Université Saint-Louis	Last Name		
INGE1243 - Conceptual physics with technical applications	First name		
	ID number		
14/01/2023 from 08 :30 to 11 :30	BBeng	11BA ☐ 12BA ☐ 13BA ☐	

The exam is 3 hours long. No graphical calculator is allowed.

We suggest you to :

— distribute your time equally between the three main questions (just a suggestion).
— spend your effort cleverly.
— read out **carefully** the different statements.
— don't hesitate to ask clarifications if something seems not clear.
— write down all physical quantities with their **physical unit** (e.g. 0.8 $[kg]$, for a mass, 17.54 $[m]$ for a length, etc.).
— answer as clearly and properly as possible (your teacher is no hieroglyphologist).

	Grade	Estimate
Q1	.../7	.../7
Q2	.../6	.../6
Q3	.../7	.../7
Total	.../20	.../20

Question 1 A race without stakes . . . /7

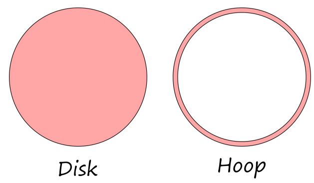
This question evaluates your ability to **look** at the same problem with **different perspectives**. It will be sequenced in four parts :

- **Dimensional analysis** is used to identify the key feature(s) of the analysed motion.
- Based on a **free body diagram**, a **vector-based** equation is solved to complement the output(s) of the first question.
- The problem is now solved thanks to the **work-energy theorem**
- Your solution serves to realize **predictions** on a real-life example with specified parameters that are, for example, a radius, a mass, a velocity etc.

Throughout your career path, you will hear many people make use of the expression **"Let us not reinvent the wheel"**. It is indeed a wise piece of advice since this breakthrough invention (credited to the Mesopotamian civilisation during the first **Neolithic** revolution - 10.000 BC) required a huge and combined intellectual effort. Nevertheless, even though this object is considered as "well mastered" by humankind, this expression does not prevent you from letting you get inspired by the beauty and elegance of the physics behind the wheel.

A seminal question when considering the rotation of wheels is the following :

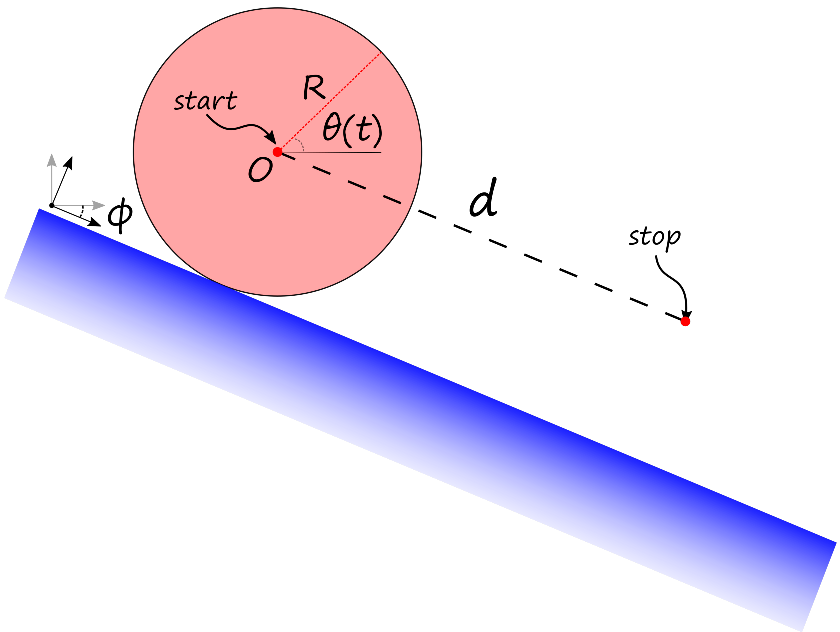
Who **wins the race** between a disk cylinder and a hoop cylinder (neglectable thickness) when both are the same radius and mass and that you make them roll on an **inclined plane** (located on Earth's surface) ?



In an experiment (where you note that the wheel **rolls without slipping**), you can show that the **time interval** required to cover **any** distance d (see the figure presented below) is smaller for the "disk-like" wheel than for the "hoop-like" wheel. The disk **wins** the race. This question aims at computing **by how much**.

To answer this question, let us go step by step.

