Change of Basis (Complex) Exercises

(1) Transform the vector $\begin{bmatrix} 2+8i \\ 4+5i \end{bmatrix}$ to the basis $\left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \right)$

(2) Transform the vector $\begin{bmatrix} 6+2i \\ 5+4i \end{bmatrix}$ to the basis $\left(\frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{5}} \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right)$

(3) Transform the vector $\begin{bmatrix} 7+7i \\ 2+8i \end{bmatrix}$ to the basis $\left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \right)$

(4) Transform the vector $\begin{bmatrix} 9+8i \\ 7+3i \end{bmatrix}$ to the basis $\left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \right)$

(5) Transform the vector $\begin{bmatrix} 7+9i \\ 7+i \end{bmatrix}$ to the basis $\left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$

(6) Transform the vector $\begin{bmatrix} 2+7i \\ 9+5i \end{bmatrix}$ to the basis $\left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \right)$

(7) Transform the vector $\begin{bmatrix} 4+4i \\ 3+8i \end{bmatrix}$ to the basis $\left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$

(8) Transform the vector $\begin{bmatrix} 9+6i \\ 9+8i \end{bmatrix}$ to the basis $\left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \right)$

(9) Transform the vector $\begin{bmatrix} 8+3i \\ 7+i \end{bmatrix}$ to the basis $\left(\frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{5}} \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right)$

(10) Transform the vector $\begin{bmatrix} 8+7i \\ 3+3i \end{bmatrix}$ to the basis $\left(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix} \right)$

Answers

(1) $\frac{1}{\sqrt{2}}\begin{bmatrix} 7+4i \\ -3+12i \end{bmatrix}$

(2) $\frac{1}{\sqrt{5}}\begin{bmatrix} 17+8i \\ 4+6i \end{bmatrix}$

(3) $\frac{1}{\sqrt{2}}\begin{bmatrix} 15+5i \\ -1+9i \end{bmatrix}$

(4) $\frac{1}{\sqrt{2}}\begin{bmatrix} 12+i \\ 6+15i \end{bmatrix}$

(5) $\frac{1}{\sqrt{2}}\begin{bmatrix} 14+10i \\ 8i \end{bmatrix}$

(6) $\frac{1}{\sqrt{2}}\begin{bmatrix} 7-2i \\ -3+16i \end{bmatrix}$

(7) $\frac{1}{\sqrt{2}}\begin{bmatrix} 7+12i \\ 1-4i \end{bmatrix}$

(8) $\frac{1}{\sqrt{2}}\begin{bmatrix} 17-3i \\ 1+15i \end{bmatrix}$

(9) $\frac{1}{\sqrt{5}}\begin{bmatrix} 23+7i \\ 6-i \end{bmatrix}$

(10) $\frac{1}{\sqrt{2}}\begin{bmatrix} 11+4i \\ 5+10i \end{bmatrix}$