Critical Scale Invariance in a Healthy Human Heart Rate

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[1,2,3]Physics of Data
[4]Physics of Matter

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1 Introduction

Probability Density Function (PDF) of detrended healthy human heart rate increments is strongly scale-invariant not only in a quiescent condition but also in a dynamic state.
A healthy human heart rate belongs to a special class of complex signals and shows the following three features:

1. Long-Range temporal correlation.
2. Non-Gaussianity of the increments PDF.
3. Multifractal scaling properties.

We can use two alternative mechanisms, both characterized by three properties mentioned above, for the heart rate complexity:

- Random Cascade Process based on the resemblance of the behavior of the structure function of heart rate increments to that of the spatial velocity differences in hydrodynamic turbulence.

- Critical Statelike Dynamics based on the resemblance of the scale-invariant properties of many systems operating near the critical point of their phase space.

Lin and Hughson reported an analogy between turbulence and heart rate dynamics of human by means of finding a similarity between structure functions of heart rate increments and the spatial velocity differences in a random cascade proposed as a model of hydrodynamic turbulence. As a matter of a fact, one of the common features is the evolution in the shape of the PDF of the increments from Gaussian at large scales to stretched exponential at smaller scales.

In our study, we demonstrate that heart rate signals do not follow the evolution in the shape of the increments' PDF characteristic for cascadelike processes, but they follow a robust scale invariance.

Moreover, there are two considerations leading us to have a conclusion:

1. We can admit that the features of a system which is at the critical point are as follows:
   - Long-Range correlation
   - Non-Gaussianity
   - Multifractality
2. Fluctuations in a system at a Critical point are associated with the scale-invariance and universal behavior of the scaling function.

Therefore, we can conclude that such robust-scale invariance in the increment PDF suggests an alternative scenario of the near critical statelike operation for the healthy heart rate dynamics.

Now, we are going to proceed with the calculations in a step-by-step procedure by means of programming in Python.

1.1 Building the cumulative time-series $B(i)$ using the detrended and normalized heartbeat series $b(i)$

We first derive the sequential heart interbeat intervals $b(i)$, where "i" is the beat number, from the long-term heart rate data analysis after implementing some modifications on the peak-detection algorithm $XQRS$, which is done in order to deal with the peaks of QRS. $XQRS$ algorithm itself has been adopted from the WaveForm DataBase library-$wfdb$. The performance of an automatic ECG analysis system relies much on the features extracted from fiducial points such as P, Q, R, S, T of which the $QRS-complex$ stands out for its large amplitude and sharp slope. Thus, QRS-complex detection has served as the fundamental step for the automatic detection of other ECG fiducial points and further analysis. The morphology of ECG varies greatly from person to person, even in different time for the same individual. In addition, ECG is generally contaminated with various noises, such as power-line interference, electrode contact noise, motion artifact, muscle contraction, and baseline wander. These factors have added challenges for automatic QRS-complex detection. Therefore, the first priority is to deal with the QRS peaks and detecting them with the help of using an appropriate algorithm.

Secondly, we make the $b(i)$ both detrended and normalized. Now, $B(i)$ can be obtained by integrating the $b(i)$ over the beat number.

$$B(i) = \Sigma_{j=1}^{i} b(j)$$

1.2 Polynomial Fit of $B(i)$

First of all the resultant $B(i)$ should be divided into sliding segments of size $2s$. Secondly, in each segment, the best 3^{rd} order polynomial is fit to the data.

1.3 Calculating the Increments (Fluctuations) from the Polynomial Fit

The differences $\Delta_s B(i) = B^*(i+s) - B^*(i)$ at a scale s are obtained by sliding over the segments, where $B^*(i)$ is a deviation from the polynomial fits. Therefore, the increments(fluctuation) from the polynomial fits can be obtained in this step.

