2001PGRE

MM

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1. Vectors, Tension, Centripetal Force, Gravity, Pendulum

$$\Sigma\vec{F} = \vec{T} - m\vec{g} \tag{1}$$

Visualize the centripetal force vector supplied by the tension of the string competing with the force of gravity at each point. This yields choice (C).

2. Static Friction, Circular Motion

$$\Sigma F = \mu mg = m\frac{v^2}{r} \tag{2}$$

$$r = \frac{v^2}{\mu g} = \frac{r^2\omega^2}{\mu g} \tag{3}$$

$$r = \frac{\mu g}{\omega^2} = \frac{\mu g}{(2\pi f)^2} \tag{4}$$

Convert the frequency supplied in RPM to cycles per second and then you can compute r with the given value of mu. Which, even with approximations, yields choice (D).

3. Kepler's Third Law (Law of Periods)

$$T^2 = \frac{4\pi^2}{G(M+m)}R^3 \tag{5}$$

$$T \propto R^{3/2} \tag{6}$$

It is helpful to remember the exact form of this law, but all you really need for this one is $T^2 \propto R^3$ to choose (D).

4. Inelastic Collisions, Momentum, Energy

$$E_0 = \frac{1}{2}2mv^2 = mv^2 \tag{7}$$

$$\Delta p = 2mv = 3mv' = 0 \tag{8}$$

$$\Rightarrow v' = \frac{2}{3}v \tag{9}$$

$$E' = \frac{1}{2}3mv'^2 = \frac{3}{2}m(\frac{2}{3}v)^2 = \frac{2}{3}mv^2 = \frac{2}{3}E_0 \tag{10}$$

Find the initial kinetic energy. Use conversation of momentum to find v'. Use this to find the final kinetic energy. Compare the difference. So 1/3 of the energy is lost after the collision. Therefore, choose (C).

5. Thermal 3D Harmonic Oscillator

$$\langle E \rangle = \frac{f}{2}kT \tag{11}$$

The problem specifies three dimensions, so there are f=3 for kinetic energy, and since this is a SHO, f=3 for potential energy, thus f=6 and:

$$\langle E \rangle = \frac{6}{2}kT = 3kT \tag{12}$$

Choose (D).

6. Monoatomic Gas Expansion Isothermal vs. Adiabatic

$$W_i = nRT \ln \frac{V_2}{V_1} = P_1V_1 \ln 2 \approx 0.693 P_1V_1 \tag{13}$$

$$W_a = \frac{1}{1-\gamma}(P_2V_2 - P_1V_1) = \frac{1}{1-\gamma}(2P_2 - P_1)V_1 \tag{14}$$

$$\Rightarrow W_a = \frac{1}{1-\gamma}(\frac{2}{2^\gamma} - 1)P_1V_1 = \frac{1}{1-\gamma}(2^{1-\gamma} - 1)P_1V_1 \tag{15}$$

$$\gamma = \frac{f+2}{f} = \frac{5}{3} \tag{16}$$

$$\Rightarrow W_a = -\frac{3}{2}\left(\frac{\sqrt[3]{2}}{2} - 1\right)P_1V_1 \approx 0.555 P_1V_1 \tag{17}$$

Thus $W_i > W_a > 0$, as in (E). Note that the problem does not ask you to prove this, only to know it. Therefore, recalling the PV diagram, this question can be answered quickly by noting an isotherm covers more area than an adiabat across the same volume difference.

7. Magnetic Field Lines: The two North poles are pointing into the page, so the field lines should repel each other as in (B).

8. Induced Charge: The conducting plane will "mirror" the positive charge Q with a negative image charge of equal magnitude, -Q. Be careful with language here and choose (D).

9. Static Charge Seance

$$\sum_i E_i = k\frac{q_i}{r^2} \tag{18}$$

But here, the charges are all positive and arranged *symmetrically* around a circle of radius r, thus the field components cancel. $E = 0$ as in (A).

10. Capacitor Energy

$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{(3\mu F)(6\mu F)}{(3\mu F + 6\mu F)} = \frac{18\mu F}{9\mu F} = 2\mu F \tag{19}$$

$$U_C = \frac{1}{2}CV^2 = \frac{1}{2}(2\mu F)(300V)^2 = 0.09J \tag{20}$$

Which is choice (A).

11. Lens Equation

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \tag{21}$$

Given $f_1 = 20cm$ and $d_o = 40cm$:

$$\frac{1}{d_i} = \frac{1}{20cm} - \frac{1}{40cm} \Rightarrow d_i = 40cm \tag{22}$$

Which puts the image 10cm to the right of lens 2, note this makes its sign negative here, thus:

$$\frac{1}{f_2} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{10cm} + \frac{1}{10cm} = \frac{1}{d_i} \tag{23}$$

$$d_i = 5cm \tag{24}$$

Therefore, the final image is 5cm to the right of the second lens. Choose (A).

12. Concave Mirror:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} \tag{25}$$

But the sign of d_o is negative by convention, so:

$$\frac{1}{d_i} = \frac{1}{f} + \frac{1}{d_o} > 0 \tag{26}$$

Which implies a virtual image is formed to the right of the mirror. Note that a concave mirror forms a virtual image whenever $d_0 < f$. Choose (E).

13. Telescope Rayleigh Criterion

$$\theta = \frac{1.22\lambda}{d} \Rightarrow d = \frac{1.22\lambda}{\theta} = \frac{1.22(6 \times 10^{-7}m)}{3 \times 10^{-5}rad} = 0.244m \tag{27}$$

Therefore, a telescope with an objective of approximately 2.5cm in diameter would suffice. Choose (B).

14. Detector Range: Note that, at all times, the fraction of radiation detected is approximately equal to the ratio of the surface area of the detector to the surface area of the radiation sphere. At first, this ratio is approximately one half. We need to find the ratio once the sample is moved to 1m away. Thus:

$$A_c = \pi r^2 = \pi(0.04m)^2 = 0.005m^2 \tag{28}$$

$$S = 4\pi R^2 = 4\pi(1m)^2 = 4\pi m^2 \tag{29}$$

$$\frac{A}{S} = \frac{0.005m^2}{4\pi m^2} = 0.0000398 = 4 \times 10^{-4} \tag{30}$$

Which is choice (C).

15. Accuracy vs. Precision: Accuracy is how close each measured value is to the real value. Precision is how close each measured value is to the average measured value. Class (A) made the most precise, if not the most accurate, measurements.

16. Sampling Rate, Uncertainty

$$\Delta\sigma_x = \frac{\sigma_x}{\sqrt{N}} \tag{31}$$

$$\sigma_x = \sqrt{\langle x \rangle} \tag{32}$$

$$\langle x \rangle = \frac{\sum x_i}{N} \tag{33}$$

With the 10 second sample:

$$\langle x \rangle = \frac{\sum x_i}{N} = \frac{20}{10} = 2 \tag{34}$$

$$\sigma_x = \sqrt{2} \tag{35}$$

$$\Delta\sigma_x = \frac{\sqrt{2}}{\sqrt{10}} = \sqrt{\frac{1}{5}} \tag{36}$$

Which is higher than $\pm 1\%$. So we need:

$$\Delta\sigma_x = 0.02 = \frac{\sqrt{2}}{\sqrt{N}} \Rightarrow N = \frac{2}{0.0004} = 5,000 \tag{37}$$

Therefore, choose (D).

17. Electron Configuration: Given phosphorus has 15 electrons, and that it's in its ground state, then they fill the subshells in this order:

$$1s^2 2s^2 2p^6 3s^2 3p^3 \tag{38}$$

In general:

	l=0	l=1	l=2	l=3
n=1	$1s^2$			
n=2	$2s^2$	$2p^6$		
n=3	$3s^2$	$3p^6$	$3d^{10}$	$3f^{14}$
n=4	$4s^2$	$4p^6$	$4d^{10}$	$4f^{14}$
n=5	$5s^2$	$5p^6$	$5d^{10}$	$5f^{14}$
n=6	$6s^2$	$6p^6$	$6d^{10}$	$6f^{14}$
n=7	$7s^2$	$7p^6$	$7d^{10}$	$7f^{14}$

Therefore, choose (B).

18. Helium Ionization Energy: Use the Bohr energy formula for Z=2 protons in Helium:

$$E_n = -\frac{Z^2 m_e e^4}{8h^2\epsilon_0^2 n^2} = \frac{Z^2 E_0}{n^2} = \frac{(2)^2 13.6eV}{1^2} = 54.4eV \tag{39}$$

Which is the energy required to remove the *last* electron. We are given the total energy required to remove *both*, thus the energy required to remove the *first* electron is:

$$\Delta E = 79.0eV - 54.4eV = 24.6eV \tag{40}$$

Choose (A).

19. Solar Nuclear Fusion

$$E = \Delta mc^2 \tag{41}$$

Is the classic formula for the energy released here, but we need rather to consider the particles involved in the process. Two hydrogen atoms combining would yield the right number of protons and electrons, but no neutrons! We need two of them, so consider: ${}^1_1\mathrm{H} + {}^1_1\mathrm{H} + {}^1_1\mathrm{H} + {}^1_1\mathrm{H} \longrightarrow {}^2_2\mathrm{He}$ Where we have four protons and four electrons combining instead. This conserves mass in the nucleus, at least. But what about the extra two electrons? Well, think of Beta plus decay, or better yet, internal conversion: $\mathrm{p} + \mathrm{e}^- \longrightarrow \mathrm{n} + \nu_e$. The reality of solar fusion is a more complex chain reaction that involves fusion of free nuclei, not atoms. Regardless, we have a sufficiently analogous model to choose (B).

