Design and Implementation of Quantum Repeaters: Insights on Quantum Entanglement Purification

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Abstract

Quantum communication is an upcoming new technology that is driving the future of information transmission and communication technologies to a new paradigm. It relies on quantum entanglement to facilitate transmission of quantum states between parties. Quantum repeaters are employed to facilitate long-distance transmission. They extend the transmission range by fragmenting the channel into multiple small segments where they perform entanglement swapping between each segment endpoints until the sender and receiver become entangled, forming a complete quantum link for communication. This research focuses on quantum entanglement purification, the protocol that ensures entangled states maintain a high fidelity above the communication channel operational threshold. Our study gives insight into the optimum purification strategy by determining at what stage and intervals the purification protocol should be executed. Moreover, optimization schemes were applied to evaluate the effects of various purification protocols. IBM Qiskit was used for the circuit implementation and simulation. The results provide a guide into future approaches to implementing practical quantum repeaters and the challenges existing and those bound to arise.

1 Introduction

Quantum communication leverages the principles of quantum mechanics to transmit quantum information between remote locations. The quantum information is encoded in a qubit - the basic unit of quantum information.

At the core of quantum communication is the principle of quantum entanglement, which gives rise to the phenomena of quantum teleportation as a new paradigm protocol for communication [1].

Currently, active experimental works in quantum communication channels are carried out either in optical fibres or free space, both of which are affected by noise during transmission. The photon intensity is attenuated exponentially with transmission distance [2]. This limits long-distance communication channels. Quantum repeaters were introduced to overcome this long-distance limitation [3].

Quantum repeaters are devices that extend the range of quantum channels. They do so by fragmenting communication channels into small segments composed of nodes or relay stations, each with a quantum repeater that teleports entangled states between adjacent nodes. We note that the terms nodes, relay stations and stations are here used interchangeably and imply the same thing unless otherwise stated. The length of the segments is chosen such that it is less than the attenuation length of the channel [4]. Entanglement is established between adjacent nodes using quantum entanglement switching protocol, eventually forming a large-scale quantum link from the sender to the receiver station [5, 2].

The main components of a quantum repeater include; quantum entanglement switching for swapping entangled states between adjacent nodes, quantum memory for storing quantum states for efficient on demand retrieval and quantum entanglement purification for enhancing the fidelity of the entangled states. These components have a few limitations arising from imperfections in the source of entangled particles, the quantum operations involved and the interconnecting communication channels [6].

Despite these limitations, quantum repeaters have been successfully demonstrated in experiments, such as Herbst et al., who managed to use a quantum repeater to teleport an entangled state, a photon, between the Canary islands of La Palma to Tenerife, a distance of about 143 km [6]. The entanglement swapping experiment used two polarization-entangled photon pairs generated in two identical spontaneous parametric down-conversion (SPDC) sources using a non-linear crystal, β-barium borate (BBO) [6].

Quantum entanglement purification protocol is a key component in quantum repeaters, necessary in first generation or near-term quantum repeaters [7]. It is essential in ensuring entangled states maintain high fidelity throughout the transmission [8], thereby compensating for loss of fidelity due to noise or imperfections in the communication channel.

The two purification protocols employed in this research are: Bennett's protocol [8] and Deutsch's protocol [9].

Performance aspects considered for the purification protocols are; fidelity of the purified Bell pair, success probability and circuit length [10]. All protocols work towards obtaining shorter circuits, achieving higher success rates and better final fidelities [10].

Quantum repeaters are necessary for future quantum communication technologies such as quantum internet [5, 3]. They will extend the range of transmission links to inter-continental global scale, powering the future of a global quantum network.

However, implementing them in the real world is a huge technological challenge. There is a lot of ongoing research into the individual components and full-scale architecture of a quantum repeater.

This research paper presents the full-scale architecture of a quantum repeater designed and implemented using quantum circuits executed on a quantum computer. We used the said quantum repeater implementation to study quantum entanglement purification.

2 Experimental Setup

2.1 Research Approach

This research paper presents a theoretical and computational approach. The architectural design of the quantum repeater is modelled based on the use of quantum optics as opposed to earth to satellite links. The conceptual implementation is however, the same.

