

```

In[*]:= (* F[1, {0}] = electron neutrino,
F[1, {1}] = muon neutrino, F[1, {2}] = tau neutrino,
F[2, {0}] = electron, F[2, {1}] = muon, F[2, {2}] = tau,
F[3, {0}] = up, F[3, {1}] = charm, F[3, {2}] = top,
F[4, {0}] = down, F[4, {1}] = strange, F[4, {2}] = bottom,
V[1] = photon, V[2] = Z, V[3] = W-, S[1] = H *)
$LoadAddOns = {"FeynArts"};
<< FeynCalc`
$FAVerbose = 0;

```

```

MakeBoxes[p1, TraditionalForm] := "\!\(\*SubscriptBox[\(p\), \((1\)]\)\)";
MakeBoxes[p2, TraditionalForm] := "\!\(\*SubscriptBox[\(p\), \((2\)]\)\)";
topology = CreateTopologies[0, 1 → 2];

```

FeynCalc 10.0.0 (dev version). For help, use the

online documentation, visit the forum and have a look at the supplied examples. The PDF-version of the manual can be downloaded here.

If you use FeynCalc in your research, please

evaluate FeynCalcHowToCite[] to learn how to cite this software.

Please keep in mind that the proper academic attribution

of our work is crucial to ensure the future development of this package!

FeynArts 3.12 (24 May 2024) patched for use with FeynCalc, for documentation see the manual or visit www.feynarts.de.

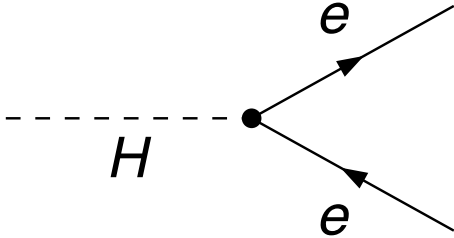
If you use FeynArts in your research, please cite

- T. Hahn, Comput. Phys. Commun., 140, 418–431, 2001, arXiv:hep-ph/0012260

```

In[ ]:= (* SM: H → lept lept *)
feynman = InsertFields[topology,
  {S[1]} → {F[2, {1}], -F[2, {1}]}, InsertionLevel → {Classes}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True, ChangeDimension → 4,
  DropSumOver → True, List → False, SMP → True, Contract → True]
FCClearScalarProducts[];
SP[p, p] = SMP["m_H"] ^ 2;
SP[p1, p1] = SMP["m_e"] ^ 2;
SP[p2, p2] = SMP["m_e"] ^ 2;
SP[p1, p2] = (SMP["m_H"] ^ 2 - 2 SMP["m_e"] ^ 2) / 2;
squareamplitude[0] = (amplitude[0] (ComplexConjugate[amplitude[0]])) //
  FeynAmpDenominatorExplicit //
  FermionSpinSum[#] & // DiracSimplify // Simplify
v
(* output = feynman diagrams, amplitude,
squared amplitude, massless squared amplitude *)

```



$$Out[] = i(\varphi(\overline{p_1}, m_e)) \left(-\frac{i e m_e}{2 m_W (\sin(\theta_W))} \right) (\varphi(-\overline{p_2}, m_e))$$

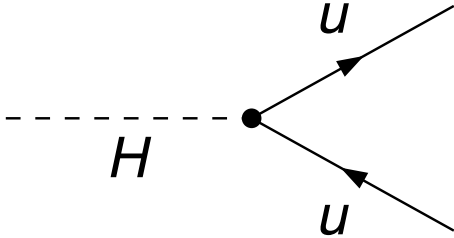
$$Out[] = -\frac{e^2 (4 m_e^4 - m_e^2 m_H^2)}{2 m_W^2 (\sin(\theta_W))^2}$$

