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On theoretical astroparticle physics:

the only particles I can tolerate.

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Abstract

In these notes, we will study.

Introduction

The standard model is the best theory we have to describe particles and interactions. It has been tested during the last decades with enormous successes. However, there are three pillars that are not yet understood: dark matter, neutrinos and matter/antimatter asymmetry.

75% of the Universe is composed by dark energy and the rest is matter. We know that baryonic matter is only the 5%, whereas the remaining 20% does not interact with the electromagnetic radiation. For this reason, it is called dark matter. Evidence of the existence of such matter can be trace into different scales: the rotational curve of galaxies, gravitational lensing of clusters of galaxies and the cosmological microwave background radiation (CMB) of the whole cosmos. Therefore, it cannot be explained by a modification of gravity, but by either particles or group of them, like primordial black holes.

Neutrinos compare in three different flavours: electronic ν_e, muonic ν_μ and tauonic ν_τ. Their oscillation means that one can transform into another flavour. It is a quantum phenomenon, due to the alignment of the Hamiltonian like in the spin case. In order to do so, it needs to have mass/energy, because if they have different Hamiltonian, they have different time evolution, and they can be recognised by that. It can only explain a small fraction of dark matter. In fact, consider the decay $e^-e^+ \to \overline{\nu}\nu$ or $e^-e^+ \to \mu^+\mu^-$. There are two time scales to consider: the decay rated $\Gamma \sim \sigma m$ and the Hubble time H. If $\Gamma > H$, the decay never happen, otherwise it does. Furthermore, if $T < m$, the decay goes out of equilibrium. Since the universe is expanding, due to redshift, the temperature is cooling down and neutrinos becomes non-relativistic. This means that at the redshift of galaxy formation $z \sim 40/50$, neutrinos where too fast although non-relativistic to explain this phenomenon.

In the early universe, there was an equilibrium between the amount of particles and antiparticles. However, a small asymmetry of order of one particle of 10^{10} particles, allows that when the temperature cooled, matter and antimatter annihilated into radiation, but a small amount of matter had no pair, so it remained and formed out Universe.

These three pillars can be studied not only looking at the early universe, but also in contemporary extreme environments. Cosmic rays produced in supernovae explosions, where the 99% of the energy is due to neutrinos, or accelerated highenergy particles in AGNs. Another observational resource is gravitational waves. They can be produced by a merger of black holes, pulsars, etc. or they can come from the early Universe, when there was a phase transition that cause a spontaneous symmetry breaking. In thermal plasma, the only contribution to mass was the thermal energy (potential like a parabola as in Higgs) but when the temperature cooled down, it becomes negligible and the normal contribution of mass arises. Suppose there is a $\mathbb{Z}_2$ symmetry where two patches where linked and then the symmetry breaks. A gravitational wave comes up when vacuum bubbles merge.

Multimessenger physics is the merge of information coming from cosmic rays and gravitational waves.

Chapter A

The Standard Model

The Standard Model is based on a gauge symmetry, i.e. it is invariant under a local symmetry which is a transformation that depends on spacetime coordinates

$$SU(3)_c \times SU(2)_l \times U(1)_y \ .$$

A.1 Covariant derivative

An aside regarding covariant derivatives. Consider a gauge transformation $SU(N)$ of the wave function

$$\psi' = U(x)\psi = \exp(iq\theta_a(x)T^a)\psi \simeq (\mathbb{I} + iq\theta^a(x)T_a)\psi \ ,$$

where $\theta_a(x)$ are the $N^2 - 1$ spacetime coordinate dependent real parameters and T_a are the $N^2 - 1$ generators. In order to have an invariant Dirac Lagrangian of the kind $\psi^\dagger \partial_\mu \psi$ under this transformation, we substitute the partial derivatives with the covariant derivative, defined as

$$D^\mu = \partial^\mu + iqA_a^\mu(x)T^a \ ,$$

where $A_a^\mu(x)$ are compensating fields. Recall that a is an index of the transformation $SU(N)$ and μ is a Lorentz index. Hence,

$$\psi^\dagger D_\mu \psi = \psi'^\dagger D'_\mu \psi' = \psi^\dagger U^{-1} D'_\mu U(x)\psi \ , \quad U^{-1} D'_\mu U(x) = D_\mu \ ,$$

which means that the covariant derivative transforms as

$$D'_\mu = U(x) D_\mu U^{-1}(x) = \exp(iq\theta_a(x)T^a) D_\mu \exp(-iq\theta_a(x)T^a) \ .$$