This study uses IBM quantum computers and IBM Qiskit library. The environment exposed to superconducting qubits in the IBM quantum computers can ideally emulate the same environment quantum repeaters will be exposed to when in real-world operation [4].

Each execution stage and protocol of the quantum repeater was translated into a modular quantum circuit that was independently executed on IBM Qiskit. The modularity of the code helped test out different purification strategies, protocols and components of the quantum repeater for better analysis. The quantum circuits were first executed on the native simulation - IBM Qiskit's QASM simulator - before being finally executed on actual IBM quantum computers.

Performance analysis was done based on the fidelity of the purified Bell pair. Optimisation schemes were applied to the entanglement purification circuits to analyse their performance.

2.2 Entanglement Generation

The circuit implementation that prepares and generates an entangled pair takes as input two qubits and performs Hadamard and Controlled-NOT unitary gate operations on them. Each EPR pair gets distributed to adjacent nodes. One pair, $|\Phi^+\rangle_{AB}$ gets to entangles A and B while the other pair, $|\Phi^+\rangle_{CD}$ gets to entangles C and D.

2.3 Quantum Entanglement Distribution

The first distribution is that of the EPR pairs $|\Phi^+\rangle_{AB}$ and $|\Phi^+\rangle_{CD}$ to their respective nodes, each node taking one of the qubits from a pair. The distribution stage that involves the quantum repeater requires the distribution of entanglement along the transmission line from sender to receiver. This entanglement distribution relies on quantum memories, entanglement purification protocols and entanglement swapping protocols to distribute entanglement between nodes from the start of the communication link to the end.

2.4 Quantum Memory

The entangled state $|\Phi^+\rangle_{AB}$ is momentarily stored in a quantum memory and only retrieved when it is needed to perform entanglement distribution between nodes B and C to get the entangled state $|\Phi^+\rangle_{BC}$.

2.5 Quantum Entanglement Purification

We constrained this research to the two common purification protocols: Bennett's protocol [8] and Deutsch's protocol [9]. Each protocol has its own complexity of implementation. They also provide varying fidelity levels and produce varying degrees of overhead during circuit operation.

Successful purification using these protocol gives measurement results as $|00\rangle$ or $|11\rangle$. Any other measurement result, either $|01\rangle$ or $|10\rangle$, indicates a failed purification operation, upon which the purification protocol needs a fresh restart.

Figure 1 and Figure 2 show the implementation of the two purification protocols.

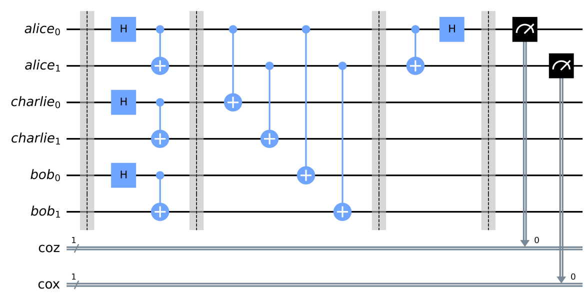


Figure 1: Quantum circuit for Bennett's purification protocol

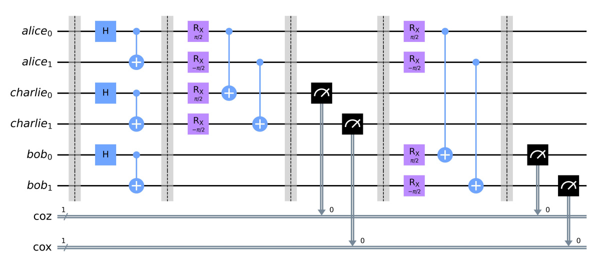


Figure 2: Quantum circuit for Deutsch's purification protocol

2.6 Quantum Entanglement Swapping

The quantum entanglement swapping circuit has a construction approach similar to the teleportation protocol. Here, we demonstrated the teleportation of an entangled qubit by entangling one qubit of Alice's Bell pair with another qubit of Bob's Bell pair. This is significant because it allows for states previously not entangled and which had never interacted with each other before to become entangled with each other. This is the guiding principle to extending the length of a quantum link.

