A first look into Quantum Logic

(Just an overview by a non-expert on the field!)

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Plan

- Motivation(s) : Physical theories and Logic.
- Classically & Quantumly.
- Bells’ Inequalities.
- Conclusions.

what is a physical theory ?

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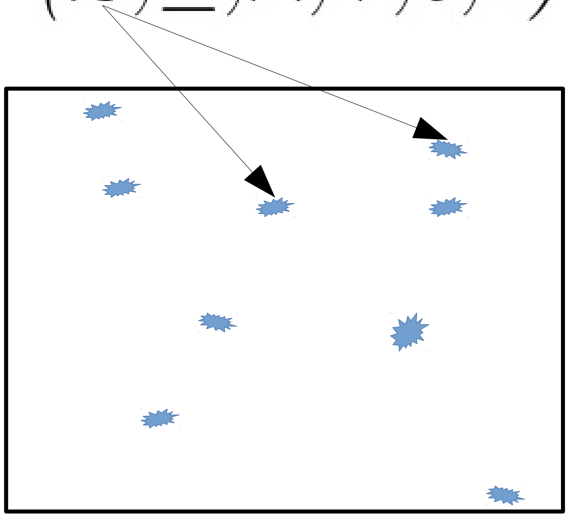
$$(\mathcal{L}, \leq, \wedge, \vee, 0, 1)$$



A Lattice is a set of propositions !

what is a physical theory ?

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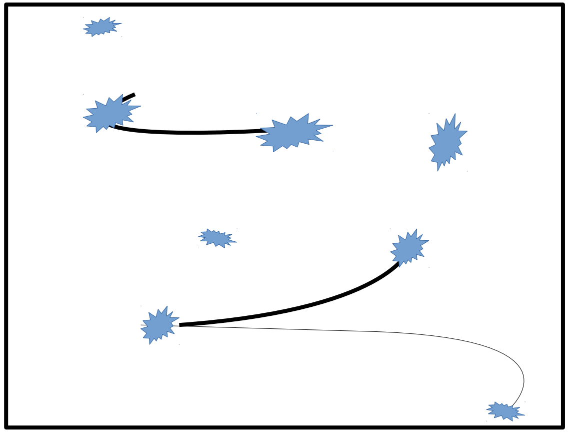

Propositions can be combined !

what is a physical theory ?

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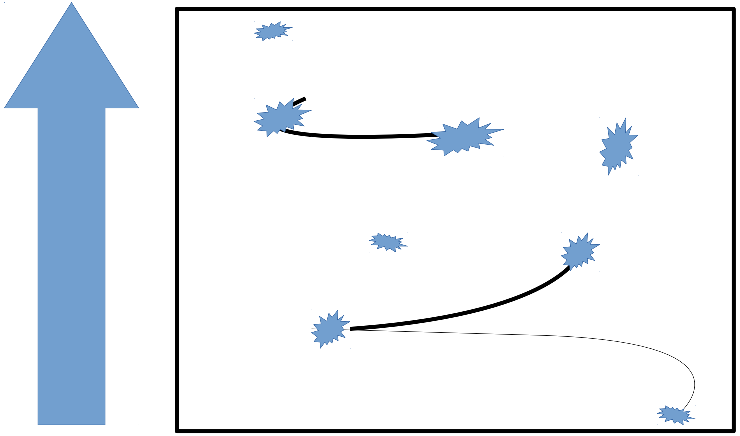

And ordered (partially) !

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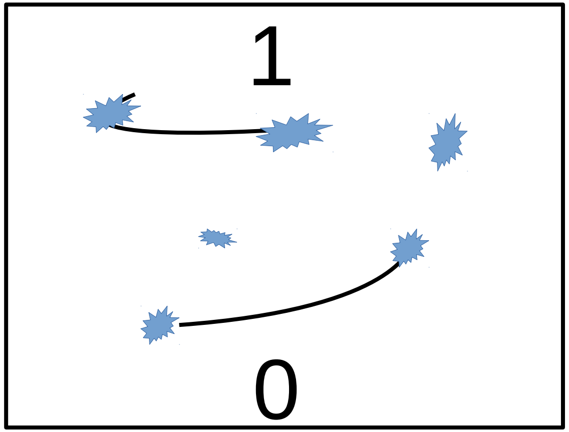

And bounded !

what else ?

Let $x \in L$. *An element* $x' \in L$ *is called the complement of* x *if* $x \vee x' = 1$ *and* $x \wedge x' = 0$.

The lattice $(L, \leq, \wedge, \vee, 0, 1)$ *is called complemented if* $(\forall) x \in L$ *has a complement in* L.