Moreover,

$$\begin{aligned}
D'^{\mu} &= \partial^{\mu} + iqA'^{\mu}_a(x)T^a = U(x)D^{\mu}U^{-1}(x) \\
&= \exp(iq\theta_b(x)T^b)(\partial^{\mu} + iqA^{\mu}_a(x)T^a)\exp(-iq\theta_b(x)T^b) \\
&= \partial_{\mu} + \exp(iq\theta_b(x)T^b)\partial^{\mu}\exp(-iq\theta_b(x)T^b) \\
&\quad + \exp(iq\theta_b(x)T^b)iqA^{\mu}_a(x)T^a\exp(-iq\theta_b(x)T^b) \\
&= \partial_{\mu} - iq\partial^{\mu}\theta_b(x)T^b + \exp(iq\theta_b(x)T^b)iqA^{\mu}_a(x)T^a\exp(-iq\theta_b(x)T^b) \ ,
\end{aligned}$$

hence,

$$\begin{aligned}
A'^{\mu}_a(x)T^a &= \exp(iq\theta_b(x)T^b)A^{\mu}_a(x)T^a\exp(-iq\theta_b(x)T^b) - \partial^{\mu}\theta_a(x)T^a \\
&= (\mathbb{I} + iq\theta_b(x)T^b)A^{\mu}_a(x)T^a(\mathbb{I} - iq\theta_b(x)T^b) - \partial^{\mu}\theta_a(x)T^a \\
&= A^{\mu}_a(x)T^a - iq\theta_b(x)A^{\mu}_a(x)(T^aT^b - T^bT^a) - \partial^{\mu}\theta_a(x)T^a \\
&= A^{\mu}_a(x)T^a - iq\theta_b(x)A^{\mu}_a(x)f^{ab}{}_cT^c - \partial^{\mu}\theta_a(x)T^a \ ,
\end{aligned}$$

and the compensating field transforms as

$$A'^{\mu}_a(x) = A^{\mu}_a(x) - iq\theta_b(x)A^{\mu}_c(x)f^{bc}{}_a - \partial^{\mu}\theta_a(x) \ .$$

More specifically, we have the following covariant derivatives

1. $U(1)$ has $D^{\mu} = \partial^{\mu} + i\frac{q_1}{2}B^{\mu}$;
2. $SU(2)$ has $D^{\mu} = \partial^{\mu} + i\frac{q_2}{2}W^{\mu}_i\sigma^i$, where $i = 1, 2, 3$;
3. $SU(3)$ has $D^{\mu} = \partial^{\mu} + i\frac{q_3}{2}G^{\mu}_{\alpha}\lambda^{\alpha}$, where $\alpha = 1, \dots 8$;

A.2 Gauge bosons

There is a number of gauge bosons equals to the number of generators. See Table A.1. After a change of basis, we find the know $\gamma, W^{\pm}, Z$.

gauge group	bosons	generators
$U(1)$	B^{μ}	-
$SU(2)$	W^{μ}_i with $i = 1, 2, 3$	Pauli matrices σ
$SU(3)$	G^{μ}_{α} with $\alpha = 1, \dots 8$	Gell-Mann matrices λ

Table A.1: Outline of the gauge bosons.

