) * u_h(i)
24      end do
25
26      ! Apply weak derivatives
27      call initialize_projection_matrix()
28      call apply_weak_derivative(flux,
29                                rhs_sigma)
30      call apply_weak_derivative(u_h, dx_u)
31      call apply_weak_derivative(T_h, dx_T)
32
33      ! Viscous and thermal terms
34      do i = 1, N
35          visc_prod(i) = (dx_u(i)**2) / T_h(i)
36          heat_grad(i) = dx_T(i) / T_h(i)
37          heat_prod(i) = (dx_T(i)**2) / (T_h(i)
38                                     **2)
39          heat_term(i) = (gamma / (gamma - 1.0d0)
40                        ) * (heat_prod(i) - heat_grad(i) /
41                            dx)
42          rhs_sigma(i) = rhs_sigma(i) - (1.0d0 /
43                                         Re) * visc_prod(i) * dx
44          rhs_sigma(i) = rhs_sigma(i) - (1.0d0 /
45                                         (Re * Pr)) * heat_term(i) * dx
46      end do
47
48      ! Divide by dx
49      do i = 1, N
50          rhs_sigma(i) = -rhs_sigma(i) / dx

```

```

44     end do
45
46     ! Boundary conditions
47     rhs_sigma(1) = 0.0d0
48     rhs_sigma(N) = 0.0d0
49
50     end subroutine compute_entropy_rhs
51
52 end module entropy_1d

```

Listing 24.7: velocity_1d.f90

24.2.1.0.8 velocity_1d

```

1  module velocity_1d
2      use states_1d
3      use projection_matrix_1d
4      implicit none
5  contains
6
7      subroutine compute_velocity(N, rho_h, m_h,
8          u_h)
9          implicit none
10
11         ! Dummy arguments
12         integer, intent(in) :: N
13         real(8), intent(in) :: rho_h(N), m_h(N)
14         real(8), intent(out) :: u_h(N)
15
16         ! Local variables
17         integer :: i
18         real(8), parameter :: eps = 1.0d-12 !
19         ! Small number to avoid division by zero
20
21         ! Compute velocity safely
22         do i = 1, N
23             if (rho_h(i) > eps) then
24                 u_h(i) = m_h(i) / rho_h(i)
25             end if
26         end do
27     end subroutine compute_velocity
28 end module velocity_1d

```

```

23         else
24             u_h(i) = 0.0d0
25         end if
26     end do
27
28     end subroutine compute_velocity
29
30 end module velocity_1d

```

Listing 24.8: EOS_1d_thermal.f90

24.2.1.0.9 EOS_1d_thermal

```

1 module eos_1d
2     use states_1d
3     implicit none
4     private
5     public :: compute_temperature, compute_eta
6
7
8
9 contains
10
11     !
12     -----
13
14     ! Compute temperature T from specific
15     ! entropy eta
16     !  $T = (\gamma - 1) * \eta$ 
17     !
18     -----
19
20     function compute_temperature(eta_h) result(
21         T_h)
22         real(8), intent(in) :: eta_h
23         real(8) :: T_h
24
25         T_h = (gamma - 1.0d0) * eta_h

```



```

20  end function compute_temperature
21
22  !
23  ! -----
24  ! Compute specific entropy eta from
25  ! temperature
26  ! eta = T / (gamma - 1)
27  !
28  ! -----
29
30  function compute_eta(T_h) result(eta_h)
31      real(8), intent(in) :: T_h
32      real(8) :: eta_h
33
34      eta_h = T_h / (gamma - 1.0d0)
35  end function compute_eta
36
37  end module eos_1d

```

Listing 24.9: time_integrator_1d.f90

24.2.1.0.10 time_integrator_1d

```

1  module time_integrator_1d
2      use mesh_1d
3      use states_1d
4      use velocity_1d
5      use eos_1d
6      use mass_1d
7      use momentum_1d
8      use entropy_1d
9      implicit none
10     private
11     public :: advance_one_step
12
13 contains
14

```

```

15  subroutine advance_one_step(dt, u_h, rhs_m,
      rho_rhs, rhs_sigma)
16      real(8), intent(in) :: dt
17      real(8), intent(INOUT) :: u_h(N), rhs_m(N)
      rho_rhs(N), rhs_sigma(N)
18      integer :: i
19
20      ! Step 1: Compute velocity
21      call compute_velocity(N, rho_h, m_h, u_h)
22
23      ! Step 2: calculate tempeture
24      do i = 1, N-1
25          T_h(i) = compute_temperature(eta_h(i))
26      end do
27      ! step three: calculate Eta
28      do i = 1, N-1
29          eta_h(i) = compute_eta(T_h(i))
30      end do
31      ! Step 4: Compute Galerkin RHS terms
32      call compute_momentum_rhs(N, rho_h, m_h,
      sigma_h, eta_h, T_h, rhs_m)
33      call compute_mass_flux(N, rho_h, u_h,
      rho_rhs)
34      call compute_entropy_rhs(N, sigma_h, u_h,
      T_h, rhs_sigma)
35
36      ! Step 5: Advance in time (Euler / RK1)
37      do i = 1, N
38          m_h(i)      = m_h(i)      + dt * rhs_m(i)
39          rho_h(i)    = rho_h(i)    + dt * rho_rhs(
      i)
40          sigma_h(i) = sigma_h(i) + dt *
      rhs_sigma(i)
41      end do
42
43      end subroutine advance_one_step
44

```

```
45 end module time_integrator_1d
```

Listing 24.10: io_1d.f90

24.2.1.0.11 io_1d

```

1 module io_1d
2   use mesh_1d
3   use states_1d
4   use velocity_1d
5   use time_integrator_1d
6   implicit none
7   private
8   public :: write_state_to_csv
9
10  contains
11
12    subroutine write_state_to_csv(filename)
13      implicit none
14      character(len=*), intent(in) :: filename
15      integer :: i
16      logical :: wait
17      real(8) :: u_h(N)
18      integer :: ierr
19
20      ! Ensure the output directory exists
21      wait = .true. ! Command should run
22                  ! synchronously
23      call execute_command_line("mkdir -p
24                                output", wait, ierr)
25      if (ierr /= 0) then
26        print *, "Error: Could not create
27                  output directory!"
28        stop
29      end if
30
31      ! Compute velocity  $u = m / \rho$ 
32      call compute_velocity(N, rho_h, m_h, u_h)

```

```

30
31      ! Open file for writing
32      open(unit=10, file=filename, status="
          replace", action="write", form="
          formatted")
33
34      ! Header
35      write(10, '(A)') 'x,rho,m,sigma,u,T,eta'
36
37      ! Data
38      do i = 1, N
39          write(10, '(F12.6,1x,F12.6,1x,F12.6,1x,
          F12.6,1x,F12.6,1x,F12.6,1x,F12.6)')
          &
40          x_nodes(i), rho_h(i), m_h(i), sigma_h
          (i), u_h(i), T_h(i), eta_h(i)
41      end do
42
43      close(10)
44      end subroutine write_state_to_csv
45
46 end module io_1d

```

Listing 24.11: momentum_1d.f90

24.2.1.0.12 1D Thermal driver

```

1 program omega_x_driver_1d
2     use mesh_1d
3     use states_1d
4     use time_integrator_1d
5     use io_1d
6     implicit none
7     real(8) :: dt, t, t_end
8     integer :: step = 1
9     real(8), dimension(N) :: u_h, rhs_m,
        rho_rhs, rhs_sigma
10    character(len=100) :: fname

```