20. Brehmsstrahlung: Is a German word that literally translates to, "braking radiation." Any charged particle will emit radiation along a continuous spectrum proportional to the square of its acceleration. This is generally governed by the Larmor Formula:

$$P = \frac{q^2\gamma^4}{6\pi\epsilon_0 c}\left(\dot{\beta}^2 + \frac{(\beta \cdot \dot{\beta})^2}{1-\beta^2}\right) \tag{42}$$

But the definition and translation suffices to choose (E).

21. Hydrogen Spectrum

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) \tag{43}$$

So for Lyman-α:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3}{4} R_H \Rightarrow \lambda_L = \frac{4}{3R_H} \tag{44}$$

And for the Balmer-α:

$$\frac{1}{\lambda_B} = R_H \left(\frac{1}{2^2} - \frac{1}{3^3} \right) = \frac{5}{36} R_H \Rightarrow \lambda_B = \frac{36}{5R_H} \tag{45}$$

We were asked to find the ratio:

$$\frac{\lambda_L}{\lambda_B} = \frac{4}{3}\frac{5}{36} = \frac{5}{27} \tag{46}$$

Which is choice (B).

22. That's No Moon: Given a small Moon orbiting a planet, and knowing its minimum and maximum radii, as well its maximal velocity, which option can we not also deduce?

$$F_G = G\frac{Mm}{r^2} = \frac{mv^2}{r} \tag{47}$$

$$T^2 = \frac{4\pi^2}{G(M+m)} a^3 \tag{48}$$

We can calculate the semi-major axis directly from the two radial extremes $a = (r+R)/2$. So choice (E) is invalid. With an approximately circular orbit or a little work, we can use the above relationships to deduce the mass of the planet M, the speed of the Moon, and the period of the orbit. Choose (A).

23. Circular Acceleration

$$\vec{a} = \vec{a}_C + \vec{a}_T \tag{49}$$

If the particle were revolving with a constant velocity, the acceleration would be centripetal only, and the difference would be 90°. But the particle is also accelerating tangentially, so the acceleration vector is 45° between the velocity vector and the centripetal acceleration. Choose (C).

24. Kinematics: The thrown stone will have a positive, constant velocity component in the x-direction, as in II. The y-component of velocity is subject to deceleration by g, so it will decrease to zero as it rises, then fall, where its sign changes to negative. Choose (C).

25. Seven Pennies.

$$I_P = I_{CM} + mh^2 = \frac{1}{2}mr^2 + m(2r)^2 = \frac{9}{2}mr^2 \tag{50}$$

For each perimeter penny, of which there are six. Adding this to the central penny, yields:

$$I = \frac{1}{2}mr^2 + \frac{54}{2}mr^2 = \frac{55}{2}mr^2 \tag{51}$$

As in choice (E).

26. Falling Rod: The obvious starting point here is:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \tag{52}$$

But consider, the GPE depends on the center of mass, and the total kinetic energy can be treated as purely rotational if one considers the moment of inertia about the pivot point:

$$Mg\frac{L}{2} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \tag{53}$$

$$\Rightarrow Mg\frac{L}{2} = \frac{1}{6}ML^2\left(\frac{v^2}{L^2}\right) = \frac{1}{6}Mv^2 \tag{54}$$

$$Mg\frac{L}{2} = \frac{1}{6}Mv^2 \Rightarrow v = \sqrt{3gL} \tag{55}$$

Choose (C).

27. Hermitian Operators: A Hermitian operator is an operator that is defined as self-adjoint. Which means it is equal to the complex conjugate of its transpose:

$$\hat{H}^{*T} = \hat{H}^\dagger = \hat{H} \tag{56}$$

In quantum mechanics, we operate on a basis to find observable eigenvalues:

$$\hat{H}\left|\psi\right\rangle = a\left|\psi\right\rangle \tag{57}$$

But this must also mean:

$$\hat{H}^\dagger\left|\psi\right\rangle = a^*\left|\psi\right\rangle \tag{58}$$

Which leads to:

$$\left(\hat{H} - \hat{H}^\dagger\right)\left|\psi\right\rangle = (a - a^*)\left|\psi\right\rangle \tag{59}$$

Which from the definition of a Hermitian operator implies that:

$$\left(\hat{H} - \hat{H}^\dagger\right) = (a - a^*) = 0 \tag{60}$$

This means, finally, that $a = a^*$. So, the eigenvalues are equal to their complex conjugates. Which can only be true if the eigenvalues are real to begin with. Keep it real and choose (A).

28. Orthogonality of Quantum States

$$|\psi_1\rangle = 5|1\rangle - 3|2\rangle + 2|3\rangle \tag{61}$$

$$|\psi_2\rangle = |1\rangle - 5|2\rangle + x|3\rangle \tag{62}$$

We are told the three states are orthonormal, so to decide if the two quantum states are orthogonal, we need to check when their inner product is equal to zero:

$$\langle\psi_1|\psi_2\rangle = 5\langle 1|1\rangle + 15\langle 2|2\rangle + 2x\langle 3|3\rangle = 0 \tag{63}$$

$$\Rightarrow 20 + 2x = 0 \Rightarrow x = 10 \tag{64}$$

Thus, the two states are orthogonal when x=10. Choose (A).

29. Expectation Value of an Operator

$$\psi = \frac{1}{\sqrt{6}}\psi_{-1} + \frac{1}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{3}}\psi_2 \tag{65}$$

Given that the above quantum superposition corresponds to three eigenstates of an operator, its expectation value is given by:

$$\langle\hat{O}\rangle = \langle\psi|\hat{O}|\psi\rangle \tag{66}$$

$$\hat{O}|\psi\rangle = \frac{1}{\sqrt{6}}(-1)\psi_{-1} + \frac{1}{\sqrt{2}}(1)\psi_1 + \frac{1}{\sqrt{3}}(2)\psi_2 \tag{67}$$

$$\left\langle\psi\middle|\hat{O}\psi\right\rangle = -\frac{1}{6} + \frac{1}{2} + \frac{2}{3} = 1 \tag{68}$$

Thus, the expectation value is equal to one, as in (C).

30. Atomic Electron Wavefunctions

$$\int_{-\infty}^{\infty} \psi(r)^*\psi(r)dr = 1 \tag{69}$$

Any wavefunction must be square-integrable and normalizable:

$$A^2\int_0^\infty |e^{-br}|^2 dr = A^2\left(-\frac{1}{2b}e^{-2br}\right)\Bigg|_0^\infty = 1 \tag{70}$$

$$A^2\left(-\frac{1}{2b}e^{-2br}\right)\Bigg|_0^\infty = -\frac{A^2}{2b}(0-1) = 1 \tag{71}$$

$$\Rightarrow A = \sqrt{2b} \tag{72}$$

So the exponential form is normalizable and therefore valid. But:

$$A^2\int_0^\infty \sin^2(br)dr = \frac{r}{2} - \frac{\sin(2br)}{4b}\Bigg|_0^\infty \rightarrow (\infty - 0) \neq 1 \tag{73}$$

And:

$$A^2 \int_0^\infty \frac{1}{r^2} dr = -A^2 \frac{1}{r}\bigg|_0^\infty \to (0 - \infty) \neq 1 \tag{74}$$

Therefore, function I is normalizable, while functions II and III are not. Choose (A).

31. Positronium: The Bohr energy levels for Hydrogen are given by:

$$E_n = -\frac{Z^2 m_e e^4}{8h^2 \epsilon_0^2 n^2} \tag{75}$$

The Bohr formula actually approximates the reduced mass as the mass of the electron, which works when paired with a proton in Hydrogen. But for positronium:

$$\mu = \frac{m_{e^-} m_{e^+}}{m_{e^-} + m_{e^+}} = \frac{(9.11)^2 \times 10^{-62} kg}{18.22 \times 10^{-31} kg} \approx \frac{81}{18} \times 10^{-31} kg \approx \frac{1}{2} m_e \tag{76}$$

So for positronium:

$$\Delta E_n = \frac{1}{2}\left(\frac{1}{1^2} - \frac{1}{3^2}\right) 13.6eV = \frac{1}{2}\left(\frac{8}{9}\right) 13.6eV \approx 6.0eV \tag{77}$$

Therefore, choose (A).

32. Relativistic Momentum

$$E^2 = (pc)^2 + \left(mc^2\right)^2 \tag{78}$$

$$E^2 = \left(2mc^2\right)^2 = 4m^2c^4 \tag{79}$$

$$p^2c^2 + m^2c^4 = 4m^2c^4 \tag{80}$$

$$p^2c^2 = 4m^2c^4 - m^2c^4 = 3m^2c^4 \Rightarrow p = \sqrt{3}mc \tag{81}$$

Which is choice (D).

