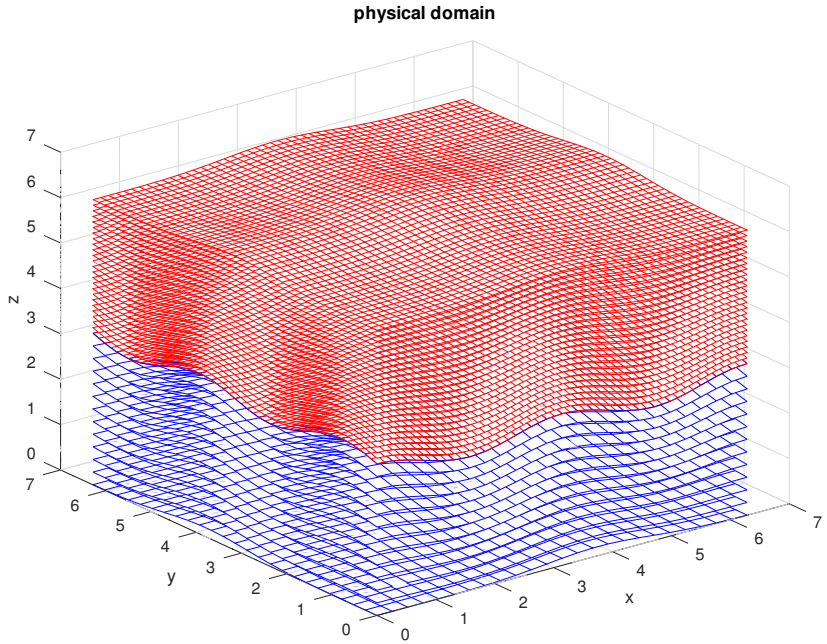
Solving Elastic Wave Equation in 3D by SBP4 with Curvilinear domain and Mesh Refinement

Lu Zhang
Mentor: Anders Petersson

CASC Division
Lawrence Livermore National Lab

Aug. 8, 2019

Geometry and space discretization



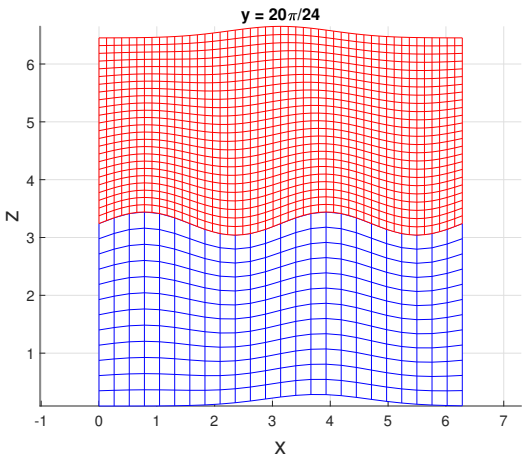
Solve:

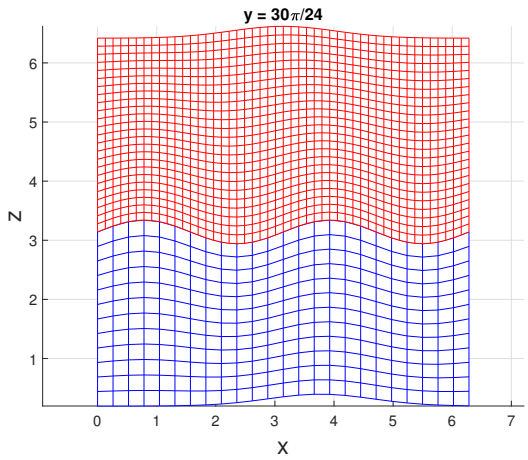
$$\rho \mathbf{u}_{tt} = \mathcal{L}\mathbf{u} + \mathbf{F},$$