$$Out[] = v$$

```

In[ ]:= (* SM: H → quark quark *)
feynman = InsertFields[topology,
  {S[1]} → {F[3, {1}], -F[3, {1}]}, InsertionLevel → {Classes}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True, ChangeDimension → 4,
  DropSumOver → True, List → False, SMP → True, Contract → True]
FCClearScalarProducts[];
SP[p, p] = SMP["m_H"] ^ 2;
SP[p1, p1] = SMP["m_u"] ^ 2;
SP[p2, p2] = SMP["m_u"] ^ 2;
SP[p1, p2] = (SMP["m_H"] ^ 2 - 2 SMP["m_u"] ^ 2) / 2;
squareamplitude2[0] = (amplitude[0] (ComplexConjugate[amplitude[0]])) //
  FeynAmpDenominatorExplicit // FermionSpinSum[#] & //
  DiracSimplify // Simplify // SUNSimplify
masslesssquareamplitude[0] =
  squareamplitude[0] // ReplaceAll[#, {SMP["m_u"] → 0}] & // Simplify
(* output = feynman diagrams, amplitude,
  squared amplitude, massless squared amplitude *)

```



Out[]:=

$$i(\varphi(\overline{p_1}, m_u)) \left(-\frac{i e m_u \delta_{\text{Col2 Col3}}}{2 m_W (\sin(\theta_W))} \right) (\varphi(-\overline{p_2}, m_u))$$

Out[]:=

$$\frac{e^2 C_A m_u^2 (m_H^2 - 4 m_u^2)}{2 m_W^2 (\sin(\theta_W))^2}$$

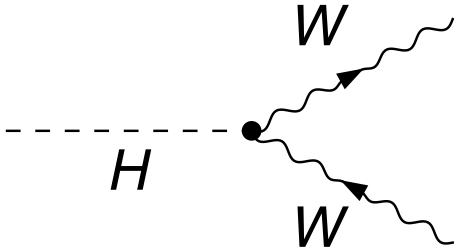
Out[]:=

$$-\frac{e^2 (4 m_e^4 - m_e^2 m_H^2)}{2 m_W^2 (\sin(\theta_W))^2}$$

```

In[ ]:= (* SM: H → W- W+ *)
feynman =
  InsertFields[topology, {S[1]} → {V[3], -V[3]}, InsertionLevel → {Classes}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True, ChangeDimension → 4,
  DropSumOver → True, List → False, SMP → True, Contract → True]
FCClearScalarProducts[];
SP[p, p] = SMP["m_H"] ^ 2;
SP[p1, p1] = SMP["m_W"] ^ 2;
SP[p2, p2] = SMP["m_W"] ^ 2;
SP[p1, p2] = (SMP["m_H"] ^ 2 - 2 SMP["m_W"] ^ 2) / 2;
squareamplitude[0] = (amplitude[0] (ComplexConjugate[amplitude[0]])) //
  FeynAmpDenominatorExplicit // DoPolarizationSums[#, p1] & //
  DoPolarizationSums[#, p2] & // Simplify
(* output = feynman diagrams, amplitude, squared amplitude *)

```



```

Out[ ]:=

$$\frac{e m_W (\vec{\epsilon}^*(p_1) \cdot \vec{\epsilon}^*(p_2))}{\sin(\theta_W)}$$


```

DoPolarizationSums : Error ! DoPolarizationSums has encountered a fatal problem and must abort the computation. The problem reads: Polarization vectors do not seem to appear in a proper way in the expression.

```

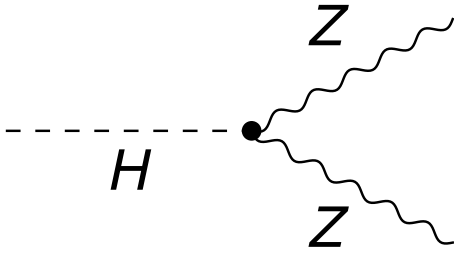
Out[ ]:=
$Aborted

```

```

In[ ]:= (* SM: H → Z Z *)
feynman =
  InsertFields[topology, {S[1]} → {V[2], V[2]}, InsertionLevel → {Classes}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True, ChangeDimension → 4,
  DropSumOver → True, List → False, SMP → True, Contract → True]
FCClearScalarProducts[];
SP[p, p] = SMP["m_H"] ^ 2;
SP[p1, p1] = SMP["m_Z"] ^ 2;
SP[p2, p2] = SMP["m_Z"] ^ 2;
SP[p1, p2] = (SMP["m_H"] ^ 2 - 2 SMP["m_Z"] ^ 2) / 2;
squareamplitude[0] = (amplitude[0] (ComplexConjugate[amplitude[0]])) //
  FeynAmpDenominatorExplicit // DoPolarizationSums[#, p1] & //
  DoPolarizationSums[#, p2] & // Simplify
(* output = feynman diagrams, amplitude, squared amplitude *)