The elementary construction of the entanglement swapping circuit contains two Bell pairs which together form a combined 4-qubit quantum state $|\psi\rangle_{ABCD} = |\Phi^{+}\rangle_{AB} \otimes |\Phi^{+}\rangle_{CD}$. (Bell-state measurement) BSM measurement is performed between the qubits in B and C. Depending on the results of the measurement, an appropriate Pauli correction operation *I, Z, X, Y* gets performed on the qubit in D [4]. The result is the projection of qubits in A and D into the state $|\Phi^{+}\rangle_{AD}$ and the entanglement between nodes B and C in the state $|\Phi^{+}\rangle_{BC}$. Teleportation can now occur directly from node A to D because the entanglement distributed to D from A maintains a complete quantum communication link not limited by spatial separation. Having executed the circuit in Figure 3 in the

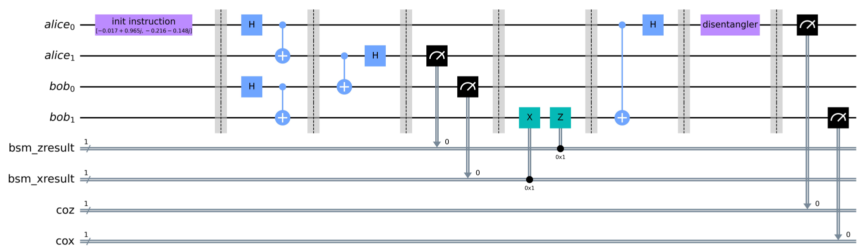


Figure 3: Quantum circuit for quantum entanglement swapping protocol

QASM simulator, the measurement results gotten were as in Figure 4. As expected, Alice's entangled qubit $alice_0$, when measured is in the state $|0\rangle$ with near 100% probability together with Bob's other qubit bob_1 with whom they are entangled. This results act as proof of a successful entanglement swapping protocol.

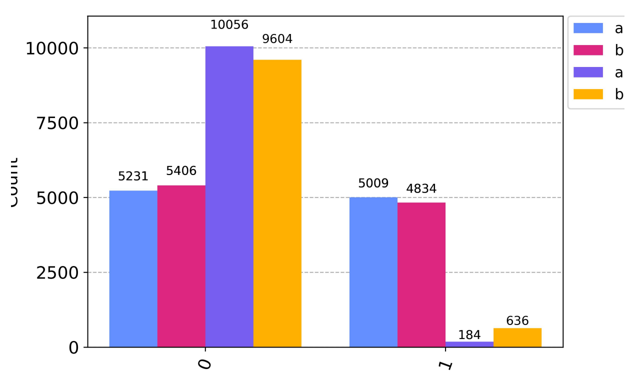


Figure 4: Results for the verification of quantum entanglement swapping

3 Results and Discussion

3.1 Complete Quantum Repeater Architecture

Combining all the necessary components, we arrived at a complete implementation of a quantum repeater and its augmenting components in a quantum network. The circuit architecture is as in Figure 5 and Figure 6.

Alice's and Bob's Bell pairs are first generated. One qubit from each Bell pair gets transmitted to Alice and Bob through a classical channel. The remaining qubits from each Bell pair get transmitted through a classical channel to quantum memory devices found on the quantum repeater. The transmission of these qubits is emulated using SWAP gates. Through heralding, a classical message is sent to the repeater indicating that Alice's and Bob's qubits are ready for swapping. The heralding helps to synchronize the swapping protocol. The qubits in the quantum memory devices get transmitted to their respective quantum channels, ready for swapping. This transmission is again represented by SWAP gates. In this circuit in Figure 5, Deutsch's purification protocol is done just before swapping.

