

```

In[*]:= (* F[1, {1}] = electron neutrino,
F[1, {2}] = muon neutrino, F[1, {3}] = tau neutrino,
F[2, {1}] = electron, F[2, {2}] = muon, F[2, {3}] = tau,
F[3, {1}] = up, F[3, {2}] = charm, F[3, {3}] = top,
F[4, {1}] = down, F[4, {2}] = strange, F[4, {3}] = bottom,
V[1] = photon, V[2] = Z, V[3] = W-, S[1] = H *)
$LoadAddOns = {"FeynArts"};
<< FeynCalc`
$FAVerbose = 0;

MakeBoxes[p1, TraditionalForm] := "\!\(\*SubscriptBox[\(p\), \ (1\)]\)\)";
MakeBoxes[p2, TraditionalForm] := "\!\(\*SubscriptBox[\(p\), \ (2\)]\)\)";
MakeBoxes[p3, TraditionalForm] := "\!\(\*SubscriptBox[\(p\), \ (3\)]\)\)";
topology = CreateTopologies[0, 1 → 3];

```

FeynCalc 10.0.0 (dev version). For help, use the

online documentation, visit the forum and have a look at the supplied examples. The PDF-version of the manual can be downloaded [here](#).

If you use FeynCalc in your research, please

evaluate `FeynCalcHowToCite[]` to learn how to cite this software.

Please keep in mind that the proper academic attribution

of our work is crucial to ensure the future development of this package!

FeynArts 3.12 (24 May 2024) patched for use with FeynCalc, for documentation see the manual or visit www.feynarts.de.

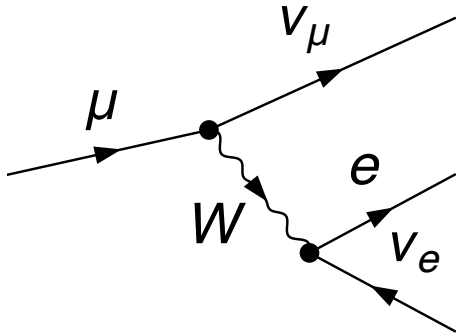
If you use FeynArts in your research, please cite

- T. Hahn, Comput. Phys. Commun., 140, 418–431, 2001, arXiv:hep-ph/0012260

```

In[*]:= (* SM: mu → nu_mu e- antinu_e *)
feynman = InsertFields[topology,
  {F[2, {2}]} → {F[1, {2}], F[2, {1}], -F[1, {1}]}, InsertionLevel → {Classes},
  Model → {SM, UnitarySM}, GenericModel → {Lorentz, UnitaryLorentz}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] =
  FCFAConvert[CreateFeynAmp[feynman, GaugeRules → {FAGaugeXi[W | Z] → Infinity}],
    IncomingMomenta → {p}, OutgoingMomenta → {p1, p2, p3},
    UndoChiralSplittings → True, ChangeDimension → 4,
    DropSumOver → True, List → False, SMP → True,
    Contract → True, DropSumOver → True, FinalSubstitutions →
      {SMP["e"] → Sqrt[8 / Sqrt[2] * SMP["G_F"] * SMP["m_W"] ^ 2 * SMP["sin_W"] ^ 2]}]
FCClearScalarProducts[];
SP[p, p] = SMP["m_mu"] ^ 2;
SP[p1, p1] = 0;
SP[p2, p2] = SMP["m_e"] ^ 2;
SP[p3, p3] = 0;
squareamplitude[0] = (amplitude[0] (ComplexConjugate[amplitude[0]])) //
  FermionSpinSum[#, ExtraFactor → 1 / 2] & // DiracSimplify // Factor
squareamplitude[1] = squareamplitude[0] // FCE // ReplaceAll[#, {p2 + p3 → 0}] & //
  FeynAmpDenominatorExplicit //
  Series[#, {SMP["m_W"], Infinity, 0}] & // Normal
(* output = feynman diagrams, amplitude,
squared amplitude, massless squared amplitude *)

