```
function cached_permute(sym::Symbol, t::TensorMap{S},
                             p1::IndexTuple{N<sub>1</sub>}, p2::IndexTuple{N<sub>2</sub>}=();
                             copy::Bool = false) where \{S, N_1, N_2\}
    cod = ProductSpace(S, N1)(map(n->space(t, n), p1))
    dom = ProductSpace(S, N<sub>2</sub>)(map(n->dual(space(t, n)), p2))
    # share data if possible
    if !copy
        if p1 === codomainind(t) && p2 === domainind(t)
            return t
        elseif has_shared_permute(t, p1, p2)
            return TensorMap(reshape(t.data, dim(cod), dim(dom)), cod, dom)
        end
    end
    # general case
    @inbounds begin
        tp = T0.cached_similar_from_indices(sym, eltype(t), p1, p2, t, :N)
        return add!(true, t, false, tp, p1, p2)
    end
end
function cached_permute(sym::Symbol, t::AdjointTensorMap{S},
                             p1::IndexTuple, p2::IndexTuple=();
                             copy::Bool = false) where \{S, N_1, N_2\}
    p1' = adjointtensorindices(t, p2)
    p2' = adjointtensorindices(t, p1)
    adjoint(cached_permute(sym, adjoint(t), p1', p2'; copy = copy))
end
scalar(t::AbstractTensorMap{S}) where {S<:IndexSpace} =</pre>
    dim(codomain(t)) == dim(domain(t)) == 1 ?
        first(blocks(t))[2][1, 1] : throw(SpaceMismatch())
.....
    _add_trivial_kernel!(α, tsrc::AbstractTensorMap, β, tdst::AbstractTensorMap,
                             p1::IndexTuple, p2::IndexTuple, fusiontreemap)
`tsrc` and `tdst` are TrivialTensorMap, thus `tsrc[]` and `tdst[]` gives the data of
the tensor map in a form of multidimensional array. `pdata = (p1..., p2...)` gives
permutation of the indices of the tensor map `tsrc`. After permutation `tsrc`
changes
`to `tsrc2` and has the same space with `tdst`.
After running, `tdst` will be replaced in-place by the tensor map `tsrc2*\alpha +
tdst*β`.
.....
function _add_trivial_kernel!(α, tsrc::AbstractTensorMap, β,
tdst::AbstractTensorMap,
                                 p1::IndexTuple, p2::IndexTuple, fusiontreemap)
    #cod = codomain(tsrc)
    #dom = domain(tsrc)
    #n = length(cod)
```

```
pdata = (p1..., p2...)
    axpby!(α, permutedims(tsrc[], pdata), β, tdst[])
    return nothing
end
0.000
    _addabelianblock!(α, tsrc::AbstractTensorMap, β, tdst::AbstractTensorMap,
        p1::IndexTuple, p2::IndexTuple, f1::FusionTree, f2::FusionTree,
fusiontreemap)
Only works for tensor maps with UniqueFusion.
Here `fusiontreemap` is a function that manipulate the fusiontree and gives the
resulting
fusiontrees as keys and coefficients as values. In the Abelian case, it only gives
one pair.
Consider the fusion tree `(f1,f2)` in `tsrc`. After the operation of `fusiontreemap`
(which result in `(f1', f2') => coeff`) and the permitation based on
`pdata = (p1..., p2...)` (result in `tsrc2`), the corresponding block of data has
the same
shape as `tdst[f1', f2']`.
After running, the block `tdst[f1', f2']` is replaced in-place by
`tsrc2*α*coeff+tdst[f1′, f2′]*β`.
function _addabelianblock!(α, tsrc::AbstractTensorMap, β, tdst::AbstractTensorMap,
        p1::IndexTuple, p2::IndexTuple, f1::FusionTree, f2::FusionTree,
fusiontreemap)
    #cod = codomain(tsrc)
    #dom = domain(tsrc)
    (f1', f2'), coeff = first(fusiontreemap(f1, f2))
    pdata = (p1..., p2...)
    @inbounds axpby!(α*coeff, permutedims(tsrc[f1, f2], pdata), β, tdst[f1', f2'])
end
.....
