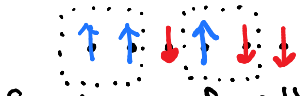
PHYS 177
winter 2019

lecture 12: the Ising model

learning goals
- use probability distribution to compute with coupled vars.

consider a simple model of ferromagnetism called the Ising model



favorable alignment | unfavorable alignment

spins sit on neighboring lattice points, can be ↑ or ↓

spin–spin coupling lowers energy if neighboring spins are aligned, increases energy if anti-aligned

call the configuration $\underline{\sigma} = \{\sigma_1, \sigma_2, \ldots, \sigma_N\}$, with $\sigma_i \in \{-1, 1\}$

then the energy $E(\underline{\sigma})$ is

$$E(\underline{\sigma}) = -\sum_{i=1}^{N-1} J \sigma_i \sigma_{i+1}$$, where J is the strength of the coupling

contributions to E are then
negative if aligned
positive if anti-aligned

σ_i \ σ_{i+1}	+1	−1
+1	−J	+J
−1	+J	−J

say that we want to know the magnetization $\langle \sigma_i \rangle$ of spin i, this is

$$\langle \sigma_i \rangle = \sum_{\underline{\sigma}} \sigma_i P(\underline{\sigma})$$

but $P(\underline{\sigma})$ is <u>not</u> a product distribution – neighboring spins are coupled!

need to sum over whole distribution to get averages

* notebook example, magnetications and correlations in coupled spin systems