33. Pion (π) Decay

$$t_0 = 10^{-8}s \tag{82}$$

If the pion were to travel a distance of 30m in its decay time:

$$v' = \frac{x'}{t_0} = \frac{30m}{10^{-8}s} = 3 \times 10^9 m/s = 10c \tag{83}$$

Which cannot be. We know the time is dilated in the lab frame:

$$t' = \gamma t_0 \tag{84}$$

For this problem, the author counts at least three unique ways to arrive at the right answer:

$$v' = \frac{s'}{\gamma t_0} \tag{85}$$

Which can be solved for velocity, but involves rearranging gamma in an unpleasant and time-consuming way to yield:

$$v' = \sqrt{\frac{\left(\frac{x'}{t_0}\right)^2}{\left(1 + \frac{1}{c^2}\left(\frac{x'}{t_0}\right)^2\right)}} \tag{86}$$

Which, if we let $\alpha = (x'/t_0)$, simplifies to:

$$v' = \frac{\alpha}{\sqrt{\left(1 + \frac{\alpha^2}{c^2}\right)}} \tag{87}$$

Using this formula:

$$v' = \frac{3 \times 10^9 m/s}{\sqrt{101}} \approx 2.985 \times 10^8 m/s \tag{88}$$

That takes way too much time for this test, and ignores elegance. Perhaps it would be better to just use spacetime invariance:

$$\Delta S^2 = (c\Delta t)^2 - (\Delta x)^2 \tag{89}$$

So we have:

$$S^2 = (ct_0)^2 - (0)^2 = c^2(10^{-8})^2 = 9 \tag{90}$$

$$S'^2 = (ct')^2 - (30)^2 = c^2 t'^2 - 900 \tag{91}$$

$$\Rightarrow 9 = c^2 t'^2 - 900 \Rightarrow t'^2 = \frac{909}{c^2} = \frac{909}{9} \times 10^{-16} \tag{92}$$

$$\Rightarrow t' = \sqrt{101} \times 10^{-8} s \tag{93}$$

$$\Rightarrow v' = \frac{x'}{t'} = \frac{30m}{\sqrt{101} \times 10^{-8} s} \approx 2.985 \times 10^8 m/s \tag{94}$$

That is better, considering the time invariance of the test parameters. However, consider that, right away, we could have recognized that the time had to dilate by a factor of 10 in order for the pion to cross the finish line:

$$\gamma = 10 = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \sqrt{1 - \frac{1}{\gamma^2}} \tag{95}$$

$$\Rightarrow \beta = \sqrt{99} \Rightarrow v' = \sqrt{99}c \Rightarrow v' \approx 2.983 \times 10^8 m/s \tag{96}$$

One pauses to wonder here. The answer is (D), but the least rigorous method gets closest to the answer. The more rigorous methods more appropriately round to $2.99 \times 10^8 m/s$. But I suppose they are closer to $2.8 \times 10^8 m/s$ than to $3.0 \times 10^8 m/s$. Regardless, the actual *limit* is $2.998 \times 10^8 m/s$, which precludes (E).

34. Frames of Reference: We are asked when $\Delta t' = 0$ in the S' frame:

$$\Delta S'^2 = (c\Delta t')^2 - (\Delta x')^2 = 0 - \Delta x'^2 \tag{97}$$

$$\Delta S^2 = (c\Delta t)^2 - (\Delta x)^2 \tag{98}$$

$$\Delta S'^2 = \Delta S^2 \Rightarrow -\Delta x'^2 = c^2 \Delta t^2 - \Delta x^2 \tag{99}$$

$$\Rightarrow \frac{\Delta x^2}{\Delta t^2} = c^2 + \frac{\Delta x'^2}{\Delta t^2} \tag{100}$$

$$\Rightarrow v = \frac{\Delta x}{\Delta t} = \sqrt{c^2 + \frac{\Delta x'^2}{\Delta t^2}} \tag{101}$$

So assuming $\Delta x'/\Delta t > 0$, $v > c$. We could have also simply noted that $\Delta S'^2 < 0$ is a spacelike interval, where v must be greater than c. As in (C). Ha.

35. Blackbody Temperature

$$j^* \equiv \frac{P}{A} = e\sigma T^4 \tag{102}$$

A blackbody is a perfect radiator, so e=1. We are told the temperature increases by a factor of three, thus:

$$j^* = \sigma(3T)^4 = 81\sigma T^4 \tag{103}$$

Which is choice (E).

36. Adiabatic Process: (A) is true by definition, an adiabatic process means that $\Delta Q = 0$. (B) is also true of an adiabatic process:

$$S = \frac{Q_i}{T_i} = \frac{Q_f}{T_f} \tag{104}$$

And since:

$$\Delta U = -\Delta W = -\int P dV \tag{105}$$

This means that the change in internal energy is indeed described by (C), and the negative sign means that the work is done *by* the gas as in (D). So that leaves $T_i \neq T_f$. (E) is false.

37. Isothermal Cycle. Going in order here:

$$W_{AB} = P\Delta V = 200kPa(V_B - 2m^3) \tag{106}$$

$$W_{BC} = nRT \ln \frac{V_C}{V_B} = P_C V_C \ln \frac{V_C}{V_B} = 1,000 \ln \frac{2}{V_B} \tag{107}$$

$$W_{CA} = P\Delta V = 0 \tag{108}$$

And because:

$$P_B V_B = P_C V_C \Rightarrow V_B = \frac{P_C V_C}{P_B} = \frac{1,000}{200} = 5m^3 \tag{109}$$

$$W_{AB} = P\Delta V = 200kPa(5m^3 - 2m^3) = 600J \tag{110}$$

$$W_{BC} = nRT \ln \frac{V_C}{V_B} = P_C V_C \ln \frac{V_C}{V_B} = 1,000 \ln \frac{2}{5} \tag{111}$$

$$\sum W = W_{AB} + W_{BC} + W_{CA} = 600J + 1,000 \ln \frac{2}{5} J + 0J \tag{112}$$

You will ideally note that $\ln(2/5) < 0 \approx -0.9$. So $\Sigma W \approx 600J - 900J = -300J$. Alternatively, you can approximate the area enclosed by using a little symmetry and noting that the sign should be negative for a counter-clockwise cycle such as ABCA. Either way gets you to (D).

38. AC Circuit. We want the capacitance necessary to minimize the reactance of the circuit. Stated simply, we want resonance here:

$$X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow \tag{113}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{114}$$

$$\Rightarrow C = \frac{1}{\omega^2 L} == \frac{1}{(1000rad/s)^2(25 \times 10^{-3}H)} = \frac{1}{25} \times 10^{-3}F = 40\mu F \tag{115}$$

Choose (D).

39. High-Pass Filters: So let's define gain as the ratio of voltage output to voltage input:

$$G = \frac{V_{out}}{V_{in}} = I\frac{R_{out}}{R_{in}} \tag{116}$$

We was the voltage output to be minimized at low frequencies and maximized at higher frequencies. Let's check:

$$G_I = I\frac{R}{R + \omega L} \tag{117}$$

$$G_{II} = I\frac{\omega L}{R + \omega L} \tag{118}$$

$$G_{III} = I\frac{R}{R + \frac{1}{\omega C}} \tag{119}$$

$$G_{IV} = I\frac{\frac{1}{\omega C}}{R + \frac{1}{\omega C}} \tag{120}$$

Check the limits as $\omega \to 0$ and $\omega \to \infty$. This shows that only II and III are high-pass filters. Note that the other two designs are inverted versions. Rendering them low-pass filters. Choose (D).

40. Inductor Voltage: Note that the voltage across an inductor is just:

$$V_L = L\frac{dI}{dt} \tag{121}$$

And the current goes from zero to $I = V/R = 10V/2\Omega = 5A$ almost instantaneously. Which maximizes the potential difference across the inductor. But immediately after, $dI/dt \to 0$, so the voltage drops off exponentially. That eliminates 3/5 choices, and 200 seconds is far too long for this process. Choose (D).

41. Maxwell's Equations and Magnetic Monopoles: Obviously, the introduction (or discovery?) of a magnetic charge would change Gauss's Law for Magnetism (II). Less obviously, this would also introduce the possibility of a magnetic current, which would change Faraday's Law (III). Choose (E).

42. Lenz's Law and the RHR: So there is a counterclockwise current in the center loop, which is moving towards loop A, and away from loop B. The magnetic flux through A is increasing in the direction of the observer. Thus, the increasing field induces a current that resists this change in A, which must be a clockwise current. On the other hand, there is decreasing magnetic flux through B in the direction of the observer, which is harder to visualize. A current will be induced to resist this reduction in flux, which must be a counterclockwise current. This is choice (C).

43. Angular Momentum Operators. Using the identities given and the identities $[A, B] = -[B, A]$ and $[AB, C] = A[B, C] + [A, C]B$:

$$[L_x L_y, L_z] = L_x [L_y, L_z] + [L_x, L_z] L_y \tag{122}$$

$$[L_x L_y, L_z] = L_x(i\hbar L_x) + (-i\hbar L_y)L_y \tag{123}$$

$$\therefore [L_x L_y, L_z] = i\hbar\left(L_x^2 - L_y^2\right) \tag{124}$$

Which is choice (D).

44. Particle in a Box. Given the familiar:

$$\phi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \tag{125}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \tag{126}$$

And the particular case that:

$$\Psi(t=0) = \frac{1}{\sqrt{14}}\left(\phi_1 + 2\phi_2 + 3\phi_3\right) \tag{127}$$

We are asked for possible measurements of E. The superposition given only has $n = 1, 2, 3$. So, the possible energies are $E_1 = E_1$, $E_2 = 4E_1$, and $E_3 = 9E_1$. Only the latter is listed as an option, so choose (D).