   _add_abelian_kernel!(α, tsrc::AbstractTensorMap, β, tdst::AbstractTensorMap,
                            p1::IndexTuple, p2::IndexTuple, fusiontreemap)
Only works for tensor maps with UniqueFusion.
For every fusion tree in `tsrc`, apply the operation `_addabelianblock!`.
function \_add\_abelian\_kernel!(\alpha, tsrc::AbstractTensorMap, \beta,
tdst::AbstractTensorMap,
                                 p1::IndexTuple, p2::IndexTuple, fusiontreemap)
    if Threads.nthreads() > 1
        nstridedthreads = Strided.get_num_threads()
        Strided.set_num_threads(1)
        Threads.@sync for (f1, f2) in fusiontrees(tsrc)
            Threads.@spawn _addabelianblock!(\alpha, tsrc, \beta, tdst, p1, p2, f1, f2,
                                                 fusiontreemap)
        end
        Strided.set_num_threads(nstridedthreads)
                                                                                  raye∠ ∪ı 11
```

```
else # debugging is easier this way
        for (f1, f2) in fusiontrees(tsrc)
             _addabelianblock!(\alpha, tsrc, \beta, tdst, p1, p2, f1, f2, fusiontreemap)
        end
    end
    return nothing
end
0.000
    \_add\_general\_kernel!(\alpha, tsrc::AbstractTensorMap, \beta, tdst::AbstractTensorMap,
                              p1::IndexTuple, p2::IndexTuple, fusiontreemap)
Works for general tensor maps.
Same operations on each fusion trees.
function \_add\_general\_kernel!(\alpha, tsrc::AbstractTensorMap, \beta,
tdst::AbstractTensorMap,
                                  p1::IndexTuple, p2::IndexTuple, fusiontreemap)
    #cod = codomain(tsrc)
    #dom = domain(tsrc)
    #n = length(cod)
    pdata = (p1..., p2...)
    if iszero(β)
        fill!(tdst, β)
    elseif \beta != 1
        mul!(tdst, β, tdst)
    end
    for (f1, f2) in fusiontrees(tsrc)
        for ((f1', f2'), coeff) in fusiontreemap(f1, f2)
            @inbounds axpy!(\alpha*coeff, permutedims(tsrc[f1, f2], pdata), tdst[f1',
f2′1)
        end
    end
    return nothing
end
const _add_kernels = (_add_trivial_kernel!, _add_abelian_kernel!,
add general kernel!)
.....
    _add!(\alpha, tsrc::AbstractTensorMap{S}, \beta, tdst::AbstractTensorMap{S, N_1, N_2},
             p1::IndexTuple{N_1}, p2::IndexTuple{N_2}, fusiontreemap) where {S, N_1, N_2}
Make the general add operation between two tensor maps. For each fusion tree of
apply the manipulation `fusiontreemap` which give new fusion trees and the
corresponding
coefficients. Apply the permutation on the data of the new fusion trees based on
`(p1..., p2...)`, where the permutation should be consistent with requirement of the
`fusiontreemap` manipulation.
Return `tdst`, whose data for each fusion tree is replaced in-place by
`(data of new fusion tree of tsrc)stlpha*coefficient + (data of fusion tree of tdst)steta`.
0.000
                                                                                     гау<del>с</del> э ог 11
```

```
function _add!(α, tsrc::AbstractTensorMap{S}, β, tdst::AbstractTensorMap{S, N1, N2},
                  p1::IndexTuple{N1}, p2::IndexTuple{N2}, fusiontreemap) where {S,
N_1, N_2
    @boundscheck begin
         all(i->space(tsrc, p1[i]) == space(tdst, i), 1:N1) ||
              throw(SpaceMismatch("tsrc = $(codomain(tsrc))←$(domain(tsrc)),
              tdst = \$(codomain(tdst)) \leftarrow \$(domain(tdst)), p1 = \$(p1), p2 = \$(p2)"))
         all(i->space(tsrc, p2[i]) == space(tdst, N_1+i), 1:N_2) ||
              throw(SpaceMismatch("tsrc = $(codomain(tsrc))←$(domain(tsrc)),
              tdst = \$(codomain(tdst)) \leftarrow \$(domain(tdst)), p1 = \$(p1), p2 = \$(p2)")
    end
    # do some kind of dispatch which is compiled away if S is known at compile time,
    # and makes the compiler give up quickly if S is unknown
    I = sectortype(S)
    i = I === Trivial ? 1 : (FusionStyle(I) isa UniqueFusion ? 2 : 3)
    if p1 == codomainind(tsrc) && p2 == domainind(tsrc)
         axpby!(\alpha, tsrc, \beta, tdst)
    else
         _add_kernel! = _add_kernels[i]
         \_add\_kernel!(\alpha, tsrc, \beta, tdst, p1, p2, fusiontreemap)
    end
    return tdst
end
    add_permute!(α, tsrc::AbstractTensorMap{S}, β, tdst::AbstractTensorMap{S, N<sub>1</sub>,
N_2},
                       p1::IndexTuple{N_1}, p2::IndexTuple{N_2}) where {S, N_1, N_2}
Replace `tdst` with `permutaion(tsrc)*\alpha + tdst*\beta`.