```

11
12 dt = 1.0d-3
13 t_end = 1.0d0
14 t = 0.0d0
15
16 call initialize_mesh()
17 call initialize_states()
18 u_h = 0.0d0
19 rhs_m = 0.0d0
20 rho_rhs = 0.0d0
21 rhs_sigma = 0.0d0  ! Explicitly initialize
    rhs_sigma here
22
23 do while (t < t_end)
24     call advance_one_step(dt, u_h, rhs_m,
        rho_rhs, rhs_sigma)
25     t = t + dt
26     print *, 't = ', t
27
28     if (mod(step, 100) == 0) then
29         write(fname, '(A,I4.4,A)') "output/
            state_", step, ".csv"
30         call write_state_to_csv(fname)
31     end if
32
33     step = step + 1
34 end do
35 end program omega_x_driver_1d

```

24.2.1.1 Plans

I have now create it in its most basic form, here are my next steps.

Break states up into constants and fields(multi-dimensional fields)

Make operators folder for projection, weak derivative, boundary conditions

Add checks(mainly energy conservation)

Upgrade time integration,(figure out)

Make different spacial discretization

More boundary conditions

Add more physics

Though, the main thing I need to work on is module registry + driver pattern.

24.3 part 3

24.3.1 Notes

24.3.1.1 Notes on module FORTRAN

I should have a main driver like:

Listing 24.12: Omegax.f90

```
1 program Omegax
2     use Driver_module
3     implicit none
4
5     call Driver_init()
6     call Driver_evolve()
7     call Driver_finalize()
8
9 end program Omegaz
```

There should also be a config file, saying which modules should be used, and a python script capable of parsing this registry file. Config would also be where parameters would be stored.

24.3.1.2 Diminsionales programming

In the config file I can have a section to input the diminsions.

Then have all variable allocatable based upon the diminsions

then have an allocation based upon such diminsions(either preprocessor or generic shape handling)

Instead of hardcoding loops, create cases for different diminsions.

Generaliz derivatives and fluxes.

Also, while physics sections should be generalized, initializing between 3D, would be beneficial.

24.3.1.3 Generalize

Due to Metriplectics dyanmics complex mathematics, a fully generalized system, like FLASH-X, is impossible. This is because it is a dynamical theory based upon Hamiltonian Mechanics, rather than forces in Newtonian.

I am thinking the way to fix this is to create a symbolic python code to be able to generate the function I need.

Also, since these would be in py

24.3.2 Plan

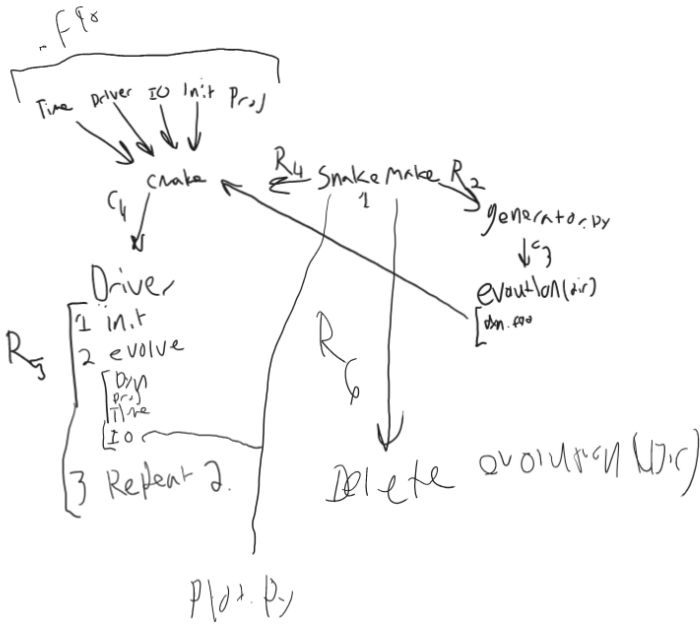
I think the steps I am going to take are:

1: Develop a python code that can generate the math I need, I would also like for it to be able to read a LaTeX code for it.

- 2: I need to look into how to create a Config file for my problem at hand
- 3: Fix integration for this new system
- 4:

24.3.3 New Structure

Here is the basic idea of what it should look like and do:



Chapter 25

Axiom

25.1 *Structure

25.2 LaTeX

25.2.1 Division of Structure

In this program I have used this structure to divide the book up:

