

# DESIGN AND IMPLEMENTATION OF MODERN QUANTUM REPEATERS FOR FUTURE QUANTUM COMMUNICATION TECHNOLOGY AND TELEPORTATION

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## ABSTRACT

Quantum communication is an upcoming new technology that is driving the future of information transmission and communication technologies to a new paradigm. It relies on quantum entanglement to facilitate transmission of quantum states between parties. Quantum repeaters are introduced to facilitate long-distance transmission. They extend the transmission range by fragmenting the channel into multiple small segments and installing quantum repeaters into each of their endpoints. The quantum repeaters perform entanglement swapping until the sender and receiver become entangled, forming a complete quantum channel for communication. The research focuses on quantum entanglement purification, the protocol that ensures entangled states maintain a high fidelity above the communication channel operational threshold. This study focused on establishing an optimum purification strategy by determining at what stage and intervals the purification protocol should be executed. Moreover, optimization schemes were applied to determine the extents of the effects of various purification protocols. IBM Qiskit was used for the circuit implementation and simulation. The results of this research provide a guide into future approaches to implementing practical quantum repeaters and the challenges existing and those bound to arise.

**Keywords** Quantum repeater · Quantum entanglement · Quantum teleportation · Quantum communication · Entanglement purification

## 1 Introduction

Quantum communication leverages the principles of quantum mechanics to transmit quantum information between remote locations. The quantum information is encoded in a qubit - the basic unit of quantum information.

At the core of quantum communication is the principle of quantum entanglement, which gives rise to the phenomena of quantum teleportation as a new paradigm protocol for communication ([Bennett et al. \[1993\]](#)).

Currently, active experimental works in quantum communication channels are carried out either in optical fibres or free space, both of which are affected by noise during transmission. The photon intensity is attenuated exponentially with transmission distance ([Ruihong and Ying \[2019\]](#)). This limits long-distance communication channels. Quantum repeaters were introduced to overcome this long-distance limitation ([Briegel et al. \[1998\]](#)).

To use quantum repeaters, the communication channels get fragmented into small segments composed of nodes or relay stations, each with a quantum repeater that extends entanglement to the adjacent nodes. We note that the terms nodes, relay stations and stations are here used interchangeably and imply the same thing unless otherwise stated. The length of the segments is chosen such that it is less than the attenuation length of the channel ([Das et al. \[2021\]](#)). Entanglement is usually established between every two adjacent nodes. Quantum entanglement switching protocol is then used to extend the entanglement along the entire link, forming a large-scale quantum link from the sender to the receiver station

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(Gisin and Thew [2007]) (Ruihong and Ying [2019]).

Each station has a quantum memory to store the entangled state before being used in the entanglement switching protocol. Quantum entanglement purification protocol is used to increase the fidelity of the entanglement (Bennett et al. [1996]). The loss of fidelity arises from noise or imperfections in the communication channel.

Quantum repeaters are necessary for future quantum communication technologies such as quantum internet (Gisin and Thew [2007], Briegel et al. [1998]). They will extend the range of transmission links to inter-continental global scale, powering the future of a global quantum network.

However, implementing them in the real world is a huge technological challenge. There is a lot of ongoing research into the individual components and full-scale architecture of a quantum repeater.

This research paper implements the full-scale architecture of a quantum repeater using quantum circuits executed on a quantum computer to study quantum entanglement purification, a key component in quantum repeaters, necessary in first generation or near-term quantum repeaters (Muralidharan et al. [2016]). It is essential in ensuring entangled states maintain high fidelity throughout the transmission.

We show results of our analysis on various entanglement purification strategies in an investigative effort to identify the optimum strategy to be applied in quantum repeaters (Kozłowski et al. [2020]). In seeking better efficiencies, we performed circuit optimizations on the purification protocols in an effort to identify if the purification protocol has an impact on the purification scheme.

## 1.1 Quantum Repeaters

Quantum repeaters are devices that extend the range of quantum channels. They teleport entangled states between adjacent nodes in a quantum channel. The main components of quantum repeaters are:

- Quantum entanglement switching for swapping entangled states between adjacent nodes.
- Quantum memory for storing quantum states for efficient on demand retrieval.
- Quantum entanglement purification for enhancing fidelity of the entangled states.

These components have a few limitations arising from imperfections in the source of entangled particles, the quantum operations involved and the interconnecting communication channels (Herbst et al. [2015]).

