APPENDIX B

Convolutional perfectly-matched layer regions and associated field updates for a three-dimensional domain

B.1 Updating E_x at convolutional perfectly-matched layer (CPML) regions

Initialization

Create new coefficient arrays for $C_{\psi exy}$ and $C_{\psi exz}$:

$$C_{\psi exy} = \Delta y C_{exhz} \quad C_{\psi exz} = \Delta z C_{exhy}.$$

Modify the coefficient arrays for C_{exhy} and C_{exhz} in the CPML regions:

$$C_{exhz} = (1/k_{ey}) C_{exhz}, \quad \text{in the } yn \text{ and } yp \text{ regions.}$$
$$C_{exhy} = (1/k_{ez}) C_{exhy}, \quad \text{in the } zn \text{ and } zp \text{ regions.}$$

Finite-difference time-domain (FDTD) time-marching loop

Update E_x in the full domain using the regular updating equation:

$$\begin{aligned} E_x^{n+1}(i,j,k) = {} & C_{exe}(i,j,k) \times E_x^n(i,j,k) \\ & + C_{exhz}(i,j,k) \times \left(H_z^{n+\frac{1}{2}}(i,j,k) - H_z^{n+\frac{1}{2}}(i,j-1,k)\right) \\ & + C_{exhy}(i,j,k) \times \left(H_y^{n+\frac{1}{2}}(i,j,k) - H_y^{n+\frac{1}{2}}(i,j,k-1)\right). \end{aligned}$$

Calculate $\psi_{exy}^{n+\frac{1}{2}}$ for the yn and yp regions:

$$\psi_{exy}^{n+\frac{1}{2}}(i,j,k) = b_{ey}\psi_{exy}^{n-\frac{1}{2}}(i,j,k) + a_{ey}\left(H_z^{n+\frac{1}{2}}(i,j,k) - H_z^{n+}(i,j-1,k)\right),$$

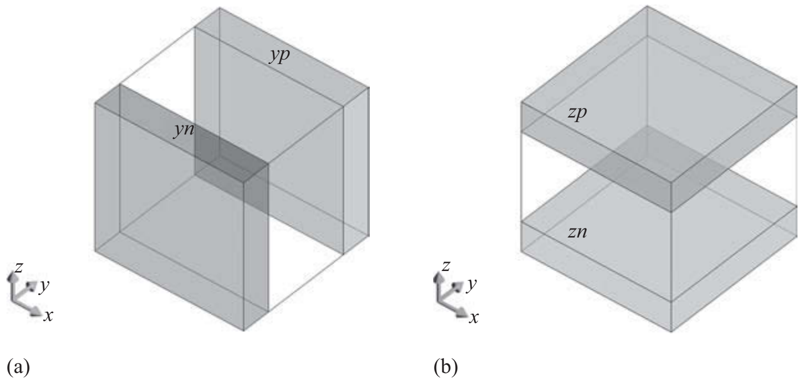


Figure B.1 CPML regions where E_x is updated: (a) E_x is updated in *yn* and *yp* using ψ_{exy} and (b) E_x is updated in *zn* and *zp* using ψ_{exz}.

where

$$a_{ey} = \frac{\sigma_{pey}}{\Delta y(\sigma_{pey}k_{ey} + \alpha_{ey}k_{ey}^2)}\left[b_{ey} - 1\right],$$

$$b_{ey} = e^{-\left(\frac{\sigma_{pey}}{k_{ey}}+\alpha_{pey}\right)\frac{\Delta t}{\varepsilon_0}}.$$

Calculate $\psi_{exz}^{n+\frac{1}{2}}$ for the *zn* and *zp* regions:

$$\psi_{exz}^{n+\frac{1}{2}}(i,j,k) = b_{ez}\psi_{exz}^{n-\frac{1}{2}}(i,j,k) + a_{ez}\left(H_y^{n+\frac{1}{2}}(i,j,k) - H_y^{n+\frac{1}{2}}(i,j,k-1)\right),$$

where

$$a_{ez} = \frac{\sigma_{pez}}{\Delta z(\sigma_{pez}k_{ez} + \alpha_{ez}k_{ez}^2)}[b_{ez} - 1],$$

$$b_{ez} = e^{-\left(\frac{\sigma_{pez}}{k_{ez}}+\alpha_{pez}\right)\frac{\Delta t}{\varepsilon_0}}.$$

