TEQS_Exercise_01

April 10, 2025

```python
[1]: import numpy as np
```

1 H.3 a)

```python
[13]: # L = 2

H_2 = -1/4 * np.matrix([
    [1, 0, 0, 0],
    [0, -1, 2, 0],
    [0, 2, -1, 0],
    [0, 0, 0, 1]
])

eigenvals2, eigenvecs2 = np.linalg.eig(H_2)

print("Eigenvalues:", eigenvals2, '\n')
print("Eigenvectors:\n", eigenvecs2, '\n')
```

```
Eigenvalues: [ 0.75 -0.25 -0.25 -0.25]

Eigenvectors:
 [[ 0.          0.          1.          0.        ]
 [-0.70710678  0.70710678  0.          0.        ]
 [ 0.70710678  0.70710678  0.          0.        ]
 [ 0.          0.          0.          1.        ]]
```

```python
[14]: # L = 3

H_3 = -1/2 * np.matrix([
    [1, 0, 0, 0, 0, 0, 0, 0],
    [0, 0, 1, 0, 0, 0, 0, 0],
    [0, 1, -1, 0, 1, 0, 0, 0],
    [0, 0, 0, 0, 0, 1, 0, 0],
    [0, 0, 1, 0, 0, 0, 0, 0],
    [0, 0, 0, 1, 0, -1, 1, 0],
    [0, 0, 0, 0, 0, 1, 0, 0],
```

```
      [0, 0, 0, 0, 0, 0, 0, 1]
])

eigenvals3, eigenvecs3 = np.linalg.eig(H_3)

print("Eigenvalues:", np.round(eigenvals3, 5), '\n')
print("Eigenvectors:\n", np.round(eigenvecs3, 5), '\n')
```

```
Eigenvalues: [ 1.  -0.  -0.5  1.  -0.5  0.  -0.5 -0.5]

Eigenvectors:
 [[ 0.       0.       0.       0.       0.       0.       1.       0.     ]
 [ 0.       0.       0.      -0.40825 -0.57735  0.70711  0.       0.     ]
 [ 0.       0.       0.       0.8165  -0.57735 -0.       0.       0.     ]
 [-0.40825  0.70711  0.57735  0.       0.       0.       0.       0.     ]
 [ 0.       0.       0.      -0.40825 -0.57735 -0.70711  0.       0.     ]
 [ 0.8165   0.       0.57735  0.       0.       0.       0.       0.     ]
 [-0.40825 -0.70711  0.57735  0.       0.       0.       0.       0.     ]
 [ 0.       0.       0.       0.       0.       0.       0.       1.     ]]
```

2 H.3 b)

```
[19]: # transformation matrix P

P = [np.matrix(eigenvecs2), np.matrix(eigenvecs3)]
H_diag = [np.diag(eigenvals2), np.diag(eigenvals3)]


# time evolution operator

def U(t, L):
    PL = P[L%2]
    HL = H_diag[L%2]

    return PL * np.exp(-1j * HL * t) * PL.T
```

```
[22]: # test call

np.round(U(1, 2), 4)
```

```
[22]: matrix([[ 0.9689+0.2474j,  0.    +0.j    ,  1.4142+0.j    ,
          1.    +0.j    ],
        [ 0.    +0.j    , -0.1497-0.2171j,  0.1186+0.4645j,
          0.    +0.j    ],
        [ 1.4142+0.j    ,  0.1186+0.4645j,  1.8503-0.2171j,
          1.4142+0.j    ],
```

```
      [ 1.    +0.j    , 0.    +0.j    , 1.4142+0.j    ,
        0.9689+0.2474j]])
```

[ ]: _____