# Solutions

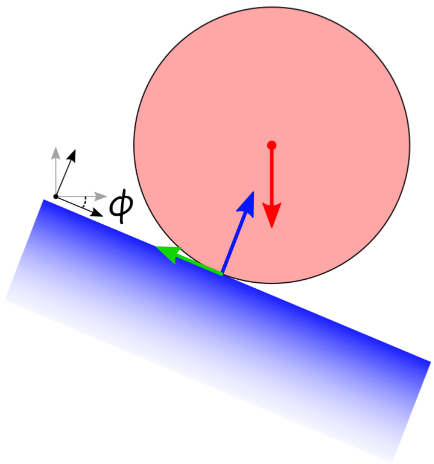
This notebook refers to the exam "INGE1243_January_2023.pdf".

# Question 1 ( /7)

### Dimensional analysis ( /2)

1. *A priori* this acceleration could depend on:
   - the radius of the object (disk or wheel), $R$ $[Length]$
   - the mass of the object, $M$ $[Mass]$
   - the Earth's gravity, $g$ $[LT^{-2}]$
   - the angle of inclination of the inclined plane, $\phi$ $[nodimension]$
2. The physical dimension of the acceleration is $[LT^2]$
3. $$[a_{CM}] = [R]^a[M]^b[g]^c$$
   This equation can be solved with $(a, b, c) = (0, 0, 1)$. From this equation, one can tell that :
   $$a_{CM} = g\ f(\phi)$$
   where $f$ is a function (to be determined) of the angle $\phi$ (both $f$ and $\phi$ having no physical dimension).

### Vector analysis ( /2)




4. - The **red** arrow on this figure represents the attraction of the Earth due to gravity. Let us call it $\vec{g}$.
   - The **blue** arrow represents the reaction of the floor. Let us call it $\vec{n}$.
   - The **green** arrow represents the friction of the floor. This force is responsible for the rotation of the wheel. Let us call it $\vec{f}$.
5. See course 5.
6. See course 5.
7. $$a_{CM} = -\frac{gsin(\phi)}{1+b}$$
   where $\phi$ is negative as it can be identified from the above figure. The acceleration is constant through time. This motion is a 1D uniform linear motion.
8. We know that
   $$a_{disk}(t) = -\frac{gsin(\phi)}{1+b_{disk}}$$
   so
   $$v_{disk}(t) = a_{disk}t$$
   and
   $$x_{disk}(t) = \frac{a_{disk}t^2}{2}$$
   where $x_{disk}$ is the distance covered during a time $t$. To cover the distance $d$, a time $t_{disk}$ of
   $$+\sqrt{\frac{2d}{a_{disk}}}$$
   is required. The same reasoning being valid for $t_{hoop}$. Leading to :
   $$\frac{t_{disk}}{t_{hoop}} = \sqrt{\frac{a_{hoop}}{a_{disk}}} = \sqrt{\frac{1+b_{disk}}{1+b_{hoop}}}$$
   Looking at your **form**, you can find that $b_{disk} = \frac{1}{2}$ and that $b_{hoop} = 1$. So,
   $$\frac{t_{disk}}{t_{hoop}} = \frac{\sqrt{3}}{2}$$

### Energy analysis ( /3)

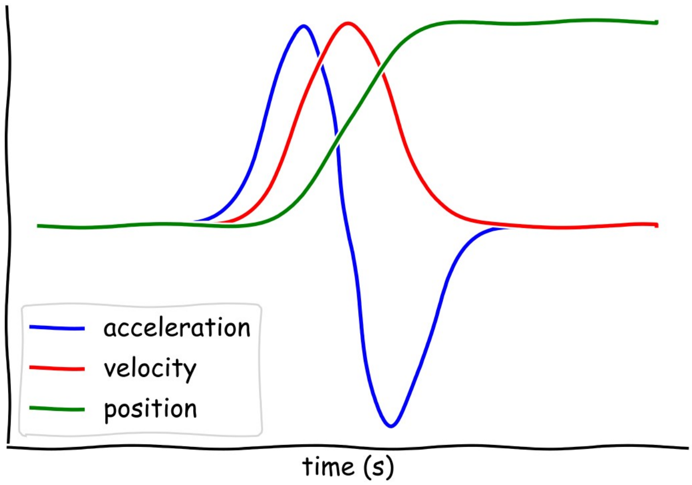
9. See course 8.
10. See course 8.

11. See course 8.

# Question 2 ( /6)

### Question A

The instantaneous acceleration is the first derivative of the instantaneous velocity. The instantaneous velocity is the first derivation of the position. The strategy is to find a curve for the velocity whose derivative is the **given** curve presented on the graph (integration). The initial condition for the velocity at $t = 0$ is **given** by $v(t = 0) = 0$ (see the **red** curve on the following figure). The position is obtained following the same strategy (with $x(t = 0) = 0$)(see the **green** curve).