Particles are singlets, doublets or triplets of the representation of this gauge group. See Table A.2. Notice that the theory is chiral, because left and right particels behave differently.

particle	$SU(3)$ (representation)	$SU(2)$ (representation)	$U(1)$ (hypercharge)
$q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix}$	3	2	1/6
u_R	3	1	2/3
d_R	3	1	$-1/3$
$l_L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$	1	2	$-1/2$
l_R	1	1	-1

Table A.2: Outline of the particles.

Suppose we have a term in the Lagrangian like $\mathcal{L} = y\bar{e}_L e_R$, where y is the Yukawa constant. There is no ν_R because it is a single of all gauge groups and it can be neglected. Notice that the dimension is not okay, since $[\mathcal{L}] = 4$ and $[\bar{e}_L] = [e_R] = 3/2$. We need to add a scalar $[H] = 1$ and the term becomes $\mathcal{L} = y\bar{e}_L H e_R$. The spin is $1/2 \otimes 1/2 = 0 \otimes 1$, which means that it can come up in singlet 0 or triplets 1. Adding the interaction/mass term that breaks the symmetry, we need to substitute the vacuum expectation value $\langle H \rangle = v_H/\sqrt{2}$. Therefore $\mathcal{L} = y\bar{e}_L v_H e_R/\sqrt{2} = m_e \bar{e}_L e_R$, where $m_e = y v_H/\sqrt{2}$ is the mass of the electron given by the Higgs boson. If the theory were not chiral, we would have a term like $\mathcal{L} = M\bar{e}_L e_R$ where M has the dimension of a mass but it does not control its scale. Therefore, the chirality controls the scale of the mass. Any mass term must depend on a spontaneous symmetry breaking and all the others must be zero. The total lagrangian contains all the possible terms of particles and gauge bosons. See Table A.3.

quarks	leptons
$q_L = \begin{bmatrix} u_L \\ d_L \end{bmatrix}, u_R, d_R \times 3$ generations	$l_L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}, e_R \times 3$ generations

vector bosons	scalar bosons
$A_\mu, Z_\mu, W^\pm_\mu, G^\alpha_\mu$	H

Table A.3: Outline of the Standard Model.

A.3 Spontaneous braking symmetry

Consider a spontaneous symmetry breaking of a global $U(1)$ group. The Lagrangian is

$$L = \partial_\mu \phi^* \partial^\mu \phi - m^2|\phi|^2 - \lambda^2|\phi|^4 = \partial_\mu \phi^* \partial^\mu \phi - V(\phi) \ .$$

We cannot have other terms otherwise the theory is not renormalisable. m is the mass term that breaks the symmetry but it is not the physical mass of the particle/field. λ is a dimensionless parameter. Suppose that ϕ is real and with the

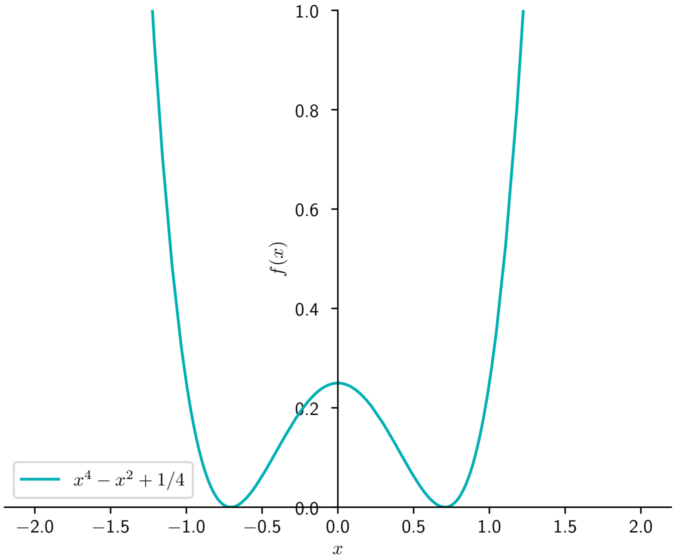

Figure A.1: A plot of the potential V as a function of ϕ. We have used $m = -1$ and $\lambda = 1$. Actually, it should be 3-dimensional since ϕ is complex and the resulting plot is a so-called Mexican hat.

symmetry breaking m^2 becomes negative $m^2 < 0$. The potential becomes

$$V(\phi) = m^2\phi^2 + \lambda\phi^4 \ .$$

See Figure A.1.