```



Out[*]=

$$\begin{aligned}
 & \frac{2 \sqrt{2} G_F m_W^2 (\varphi(\bar{p}_2, m_e) \cdot \bar{\gamma}^{1 \text{ or } 2} \cdot \bar{\gamma}^7 \cdot (\varphi(-\bar{p}_3)) (\varphi(\bar{p}_1)) \cdot \bar{\gamma}^{1 \text{ or } 2} \cdot \bar{\gamma}^7 \cdot (\varphi(\bar{p}, m_\mu)))}{(\bar{p}_2 + \bar{p}_3)^2 - m_W^2} - \\
 & \frac{2 \sqrt{2} G_F (\varphi(\bar{p}_2, m_e) \cdot (\bar{\gamma} \cdot (\bar{p}_2 + \bar{p}_3)) \cdot \bar{\gamma}^7 \cdot (\varphi(-\bar{p}_3)) (\varphi(\bar{p}_1)) \cdot (\bar{\gamma} \cdot (-\bar{p}_2 - \bar{p}_3)) \cdot \bar{\gamma}^7 \cdot (\varphi(\bar{p}, m_\mu)))}{(\bar{p}_2 + \bar{p}_3)^2 - m_W^2}
 \end{aligned}$$

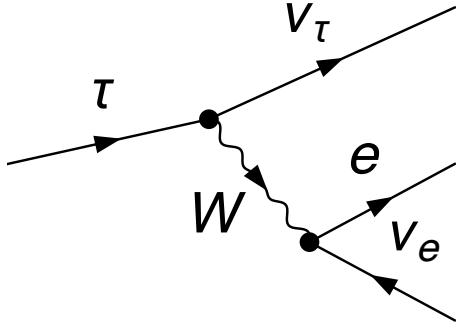
Out[*n*] =

$$16 G_F^2 \frac{1}{(\overline{p_2} + \overline{p_3})^2 - m_W^2} \\ (-2 m_e^2 (\overline{p} \cdot \overline{p_1}) (\overline{p_2} \cdot \overline{p_3})^2 - 2 m_e^2 m_W^2 (\overline{p} \cdot \overline{p_3}) (\overline{p_1} \cdot \overline{p_2}) - 2 m_e^2 m_W^2 (\overline{p} \cdot \overline{p_2}) (\overline{p_1} \cdot \overline{p_3}) + 2 m_e^2 m_W^2 (\overline{p} \cdot \overline{p_1}) (\overline{p_2} \cdot \overline{p_3}) + \\ m_e^4 (-(\overline{p} \cdot \overline{p_1}) (\overline{p_2} \cdot \overline{p_3}) + 2 m_e^2 (\overline{p} \cdot \overline{p_2}) (\overline{p_1} \cdot \overline{p_2}) (\overline{p_2} \cdot \overline{p_3}) + 2 m_e^2 (\overline{p} \cdot \overline{p_3}) (\overline{p_1} \cdot \overline{p_2}) (\overline{p_2} \cdot \overline{p_3}) + 2 m_e^2 (\overline{p} \cdot \overline{p_2}) \\ (\overline{p_1} \cdot \overline{p_3}) (\overline{p_2} \cdot \overline{p_3}) + 2 m_e^2 (\overline{p} \cdot \overline{p_3}) (\overline{p_1} \cdot \overline{p_3}) (\overline{p_2} \cdot \overline{p_3}) - 4 m_e^2 m_W^2 (\overline{p} \cdot \overline{p_3}) (\overline{p_1} \cdot \overline{p_3}) + 4 m_W^4 (\overline{p} \cdot \overline{p_3}) (\overline{p_1} \cdot \overline{p_2}))$$