### Question B

1. Path 4
2. The bowling ball is inside the plane so it has the same initial velocity:

$$\vec{v}(t = 0) = V\mathbf{e}_x$$

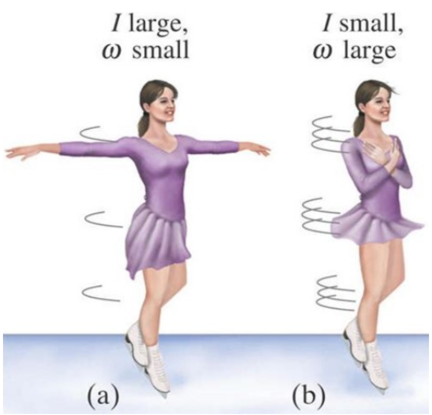
3. We face a **projectile motion** since the only force is the constant force due to gravity and that the ball is shooted at a given angle ($\theta = 0$) with an initial velocity.

### Question C

The key word in this question is that the path is **frictionless**. In such situation, the work-energy theorem can be applied to answer. Indeed, the $Work$ is simply $mg\Delta h$ where $m$ is the mass of the girl, $g$, the acceleration at the surface of the Earth and $\Delta h$, the difference in height between the starting and the ending point.

The change in kinetic energy (starting from rest) is equal to the work **no matter the path** (answer 5).

### Question D




Suppose that the iceskater, presented in (a) on the figure, rotates at an angular $\omega_a$. In (a), the inertia $I_a$ is proportional to $R_a^2$. When drawing her arms in, the radius becomes $R_b$ (smaller than $R_a$) thus, the inertia $I_b$ is smaller than the inertia $I_a$. The key observation is that, with no external torque applied, the **angular momentum is conserved between both configurations**.

$$L_a = I_a\omega_a = L_b = I_b\omega_b$$

For all these equality to be valid $\omega_b$ has to **increase** (answer 1).

### Question E

Using the right hand rule, we have:

1. Arrow 1 (the proton carries a positive charge).
2. Arrow 3 (the electron carries a negative charge).
3. The magnetic field should be oriented **inside** the plane for the force to be oriented that way.

### Question F (erratum: named G)

The period of the simple pendulum in the small angle approximation is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

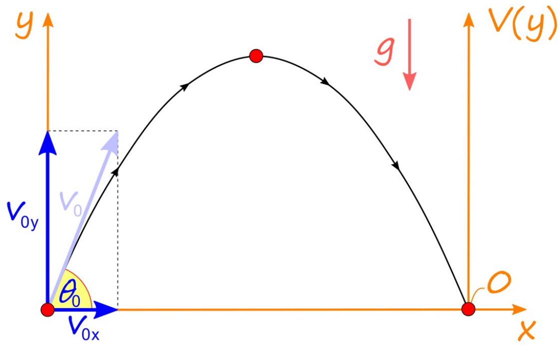
1. T remains unchanged.
2. T remains unchanged.
3. T gets multiplied by a factor $\sqrt{4}$ (=2).
4. T remains unchanged.
5. In such situation, the acceleration at the surface of the Earth would be multiplied by 4 (since it is proportional to the **inverse square** of the radius). The "new g" being multiplied by 4, the period gets divided by a factor $\sqrt{4}$ (=2).

## Question 3

### Question A (retrieved from the mock exam)

For additionnal information, consult the exercice session 3 and the course 7.

Considering the following figure :




The range $R$ of a projectile is given by:

$$R = \frac{v_0^2}{g}sin(2\theta_0)$$

where $v_0$ is the magnitude of the initial velocity and $\theta_0$ the initial shooting angle.

Note also that the total time spent in the air is:

$$T = \frac{2v_0 sin(\theta_0)}{g}$$

1. The initial spped is obtained by solving:

$$60 = \frac{v_0^2}{9.81}sin(90°)$$

which gives us $v_0 = 24.26$ m/s (and thus, not asked but worth computing, $T = 3.49\ s$)

2. With such initial velocity, the horizontal components $v_{0x}$ (concerved up to the brief gust of wind) is given by:

$$v_{0x} = 24.26 \times cos(45°) = 17.15\ m/s$$

At the maximum height (the ball already covered 30 m), $v_{0x}$ is reduced by $1.5\ m/s$

The "new" motion, from that point, is a projectile motion with an initial velocity of **15.65 m/s** and an initial shooting angle of 0°.

This gust of wind, since it affects only the horizontal direction, has no effect on the amount of time $T$ that the ball spends in the air.

Half of this time is the time spent on the second half of the trajectory (=3.49/2=1.74).

