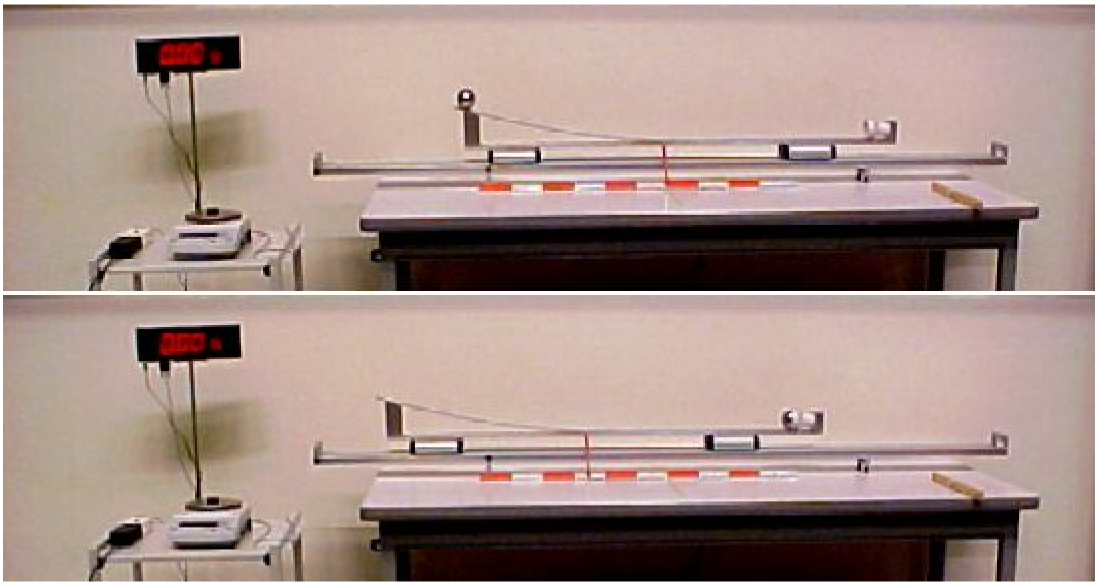
Centre of mass

Aim: To show that when there are only internal forces acting in a body, the centre of mass does not move.

Subjects: 1D40 (Motion of the Centre of Mass)

Diagram:



Equipment:

- Track (2.2m, PASCO ME-9452), levelled.
- Two carts (PASCO ME-9454).
- Bent rail track on frame, with "ball catcher" (plastic coffee cup) and a pointer fixed to it (see Diagram). The bent rail track is fixed **firmly** to the two carts. Take also care that the two carts are nicely in line.
- Steel ball (m=1kg).
- Graduated ruler, l=1m.
- Small wooden beam.
- Balance with large display.

Safety:

- The steel ball is heavy. Handle it with attention so it will not drop on the floor (on your feet).

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Presentation: The set-up is shown to the students.
Then the mass of the cart-assembly is measured by the balance (4kg) and also that of the steel ball (1kg).Next, the centre of mass of the cart-assembly is determined by balancing this assembly on the small wooden beam. Then the assembly is set at rest in the middle of the levelled cart-track and the pointer is fixed to the assembly in its centre of mass. Furthermore this pointer-position is marked on the table by placing a piece of chalk upright.

