

lecture 8: probability and random numbers

statistical mechanics is actually all about probability theory - more on that next time

today: what is a random number?

learning goals

- give a mathematical definition of random numbers
- use simple random number generation in Python

what properties does a random number have? examples of random numbers?

random number comes from a distribution $p(x)$,
with x real-valued or taking values from a finite set of possibilities

computer generation of both types of RNGs, example "PRNG
and criteria for "randomness"

* notebook example

law of large numbers: as number of samples increases, empirical average approaches the true average

but how far away? assume finite variance, $\text{var}(x) = \langle (x - \langle x \rangle)^2 \rangle = \sigma^2$
then using $\text{var}(x_1 + x_2 + \dots) = \text{var}(x_1) + \text{var}(x_2) + \dots$, we have

$$\text{var}\left(\frac{1}{n}(x_1 + x_2 + \dots)\right) = \frac{1}{n^2} \text{var}(x_1 + x_2 + \dots) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

convergence toward mean goes like $1/\sqrt{n}$!

* notebook example: estimating π with Monte Carlo