```
\part{}  
\chapter{}  
\section{}  
\subsection{}  
\subsubsection{}  
\subsubsubsection{} % custom made  
\paragraph  
\subparagraph
```

I may have to learn new ways to divide in the future, but I will try to avoid that.

25.2.2 Commands

Here are some commands that I will have here for reference:

`\begin{verbatim}` writes things exactly, used mostly for writing LaTeX commands in this method

`\label{}` → Set a reference point

`\ref{}` → Refer to numbered items

`\pageref{}` → Refer to page number

`\nameref{}` (requires hyperref) → Refer to name/title of chapter/section

`\autoref{}` (requires hyperref) → Smart referencing (automatically adds `\Chapter`", `\Section`")

`\begin{equation} ... \end{equation}` → Numbered equations

`\[... \]` → Displayed, unnumbered equations

`$... $` → Inline math

`\frac{a}{b}`, `\sqrt{x}`, `\sum`, `\int` → Standard math symbols

`\begin{align} ... \end{align}` (from amsmath)
→ Multi-line aligned equations

`\begin{cases} ... \end{cases}` → Piecewise functions

`\bra{}`, `\ket{}`, `\braket{}` →

Use physics package for quantum notation

`\tensor{}` → Use tensor package
for tensor indices

`\dv{}`, `\p dv{}` → Derivatives (from physics)
`\begin{figure}[h!] ... \end{figure}` →
Insert figures

`\includegraphics[width=0.8\textwidth]{file}`
→ Image inclusion (graphicx)

`\caption{}` / `\label{}` → Caption and reference

`\begin{table}[h!] ... \end{table}` → Tables

I also have `\begin{lstlisting}` to write both FORTRAN
and make. I will likely add python and Julia

Part IV

Essays

Chapter 26

Introduction

This part is used as a collection of essays on whatever topic. Whether it be philosophy, physics, economic, whatever.

Part V

Journaling

Chapter 27

***Introduction**

Chapter 28

Plans

28.1 Now

28.1.1 Self-Education

elf-education has always been a great value of mine. As mentioned earlier, I became obsessed with learning Einstein's field equation in 3-4th grade. Beyond that I have spent much of my time reading books, watching YouTube videos, and more about topics that interest me. Though, late in 8th grade, this desire to teach myself grew a great deal. Before I can go about my future goals for teaching myself I must first go over what I have already down.(This list is not full, mainly lectures finished and books from lectures, this does not include books not finished, non-lecture based video education, research papers read, projects where I learned things for the project alone, and other similar programs)

Mathematics:

- AP Calculus AB and BC - Khan Academy

- Multivariable Calculus
- Khan Academy
- 18.03 Differential Equations - MIT OpenCourseWare
- Gilbert Strang on linear algebra - MIT OpenCourseWare
- Vector Calculus - Trevor Bazett
- Tensor Analysis - eigenchris
- Tensor Calculus - eigenchris
- Crash Course in Complex Analysis - Steve Burton
- Introduction to Applied Numerical Analysis - Richard W. Hamming
- Symplectic geometry & classical mechanics - Tobias Osborne
- Lie groups, algebras, brackets - Mathemaniac (more conceptual than mathematical)
- Differential geometry - Robert Davie
- Calculus of variations - Faculty of Khan
- Working on
 - Advanced Analytic Methods in Continuum Mathematics: Fundamentals for Science and Engineering - Hung Cheng
 - Discrete Differential Geometry - CMU
 - Differential Geometry for students of Numerical Electrodynamics - Alain Bossavit

Physics:

- 8.02 Physics II - MIT OpenCourseWare
- 8.03 Physics III - MIT OpenCourseWare
- 8.033 Relativity - MIT OpenCourseWare
- General Relativity - Stanford Online
- Introduction to plasma physics - USYD senior plasma physics lectures
- Introduction to electromagnetism - Griffiths
- Computational physics - Mark Newman
- 8.224 Exploring Black Holes: General Relativity and astrophysics - MIT OpenCourseWare
- Classical Dynamics of Particles and Systems - Thornton Marion
- Introduction to cosmology - Stanford Online
- Introduction to fusion energy and plasma physics course - PPPL
- Seminar: Fusion and plasma physics - MIT OCW
- Statistical Mechanics - Stanford Online
- Hamiltonian description for magnetic field lines in fusion plasmas: A tutorial - AIP
- Fusion economics: power density, materials and maintenance - PPPL Frontiers Colloquia
- Flash-X code tutorial, a users perspective - University of Chicago(skimmed)

- Flash-X user guide - Flash-x
- Flash4 User support(skimmed)
- Radiative Procureses in Astrophysical Phenomena- George B. Rybicki, Alan P. Lightman
- Goldstien Classical Mechanics Lectures - Prof. Jacob Linder
- PiTP 2016 - Institute of Advanced study
- 2024 PPPL Graduate Summer School
- Hamiltonian description of the ideal Fluid - P.J.Morrison
- Quantum Mechanics with Applications - David B. Beard & George B. Beard
- "What IS Structure, How Do You Create or Recognize It, and How Can You Use It? or Metriplectic Dynamics: Using the 4-Bracket for Constructing Thermodynamically Consistent Models." - workshop Geometric Mechanics Formulations for Continuum Mechanics
- "On an Inclusive Curvature-Like Framework for Describing Dissipation: Metriplectic 4-Bracket Dynamics." - workshop Infinite Dimensional Geometry and Fluids
- Haven't finished
 - Theoretical Physics - Georg Joos
 - Advanced MHD with Applications to Laboratory and Astrophysical Plasmas - Cambridge
 - Classical Mechanics: A Modern Perspective - Sudarshan and N. Mukunda

- The Interpretation of Structure From Motion
- Ullman, S.
- Radiative Processes in Astrophysics - George
B. Rybicki & Alan P. Kightman
- Symplectic geometry & classical mechanics -
Tobias Osborne

Coding:

- Computational physics - Mark Newman
- Python Numerical Methods - Berkeley
- Applied Numerical Methods - Crice Carnahan, H.A
Luther, James O.Wilkes

Other:

- Management in engineering - MITOpenCourseware
- Principles of microeconomics - MITOpenCourse-
ware
- Dynamic leadership: using improvisation in busi-
ness - OpenCourseware
- Logic 1 - OpenCourseware
- Policy for science, technology, and innovation -
MITX
- Reducing The Danger Of Nuclear Weapons And
Proliferation- MITOpenCourseware

28.1.2 Research

- Egg drop simulation
- Numerical Methods for 3D compressed Plasma using Lattice Boltzmann(paper written)
- Simulated Annealing for plasma thrusters
- High Altitude EMP for primordial black holes(never finished)
- Flash-x simulations:
 - Basic Sedov
 - Running simple MHD simulation of my own design
- A Hamiltonian Framework on ICF Implosions Rocket Equation Based on Rayleigh-Taylor Instabilities
- Contributed to PlasmaPY
- Relativistic Rocket Equation(Newtonian)
- Relativistic rocket equation(Hamiltonian)
- Finite difference MHD Model in Fortran
- Computes velocity based upon Navier Stokes equation adapted to magnetic and electric fields
- Outputs graphical color field of velocity
- Asteroid game
 - Classical
 - Time
 - Survival

- Newtonian(player and asteroids have gravity)
 - Dark Matter(Invisiable gravity fields)
 - Relativistic(time dilation, spacial contraction, black holes, etc)
 - Proximal Policy Optimization “AI” played game
- Hamiltonian based plasma thruster Fortran Numerical Code
- N body simulation
- Fluid based N-body simulation
- Death Star
 - Basic gravitational ‘planet’
 - MHD Plasma beam hitting
 - Basic evaporation and kinetic energy transfer
- OpenMHD usage
- Usage Gkyl
- Omega-X project
 - Inspired by FLASH-X, attempting to build a multi-physics plasma physics codebase using Metriplectic forumism rather than Lagrandian and Newtonian
 - I have successfully implemented the 1D ther-mofluid metriplectic discretization onto a discontinuous Galerkin subspace using an L^2 projection. Integrated using implicit midpoint rule.
 - Working on module registry & driver pattern

28.1.3 Other activities

I also had some other activities that I have done in high school, I won't go too much in them.

- SVA
- Boy Scouts
- Church activities
- JROTC
- FCA
- OA
- Etc

The OA, Order of the Arrow, I will continue into college.

28.2 Pre-University

28.2.1 Independent Learning

One thing I need to do is nail calculus completely. I am going to be taking the AP calc BC test without taking the course.

Beyond that, I am completing differential geometry. Once done I will finish Symplectic geometry. Then lie (brackets, algebra, group, and theory)

I also want to get into functional analysis.

I should also learn C++ beyond what I have done in the past.

28.3 Undergrad

28.3.1 Classes

28.3.1.1 Requirments

28.3.1.1.1 General

I have tested out of many of the general education requirements, so here is the remaining bit:

- Written communication(pick 1)
 - Honors ethical theory
 - Haslam knowledge
- Oral communication
 - A survey in Contemporary Physics
- Arts and humanities(pick 2)
 - Haslam Knowledge
 - honors intro to philosophy
 - Professional responsibility
- Civilization(pick 2)
 - Honors development of Western civilization
 - Honors history of world civilizations
 - University Honors
 - World Religions

28.3.1.1.2 Academic Physics Major

- Prerequisites
 - Intro Computer Science
 - Differential Equations 1
 - Matrix Algebra
- Core
 - Fundamentals of modern physics
 - Mechanics 1
 - Thermal Physics
 - Electronics Lab
 - Intro to quantum mechanics
 - Modern optics
 - Electricity and Magnetism
 - Survey in modern physics
 - Modern Lab
- Academic
 - Mechanics 2
 - Intro to quantum 2
 - Electricity and magnetism 2
