3.1 Meantimer Technique

The meantimer formulas are relations among the drift times produced by a track in consecutive layers of a superlayer (t_i) and the maximum drift time (T_{max}) in a semi-cell (i.e. half cell), under the assumption of a constant drift velocity. Even with small deviations from this assumption, as in the case of the DTs, the average of the meantimer distribution contains information about the average drift velocity in different regions of the cell, since it is computed using drift times produced by hits all over the gas volume. The mathematical expression of the meantimer relation depends on the track angle and on the pattern of cells hit by the track. In the easiest case the track crosses a semi-column of cells i.e. the interested wires are at the same position for each couple of staggered cells. In this simple case the correspondent meantimer relation is

$$T_{max} = (t_1 + t_3)/2 + t_2 \tag{6}$$

The meantimer relations for different track angles and patterns of hit cells are listed in Table 1, using the naming convention illustrated in Fig. 4. It should be noted that not all the track geometrical configurations can be used because in some cases the relation between drift times is independent of T_{max}.

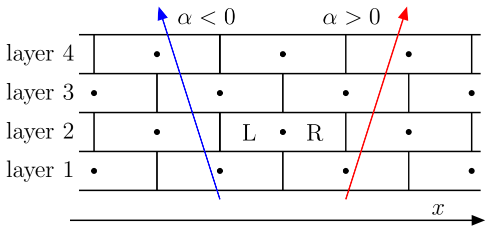

Figure 4: Schematic of a superlayer showing the track segment angle convention and the pattern of semi-cells crossed by the track.

The proper meantimer formula is chosen among those listed in Table 1 track by track, using the direction and position information provided by the three-dimensional segments in a superlayer. This implies an iterative calibration procedure, starting with values of the drift velocity and of t_{trig} that already result in efficient pattern recognition and segment reconstruction.

The meantimer is normally computed superlayer by superlayer, assuming the same effective drift velocity in all layers. It may be interesting, however, to calibrate the average drift velocity with finer granularity to take into account possible local variations within the layer quadruplet due to magnetic field inhomogeneities.
The mechanical precision of the wire and layer positions inside the superlayers is of the order of 100μm and it should be known to 10μm after the first alignment procedure. This precision corresponds to a bias of 1.8 ns (0.18 ns) on the measured drift times and it causes a different uncertainty on the T_{max} depending on the formula, the consequent error on the drift velocity is of the order of 1% (0.1%) or less.

In Section 3.1.1 the various steps of the drift velocity calibration procedure are listed.

3.1.1 Calibration Procedure

The calibration procedure of the drift velocity consists of the following steps:

- a Gaussian is fit to the meantimer distribution for each track pattern j to estimate the mean value T^j_{max}, the standard deviation σ^j_T, and the error on the mean $\sigma^j_T/\sqrt{N_j}$ (where N_j is the number of entries in the distribution);
- the weighted average of the values of T^j_{max} is computed where the weights are taken as $N_j/(\sigma^j_T)^2$:

$$\langle T_{max} \rangle = \frac{\sum_j \frac{T^j_{max}}{(\sigma^j_T)^2} N_j}{\sum_j \frac{N_j}{(\sigma^j_T)^2}}. \tag{7}$$

  This accounts for the relative importance of the different cell patterns in the computation of the maximum drift time.

- once $\langle T_{max}\rangle$ is computed it is straightforward to find the average drift velocity through the relation:

$$v_{drift} = \frac{L/2}{\langle T_{max}\rangle}; \tag{8}$$

where L is the width of the cell. The effective drift velocity computed for each superlayer is then stored in a database to be used by both the HLT and the off-line hit reconstruction.

3.2 Estimate of the Cell Resolution

The meantimer technique allows the estimation of the cell resolution and hence the uncertainties on the reconstructed distance.

The standard deviation of the meantimer distribution (σ_T^j) is a measurement of the resolution of T_{max}^j. It can be therefore used to estimate the uncertainty on the measurement of the drift times (σ_t^j) with a relation that depends on the particular formula used to compute the meantimer. In the case of tracks crossing a semi-column of cells (123LRL or 123RLR), given the meantimer relation in Table 1, the time resolution can be computed as

$$\sigma_t^j = \sqrt{\frac{2}{3}} \cdot \sigma_T^j, \tag{9}$$

which is valid under the assumption that the uncertainties are the same for all three layers used in the meantimer computation.