Add the CPML auxiliary term to E_x in the *yn* and *yp* regions:

$$E_x^{n+1} = E_x^{n+1} + C_{\psi exy} \times \psi_{exy}^{n+\frac{1}{2}}.$$

Add the CPML auxiliary term to E_x in the *zn* and *zp* regions:

$$E_x^{n+1} = E_x^{n+1} + C_{\psi exz} \times \psi_{exz}^{n+\frac{1}{2}}.$$

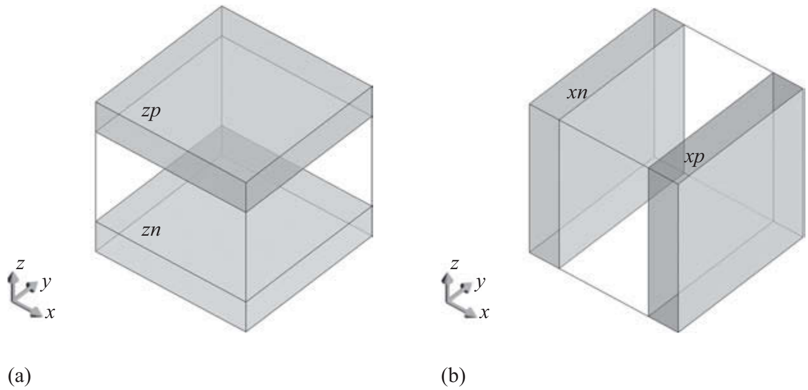


Figure B.2 CPML regions where E_y is updated: (a) E_y is updated in *zn* and *zp* using ψ_{eyz} and (b) E_y is updated in *xn* and *xp* using ψ_{eyx}.

B.2 Updating E_y at CPML regions

Initialization

Create new coefficient arrays for $C_{\psi eyz}$ and $C_{\psi eyx}$:

$$C_{\psi eyz} = \Delta z C_{eyhx} \quad C_{\psi eyx} = \Delta x C_{eyhz}.$$

Modify the coefficient arrays for C_{eyhz} and C_{eyhx} in the CPML regions:

$$C_{eyhx} = (1/k_{ez})C_{eyhx}, \quad \text{in the } zn \text{ and } zp \text{ regions.}$$
$$C_{eyhz} = (1/k_{ey})C_{eyhz}, \quad \text{in the } xn \text{ and } xp \text{ regions.}$$

FDTD time-marching loop

Update E_y in the full domain using the regular updating equation:

$$\begin{aligned} E_y^{n+1}(i,j,k) = {} & C_{eye}(i,j,k) \times E_y^n(i,j,k) \\ & + C_{eyhx}(i,j,k) \times \left(H_x^{n+\frac{1}{2}}(i,j,k) - H_x^{n+\frac{1}{2}}(i,j,k-1)\right) \\ & + C_{eyhz}(i,j,k) \times \left(H_z^{n+\frac{1}{2}}(i,j,k) - H_z^{n+\frac{1}{2}}(i-1,j,k)\right). \end{aligned}$$

Calculate $\psi_{eyz}^{n+\frac{1}{2}}$ for the *zn* and *zp* regions:

$$\psi_{eyz}^{n+\frac{1}{2}}(i,j,k) = b_{ez}\psi_{eyz}^{n-\frac{1}{2}}(i,j,k) + a_{ez}\left(H_x^{n+\frac{1}{2}}(i,j,k) - H_x^{n+\frac{1}{2}}(i,j,k-1)\right),$$

where

$$a_{ez} = \frac{\sigma_{pez}}{\Delta z(\sigma_{pez}k_{ez} + \alpha_{ez}k_{ez}^2)}[b_{ez} - 1],$$

$$b_{ez} = e^{-\left(\frac{\sigma_{pez}}{k_{ez}} + \alpha_{pez}\right)\frac{\Delta t}{\varepsilon_0}}.$$

Calculate $\psi_{eyx}^{n+\frac{1}{2}}$ for the xn and xp regions:

$$\psi_{eyx}^{n+\frac{1}{2}}(i,j,k) = b_{ex}\psi_{eyx}^{n-\frac{1}{2}}(i,j,k) + a_{ex}\left(H_z^{n+\frac{1}{2}}(i,j,k) - H_z^{n+\frac{1}{2}}(i-1,j,k)\right),$$

where

$$a_{ex} = \frac{\sigma_{pex}}{\Delta x(\sigma_{pex}k_{ex} + \alpha_{ex}k_{ex}^2)}[b_{ex} - 1],$$

$$b_{ex} = e^{-\left(\frac{\sigma_{pex}}{k_{ex}} + \alpha_{pex}\right)\frac{\Delta t}{\varepsilon_0}}.$$