@propagate_inbounds function add_permute!(α, tsrc::AbstractTensorMap{S},
                                                   β, tdst::AbstractTensorMap{S, N<sub>1</sub>, N<sub>2</sub>},
                                                   p1::IndexTuple{N<sub>1</sub>},
                                                   p2::IndexTuple{N<sub>2</sub>}) where {S, N<sub>1</sub>, N<sub>2</sub>}
     _add!(α, tsrc, β, tdst, p1, p2, (f1, f2)->permute(f1, f2, p1, p2))
end
0.00
    add braid!(\alpha, tsrc::AbstractTensorMap{S}, \beta, tdst::AbstractTensorMap{S, N_1, N_2},
              p1::IndexTuple{N<sub>1</sub>}, p2::IndexTuple{N<sub>2</sub>}, levels::IndexTuple) where {S,
N_1, N_2
Replace `tdst` with `braid(tsrc)*\alpha + tdst*\beta`.
@propagate inbounds function add braid!(α, tsrc::AbstractTensorMap{S},
                                                   β, tdst::AbstractTensorMap{S, N<sub>1</sub>, N<sub>2</sub>},
                                                   p1::IndexTuple{N<sub>1</sub>},
                                                   p2::IndexTuple{N<sub>2</sub>},
                                                   levels::IndexTuple) where {S, N<sub>1</sub>, N<sub>2</sub>}
    length(levels) == numind(tsrc) ||
         throw(ArgumentError("incorrect levels $levels for tensor map
                                                                                            гау<del>с</del> 4 от 11
```

```
$(codomain(tsrc)) ← $(domain(tsrc))"))
    levels1 = TupleTools.getindices(levels, codomainind(tsrc))
    levels2 = TupleTools.getindices(levels, domainind(tsrc))
    _add!(α, tsrc, β, tdst, p1, p2, (f1, f2)->braid(f1, f2, levels1, levels2, p1,
p2))
end
.....
    add_transpose!(α, tsrc::AbstractTensorMap{S}, β, tdst::AbstractTensorMap{S, N<sub>1</sub>,
N_2},
                      p1::IndexTuple{N_1}, p2::IndexTuple{N_2}) where {S, N_1, N_2}
Replace `tdst` with `transpose(tsrc)*\alpha + tdst*\beta`.
?????? It seems that transpose is the same with permutation...
@propagate_inbounds function add_transpose!(α, tsrc::AbstractTensorMap{S},
                                                β, tdst::AbstractTensorMap{S, N<sub>1</sub>, N<sub>2</sub>},
                                                p1::IndexTuple{N<sub>1</sub>},
                                                p2::IndexTuple{N<sub>2</sub>}) where {S, N<sub>1</sub>, N<sub>2</sub>}
    _add!(α, tsrc, β, tdst, p1, p2, (f1, f2)->transpose(f1, f2, p1, p2))
end
.....
    add!(α, tsrc::AbstractTensorMap{S}, β, tdst::AbstractTensorMap{S},
             p1::IndexTuple, p2::IndexTuple) where {S}
Replace `tdst` with `permutaion(tsrc)*\alpha + tdst*\beta`.