Despite the limitations, the use of quantum repeaters is still possible as demonstrated by Herbst et al., who managed to use a quantum repeater to teleport an entangled state, a photon, between the Canary islands of La Palma to Tenerife, a distance of about 143 km (Herbst et al. [2015]). The entanglement swapping experiment used two polarization-entangled photon pairs generated in two identical spontaneous parametric down-conversion (SPDC) sources using a non-linear crystal,  $\beta$ -barium borate (BBO) (Herbst et al. [2015]).

## 1.2 Quantum Repeater Purification Protocols

Purification has to be done to keep the states at high fidelity. The three popular purification protocols are:

- Bennett’s protocol (Bennett et al. [1996]). Both Alice and Bob apply Controlled-NOT operations between the two qubits of the Bell pair, then measuring in computational basis and transmitting the results over a classical channel between the nodes (Das et al. [2021]). Purification is a success if the measurements agree and the resulting state is kept.
- Deutsch’s protocol (Deutsch et al. [1996]). The states get represented as Bloch vectors in a Bloch sphere. Alice performs a rotational  $R_x(\pi/2)$  on her qubits while Bob performs the inverse rotation  $(-\pi/2)$ . The rest of the process then proceeds as in Bennett’s protocol.
- Multi-qubit entanglement purification. The purification protocol is executed on multiple qubits simultaneously (Das et al. [2021]).

Performance aspects considered for purification protocols are; fidelity of the purified Bell pair, success probability and circuit length (Krastanov et al. [2019]). All protocols work towards obtaining shorter circuits, achieving higher success rates and better final fidelities (Krastanov et al. [2019]).

## 2 Experimental Setup

### 2.1 Research Approach

This research paper presents a theoretical and computational approach. The architectural design of the quantum repeater is modelled based on the use of quantum optics as opposed to earth to satellite links. The conceptual implementation is

however, the same.

The study uses simulation platforms and currently available quantum computers, such as those from IBM. The environment exposed to superconducting qubits in the IBM quantum computers can ideally emulate the same environment quantum repeaters will be exposed to when in real-world operation (Das et al. [2021]).

## 2.2 Design and Implementation Approach

Each execution stage and protocol of the quantum repeater was translated into a modular quantum circuit that was independently executed on IBM Qiskit. The modularity of the code helped test out different purification strategies, protocols and components of the quantum repeater for better analysis. The quantum circuits were first executed on the native simulation - IBM Qiskit's QASM simulator - before being finally executed on actual IBM quantum computers. Performance analysis was done based on the fidelity of the purified Bell pair. Optimisation schemes were applied to the entanglement purification circuits to analyse their performance.

## 2.3 Entanglement Generation

The circuit implementation that prepares and generates an entangled pair takes in two qubits as input and performs Hadamard and Controlled-NOT unitary gate operations on them. Each EPR pair gets distributed to adjacent nodes. One pair,  $\Phi^+_{AB}$  gets to entangles A and B while the other pair,  $\Phi^+_{CD}$  gets to entangles C and D.

## 2.4 Quantum Entanglement Distribution

The first distribution is that of the EPR pairs  $\Phi^+_{AB}$  and  $\Phi^+_{CD}$  to their respective nodes, each node taking one of the qubits from a pair. The distribution stage that involves the quantum repeater requires the distribution of entanglement along the transmission line from sender to receiver. This entanglement distribution relies on quantum memories, entanglement purification protocols and entanglement swapping protocols to distribute entanglement between nodes from the start of the communication link to the end.

## 2.5 Quantum Memory

The entangled state  $\Phi^+_{AB}$  is momentarily stored in a quantum memory and only retrieved when it is needed to perform entanglement distribution between nodes B and C to get the entangled state  $\Phi^+_{BC}$ .

## 2.6 Quantum Entanglement Purification

This is a crucial component of near-term quantum repeaters. We constrained this research to the two common purification protocols: Bennett's protocol (Bennett et al. [1996]) and Deutsch's protocol (Deutsch et al. [1996]). The protocols can be extended to accommodate multi-qubit purification (Das et al. [2021]). Each protocol has its own complexity of implementation. They also provide varying fidelity levels and produce varying degrees of overhead during circuit operation.

Successful purification using these protocol gives measurement results as 00 or 11. Any other measurement result, either 01 or 10, indicates a failed purification operation, upon which the purification protocol needs a fresh restart.

Figure 1 shows the implementation of Bennett's protocol. Figure 2 shows the implementation of Deutsch's protocol.

