

# Module `solver`

## Classes

```
class EquationOfState
```

Class to represent an equation of state.

### Attributes

**number\_densities** : NumPy array  
NumPy array that stores number densities in  $\text{cm}^{-3}$ .

**pressures** : NumPy array  
NumPy array that stores pressures in  $\text{dyne/cm}^2$ .

**energy\_densities** : NumPy array  
NumPy array that stores energy densities in  $\text{erg/cm}^3$ .

### Methods

`load_from_file(file_path)` Read data from a file and store number densities, pressures and energy densities in corresponding NumPy arrays. `plot()` Plot energy densities versus pressure. `interpolate()` Interpolate using cubic spline and return the equation of state in the form of energy density as a function of pressure. `get()` Return NumPy arrays containing pressure and energy density.

Initialise NumPy arrays to store number densities, pressures and energy densities.

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### Methods

```
def get(self)
```

Return NumPy arrays containing pressures and energy densities.

### Returns

tuple  
NumPy arrays of pressures and energy densities.

```
def interpolate(self)
```

Interpolate pressures and energy densities into an equation of state using cubic spline (imported from SciPy).

### Returns

CubicSpline  
CubicSpline representation of the equation of state, which gives energy density as output once

pressure is given as input.

## Raises

ValueError

No data is loaded to interpolate.

```
def load_from_file(self, file_path)
```

Open file given its path, read data and store quantities in corresponding NumPy arrays. The first row is skipped because it is the header. The delimiter must be comma for csv files. In the first column there must be number densities in  $1/\text{cm}^3$ , in the second column there must be pressures in  $\text{dyne}/\text{cm}^2$ , in the third column there must be energy densities in  $\text{erg}/\text{cm}^3$ .

## Parameters

**file\_path** : str

Path of the file containing number densities, pressures and energy densities in respective columns.

## Raises

InputError

The file cannot be opened.

ValueError

The file is empty. Pressures are not strictly increasing.

```
def plot(self)
```

Plot energy density versus pressure using logarithmic scale for both x and y axis with a grid.

## Raises

ValueError

No data is loaded to plot.

```
class SolverRangePressure (eos, relativity_corrections=True)
```

Class to solve numerically the system of ordinary differential equations given a range of initial pressures. It uses the class SolverSinglePressure for each initial pressure.

## Attributes

**eos** : CubicSpline

A CubicSpline representation of the equation of state.

**relativity\_corrections** : bool

A Boolean variable to determined whether to include general relativity corrections. If True the TOV equation is considered, if False the Newtonian equation is considered.

**radii** : NumPy array

NumPy array that stores radii in km.

**masses** : NumPy array

NumPy array that stores masses in solar masses.

**initial\_pressures** : NumPy array

NumPy array that stores initial pressures in dyne/cm<sup>2</sup>.

## Methods

**solve(step\_r, initial\_pressures)** Integrate the system until pressure drops to zero given the initial pressure and radius step. Store radii, masses and pressures in corresponding NumPy arrays. **print\_max\_mass()** Print the maximum mass, its radius and central pressure. **get()** Return NumPy arrays containing radius, mass and initial pressure. **plot\_MRvsP()** Plot mass and radius versus initial pressure. **plot\_RvsM()** Plot radius versus mass.

Take an equation of state and an option for relativity corrections and copy them to attributes of the class. Initialise NumPy arrays to store radii in km, pressures in dyne/cm<sup>2</sup> and masses in solar masses.

## Parameters

**eos** : CubicSpline

A CubicSpline representation of the equation of state.

**relativity\_corrections** : bool

A Boolean variable to determined whether to include general relativity corrections. If True the TOV equation is considered, if False the Netwonian equation is considered.

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## Methods

```
def get(self)
```

Return the NumPy arrays of radius, mass and initial pressure.

### Returns

tuple

NumPy array of radius in km, mass in solar masses and initial pressure in dyne/cm<sup>2</sup>.

### Raises

ValueError

No data is computed to get

```
def plot_MRvsP(self)
```

Plot mass and radius versus initial pressure. Show two subplots: radius versus initial pressure and mass versus initial pressure in logarithmic scal for the x axis with a grid.

## Raises

ValueError

No data is computed to plot

```
def plot_RvsM(self)
```

Plot radius versus mass with a grid.

## Raises

ValueError

No data is computed to plot

```
def print_max_mass(self)
```

Print the maximum mass, its corresponding radius and central pressure.

## Raises

ValueError

No data is computed to print.

```
def solve(self, step_r, initial_pressures)
```

Solve the system for a range of initial pressures. Using the class SolverSinglePressure, compute the mass and the radius and store them for each initial pressure.

## Parameters

**step\_r** : float

Radius step in cm.

**initial\_pressures** : NumPy array

NumPy array of initial pressures in dyne/cm<sup>2</sup>.

## Raises

ValueError

If radius step is negative. If initial pressure is negative.

```
class SolverSinglePressure (eos, relativity_corrections=True)
```

Class to solve numerically the system of ordinary differential equations given a single initial pressure. Implement the 4th-order Runge-Kutta with a breaking condition when the pressure becomes zero or negative, because it means that the surface of the object is reached.

## Attributes

**radii** : NumPy array

NumPy array that stores radii in km.