At an horizontal speed of $17.15\ m/s$, this corresponds to a distance of:

$$d = 15.65 \times 1.74 = 27.23$$

The total distance is then $30 + 27.23 = 57.23\ m.$

### Question B

The terminal velocity of the squirrel is reached when the drag force $F_D$ equals the weight of the squirrel $mg$.

$$\frac{1}{2}\rho C_D A V_T^2 = mg$$

which means:

$$V_T = \sqrt{\frac{2mg}{\rho C_D A}}$$

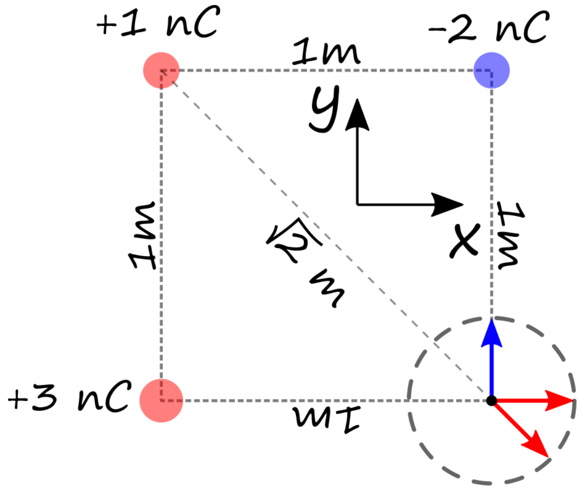
where $m = 0.56\ kg$, $A = 0.06\ m^2$, $\rho = 1.2\ kg/m^3$, $C_D = 0.7$ and $g = 9.81\ m/s^2$.

We have:

$$V_T = 14.76\ m/s$$

### Question C

Based on the following figure, let us first compute the electric field caused by each charge at the location of the last corner and then sum them.




Electric field due to the first **positive** charge $\vec{E}_1$ :

The distance separating the first corner from the last one is $\sqrt{2}$. Let us call $\mathbf{e}_*$, the unit vector aligned with $\vec{E}_1$. It is expressed, in the $\mathbf{e}_x$, $\mathbf{e}_y$ coordinate system as:

$$\mathbf{e}_* = \frac{\sqrt{2}}{2}\mathbf{e}_x - \frac{\sqrt{2}}{2}\mathbf{e}_y$$

We have that:

$$\vec{E}_1 = \frac{k * (+1) *\ 10^{-9}}{(\sqrt{2})^2} * \left(\frac{\sqrt{2}}{2}\mathbf{e}_x - \frac{\sqrt{2}}{2}\mathbf{e}_y\right)$$

Electric field due to the second **negative** charge $\vec{E}_2$ :

The distance separating the first corner from the last one is $1$. Let us call $\mathbf{e}_*$, the unit vector aligned with $\vec{E}_2$. It is expressed, in the $\mathbf{e}_x$, $\mathbf{e}_y$ coordinate system as:

$$\mathbf{e}_* = -\mathbf{e}_y$$

We have that:

$$\vec{E}_2 = \frac{k * (-2) *\ 10^{-9}}{1^2} * (-\mathbf{e}_y)$$

Electric field due to the third **positive** charge $\vec{E}_3$ :

The distance separating the first corner from the last one is $1$. Let us call $\mathbf{e}_*$, the unit vector aligned with $\vec{E}_2$. It is expressed, in the $\mathbf{e}_x$, $\mathbf{e}_y$ coordinate system as:

$$\mathbf{e}_* = \mathbf{e}_x$$

We have that:

$$\vec{E}_3 = \frac{k * (+3) *\ 10^{-9}}{1^2} * (\mathbf{e}_x)$$

We now have to compute the vector sum:

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

A frequent mistake is to only sum the amplitudes (scalar sum). The following relation has no meaning:

$$E_{tot} = \frac{k * 10^{-9}}{(\sqrt{2})^2} - \frac{2k * 10^{-9}}{(1)^2} + \frac{3k * 10^{-9}}{(1)^2}$$

$$\vec{E}_{tot} = k * 10^{-9} \left[ \left( \frac{\sqrt{2}}{4} + \frac{3}{1} \right) \mathbf{e}_x - \left( \frac{\sqrt{2}}{4} - \frac{2}{1} \right) \mathbf{e}_y \right]$$

We now that:

$$k * 10^{-9} = 9$$

We have:

$$E_x = 30.1819$$

and

$$E_y = 14.8180$$

so,

$$|\vec{E}_{tot}| = \sqrt{E_x^2 + E_y^2} = 33.6232 \left( \frac{N}{C} \right)$$

and

$$\theta = tan^{-1} \left( \frac{E_y}{E_x} \right) = 26.1489°$$

### Question D

1. $a = R\omega^2$ with $\omega = \frac{2\pi}{T}$ where $T$ is the duration of the sideral month $(27.3 * 24 * 60 * 60\ s)$ and $R = 384000000\ m$. Hence, $a = 0.00272482\ m/s^2$.
2. $a = G\frac{M_{Earth}}{R^2} = 0.00270136\ m/s^2$

Entrée [ ]: