PHYS 177
winter 2019

lecture 11: entropy

learning goals
- recall the probabilistic definition of entropy
- understand that macroscopic behavior results from the compromise between energy and entropy

last time we considered a minimal system: a single spin in a magnetic field, tradeoff between E and T

\# of states is also important for behavior, quantified by entropy

$$S = -k_B T \sum_{\sigma} P(\sigma) \log P(\sigma)$$

← high when probabilities uniform, low when concentrated

if all states equally probable, then we have

$$S = -k_B T \sum_{\sigma} \frac{1}{N} \log \frac{1}{N} = -k_B T \log 1/N = k_B T \log N$$

where N is the number of states, so $e^{\beta S}$ measures "effective # of states"

consider a system with two energy levels: E_A and E_B
assume there are N_A A states and N_B B states, then

$$P(A) = \sum_{\sigma \in A} \frac{e^{-\beta E_A}}{Z} = N_A \frac{e^{-\beta E_A}}{Z}$$

$$= \frac{1}{Z} \exp\left(-\beta E_A - \log \frac{1}{N_A}\right)$$

$$= \frac{1}{Z} \exp\left(-\beta\left(E_A + \frac{1}{\beta} \sum_{\sigma \in A} \frac{1}{N_A} \log \frac{1}{N_A}\right)\right)$$

$P_A(\sigma)$

$$= \frac{1}{Z} \exp\left(-\beta\left(E_A - S_A\right)\right) = \frac{e^{-\beta F_A}}{Z}$$

F_A is the free energy of state A, $E\downarrow$ and $S\uparrow$ → $F\downarrow$, higher probability

so macroscopic properties can be dominated by high energy states if there are many of them, and if temperature is high

* notebook example, magnetization free energies

if time/interest: connection w/ information theory