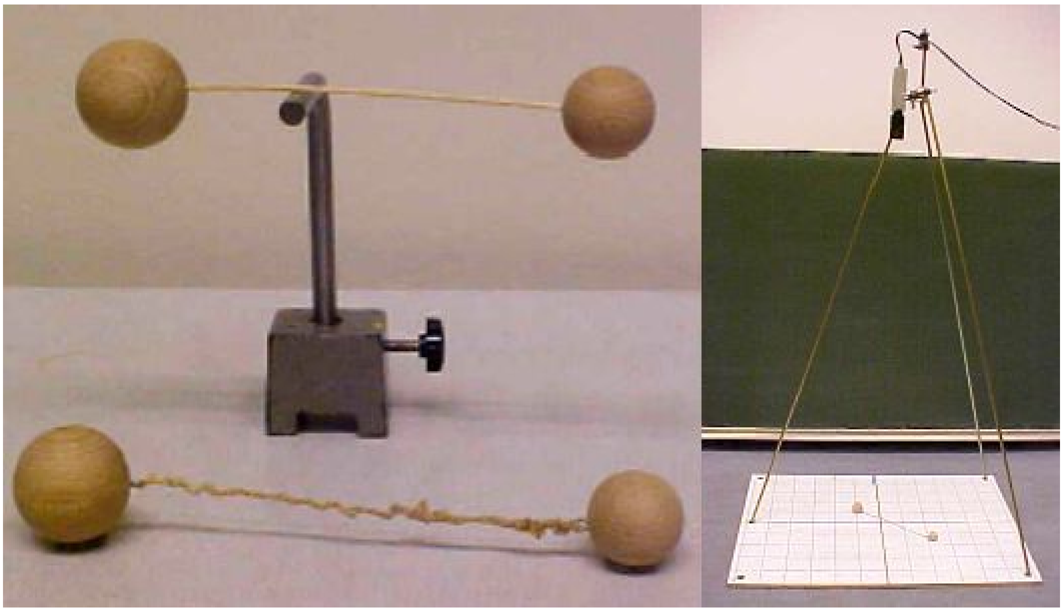
Centre of rotation

Aim: To show that a free rotating body rotates around its centre of mass.

Subjects: 1D40 (Motion of the Centre of Mass)
1Q60 (Rotational Stability)

Diagram:



Equipment:

- Two different wooden spheres (d_1=5cm and d_2=4cm), connected by a light stick.
- Two different wooden spheres (d_1=5cm and d_2=4cm), connected by a rubber band (length unwound around 50cm).
- Board with square grid (10×10 cm^2).
- Video camera on tripod (see Diagram).
- Beamer to project image of square grid to the audience.

Safety:

-

Centre of rotation

Presentation: The two spheres connected to each other by means of the light stick are balanced (see Diagram) to show where the centre of mass (CM) is located. As can be observed, the location of the CM divides the distance (*d*) between the two centres of the spheres in roughly $^1/_3 d$ and $^2/_3 d$.
Then the system of the two spheres connected by the rubber band is taken. The rubber band is twisted (see Diagram and Remarks). The system is placed on the board with the grid and then left by itself. The combination starts to rotate while unwinding the twisted rubber band. As can be observed, the combination rotates around a fixed point. This can be recognized as the CM shown in the first part of the demonstration. During the rotation the distance between the spheres increases, but its centre of rotation keeps the ratio $^1/_3$ - $^2/_3$.

Explanation: 1.
The total mass is located at the CM. No external forces are acting, so the CM has to remain at its position on the board as Newton's first law tells us. (When the system should rotate around any other point but the CM, the CM performs a rotation and an external torque should be needed for that.)