It is highly important to know that scaling analysis of the qth-order Partition Function $Z_q(s)$ is one of the widely used methods to characterize fractal signals with long-range power-law correlations and multiscaling(multifractal) properties of the amplitude. Here, the Partition Function $Z_q(s) =< |B(i+s) - B(i)|^q >$, the Statistical Average, in the context of hydrodynamic turbulence is called Structure Function.

For fractal signals, structure function obeys the power law with s as
$Z_q(s) \sim s^{\zeta}(q)$ where $\zeta(q)$ is the scaling exponent.

For the multifractal signals, these scaling exponents $\zeta(q)$ exhibit non-linear dependency on q (order of the partition function).

1.4 Gaussian and Non-Gaussian fitting process of the increment PDF after its establishment

Increment's PDF is created by means of utilization of PDF creation in Python. Now, it's the time for going through the fitting process. The best fit should be chosen between Gaussian and Non-Gaussian ones! According to our reference guides, the Castaing fit is preferred as a Non-Gaussian fit. First of all, we have to characterize the non-Gaussian PDF meaning that we have to understand how different the PDF is from a Gaussian distribution.

1.4.1 Characterization of a Non-Gaussian PDF

We first obtain the standardized PDFs (the variance has been set to one) of $\Delta_s B(i)$, and then use two following methods:
Parameter estimation based on Castaing Equation introduced in the study of

hydrodynamic turbulence:

$$\Pi_s(x) = \int P_L(\frac{x}{\sigma}) G_{s,L}(ln\sigma)\, d(ln\sigma)$$

Intermittency only comes from the large distribution of s ($L > s$). For a fixed value of s it is thus simpler to assume that any quantity has a Gaussian distribution. Such a distribution is commonly defined by its variance σ. The distribution of x for a fixed s, is therefore given by

$$P_s(x) = \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{x^2}{2\sigma^2}) \equiv P_L(\frac{x}{\sigma})$$

Introducing now the intermittency through the fluctuations of σ, we postulate that σ has a log-normal distribution

$$Q_\lambda(\sigma)d\sigma = \frac{1}{\lambda\sqrt{2\pi}} exp(-\frac{ln^2(\sigma/\sigma_0)}{2\lambda^2}) d(ln\sigma)$$

and we can define $G_{s,L}$ as:

$$G_{s,L}(ln\sigma) \equiv \frac{1}{\sqrt{2\pi}\lambda} exp(-\frac{ln^2\sigma}{2\lambda^2})$$

where σ_0 is the probable variance of x.

Now, we can obtain the $\Pi_s(x)$ by the multiplication of $P_s(x)$ and $Q(\sigma)d\sigma$:

$$\Pi_s(x) = \int P_s(x) Q_\lambda(\sigma) d\sigma$$

$$\Pi_s(x) = \int \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{x^2}{2\sigma^2}) \frac{1}{\lambda\sqrt{2\pi}} exp(-\frac{ln^2(\sigma/\sigma_0)}{2\lambda^2}) d(ln\sigma)$$

where P_L is the increment PDF at a large scale $L > s$, and the kernel $G_{s,L}(ln\sigma)$ determines the nature of the cascade-type multiplicative process. We assume that P_L and $G_{s,L}$ are both Gaussian.
Now, we can estimate the parameter λ^2 which perfectly characterizes the scale-dependence of the multiscale PDF. The fitting parameter, λ^2, is estimated by the curve-fitting process done by means of *scipy.optimize.least squares*.

Within the multiplicative cascade picture originally introduced as a model of hydrodynamic turbulence, the parameter λ^2 can be interpreted as being proportional to the number of cascade steps and is known to decrease linearly with *logs* for the self-similar cascade process.
It is demonstrated that the shape of the PDF is more sensitive to the variance of $G_{s,L}$ than to its exact shape.

Note: Fitting of Castaing equation is useful in characterizing intermittent and non-Gaussian fluctuations of a stochastic process different from the cascade process.

