LOGISTICS: no class Tue!
WILL STILL HAVE SHORT HW.

QUESTIONS?

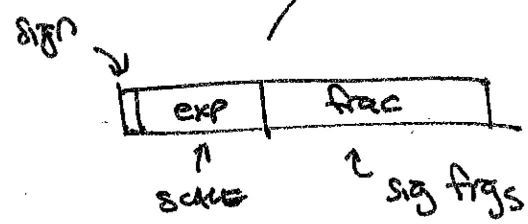
- installation?
- HW?

QUENTIN'S Q:
HOW TO SUBMIT VIA GITHUB

PUZZLE from LAST TIME:

1E15 − 1 000 000 000 000 001.28456 = 1.25

① 3 sig figs → consistent w/ ~10^{16} precision



sig figs: 52 bits
$2^{52} \sim 5 \times 10^{15}$

so that makes sense.

② Why 1.25 vs 1.23 or 1.24

correct decimal rounding (1.23)

if we rounded up (1.24)

RECALL: All numbers are stored in binary

eg. $(101.101)_2$ ← binary number

1×2^2, 0×2^1, 1×2^0, 1×2^{-1}, 0×2^{-2}, $1 \times \boxed{2^{-3}}$

$2^{-3} = \frac{1}{8} = 0.125$

SO: $(0.001)_2 = \boxed{0.125}$

so #'s that are easy to represent in binary:

$(0.0001)_2 = 1/2^4 = 0.0625$

$(0.0010)_2 = 1/2^3 = 0.125$

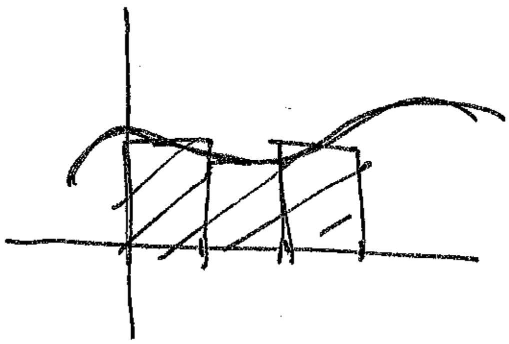
$(0.0011)_2 = \frac{1}{2^3} + \frac{1}{2^4} = 0.1875$

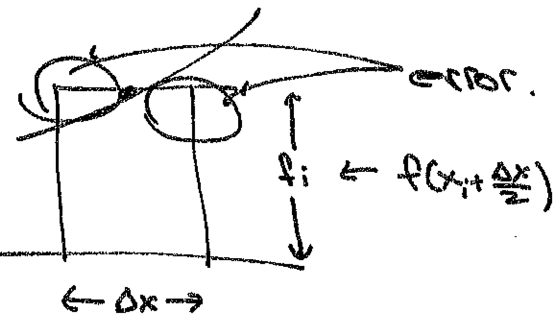
to this precision in binary, $\boxed{.123}$ is closest to $\boxed{0.125} = (0.001)_2$

(we were glib about the exponential - not the interesting part — as an exercise you can work it out.)

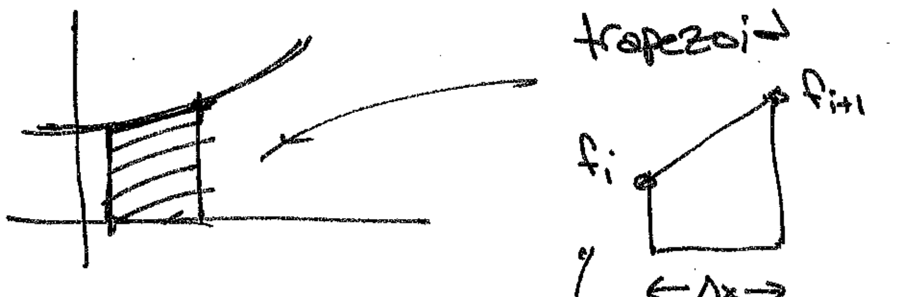
Reminder: INTEGRATION
↳ see Lec 4 Jupyter notes on GitHub

INTELLIGENT APPROXIMATIONS (?)





smarter?