There are two extrema. In fact

$$0 = \frac{dV}{d\phi} = 2m^2\phi + 4\lambda\phi^3 \ , \quad \phi = 0 \vee \phi = \sqrt{-\frac{m^2}{2\lambda}} = \frac{v_\phi}{\sqrt{2}} \ .$$

Computing the second derivative, we find

$$\frac{d^2V}{d\phi^2} = 2m^2 + 12\lambda\phi^2 \ , \quad \left.\frac{d^2V}{d\phi^2}\right|_{\phi=0} = 2m^2 < 0 \, , \quad \left.\frac{d^2V}{d\phi^2}\right|_{\phi=\sqrt{-\frac{m^2}{2\lambda}}} = 2m^2 - 12\frac{m^2}{2} = -4m^2 > 0 \, ,$$

which means that $\phi = 0$ is a maximum and $\phi = v_\phi/\sqrt{2}$ is a minimum. It is better to (second) quantise the theory using as vacuum the minimum, called $|\phi|_{vev}$. Since ϕ is complex, we can decompose it into two real fields $\phi = (v_\phi + \phi_1 + i\phi_2)/\sqrt{2}$. The potential becomes

$$\begin{aligned}
V(\phi) &= \frac{m^2}{2}|\phi|^2 + \frac{\lambda}{4}|\phi|^4 = \frac{m^2}{2}(v_\phi + \phi_1 + i\phi_2)(v_\phi + \phi_1 - i\phi_2) \\
&\quad + \frac{\lambda}{4}((v_\phi + \phi_1 + i\phi_2)(v_\phi + \phi_1 - i\phi_2))^2 \\
&= \frac{m^2}{2}((v_\phi + \phi_1)^2 + \phi_2^2) + \frac{\lambda}{4}((v_\phi + \phi_1)^2 + \phi_2^2)^2 \\
&= \frac{m^2}{2}((v_\phi + \phi_1)^2 + \phi_2^2) + \frac{\lambda}{4}((v_\phi + \phi_1)^4 + 2(v_\phi + \phi_1)^2\phi_2^2 + \phi_2^4) \\
&= \frac{m^2}{2}v_\phi^2 + \frac{m^2}{2}\phi_1^2 + m^2 v_\phi\phi_1 + \frac{m^2}{2}\phi_2^2 + \frac{\lambda}{4}v_\phi^4 + \frac{\lambda}{4}\phi_1^4 + \lambda v_\phi^3\phi_1 + \lambda v_\phi\phi_1^3 + \frac{6}{4}\lambda v_\phi^2\phi_1^2 \\
&\quad + \frac{\lambda}{2}v_\phi^2\phi_2^2 + \frac{\lambda}{2}\phi_1^2\phi_2^2 + \lambda v_\phi\phi_1\phi_2^2 + \frac{\lambda}{4}\phi_2^4 \ .
\end{aligned}$$

Now, in order to study the mass term, we are interested in terms containing only ϕ_1^2 and ϕ_2^2

$$\begin{aligned}
V(\phi) &= \frac{m^2}{2}\phi_1^2 + \frac{m^2}{2}\phi_2^2 + \frac{3}{2}\lambda v_\phi^2\phi_1^2 + \frac{\lambda}{2}v_\phi^2\phi_2^2 \\
&= \frac{m^2}{2}\phi_1^2 + \frac{m^2}{2}\phi_2^2 - \frac{3}{2}\lambda\frac{m^2}{\lambda}\phi_1^2 - \frac{\lambda}{2}\frac{m^2}{\lambda}\phi_2^2 = -m^2\phi_1^2 \ .
\end{aligned}$$

This means that ϕ_1 is massive $m_1 = \sqrt{-m^2}$ and ϕ_2 represents a massless boson $m_2 = 0$, because there are no terms $m^2\phi_2^2$. A Goldstone boson is a massless degree of freedom that comes up whenever there is spontaneous break of the symmetry and not when you add explicitly a term in the Lagrangian. In the Standard Model there are two accidental global symmetrues (lepton number) that give rise to pseudo-Goldstone numbers, called axions, that couple with fermions.