Add the CPML auxiliary term to E_y in the zn and zp regions:

$$E_y^{n+1} = E_y^{n+1} + C_{\psi eyz} \times \psi_{eyz}^{n+\frac{1}{2}}.$$

Add the CPML auxiliary term to E_y in the xn and xp regions:

$$E_y^{n+1} = E_y^{n+1} + C_{\psi eyx} \times \psi_{eyx}^{n+\frac{1}{2}}.$$

B.3 Updating E_z at CPML regions

Initialization

Create new coefficient arrays for $C_{\psi ezx}$ and $C_{\psi ezy}$:

$$C_{\psi ezx} = \Delta x C_{ezhy} \quad C_{\psi ezy} = \Delta y C_{eyhx}.$$

Modify the coefficient arrays for C_{ezhx} and C_{ezhy} in the CPML regions:

$$C_{ezhy} = (1/k_{ex})C_{ezhy}, \quad \text{in the } xn \text{ and } xp \text{ regions.}$$

$$C_{ezhx} = (1/k_{ez})C_{ezhx}, \quad \text{in the } yn \text{ and } yp \text{ regions.}$$

FDTD time-marching loop

Update E_z in the full domain using the regular updating equation:

$$\begin{aligned} E_z^{n+1}(i,j,k) &= C_{eze}(i,j,k) \times E_z^n(i,j,k) + C_{ezhy}(i,j,k) \times \left(H_y^{n+\frac{1}{2}}(i,j,k) - H_y^{n+\frac{1}{2}}(i-1,j,k)\right) \\ &\quad + C_{ezhx}(i,j,k) \times \left(H_x^{n+\frac{1}{2}}(i,j,k) - H_x^{n+\frac{1}{2}}(i,j-1,k)\right) \end{aligned}$$

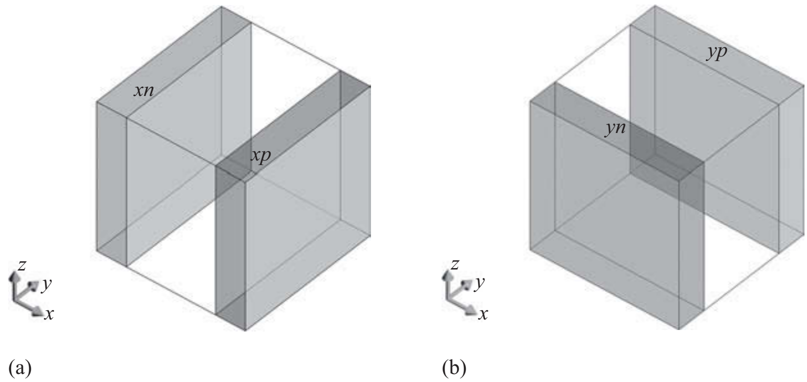


Figure B.3 CPML regions where E_z is updated: (a) E_z is updated in *xn* and *xp* using ψ_{ezx} and (b) E_z is updated in *yn* and *yp* using ψ_{ezy}.

Calculate $\psi_{ezx}^{n+\frac{1}{2}}$ for the *xn* and *xp* regions:

$$\psi_{ezx}^{n+\frac{1}{2}}(i,j,k) = b_{ex}\psi_{ezx}^{n-\frac{1}{2}}(i,j,k) + a_{ex}\left(H_y^{n+\frac{1}{2}}(i,j,k) - H_y^{n+\frac{1}{2}}(i-1,j,k)\right),$$

where

$$a_{ex} = \frac{\sigma_{pex}}{\Delta x(\sigma_{pex}k_{ex} + \alpha_{ex}k_{ex}^2)}[b_{ex} - 1],$$

$$b_{ex} = e^{-\left(\frac{\sigma_{pex}}{k_{ex}} + \alpha_{pex}\right)\frac{\Delta t}{\varepsilon_0}}.$$

Calculate $\psi_{ezy}^{n+\frac{1}{2}}$ for the *yn* and *yp* regions:

$$\psi_{ezy}^{n+\frac{1}{2}}(i,j,k) = b_{ey}\psi_{ezy}^{n-\frac{1}{2}}(i,j,k) + a_{ey}\left(H_x^{n+\frac{1}{2}}(i,j,k) - H_x^{n+\frac{1}{2}}(i,j-1,k)\right),$$

where

$$a_{ey} = \frac{\sigma_{pey}}{\Delta y(\sigma_{pey}k_{ey} + \alpha_{ey}k_{ey}^2)}\left[b_{ey} - 1\right],$$

$$b_{ey} = e^{-\left(\frac{\sigma_{pey}}{k_{ey}} + \alpha_{pey}\right)\frac{\Delta t}{\varepsilon_0}}.$$