- Recommended
 - Mathematical Methods for scientist
 - Partial Differential Equations
 - Complex Variables
- Thesis in physics

28.3.1.1.3 Applied Maths Major

- Computer Science
- Core
 - Diff equations
 - Matrix Algebra
 - Intro to abstract mathematics
 - Math proficiency
- Breath
 - Honors abstract algebra 1
 - Honors Analysis 1
 - Numerical Analysis
 - Stochastic processes
- Applied breath(physics courses)
- Depth
 - Numerical Algebra

28.3.1.1.4 Computer Science Minor

- Intro comp sci
- data structures
- Logic design
- Numerical Algorithms
- Numerical Analysis
- Numerical Algebra
- Parallel Programming

28.3.1.1.5 Electives

- PHYS 441 - Introduction to Computational Physics
- PHYS 494 - Special Topics in Physics
- 2nd PHYS 493 - Research and Independent Study
- PHYS 531 - Mechanics(Maybe, graduate)
- PHYS 541 - Electromagnetic Theory(Maybe)
- PHYS 571 - Mathematical Methods in Physics I(maybe)
- PHYS 573 Numerical methods in physics(maybe)
- PHYS 615 Astrophysics(maybe)
- PHYS 643 Computational Physics(maybe)
- MATH 462 - Differential Geometry
- MATH 467 - Honors: Topology
- MATH 536 - Partial Differential Equations II
- MATH 567 - Riemannian Geometry I
- MATH 568 - Riemannian Geometry 2
- MATH 572 - Numerical Analysis II
- MATH 577 - Optimization
- MATH 579 - Seminar in Numerical Mathematics

28.3.1.1.6 Total

- Honors ethical theory(x)
- honors intro to philosophy(x)
- Professional responsibility(x)
- Honors development of Western civilization(x)
- Honors history of world civilizations(x)
- Intro Computer Science(x)
- Differential Equations 1(x)
- Matrix Algebra(x)
- Fundamentals of modern physics(x)
- Mechanics 1(x)
- Thermal Physics(x)
- Electronics Lab(x)
- Intro to quantum mechanics(x)
- Modern optics(x)
- Electricity and Magnetism(x)
- Survey in modern physics(x)
- Modern Lab(x)
- Mechanics 2(x)
- Intro to quantum 2 x
- Electricity and magnetism 2 x

- Mathematical Methods for scientist(x)
- Partial Differential Equations x
- Complex Variables x
- Thesis in physics (x)
- Intro to abstract mathematics x
- Honors abstract algebra 1(x)
- Honors Analysis 1 (x)
- Numerical Analysis x
- Stochastic processes (x)
- Numerical Algebra(x)
- data structures(x)
- Logic design(x)
- Parallel Computing (x)
- 35(-1, -4, maybe) main(13-15 electives)
- MATH 462 - Differential Geometry(1)(x)
- MATH 467 - Honors: Topology(2)(x)
- PHYS 441 - Introduction to Computational Physics(15)
- PHYS 493 - Research and Independent Study(3)(x)
- PHYS 531 - Classical Mechanics(4)(x)
- PHYS 541 - Electromagnetic Theory(5)(x)
- PHYS 571 - Mathematical Methods in Physics I(6)(x)

- PHYS 573 - Numerical Methods in Physics(7)(x)
- MATH 515 - Analytical Applied Mathematics I(8)(x)
- MATH 567 - Riemannian Geometry I(9) x
- MATH 568 - Riemannian Geometry II(10) x
- MATH 569 - Seminar in Topology and Geometry(11)
- MATH 574 - Finite Element Methods(12)
- MATH 578 - Introduction to Scientific Computing(13) (x)
- MATH 585 - Optimal Control Theory(14)
- PHYS 599 - Seminars(x)
- PHYS 593 - Independent Study(x)

28.3.1.2 Schedule

physics general math comp sci

28.3.1.3 First Year

1. Honors ethical theory
2. Honors intro to philosophy
3. Professional Responsibility
4. HR: Western Civilization
5. HR: World civilization
6. Intro computer science

7. Diff equations 1
8. Matrix Algebra
9. Fundamentals in modern physics
10. mechanics 1
11. Intro to abstract mathematics
12. PHYS 503 - Physics Colloquium

28.3.1.4 Second Year

1. Thermal Physics
2. Electronics Lab
3. Intro to quantum mechanics
4. Modern optics
5. Electricity and Magnetism
6. Survey in modern physics
7. Modern Lab
8. Mechanics 2
9. Intro to quantum 2
10. Electricity and magnetism 2
11. Partial Differential Equations
12. Numerical Analysis

28.3.1.5 Third year

1. Honors abstract algebra 1
2. Honors Analysis 1
3. Stochastic Processes
4. Numerical Algebra
5. Mathematical Methods in physics
6. Research & independent study
7. Differential Geometry
8. Honors: Topology
9. Classical Mechanics, grad
10. PHYS 541 - Electromagnetic Theory, grad
11. MATH 515 - Analytical Applied Mathematics I
12. Independent study and research

28.3.1.6 Fourth year

1. Thesis in physics
2. MATH 567 - Riemannian Geometry I
3. MATH 568 - Riemannian Geometry II
4. Numerical methods in physics

5. data structures
6. Logic design
7. Parallel Computing
8. PHYS 599 - Seminars
9. PHYS 513 - Problems in Theoretical Physics I
10. MATH 578 - Introduction to Scientific Computing
11. MATH 534 - Calculus of Variations
12. MATH 537 - Mathematical Principles of Continuum M

28.3.1.7 Others

1. PHYS 514 - Problems in Theoretical Physics II
2. MATH 585 - Optimal Control Theory
3. MATH 574 - Finite Element Methods
4. PHYS 615 - Astrophysics and Cosmology(grad required)
5. PHYS 643 - Computational Physics(grad required)
6. MATH 515 - Analytical Applied Mathematics I
7. MATH 571 - Numerical Analysis I
8. MATH 572 - Numerical Analysis II

28.3.1.8 Possible summer

1. Graduate Reading in Mathematics
2. Research and independent study
3. graduate reading in physics
4. Special Problems

28.4 *Grad**28.5 *Early career****28.6 *Late Career**

Chapter 29

Personal-Analysis

"The unexamined life is not worth living" — Socrates

29.1 Introduction

I will use this journaling of sorts to examine myself and my own personality.

29.2 Disgrace and Pride

(if anyone else reads this, don't pay too much mind. I use words differently than their actual meaning. I don't word things very well. I was simply trying to capture my own indescribable and esoteric and possibly failable thoughts in this moment on this topic, my actual feelings are very different than what can be interpreted through the word choice.)(Additional update: upon later analysis, I have found that these ideas of pride and disgrace are for the most part the idea of seeing my earlier mentioned "personal values" in a light

very similar to traditional morality. This doesn't fully explain it and I will continue to work on refining these ideas beyond the esoteric concentration currently presented.)

Why is it that you are so accustomed to the use of disgrace in your speech and why does it effect you in such a manner. The idea of 'disgracing' yourself is so vital to your worldview, it effects everything; morals, interactions with other, and so much more. It isn't even like you are all that effected by the thoughts of others. In fact the most abnormal and extreme feelings of self-disgust and disgrace are related to a refec-tion of the thoughts of other. Why is it that I feel like to see other, value their opinions, and let them influence men to disgrace myself. It is an odd thing, yet it is so very natural. It influences everything, and it is so much more extreme when I am in isolation as I am now. What is it about the concept of being influenced by others to be so repulsive, so disgusting that I won't just let it influence my actions but I will try to persuade others my way is best with pride. With this eternal pride that locks me in, says that I am always all of the way in all things. There are so many things to think about in this discussion so I will try to take it piece by piece and hopefully add some things.(also, what it with the strange switching from dialogue with an external, internal, and this explanation style?)”

Well, disgust is a natural motivating force. It motivates far better than most, it is consuming. It involves fear, pride, righteous anger, annoyance, and so many more extreme and strong emotions. It motives far more than most, and it comes so easily, especially for someone like me. There is no greater fear than falling in ones own eyes. My disgust of others is just who I am,

it is what makes me who I am. It disconnects me from the petty emotions around me. I mean, can people really say that those are better; to be insecure, jealous, vapid, empty, etc? Is it really better? Take insecurity, it is such a strong emotion that consumes people so very easily, making them bicker and fight, making them claw at each other with their broken paws that do very little besides hurt themselves and hurt others by their own volition of nonsensical blight. My 'pride' can rise me above that. If I do not see them as anything, then for what reason do I have to be insecure about this vapid collection of dust. Now I am not truly some raging narcissist who sees no one beyond and object for my own use. I see people as they are, I care for others, value them. I just simply do not value them in the way others prescribe them. I don't see them as threats, competition, just broken monkeys in need of assistance (and that I am the same way and can occasionally use assistance myself, just less than most.) I still see them, I make friends just as easy, if not easier than most due to this.

This disgust is a strengthening endeavor that assists me in many ways. My self-disgust pushes me further. It is what led me to teach myself calculus in elementary, and has brought me here now with graduate level knowledge, research, and so much more. It has disconnected me from the vapid desires of others in their looks and whatever other nonsense. (though I will add, this disgust is not some obscene emotional self-distrain but rather a more abstract understanding that doesn't truly make me feel bad, just conscious of 'wrong doing.' More so like a passion against, I feel no negative emotions about myself)