## 2.7 Quantum Entanglement Swapping

The design and construction of the quantum entanglement swapping circuit follows a similar structure to that of the teleportation protocol. In this case however, we demonstrated the teleportation of an entangled qubit. One qubit of Alice's Bell pair become entangled with another qubit of Bob's Bell pair. This is significant in a quantum repeater because it allows for states previously not entangled and which had never interacted with each other before to become entangled with each other. This is the guiding principle to extending the length of a quantum link.

The elementary construction of the entanglement swapping circuit contains two Bell pairs which together form a combined 4-qubit quantum state  $\psi_{ABCD} = \Phi^+_{AB} \otimes \Phi^+_{CD}$ . (Bell-state measurement) BSM measurement is performed between the qubits in B and C. Depending on the results of the measurement, an appropriate Pauli correction operation  $I, Z, X, Y$  gets performed on the qubit in D (Das et al. [2021]). The result is the projection of qubits in A and D into the state  $\Phi^+_{AD}$  and the entanglement between nodes B and C in the state  $\Phi^+_{BC}$ . Teleportation can now occur directly from node A to D because the entanglement distributed to D from A maintains a complete quantum communication link not limited by spatial separation. Having executed the circuit in Figure 3 in the QASM simulator, the measurement results gotten were as in Figure 4. As expected, Alice's entangled qubit  $alice_0$ , when measured is in the state 0 with

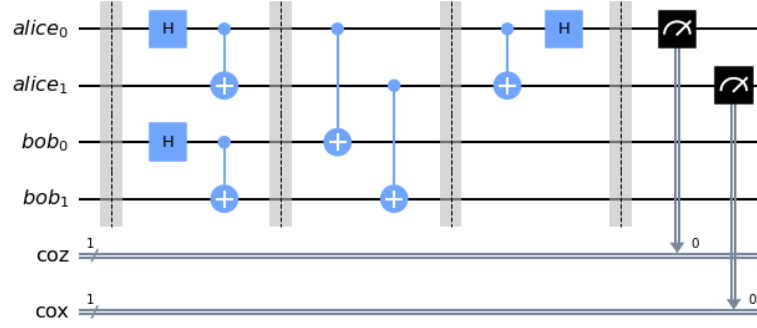


Figure 1: Quantum circuit for Bennett's purification protocol

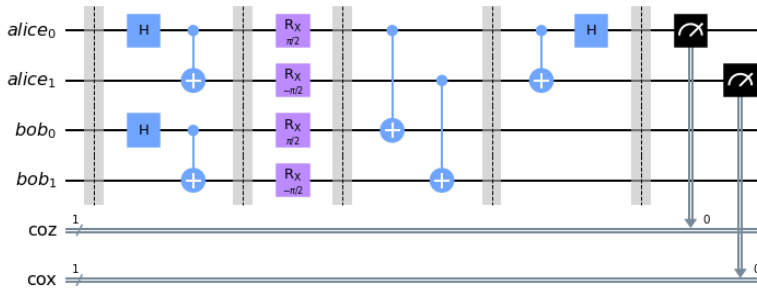


Figure 2: Quantum circuit for Deutsch's purification protocol

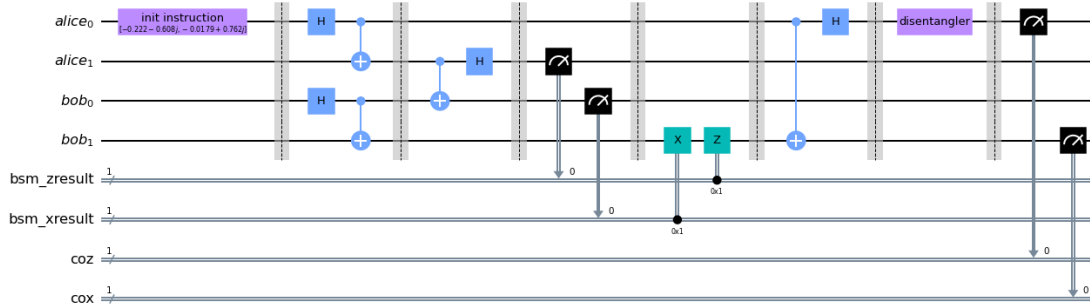


Figure 3: Quantum circuit for quantum entanglement swapping protocol

near 100% probability together with Bob's other qubit  $bob_1$  with whom they are entangled. This results act as proof of a successful entanglement swapping protocol.