@propagate_inbounds function add!(α, tsrc::AbstractTensorMap{S},
                                       β, tdst::AbstractTensorMap{S},
                                       p1::IndexTuple, p2::IndexTuple) where {S}
    I = sectortype(S)
    if BraidingStyle(I) isa SymmetricBraiding
        add_permute!(α, tsrc, β, tdst, p1, p2)
    else
        throw(ArgumentError("add! without levels is defined only if
                                   `BraidingStyle(sectortype(...)) isa
SymmetricBraiding`"))
    end
end
.....
    add!(α, tsrc::AbstractTensorMap{S}, β, tdst::AbstractTensorMap{S},
             p1::IndexTuple, p2::IndexTuple, levels::IndexTuple) where {S}
Replace `tdst` with `braid(tsrc)*\alpha + tdst*\beta`.
@propagate_inbounds function add!(α, tsrc::AbstractTensorMap{S},
                                       β, tdst::AbstractTensorMap{S},
                                       p1::IndexTuple, p2::IndexTuple,
                                       levels::IndexTuple) where {S}
    add_braid!(α, tsrc, β, tdst, p1, p2, levels)
                                                                                       гау<del>с</del> э ог 11
```

```
end
.....
    trace!(α, tsrc::AbstractTensorMap{S}, β, tdst::AbstractTensorMap{S, N<sub>1</sub>, N<sub>2</sub>},
             p1::IndexTuple{N<sub>1</sub>}, p2::IndexTuple{N<sub>2</sub>},
             q1::IndexTuple{N_3}, q2::IndexTuple{N_3}) where {S, N_1, N_2, N_3}
function trace!(α, tsrc::AbstractTensorMap{S}, β, tdst::AbstractTensorMap{S, N1,
N<sub>2</sub>},
                 p1::IndexTuple{N<sub>1</sub>}, p2::IndexTuple{N<sub>2</sub>},
                 q1::IndexTuple{N_3}, q2::IndexTuple{N_3}) where {S, N_1, N_2, N_3}
    if !(BraidingStyle(sectortype(S)) isa SymmetricBraiding)
        throw(SectorMismatch("only tensors with symmetric braiding rules can be
contracted;
                                   try `@planar` instead"))
    end
    @boundscheck begin
        all(i->space(tsrc, p1[i]) == space(tdst, i), 1:N1) ||
             throw(SpaceMismatch("trace: tsrc = $(codomain(tsrc))←$(domain(tsrc)),
                      tdst = (codomain(tdst)) \leftarrow (domain(tdst)), p1 = (p1), p2 =
$(p2)"))
        all(i->space(tsrc, p2[i]) == space(tdst, N_1+i), 1:N_2) ||
             throw(SpaceMismatch("trace: tsrc = $(codomain(tsrc))←$(domain(tsrc)),
                      tdst = (codomain(tdst)) \leftarrow (domain(tdst)), p1 = (p1), p2 =
$(p2)"))
        all(i\rightarrow space(tsrc, q1[i]) == dual(space(tsrc, q2[i])), 1:N<sub>3</sub>) ||
             throw(SpaceMismatch("trace: tsrc = $(codomain(tsrc))←$(domain(tsrc)),
                      q1 = \$(q1), q2 = \$(q2)")
    end
    I = sectortype(S)
    if I === Trivial
        cod = codomain(tsrc)
        dom = domain(tsrc)
        n = length(cod)
        pdata = (p1..., p2...)
        T0._trace!(α, tsrc[], β, tdst[], pdata, q1, q2)
    # elseif FusionStyle(I) isa UniqueFusion
    # TODO: is it worth multithreading UniqueFusion case for traces?
    else
        cod = codomain(tsrc)
        dom = domain(tsrc)
        n = length(cod)
        pdata = (p1..., p2...)
        if iszero(β)
             fill!(tdst, β)
        elseif \beta != 1
             mul!(tdst, β, tdst)
        end
        r1 = (p1..., q1...)
        r2 = (p2..., q2...)