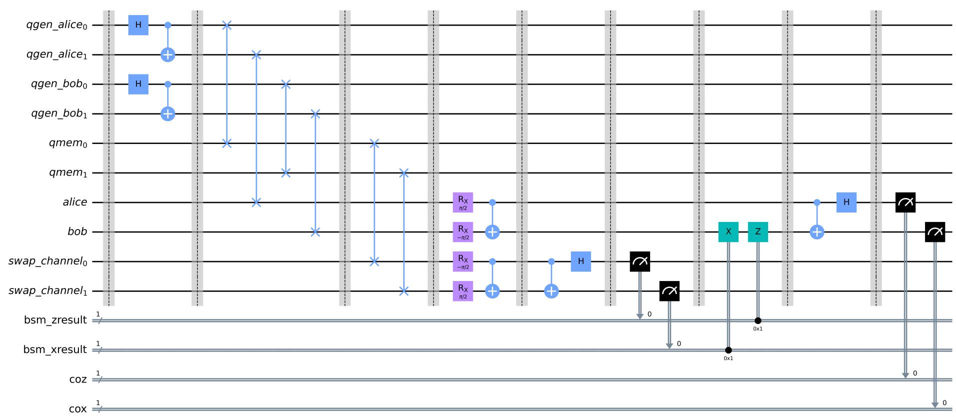


Figure 5: Quantum circuit for the full complete quantum repeater, implementing Deutsch's purification protocol just before the swapping protocol stage. *qgen_alice* and *qgen_bob* represents the modules generating Alice's and Bob's entangled qubits respectively. *qmem* represents quantum memory devices present in a quantum repeater. The transmission of qubits through classical channels to either quantum channels of quantum memory is emulated using SWAP gates.

Thereafter entanglement swapping protocol is done. Finally, Alice's and Bob's qubits are measured out in the Bell basis.

The circuit architecture demonstrates the quantum repeater protocol as it is to be implemented in the real world upon deployment. Using this quantum repeater circuit, we moved to investigate our

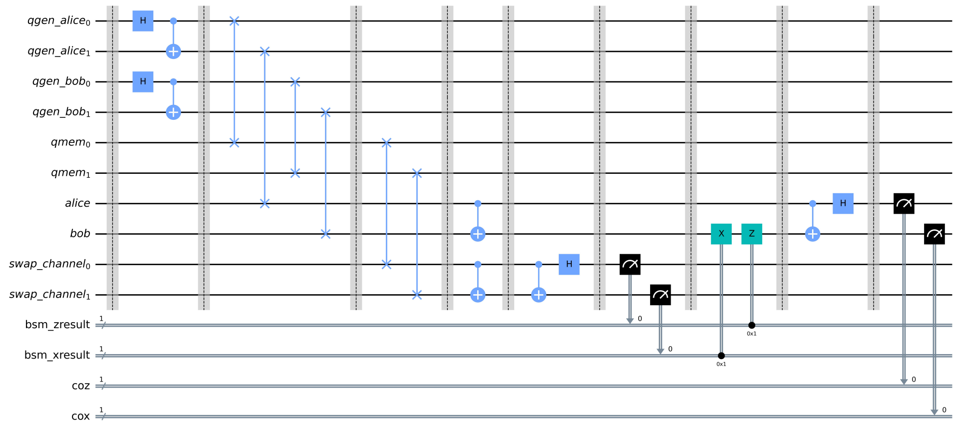


Figure 6: Quantum circuit for the full complete quantum repeater, implementing Bennett's purification protocol just before the swapping protocol stage.

remaining objectives regarding purification strategies and optimization schemes.

3.2 Purification Strategy

[A number of] experimental simulations were carried out to determine the optimum purification strategy. Identifying an optimum purification strategy for near-term quantum repeaters means better efficiency of operation of future practical quantum repeaters.

In total, five strategies were tested: purification after Bell-pairs are created, purification after every swap, entanglement swapping alternating purification that is, before and after swapping, repeated purification after every swap and a custom purification strategy aiming at a suitable combination of various steps.

Qiskit provides a noise model module that we used to create the a simplified approximate noise model based on the properties of real quantum computer devices. The errors due to the noise model are sufficient to emulate real world errors.

Figure 7, shows the results of the impact purification had on different stages in the quantum repeater protocol. The considered stages are: distribution stage denoted *dist*, first swap denoted *aswap1*, second swap denoted *aswap2* and eventually the end of the node.