### 3 Results and Discussion

#### 3.1 Complete Quantum Repeater Architecture

Combining all the necessary components, we arrived at a complete implementation of a quantum repeater and its augmenting components in a quantum network. The circuit architecture is as in Figure 5 and Figure 6.

Alice's and Bob's Bell pairs are first generated. One qubit from each Bell pair gets transmitted to Alice and Bob through a classical channel. The remaining qubits from each Bell pair get transmitted through a classical channel to quantum memory devices found on the quantum repeater. The transmission of these qubits is emulated using SWAP gates. Through heralding, a classical message is sent to the repeater indicating that Alice's and Bob's qubits are ready for

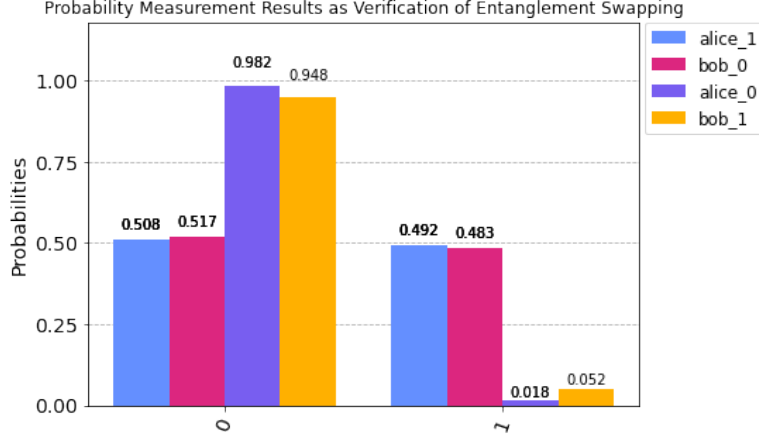


Figure 4: Results for the verification of quantum entanglement swapping

swapping. The heralding helps to synchronize the swapping protocol. The qubits in the quantum memory devices get transmitted to their respective quantum channels, ready for swapping. This transmission is again represented by SWAP gates. In this circuit in Figure 5, Deutsch's purification protocol is done just before swapping. Thereafter entanglement

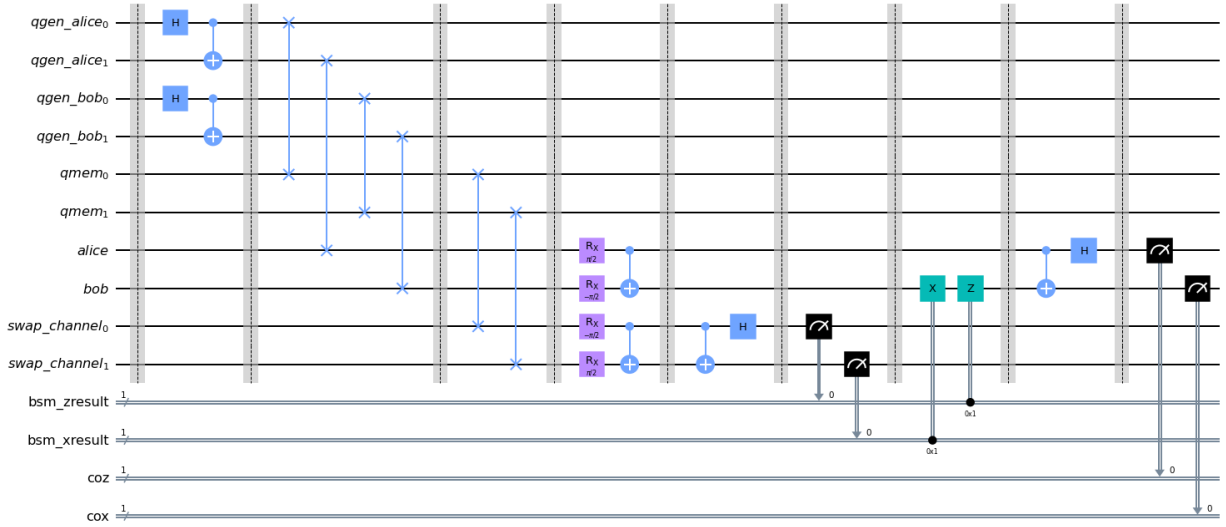


Figure 5: Quantum circuit for the full complete quantum repeater, implementing Deutsch's purification protocol just before the swapping protocol stage. *qgen\_alice* and *qgen\_bob* represents the modules generating Alice's and Bob's entangled qubits respectively. *qmem* represents quantum memory devices present in a quantum repeater. The transmission of qubits through classical channels to either quantum channels of quantum memory is emulated using SWAP gates.

swapping protocol is done. Finally, Alice's and Bob's qubits are measured out in the Bell basis. The circuit architecture demonstrates the quantum repeater protocol as it is to be implemented in the real world upon deployment. Using this quantum repeater circuit, we moved to investigate our remaining objectives regarding purification strategies and optimization schemes.