45. Quantum Harmonic Oscillator. Given:

$$H|n\rangle = \hbar\omega\left(n+\frac{1}{2}\right)|n\rangle \tag{128}$$

$$|\psi\rangle = \frac{1}{\sqrt{14}}|1\rangle - \frac{2}{\sqrt{14}}|2\rangle + \frac{3}{\sqrt{14}}|3\rangle \tag{129}$$

We are asked for the expectation value of the energy, which is just:

$$\langle H\rangle = \langle\psi|H\psi\rangle = \frac{\hbar\omega}{14}\left(1^2\frac{3}{2}+2^2\frac{5}{2}+3^2\frac{7}{2}\right) = \frac{43}{14}\hbar\omega \tag{130}$$

Which is choice (C).

46. De Broglie Wavelength. Given a particle with K=E, it has an initial de Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \tag{131}$$

After entering the region of potential energy V, $E' = E - V$:

$$\lambda' = \frac{h}{\sqrt{2m(E-V)}} \tag{132}$$

In order to relate the two, notice:

$$h = \lambda\sqrt{2mE} \tag{133}$$

$$\Rightarrow \lambda' = \lambda\frac{\sqrt{2mE}}{\sqrt{2m(E-V)}} = \lambda\sqrt{\frac{E}{E-V}} \tag{134}$$

$$\therefore \lambda' = \frac{\lambda}{\sqrt{1-\frac{V}{E}}} \tag{135}$$

Which is choice (E).

47. Entropy. Given an ideal gas enclosed in a thermally-insulated system that is allowed to expand to twice its volume, what is the change in entropy?

$$dS = \frac{dQ}{T} \tag{136}$$

We can treat this as an isothermal process because there is no change in the interal energy of the system, thus:

$$dU = 0 = dQ - dW = TdS - PdV \Rightarrow S = \int\frac{dQ}{T} = \int\frac{P}{T}dV \tag{137}$$

$$PV = nRT \Rightarrow P = \frac{nRT}{V} \tag{138}$$

$$S = \frac{nRT}{T}\int_V^{2V}\frac{dV}{V} = nR\ln 2 \tag{139}$$

Which is the thermodynamic approach. There is also the statistical mechanical approach:

$$S = k_B \ln\Omega \tag{140}$$

The thing to consider here is multiplicity. Consider if the system in question were one atom confined to a volume equal to its size, then there would only be one possible microstate, which interestingly would mean that $S = k\ln 1 = 0$. This, is the third law of thermodynamics, $S = 0 \Rightarrow T = 0$. Once we raise the barrier to double the volume, there would be two possible microstates. Of course, there are actually N particles, but we can deduce that $\Omega = 2^N$ for this system. So:

$$S = k_B \ln 2^N = Nk_B \ln 2 \equiv nR\ln 2 \tag{141}$$

Rest well, dear Ludwig. The choice is (B).

48. Ratio of Root-Mean-Square Velocity. We are given $m(O2) = 32u$ and $m(N2) = 28u$:

$$v_{rms} = \sqrt{\frac{3kT}{m}} \tag{142}$$

$$\frac{v_{rms}(N2)}{v_{rms}(O2)} = \sqrt{\frac{3kT}{m_{N2}}}\sqrt{\frac{m_{02}}{3kT}} = \sqrt{\frac{32}{28}} = \sqrt{\frac{8}{7}} \tag{143}$$

Which is choice (C).

49. Maxwell-Boltzmann Partition Function. Given there are two states, each with a degeneracy of 2, of energies ϵ and 2ϵ:

$$Z = \sum_i^N g_i e^{-E_i/kT} = 2\left[e^{-\epsilon/kT} + e^{-2\epsilon/kT}\right] \tag{144}$$

Which is choice (E).

50. Frozen Notes. Given a pipe with resonant frequency of 440Hz at standard temperature, if it is cold enough that the speed of sound is 3% lower than standard, what is the new frequency? Well:

$$v = \lambda f \Rightarrow f = \frac{v}{\lambda} \tag{145}$$

We just use the fact that frequency is proportional to the speed of sound, and note that the wavelength for this instrument actually cannot change:

$$f' = \frac{v'}{\lambda_0} = (0.97)\frac{v}{\lambda} = (0.97)f_0 = (0.97)(440Hz) = 427Hz \tag{146}$$

Which is choice (B).

51. Polarizing Filters. Three polarizing filters are arranged such that each is at an angle of 45° to the next. Importantly, the first filter will reduce the intensity of *unpolarized* light by 1/2. Then, the intensity is governed by:

$$I = I_0 \cos^2 \theta \tag{147}$$

So $I_1 = I_0/2$ and then:

$$I_2 = I_1 \cos^2 (45°) = \frac{I_0}{2}\left(\frac{1}{2}\right) = \frac{I_0}{4} \tag{148}$$

$$I_3 = I_2 \cos^2 \theta = \frac{I_0}{4} \cos^2 (45°) = \frac{I_0}{4}\left(\frac{1}{2}\right) = \frac{I_0}{8} \tag{149}$$

Which is the maximum intensity allowed by this system and choice (B). Note that the wording is intentionally misleading you. I suggest drawing a picture to clarify.

52. Body-Centered Cubic Bravais Lattice. To keep this as simple as possible, there are three types of cubic unit cells: cubic, body-centered, and face-centered. Just remember that the volumes of their primitive unit cells are equal to $V = a^3/N$, where N is the number of atoms per unit cell for each type, which goes $N = 1, 2, 4$, respectively. Here $V = a^3/2$. Which is choice (C).

53. Undoped Semiconductor Temperature. Let's review them all. *Metals* are our usual *conductors* and their resistivity (ρ) rises with T. Most *insulators* are the inverse, and their resistivity falls with T. *Superconductors* are conductors with a similar dependence on T above a certain point, called the *critical temperature*, but at that point their resistance falls abruptly to zero. *Semiconductors* start off with a relatively high resistance that falls exponentially with temperature. (B) matches this latter definition.

54. Impulse.

$$I = \int F dt \tag{150}$$

But this is made easy for us since we can just deduce the area of the graph geometrically to find the impulse: $1/2(2N \cdot 2s) = 1/2(4N \cdot s) = 2kg \cdot m/s$. Which is choice (C).

55. Inelastic Collision. One particle of mass m moving in the x-direction collides with a particle of mass 2m, initially at rest, which then divides into two particles of mass m that travel along parallel to the x-direction at equal angles to the initial velocity of the first particle. Then:

$$mv = mv_x = 2mv'_x \Rightarrow v'_x = \frac{v_x}{2} \tag{151}$$

But clearly, the velocity of each particle has a non-zero y-component, so:

$$|\vec{v}| = \frac{v_x}{2} + v_y > \frac{v}{2} \tag{152}$$

56. Balloon Ride. We want to lift a mass of 300kg with a helium balloon. We're given that $\rho_A = 1.29kg/m^3$ and $\rho_H = 0.18kg/m^3$. To figure out how much helium we need, we use Archimedes' principle:

$$F_B = F_D = \rho g V \tag{153}$$

$$W = mg = \rho_A g V \Rightarrow V = \frac{m}{\rho_A} = \frac{300kg}{1.29lkg/m^3} = 233m^3. \tag{154}$$

Gives the volume of helium required to displace enough air to lift the weight, but it also must carry its own weight:

$$m_H g = \rho_H g V \Rightarrow m_H = \rho_H V_H = (0.18kg/m^3)(233m^3) = 42kg \tag{155}$$

$$V = \frac{m}{\rho_A} = \frac{42kg}{1.29kg/m^3} = 33m^3 \tag{156}$$

$$V_T = 233m^3 + 33m^3 = 266m^3 \approx 270m^3 \tag{157}$$

Of course one then asks, but what about the weight of *that* volume? Well, we could continue, but the successive terms start dropping off rapidly enough that what we already have is a good approximation. Choose (D).

57. Up Against The Wall. The water is flowing into the wall and then outward across its surface. We are asked for the form of the force against the wall. Well, this looks like a job for Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \tag{158}$$

We're basically looking at a top-down view of a laminar flow where $h_1 = h_2$, so we can get rid of the potential energy terms, and note that $P_1 = P_2$:

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = 0 \tag{159}$$

$$P = \frac{1}{2}\rho v^2 = \frac{F}{A} \Rightarrow F = \frac{1}{2}\rho v^2 A \tag{160}$$

Which is of the same form as (A). Also note that dimensional analysis will work on this problem.

58. Proton Pinball. A proton is accelerated by a potential difference of V giving it a velocity in the +z-direction. There is a uniform E in the +x-direction and an uniform B in the +y-direction. The trajectory is not affected. How would it be affected if the potential difference were increased to 2V?

$$F_L = e\left(\mathbf{v} \times \mathbf{B} + \mathbf{E}\right) = evB\sin\theta(\hat{z} \times \hat{y}) + eE\cos\theta\hat{x} = 0 \tag{161}$$

$$\Rightarrow -vB\hat{x} = E\hat{x} \tag{162}$$

But if the proton accelerates through 2V first:

$$\frac{1}{2}mv^2 = e(2V) \Rightarrow v^2 = \frac{4eV}{m_e} \Rightarrow v' = 2v \tag{163}$$

$$\Rightarrow -2vB\hat{x} > E\hat{x} \tag{164}$$

Therefore there is a net acceleration in the -x direction, as in choice (B).

59. LC Circuit.

$$L\frac{d^2Q}{dt^2} + \frac{1}{C}Q = 0 : m\frac{d^2x}{dt^2} + kx = 0 \tag{165}$$

Therefore, self-inductance is analogous to inertia, capacitance is analogous to the inverse of a spring constant, and charge is analogous to position. Choose (B).

