

9.2 Bell State Problem Info

(July 4, 2020)

The X operator is: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

It's eigenvectors are: $|+x\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $|-x\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The dimension 4 (2 bit) X basis is:

$$|+x\rangle \otimes |+x\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$|+x\rangle \otimes |-x\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

$$|-x\rangle \otimes |+x\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

$$|-x\rangle \otimes |-x\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

The 45 degree operator is: $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

It's eigenvectors are: $|+45\rangle = \begin{bmatrix} 0.924 \\ 0.383 \end{bmatrix}$, $|-45\rangle = \begin{bmatrix} -0.383 \\ 0.924 \end{bmatrix}$

The dimension 4 (2 bit) 45 degree basis is:

$$|+45\rangle \otimes |+45\rangle = \begin{bmatrix} 0.924 \\ 0.383 \end{bmatrix} \otimes \begin{bmatrix} 0.924 \\ 0.383 \end{bmatrix} = \begin{bmatrix} 0.854 \\ 0.354 \\ 0.354 \\ 0.147 \end{bmatrix}$$

$$|+45\rangle \otimes |-45\rangle = \begin{bmatrix} 0.924 \\ 0.383 \end{bmatrix} \otimes \begin{bmatrix} -0.383 \\ 0.924 \end{bmatrix} = \begin{bmatrix} -0.354 \\ 0.854 \\ -0.147 \\ 0.354 \end{bmatrix}$$

$$|-45\rangle \otimes |+45\rangle = \begin{bmatrix} -0.383 \\ 0.924 \end{bmatrix} \otimes \begin{bmatrix} 0.924 \\ 0.383 \end{bmatrix} = \begin{bmatrix} -0.354 \\ -0.147 \\ 0.854 \\ 0.354 \end{bmatrix}$$

$$|-45\rangle \otimes |-45\rangle = \begin{bmatrix} -0.383 \\ 0.924 \end{bmatrix} \otimes \begin{bmatrix} -0.383 \\ 0.924 \end{bmatrix} = \begin{bmatrix} 0.147 \\ -0.354 \\ -0.354 \\ 0.854 \end{bmatrix}$$

Bell 1 through 4 in the Z basix

$$\frac{1}{\sqrt{2}}|+z, +z\rangle + \frac{1}{\sqrt{2}}|-z, -z\rangle$$

$$\frac{1}{\sqrt{2}}|+z, +z\rangle - \frac{1}{\sqrt{2}}|-z, -z\rangle$$

$$\frac{1}{\sqrt{2}}|+z, -z\rangle - \frac{1}{\sqrt{2}}|-z, +z\rangle$$

$$\frac{1}{\sqrt{2}}|+z, -z\rangle + \frac{1}{\sqrt{2}}|-z, +z\rangle$$

Bell 1 through 4 in the X basix

$$\frac{1}{\sqrt{2}}|+x, +x\rangle + \frac{1}{\sqrt{2}}|-x, -x\rangle$$

$$\frac{1}{\sqrt{2}}|+x, -x\rangle + \frac{1}{\sqrt{2}}|-x, +x\rangle$$

$$\frac{1}{\sqrt{2}}|+x, +x\rangle - \frac{1}{\sqrt{2}}|-x, -x\rangle$$

$$- \frac{1}{\sqrt{2}}|+x, -x\rangle + \frac{1}{\sqrt{2}}|-x, +x\rangle$$

Bell 1 through 4 in the 45° basis

$$\frac{1}{\sqrt{2}}|+45, +45\rangle + \frac{1}{\sqrt{2}}|-45, -45\rangle$$

$$\frac{1}{2}|+45, +45\rangle - \frac{1}{2}|+45, -45\rangle - \frac{1}{2}|-45, +45\rangle - \frac{1}{2}|-45, -45\rangle$$

$$\frac{1}{2}|+45, +45\rangle \frac{1}{2}|+45, -45\rangle - \frac{1}{2}|-45, +45\rangle - \frac{1}{2}|-45, -45\rangle$$

$$\frac{1}{\sqrt{2}}|+45, -45\rangle + \frac{1}{\sqrt{2}}|-45, +45\rangle$$