Here is a sketch of the situation :



The key physical quantity in this problem is the linear acceleration of the center of mass of the wheel (written $\vec{a}_{CM}$).

Dimensional analysis (/2)

1. *A priori*, could you list the different **relevant** physical quantities that the magnitude of this acceleration (written a_{CM}) could depend on ? Name them and identify them with their physical dimension. Hint : It could, for example, depend on the radius R whose physical dimension is $[L]$. You can use the usual symbol $[M]$ standing for "mass", $[L]$ for "length" and $[T]$ for "time.
2. What is the physical dimension of this acceleration ?
3. Based on a **dimensional analysis** equation, write down an equation for the magnitude of this acceleration (using the quantities you listed in sub-question 1).

Vector analysis (/2)

4. On the figure presented above, draw and name all the forces acting **on** the wheel at their correct point of application.
5. In the frame of reference tilted by an angle ϕ (see figure), write down (with appropriate **vector notations**) the **net force (written $\vec{F}$) acting on the wheel**. Relate its 'x' and 'y' components to the magnitude of the acceleration of the center of mass and also to R and $\theta(t)$ (shown on the figure presented above).
6. In the **same** frame of reference, write down (with appropriate **vector notations**) the **net torque (written $\vec{\tau}$)** about the center of the wheel. Relate it to the rate of change of the angular momentum.
7. Thanks to these equations (results of the questions 5 and 6), express the acceleration of the center of mass of the wheel. Does it depend on time ? What type of motion is this ?
8. For a given distance d and a fixed angle ϕ, starting from rest ($x(t=0)=0$ and $v(t=0)=0$), you can compute the time required for the **disk** to cover that d (let us call it t_{disk}). You can do the same for the **hoop** (on the same distance d)(t_{hoop}). Express the ratio and give its numerical value :

$$\frac{t_{disk}}{t_{hoop}}$$

Energy analysis (/3)

To solve this problem a little faster (and with much more elegance), let us use the **work-energy theorem**, written as follow :

$$\Delta K.E. = Work$$

9. Express the $Work$ realised by the relevant(s) force(s) on a distance d.
10. Express the change in Kinetic Energy ($\Delta K.E.$) (starting from rest $x(t=0)=0$ and $v(t=0)=0$).
11. From the equation of the **work-energy theorem**, obtain the acceleration a_{CM} (hint 1 : use the time derivative on both sides of this equation obtained in question 10) (hint 2 : it should be identical to the result obtained in question 7).

Solutions to Question 1

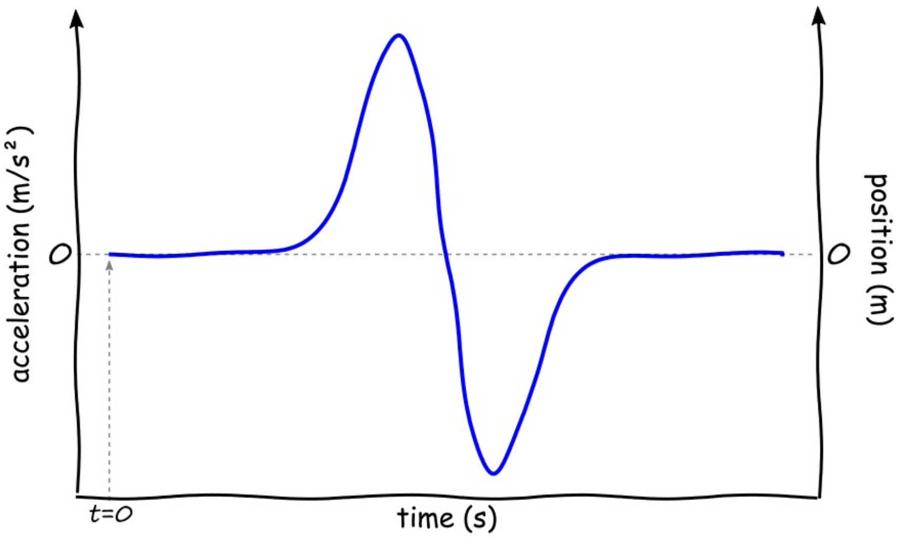
Solutions to Question 1

Question 2 Conceptual physics . . . /6

This question evaluates your ability to **identify** the relevant **concepts** required to explain an observation and present a **justification** (a few lines of text with, possibly, relevant equations - this is what is meant by 'justify'. Be as concise as possible, do not reinvent the wheel ;)).