Consider now a spontaneous symmetry breaking of a gauge/local $U(1)$ group. The Lagrangian is

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - m^2|\phi|^2 - \lambda^2|\phi|^4 = \partial_\mu\phi^*\partial^\mu\phi - V(\phi) \ .$$

Treatment of the potential is the same as before. Consider now the kinetic term with the covariant derivative

$$\begin{aligned}
K &= (D_\mu\phi)^* D^\mu\phi = (\partial_\mu\phi^* - iqA_\mu\phi^*)(\partial^\mu\phi + iqA^\mu\phi) \\
&= \partial_\mu\phi^*\partial^\mu\phi + q^2A_\mu A^\mu|\phi|^2 + iqA^\mu(\partial_\mu\phi^*\phi - \partial_\mu\phi\phi^*) \ .
\end{aligned}$$

Since we have a degree of freedom to choose, we use the unitary gauge $\alpha = -\tan\phi_2/v_\phi + \phi_1$, so that the Goldstone boson ϕ_2 disappears. The gauge boson mass becomes

$$K = \frac{q^2v_\phi^2}{2}A_\mu A^\mu = m^2A_\mu A^\mu \ .$$

This means that a massless boson A_μ acquire mass after the spontaneous symmetry breaking.

A.4 Higgs boson

Consider the potential part of the Lagrangian for the Higgs scalar field

$$\mathcal{L} = -m^2 H^\dagger H + \lambda (H^\dagger H)^2 \ .$$

where H is a matrix

$$H = \begin{bmatrix} h_+ \\ h_0 \end{bmatrix} \ .$$

+

Regarding the potential for h_0, the same procedure of before is valid with $m^2 < 0$ and $\langle h_0 \rangle = v_H/\sqrt{2} = \sqrt{m^2/2\lambda}$.

The kinetic part of the Lagrangian is

$$(D_\mu H)^\dagger D^\mu H \ ,$$

where D_μ is the covariant derivative associated to the gauge group $SU(2) \times U(1)$

$$D_\mu = \partial_\mu + i\frac{q_1}{2} B^\mu + i\frac{q_2}{2} W_i^\mu \sigma^i \ .$$

After spontaneous symmetry breaking, for $h_0 = (v_H + \tilde{h}_{0,1} + \tilde{h}_{0,2})$ where $\tilde{h}_{0,2}$ is a Goldstone boson that disappears in unitary gauge, $\tilde{h}_{0,1}$ is the real physical scalar field and $\tilde{h}_{0,1}, h_+$ get eaten up by 3 massive Goldstone bosons. Therefore, with

$$\langle H \rangle = \begin{bmatrix} 0 \\ v_H/\sqrt{2} \end{bmatrix} \ ,$$

we obtain, neglecting the pure kinetic term ∂_μ,

$$\begin{aligned} D_\mu H &= \frac{i}{2}(q_1 B^\mu \mathbb{I}_2 + q_2 W_1^\mu \sigma^1 + q_2 W_2^\mu \sigma_2 + q_2 W_3^\mu \sigma^3) H \\ &= \frac{i}{2} \begin{bmatrix} q_1 B^\mu + q_2 W_3^\mu & q_2 W_1^\mu - i q_2 W_2^\mu \\ q_2 W_1^\mu + i q_2 W_2^\mu & q_1 B^\mu - q_2 W_3^\mu \end{bmatrix} \begin{bmatrix} 0 \\ v_H/\sqrt{2} \end{bmatrix} = i\frac{v_H}{2\sqrt{2}} \begin{bmatrix} q_2 W_1^\mu - i q_2 W_2^\mu \\ q_1 B^\mu - q_2 W_3^\mu \end{bmatrix} \end{aligned}$$

and

$$(D_\mu H)^\dagger = -i\frac{v_H}{2\sqrt{2}} \begin{bmatrix} q_2 W_1^\mu + i q_2 W_2^\mu & q_1 B^\mu - q_2 W_3^\mu \end{bmatrix} \ .$$