Though I must acknowledge that this does not make

me better, it makes me better within my own eyes, my own values, not others. My values have no greater truth than that they are my own. So my disgust should always remain abstract, looking down at the ideas not the people. I have done great at this, never prescribing habits or concepts to people. Though as of late I have had trouble with this, seeing people beyond the moment. Being conscious of their deniers and thoughts, it can create some discontent. I should remain as I was, seeing people only in the instant. Nothing more. I am the only conscious being within the confines of my own mind, for it is better this way. You cannot accuse a rock of moral failure, only a man. For which I am the only man that I see, the only man that I know, I am to constituent for the moral blame. The disgust should be within me and me alone seeing only me. For I am the only one held to my values and the only one who could suffer the breaking of them. For they are my values alone, that is the purpose of the disconnect. Why my values must be mine alone, for I am the only one to be connected to them, the only man in my eyes.

For this I also hold the burden of thought and reason. The thought behind my morals, my values, my beliefs, my religion, my reality. I see the world as my land for the conquest of my own knowledge to build up it all. That is the whole purpose of this book, to fully disconnect, to find myself fully and fully alone. Because, in the end that is all that matters in the abstract. For sure, I love my family, my friends, and my fellow man, but I love myself in the way that one can only love oneself, the expectation of my own measurement, the pride in my achievements, the disgust in my faults, the understanding of my beliefs. For one cannot love others without loving oneself, for love is derived

from ones understanding of their own values, who they are in all of reality.

Now why are these feelings and thoughts all the more present in isolation. For when with others, those are my real feelings. In isolation I attempt to derive and find, deriving and finding the strange and unwieldy emotions of the mind does not come truly and with the same accuracy of that of physics. It comes in strange botches of thought that don't mean what they literally mean but can be described through thoughtful examination of the words, other words, and the actions of the person.

When did I become this, so thoughtful behind it all. Seeing the world beyond the material, seeing my thoughts beyond the exact. Maybe I really am chaining, in a way that must lead to the changing of even my most basic assumptions. (this last paragraph is really stupid)

29.3 On Ayn Rand

It would be obvious to admit that this section would be an analysis and critique of Ayn Rand's ideas, but like the more erudite among you will notice that this would go against the structure of this chapter. This is exactly correct, this will rather be an analysis about my propensity towards Ayn Rand's ideas.

While this may seem useless and without purpose, it isn't. It has been a strange psychological question about my enjoyment of Ayn Rand's novels even though I starkly disagree with actual philosophy and the fact that her books lack the many of the general characteristics that generally allow me to enjoy such novels. So

what is it?

There are several reasons, though I will start with the most fundamental. Simply put, I see myself within the characters. Most particularly Howard Roark. Many of the descriptions of his won emotions and others description of me mirror is a strange way. While not absolutely similar, it builds off in a way more closely connected than any other fictional character. How he exists as an independent entity, not noticing others but living by their moral code not out of other means but as its own mean. Because of integrity above all. The way he is an act of moral striving rather than a disgusting abstraction for those even more disgusting to connect to. He lacks those pathetic neurotic tendencies that those around me let control and give authority over them.

Beyond that, he gives me ways to articulate my own personal feelings in a way that I have never seen before. Being "to proud to boast" by seeing both the criticism and complements of the world to be equally insignificant because it does not come from myself. To find pride, value, truth, within myself not within the pathetic world beneath me. Seeing another person see how pathetic the social validation games are, not is disdain for others but as seeing it beneath myself.

The way happiness is the his natural order rather than some far flung ideal that is beyond, that negative emotions seem dulled by their pathetic attempt. That he is truly happy at all times, just as I am.

The way compromise, even in the slightest way seems to be evil. That this concept has been a driving force in my life thoroughly. Because it is a self-betrayal, a betrayal that can't even be thought about. An evil beyond belief and idea. Even a white lie, a broken ideal

without real backing, a principle made as a child, and so much more seem evil and I can't figure out how others live with it. Though these compromises are never entertained long enough to feel anything beyond the knowledge of it.

The way others see him as cold and arrogant despite him clearly not being, due to there misunderstanding of him. Because he is independent, because he doesn't care about friends and what insignificant interactions they had, what their friend's did to annoy them. That they don't care about philosophy, physics, mathematics.

How he is both happy in isolation and with others. Equally, because the existence of others doesn't have that effect on him.

The way his creative and logical thoughts of Philosophy, physics, mathematics, and other intellectual topics are all that matter, well beyond the existence so often people confine themselves to.

Finally, he lacks all neurotic tendencies. No desire for complements, praise, people to soften their words, bend down. These neurotic tendencies are found everywhere, in everything. People, fictional characters, and others. Though I feel such thoughts so rarely. It is a impeded idea in almost all of fictional by its own virtue, but I never see it within my own mind. He like me is truly beyond these petty neuroticism, no capacity for them at all. No vulnerabilities, no anxieties, no insecurities, no of it.

29.4 On the Pursuit of Thought

NFC, or Need for Cognition is a psychological concept seen clearly in this book. Though it may not be obvious to the extreme extent it is true.

Much of my ideas of philosophy, ethics, physics, and more may seem like a desire to know reality in its greatest extent (and this is certainly true), but in its most basic and primate way, it is my need to think.

I love thinking, it is my favorite thing. This is what drives me, my desire to satisfy that part of me. I think about anything complex enough; philosophy, ethics, physics, mathematics, coding, economics, political philosophy, literary analysis, world building, international relations theory, geopolitics, formal analysis, psychology, meta-cognition, and so much more. The more thinking required the better.

In fact, I love it the most when it is beyond me. When it takes me weeks to not even fully understand what questions to ask, when it feels beyond my comprehension, when I think for days and go nowhere. I love it, the scavenger to knowledge, then to actually get it. For it to all fall into place just as it should.

Now very little does this, in fact most things just come. They are understood intuitively. Seem to be basic to even consider. Even some of my other hobbies like psychology, politics, and such seem to be basic, and most of the other things beyond my intellectual hobbies seem so basic as to not even give it time at all.

Another funny consequence, is despite my almost compulsion to efficiency, I still am drawn to complexity. Despite my normally physicalistic and literal tendencies I am also drawn to the abstract. I have found this

most easily seen in my obsession with using higher order math. I use tensor calculus, hamiltonian mechanics, geometric identities, when similar methods can do it. I obsess over these ideas when there are more practical matters.

29.5 Nietzsche's Sovereign Man and Morality

Upon further reflection, my chapter on disgrace and pride has much connection with Nietzsche's "sovereign man." Here I will first explain what that is, its connection, and divergence.

Before I begin, one thing with Nietzsche's writing is that it is up to controversial interpretation. Some say the sovereign man is sincere, other ironic, others meta-ironic, others see it simply as a literary device, and there are still other interpretations out there. Luckily though, for the purposes of this exercise it doesn't matter Nietzsche's intentions, only my intentions and ideas. For instance, Nietzsche's reasoning for presenting the sovereign individual differ from mine; which I will present later.