60. Gauss's Law.

$$\oint E \cdot dS = \frac{1}{\epsilon_0}\int \rho dV \tag{166}$$

The electric flux through the surface is equal to the charge enclosed by the volume, which here is equal to the surface charge density on the surface area of the circle where the sphere intersects with the Gaussian sphere:

$$\oint E \cdot dS = \phi_E = \frac{1}{\epsilon_0}\int \rho dV = \frac{1}{\epsilon_0}\int \sigma dA \tag{167}$$

But we have to find the radius of that circle from the geometry:

$$R^2 = x^2 + \ell^2 \Rightarrow \ell^2 = R^2 - x^2 \tag{168}$$

$$\Rightarrow \phi_E = \frac{1}{\epsilon_0}\int \sigma dA = \frac{\sigma}{\epsilon_0}\pi\ell^2 \tag{169}$$

$$\therefore \phi_E = \frac{\pi\left(R^2 - x^2\right)\sigma}{\epsilon_0} \tag{170}$$

As in choice (D).

61. Through the Looking Glass. Here we have an electromagnetic plane wave incident on a *perfect* conductor. We're given:

$$E = E_0 \cos(kx - \omega t) \tag{171}$$

Suppose this component is polarized in the +y-direction, while the B field is polarized in the +z-direction. Then we can model this problem with a Poynting vector in the +x-direction:

$$\vec{S} = \frac{1}{\mu_0}E\hat{y} \times B\hat{z} \tag{172}$$

Because this a perfect conductor, the Poynting vector is reflected:

$$\vec{S}_R = -\vec{S}_I = -\frac{1}{\mu_0} E\hat{y} \times B\hat{z} \tag{173}$$

Then the incident wave interferes with the reflected wave:

$$S_T = S_I + S_R = \frac{1}{\mu_0}\left[(E_0 \cos{(kx - \omega t)}\hat{y} - E_0 \cos{(kx - \omega t)}\hat{y}) + (B\hat{z} + B\hat{z})\right] \tag{174}$$

$$\Rightarrow S_T = 0\hat{y} + 2B\hat{y} = 2B\hat{z} \tag{175}$$

And we are really only looking at x=0, and recalling that $c = E/B$:

$$S_T(x=0) = 2B\hat{z} = \left(\frac{2E_0}{c}\right)\cos{(\omega t)}\hat{z} \tag{176}$$

So, we've found that at the left of the conductor approaching $x = 0$, that $E = 0$ and $B = 2E/c$. Which is choice (C).

62. Cyclotron. Given $q = 2e$, $B = \pi/4$, and $f = 1,600Hz$. If the particle moves in a cyclotron trajectory in this magnetic field, what is its mass?

$$F = \frac{mv^2}{r} = qv \times B = qvB \Rightarrow m\omega = qB \tag{177}$$

$$\Rightarrow m = \frac{qB}{\omega} = \frac{2eB}{2\pi f} = \frac{e(\frac{\pi}{4}T)}{\pi f} = \frac{e \cdot T}{4f} = \frac{(1.6 \times 10^{-19} C\dot{T})}{4(1,600Hz)} \tag{178}$$

$$\therefore m = 2.5 \times 10^{-23} kg \tag{179}$$

Which is (A).

63. Wien Displacement Law.

$$\lambda_{Max} = \frac{b}{T} = \frac{0.0029m \cdot K}{T} \tag{180}$$

We can approximate the peak wavelength from the graph here as $2\mu m$, so:

$$T = \frac{0.0029m \cdot K}{\lambda} = \frac{0.0029m \cdot K}{2\mu m} = 1,500K \tag{181}$$

Which is choice (D).

64. Electromagnetic Radiation. We're asked which choice is false. (A) seems questionable, since we'd expect nuclear phenomena to emit higher energy radiation like x-rays or gamma. (B) This is fundamentally true, since the energy absorbed and emitted by the electrons in atoms is quantized. (C) True. (D) True. (E) Molecules have a lower "resolution" than atoms, which have lines, not bands. Choose (A).

65. Einstein Heat Capacity.

$$C = 3kN_A \left(\frac{h\nu}{kT}\right)^2 \frac{e^{h\nu/kT}}{\left(e^{h\nu/kT} - 1\right)^2} \tag{182}$$

We're asked what happens to this formula at high temperatures, but taking the limit of this formula directly leads to an indeterminate form. For small x we can approximate the exponential terms with $e^x \approx 1 + x$:

$$C = 3kN_A \left(\frac{h\nu}{kT}\right)^2 \frac{1 + \frac{h\nu}{kT}}{\left(\frac{h\nu}{kT}\right)^2} = 3kN_A \left(1 + \frac{h\nu}{kT}\right) \tag{183}$$

$$\lim_{T\to\infty} 3kN_A \left(1 + \frac{h\nu}{kT}\right) = 3kN_A \tag{184}$$

Which is choice (D).

66. Decay Rates. In general:

$$\frac{d\Gamma}{dt} = -k\Gamma \Rightarrow \frac{d\Gamma}{\Gamma} = -kdt \Rightarrow \ln\left(\frac{\Gamma}{\Gamma_0}\right) = -kt \Rightarrow \Gamma(t) = \Gamma_0 e^{-kt} \tag{185}$$

We want:

$$\frac{\Gamma}{\Gamma_0} = \frac{1}{2} = e^{-kt} \Rightarrow \ln\frac{1}{2} = -kt \Rightarrow k = -\frac{\ln(1/2)}{t} \tag{186}$$

$$\therefore \frac{d\Gamma}{dt} = -k\Gamma = -\frac{\ln(1/2)}{t}\Gamma \tag{187}$$

We simply sum the decay rates to get the total:

$$\frac{d\Gamma}{dt} = \frac{d\gamma}{dt} + \frac{d\beta}{dt} = \frac{\ln(1/2)}{24}\gamma + \frac{\ln(1/2)}{36}\beta = \frac{\ln(1/2)}{t}\Gamma \tag{188}$$

From which we can solve for the total decay time:

$$\frac{1}{24} + \frac{1}{36} = \frac{1}{t} \Rightarrow t = \frac{864}{60} = 14.4min \tag{189}$$

Which is choice (D).

67. Nuclear Binding Energy. Given that ^{238}U has a binding energy of $\epsilon \equiv 7.6MeV$ per nucleon, and it undergoes fission leaving each half with $100MeV$ of kinetic energy each, what is the binding energy per nucleon, x, of the daughter nuclei?

$$E_i = -U_i = -N\epsilon = -240(7.6MeV) \tag{190}$$

$$E_f = -U_f + T = -Nx + 200MeV \tag{191}$$

$$\Rightarrow -U_f = -U_i - T \Rightarrow -Nx = -N\epsilon - T \tag{192}$$

$$\Rightarrow x = \epsilon + \frac{T}{N} = 7.6MeV + \frac{200MeV}{240} = 8.4MeV \tag{193}$$

Which is closest to choice (E).

68. Nuclear Decay Processes. Given:

$$^{7}_{4}\text{Be} \rightarrow ^{7}_{3}\text{Li} \tag{194}$$

An alpha decay (A) would look like:

$$^{7}_{4}\text{Be} \rightarrow ^{3}_{2}\text{He} + ^{4}_{2}\text{He}^{+} \tag{195}$$

That's not it. Then we consider if it emits an electron (B), neutron (C), or positron (D). Well, the nucleus has lost a neutron, but it's gained a proton. So (C) is out. Let's review the two types of β decay:

$$\beta^{-} : n \rightarrow p + e^{-} + \bar{\nu}_e \tag{196}$$

$$\beta^{+} : p \rightarrow n + e^{+} + \nu_e \tag{197}$$

An electron is emitted in β^{-} decay, which would convert the neutron to a proton, but there would be an antineutrino. Emitting an electron only implies ionization of the atom, not nuclear decay. So it's not (B). A positron could only be emitted in β^{+}, and with a neutrino, but that converts a proton into a neutron. So it's not (D). By deduction, we know it must be (E), which looks like this:

$$p + e^{+} \rightarrow n + \nu_e \tag{198}$$

69. Thin-Film Interference. It's important to recognize that the phase of the light incident on the glass slide will shift by 180°, because the glass has a higher index of refraction than the oil. This means that we want to use the following form for *constructive* interference:

$$2t\cos\theta = \frac{m\lambda}{n} \tag{199}$$

We're told that the maximum strength of reflection occurs near normal incidence, so let $\theta = 0$. Then:

$$2t = \frac{480nm}{1.2} = 400nm \Rightarrow t = 200nm \tag{200}$$

Which is choice (B)

70. The Double Slit Experiment. We're asked to determine what happens to the spacing between fringes if the frequency of the light is doubled. Let's review that for a double slit, the maximum condition (it's the opposite for a single-slit) is given by:

$$d\sin\theta = m\lambda \tag{201}$$

And that if the distance between the slits and the "detector," D, is much greater than the distance between the slits, d, then:

$$\sin\theta \approx \tan\theta \approx \theta \approx \frac{y}{D} \tag{202}$$

Where y is the displacement of the fringes from the center axis. Which yields:

$$d\sin\theta = d\frac{y}{D} = m\lambda \Rightarrow y = \frac{m\lambda D}{d} \tag{203}$$

Which shows us that if we double the frequency:

$$y = \frac{m\lambda D}{d} = \frac{mcD}{fd} \tag{204}$$

We will reduce the spacing between fringes by a factor of 2, so 1.0mm is divided in half to 0.5mm, as in choice (B).