```



Out[]=

$$\frac{e m_W (\vec{\epsilon}^*(p_1) \cdot \vec{\epsilon}(p_2))}{(\cos(\theta_W))^2 (\sin(\theta_W))}$$

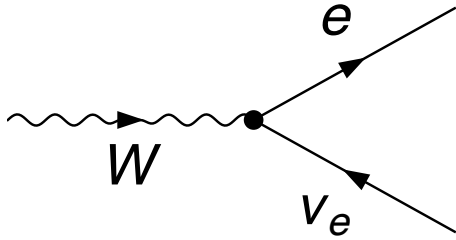
Out[]=

$$\frac{e^2 m_W^2 (-4 m_H^2 m_Z^2 + m_H^4 + 12 m_Z^4)}{4 m_Z^4 (\cos(\theta_W))^4 (\sin(\theta_W))^2}$$

```

In[ ]:= (* SM: W- → e- νe *)
feynman = InsertFields[topology,
  {V[3]} → {F[2, {1}], -F[1, {1}]}, InsertionLevel → {Classes}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True, ChangeDimension → 4,
  DropSumOver → True, List → False, SMP → True, Contract → True]
FCClearScalarProducts[];
SP[p, p] = SMP["m_W"] ^ 2;
SP[p1, p1] = SMP["m_e"] ^ 2;
SP[p2, p2] = 0;
SP[p1, p2] = (SMP["m_W"] ^ 2 - SMP["m_e"] ^ 2) / 2;
SP[p, p1] = SP[p1, p1] + SP[p2, p1];
SP[p, p2] = SP[p1, p2] + SP[p2, p2];
squareamplitude[0] = (amplitude[0] (ComplexConjugate[amplitude[0]])) //
  FeynAmpDenominatorExplicit // FermionSpinSum[#] & // DiracSimplify //
  DoPolarizationSums[#, p, ExtraFactor → 1/3] & // Simplify
masslesssquareamplitude[0] =
  squareamplitude[0] // ReplaceAll[#, {SMP["m_e"] → 0}] & // Simplify
(* output = feynman diagrams, amplitude,
squared amplitude, massless squared amplitude *)

```



$$Out[] = \frac{e(\varphi(\vec{p}_1, m_e)) \cdot (\vec{\gamma} \cdot \vec{E}(p)) \cdot \vec{\gamma}^7 \cdot (\varphi(-\vec{p}_2))}{\sqrt{2} (\sin(\theta_W))}$$

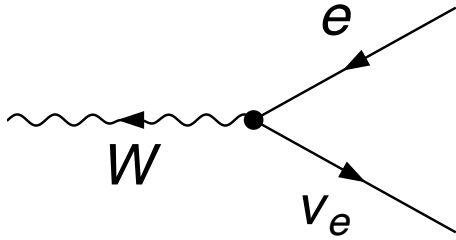
$$Out[] = \frac{e^2 (m_e^2 m_W^2 + m_e^4 - 2 m_W^4)}{6 m_W^2 (\sin(\theta_W))^2}$$

$$Out[] = \frac{e^2 m_W^2}{3 (\sin(\theta_W))^2}$$

```

In[ ]:= (* SM: W+ → e+ nu *)
feynman = InsertFields[topology,
  {-V[3]} → {-F[2, {1}], F[1, {1}]}, InsertionLevel → {Classes}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True, ChangeDimension → 4,
  DropSumOver → True, List → False, SMP → True, Contract → True]
FCClearScalarProducts[];
SP[p, p] = SMP["m_W"] ^ 2;
SP[p1, p1] = SMP["m_e"] ^ 2;
SP[p2, p2] = 0;
SP[p1, p2] = (SMP["m_W"] ^ 2 - SMP["m_e"] ^ 2) / 2;
SP[p, p1] = SP[p1, p1] + SP[p2, p1];
SP[p, p2] = SP[p1, p2] + SP[p2, p2];
squareamplitude[0] = (amplitude[0] (ComplexConjugate[amplitude[0]])) //
  FeynAmpDenominatorExplicit // FermionSpinSum[#] & // DiracSimplify //
  DoPolarizationSums[#, p, ExtraFactor → 1 / 3] & // Simplify
masslesssquareamplitude[0] =
  squareamplitude[0] // ReplaceAll[#, {SMP["m_e"] → 0}] & // Simplify
(* output = feynman diagrams, amplitude,
squared amplitude, massless squared amplitude *)