Question A (/1)

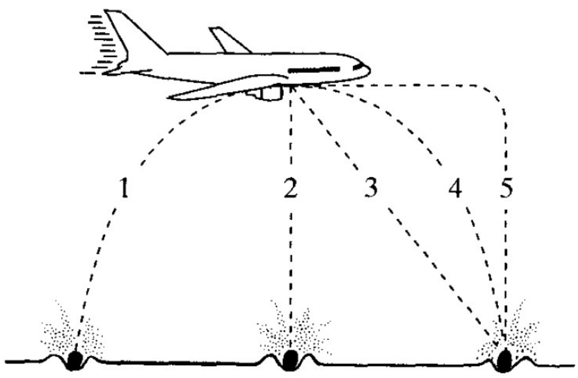
Considering the following time evolution of the acceleration of an object on a 1D line and with both the position and velocity at $t = 0$ being zero, draw **qualitatively** (on the same graph) the curve of the **position** with respect to time. Justify your strategy.



Question B (/1)

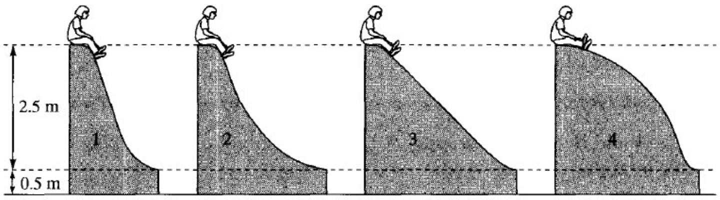
A bowling ball accidentally falls out of the cargo bay of an airliner as it flies (with a speed V) along in a horizontal direction. As observed by a person standing on the ground and viewing the plane as in the figure below :

1. Which of the paths 1-5 would the bowling ball most closely follow after leaving the airplane (neglect air friction) ?
2. What is the initial speed of the bowling ball ?
3. What type of motion does the bowling ball follow ? Justify your answer.



Question C (/1)

A young girl wishes to select one of the frictionless playground slides illustrated below to give her the greatest possible speed when she reaches the bottom of the slide.



Which of the slides illustrated in the diagram above should she choose ?

1. Slide 1
2. Slide 2
3. Slide 3
4. Slide 4
5. It does not matter, her speed would be the same for each slide.

Justify your choice.

Question D (/1)

An ice-skater spins about a vertical axis through her body with her arms held out. As she draws her arms in, her angular velocity :

1. increases.
2. decreases.
3. remains the same.
4. need more information

Note : The inertia of an ice-skater is of the form bmR^2 where b is a constant with no physical dimensions

Justify your choice.

Question E (/1)

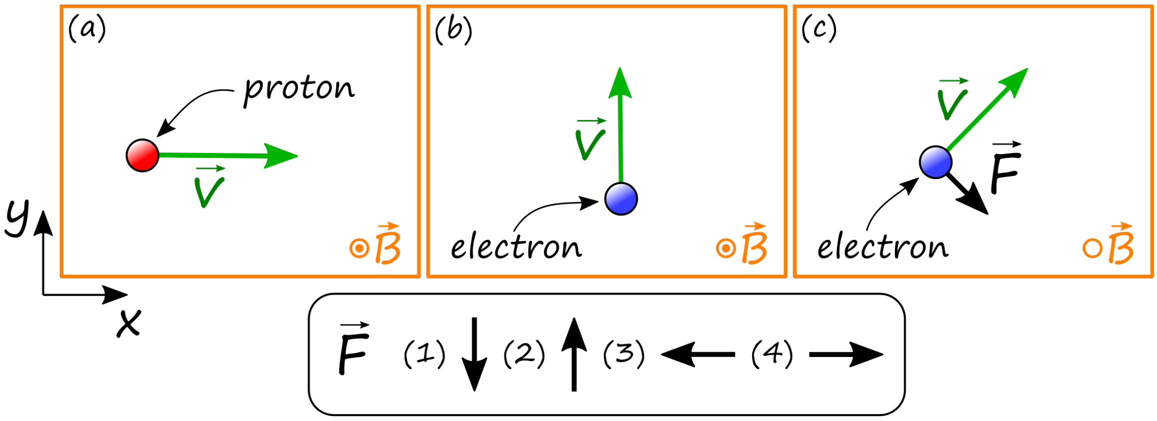
(No justification required for this question) Considering the **Lorentz force**, expressed as :

$$\vec{F} = q(\vec{v} \times \vec{B})$$

where q is the electric charge of the particle, $\vec{v}$ is the velocity and $\vec{B}$ is the magnetic field.