Add the CPML auxiliary term to E_z in the *xn* and *xp* regions:

$$E_z^{n+1} = E_z^{n+1} + C_{\psi ezx} \times \psi_{ezx}^{n+\frac{1}{2}}.$$

Add the CPML auxiliary term to E_z in the *yn* and *yp* regions:

$$E_z^{n+1} = E_z^{n+1} + C_{\psi ezy} \times \psi_{ezy}^{n+\frac{1}{2}}.$$

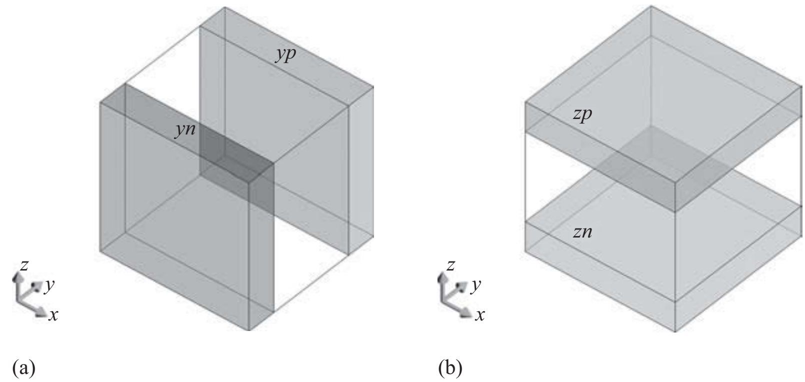


Figure B.4 CPML regions where H_x is updated: (a) H_x is updated in *yn* and *yp* using ψ_{hxy} and (b) H_x is updated in *zn* and *zp* using ψ_{hxz}.

B.4 Updating H_x at CPML regions

Initialization

Create new coefficient arrays for $C_{\psi hxy}$ and $C_{\psi hxz}$:

$$C_{\psi hxy} = \Delta y C_{hxez} \quad C_{\psi hxz} = \Delta z C_{hxey}.$$

Modify the coefficient arrays for C_{hxey} and C_{hxez} in the CPML regions:

$$C_{hxez} = (1/k_{my})C_{hxez,} \text{ in the } yn \text{ and } yp \text{ regiones.}$$

$$C_{hxey} = (1/k_{mz})C_{hxey,} \text{ in the } zn \text{ and } zp \text{ regiones.}$$

FDTD time-marching loop

Update H_x in the full domain using the regular updating equation:

$$\begin{aligned} H_x^{n+\frac{1}{2}}(i,j,k) = {} & C_{hxh}(i,j,k) \times H_x^{n-\frac{1}{2}}(i,j,k) \\ & + C_{hxez}(i,j,k) \times \left(E_z^n(i,j+1,k) - E_z^n(i,j,k)\right) \\ & + C_{hxey}(i,j,k) \times \left(E_y^n(i,j,k+1) - E_y^n(i,j,k)\right). \end{aligned}$$

Calculate ψ_{hxy}^n for the *yn* and *yp* regions:

$$\psi_{hxy}^n(i,j,k) = b_{my}\psi_{hxy}^{n-1}(i,j,k) + a_{my}\left(E_z^n(i,j+1,k) - E_z^n(i,j,\ \ k)\right),$$

where

$$a_{my} = \frac{\sigma_{pmy}}{\Delta y(\sigma_{pmy}k_{my} + \alpha_{my}k_{my}^2)}\left[b_{my} - 1\right],$$

$$b_{my} = e^{-\left(\frac{\sigma_{pmy}}{k_{my}} + \alpha_{pmy}\right)\frac{\Delta t}{\mu_0}}.$$

Calculate ψ^n_{hxz} for the zn and zp regions:

$$\psi^n_{hxz}(i,j,k) = b_{mz}\psi^{n-1}_{hxz}(i,j,k) + a_{mz}\big(E^n_y(i,j,k+1) - E^n_y(i,j,k)\big),$$

where

$$a_{mz} = \frac{\sigma_{pmz}}{\Delta z(\sigma_{pmz}k_{mz} + \alpha_{mz}k^2_{mz})}[b_{mz} - 1],$$

$$b_{mz} = e^{-\left(\frac{\sigma_{pmz}}{k_{mz}}+\alpha_{pmz}\right)\frac{\Delta t}{\mu_0}}.$$