Out[*n*] =

$$64 G_F^2 (\overline{p} \cdot \overline{p_3}) (\overline{p_1} \cdot \overline{p_2})$$

```
In[n] := (* SM: tau -> nu_tau e- antinu_e *)
feynman = InsertFields[topology,
  {F[2, {3}]} -> {F[1, {3}], F[2, {1}], -F[1, {1}]}, InsertionLevel -> {Classes},
  Model -> {SM, UnitarySM}, GenericModel -> {Lorentz, UnitaryLorentz}};
Paint[feynman, Numbering -> None, SheetHeader -> False,
  ColumnsXRows -> {4, 1}, ImageSize -> {1032, 256}];
amplitude[0] =
  FCFAConvert[CreateFeynAmp[feynman, GaugeRules -> {FAGaugeXi[W | Z] -> Infinity}],
    IncomingMomenta -> {p}, OutgoingMomenta -> {p1, p2, p3},
    UndoChiralSplittings -> True, ChangeDimension -> 4,
    DropSumOver -> True, List -> False, SMP -> True,
    Contract -> True, DropSumOver -> True, FinalSubstitutions ->
      {SMP["e"] -> Sqrt[8 / Sqrt[2] * SMP["G_F"] * SMP["m_W"] ^ 2 * SMP["sin_W"] ^ 2]}]
FCClearScalarProducts[];
SP[p, p] = SMP["m_tau"] ^ 2;
SP[p1, p1] = 0;
SP[p2, p2] = SMP["m_e"] ^ 2;
SP[p3, p3] = 0;
squareamplitude[0] = (amplitude[0] (ComplexConjugate[amplitude[0]])) //
  FermionSpinSum[#, ExtraFactor -> 1 / 2] & // DiracSimplify // Factor
squareamplitude[1] = squareamplitude[0] // FCE // ReplaceAll[#, {p2 + p3 -> 0}] & //
  FeynAmpDenominatorExplicit //
  Series[#, {SMP["m_W"], Infinity, 0}] & // Normal
(* output = feynman diagrams, amplitude,
squared amplitude, massless squared amplitude *)
```



Out[*]=

$$-\frac{2\sqrt{2}G_F m_W^2 (\varphi(\bar{p}_2, m_e)) \cdot \bar{\gamma}^{\text{Lor}2} \cdot \bar{\gamma}^7 \cdot (\varphi(-\bar{p}_3)) (\varphi(\bar{p}_1)) \cdot \bar{\gamma}^{\text{Lor}2} \cdot \bar{\gamma}^7 \cdot (\varphi(\bar{p}, m_\tau))}{(\bar{p}_2 + \bar{p}_3)^2 - m_W^2} -$$

$$\frac{2\sqrt{2}G_F (\varphi(\bar{p}_2, m_e)) \cdot (\bar{\gamma} \cdot (\bar{p}_2 + \bar{p}_3)) \cdot \bar{\gamma}^7 \cdot (\varphi(-\bar{p}_3)) (\varphi(\bar{p}_1)) \cdot (\bar{\gamma} \cdot (-\bar{p}_2 - \bar{p}_3)) \cdot \bar{\gamma}^7 \cdot (\varphi(\bar{p}, m_\tau))}{(\bar{p}_2 + \bar{p}_3)^2 - m_W^2}$$

Out[*]=

$$16G_F^2 \frac{1}{(\bar{p}_2 + \bar{p}_3)^2 - m_W^2}^2$$

$$(-2m_e^2 (\bar{p} \cdot \bar{p}_1) (\bar{p}_2 \cdot \bar{p}_3)^2 - 2m_e^2 m_W^2 (\bar{p} \cdot \bar{p}_3) (\bar{p}_1 \cdot \bar{p}_2) - 2m_e^2 m_W^2 (\bar{p} \cdot \bar{p}_2) (\bar{p}_1 \cdot \bar{p}_3) + 2m_e^2 m_W^2 (\bar{p} \cdot \bar{p}_1) (\bar{p}_2 \cdot \bar{p}_3) +$$

$$m_e^4 (-(\bar{p} \cdot \bar{p}_1) (\bar{p}_2 \cdot \bar{p}_3) + 2m_e^2 (\bar{p} \cdot \bar{p}_2) (\bar{p}_1 \cdot \bar{p}_2) (\bar{p}_2 \cdot \bar{p}_3) + 2m_e^2 (\bar{p} \cdot \bar{p}_3) (\bar{p}_1 \cdot \bar{p}_2) (\bar{p}_2 \cdot \bar{p}_3) + 2m_e^2 (\bar{p} \cdot \bar{p}_2) (\bar{p}_1 \cdot \bar{p}_3) (\bar{p}_2 \cdot \bar{p}_3) + 2m_e^2 (\bar{p} \cdot \bar{p}_3) (\bar{p}_1 \cdot \bar{p}_3) (\bar{p}_2 \cdot \bar{p}_3) - 4m_e^2 m_W^2 (\bar{p} \cdot \bar{p}_3) (\bar{p}_1 \cdot \bar{p}_3) + 4m_W^4 (\bar{p} \cdot \bar{p}_3) (\bar{p}_1 \cdot \bar{p}_2))$$