        for (f1, f2) in fusiontrees(tsrc)
```

гаус о от 11

```
for ((f1', f2'), coeff) in permute(f1, f2, r1, r2)
                 f1'', g1 = split(f1', N<sub>1</sub>)
                f2'', g2 = split(f2', N<sub>2</sub>)
                 if q1 == q2
                     coeff *= dim(g1.coupled)/dim(g1.uncoupled[1])
                     for i = 2:length(g1.uncoupled)
                         if !(q1.isdual[i])
                             coeff *= twist(g1.uncoupled[i])
                         end
                     end
                     T0._trace!(α*coeff, tsrc[f1, f2], true, tdst[f1'', f2''],
pdata, q1, q2)
                end
            end
        end
    end
    return tdst
end
# TODO: contraction with either A or B a rank (1, 1) tensor does not require to
# permute the fusion tree and should therefore be special cased. This will speed
# up MPS algorithms
function contract!(α, A::AbstractTensorMap{S}, B::AbstractTensorMap{S},
                     β, C::AbstractTensorMap{S},
                     oindA::IndexTuple{N1}, cindA::IndexTuple,
                     oindB::IndexTuple{N2}, cindB::IndexTuple,
                     p1::IndexTuple, p2::IndexTuple,
                     syms::Union{Nothing, NTuple{3, Symbol}} = nothing) where {S,
N_1, N_2
    # find optimal contraction scheme
    hsp = has shared permute
    ipC = TupleTools.invperm((p1..., p2...))
    oindAinC = TupleTools.getindices(ipC, ntuple(n->n, N_1))
    oindBinC = TupleTools.getindices(ipC, ntuple(n->n+N1, N2))
    qA = TupleTools.sortperm(cindA)
    cindA' = TupleTools.getindices(cindA, qA)
    cindB' = TupleTools.getindices(cindB, qA)
    qB = TupleTools.sortperm(cindB)
    cindA'' = TupleTools.getindices(cindA, qB)
    cindB' = TupleTools.getindices(cindB, gB)
    dA, dB, dC = dim(A), dim(B), dim(C)
    # keep order A en B, check possibilities for cind
    memcost1 = memcost2 = dC*(!hsp(C, oindAinC, oindBinC))
    memcost1 += dA*(!hsp(A, oindA, cindA')) +
                dB*(!hsp(B, cindB', oindB))
    memcost2 += dA*(!hsp(A, oindA, cindA'')) +
                dB*(!hsp(B, cindB'', oindB))
    # reverse order A en B, check possibilities for cind
    memcost3 = memcost4 = dC*(!hsp(C, oindBinC, oindAinC))
    memcost3 += dB*(!hsp(B, oindB, cindB')) +
                                                                                   гау<del>с</del> / от 11
```

```
dA*(!hsp(A, cindA', oindA))
    memcost4 += dB*(!hsp(B, oindB, cindB'')) +
                dA*(!hsp(A, cindA'', oindA))
    if min(memcost1, memcost2) <= min(memcost3, memcost4)</pre>
        if memcost1 <= memcost2</pre>
            return _contract!(\alpha, A, B, \beta, C, oindA, cindA', oindB, cindB', p1, p2,
syms)
        else
            return _contract!(α, A, B, β, C, oindA, cindA´´, oindB, cindB´´, p1,
p2, syms)
        end
    else
        p1' = map(n->ifelse(n>N_1, n-N_1, n+N_2), p1)
        p2' = map(n->ifelse(n>N_1, n-N_1, n+N_2), p2)
        if memcost3 <= memcost4</pre>
            return _contract!(α, Β, Α, β, C, oindB, cindB', oindA, cindA', p1',
p2', syms)
            return _contract!(α, Β, Α, β, C, oindB, cindB'', oindA, cindA'', p1',
p2', syms)
        end
    end
end
function _contract!(α, A::AbstractTensorMap{S}, B::AbstractTensorMap{S},
                     β, C::AbstractTensorMap{S},
                     oindA::IndexTuple{N1}, cindA::IndexTuple,
                     oindB::IndexTuple{N2}, cindB::IndexTuple,
                     p1::IndexTuple, p2::IndexTuple,
                     syms::Union{Nothing, NTuple{3, Symbol}} = nothing) where {S,
N_1, N_2
    if !(BraidingStyle(sectortype(S)) isa SymmetricBraiding)
        throw(SectorMismatch("only tensors with symmetric braiding rules can be
contracted;
                                 try `@planar` instead"))
    end
    copyA = false
    if BraidingStyle(sectortype(S)) isa Fermionic
        for i in cindA
            if !isdual(space(A, i))
                 copyA = true
            end
        end
    end
    if syms === nothing
        A' = permute(A, oindA, cindA; copy = copyA)
        B' = permute(B, cindB, oindB)
    else
        A' = cached_permute(syms[1], A, oindA, cindA; copy = copyA)
        B' = cached_permute(syms[2], B, cindB, oindB)
    if BraidingStyle(sectortype(S)) isa Fermionic
        for i in domainind(A')
                                                                                   raye o ur l1
```

```
if !isdual(space(A', i))
                A' = twist!(A', i)
            end
        end
    end
    ipC = TupleTools.invperm((p1..., p2...))