71. Relativistic Doppler Shift. Given $\lambda_S = 121.5nm$ and that $\lambda_R = 607.5nm$. What is the celestial object's velocity relative to an observer on Earth?

$$\lambda_R = \lambda_S\sqrt{\frac{1+\beta}{1-\beta}} \tag{205}$$

$$\frac{\lambda_R}{\lambda_S} = \frac{607.5nm}{121.5nm} \approx 5 = \sqrt{\frac{1+\beta}{1-\beta}} \tag{206}$$

$$\Rightarrow \beta = \frac{12}{13} \Rightarrow v = \frac{12}{13}c \approx 2.8\times 10^8 m/s \tag{207}$$

Since the wavelength of observed light is lengthened, we say that it is *redshifted*, which means it is moving away from us. This is choice (D).

72. Masses on a Spring on a String. Let's call the top block A and the bottom one B, and their masses are equal. At first, the system is in a static equilibrium described by:

$$\Sigma F_A = T - mg - kx = 0 \tag{208}$$

$$\Sigma F_B = kx - mg = 0 \tag{209}$$

From the state of B, we see that $kx = mg$. Then, when the string is cut, the tension goes to zero:

$$\Sigma F_A = -mg - kx = -2mg = ma_A \Rightarrow a_A = -2g \tag{210}$$

The problem defined the downwards direction as positive, but the magnitude is the same as choice (E).

73. Static Friction. Given $m_A = 16.0kg$, $m_B = 4.0kg$, and $\mu = 0.5$ We need the static friction against block B to be equal to at least its weight:

$$\Sigma F_{By} = fr - W = \mu N - m_B g = 0 \tag{211}$$

$$\mu m_B a_x = m_B g \Rightarrow a_x = \frac{g}{\mu} = 20m/s^2 \tag{212}$$

So we know the required acceleration in the x-direction in order achieve a normal force that will produce enough friction to balance weight. But in order to produce this acceleration, the force F needs to be:

$$F = (m_A + m_B)a_X = (20kg)(20m/s^2) = 400N \tag{213}$$

Which is choice (D).

74. Langrangian Mechanics. Given the Lagrangian:

$$L = a\dot{q}^2 + bq^4 \tag{214}$$

We can derive the equation of motion using the Euler-Lagrange equation:

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) \tag{215}$$

$$\Rightarrow 4bq^3 = 2a\ddot{q} \Rightarrow \ddot{q} = \frac{2b}{a}q^3 \tag{216}$$

The greatest danger here is falling for the sign trap in choice (C). Because we know $L = T - V$, but take it as given, and think of it as $L = T-(-V) = T + V$. Choose (D).

75. Matrix Revolutions... Given:

$$\begin{pmatrix} a'_x \\ a'_y \\ a'_z \end{pmatrix} = \begin{bmatrix} 1/2 & \sqrt{3}/2 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} \tag{217}$$

We're told that this is a linear transformation of a vector in frame S to that same vector corresponding to a new basis in frame S'. We're asked how this transforms the *reference frame*, not the vector itself. Well, technically, we should recognize that this as a rotation of coordinates of the form:

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{218}$$

If you recall this then you can just compare the matrices and note that this transformation maps to the unit circle as such:

$$(x, y) = (r\cos\theta, r\sin\theta) : (1/2, -\sqrt{3}/2) \tag{219}$$

Which represents a 60° clockwise rotation *of the vector*. Therefore, the reference frame rotates 60° counterclockwise. Alternatively, if you don't recognize that, you can deduce this same fact if you apply the transformation given to a simple vector such as (1,0,0). Finally, suppose you don't know about the transformation form above and that you forget the unit circle. Well, take a unit vector (1,1,0) in the xy-plane and notice that:

$$(a'_x, a'_y, a'_z) = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)\hat{a_x} + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)\hat{a_y} + (1)\hat{a_z} \approx 1.4\hat{a_x} - 0.4\hat{a_y} + 0\hat{a_z} \tag{220}$$

Applying this to your unit vector, you again notice it rotates clockwise by about 60°. Which, again, means that the *reference frame* rotates counterclockwise. Therefore, the choice is (E).

76. Conduction Electrons. Why is the average kinetic energy of electrons in metals generally much higher than KT? (A) assuming both were free, electrons and atoms would have the same degrees of freedom. Some atoms would have higher degrees of freedom that wouldn't apply to electrons. Our electrons are bound to the metal, so no. (B) Generally, the electrons in a metal are at approximately the same temperature as the metal itself. (C) Fermi was a smart guy, and electrons are Fermions, this sounds promising. (D) We shouldn't need to calculate the drift velocity here to know that it is probably much less than c, which it usually is. (E) They may or may not interact strongly, but that doesn't tell us how much kinetic energy they have here. Going back to (C), electrons do form a degenerate Fermi gas in metals when they are below the Fermi temperature, which is much higher than room temperature for metals, on the order of 10^4K. This means that the average energy of the conduction electrons is close to the Fermi energy $\mathcal{E}_F = k_B T_F$. Which is much higher than kT at room temperature. So the choice is (C).

77. Maxwell-Boltzmann Ratio. Given an ensemble of systems with two possible states at thermal equilibrium at $kT = 0.0025eV$, state A has a 0.1eV higher energy than state B. What is the ratio of systems in state A to state B? Given this follows the Maxwell-Boltzmann Distribution:

$$Z = \sum_i e^{-\epsilon_i/kT} \tag{221}$$

We can arbitrarily pick two energies with a difference of $0.1eV$, and then divide them to find the ratio:

$$\frac{Z_A}{Z_B} = \frac{e^{-1.1eV/0.0025eV}}{e^{-1.0eV/0.0025eV}} = e^{-0.1/0.0025} = e^{-4} \tag{222}$$

Which is choice (E).

78. Muon Decay. Given that a muon decay does look like this:

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e \tag{223}$$

We're asked why it cannot decay into just an electron and a single (unspecified) neutrino. Muons, electrons, and neutrinos are leptons, and each family of leptons is conserved independently. So the number of each type of lepton is conserved: $\Delta L_e = 0$ and $\Delta L_\mu = 0$. Where particles have a value of 1 or -1 for antiparticles. If the muon decays into a single electron and one of *any* of the four types of relevant neutrinos, lepton conservation would be violated. So the choice is (E).

79. Relativistic Energy-Momentum. Given that a particle has a total energy of $10GeV$ and a momentum of $8GeV/c$, we want to find its rest mass. Well, we can just plug the numbers into the standard formula:

$$E^2 = (pc)^2 + (mc^2)^2 \tag{224}$$

$$(10GeV)^2 = (8GeV)^2 + (mc^2)^2 \Rightarrow m = 6GeV/c^2 \tag{225}$$

Which is choice (D).

80. Lorentz Refraction. Given that a tube of water is moving at $1/2c$, and that the index of refraction in water is $n = 4/3$, what is the velocity of light that travels through the tube relative to the lab frame? Well first, the light is refracted in the water so that:

$$u = \frac{c}{n} = \frac{c}{4/3} = \frac{3c}{4} \tag{226}$$

So the observed velocity u' of the light moving at u in its rest frame, traveling at a relative velocity v, is given by:

$$u' = \frac{u+v}{1+\frac{uv}{c^2}} = \frac{(3c/4)+(c/2)}{1+\frac{(3c/4)(c/2)}{c^2}} = \frac{10}{11}c \tag{227}$$

Which is choice (D).

81. Orbital Angular Momentum Eigenstates. Given the eigenfunction is of the form $Y_\ell^m(\theta,\phi)$, and has eigenvalues of $6\hbar^2$ and $-\hbar$ for the operators L^2 and L_z, respectively, we're asked to choose which state matches. For the total angular momentum operator, that means:

$$L^2\psi = 6\hbar^2\psi \tag{228}$$

This must mean that:

$$L^2 = \hbar^2\ell(\ell+1) = 6\hbar^2 \tag{229}$$

Which implies that $\ell = -3$ or $\ell = 2$. Similarly, for the projection:

$$L_z\psi = -\hbar\psi \tag{230}$$

Where $L_z = m_\ell\hbar$. This implies that $m = -1$. The only choice that fits these parameters is (B).

82. Spin Triplet Eigenstate. Given that $|\alpha\rangle \equiv |\uparrow\rangle$ and $|\beta\rangle \equiv |\downarrow\rangle$, which are valid eigenfunctions of a two-electron atom in a triplet state (^3S)?

$$\text{I. } |\alpha\rangle_1 |\alpha\rangle_2 \tag{231}$$

$$\text{II. } \frac{1}{\sqrt{2}} \left(|\alpha\rangle_1 |\beta\rangle_2 - |\alpha\rangle_2 |\beta\rangle_1 \right) \tag{232}$$

$$\text{III. } \frac{1}{\sqrt{2}} \left(|\alpha\rangle_1 |\beta\rangle_2 + |\alpha\rangle_2 |\beta\rangle_1 \right) \tag{233}$$

Note that the spectroscopic notation for the triplet state:

$$^3S \Rightarrow 2s+1 = 3 \Rightarrow s = 1 \tag{234}$$

One might recall that there are *three* allowed triplet combinations, in $|sm\rangle$ notation:

$$|1+1\rangle = \uparrow\uparrow \tag{235}$$

$$|10\rangle = \frac{1}{\sqrt{2}} \left(\uparrow\downarrow + \downarrow\uparrow \right) \tag{236}$$

$$|1-1\rangle = \downarrow\downarrow \tag{237}$$

And *one* allowed singlet combination:

$$|00\rangle = \frac{1}{\sqrt{2}} \left(\uparrow\downarrow - \downarrow\uparrow \right) \tag{238}$$

Where the singlet state is antisymmetric and the triplet states are symmetric. If not, one can apply a spin ladder operator to each state, where if it is a triplet state, it will give you another triplet state, and if it is a singlet state, it will give you zero.

$$S_- \equiv \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \tag{239}$$

$$S_+ \equiv \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{240}$$

$$|\uparrow\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{241}$$

$$|\downarrow\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{242}$$

In doing so, one must apply the operator to each each particle in each term in turn, for example:

$$S_- |\alpha\rangle_1 |\alpha\rangle_2 = S_-^{(1)} |\alpha\rangle_1 |\alpha\rangle_2 + |\alpha\rangle_1 S_-^{(2)} |\alpha\rangle_2 \tag{243}$$

$$\Rightarrow |\beta\rangle_1 |\alpha\rangle_2 + |\alpha\rangle_1 |\beta\rangle_2 \tag{244}$$

So state I becomes state III. Applying the raising operator to state III yields state I, and the lowing operator gives its mirror state $|\downarrow\rangle\,|\downarrow\rangle$. Applying either ladder operator to state II, however, yields zero. So I and III are triplet states and II is the singlet state, as in choice (D).

