H.1] $D_H = D_T + D_V \quad , \dot{z} = \{z, H\} \quad , H = T(p) + V(q) \quad , z = \binom{q}{p}$

$$D_T z = \{z, T\} \quad , \quad D_V z = \{z, V\} = \sum_i \left(\frac{\partial z}{\partial q_i} \cancel{\frac{\partial V}{\partial p_i}} - \frac{\partial z}{\partial p_i} \frac{\partial V}{\partial q_i} \right)$$

$$D_T z \overset{!}{=} + \sum_i \frac{\partial z}{\partial q_i} \frac{\partial T}{\partial p_i} \qquad D_V z \overset{!}{=} - \sum_i \frac{\partial z}{\partial p_i} \frac{\partial V}{\partial q_i}$$

a) $D_T^2 z = D_T \{z, T\} = \{\{z, T\}, T\}$

$$\overset{!}{=} \sum_i \frac{\partial \{z, T\}}{\partial q_i} \frac{\partial T}{\partial p_i}$$

$$\overset{!}{=} \sum_i \frac{\partial}{\partial q_i} \left(\sum_j \frac{\partial z}{\partial q_j} \frac{\partial T}{\partial p_j} \right) \frac{\partial T}{\partial p_i}$$

$$\overset{!}{=} \sum_{ij} \left(\underset{\uparrow}{\frac{\partial^2 z}{\partial q_i \partial q_j}} \frac{\partial T}{\partial p_j} + 0 \right) \frac{\partial T}{\partial p_i} = 0$$

=0 SINCE q_i's ARE CANONICAL

$$D_V^2 z = \{\{z, V\}, V\}$$

$$\overset{!}{=} -\sum_i \frac{\partial}{\partial p_i} \left(-\sum_j \frac{\partial z}{\partial p_j} \frac{\partial V}{\partial q_j} \right) \frac{\partial V}{\partial q_i}$$

$$\overset{!}{=} \sum_{ij} \left(\underset{\uparrow \; =0}{\frac{\partial^2 z}{\partial p_i \partial p_j}} \frac{\partial V}{\partial q_j} + 0 \right) \frac{\partial V}{\partial q_i} = 0$$

b) $T = \frac{p^2}{2m} \Rightarrow D_T z = \sum_i \frac{\partial z}{\partial q_i} \frac{p_i}{m}$

$$[D_V, D_T] z = \{D_T z, V\} - \{D_V z, T\} \neq$$

$$[D_V, [D_V, D_T]] z = D_V^2 D_T z - \cancel{D_V D_T D_V z} - D_V D_T D_V z + \underbrace{D_T D_V^2 z}_{=0}$$

$$\overset{!}{=} D_V^2 D_T z - 2 D_V D_T D_V z$$

$$[D_V, D_V, [D_V, D_T]]] z = D_V^3 D_T z - 2 D_V^2 D_T D_V z - D_V^2 D_T D_V z + \underbrace{2 D_V D_T D_V^2 z}_{=0}$$

$$\overset{!}{=} D_V^3 D_T z - 3 \underbrace{D_V^2 D_T D_V z}_{\sum_{ij} \frac{\partial}{\partial q_i}\left(-\frac{\partial z}{\partial p_j} \frac{\partial V}{\partial q_j}\right) \frac{\partial T}{\partial p_i} = -\sum_{ij} \frac{\partial z}{\partial p_j} \frac{\partial^2 V}{\partial q_i \partial q_j} \underbrace{\frac{\partial T}{\partial p_i}}_{= \frac{p_i}{m}}}$$

$$D_V D_T z = -\sum_{ij} \frac{\partial z}{\partial q_j} \underbrace{\frac{\partial^2 T}{\partial p_i \partial p_j}}_{= \delta_{ij} \frac{1}{m}} \frac{\partial V}{\partial q_i}$$

$$D_V^3 D_T z = \Big\{\Big\{-\sum_i \frac{1}{m} \underbrace{\frac{\partial z}{\partial q_i}}_{=1} \frac{\partial V}{\partial q_i}, V\Big\}, V\Big\}$$

$$= -\sum_j \frac{\partial}{\partial p_j} \underbrace{\Big\{-\sum_i \frac{1}{m}\frac{\partial V}{\partial q_i}, V\Big\}}_{= \sum_{ik} \frac{\partial}{\partial p_k}\left(\frac{1}{m}\frac{\partial V}{\partial q_i}\right)\frac{\partial V}{\partial q_k}} \frac{\partial V}{\partial q_j}$$

$$= -\sum_{ijk} \frac{\partial}{\partial p_j}\left(\frac{\partial}{\partial p_k}\left(\frac{1}{m}\frac{\partial V}{\partial q_i}\right)\frac{\partial V}{\partial q_k}\right)\frac{\partial V}{\partial q_j}$$

$$= -\sum_{ijk} \frac{\partial}{\partial p_j} \frac{1}{m}\left(\cancel{\partial}\, 0 + 0\right)\frac{\partial V}{\partial q_j} = 0$$

$$D_V^2 D_T D_V z = \Big\{\Big\{-\sum_{ij} \frac{\partial z}{\partial p_j}\frac{\partial^2 V}{\partial q_i \partial q_j}\frac{p_i}{m}, V\Big\}, V\Big\}$$

$$= -\sum_k \frac{\partial}{\partial p_k} \underbrace{\Big\{-\sum_{ij}\frac{\partial z}{\partial p_j}\frac{\partial^2 V}{\partial q_i \partial q_j}\frac{p_i}{m}, V\Big\}}_{= +\sum_{ijl} \frac{\partial}{\partial p_l}\left(\frac{\partial^2 V}{\partial q_i \partial q_j}\frac{p_i}{m}\right)\frac{\partial V}{\partial q_l}} \frac{\partial V}{\partial q_j}$$

$$\underbrace{\frac{\partial}{\partial p_l}\left(\frac{\partial^2 V}{\partial q_i \partial q_j}\frac{p_i}{m}\right)}_{\cancel{=0} = \frac{\partial^2 V}{\partial q_i \partial q_j}\frac{1}{m}\delta_{il}}$$

$$= -\sum_k \frac{\partial}{\partial p_k}\left(\sum_{ij} \cancel{\frac{\partial}{\partial p_i}} \frac{\partial^2 V}{\partial q_i \partial q_j}\frac{1}{m}\frac{\partial V}{\partial q_i}\right)\frac{\partial V}{\partial q_j}$$

$$= -\sum_{ijk}\left(\frac{\partial^2 V}{\partial q_i \partial q_j}\frac{1}{m}\cdot 0 + 0\right)\frac{\partial V}{\partial q_j} = 0$$

$$\Rightarrow [D_V, [D_V, [D_V, D_T]]] = 0$$