Contents

Chapter 1

Electromagnetic waves

1.1 Electromagnetic theory

1.1.1 Macroscopic Maxwell equations

Theorem 1.1.1 (Gauss' law). *Let ω be a solid and $\rho(\mathbf{r}) : \Omega \longrightarrow \mathbb{R}$ its charge density distribution. Then,*

$$\oint_{\partial\Omega} \langle \mathbf{E}, \mathrm{d}\mathbf{s} \rangle_I = \frac{1}{\epsilon} \int_{\Omega} \rho(\mathbf{r})\, \mathrm{d}v\,, \qquad \operatorname{div} \mathbf{E} = \frac{\rho(\mathbf{r})}{\epsilon}. \tag{1.1}$$

Definition 1.1.1.

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}. \tag{1.2}$$

Theorem 1.1.2 (Gauss' law in media). *Let Ω be a dipolar solid and $\rho_f(\mathbf{r}) : \Omega \longrightarrow \mathbb{R}$ its free charge density distribution. Then,*

$$\oint_{\partial\Omega} \langle \mathbf{D}, \mathrm{d}\mathbf{s} \rangle_I = \int_{\Omega} \rho_f(\mathbf{r})\, \mathrm{d}v\,, \qquad \operatorname{div} \mathbf{D} = \rho_f(\mathbf{r}). \tag{1.3}$$

Theorem 1.1.3 (Gauss' law for magnetism). *Let Ω be a solid. Then,*

$$\oint_{\partial\Omega} \langle \mathbf{B}, \mathrm{d}\mathbf{s} \rangle_I = 0, \qquad \operatorname{div} \mathbf{B} = 0. \tag{1.4}$$

Theorem 1.1.4 (Faraday's law for magnetism). *Let Σ be a surface formed by a circuit $\Gamma = \partial\Sigma$. Then,*

$$\oint_{\Gamma} \langle \mathbf{E}, \mathrm{d}\mathbf{r} \rangle_I = -\frac{\mathrm{d}}{\mathrm{d}t} \int_{\Sigma} \langle \mathbf{B}, \mathrm{d}\mathbf{s} \rangle_I, \qquad \operatorname{curl} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \tag{1.5}$$

Theorem 1.1.5 (Ampère's law). *Let Σ be a surface formed by a circuit $\Gamma = \partial\Sigma$. Then,*

$$\oint_{\Gamma} \langle \mathbf{B}, \mathrm{d}\mathbf{r} \rangle_I = \mu_0 \int_{\Sigma} \langle \mathbf{J}, \mathrm{d}\mathbf{s} \rangle_I, \qquad \operatorname{curl} \mathbf{B} = \mu_0 \mathbf{J}. \tag{1.6}$$

Theorem 1.1.6 (Ampère's law for variables fields). *Let Σ be a surface formed by a circuit $\Gamma = \partial\Sigma$. Then,*

$$\operatorname{curl} \mathbf{B} = \mu_0 \left[\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right] \tag{1.7}$$

Definition 1.1.2.

$$\mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}. \tag{1.8}$$

Theorem 1.1.7 (Ampère's law in media). *Let Σ be a surface formed by a circuit $\Gamma = \partial\Sigma$ and $\mathbf{J}_f : \Sigma \longrightarrow \mathbb{R}^3$ its free current distribution. Then,*

$$\operatorname{curl} \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}. \tag{1.9}$$

Proposition 1.1.8. *If $\mathbf{P}, \mathbf{M}$ ad $\mathbf{J}_f$ are linear, then the equations are linear and superposition principle holds.*

1.1.2 Boundary conditions

1.1.3 Lorentz force

1.1.4 Material response

Proposition 1.1.9. *Let Ω be an homogeneous, isotropus and linear solid with $\rho_f(\mathbf{r}), \sigma_f(\mathbf{r}) = 0$. Then,*

$$\operatorname{div} \mathbf{E} = 0, \qquad \operatorname{div} \mathbf{B} = 0, \qquad \operatorname{curl} \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}, \qquad \operatorname{curl} \mathbf{B} = \mu\epsilon \frac{\partial \mathbf{E}}{\partial t}. \tag{1.10}$$

1.1.5 Energetic relations

Proposition 1.1.10. *Magnetic energy density distribution*

$$\eta_m = \frac{\mu}{2}\|\mathbf{H}\|^2 \approx \frac{\mu_0}{2}\|\mathbf{H}\|^2. \tag{1.11}$$

Proposition 1.1.11. *Electric energy density distribution for non-absorving media*

$$\eta_e = \frac{1}{2}\langle \mathbf{E}, \epsilon_0\mathbf{E} + \mathbf{P}\rangle_I = \frac{\epsilon}{2}\|\mathbf{E}\|^2. \tag{1.12}$$

Definition 1.1.3. Poynying vector

$$\mathbf{S} \coloneqq \mathbf{E} \times \mathbf{H} = \frac{1}{\mu}\mathbf{E} \times \mathbf{B}. \tag{1.13}$$

1.2 Wave equation in dielectric media

Proposition 1.2.1. *Let Ω be an homogeneous, isotropus and linear solid with $\rho_f(\mathbf{r}) = 0$. Then,*

$$\operatorname{div}\mathbf{E} = 0, \qquad \operatorname{div}\mathbf{B} = 0, \qquad \operatorname{curl}\mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}, \qquad \operatorname{curl}\mathbf{B} = \mu\sigma\mathbf{E} + \mu\epsilon\frac{\partial \mathbf{E}}{\partial t}. \tag{1.14}$$

Proposition 1.2.2. *Let Ω be an homogeneous, isotropus and linear solid with $\rho_f(\mathbf{r}) = 0$. Then,*

$$\nabla^2\mathbf{E} = \mu\sigma\frac{\partial \mathbf{E}}{\partial t} + \mu\epsilon\frac{\partial^2 \mathbf{E}}{\partial t^2}, \tag{1.15}$$

and if $\sigma(\mathbf{r}) = 0$, then

$$\nabla^2\mathbf{E} = \mu\epsilon\frac{\partial^2 \mathbf{E}}{\partial t^2}. \tag{1.16}$$

Proposition 1.2.3. *Let*

Chapter 2

Extra

2.1 Physical constants

Definition 2.1.1. Physical constants

$$
\begin{gathered}
q_e = 1{,}602 \times 10^{-19}\,\mathrm{C}, \\
h = 6{,}62 \times 10^{-34}\,\mathrm{kg}, \\
\epsilon_0 = 8{,}8542 \times 10^{-12}\,\mathrm{C}^2\,\mathrm{N}\,\mathrm{m}^2, \\
\mu_0 = 4\pi \times 10^{-7}\mathrm{N}^2\,\mathrm{s}^2\,\mathrm{C}^{-2}
\end{gathered}
$$