Sorin Nadaban (2021)
arXiv: 1211.5627 [math-ph]

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$(L, \leq, \wedge, \vee, 0, 1, \perp)$ *is called orthocomplemented lattice if* $(\forall) x, y \in L$ *we have:*

$$x^{\perp^{\perp}} = x \qquad x \leq y \Rightarrow y^{\perp} \leq x^{\perp} \qquad x \wedge x^{\perp} = 0$$

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An orthomodular lattice is an orthocomplemented lattice such that $x \leq y \Rightarrow x \vee (x^{\perp} \wedge y) = y, (\forall) x, y \in L$ (orthomodular law).

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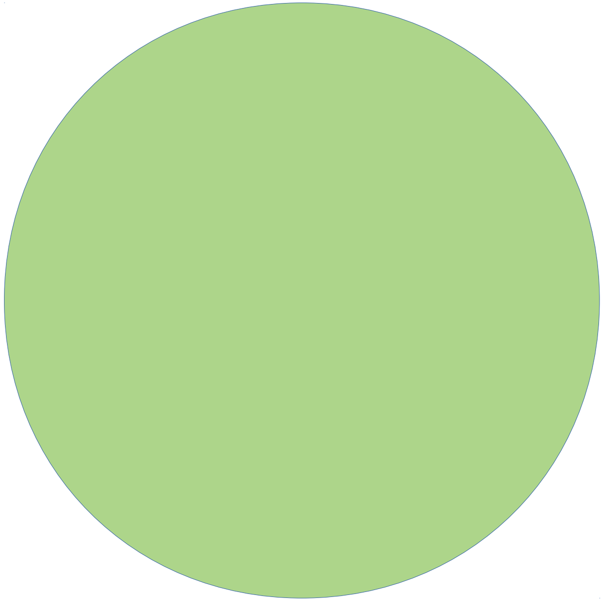
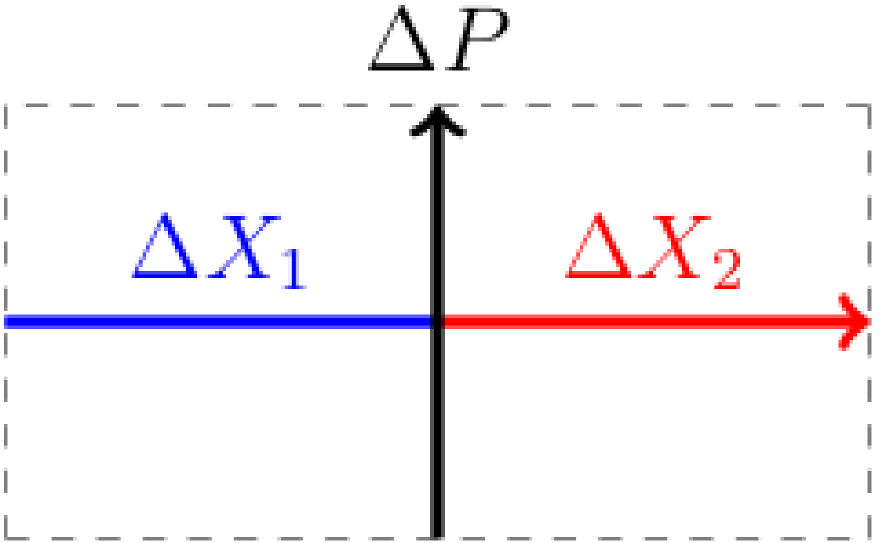
Classically



$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z).$$

If L is Boolean algebra, than L is modular, orthomodular and orthocomplemented.

Quantumly



$$x \wedge (y \vee z) \neq (x \wedge y) \vee (x \wedge z).$$

arXiv: 1211.5627
J.Von-Neumann G.Birkhoff (1936)

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Bell Inequalities

For any two proposition A,B in a lattice $\mathcal{L}$, we define

$$d(A,B) = P(A \vee B) - P(A \wedge B)$$

Satisfying the following triangle inequality which are closely related to Bell inequalities

$$|d(A,B) - d(A,C)| \leq d(B,C) \leq d(A,B) + d(A,C).$$

The point is, if we consider a Boolean structure, the inequality hold true, but a non-Boolean structure will break the inequality as QM do with Bell's inequalities !

E. Santos, Physics letters A 115.8 (1986)

arXiv: quant-ph/0207062

S. Pulmannová and V. Majernik (1992)

Conclusions

- Classical and Quantum systems have both an *orthocomplemented structure* and the *distributive property* provides the essential difference between them.

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Sorin Nadaban (2021) Jaroslaw Pykacz (1993)

- Many works have been done to generalize *quantum logic to fuzzy quantum logic* but also to be related to *Non-Commutative Geometry* (throught a W*-algebra).

Marchetti and R. Rubele (2007)

Thanks for your Attention !

Your questions are more than welcomed :)