1.5 Testing the Scale-invariance of the PDFs by means of Collapse Plot Utilization

Collapse data or Collapse plot is for checking any variances of the PDF between two Gaussian-like plots in order to understand the correlation, anti-correlation or absence of any correlations. In order to do that, one of the procedures is going to be mentioned below;

1.5.1 Calculation of the Hurst Exponent

We know that for fractal signals, structure function obeys the power law with s as $Z_q(s) \sim s^{\zeta}(q)$.

$$Z_q(s) =< |B(i+s) - B(i)|^q > \tag{1}$$

If the partition function (structure function) $Z_q(s)$ is of a second order $q = 2$, the scaling exponent $\zeta(q)$ is defined as $2H$, where H is called Hurst exponent meaning that $Z_2(s) \sim s^{2\mathbf{H}}$.
Conventionally, the mean square displacement $Z_2(s)$ is related to the so-called Hurst exponent H.
H represents the long-range power law correlation properties of the signal. There are three conditions for the modulus of H:

- if $H > 0.5$: The signal is correlated.
- if $H < 0.5$: The signal is anti-correlated
- if $H = 0.5$: There is no correlation and the signal $b(i)$, the increment of the $B(i)$ analyzed, is uncorrelated with noise.

1.6 DATABASEs

There are three experimental databases that our group has chosen to do the data analysis on:

1. MIT-BIH Normal Sinus Rhythm Database [1]
2. Fantasia Database [2]
3. BIDMC Congestive Heart Failure Database [3]

[1] https://physionet.org/content/nsrdb/1.0.0/
[2] https://physionet.org/content/fantasia/1.0.0/
[3] https://physionet.org/content/chfdb/1.0.0/

1.6.1 MIT-BIH Normal Sinus Rhythm Database Description

This database includes 18 long-term ECG recordings of subjects referred to the Arrhythmia Laboratory at Boston's Beth Israel Hospital (now the Beth Israel Deaconess Medical Center). Subjects included in this database were found to have had no significant arrhythmias; they include 5 men, aged 26 to 45, and 13 women, aged 20 to 50.

1.6.2 Fantasia Database Description

Twenty young (21 - 34 years old) and twenty elderly (68 - 85 years old) rigorously-screened healthy subjects underwent 120 minutes of continuous supine resting while continuous electrocardiographic (ECG), and respiration signals were collected; in half of each group, the recordings also include an uncalibrated continuous non-invasive blood pressure signal. Each subgroup of subjects includes equal numbers of men and women.
All subjects remained in a resting state in sinus rhythm while watching the movie Fantasia (Disney, 1940) to help maintain wakefulness. The continuous ECG, respiration, and (where available) blood pressure signals were digitized at 250 Hz. Each heartbeat was annotated using an automated arrhythmia detection algorithm, and each beat annotation was verified by visual inspection.
Records f1y01, f1y02, ... f1y10 and f2y01, f2y02, ... f2y10) were obtained from the young cohort, and records f1o01, f1o02, ... f1o10 and f2o01, f2o02, ... f2o10) were obtained from the elderly cohort. Each group of subjects includes equal numbers of men and women. Each record includes ECG and respiration, and half of those in each group (the f2* records) include a blood pressure waveform, as noted above.

1.6.3 BIDMC Congestive Heart Failure Database Description

This database is specially for testing the unhealthy cases. It includes long-term ECG recordings from 15 subjects (11 men, aged 22 to 71, and 4 women, aged 54 to 63) with severe congestive heart failure. The individual recordings are each about 20 hours in duration, and contain two ECG signals each sampled at 250 samples per second with 12-bit resolution over a range of ± 10 millivolts. The original analog recordings were made at Boston's Beth Israel Hospital (now the Beth Israel Deaconess Medical Center) using ambulatory ECG recorders with a typical recording bandwidth of approximately 0.1 Hz to 40 Hz. Annotation files (with the suffix .ecg) were prepared using an automated detector and have not been corrected manually.