```



$$Out[] = \frac{e(\varphi(\vec{p}_2)) \cdot (\vec{\gamma} \cdot \vec{E}(p)) \cdot \vec{\gamma}^7 \cdot e(\varphi(-\vec{p}_1, m_e))}{\sqrt{2} (\sin(\theta_W))}$$

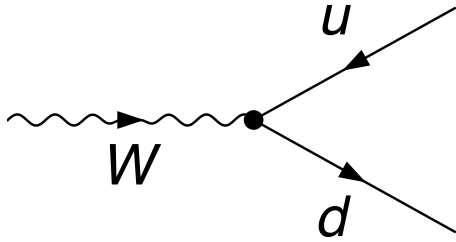
$$Out[] = -\frac{e^2 (m_e^2 m_W^2 + m_e^4 - 2 m_W^4)}{6 m_W^2 (\sin(\theta_W))^2}$$

$$Out[] = \frac{e^2 m_W^2}{3 (\sin(\theta_W))^2}$$

```

In[ ]:= (* SM: W- → d antiu *)
feynman = InsertFields[topology,
  {V[3]} → {-F[3, {1}], F[4, {1}]}, InsertionLevel → {Classes}};
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True, ChangeDimension → 4,
  DropSumOver → True, List → False, SMP → True, Contract → True]
FCClearScalarProducts[];
SP[p, p] = SMP["m_W"] ^ 2;
SP[p1, p1] = SMP["m_u"] ^ 2;
SP[p2, p2] = SMP["m_d"] ^ 2;
SP[p1, p2] = (SMP["m_W"] ^ 2 - SMP["m_u"] ^ 2 - SMP["m_d"] ^ 2) / 2;
SP[p, p1] = SP[p1, p1] + SP[p2, p1];
SP[p, p2] = SP[p1, p2] + SP[p2, p2];
squareamplitude[0] =
  (amplitude[0] (ComplexConjugate[amplitude[0]])) // SUNSimplify //
  FermionSpinSum // DiracSimplify //
  DoPolarizationSums[#, p, ExtraFactor → 1 / 3] & // Simplify
masslessquareamplitude[0] = squareamplitude[0] //
  ReplaceAll[#, {SMP["m_u"] → 0, SMP["m_d"] → 0}] & // Simplify
(* output = feynman diagrams, amplitude,
  squared amplitude, massless squared amplitude *)

```



$$Out[] = \frac{e \delta_{Col2 Col3} (\varphi(\vec{p}_2, m_d)) \cdot (\vec{\gamma} \cdot \vec{E}(p)) \cdot \vec{\gamma}^7 \cdot (\varphi(-\vec{p}_1, m_u))}{\sqrt{2} (\sin(\theta_W))}$$

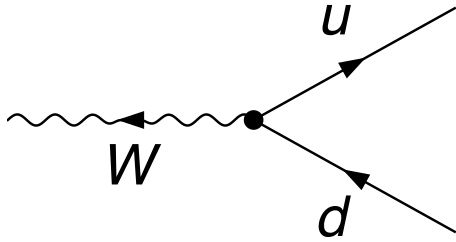
$$Out[] = - \frac{e^2 C_A (m_d^2 (m_W^2 - 2 m_u^2) + m_d^4 + m_u^2 m_W^2 + m_u^4 - 2 m_W^4)}{6 m_W^2 (\sin(\theta_W))^2}$$