Hence,

$$\begin{aligned} (D_\mu H)^\dagger D_\mu H &= \frac{v_H^2}{8} \begin{bmatrix} q_2 W_1^\mu + i q_2 W_2^\mu & q_1 B^\mu - q_2 W_3^\mu \end{bmatrix} \begin{bmatrix} q_2 W_1^\mu - i q_2 W_2^\mu \\ q_1 B^\mu - q_2 W_3^\mu \end{bmatrix} \\ &= \frac{v_H^2}{8} \Big((q_2 W_1^\mu + i q_2 W_2^\mu)(q_2 W_1^\mu - i q_2 W_2^\mu) + (q_1 B^\mu - q_2 W_3^\mu)^2 \Big) \\ &= \frac{v_H^2}{8} \Big(q_2^2 (W_1^\mu)^2 + q_2^2 (W_2^\mu)^2 + q_1^2 (B^\mu)^2 + q_2^2 (W_3^\mu)^2 + 2 q_1 q_2 B_\mu W_3^\mu \Big) \ . \end{aligned}$$

However, it is better to diagonalise and make a change of basis of (W_3^μ, B^μ). In fact, introducing the Weinberg angle

$$\sin\theta_W = \frac{q_1}{\sqrt{q_1^2+q_2^2}} \ , \quad \cos\theta_W = \frac{q_2}{\sqrt{q_1^2+q_2^2}} \ ,$$

we find

$$\begin{bmatrix} A^\mu \\ Z^\mu \end{bmatrix} = \begin{bmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{bmatrix} \begin{bmatrix} B^\mu \\ W_3^\mu \end{bmatrix} \ , \quad \begin{bmatrix} B^\mu \\ W_3^\mu \end{bmatrix} = \begin{bmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{bmatrix} \begin{bmatrix} A^\mu \\ Z^\mu \end{bmatrix} \ ,$$

or

$$B^\mu = A^\mu \cos\theta_W + Z^\mu \sin\theta_W \ , \quad W_3^\mu = -A^\mu \sin\theta_W + Z^\mu \cos\theta_W \ .$$

Hence,

$$\begin{aligned}
&q_1^2(B^\mu)^2 + q_2^2(W_3^\mu)^2 + 2q_1q_2B_\mu W_3^\mu \\
&= q_1^2(A^\mu)^2\cos^2\theta_W + q_1^2(Z^\mu)^2\sin^2\theta_W + 2q_1^2A^\mu Z_\mu \cos\theta_W\sin\theta_W \\
&\quad + q_2^2(A^\mu)^2\sin^2\theta_W + q_2^2(Z^\mu)^2\cos^2\theta_W - 2q_2^2A^\mu Z_\mu\cos\theta_W\sin\theta_W \\
&\quad - 2q_1q_2(A^\mu)^2\cos\theta_W\sin\theta_W + 2q_1q_2(Z^\mu)^2\cos\theta_W\sin\theta_W \\
&\quad - 2q_1q_2A_\mu Z^\mu\cos^2\theta_W + 2q_1q_2A_\mu Z^\mu\sin^2\theta_W \\
&= (A^\mu)^2\frac{q_1^2q_2^2}{q_1^2+q_2^2} + (Z^\mu)^2\frac{q_1^4}{q_1^2+q_2^2} + 2A^\mu Z_\mu\frac{q_1^3q_2}{q_1^2+q_2^2} + (A^\mu)^2\frac{q_2^2q_1^2}{q_1^2+q_2^2} \\
&\quad + (Z^\mu)^2\frac{q_2^4}{q_1^2+q_2^2} - 2A^\mu Z_\mu\frac{q_1q_2^3}{q_1^2+q_2^2} - 2(A^\mu)^2\frac{q_1^2q_2^2}{q_1^2+q_2^2} \\
&\quad + 2(Z^\mu)^2\frac{q_1^2q_2^2}{q_1^2+q_2^2} - 2A_\mu Z^\mu\frac{q_1q_2^3}{q_1^2+q_2^2} + 2A_\mu Z^\mu\frac{q_1^3q_2}{q_1^2+q_2^2} \ .
\end{aligned}$$

