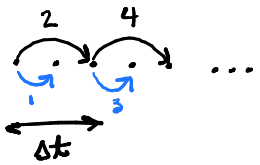
PHYS 171
winter 2019

lecture 7.5: leapfrog method

learning goals
- appreciate situational applications for different numerical integration methods

another way to express RK method

$$x(t+\tfrac{\Delta t}{2}) = x(t) + \tfrac{\Delta t}{2} f(x(t),t)$$
$$x(t+\Delta t) = x(t) + \Delta t\, f(x(t+\tfrac{\Delta t}{2}), t+\tfrac{\Delta t}{2})$$



what happens if we reverse time?

$$x(t-\tfrac{\Delta t}{2}) = x(t) - \tfrac{\Delta t}{2} f(x(t),t)$$
$$x(t-\Delta t) = x(t) - \Delta t\, f(x(t-\tfrac{\Delta t}{2}), t-\tfrac{\Delta t}{2})$$

compare, not the same

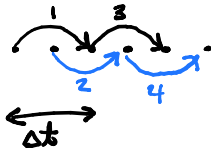
now substitute in $t = t + \frac{3\Delta t}{2}$

$$x(t+\Delta t) = x(t+\tfrac{3\Delta t}{2}) - \tfrac{\Delta t}{2} f(x(t+\tfrac{3\Delta t}{2}), t+\tfrac{3\Delta t}{2})$$
$$x(t+\tfrac{\Delta t}{2}) = x(t+\tfrac{3\Delta t}{2}) - \Delta t\, f(x(t+\Delta t), t+\Delta t)$$

leapfrog method <u>preserves</u> time reversal symmetry

$$x(t+\tfrac{\Delta t}{2}) = x(t) + \tfrac{\Delta t}{2} f(x(t),t) \leftarrow \text{start w/ Euler, then leapfrog}$$

$$x(t+\Delta t) = x(t) + \Delta t\, f(x(t+\tfrac{\Delta t}{2}), t+\tfrac{\Delta t}{2})$$
$$x(t+\tfrac{3\Delta t}{2}) = x(t+\tfrac{\Delta t}{2}) + \Delta t\, f(x(t+\Delta t), t+\Delta t)$$



check time reversal

$$x(t-\Delta t) = x(t) - \Delta t\, f(x(t-\tfrac{\Delta t}{2}), t-\tfrac{\Delta t}{2})$$
$$x(t-\tfrac{3\Delta t}{2}) = x(t-\tfrac{\Delta t}{2}) - \Delta t\, f(x(t-\Delta t), t-\Delta t)$$

same!

substitute $t = t + \frac{3\Delta t}{2}$

$$x(t+\tfrac{\Delta t}{2}) = x(t+\tfrac{3\Delta t}{2}) - \Delta t\, f(x(t+\Delta t), t+\Delta t)$$
$$x(t) = x(t+\Delta t) - \Delta t\, f(x(t+\tfrac{\Delta t}{2}), t+\tfrac{\Delta t}{2})$$