    oindAinC = TupleTools.getindices(ipC, ntuple(n->n, N1))
    oindBinC = TupleTools.getindices(ipC, ntuple(n->n+N1, N2))
    if has_shared_permute(C, oindAinC, oindBinC)
        C' = permute(C, oindAinC, oindBinC)
        mul!(C', A', B', \alpha, \beta)
    else
        if syms === nothing
            C' = A'*B'
        else
            p1' = ntuple(identity, N_1)
            p2' = N_1 + ntuple(identity, N_2)
            TC = eltype(C)
            C' = T0.cached_similar_from_indices(syms[3], TC, oindA, oindB, p1',
p2', A, B,
                                                       :N, :N)
            mul!(C', A', B')
        end
        add! (\alpha, C', \beta, C, p1, p2)
    end
    return C
end
# Add support for cache and API (`@tensor` macro & friends) from
  TensorOperations.jl:
# compatibility layer
function TensorOperations.memsize(t::TensorMap)
    s = 0
    for (c, b) in blocks(t)
        s += sizeof(b)
    end
    return s
end
TensorOperations.memsize(t::AdjointTensorMap) = TensorOperations.memsize(t')
function TO.similarstructure_from_indices(T::Type, p1::IndexTuple, p2::IndexTuple,
        A::AbstractTensorMap, CA::Symbol = :N)
    if CA == :N
        _similarstructure_from_indices(T, p1, p2, A)
    else
        p1 = adjointtensorindices(A, p1)
        p2 = adjointtensorindices(A, p2)
        _similarstructure_from_indices(T, p1, p2, adjoint(A))
    end
end
function TO.similarstructure_from_indices(T::Type, poA::IndexTuple, poB::IndexTuple,
        p1::IndexTuple, p2::IndexTuple,
        A::AbstractTensorMap, B::AbstractTensorMap,
        CA::Symbol = :N, CB::Symbol = :N)
                                                                                   гау<del>с</del> э от 11
```

```
if CA == :N && CB == :N
        _similarstructure_from_indices(T, poA, poB, p1, p2, A, B)
    elseif CA == :C && CB == :N
        poA = adjointtensorindices(A, poA)
        _similarstructure_from_indices(T, poA, poB, p1, p2, adjoint(A), B)
    elseif CA == :N && CB == :C
        poB = adjointtensorindices(B, poB)
        _similarstructure_from_indices(T, poA, poB, p1, p2, A, adjoint(B))
    else
        poA = adjointtensorindices(A, poA)
        poB = adjointtensorindices(B, poB)
        _similarstructure_from_indices(T, poA, poB, p1, p2, adjoint(A), adjoint(B))
    end
end
function _similarstructure_from_indices(::Type{T}, p1::IndexTuple{N1},
p2::IndexTuple{N<sub>2</sub>},
        t::AbstractTensorMap{S}) where {T, S<:IndexSpace, N1, N2}
    cod = ProductSpace{S, N1}(space.(Ref(t), p1))
    dom = ProductSpace{S, N<sub>2</sub>}(dual.(space.(Ref(t), p2)))
    return dom→cod
end
function _similarstructure_from_indices(::Type{T}, oindA::IndexTuple,
oindB::IndexTuple,
        p1::IndexTuple{N<sub>1</sub>}, p2::IndexTuple{N<sub>2</sub>},
        tA::AbstractTensorMap{S}, tB::AbstractTensorMap{S}) where {T,
S<:IndexSpace, N<sub>1</sub>, N<sub>2</sub>}
    spaces = (space.(Ref(tA), oindA)..., space.(Ref(tB), oindB)...)