From observation, entanglement distribution has little effect on the fidelity. The fidelity of the Bell-pairs takes a hit during entanglement swapping and readout at the end of the node. As expected, the fidelity is lowest at the end node, during readout. Of interest are the results gotten during the after the swapping. The differences in the fidelities from using different approaches are not dramatically huge. However, the most promising approach revolves around either performing repeated purification

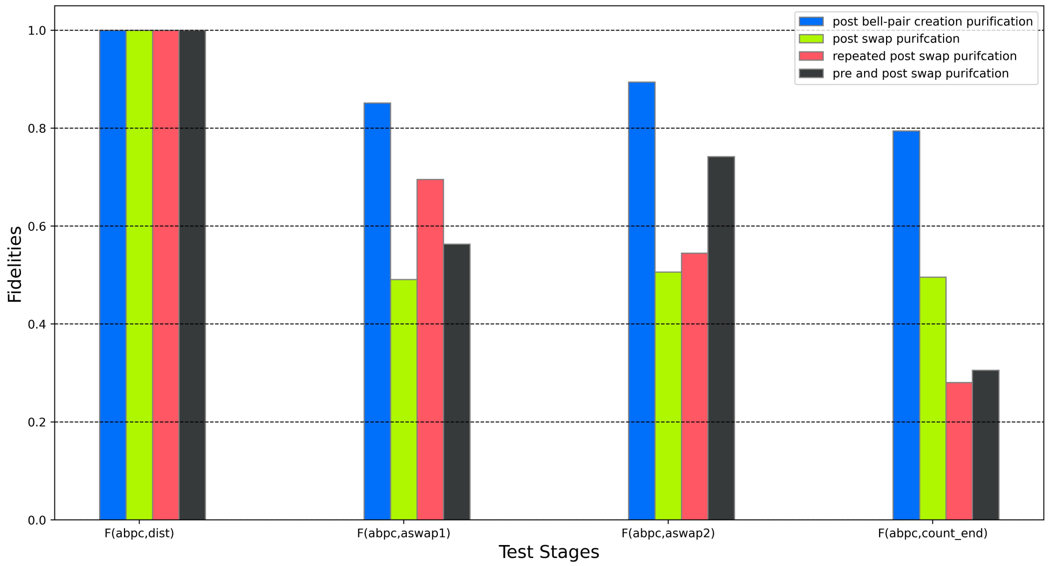


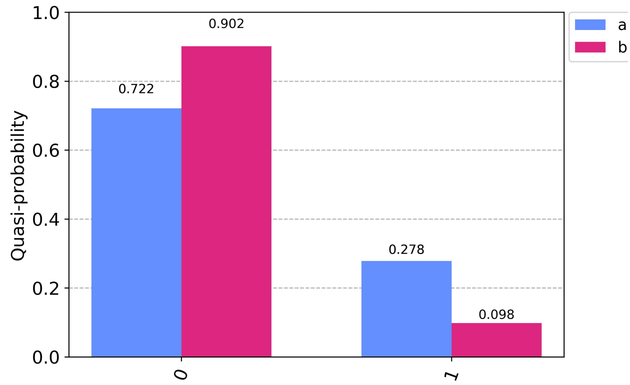
Figure 7: Effect of purification on fidelity across different purification strategies. The results were obtained from a circuit implementing one intermediate quantum repeater and hence two entanglement swapping procedures, labelled as *aswap1* amd *aswap2*.

after every swap or performing purification alternatingly with the swaps.

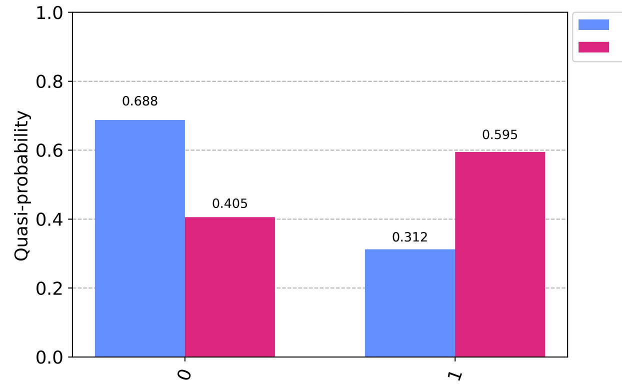
On deeper analysis, the approach of performing repeated purification creates a bottleneck on the circuit, reducing its bandwidth. The extra circuitry overhead has the potential of not being as efficient at larger scopes. We also performed a verification test for the quantum repeater protocol as seen in Figure 3.2. The probability of measuring 0 at the end node using the reverse initialization procedure varies depending on the purification strategy. The expectation was that we would get 100% probability on the state 0 which is what all qubits are initialized to at the beginning. These results mean that the quantum repeater protocol takes on a probabilistic nature with the effects of noise still at play.