**masses** : NumPy array

NumPy array that stores masses in solar masses.

**pressures** : NumPy array

NumPy array that stores pressures in dyne/cm<sup>2</sup>.

## Methods

`runge_kutta_4th_step(r, dr, p, m)` Implement the Runge-Kutta 4th order algorithm for a single step to solve numerically a system of two differential equations, according to current pressure, mass, radius and radius step. `solve(step_r, p0)` Solve the system until pressure drops to zero given the initial pressure and radius step. Store radii, masses and pressures in corresponding NumPy arrays. `print_mass_radius()` Print the mass and the radius of the object. `get()` Return radius, mass and pressure NumPy arrays. `plot()` Plot mass and radius versus pressure.

Initialize the class `TOVSystem` with an equation of state and an option for relativity corrections.

## Parameters

**eos** : CubicSpline

A CubicSpline representation of the equation of state.

**relativity\_corrections** : bool

A Boolean variable to determined whether to include general relativity corrections. If True the TOV equation is considered, if False the Netwonian equation is considered.

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## Methods

```
def get(self)
```

Return radius, mass and pressure NumPy arrays.

## Returns

tuple

NumPy array of radius in km, mass in solar masses and pressures in dyne/cm<sup>2</sup>.

## Raises

ValueError

No data is computed to get.

```
def plot(self)
```

Plot mass and radius versus pressure. Show two subplots: radius versus pressure and mass versus

pressure with a grid.

## Raises

ValueError

No data is computed to plot.

```
def print_mass_radius(self)
```

Print the radius in km and mass in solar masses of the object, rounded to fourth significant digits.

## Raises

ValueError

No data is computed to print.

```
def runge_kutta_4th_step(self, r, dr, p,  
m)
```

Perform a single step of the 4th-order Runge-Kutta algorithm. Given the current radius, pressure, mass and radius step, compute the 4 coefficients of the algorithm, using the equations in the methods of TOVSystem, and then compute and return the new calculated value of pressure and mass.

## Parameters

**r** : float

Current radius in cm.

**dr** : float

Radius step in cm.

**p** : float

Current pressure in dyne/cm<sup>2</sup>.

**m** : float

Current mass in g.

## Returns

tuple

NumPy arrays of pressures and masses.

```
def solve(self, step_r, p0)
```

Apply the 4th-order Runge-Kutta step using a while loop for the given initial pressure until pressure drops to zero. For every iteration, update the radius, pressure and mass values in the corresponding NumPy arrays. Change unit of measure of mass from g to solar masses and radius from cm to km.

## Parameters

**step\_r** : float

Radius step in cm.

**p0** : float

Initial central pressure in dyne/cm<sup>2</sup>.

## Raises

ValueError

If radius step is negative. If initial pressure is negative.

```
class TOVSystem (eos, relativity_corrections=True)
```

Class to represent the system of ordinary differential equations for a spherically symmetric physical object which is in static gravitational equilibrium. The system is composed in 3 equations: the mass equation, the pressure equation and the equation of state. The pressure equation can be either Newtonian or relativistic (the latter called properly TOV equation).

## Attributes

**eos** : CubicSpline

A CubicSpline representation of the equation of state.

**relativity\_corrections** : bool

A Boolean variable to determined whether to include general relativity corrections. If True the TOV equation is used, if False the Newtonian equation is used.

## Methods

**check\_physical\_constraints**(r, p, m) Control if physical quantities, like mass, pressure and radius, are positive.

**dmdr**(r, p, m) Compute and return the mass equation accordingly to current radius, pressure and mass.

**dpdr**(r, p, m) Compute and return the pressure equation accordingly to current radius, pressure, mass. It can consider relativity corrections if attribute relativity\_corrections is True.

Take an equation of state and an option for relativity corrections and copy them as attributes of the class.

Parameters: **eos** : EquationOfState A CubicSpline representation of the equation of state. **relativity\_corrections** : bool A Boolean variable to determined whether to include general relativity corrections. If True the TOV equation is used, if False the Newtonian equation is used.

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## Methods

```
def check_physical_constraints(self, r, p, m)
```

Check the physical constraints for radius, pressure and mass, which must be all positive. When pressure is negative, return False to signal that the surface of the object is reached and integration has to stop.

## Parameters

**r** : float

Radius in cm.

**p** : float

Pressure in dyne/cm<sup>2</sup>.

**m** : float

Mass in g.

## Raises

ValueError

If radius is negative. If mass is negative.

## Returns

bool

True if all physical constraints are satisfied, False if pressure is negative.

```
def dmdr(self, r, p, m)
```

Compute the mass equation (dm/dr) based on the current radius, pressure and mass.

## Parameters

**r** : float

Radius in cm.

**p** : float

Pressure in dyne/cm<sup>2</sup>.

**m** : float

Mass in g.

## Returns

float

Mass equation of the system (dm/dr).

```
def dpdr(self, r, p, m)
```

Compute the pressure equation (dp/dr) based on the current radius, pressure and mass. If relativity corrections are included, return the TOV equation, otherwise return the Newtonian pressure equation.



## Parameters

**r** : float  
Radius in cm.

**p** : float

Pressure in dyne/cm<sup>2</sup>.

**m** : float  
Mass in g.

## Returns

float  
Pressure equation of the system (dp/dr).

## Classes

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