### 3.2 Purification Strategy

A number of experimental simulations were carried out to determine the optimum purification strategy. Identifying an optimum purification strategy for near-term quantum repeaters means better efficiency of operation of future practical quantum repeaters.

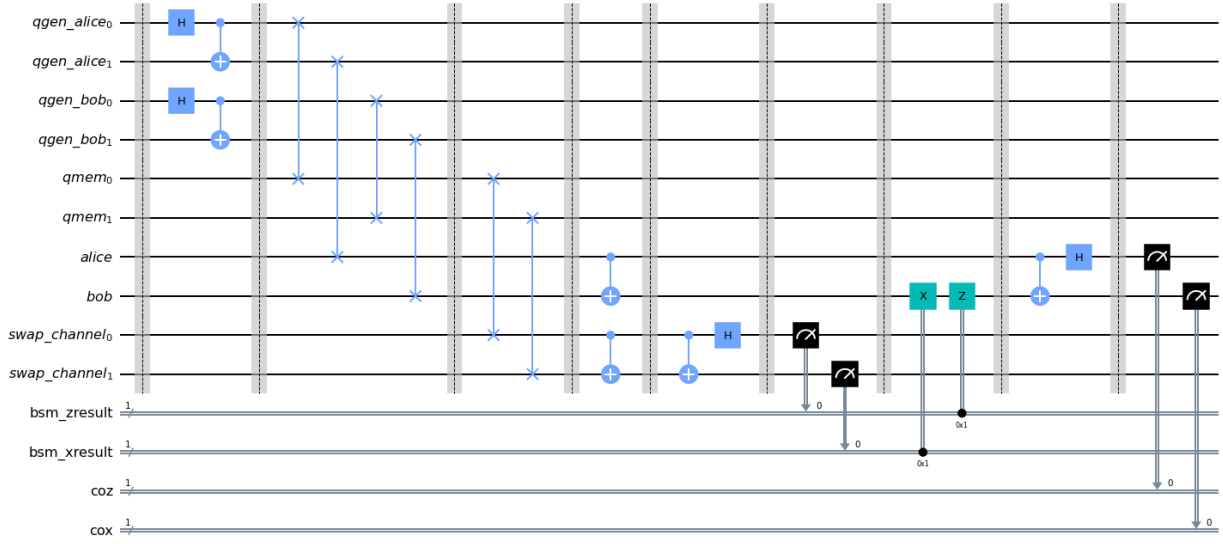


Figure 6: Quantum circuit for the full complete quantum repeater, implementing Bennett's purification protocol just before the swapping protocol stage.

In total, five strategies were tested: purification after Bell-pairs are created, purification after every swap, entanglement swapping alternating purification that is, before and after swapping, repeated purification after every swap and a custom purification strategy aiming at a suitable combination of various steps.

Qiskit provides a noise model module that we used to create the a simplified approximate noise model based on the properties of real quantum computer devices. The errors due to the noise model are sufficient to emulate real world errors.

Figure 7, shows the results of the impact purification had on different stages in the quantum repeater protocol. The considered stages are: distribution stage denoted *dist*, first swap denoted *aswap1*, second swap denoted *aswap2* and eventually the end of the node.

From observation, entanglement distribution has little effect on the fidelity. The fidelity of the Bell-pairs takes a hit during entanglement swapping and readout at the end of the node. As expected, the fidelity is lowest at the end node, during readout. Of interest are the results gotten during the after the swapping. The differences in the fidelities from using different approaches are not dramatically huge. However, the most promising approach revolves around either performing repeated purification after every swap or performing purification alternatingly with the swaps.

On deeper analysis, the approach of performing repeated purification creates a bottleneck on the circuit, reducing its bandwidth. The extra circuitry overhead has the potential of not being as efficient at larger scopes. We also performed a verification test for the quantum repeater protocol as seen in Figure ?? . The probability of measuring 0 at the end node using the reverse initialization procedure varies depending on the purification strategy. The expectation was that we would get 100% probability on the state 0 which is what all qubits are initialized to at the beginning. These results mean that the quantum repeater protocol takes on a probabilistic nature with the effects of noise still at play.//

The noise results in the observed results event in the states 1 and lower percentages of the states 0.

