Linear Algebra Notes

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Abstract

This is going to contain a lot of linear algebra. This is just a draft for proof of concept.

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Linear Algebra

1 Introduction

1.1 Problem 1

Let's row-reduce this pair of linear equations

$$x_1 + 5x_2 = 7$$

$$-2x_1 - 7x_2 = -5$$

The augmented matrix form is thus:

$$\left(\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right)$$

If we add twice the first row to the second row, we get:

$$2R_1 + R_2 \rightarrow \left(\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right)$$

If we divide the entire second row by 3, we get:

$$\frac{1}{3} \cdot R_2 \to \left(\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array} \right)$$

If we add a negative factor of 5 of the second row to the first row, we get:

$$-5R_2 + R_1 \to \left(\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array}\right)$$

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$$x_1 = -8$$

$$x_2 = 3$$

1.2 Problem 2

$$2x_1 + 4x_2 = -4$$
$$5x_1 + 7x_2 = 11$$

This gives us the augmented matrix: $\begin{pmatrix} 2 & 4 & | & -4 \\ 5 & 7 & | & 11 \end{pmatrix}$

Solving for the first leading 1:

$$\frac{1}{2} \cdot R_2 \to \left(\begin{array}{cc|c} 1 & 2 & -2 \\ 5 & 7 & 11 \end{array} \right)$$

Make all entries below above and below zero:

$$-5 \cdot R_1 + R_2 \rightarrow \left(\begin{array}{cc|c} 1 & 2 & -2 \\ 0 & -3 & 21 \end{array}\right)$$

Solving for the second leading 1:

$$-\frac{1}{3} \cdot R_2 \to \left(\begin{array}{cc|c} 1 & 2 & -2 \\ 0 & 1 & -7 \end{array}\right)$$

Make all entries below above and below zero:

$$-2R_2 + R_1 \rightarrow \left(\begin{array}{cc|c} 1 & 0 & 12 \\ 0 & 1 & -7 \end{array}\right)$$

Thus:

$$x_1 = 12$$

$$x_2 = -7$$