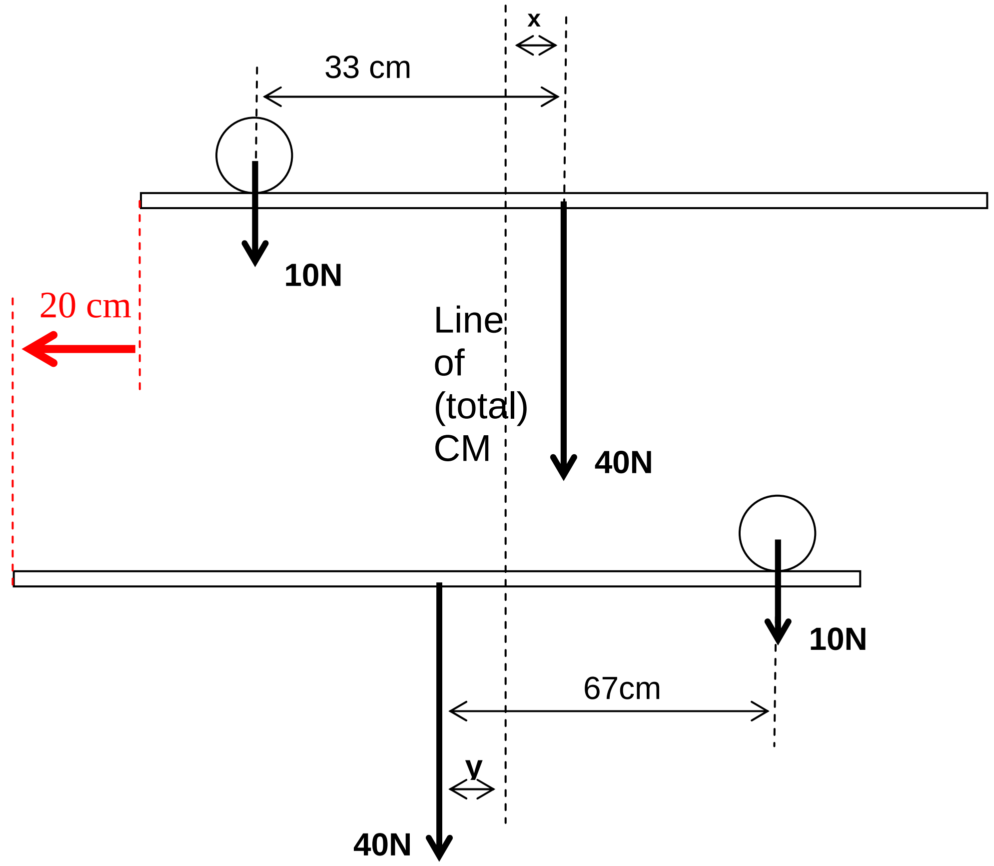

Figure 1

By hand the steel ball is placed at the slope of the bent track. We place it about 33 cm away from the point of reference on the assembly. With our other hand we hold the cart-assembly in its rest position. Then we let go, and the ball rolls down the bent track. The whole assembly is moving, and the steel ball is captured in the plastic coffee cup (a small piece of modelling clay at the low side at the entrance of the cup takes care that the steel ball remains in the cup). This plastic cup is 67 cm away from the centre of mass of the cart-assembly.
When the ball is captured, the whole assembly is immediately at rest again. The difference is that the whole assembly is shifted (about) 20cm to the side where the ball came from (see the pictures in the Diagram, where the actual displacement is larger due to the higher starting point of the steel ball).

Explanation: Our steel ball has a mass of 1kg. The cart-assembly (with carts, track and cup etc.) has a mass of 4kg. With our starting position at 33cm away from the point of reference, this gives a sketch of the situation as shown in the first picture of Figure 1.

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Calculating the distances **x** and **y** gives the displacement of the cart-assembly:
With *m* being the mass of the steel ball and *4m* the mass of the cart-assembly, the centre of mass is at the position indicated by the dotted line, because by definition the centre of mass is positioned at $R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$, relative to some origin. When that origin is positioned at the dotted line, so *R=0*, we have: $R = \frac{1(x-.33)+4(x)}{1+4} = 0$, making $x = 6.6cm$. When the steel ball is released there are no external forces acting. This means that the centre of mass will not move and so the second figure shows the situation at the end and the displacement of the steel ball and cart-assembly.
Calculating again the position of the centre of mass in the same way:
$R = \frac{1(.67-y)-4(y)}{1+4} = 0$, yielding $y = 13.5cm$. So the total displacement of the assembly equals *6.6+13.5=20.1cm*. This is confirmed in our demonstration.

Remarks:

- Take care that when you release the steel ball that you impart no horizontal momentum to the track. **Practice before performing the demonstration is necessary!**
- In our demonstration the reference pointer is fixed at the centre of mass of the track assembly, but this point of reference can of course be situated at any other position. Our choice makes calculations relatively easy.
- When the ball hits the cup the whole assembly should come to a full stop. But very often the assembly moves back just a little. This is caused by the friction of the carts on the track: the direction of that friction force is opposite to the displacement of the assembly. The effect of this external force becomes visible when the ball comes to a rest in the cup.

Sources:

- Mansfield, M and O'Sullivan, C., Understanding physics, pag. 128-130.
- McComb,W.D., Dynamics and Relativity, pag. 103-105.