$$Out[] = \frac{e^2 C_A m_W^2}{3 (\sin(\theta_W))^2}$$


```

In[ ]:= (* SM: W+ → u antid *)
feynman = InsertFields[topology,
  {-V[3]} → {F[3, {1}], -F[4, {1}]}, InsertionLevel → {Classes}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True, ChangeDimension → 4,
  DropSumOver → True, List → False, SMP → True, Contract → True]
FCClearScalarProducts[];
SP[p, p] = SMP["m_W"] ^ 2;
SP[p1, p1] = SMP["m_u"] ^ 2;
SP[p2, p2] = SMP["m_d"] ^ 2;
SP[p1, p2] = (SMP["m_W"] ^ 2 - SMP["m_u"] ^ 2 - SMP["m_d"] ^ 2) / 2;
SP[p, p1] = SP[p1, p1] + SP[p2, p1];
SP[p, p2] = SP[p1, p2] + SP[p2, p2];
squareamplitude[0] =
  (amplitude[0] (ComplexConjugate[amplitude[0]])) // SUNSimplify //
  FermionSpinSum // DiracSimplify //
  DoPolarizationSums[#, p, ExtraFactor → 1 / 3] & // Simplify
masslesssquareamplitude[0] = squareamplitude[0] //
  ReplaceAll[#, {SMP["m_u"] → 0, SMP["m_d"] → 0}] & // Simplify
(* output = feynman diagrams, amplitude,
  squared amplitude, massless squared amplitude *)

```



$$Out[] = \frac{e \delta_{Col2 Col3} (\varphi(\vec{p}_1, m_u)) \cdot (\vec{\gamma} \cdot \vec{\epsilon}(p)) \cdot \vec{\gamma}^7 \cdot (\varphi(-\vec{p}_2, m_d))}{\sqrt{2} (\sin(\theta_W))}$$

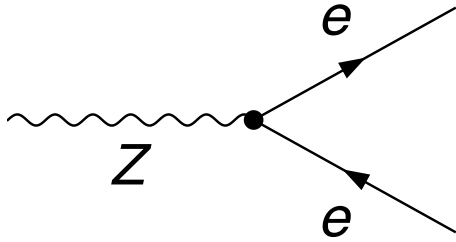
$$Out[] = \frac{e^2 C_A (m_d^2 (m_W^2 - 2 m_u^2) + m_d^4 + m_u^2 m_W^2 + m_u^4 - 2 m_W^4)}{6 m_W^2 (\sin(\theta_W))^2}$$

$$Out[] = \frac{e^2 C_A m_W^2}{3 (\sin(\theta_W))^2}$$

```

In[ ]:= (* SM: Z → e- e+ *)
feynman = InsertFields[topology,
  {V[2]} → {F[2, {1}], -F[2, {1}]}, InsertionLevel → {Classes}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True, ChangeDimension → 4,
  DropSumOver → True, List → False, SMP → True, Contract → True]
FCClearScalarProducts[];
SP[p, p] = SMP["m_Z"] ^ 2;
SP[p1, p1] = SMP["m_e"] ^ 2;
SP[p2, p2] = SMP["m_e"] ^ 2;
SP[p1, p2] = (SMP["m_Z"] ^ 2 - 2 SMP["m_e"] ^ 2) / 2;
SP[p, p1] = SP[p1, p1] + SP[p2, p1];
SP[p, p2] = SP[p1, p2] + SP[p2, p2];
squareamplitude[0] = (amplitude[0] (ComplexConjugate[amplitude[0]])) //
  FeynAmpDenominatorExplicit // FermionSpinSum[#] & // DiracSimplify //
  DoPolarizationSums[#, p, ExtraFactor → 1 / 3] & // Simplify
masslesssquareamplitude[0] =
  squareamplitude[0] // ReplaceAll[#, {SMP["m_e"] → 0}] & // Simplify
(* output = feynman diagrams, amplitude,
  squared amplitude, massless squared amplitude *)

```



Out[]:=

$$i(\varphi(\overline{p_1}, m_e)) \cdot \left(\frac{i e (\sin(\theta_W)) (\overline{\gamma} \cdot \vec{\varepsilon}(p)) \cdot \overline{\gamma}^6}{\cos(\theta_W)} + \frac{i e \left((\sin(\theta_W))^2 - \frac{1}{2} \right) (\overline{\gamma} \cdot \vec{\varepsilon}(p)) \cdot \overline{\gamma}^7}{(\cos(\theta_W)) (\sin(\theta_W))} \right) \cdot (\varphi(-\overline{p_2}, m_e))$$