At this stage of research we cannot make definitive claims onto the best or optimum strategy for conducting purification. However, our results provide enough insight into understanding the purification protocol and we recommend tailoring to purification strategy to the needs of the quantum network.

### 3.3 Purification Optimization Scheme

The purification circuits underwent two levels of optimization - a light optimization scheme and a heavy optimization scheme. The purification optimization scheme was carried out for all purification strategies tested out earlier using the two major purification protocols - Bennett's and Deutsch's protocols.

Each optimization scheme was implemented in the circuits for both purification protocols and measurement results obtained over several iterations before averaging them out - to get quasi probabilistic results. To get a clear picture of the differences in the results of the purification protocols, their percentage differences were plotted out. This procedure was repeated for all purification strategies considered in this research. The consequent results are as seen in Figure 9

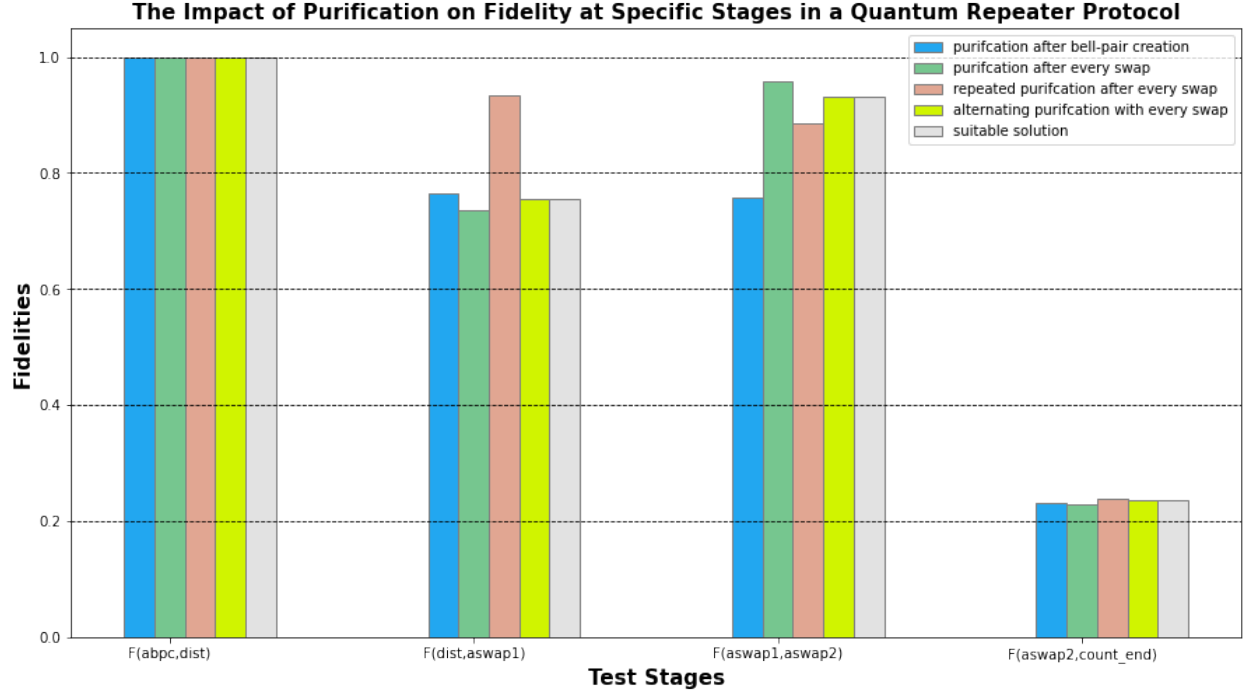


Figure 7: Effect of purification on fidelity across different purification strategies. The results were obtained from a circuit implementing one intermediate quantum repeater and hence two entanglement swapping procedures, labelled as *aswap1* and *aswap2*.

The classical bits represent the classical states that can be measured by Alice and Bob at the end of the communication channel. The lower the percentage values, the lower the difference of results obtained from the purification protocols. This is to mean, near zero percentages show that the purification protocols under consideration would give the same measurement results.

The results from this experiment clearly show and confirm that the choice of purification protocol has on the average minimal impact on the overall purification optimization scheme used in the purification circuits.

However minimal the differences, they should still be monitored in real life operation since some purification strategies do have higher percentage values such as Figure 9 (b).