Pay attention that this symbol - $\odot$ - means that a vector points **outside** the sheet of paper (towards you when facing the questionnaire) and this one - $\otimes$ - **inside**.

Consider the following figures (where the magnetic field $\vec{B}$ is **uniform** in the orange plan) :



(the lengths of the arrows are arbitraries)

By which of the arrows is the force acting on the particle ($\vec{F}$) best represented :

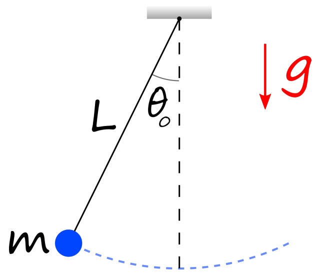
1. For the proton of figure (a)
2. For the electron in figure (b)

Your options :
- Arrow 1
- Arrow 2
- Arrow 3
- Arrow 4
- None of the arrows, the force is not within the orange plane.

3. Considering the velocity ($\vec{v}$) and the force ($\vec{F}$) acting on the electron, as illustrated in (c), is the magnetic field $\vec{B}$ pointing **outside** or **inside** the plane ?

Question G (/1)

A simple pendulum consists in a bob of mass m attached to a string of length L as illustrated on the figure below.



Considering that it takes a period T to complete a swing (the **small angle** approximation is considered as valid) and that the pendulum is located on Earth's surface.

By what factor should we multiply T if :

1. The mass gets multiplied by four.
2. The initial angle (θ_0) gets multiplied by four (the small angle approximation remaining valid).
3. The length gets multiplied by four.
4. Both the mass, the length and Earth gravity get multiplied by four.
5. The Earth's radius gets divided by two (all other things remaining unchanged, the Earth keeps the same mass, etc.).

Justify all your answers.

Solutions to Question 2

Solutions to Question 2

Question 3 Exercises . . . /7

This question evaluates your ability to **solve** problems with numerical values expected (**form and calculator available**).

Question A (/2)

A football player punts the ball at an angle of 45°. Without an effect from the wind, the ball would travel 60.0 m horizontally (neglect air friction).

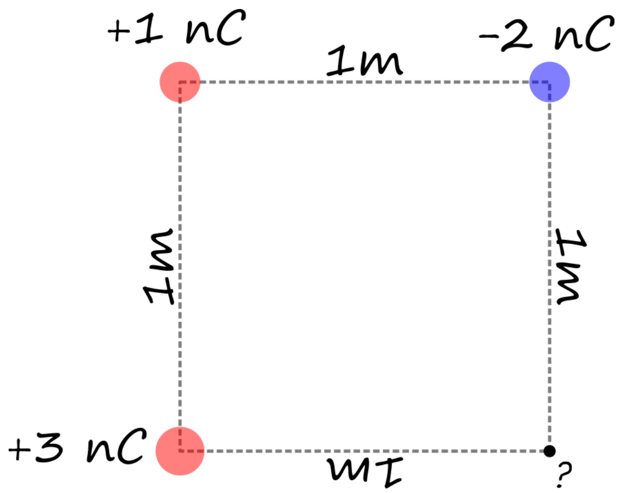
1. What is the initial speed of the ball ?
2. When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally ?

Question B (/1.5)

A 560-g squirrel with a surface area (facing the motion) of $600cm^2$ falls from a 5 meters tree to the ground (this distance is sufficient for the squirrel to reach its terminal velocity). Compute this terminal velocity (use a drag coefficient of 0.7).

Question C (/2)

Three point charges, of respectively +1, -2 and +3 nano-coulombs, are located on a plane as depicted on the following figure :



Suppose that a positive charge of +1 nano-coulombs is placed at the last corner of the dashed lines square. Find the magnitude and orientation of net force acting on this charge.

Question D (/1.5)

The Moon's nearly **circular** orbit about the earth has a radius (distance between both centers) of about $R = 384000km$ and the Moon appears to move completely around the celestial sphere once in about 27.3 days as observed from the Earth. This is called a sidereal month. It represents the orbital period of the Moon around the Earth.