The noise results in the observed results event in the states 1 and lower percentages of the states 0.

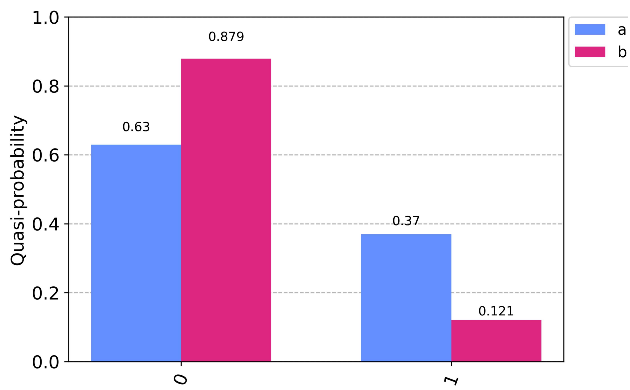
At this stage of research we cannot make definitive claims onto the best or optimum strategy for conducting purification. However, our results provide enough insight into understanding the purification protocol and we recommend tailoring to purification strategy to the needs of the quantum network.



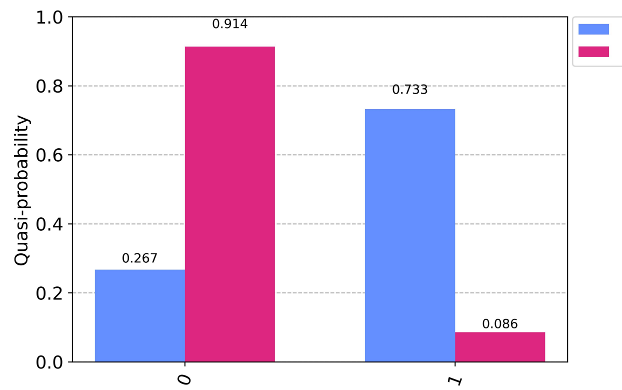
(a) Post Bell-pair production purification strategy



(b) Post swap purification strategy



(c) Pre-and-post swap purification strategy



(d) Repeated post swap purification strategy

Figure 8: Results verifying the success of the teleportation of entanglement in the quantum repeater protocol between Alice and Bob under different purification strategies as indicated in each graph. Ideally, there should be higher percentages in the 0 state. Errors inherent in the circuit and from noise result in varied results and probabilities in the 1 state.

3.3 Purification Optimization Scheme

The purification circuits underwent two levels of optimization - a light optimization scheme and a heavy optimization scheme. The purification optimization scheme was carried out for all purification strategies tested out earlier using the two major purification protocols - Bennett's and Deutsch's protocols.

Each optimization scheme was implemented in the circuits for both purification protocols and measurement results obtained over several iterations before averaging them out - to get quasi probabilistic results. To get a clear picture of the differences in the results of the purification protocols, their percentage differences were plotted out. This procedure was repeated for all purification strategies considered in this research. The consequent results are as seen in Figure 9

The classical bits represent the classical states that can be measured by Alice and Bob at the end of the communication channel. The lower the percentage values, the lower the difference of results obtained from the purification protocols. This is to mean, near zero percentages show that the purification protocols under consideration would give the same measurement results.

The results from this experiment clearly show and confirm that the choice of purification protocol has on the average minimal impact on the overall purification optimization scheme used in the purification circuits.

However minimal the differences, they should still be monitored in real life operation since some purification strategies do have higher percentage values such as Figure 9 (b).

The most important bits are the 00 since they represent the expected ideal measurement results, with the rest of the bits representing states that are the result of noise in the circuit. Note also that in the bits 00, the percentage differences are quite minimal.

4 Conclusions and Suggestions

This research concludes with a quantum repeater design setup and implementation that provides insight into determining the optimum strategy for conducting entanglement purification.

An important observation is that the great deal of noise in our circuit simulations will transfer to practical quantum repeaters, making the practical implementation a technological hardship.

The results from the purification strategy test did not provide a definitive claim on the best purification strategy to use. Therefore, it is recommended that one tailors the purification strategy to the needs of the quantum network.