    cod = ProductSpace{S, N1}(getindex.(Ref(spaces), p1))
    dom = ProductSpace{S, N<sub>2</sub>}(dual.(getindex.(Ref(spaces), p2)))
    return dom→cod
end
TO.scalar(t::AbstractTensorMap) = scalar(t)
function T0.add!(\alpha, tsrc::AbstractTensorMap{S}, CA::Symbol, \beta,
    tdst::AbstractTensorMap{S, N1, N2}, p1::IndexTuple, p2::IndexTuple) where {S,
N_1, N_2
    if CA == :N
        p = (p1..., p2...)
        pl = TupleTools.getindices(p, codomainind(tdst))
        pr = TupleTools.getindices(p, domainind(tdst))
        add!(α, tsrc, β, tdst, pl, pr)
    else
        p = adjointtensorindices(tsrc, (p1..., p2...))
        pl = TupleTools.getindices(p, codomainind(tdst))
        pr = TupleTools.getindices(p, domainind(tdst))
        add!(\alpha, adjoint(tsrc), \beta, tdst, pl, pr)
    end
    return tdst
end
```

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```
function TO.trace!(α, tsrc::AbstractTensorMap{S}, CA::Symbol, β,
    tdst::AbstractTensorMap{S, N1, N2}, p1::IndexTuple, p2::IndexTuple,
    q1::IndexTuple, q2::IndexTuple) where {S, N<sub>1</sub>, N<sub>2</sub>}
    if CA == :N
        p = (p1..., p2...)
        pl = TupleTools.getindices(p, codomainind(tdst))
        pr = TupleTools.getindices(p, domainind(tdst))
        trace! (\alpha, tsrc, \beta, tdst, pl, pr, q1, q2)
    else
        p = adjointtensorindices(tsrc, (p1..., p2...))
        pl = TupleTools.getindices(p, codomainind(tdst))
        pr = TupleTools.getindices(p, domainind(tdst))
        q1 = adjointtensorindices(tsrc, q1)
        q2 = adjointtensorindices(tsrc, q2)
        trace!(α, adjoint(tsrc), β, tdst, pl, pr, q1, q2)
    end
    return tdst
end
function TO.contract!(\alpha,
    tA::AbstractTensorMap{S}, CA::Symbol,
    tB::AbstractTensorMap{S}, CB::Symbol,
    β, tC::AbstractTensorMap{S, N<sub>1</sub>, N<sub>2</sub>},
    oindA::IndexTuple, cindA::IndexTuple,
    oindB::IndexTuple, cindB::IndexTuple,
    p1::IndexTuple, p2::IndexTuple,
    syms::Union{Nothing, NTuple{3, Symbol}} = nothing) where \{S, N_1, N_2\}
    p = (p1..., p2...)
    pl = ntuple(n->p[n], N_1)
    pr = ntuple(n->p[N_1+n], N_2)
    if CA == :N && CB == :N
        contract!(\alpha, tA, tB, \beta, tC, oindA, cindA, oindB, cindB, pl, pr, syms)
    elseif CA == :N && CB == :C
        oindB = adjointtensorindices(tB, oindB)
        cindB = adjointtensorindices(tB, cindB)
        contract!(\alpha, tA, tB', \beta, tC, oindA, cindA, oindB, cindB, pl, pr, syms)
    elseif CA == :C && CB == :N
        oindA = adjointtensorindices(tA, oindA)
        cindA = adjointtensorindices(tA, cindA)
        contract!(\alpha, tA', tB, \beta, tC, oindA, cindA, oindB, cindB, pl, pr, syms)
    elseif CA == :C && CB == :C
        oindA = adjointtensorindices(tA, oindA)
        cindA = adjointtensorindices(tA, cindA)
        oindB = adjointtensorindices(tB, oindB)
        cindB = adjointtensorindices(tB, cindB)
        contract!(\alpha, tA', tB', \beta, tC, oindA, cindA, oindB, cindB, pl, pr, syms)
    else
        error("unknown conjugation flags: $CA and $CB")
    end
    return tC
end
```