Now to actually begin with the analysis. Who is the sovereign individual? The sovereign individual is the archetype of Nietzsche's will to power (the *Übermensch* later takes its place as a further extreme but I care little for this idea.) It is an implied ideal in which a person creates mastery over their own life. To live their life in accordance with their values, that they themselves had created rather than inherited. They do this with their mastery over their own impulses; a mastery so extreme

that they eventually shape their impulses into however they consciously wish. Further more, these values and 'morals' are created through anesthetic ideas, creativity, future-bound, and affirmation to the power of life rather than traditional fear. Finally another key part in is surprisingly forgiveness, though not in the traditional sense. Rather than forgiveness caused by God or some other values it is a combination of the acknowledgment of the fact that most people are incapable of true moral thought, self-mastery, and to let go of their resentments and impulses; another key idea is simply the fact that not forgiving hurts the sovereign individual, Nietzsche's suggests that it is better to simply forget about the trespasses rather than hold on to resentment and call it responsibility to forgive when it is hidden resentment. Essentially the sovereign individual is the archetype of the promise made flesh(or word)

Now how does this relate? Well before I get too pride and disgrace, let's explore the philotimic virtue. The philotimic virtue highly mirrors the concept of creating your own values, setting your life and soul in pursuit of them, and finally holding yourself to them in a way that can be seen as fanatic to the outside. Also, creating these values individual of the existence of others(though the philotimic virtue includes the will and thought of God to yet be higher than the will to power of myself.) To live your life as you will it so. T

Now disgrace, the idea that my personal values that I have willed hold the same good and evil. That to betray them is evil. But both ideas have a similar theme, by not hiding from it but rather carrying the burden is guilt erased. By taking the conscious action of purifying oneself one becomes worthy of forgiveness(not God's forgiveness but my personal one) and because

I have achieved this, while I carry momentary understandings of mistakes unlike the sovereign man, I hold no returning 'guilt' or resentment of any kind. No neurotic tendencies, no projections onto other, none.

Pride, this as mentioned earlier is the same as the earlier mentioned philotimic virtue. The idea of creation and happiness as a norm that has been willed by me as an action of my pursuit of virtue and creation. This is where the disgrace truly comes in, it is reabsorbed, not as pain but as energy for the recreation of the self in the form of higher values.

Another key idea is that of forgiveness. Once again there is the connection between my idea and his. The fact that for the self; moral, ethical, value-driven, etc is not a same thing. It is a rapture of the very existence, but rather than waste energy on being sorry for oneself, one must use that energy on self-perfection. Though, forgiveness of others is another story. Other people are not capable of moral action, they are slaves to impulses, temperament, social constraints and much more. While people claim to be moral, much of there actions disgrace their ideals, their intentions align with physiological temperaments rather than values, and they abandon so much so easily. Their transgressions should be forgotten, or better yet not noticed at all.

Though, I am sure you are thinking about the differences(I mean I have been sprinkling them about these section) and while they are important to some extent. I don't want to go though a point by point refutation of Nietzsche ideas, especially when this analogy of the sovereign man is meant more an a psychological analogy to help explain intuitively my own personality rather than philosophical affirmation.(I disagree with Nietzsche on much)

One thing I will say though is the difference in end goal. For me, the sovereign man in a more refined sense(axiomatically driven, recursively made, God fearing, etc) is the final form. The *Urbarmensch* is useless. The chaotic form of disparaging logic, reason, and God in forming morality disbanded the entire project and is just as disgusting as the slave morality of others.

29.6 Half-Growth and Half-Death

While many proclaim that my ideas of seeing people not as thinking being, not as moral agents is misanthropic and anti-social. That I should have more respect for others, that this lack of respect can lead to later personal problems with others. Though I disagree, first I will go over how I have seen others interact that makes me disagree, and my own personal evidence from my falterations with these values.(one thing I will add, is once again these are not true literal beliefs but analogies to explore esoteric identities.)

Think of how people treat children, then people they deem as equals, then themselves, and then those they deem as higher than them when they perceive a slight.

The child's slight is either ignored or the person takes conscious action to help them, not by expecting moral action as they would with others but by understanding the child's abilities and inabilities and working around them in a kind manner.(Bar extreme causes or grotesque individuals)

With others there exist a range of reactions to slights, but their is a clear differentiator. They expect others that are 'equals' to exist in a semi-moral manner(this is because most people hardly understand moral manner

to begin with). They try to act in kindness and understand others faults and temperaments but when push comes to shove they resent others, they expect from others. This can come in a wide variety of ranges: resentment, anger, fear, annoyance, instability, and so much more. These reactions are almost always unproductive and hurt relationships, people and such. They also use the perceived moral responsibility to negate their own moral responsibility in many cases (under my analysis, this is where most pain received by the affliction of others comes from, either consciously or unconsciously)

Then from here it is easy to see that the extremes of these go up and up.

These are where a majority of the petty, vapid, and pathetic emotions I wrote about earlier come from.

Now what I suggest isn't as radical as it seems, it is just shifting the average person closer to where others see children, no one other than God as above me. That I intentionally analysis and 'handle' other people. I understand them both to better see them as pathetic not in a disgusting way but like a child. Also, to better handle them for interactions with them.

Though, one thing I didn't include in my earlier writing is that some people do come closer. Select people I know that I am close to enjoy (or don't depending on how you look at it) the responsibility of a human and conscious life within my own ideas.

Now, where does my growth and death come from? Well in recent time I have faltered, I have subconsciously found that I do expect things from others. These is so terrible in my eyes fro several reasons. One, as I have mentioned earlier, this 'equality' is the source of much

of our troubles in the modern age void of true horrors. Second, for my personality and moral theory this effects me in a much more extreme sense than it does other. From a shallow perspective I am an extreme puritanical person who is obviously very prone to disdain. Beyond that I hold some values in high regards that many don't hold at all. Though, more truly than that, my in my moral theory intentions mean everything and so very few people have truly pure intentions. Now I don't mean everyone is a selfish jerk(though many are) but beyond that many are husks of people that only follow moral theory because they don't comprehend any other way, others hedonists that find minor moral action as easier, others do it because of insecurities, and so on and so forth. The concept of doing things for the mere fact they are moral/logical is lost on almost every person you will ever meet.

Here I will give an illustration. Last week(as of writing this section) I had an interaction. Someone had very condescendingly given me 'advice.' Telling me in a clearly condescending and antagonistic tone to remember to unroll my selves I had rolled up to wash my hands.(this is also after of many other similar interactions with this person) Now because I knew that to explain the ethical ideas of attempting to 'assert' fake authority on another in such efforts was immoral due to the fact that on a psychological basis that such things could very easily be comprehended as attacks on their own self-determination and competency. Also, that such actions could easily be interpreted(and likely truly) as things like need for superiority, low self-esteem, extreme lack of social awareness, control issues, projection of their own incompetence, desire for a reaction, or malignant/grandiose/competitive narcissism.