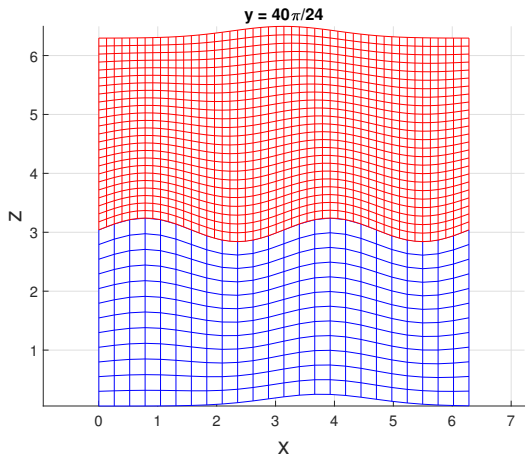
with

$$\mathcal{L}\mathbf{u} = \sum_{j=1}^{3} \partial_1(M_{1j}\partial_j \mathbf{u}) + \partial_2(M_{2j}\partial_j \mathbf{u}) + \partial_3(M_{3j}\partial_j \mathbf{u}).$$

Geometry and Space Discretization







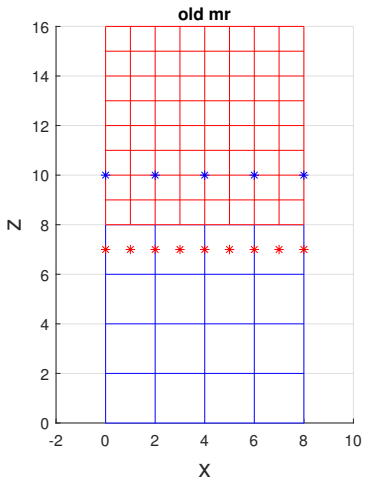
geometry :

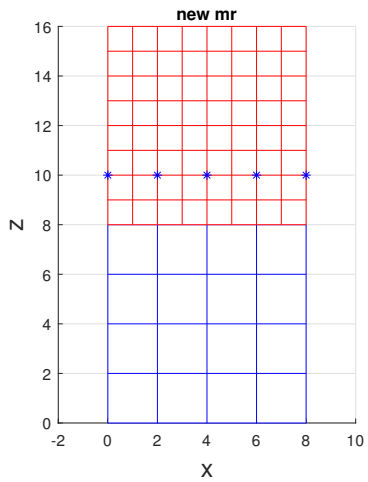
- top : $f_t(r_1, r_2)$
- bottom : $f_b(r_1, r_2)$
- interface : $f_i(r_1, r_2)$

fine domain : $x = r_1 L_1, y = r_2 L_2, z = r_3 f_t(r_1, r_2) + (1 - r_3) f_i(r_1, r_2)$

coarse domain : $x = r_1 L_1, y = r_2 L_2, z = r_3 f_i(r_1, r_2) + (1 - r_3) f_b(r_1, r_2)$

Mesh Refinement Interface





Interface condition:

- continuity in displacement: $\mathbf{u}_f\big|_\Gamma = \mathcal{P}(\mathbf{u}_c\big|_\Gamma)$
- continuity in normal stress :

$$\sum_{j=1}^{3} M_{3j}^c \partial_j \mathbf{u}_c\big|_\Gamma = \mathcal{R}\Big(\sum_{j=1}^{3} M_{3j}^f \partial_j \mathbf{u}_f\big|_\Gamma - h_f w_1 \eta\Big),$$

with

$$\eta = \rho_f\big|_\Gamma \mathcal{P}\Big((\rho_c)^{-1}\tilde{G}_c(\mu,\lambda)\tilde{u}_c\big|_\Gamma\Big) - G_c(\mu,\lambda)\mathbf{u}_f\big|_\Gamma$$

Errors and Convergence Rate

material:

- $\rho = 2 + \sin(x + 0.3)\sin(y + 0.3)\sin(z - 0.2)$
- $\mu = 3 + \sin(3x + 0.1)\sin(3y + 0.1)\sin(z)$
- $\lambda = 21 + \cos(x + 0.1)\cos(y + 0.1)\sin^2(3z)$

coarse mesh : $2h$; fine mesh : h

2h	L^c_∞	L^f_∞	L^2
$2\pi/24$	3.9702e-03	3.8742e-03	1.7834e-03
$2\pi/48$	2.6360e-04 (3.91)	2.6921e-04 (3.85)	1.0718e-04 (4.06)
$2\pi/96$	1.8563e-05 (3.83)	1.6638e-05 (4.02)	6.3739e-06 (4.07)

Iterative Solver

solve :

$$Ax = b$$

tol : $1e-7$

- Conjugate Gradient method : around 44 iterations
- Preconditioned Conjugate Gradient method : around 9 iterations
- Block Jacobian method : around 13 iterations