Add the CPML auxiliary term to H_x in the yn and yp regions:

$$H_x^{n+\frac{1}{2}} = H_x^{n+\frac{1}{2}} + C_{\psi hxy} \times \psi^n_{hxy}.$$

Add the CPML auxiliary term to H_x in the zn and zp regions:

$$H_x^{n+\frac{1}{2}} = H_x^{n+\frac{1}{2}} + C_{\psi hxz} \times \psi^n_{hxz}.$$

B.5 Updating H_y at CPML regions

Initialization

Create new coefficient arrays for $C_{\psi hyz}$ and $C_{\psi hyx}$:

$$C_{\psi hyz} = \Delta z C_{hyex} \quad C_{\psi hyx} = \Delta x C_{hyez}.$$

Modify the coefficient arrays for C_{hyez} and C_{hyex} in the CPML regions:

$$C_{hyex} = (1/k_{mz})C_{hyex}, \quad \text{in the } zn \text{ and } zp \text{ regions.}$$
$$C_{hyez} = (1/k_{mx})C_{hyez}, \quad \text{in the } xn \text{ and } xp \text{ regions.}$$

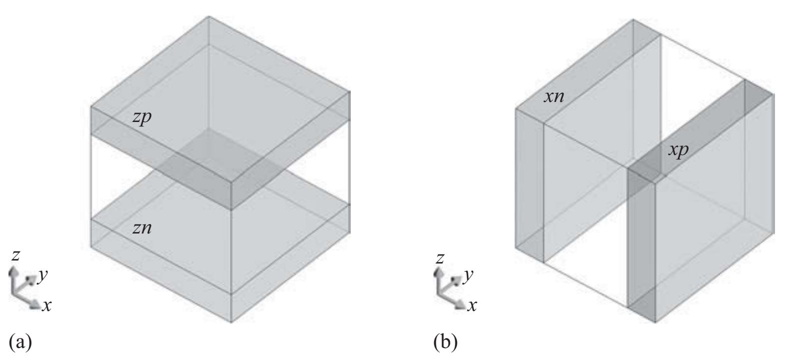


Figure B.5 CPML regions where H_y is updated: (a) H_y is updated in zn and zp using ψ_{hyz} and (b) H_y is updated in xn and xp using ψ_{hyx}.

FDTD time-marching loop

Update H_y in the full domain using the regular updating equation

$$H_y^{n+\frac{1}{2}}(i,j,k) = C_{hyh}(i,j,k) \times H_y^{n-\frac{1}{2}}(i,j,k) + C_{hyex}(i,j,k) \times \left(E_x^n(i,j,k+1) - E_x^n(i,j,k)\right) + C_{hyez}(i,j,k) \times \left(E_z^n(i+1,j,k) - E_z^n(i,j,k)\right).$$

Calculate ψ_{hyz}^n for the *zn* and *zp* regions:

$$\psi_{hyz}^n(i,j,k) = b_{mz}\psi_{hyz}^{n-1}(i,j,k) + a_{mz}\left(E_x^n(i,j,k+1) - E_x^n(i,j,k)\right),$$

where

$$a_{mz} = \frac{\sigma_{pmz}}{\Delta z(\sigma_{pmz}k_{mz} + \alpha_{mz}k_{mz}^2)}[b_{mz} - 1],$$
$$b_{mz} = e^{-\left(\frac{\sigma_{pmz}}{k_{mz}} + \alpha_{pmz}\right)\frac{\Delta t}{\mu_0}}.$$

Calculate ψ_{hyx}^n for the *xn* and *xp* regions:

$$\psi_{hyx}^n(i,j,k) = b_{mx}\psi_{hyx}^{n-1}(i,j,k) + a_{mx}\left(E_z^n(i+1,j,k) - E_z^n(i,j,k)\right),$$

