Section 10 Exercises

(July 9, 2020)

(1) The $\hat{Z}$ operator

$$\hat{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \text{ changes the phase between the parts of a superposition:}$$

$$\hat{Z}\,(\,\alpha|0\rangle + \beta|1\rangle\,) = \alpha|0\rangle - \beta|1\rangle$$

Work this out in matrix/vector form to make sure you understand how that happens.

(2) Two-bit Zs

Given a two-bit state, create two operators. One that changes the phase of the first bit and one that changes the phase of the second bit.

(3) Xs and Zs

Given a two-bit state, create the following operators by tensoring X and Z matrices:

1. Flip the first bit and change the phase of the second.
2. Flip the second bit and change the phase of the first.
3. First flip, *then* change the phase of the first bit.
4. First change the phase, *then* flip the second bit.

(4) Three bit operators

By tensoring X, Z, and CNOT operators, construct three-bit operators which will:

1. Flip the 2nd (middle) bit.
2. Flip the 1st bit and change the phase of the 3rd bit.
3. Flip the middle bit only if the 1st bit is one.
4. Flip the 3rd bit only if the middle bit is one.