Out[*]=

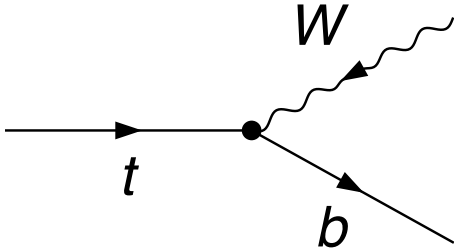
$$64G_F^2 (\bar{p} \cdot \bar{p}_3) (\bar{p}_1 \cdot \bar{p}_2)$$

In[*]:= **topology = CreateTopologies[0, 1 → 2];**

```

In[ ]:= (* SM: t → W+ b *)
feynman = InsertFields[topology,
  {F[3, {3}]} → {-V[3], F[4, {3}]}, InsertionLevel → {Classes},
  Model → {SM, UnitarySM}, GenericModel → {Lorentz, UnitaryLorentz}];
Paint[feynman, Numbering → None, SheetHeader → False,
  ColumnsXRows → {4, 1}, ImageSize → {1032, 256}];
amplitude[0] = FCFAConvert[CreateFeynAmp[feynman], IncomingMomenta → {p},
  OutgoingMomenta → {p1, p2}, UndoChiralSplittings → True,
  ChangeDimension → 4, DropSumOver → True, List → False, SMP → True,
  Contract → True, DropSumOver → True, FinalSubstitutions →
  {SMP["e"] → Sqrt[8 / Sqrt[2] * SMP["G_F"] * SMP["m_W"] ^ 2 * SMP["sin_W"] ^ 2]}]
FCClearScalarProducts[];
SP[p, p] = SMP["m_tau"] ^ 2;
SP[p1, p1] = SMP["m_W"] ^ 2;
SP[p2, p2] = SMP["m_b"] ^ 2;
SP[p1, p2] = (SMP["m_tau"] ^ 2 - SMP["m_W"] ^ 2 - SMP["m_b"] ^ 2) / 2;
SP[p, p1] = SP[p1, p1] + SP[p1, p2];
SP[p, p2] = SP[p1, p2] + SP[p2, p2];
squareamplitude[0] = (amplitude[0] (ComplexConjugate[amplitude[0]])) //
  DoPolarizationSums[#, p1] & // FermionSpinSum[#, ExtraFactor → 1 / 2] & //
  DiracSimplify // Simplify // SUNSimplify
masslessquareamplitude[0] =
  squareamplitude[0] // ReplaceAll[#, {SMP["m_b"] → 0}] & // Simplify
(* output = feynman diagrams, amplitude,
  squared amplitude, massless squared amplitude *)

```



$$Out[] = \frac{2^{3/4} \delta_{\text{Col1 Col3}} \sqrt{G_F m_W^2 (\sin(\theta_W))^2} (\varphi(\vec{p}_2, m_b)) \cdot (\vec{\gamma} \cdot \vec{E}^*(p_1)) \cdot \vec{\gamma}^7 \cdot (\varphi(\vec{p}, m_t))}{\sin(\theta_W)}$$

$$Out[] = \sqrt{2} C_A G_F (-2 m_b^2 m_\tau^2 + m_b^2 m_W^2 + m_b^4 + m_\tau^4 + m_\tau^2 m_W^2 - 2 m_W^4)$$

$$Out[] = \sqrt{2} C_A G_F (m_\tau^4 + m_\tau^2 m_W^2 - 2 m_W^4)$$

```
In[*]:= Print["\tCPU Time used: ", Round[N[TimeUsed[]], 4], 0.001], " s."];  
CPU Time used: 3.94 s.
```

```
In[*]:= FeynCalcHowToCite[]
```

- V. Shtabovenko, R. Mertig and F. Orellana, arXiv:2312.14089.
- V. Shtabovenko, R. Mertig and F. Orellana,
Comput.Phys.Commun. 256 (2020) 107478, arXiv:2001.04407.
- V. Shtabovenko, R. Mertig and F. Orellana,
Comput.Phys.Commun. 207 (2016) 432–444, arXiv:1601.01167.
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun. 64 (1991) 345–359.

```
Out[*]=
```

Null