The results from the purification optimization scheme give conclusive results that the choice of purification protocols has minimal effect on the overall optimization scheme and no optimization scheme favours a certain purification protocol.

The study recommends that more research into quantum repeaters should focus on tackling noise, while still in the heralding era before implementing quantum error correction into practical quantum repeaters.

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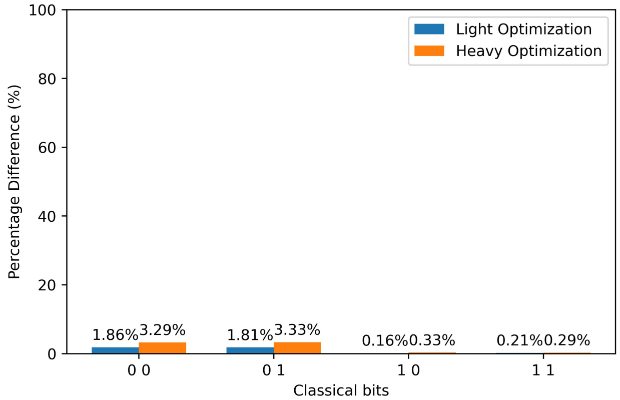
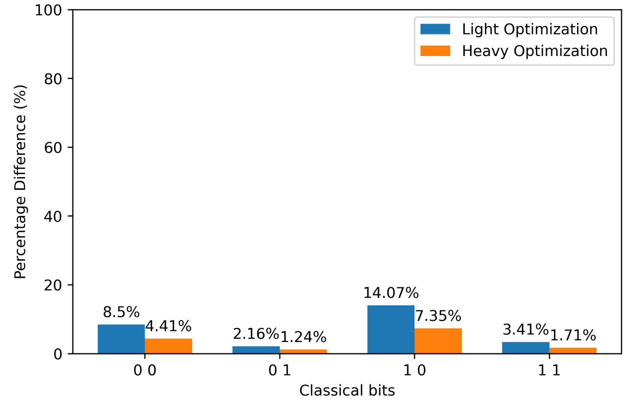
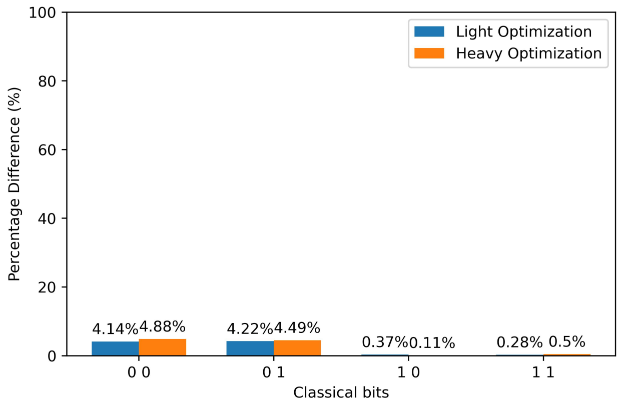


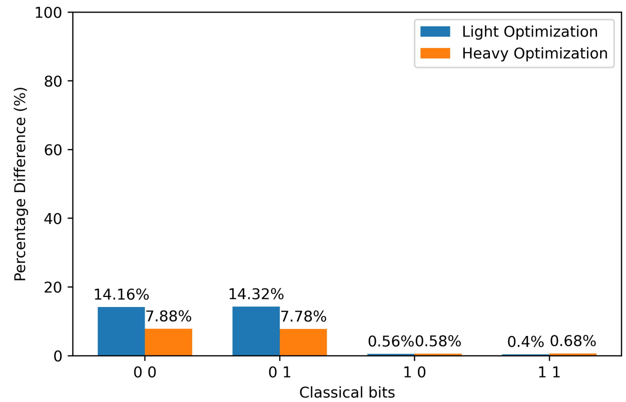
(a) Post Bell-Pair



(b) Post swap



(c) Pre and Post Swap



(d) Repeated Post Swap

Figure 9: The results of experiments testing the differences of various purification protocols under different optimization schemes. The experiments were carried out for different purification strategies. (a) was done in the Post Bell-pair production, (b) in the Post entanglement swap, (c) in the Pre and Post entanglement swap and (d) in the Repeated Post entanglement swap purification strategies.

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