83. Pauli Matrices. Given that $-\frac{1}{2}\hbar$ is an eigenvalue of the operator S_x, we are asked which state is the corresponding eigenfunction. Given the Pauli matrix:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{245}$$

This matrix actually defines the S_x operator:

$$S_x = \frac{\hbar}{2}\sigma_x \tag{246}$$

We're presented states in arrow notation, but the matrix operator works best in spinor notation, where:

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{247}$$

$$|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{248}$$

Operating on either of these independent states with this matrix will not yield the factor of -1 we need in the eigenvalue. So it must be a linear combination, and it's definitely not (A). Also, there are no imaginary numbers in this matrix (there are in σ_y), so it's probably not (D) or (E). That leaves (B):

$$|\uparrow\rangle + |\downarrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{249}$$

Or (C):

$$|\uparrow\rangle - |\downarrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{250}$$

Operating on (B) won't change it (prove this to yourself). Let's see what happens to (C):

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} = -\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{251}$$

Indeed, state (C) is the eigenfunction here that corresponds to an eigenvalue of $-\hbar/2$ for the S_x operator, or more simply, -1 for the σ_x matrix. Therefore, choose (C).

84. Electric Dipole Transitions. There is, derived from perturbation theory, a set of rules about the allowed transitions here:

$$\Delta\ell = \pm 1 \tag{252}$$

$$\Delta m_\ell = 0, \pm 1 \tag{253}$$

$$\Delta j = 0, \pm 1 \tag{254}$$

Transition A has $\Delta\ell = 0$, which is verboten. Transitions B and C have $\Delta\ell = -1$, which is allowed. B has $\Delta j = -1$, and C has $\Delta j = 0$. These are both allowed. Therefore, choose (D).

85. Nichrome Wire Voltage Divider. We want to know the voltage at the junction of two wires, the first twice as long as the second, and the second twice as thick as the first. There is an $8V$ potential applied at the end of the first, and $1V$ at the second. We can calculate the relative resistance of each section with:

$$R = \frac{\rho L}{A} \tag{255}$$

$$R_1 = \frac{2\rho L}{A} \tag{256}$$

$$R_2 = \frac{\rho L}{2A} \tag{257}$$

$$R_T = R_1 + R_2 = \frac{5\rho L}{2A} \tag{258}$$

Which further yields:

$$I = \frac{\Delta V}{R_T} = \frac{7V}{(5\rho L/2A)} = \frac{14VA}{5\rho L} \tag{259}$$

One way to get the voltage at the junction is take the voltage drop across the "first" wire, and subtract it from the starting potential:

$$V_1 = IR_1 = \frac{14VA}{5\rho L}\frac{2\rho L}{A} = \frac{28}{5}V \tag{260}$$

$$V_j = 8V - \frac{28}{5}V = 2.4V \tag{261}$$

This is the answer we want, which is choice (A). Note that we could have also used the voltage divider equation directly, but we would have to apply it twice, once from each end, because this circuit is not grounded, and you wind up with $V_j = 2.24V$. Presumably this is why they use the phrase, "most nearly."

86. Rotating Wire Coil. The normal vector to the surface area of the coil is initially facing in the y-direction, so the magnetic field is given by:

$$B_0 \sin \omega t \tag{262}$$

And the current will be given by:

$$I = \frac{\mathcal{E}}{R} = \frac{N}{R}\frac{d\Phi_B}{dt} \tag{263}$$

$$\frac{N}{R}\frac{d\Phi_B}{dt} = \frac{N}{R}B_0\pi r^2\frac{d}{dt}\sin\omega t = \frac{N}{R}B_0\pi r^2\omega\cos\omega t \tag{264}$$

Plugging in the numbers yields:

$$\frac{15}{9\Omega}(0.5T)\pi(0.01m)^2\cos\omega t \approx 0.0225\cos\omega t A \tag{265}$$

Which, in milliamps, is closest to choice (E).

87. Gaussian Spheres. The charge q is enclosed in one of two Gaussian shells, each with charge Q on their surface. We want the force on the charge q. First, note that the force from the shell enclosing it will cancel by symmetry. Then we just need the force on q from the other shell, as a function of distance:

$$F_C = k\frac{qQ}{r^2} = k\frac{qQ}{(10d - d/2)^2} = k\frac{qQ}{(19d/2)^2} \tag{266}$$

$$\Rightarrow F_C = k\frac{qQ}{(361/4)d^2} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{(361/4)d^2} = \frac{qQ}{361\pi\epsilon_0 d^2} \tag{267}$$

All of the charges are positive, so the net force will be to the left. Which is choice (A).

88. Biot-Savart Law. The curved arc of wire is will contribute to the magnetic field at P, according to:

$$B = \int\frac{\mu_0 I d\ell \times \hat{r}}{4\pi r^2} \tag{268}$$

Where $d\ell = R\theta$ is the arc length of the curved wire. This length is constant, and always perpendicular to the radial vector, so:

$$B = \int\frac{\mu_0 I d\ell \times \hat{r}}{4\pi r^2} = \frac{\mu_o I\ell}{4\pi r^2} = \frac{\mu_0 IR\theta}{4\pi R^2} = \frac{\mu_0 I\theta}{4\pi R} \tag{269}$$

Which is choice (C).

89. Merry-Go-Round. We want the final angular velocity of the system once the foundling, initially at the edge of the merry-go-round, has walked into the center. We just need to use conservation of momentum and calculate the moments of inertia:

$$L_0 = (I_{MG} + I_C)\,\omega_0 \tag{270}$$

$$I_{MG} = \frac{1}{2}Mr^2 = \frac{1}{2}(200kg)(2.5m)^2 = 625kgm^2 \tag{271}$$

$$I_C = mr^2 = (40kg)(2.5m)^2 = 250kgm^2 \tag{272}$$

Once the child is at the center, $r = 0$, so his moment of inertia disappears. Then:

$$L_0 = L_F \Rightarrow \omega' = \frac{I_{MG} + I_C}{I_{MG}}\omega_0 = \frac{875kgm^2}{625kgm^2}(2.0rad/s) = 2.8rad/s \tag{273}$$

Which is choice (E).

90. Series vs. Parallel Springs. We have two identical masses on springs, the first on two identical springs in parallel, the second on identical springs attached in series. We're asked for the ratio of the period of the first to the period of the second. Starting with each system's potential energy:

$$U_1 = \frac{1}{2}2kx^2 = kx^2 \tag{274}$$

$$U_2 = \frac{1}{2}2k(x/2)^2 = \frac{1}{4}kx^2 \tag{275}$$

Then:

$$F_1 = \frac{dU_1}{dx} = 2kx = m\ddot{x} \tag{276}$$

$$F_2 = \frac{dU_2}{dx} = \frac{1}{2}kx = m\ddot{x} \tag{277}$$

Which means that:

$$\omega_1 = \sqrt{\frac{2k}{m}} = 2\pi f_1 = \frac{2\pi}{T_1} \tag{278}$$

$$\omega_2 = \sqrt{\frac{k}{2m}} = 2\pi f_2 = \frac{2\pi}{T_2} \tag{279}$$

$$T_1 = 2\pi\sqrt{\frac{m}{2k}} \tag{280}$$

$$T_2 = 2\pi\sqrt{\frac{2m}{k}} \tag{281}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{2\pi}{2\pi}\sqrt{\frac{m}{2k}}\sqrt{\frac{k}{2m}} = \sqrt{\frac{1}{4}} = \frac{1}{2} \tag{282}$$

Which is choice (A).

91. Rotational Inertia. Given that the cylinder has mass M, heigh H, and radius R, and its final translation velocity given by $v = \sqrt{8gH/7}$, we want the moment of inertia of the cylinder. We can use conservation of energy, where:

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega \tag{283}$$

$$\omega = \frac{v}{R} \tag{284}$$

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\frac{v}{R} \tag{285}$$

$$MgH = \frac{1}{2}M\frac{8gH}{7} + \frac{1}{2}I\frac{8gH}{7R^2} = \frac{4}{7}MgH + \frac{4}{7}gH\frac{I}{R^2} \tag{286}$$

$$\Rightarrow M - \frac{4}{7}M = \frac{4}{7}\frac{I}{R^2} = \frac{3}{7}M \tag{287}$$

$$\Rightarrow I = \frac{7}{4}\frac{3}{7}MR^2 = \frac{3}{4}MR^2 \tag{288}$$

Which is choice (B).