Out[]:=

$$\frac{e^2 \left(m_e^2 (16 (\sin(\theta_W))^4 - 8 (\sin(\theta_W))^2 - 1) + m_Z^2 (8 (\sin(\theta_W))^4 - 4 (\sin(\theta_W))^2 + 1) \right)}{6 (\cos(\theta_W))^2 (\sin(\theta_W))^2}$$

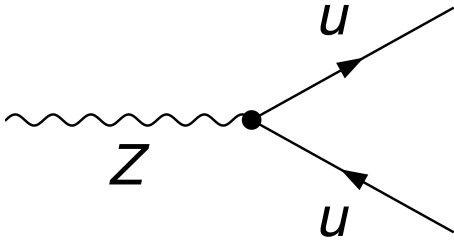
Out[]:=

$$\frac{e^2 m_Z^2 (8 (\sin(\theta_W))^4 - 4 (\sin(\theta_W))^2 + 1)}{6 (\cos(\theta_W))^2 (\sin(\theta_W))^2}$$

```

In[ ]:= (* SM: Z → u antiu *)
feynman = InsertFields[topology,
  {V[2]} → {F[3, {1}], -F[3, {1}]}, InsertionLevel → {Classes}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True, ChangeDimension → 4,
  DropSumOver → True, List → False, SMP → True, Contract → True]
FCClearScalarProducts[];
SP[p, p] = SMP["m_Z"] ^ 2;
SP[p1, p1] = SMP["m_u"] ^ 2;
SP[p2, p2] = SMP["m_u"] ^ 2;
SP[p1, p2] = (SMP["m_Z"] ^ 2 - 2 SMP["m_u"] ^ 2) / 2;
SP[p, p1] = SP[p1, p1] + SP[p2, p1];
SP[p, p2] = SP[p1, p2] + SP[p2, p2];
squareamplitude[0] =
  (amplitude[0] (ComplexConjugate[amplitude[0]])) // SUNSimplify //
  FermionSpinSum // DiracSimplify //
  DoPolarizationSums[#, p, ExtraFactor → 1 / 3] & // Simplify
masslessquareamplitude[0] =
  squareamplitude[0] // ReplaceAll[#, {SMP["m_u"] → 0}] & // Simplify
(* output = feynman diagrams, amplitude,
  squared amplitude, massless squared amplitude *)

```



$$Out[] = i(\varphi(\vec{p}_1, m_u)) \left(\frac{i e \delta_{\text{Col2 Col3}} \left(\frac{1}{2} - \frac{2}{3} (\sin(\theta_W))^2 \right) (\vec{\gamma} \cdot \vec{\epsilon}(p)) \cdot \vec{\gamma}^7}{(\cos(\theta_W)) (\sin(\theta_W))} - \frac{2 i e \delta_{\text{Col2 Col3}} (\sin(\theta_W)) (\vec{\gamma} \cdot \vec{\epsilon}(p)) \cdot \vec{\gamma}^6}{3 (\cos(\theta_W))} \right) \cdot (\varphi(-\vec{p}_2, m_u))$$

$$Out[] = \frac{e^2 C_A (m_u^2 (64 (\sin(\theta_W))^4 - 48 (\sin(\theta_W))^2 - 9) + m_Z^2 (32 (\sin(\theta_W))^4 - 24 (\sin(\theta_W))^2 + 9))}{54 (\cos(\theta_W))^2 (\sin(\theta_W))^2}$$

$$Out[] = \frac{e^2 C_A m_Z^2 (32 (\sin(\theta_W))^4 - 24 (\sin(\theta_W))^2 + 9)}{54 (\cos(\theta_W))^2 (\sin(\theta_W))^2}$$

```
In[*]:= Print["\tCPU Time used: ", Round[N[TimeUsed[]], 4], 0.001], " s."];
CPU Time used: 4.122 s.
```

```
In[*]:= FeynCalcHowToCite[]
```

- V. Shtabovenko, R. Mertig and F. Orellana, arXiv:2312.14089.
- V. Shtabovenko, R. Mertig and F. Orellana,
Comput.Phys.Commun. 256 (2020) 107478, arXiv:2001.04407.
- V. Shtabovenko, R. Mertig and F. Orellana,
Comput.Phys.Commun. 207 (2016) 432–444, arXiv:1601.01167.
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991) 345–359.

```
Out[*]=
```

Null