Though I instead let it slide and said thank you, I didn't feel it would be worth it or would give any results on the matter. Now I didn't expect them to realize their fault or anything of that extreme. I expected them to say your welcome either as a empty social connection or an understanding of the fact that I understood their game and they would either give up or hopefully be filled with self-disgust over the conscious acknowledgment of why they did what they did. Instead they didn't make eye contact and instead "hmf" at me. I still had the rational ability to understand that this person was either too stupid or immoral to understand whatever I wanted to say on the matter, so I decided not to. Though regardless I was filled with disdain. I was disgusted by such extreme evil. I know for many this doesn't seem like evil, but in the eyes of intentions it can only be logically assumed as. Those of lacking ability of intelligence to react in moral manners wouldn't make such an extreme mistake, those powered by resentment or anger would likely be filled with shame of such actions, and so on and so forth. Likely the only remaining interpretations would be that of malignant/competitive narcissism, desire for the reaction/pain of others, and other similar grotesque ideas. Now while I see the earlier stated ideas as immoral, these take special places, especially for someone like her who has the capacity of living in dignity.

Now this isn't some one shot, lately I have been more and more likely to to feel disdain for others. I don't know if it is puberty, social connection, increases in closeness with others, or whatever. All I know is that while this is something that so many other have proclaimed as what would be the greatest thing to my self-perfection is rather the worst thing.

29.7 Lack of Aesthetic

When I am referring to aesthetics in this section, I do not mean philosophical aesthetics, as in valuing something over another; I mean traditional ideas of beauty in music, art, and natural affairs.

In this sense of the word, I lack almost all aesthetic values. I have no favorite color, no concept on beauty in most things, and no care for musics, visual art, or other 'artistic' concepts.

This doesn't mean I have none. I have some interest in the artistic understanding in complex and intriguing literature, I can also be temporary incapsed by complex art/music(though only as long as it takes for me to understand it, and if it is too abstract as to be useless I have little or no care for it); beyond that I can find beauty in mathematics, logic, efficiency, ideas, plans.

In relation with that I have no feelings of sentimentality, meek emotions, and other similar emotions.

Now, I have always naturally found all of these things as beneath me and childish(and still do to some extent) though I have been taking time to try to expand my horizons.

As I have been doing this, I have noticed some changes. I have thought in more lofty ways than before, been more open to some experiences, and even engage with emotions in a way previously unheard of. Now here I will leave with this, but later I will explore this in more detail. What is actually causing this change, Explore what the change actually is rigorously, and whether is it actually a good or bad thing?

29.8 Boredom

Just as NFC is a primary driver in my life, so is aversion to boredom.

The most obvious connection is simply for my NFC is grown and in a feedback loop with my aversion to boredom. I satisfy my boredom with challenging cognitive tasks. This is one of if not the largest driver in my self-education, research projects, writing this book, and so much more are driven by aversion to boredom.

A little beyond that is ambition. Just as I choice challenging cognitive tasks to satisfy my boredom, so do I choice other challenging tasks. Leading, planning, mentorship, responsibility, work, physical activity, and more. This is what pushes me in SVA, Slack, JROTC, Scouts, OA, and much more. These activities satisfy my boredom.

I will go even further and say this desire for challenge is primary from aversion to boredom, not fear, perfectionism, and other traditional psychological reasons for people pushing themselves further than most.

Now, my aversion to boredom isn't just in ambition and other productive means. The most obvious is in watching TV or non-educational videos. Though this is largely to simple to take any time analyzing.

Though there are other things to analysis, relations. First I will go over casual relations.

For some reason, as I will discuss in more detail later on, I switch between 'extroversion' and 'introversion' in a sense. When I am away from others I hold no desire to be with them, I satisfy my boredom through abstract thought, arguments with myself, physics, and other similar things. That this feels the most natural

thing. Though when I am with others this switches. Getting lost in thought no longer feels natural but rather challenging and hard to focus in the same way. I naturally feel the way to satisfy my boredom is through conversation with others. It is an interesting development that I will likely look into further.

29.9 Why Physics

While I have many interest; math, coding, economics, psychology, philosophy, logic, and many more. Though, one interest stands above and beyond the others. Physics. Ever since I was a kid I have been obsessed with physics.

Now I am sure I can come up with some lofty reason on how physics is the nature of the universe in its purest form, or the most fundamental science of them all. While I am sure that it is part of the reason I like physics so much, it is not one of the primary.

The most easily observed is its balance of abstract thinking and practicality. It is one of the most abstract and purest forms of logic other than pure maths and some forms of philosophy, but unlike the others it has a more direct connection to real-world application. For instance, my work is working towards fusion reactors, something I think once working will be one of the most influential and greatest creations of the modern age. To combine both reduction to axioms and analytics and contructionism of creating something practical.

Next is the challenge. Not only is physics itself extremely challenging, with some aspects of it taking months of studying to even comprehend it, it also combines many other hard disciplines; math, coding, engineering, and even metaphysics as times. This challenge is

exhilarating and as mentioned elsewhere is one of the things that i most desire.

On combining other things, physics is a multi-disciplinary subject. Combining and using many of my other hobbies.

There are many more reasons but these are the main ones.

29.10 Lack of Resistance

As developed throughout this book, especially in "On the Pursuit of Thought" the idea of challenge as a goal.

The joy of an intellectual challenge specifically. To mull over a topic for hours. To have something beyond current comprehension. Something that doesn't make sense. Then all of a sudden explodes, not only to explain itself, but making connections all over. Like a flood.

This is my greatest joy, what I live for. Though, even as a teenager I am coming to limits. I will examine this in parts, first through my self-education and then through external world interactions.

What is going on personally is I fear that I won't feel this feeling. Many topics like economics, psychology, geopolitics, history, philosophy, literature, and more don't give this rush anymore.

As an example I will examine literature. In the past year I have found interest in literature, a topic I had previously missed. It was enticing, learning about the usages of symbolic characters(and figuring out who was and what was), setting as a character, philosophy through stories, and so much more. I had that excite-

ment, though lately I don't. While there is still more to learn, it isn't the same. Everything new, is obvious. No requirements of complex thought when the workings are already there, only new information to be feed into the pre-made algorithm.

Now, I still have some exiting things in the above fields and much, much more in abstract mathematics and physics, but I am only 16 at the moment. If I am already this far along now, what is to say in 10 years there will be much left, what about 40? I have already surpassed all traditional classes and all that is left is research articles and manuscripts, these will leave me for quite some time, but who knows how long that will be.

This also continues in my personal life, school has the intellectual engagement of watching paint dry, talking to people is a burden, leadership roles still have excitement but have less and less return,

29.11 Am I Misanthropic?*

My misanthropic like tendencies are clear throughout this book, but am I really misanthropic? No, well... maybe a little.

I do have many friends, friends that I enjoy. I like talking to people. I am truly an extrovert in nature.

Though, beneath that there is a disconnect. I don't truly enjoy talking but rather staving off boredom. While in isolation I generate distrust and disgust of others. Many people I dislike more than I dislike boredom and everyday the percentage of people within that camp grows.

29.12 Depth of Feeling

All of this discussion on emotions and psychology can leave a reader with the thoughts of a sensitive and emotional boy, but this is far from the case.

While I use extreme words to convey my ideas, the extremity is not there. All my life my feelings have been dulled. While others are overwhelmed by feelings, for me they are hard to observe. Like they don't fully touch me.