We are interested in mass terms in which the fields are quadratic

$$\begin{aligned}
\mathcal{L} &= \frac{v_H^2}{8}\Big(q_2^2(W_1^\mu)^2 + q_2^2(W_2^\mu)^2 + (A^\mu)^2\frac{q_1^2q_2^2}{q_1^2+q_2^2} + (Z^\mu)^2\frac{q_1^4}{q_1^2+q_2^2} \\
&\qquad + (A^\mu)^2\frac{q_2^2q_1^2}{q_1^2+q_2^2} + (Z^\mu)^2\frac{q_2^4}{q_1^2+q_2^2} - 2(A^\mu)^2\frac{q_1^2q_2^2}{q_1^2+q_2^2} + 2(Z^\mu)^2\frac{q_1^2q_2^2}{q_1^2+q_2^2}\Big) \\
&= \frac{v_H^2}{8}\Big(q_2^2(W_1^\mu)^2 + q_2^2(W_2^\mu)^2 + \frac{q_1^4+2q_1^2q_2^2+q_2^4}{q_1^2+q_2^2}(Z_2^\mu)^2\Big) \\
&= \frac{v_H^2}{8}\Big(q_2^2(W_1^\mu)^2 + q_2^2(W_2^\mu)^2 + (q_1^2+q_2^2)(Z_2^\mu)^2\Big) \ .
\end{aligned}$$

Last passage is to make a change also for W_1 and W_2 in

$$W_\pm^\mu = \frac{W_1^\mu \mp iW_2^\mu}{\sqrt{2}} \ , \quad W_1^\mu = \frac{W_+^\mu + W_-^\mu}{\sqrt{2}} \ , \quad W_2^\mu = \frac{-W_+^\mu + W_-^\mu}{\sqrt{2}i} \ ,$$

so that, recalling that in the complex case the square is the norm

$$q_2^2(W_1^\mu)^2+q_2^2(W_2^\mu)^2=\frac{q_2^2}{2}(W_+^\mu)^2+\frac{q_2^2}{2}(W_-^\mu)^2+q_2^2W_+^\mu W_{\mu-}+\frac{q_2^2}{2}(W_+^\mu)^2+q_2^2(W_-^\mu)^2+\frac{q_2^2}{2}W_+^\mu W_{\mu-}$$
$$=q_2^2(W_+^\mu)^2+q_2^2(W_-^\mu)^2+2q_2^2W_+^\mu W_{\mu-}\ .$$

Putting together, for the mass terms we find

$$\mathcal{L}=\frac{v_H^2}{8}\left(q_2^2(W_+^\mu)^2+q_2^2(W_-^\mu)^2+(q_1^2+q_2^2)(Z_2^\mu)^2\right)=\frac{v_H^2}{8}\left(q_2^2(W_+^\mu)^2+q_2^2(W_-^\mu)^2+(q_1^2+q_2^2)(Z_2^\mu)^2\right)\ ,$$

which means that the mass of the bosons are

$$m_A=0\ ,\quad m_Z=v_H\sqrt{\frac{q_1^2+q_2^2}{8}}\ ,\quad m_{W_\pm}=v_H\frac{q_2}{\sqrt{8}}\ .$$

A_μ, associated to the electromagnetic field (photon) is massless, whereas Z and $W^\pm$ associated to the weak interaction are massive.

Chapter B

QFT

Chapter C

Cosmology