1. Using the **uniform circular motion** equations, determine the acceleration of the Moon towards the Earth.

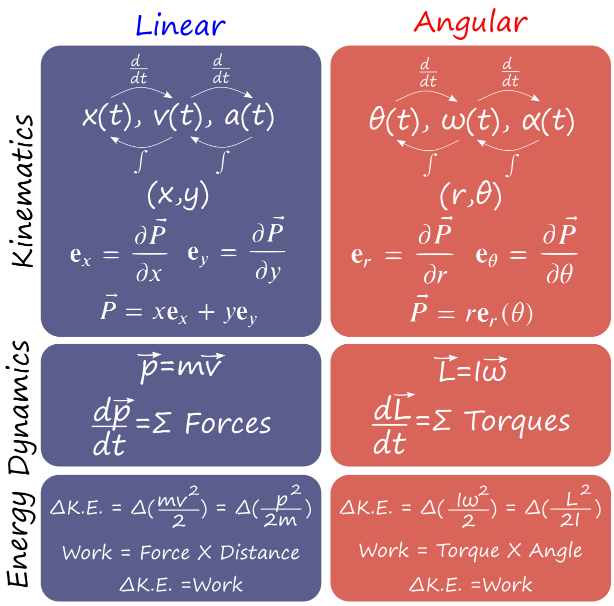
Knowing that $m_{Moon} = 7.348 \times 10^{22}$ [kg] is the mass of the Moon, $m_{Earth} = 5.972 \times 10^{24}$ [kg] is the mass of the Earth.

2. Using the expression of the gravitational force, determine the acceleration of the Moon towards the Earth.

Solutions to Question 3

Solutions to Question 3

Form



Symbol	Important constant name	Approximative value (to use)
c	Speed of light in vacuum	3×10^8 m/s
G	Gravitational constant	6.67×10^{-11} $N \cdot m^2/kg^2$
g	Gravity acceleration	9.81 m/s^2
k	Coulomb force constant	9×10^9 $N \cdot m^2/C^2$
q	Charge of the electron	-1.6×10^{-19} C
$\mathbf{m}_e$	electron mass	9.11×10^{-31} kg
$\mathbf{m}_p$	proton mass	1.67×10^{-27} kg
$\mathbf{m}_E$	Earth mass	5.97×10^{24} kg
$\mathbf{R}_E$	Earth radius (average)	6.37×10^6 m

Peculiar forces and torques

Gravitational force magnitude

$$F = G\frac{m_1 m_2}{R^2}$$

where G is the gravitational constant, m_1 the mass of the first particle, m_2 the mass of the second particle and R, the distance between the two.

Coulomb's law (force magnitude)

$$F = k\frac{|q_1 q_2|}{R^2}$$

where k is the Coulomb force constant, q_1 the charge of the first particle, q_2 the charge of the second particle and R, the distance between the two.

Lorentz force (vector)

$$\vec{F} = q(\vec{v} \times \vec{B})$$

where q is the electric charge of the particle, $\vec{v}$ is the velocity of the particle and $\vec{B}$ is the magnetic field.

Drag force (magnitude)

$$F_d = \frac{1}{2} c_d \rho v^2 A$$

where c_d is drag coefficient, A is the area of the object facing the fluid, v is the velocity of the object and ρ is the density of the fluid (e.g. air density : $1.2\ kg/m^3$).

Torque - Magnet in a magnetic field (vector)

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

where $\vec{\mu}$ is the magnetic moment and the magnetic field $\vec{B}$. (The potential energy associated being $P.E. = -\vec{\mu} \cdot \vec{B}$)

Peculiar laws

Uniform circular motion

Magnitude of the centripetal acceleration is :

$$a = \frac{V^2}{R}$$

which is the acceleration of an object in a circle of radius R at a speed V. The speed V is related to the angular velocity ω through $V = R\omega$. In the uniform circular motion ω is constant and defined by $\omega = \frac{2\pi}{T}$ with T, the time period to complete a **full turn**.

Peculiar physical properties

The inertia I is :

— For the hoop :

$$I = mR^2$$

— For the disk :

$$I = \frac{1}{2} mR^2$$

This is of the form bmR^2 where b is a constant with no physical dimensions (for the hoop, $(b = 1)$ and for the disk, $(b = \frac{1}{2})$. This is a frequent form for object presenting a rotational symmetry.

— For the simple pendulum :

$$I = mL^2$$