The most important bits are the 00 since they represent the expected ideal measurement results, with the rest of the bits representing states that are the result of noise in the circuit. Note also that in the bits 00, the percentage differences are quite minimal.

## 4 Conclusion

This research concludes with a quantum repeater design setup and implementation that provides insight into determining the optimum strategy for conducting entanglement purification.

An important observation is that the great deal of noise in our circuit simulations will transfer to practical quantum repeaters, making the practical implementation a technological hardship.

The results from the purification strategy test did not provide a definitive claim on the best purification strategy to use. Therefore, it is recommended that one tailors the purification strategy to the needs of the quantum network.

The results from the purification optimization scheme give conclusive results that the choice of purification protocols has minimal effect on the overall optimization scheme and no optimization scheme favours a certain purification protocol. The study recommends that more research into quantum repeaters should focus on tackling noise, while still in the heralding era before implementing quantum error correction into practical quantum repeaters.

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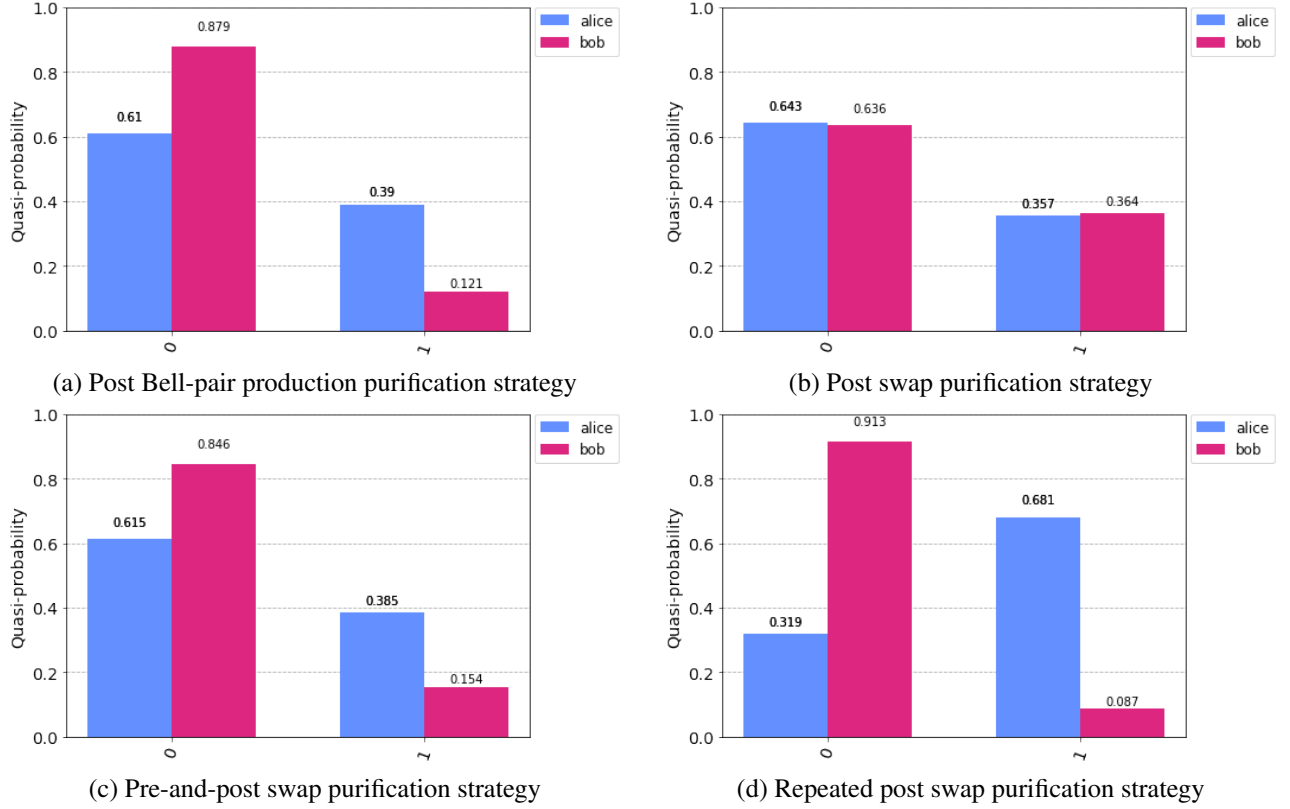


Figure 8: Results verifying the success of the teleportation of entanglement in the quantum repeater protocol between Alice and Bob under different purification strategies as indicated in each graph. Ideally, there should be higher percentages in the 0 state. Errors inherent in the circuit and from noise result in varied results and probabilities in the 1 state.