This isn't true for all emotions; my passion, excitement, fanaticism, and need for cognition are all real. This doesn't take away from my thesis.

Beyond these, I have always required conscious effort to hold on to emotions. While this may sound like lamentations, it is not. It is a great thing to forget anger and pain because of their insignificance. To never feel overwhelmed or anxious, to have control over my thoughts and actions, I am in control. I love that I am this way.

There was once a time that this made me feel inhuman. While others talked about their emotions, though that feeling like all has passed. Though, don't take this too far, I am by no means some robot devoid of all emotions, just simply that for any population with variance some people will be above or below average at things. For me I am simply far enough that observing others like me is a rare enough occurrence to think it truly is rare (but it really isn't.)

29.13 Language Usage*

Before I actually begin upon the main idea of this section I feel the best way to introduce this is to explain the inspiration of this section.

The inspiration is the oddest place possible; Ted Kaczynski, or the Unabomber.

I remember when I was a kid I always hated the idiom "Have your cake and eat it too." The reason for this is due to the fact that the have your cake is placed first, and this fit(in a way that doesn't fit with the idiom.), because you must have your cake to eat it. So, I started to switch it around to "Eat your cake and have it to" though never really liked that. Until I read that part of the reason the Unabomber was caught was due to his strange phrase "Eat your cake, and have it all the same." When I had read this my first thought was insult that I hadn't thought of that myself.

Once I thought of that I realized more connection. Some more 'normal.' Starting with a thoughtfully researched and observed effect being the fact that highly educated members of fields like physics, mathematics, logisticians, computer science, and philosophy use mathematical and logic terms in regular speech. This is because their thorough and clear definitions have clear applications. Axioms, manifolds, prior, bias(in a mathematical sense), fallacy, paradox, induction, inference, deduction abduction, ambiguity, equivocation, disjunction, mapping, inversion, duality, convergence, divergence, topology, entropy, singularity, resonance, phase transition, field, fractal, lattice, span, gradient, symmetry breaking, null, second order, and more. This isn't that odd considering that it is more wide spread,

I mean a couple of those terms described above have even introduced themselves into colloquial speech(though used slightly differently.)

Another is using words that were created in academia, then introduced into regular speech that then changed it, in their original usage.

Though some are a bit stranger, mainly the fact that I simply create my own definitions for words or even create my own words.

So lets go through these one by one.

The first is very simple, these words are very useful in regular speech, convey abstract ideas easily, and can be applied easily. The mere fact that I understand them makes it only logical that I use it.

Next, the usage of academic definitions has a similar requirement. The fact that these academic definitions are generally more completely and logically defined than generally used word.

Finally, the changing of words and creation has a similar efficiency and logic.

Though, this brings new questions, why do I feel so comfortable with this, what criteria do I use for greater 'logic,' what is with this obsession with logic, why don't I just pick either change current words or create new ones instead of do a bit of both, and much more.

...

29.14 Euphesus*

As I mentioned earlier, there has been change in me as of recent, and it has recently come to head with the advent of 'dating'(not actually dating the girl formally[will explain more later]Also not completely sure it is 'coming to head' just more extreme than before).

What I mean, is for the first time in a long time I feel conflicted. As strange as it may sound, I have been complete for a while now. I never feel internal conflict, neuroticism, second guessing myself, or anything of that such. I have for a very long time clearly defined my morals and values and thus never needed to second guess myself. While I am not perfect, the few times I make mistakes, I clearly identify them after the fact and rectify them. There is no lingering guilt nor do I go back and forth between what I should have done or should do for the future.

Now, in the last year or so, I have had some rare conflicts; many of whom I have talked about in earlier sections. Though with this ambiguous relationship, it has become an almost daily thing.

I went back and forth between whether or not I should ask her out in the first place. When I did I had a lingering regret about doing it over text rather than in-person. Later when texting her after the fact, I felt the inexplicable desire to talk to her, without any real end-desire other than the conversation. First, I felt conflicted because I have never felt the such a desire, in fact I often was disgusted by others for this desire. It always seemed beneath me, while it wasn't against my values, for someone who has always been so 'complete' and whose entire personality and temperaments are

recursively defined by myself the mere fact of something new is insane upon the face of it. It is like a self-betrayal which it took me a while to figure out what the betrayal was in the first place. Furthermore, this caused me to start questioning thing, why I didn't desire to talk to my friends about nothing in particular, should I start doing it, etc. It even lead me to try it. Then this started to make me think about how some random girl I am not even formally dating, has caused me to alter my own behavior. Just slightly, but nobody alters me even slightly.

Once I had finally gone through all of that and moved on, I decided I would talk to her over text. During that time I neurotically tried to when would be a good time and what to say. Once again, I don't do that. I don't care what other people think of me, I don't care if the message wasn't perfect, I don't feel neurotic over anything, certainly not a person. Yet I did, I thought about it. Finally, I texted her, it went nice.

I decided I should keep in touch, because I wouldn't see her in person for a month. So I decided to reduce ambiguity I would simply have a schedule of when to initiate a conversation. The number of days between initiations was meaningless and I knew I just needed some arbitrary number, yet I thought about what would be an embarrassingly long amount of time.

Later, came the day for my next initiated conversation. and once again I spent way to long figuring out what to say, but when I did, she hadn't responded. I didn't think much of it, due to the fact there were dozens of possible explanations for the lack of communication that were fine. That was until the next day, when she responded on a group text, but continued to ignore the individual one. Now even still, hours later

I am still thinking of it. Still conflicted, wondering if asking her out was the right choice, or my number, or what I said. Wondering what my next steps should be.

While yes, I understand these emotions are normal for an adolescent boy of my age, but they aren't for me. Not only am I rarely conflicted about anything. I am much less about people. I am the kind of person who interacts with people for amusement or out of responsibility; deep down I don't care other people in the way required to feel these neurotic thoughts and emotions.

It is so strange, having the last couple years of my identity and foundation so clearly defined for everything. Only to bring out something new, something that doesn't fit. Now I don't know what to do; stay rigid or change (but change what)

Over the last month or so I have gotten over the visceral reaction. Now that I am back from my trip we can see each other in person, but haven't very much.

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2025-11-27

ajsp4

November 29, 2025

Contents

29.15 Links to Book Chapters

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This is my first journal write using EMACS, for that has been my project at the moment. Creating a doom Emacs with Org-mode enhancements, AUCTeX/pdf tools. Org-roam(graph+server), Org-bable for multiple languages(FOR python, Julia, Jupyter, and shell), LSP for coding, Magit(for git), projectile/treemacs(for using projects), a daily journal that auto inserts into my \LaTeX book, and create Org nodes and graphs for chapters and journals.

For the most part I have this set up. I need to edit AUCTeX so that it uses latexmc rather than latex, so that it will run multiple times for the book-like structure. I think I have more dependencies I need for the languages. I still have more to learn to get comfortable with this program. Also, I am not sure that the program auto inserts my journals.

```
print("2+2 =", str(2+2))
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