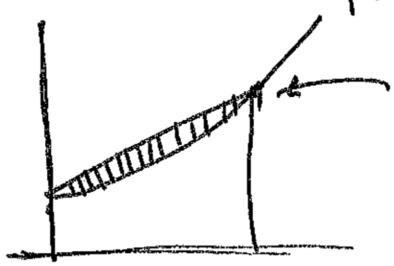
$A = \underbrace{\frac{1}{2}(f_i + f_{i+1})}_{\text{avg height}} \Delta x$

(APPROXIMATES DERIVATIVE)

note: "smarter" approx can be dumber!!

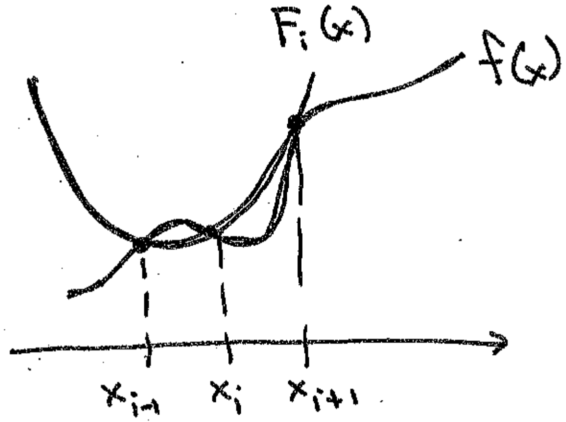
eg if function is convex

$f \sim ax^2 + b$



trapezoid always overestimates area.

try to be smarter:



approx $f(x)$ around x_i by a parabola, $F_i(x)$

WE KNOW THE AREA UNDER A PARABOLA.

can just write closed form formula & use that in my code.

trick: use coordinate $y \equiv x - x_i$

st $x_i \rightarrow y = 0$

$$F(y) = Ay^2 + By + C$$

can find these by sampling f

$$\int_{-\Delta x}^{\Delta x} dy\ F = \frac{1}{3}Ay^3 + \frac{1}{2}By^2 + Cy \Big|_{-\Delta x}^{\Delta x}$$

$$= \frac{2}{3}A\,\Delta x^3 + 2C\,\Delta x$$

sample @ $f(x_{i-1})$, $f(x_i)$, $f(x_{i+1})$

$$f(x_{i-1}) = F(-\Delta x) = A\,\Delta x^2 - B\Delta x + C$$

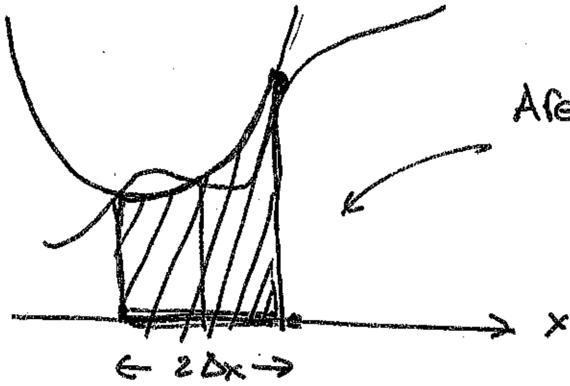
$$f(x_{i+1}) = F(\Delta x) = \quad '' \quad + B\Delta x + C$$

$$\boxed{f(x_i) = F(0) = C}$$

$$f(x_{i-1}) + f(x_{i+1}) = 2A(\Delta x)^2 + 2f(x_i)$$

$$\boxed{A = \frac{1}{2(\Delta x)^2}\left(f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)\right)}$$

thus we estimate



$$\boxed{-2f_i}$$

$$\text{Area}_i = \frac{2}{3}\left(\frac{1}{2\Delta x^2}\left(f_{i+1}+f_{i-1}\right)\right)\Delta x^3 + 2f(x_i)\,\Delta x$$

sanity check: all terms are $\Theta(\Delta x)$

$$\text{Area}_i = \frac{1}{3}\Delta x\left[\, f_{i-1} + 4f_i + f_{i+1}\,\right]$$

$$\frac{2}{3}\left(\frac{1}{2(\Delta x)^2}(-2f_i)\right)\Delta x^3 + 2f_i\,\Delta x$$

$$= \frac{\Delta x}{3}(-2f_i) + \frac{6}{3}\Delta x\, f_i$$

$$= \frac{4\Delta x}{3} f_i$$

can now code this up.

CAN YOU SEE HOW THIS GENERALIZES?