Since the cell resolution depends on the track angle, an average effective value is computed by averaging the different values obtained for the contributing cell patterns weighted on the number of entries in each meantimer histogram (N_j):

$$\langle\sigma_t\rangle = \frac{\sum_j \sigma_t^j \cdot N_j}{\sum_j N_j}. \tag{10}$$

The resolution of the reconstructed distance is therefore given by:

$$\sigma_d = v_{drift} \cdot \langle\sigma_t\rangle. \tag{11}$$

This value is used during the reconstruction to assign the uncertainties to the one-dimensional RecHits in the gas volume. These uncertainties include the effect of the cell non-linearities (as those shown in Fig.2) only on average, therefore their dependence on the distance from the wire cannot be taken into account with this method.

4 Interplay of Meantimer Computation and Time Pedestals Determination

Reconstruction using a constant drift velocity requires both the calibration of the time pedestals needed for synchronization and of the average drift velocity. These two tasks are not independent since on one hand the computation of the meantimer requires knowledge of the time pedestals and on the other hand fine tuning of t_{trig} is based on analysis of the residuals, which are directly affected by a mis-calibration of the drift velocity.

If the determination of t_{trig} is affected by a systematic shift Δt:

$$t'_{trig} = t_{trig} + \Delta t, \tag{12}$$

the meantimer will be consequently biased by a quantity that depends on the particular formula among those in Table 1. In the case of tracks crossing a semi-column (Table 1: 123LRL or 123RLR) we can evaluate the effect on T_{max} as

$$T'_{max} = T_{max} - 2\Delta t. \tag{13}$$

In a simplified scenario where this particular pattern is the one determining the meantimer calculation ($\langle T_{max}\rangle \approx T'_{max}$) the bias on t_{trig} determination will result in a mis-calibration of the drift velocity Δv_{drift}, which can be estimated as

$$\begin{aligned} v_{drift} + \Delta v_{drift} &= \frac{L}{2 \cdot T'_{max}} \\ &= \frac{L}{2 \cdot (T_{max} - 2\Delta t)}. \end{aligned} \tag{14}$$

To first order, this is equivalent to the following requirement:

$$2v_{drift}\Delta t - T_{max}\Delta v_{drift} = 0, \quad (15)$$

which can be considered as a calibration condition: all values of drift velocity and time pedestal that satisfy this relation will not affect the mean value of the residuals. This is strictly true only for small variations around the "optimal" values of t_{trig} and v_{drift} since larger fluctuations may affect pattern recognition efficiency and segment building. Lacking an external system for the track measurement, the segment is used as a reference for the computation of the residuals of the reconstructed drift distance.

The main sources of uncertainty in the determination of the time pedestal are the fluctuations in the mean value $\langle t \rangle$ and in the σ of the fit in the different layers of a superlayer: the intrinsic statistical error, the presence of noise before the drift time box (evidenced, e.g., by the entries shown in Fig. 1 before the starting point of the drift time box), the finite step size of the TDC (0.78 ns), and the fact that the distribution is not perfectly described by Eq. 4, which together limit the accuracy of t_{trig} determination to about 1 ns. Further systematic uncertainties come from the uncertainty of the drift velocity, as demonstrated by Eq. 15, therefore higher accuracy can only be achieved using a procedure for fine tuning of the time pedestal independent of the drift velocity.

An alternative approach consists in using the different dependences of t_{trig} mis-calibration of the various meantimer formulas listed in Table 1 to calibrate the pedestal. The differences among the values of T_{max} computed using different formulas can be used to measure the value of the mis-calibration Δt once the dependence of the meantimer on the track impact angle is well under control. This would allow t_{trig} to be tuned without relying on the residual distribution and therefore without depending on the calibration precision of the drift velocity. This alternative approach will be investigated in the future.

5 Conclusions

The calibration task is fundamental to the local reconstruction: the knowledge of the time pedestal is an unavoidable prerequisite for the computation of the drift distance, while the calibration of the average drift velocity determines the accuracy of the reconstruction.

For this reason, a robust calibration procedure has been developed with the goal of satisfying the requirements imposed by all possible running conditions: dedicated cosmic runs, test beams, and pp-collision data.

The calibration algorithms described in the present document have been tested both on the simulation and on real data acquired during commissioning, the MTCC, and the 2004 test beam. Additional documents are presently in preparation regarding these subjects.

Using the tools developed for the calibration and synchronization procedure we also studied the effect of possible mis-calibration of the pedestals and of the drift velocity on the muon track fit and thus eventually on higher level reconstructed quantities. We applied these to analysis of the systematic uncertainties while studying the physics reach of the experiment as documented in [7].

Further optimization is still possible. In particular, the accuracy of the current procedure is limited by the interdependence of the time pedestal and the drift velocity used in the reconstruction. Other methods for fine tuning of t_{trig} are under study; a procedure based on the usage of different meantimer formulas to estimate the best value of the time pedestal is the most promising.