where

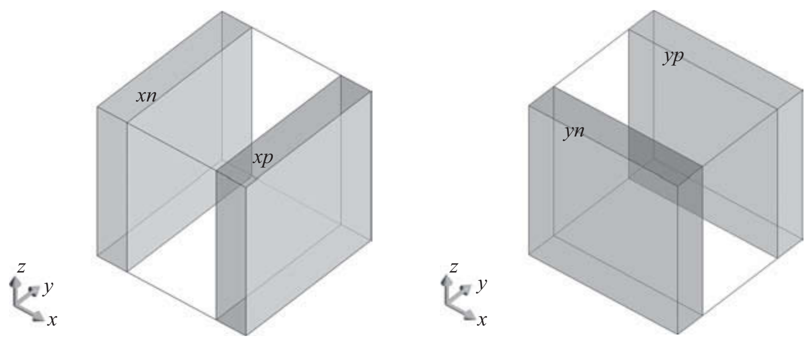
$$a_{mx} = \frac{\sigma_{pmx}}{\Delta x(\sigma_{pmx}k_{mx} + \alpha_{mx}k_{mx}^2)}[b_{mx} - 1],$$
$$b_{mx} = e^{-\left(\frac{\sigma_{pmx}}{k_{mx}} + \alpha_{pmx}\right)\frac{\Delta t}{\mu_0}}.$$

Add the CPML auxiliary term to H_y in the *zn* and *zp* regions:

$$H_y^{n+\frac{1}{2}} = H_y^{n+\frac{1}{2}} + C_{\psi hyz} \times \psi_{hyz}^n.$$

Add the CPML auxiliary term to H_y in the *xn* and *xp* regions:

$$H_y^{n+\frac{1}{2}} = H_y^{n+\frac{1}{2}} + C_{\psi hyx} \times \psi_{hyx}^n.$$



(a) (b)

Figure B.6 CPML regions where H_z is updated: (a) H_z is updated in *xn* and *xp* using ψ_{hzx} and (b) H_z is updated in *yn* and *yp* using ψ_{hzy}.

B.6 Updating H_z at CPML regions

Initialization

Create new coefficient arrays for $C_{\psi hzx}$ and $C_{\psi hzy}$:

$$C_{\psi hzy} = \Delta x C_{hzey} \quad C_{\psi hzy} = \Delta y C_{hzex}.$$

Modify the coefficient arrays for C_{hzex} and C_{hzey} in the CPML regions:

$$C_{hzey} = (1/k_{mx})C_{hzey}, \quad \text{in the } xn \text{ and } xp \text{ regions.}$$

$$C_{hzex} = (1/k_{my})C_{hzex}, \quad \text{in the } yn \text{ and } yp \text{ regions.}$$

FDTD time-marching loop

Update H_z in the full domain using the regular updating equation:

$$\begin{aligned} H_z^{n+\frac{1}{2}}(i,j,k) = C_{hzh}(i,j,k) \times H_z^{n-\frac{1}{2}}(i,j,k) \\ + C_{hzey}(i,j,k) \times \left(E_y^n(i+1,j,k) - E_y^n(i,j,k)\right) \\ + C_{hzex}(i,j,k) \times \left(E_x^n(i,j+1,k) - E_x^n(i,j,k)\right). \end{aligned}$$

Calculate ψ_{hzx}^n for the xn and xp regions:

$$\psi_{hzx}^n(i,j,k) = b_{mx}\psi_{hzx}^{n-1}(i,j,k) + a_{mx}\left(E_y^n(i+1,j,k) - E_y^n(i,j,k)\right),$$

where

$$a_{mx} = \frac{\sigma_{pmx}}{\Delta x(\sigma_{pmx}k_{mx} + \alpha_{mx}k_{mx}^2)}[b_{mx} - 1],$$

$$b_{mx} = e^{-\left(\frac{\sigma_{pmx}}{k_{mx}} + \alpha_{pmx}\right)\frac{\Delta t}{\mu_0}}.$$

Calculate ψ_{hzy}^n for the yn and yp regions:

$$\psi_{hzy}^n(i,j,k) = b_{my}\psi_{hzy}^{n-1}(i,j,k) + a_{my}\left(E_y^n(i,j+1,k) - E_y^n(i,j,k)\right),$$

where

$$a_{my} = \frac{\sigma_{pmy}}{\Delta y(\sigma_{pmy}k_{my} + \alpha_{my}k_{my}^2)}\left[b_{my} - 1\right],$$

$$b_{my} = e^{-\left(\frac{\sigma_{pmy}}{k_{my}} + \alpha_{pmy}\right)\frac{\Delta t}{\mu_0}}.$$

Add the CPML auxiliary term to H_z in the xn and xp regions:

$$H_z^{n+\frac{1}{2}} = H_z^{n+\frac{1}{2}} + C_{\psi hzx} \times \psi_{hzx}^n.$$

Add the CPML auxiliary term to H_z in the yn and yp regions:

$$H_z^{n+\frac{1}{2}} = H_z^{n+\frac{1}{2}} + C_{\psi hzy} \times \psi_{hzy}^n.$$