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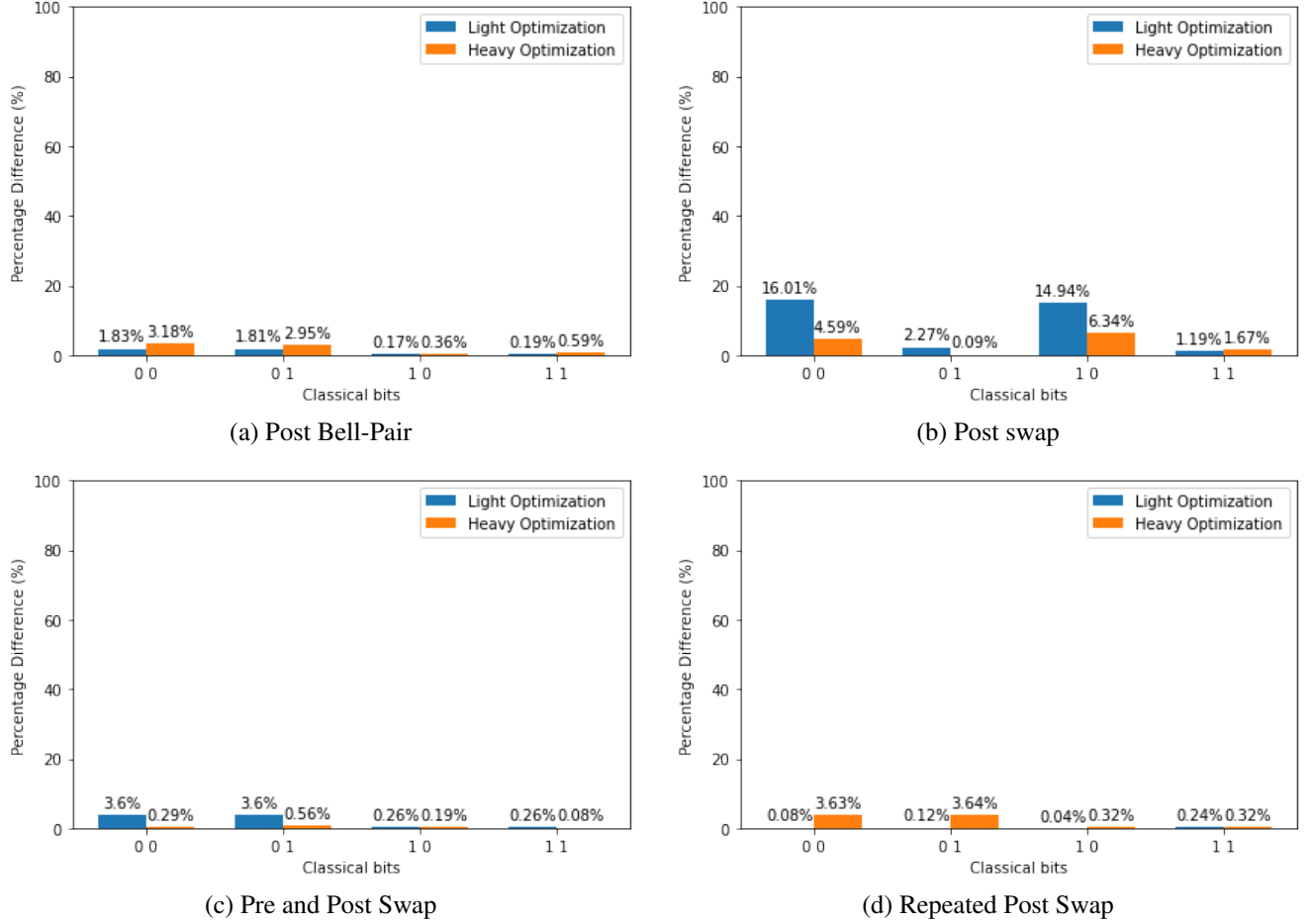


Figure 9: The results of experiments testing the differences of various purification protocols under different optimization schemes. The experiments were carried out for different purification strategies. (a) was done in the Post Bell-pair production, (b) in the Post entanglement swap, (c) in the Pre and Post entanglement swap and (d) in the Repeated Post entanglement swap purification strategies.

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