Table 1: Meantimer equations for different track angles and patterns of hit semi-cells. The definition of the sign of the segment angle α is given in Fig. 4. The pattern is defined through four labels, one for each layer: L and R stand for left and right semi-cells, respectively. The label enclosed in parentheses refers to the layer not directly used in the T_{max} computation. Where necessary the relative positions of the hit wires of the first and the last layers in the chamber RF (x_1, x_4) are also shown. The time t_i is the measurement in the cell belonging to layer i.

ID	**Meantimer formula**	**Segment direction**	**Semi-cell pattern**
	Layers 1-2-3		
123LRL	$T_{max} = (t_1 + t_3)/2 + t_2$	all α	LRL(L/R)
123RLR			RLR(L/R)

Table 1: (continued)

ID	Meantimer formula	Segment direction	Semi-cell pattern
123LLR	$T_{max} = (t_3 - t_1)/2 + t_2$	$\alpha > 0$	LLR(L/R)
123RRL		$\alpha < 0$	RRL(L/R)
123LRR	$T_{max} = (t_1 - t_3)/2 + t_2$	$\alpha > 0$	LRR(L/R)
123RLL		$\alpha < 0$	RLL(L/R)
	Layers 1-2-4		
124LRR(1)	$T_{max} = 3t_2/2 + t_1 - t_4/2$	all α	LR(L)R $x_4 < x_1$
124RLL(1)			RL(R)L $x_4 > x_1$
124LLR	$T_{max} = 3t_2/2 - t_1 + t_4/2$	$\alpha > 0$	LL(L/R)R
124RRL		$\alpha < 0$	RR(L/R)L
124LLL(1)	$T_{max} = 3t_2/2 - t_1 - t_4/2$	$\alpha > 0$	LL(R)L
124LRR(2)			LR(R)R
124RRR(1)		$\alpha < 0$	RR(L)R
124RLL(2)			RL(L)L
124RRR(2)	$T_{max} = -3t_2/2 + t_1 + t_4/2$	$\alpha > 0$	RR(L)R
124LLL(2)		$\alpha < 0$	LL(R)L
124RLR	$T_{max} = 3t_2/2 + t_1 + t_4/2$	$\alpha > 0$	RL(L/R)R
124LRL		$\alpha < 0$	LR(L/R)L
124LRL	$T_{max} = 3t_2/4 + t_1/2 + t_4/4$	$\alpha > 0$	LR(L/R)L
124RLR		$\alpha < 0$	RL(L/R)R
124LRR(3)	$T_{max} = 3t_2/4 + t_1/2 - t_4/4$	$\alpha > 0$	LR(R)R $x_4 > x_1$
124RLL(3)		$\alpha < 0$	RL(L)L $x_4 < x_1$
	Layers 1-3-4		
134LLR(1)	$T_{max} = 3t_3/2 + t_4 - t_1/2$	all α	LRLR $x_4 < x_1$
134RRL(1)			RLRL $x_4 > x_1$
134LRR	$T_{max} = 3t_3/2 - t_4 + t_1/2$	$\alpha > 0$	L(L/R)RR
134RLL		$\alpha < 0$	R(L/R)LL
134RRR(1)	$T_{max} = 3t_3/2 - t_4 - t_1/2$	$\alpha > 0$	R(L)RR
134LLL(1)		$\alpha < 0$	L(R)LL
134LLL(2)	$T_{max} = -3t_3/2 + t_4 + t_1/2$	$\alpha > 0$	L(R)LL
134RRR(2)		$\alpha < 0$	R(L)RR
134LRL	$T_{max} = 3t_3/2 + t_4 + t_1/2$	$\alpha > 0$	L(L/R)RL
134RLR		$\alpha < 0$	R(L/R)LR
134RLR	$T_{max} = 3t_3/4 + t_4/2 + t_1/4$	$\alpha > 0$	R(L/R)LR
134LRL		$\alpha < 0$	L(L/R)RL
134LLR(2)	$T_{max} = 3t_3/4 + t_4/2 - t_1/4$	$\alpha > 0$	LLLR $x_4 > x_1$
134RLR(2)		$\alpha < 0$	RRRL $x_4 < x_1$

Table 1: (continued)

ID	Meantimer formula	Segment direction	Semi-cell pattern
		Layers 2-3-4	
234RLR	$T_{max} = (t_2 + t_4)/2 + t_3$	all α	(L/R)RLR
234LRL			(L/R)LRL
234LRR	$T_{max} = (t_2 - t_4)/2 + t_3$	$\alpha > 0$	(L/R)LRR
234RLL		$\alpha < 0$	(L/R)RLL
234LLR	$T_{max} = (t_4 - t_2)/2 + t_3$	$\alpha > 0$	(L/R)LLR
234RRL		$\alpha < 0$	(L/R)234RRL