92. Spring Hamiltonian. Two equal masses are attached to either end of a spring and we are asked for the Hamiltonian of the system. Of course, $H = T + U$, and $T = p^2/2m$. The only trick here is recognizing that there is only one spring and thus only one source of potential energy:

$$H = T + U = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}k(\ell - \ell_0)^2 \tag{289}$$

Which is choice (E).

93. Bohr Radius. Given that the ground-state radial wavefunction for hydrogen is:

$$\frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \tag{290}$$

We're asked which is the most likely radius... Mathematically, we can determine this if we note that the probability is given by:

$$P = \int |\psi|^2 dV = \frac{4\pi}{\pi a_0^3} \int e^{-2r/a_0} r^2 dr \tag{291}$$

We don't need to solve this integral, we can just accept that the probability will be proportional to its integrand, and then we can set its derivative equal to zero to find the extrema of r:

$$\frac{dP}{dr} = e^{-2r/a_0} 2r - \frac{2}{a_0} e^{-2r/a_0} r^2 = 0 \tag{292}$$

$$\Rightarrow r = a_0 \tag{293}$$

Which is, of course, choice (C). This derivation is basically the definition of the Bohr radius, which one might simply recall.

94. Perturbation Theory. We're given the raising and lowering operators for the quantum harmonic oscillator:

$$a |n\rangle = \sqrt{n} |n-1\rangle \tag{294}$$

$$a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle \tag{295}$$

And then asked for the first-order shift in the $n = 2$ energy level due to the perturbation:

$$\Delta H = V \left(a + a^\dagger\right)^2 \tag{296}$$

So the shift in energy is the expectation value of the perturbation:

$$E_2^1 = \langle n| \Delta H |n\rangle = \langle 2| V \left(a + a^\dagger\right)^2 |2\rangle \tag{297}$$

$$E_2^1 = \langle 2| V \left(a^2 |2\rangle + a a^\dagger |2\rangle + a^\dagger a |2\rangle + a^{\dagger 2} |2\rangle\right) \tag{298}$$

This could get messy, or we can recognize that the a^2 and $a^{\dagger 2}$ terms will have orthogonal bases and thus reduce to zero. Thus, this simplifies to:

$$E_2^1 = \langle 2| V \left(aa^\dagger |2\rangle + a^\dagger a |2\rangle \right) \tag{299}$$

$$aa^\dagger |2\rangle = a\sqrt{3}|3\rangle = 3|2\rangle \tag{300}$$

$$a^\dagger a |2\rangle = a^\dagger \sqrt{2}|1\rangle = 2|2\rangle \tag{301}$$

$$\Rightarrow E_2^1 = \langle 2| V (3|2\rangle + 2|2\rangle) = V (3\langle 2|2\rangle + 2\langle 2|2\rangle) \tag{302}$$

$$\therefore E_2^1 = 5V \tag{303}$$

Which is choice (E).

95. Dielectric Constant. The dielectric constant K is actually the *relative permittivity*:

$$K \equiv \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \tag{304}$$

Where ε_0 is the usual vacuum permittivity and ε is the permittivity of the medium. So the expression:

$$E' = \frac{E_0}{K} \equiv \frac{E_0}{(\varepsilon/\varepsilon_0)} = \frac{\varepsilon_0 E_0}{\varepsilon} = \frac{E_0}{\varepsilon_r} \tag{305}$$

For example:

$$\frac{E_0}{\varepsilon_r} = \frac{Q}{4\pi\varepsilon_0 R^2}\frac{\varepsilon_0}{\varepsilon} = \frac{Q}{4\pi\varepsilon R^2} \tag{306}$$

Will give us the electric field in the medium. Therefore, the choice is (A).

96. Pulsating Charged Sphere. We're asked, what is the total power radiated by a charged sphere, expanding and contracting between R_1 and R_2 at frequency f? Yes, this *is* a trick question. There is none. How do we know? Well, an accelerating electric charge will emit radiation according to the Larmor formula:

$$P = \frac{2}{3}\frac{Q^2}{4\pi\varepsilon_0 c}\left(\frac{a}{c}\right)^2 \tag{307}$$

Which is proportional to Q^2, not Q. And, one can determine that if the charge were accelerating in a cyclotron, it would be proportional to f^4, not f^2. And the ratio R_2/R_1 is irrelevant, since the electric field would rather depend on the difference. But the real problem here is symmetry, if electromagnetic radiation were radiating outward, the Poynting vector would point in the radial direction:

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \tag{308}$$

But the electric field is, at all times, pointing in the radial direction. Where would the magnetic field point? Indeed, the power per solid unit angle is given by:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c}\frac{\sin^2(\theta)a^2}{c^2} \tag{309}$$

Where θ is the angle between the acceleration vector and the observed radiation. The acceleration is only oscillatory in the radial direction, equally inwards and outwards. Therefore, by symmetry, there is no power radiated here. As in choice (E).

97. Angular Refraction. This is a more technical application of Snell's Law. We want to find $\delta\theta'$. The light has wavelength centered around λ with spread $\delta\lambda$. It ravels from a vacuum into glass with $n(\lambda)$. Alright then, the familiar:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{310}$$

Becomes:

$$(1)\sin\theta = n(\lambda)\sin\theta' \Rightarrow n(\lambda) = \frac{\sin\theta}{\sin\theta'} \tag{311}$$

The answers should clue you in that we need to take a derivative:

$$\frac{dn(\lambda)}{d\lambda} = \sin\theta(-1)\cos\theta'\sin^{-2}\theta'\frac{d\theta'}{d\lambda} = \frac{-\sin\theta\cos\theta'}{\sin^2\theta'}\frac{d\theta'}{d\lambda} \tag{312}$$

$$\Rightarrow \frac{dn(\lambda)}{d\lambda} = \frac{-\sin\theta\cos\theta'}{\sin\theta'\sin\theta'}\frac{d\theta'}{d\lambda} = \frac{-n(\lambda)}{\tan\theta'}\frac{d\theta'}{d\lambda} \tag{313}$$

$$\frac{dn(\lambda)}{d\lambda} = \frac{-n(\lambda)}{\tan\theta'}\frac{d\theta'}{d\lambda} \tag{314}$$

$$\Rightarrow \frac{d\theta'}{d\lambda} = -\frac{\tan\theta'}{n(\lambda)}\frac{dn(\lambda)}{d\lambda} \tag{315}$$

$$\therefore \delta\theta' = \left|\frac{\tan\theta'}{n(\lambda)}\frac{dn(\lambda)}{d\lambda}\delta\lambda\right| \tag{316}$$

Which is choice (E).

98. Statistical Mechanics. Supposing that a system in thermal equilibrium has a quantum state i, and has energyE_i. We're given the expression:

$$\frac{\sum_i E_i e^{-E_i/kT}}{\sum_i e^{-E_i/kT}} \tag{317}$$

And asked what it means. Well, let's say we had one state with energy ϵ:

$$\frac{\sum_i \epsilon e^{-\epsilon/kT}}{\sum_i e^{-\epsilon/kT}} = \epsilon e^0 = \epsilon \tag{318}$$

So this expression yields the energy, indeed it is of the form of the equation for the average energy (where $kT \equiv \beta$):

$$\langle E \rangle = -\frac{1}{Z}\left(\frac{\partial Z}{\partial \beta}\right) \tag{319}$$

Therefore, we can safely choose (A).

99. Pair Production. Here, a photon strikes an electron at rest, and creates an electron-positron pair in the process. If the photon is destroyed and all three leptons move off in the initial direction of the photon, what is its initial energy?

$$E^2 = (pc)^2 + (mc^2)^2 \tag{320}$$

But be careful because:

$$E_0 = E + mc^2 \Rightarrow E_0^2 = E^2 + 2Emc^2 + m^2c^4 \tag{321}$$

Where $E = pc$. But the initial momentum from the single photon will equal the total momentum of the three leptons after, so we can just say:

$$E_F^2 = E^2 + (3mc^2)^2 = E^2 + 9m^2c^4 \tag{322}$$

$$E_0^2 = E_F^2 \Rightarrow E^2 + 2Emc^2 + m^2c^4 = E^2 + 9m^2c^4 \tag{323}$$

$$\Rightarrow 2Emc^2 + m^2c^4 = 9m^2c^4 \Rightarrow 2Emc^2 = 8m^2c^4 \tag{324}$$

$$\therefore E = 4mc^2 \tag{325}$$

Which is choice (D).

100. Michelson Interferometer. we're given the set up of the experiment such that the distance d is the same for the green light and the red light, where $m_G = 100,000$ fringes are produced for the green light and $m_R = 85,865$ fringes are produced by the red light of wavelength $\lambda_R = 632.82nm$. We're asked to find the wavelength of the green light. For a Michelson interferometer, these quantities are related by:

$$d = \frac{m\lambda}{2} \tag{326}$$

So, there are actually two straightforward ways to solve this, the obvious being calculating the distance from the red light and plugging it back in to get λ_G. Or you can note that the distance is the same and use the ratio to skip calculating the distance altogether:

$$\frac{d_G}{d_R} = 1 = \frac{m_G\lambda_G/2}{m_R\lambda_R/2} \tag{327}$$

$$\Rightarrow \lambda_G = \frac{m_R\lambda_R}{m_G} = \frac{(85,865)(632.82nm)}{(10,000)} = 543.37nm \tag{328}$$

Which is choice (B).