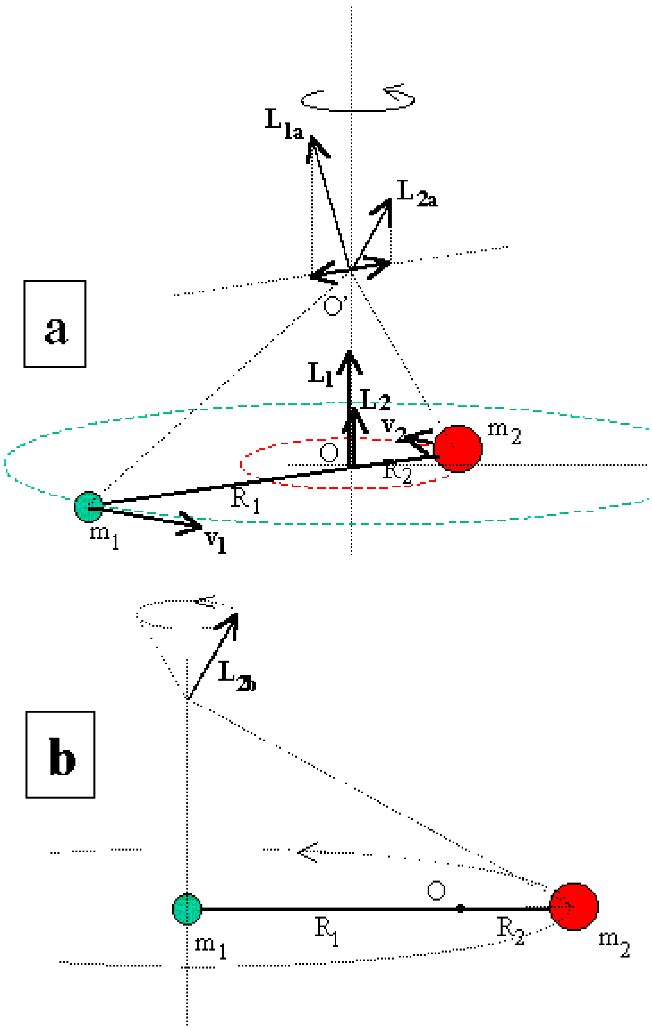


Figure 1

Centre of rotation

2.
When the body rotates it does so around an axis perpendicular to *d*. When no external forces are acting, the angular momentum vector $\vec{L}$ remains constant. The body in our demonstration consists of two masses: m_1 and m_2 (see Figure 1a).
$\vec{L} = \vec{L}_1 + \vec{L}_2$; $\vec{L} = \vec{R}_1 \times m_1\vec{v}_1 + \vec{R}_2 \times m_2\vec{v}_2$.
The axis of rotation characterizes itself by the fact that, at any position on this axis, $\vec{L}$ will have the same magnitude and direction (at O' the horizontal components of L_{1a} and L_{2a} cancel, and their vertical components add up to $\vec{L}$: see Sources). This means that only one axis of rotation is possible. When, for instance, an axis of rotation is chosen passing through the centre of m_1, then the total angular momentum adds up to L_{2b} (see Figure 1b). And so, being constant in magnitude, its direction constantly changes (describing a cone). Such a situation needs an external torque.
In our demonstration, there is no external torque and the sphere-system rotates around an axis somewhere between the two spheres (Figure 2). At O, $\vec{L} = \vec{R}_1 \times m_1\vec{v}_1 + \vec{R}_2 \times m_2\vec{v}_2$, directed along the axis of rotation. At O', the horizontal components of L_1 and L_2 need to cancel in order to keep L along the axis of rotation.

$$\left|\vec{L}_1\right| = \frac{R_1 m_1 v_1}{\cos\alpha}$$

$$L_{1h} = \frac{R_1 m_1 v_1}{\cos\alpha}\sin\alpha = R_1 m_1 v_1 \tan\alpha = m_1 v_1 y$$

In the same way: $L_{2h} = m_2 v_2 y$. $v=\omega R$, so $L_{1h} = m_1\omega R_1 y$ $L_{1h} = m_1\omega R_1 y$ and $L_{2h} = m_2\omega R_2 y$ $L_{2h} = m_2\omega R_2 y$. These two are equal when $m_1R_1=m_2R_2$, and this is so when the axis of rotation passes through the CM.

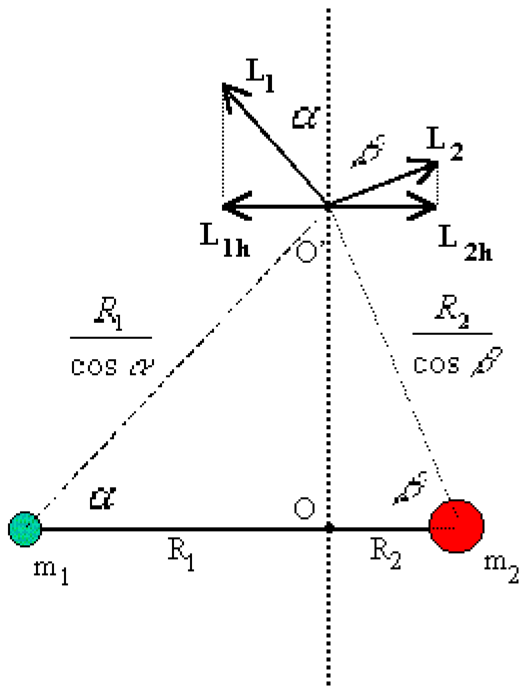


Figure 2

Centre of rotation

Remarks:

- The easiest way to wind the rubber band is done by holding the small sphere in your hand and whirling the large sphere round over the ground.
- The wooden spheres having diameters of 4- and 5cm will have a mass ratio of 1 to 1.95, so very close to a factor 2.
- See also the demonstration "Dumbbell" in this database.

Sources:

- Mansfield, M and O'Sullivan, C., Understanding physics, pag. 139-141.
- PSSC, College Physics, pag. 352-354.