

UNIVERSITY OF NAIROBI

**DESIGN AND IMPLEMENTATION OF MODERN QUANTUM
REPEATERS FOR FUTURE QUANTUM COMMUNICATION
TECHNOLOGY AND TELEPORTATION**

BY

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ABSTRACT

Quantum communication is a new technological paradigm driving the future of information transmission and communication technologies. Founded on fundamental quantum mechanics, it relies on quantum entanglement as a resource in maintaining communication links with the quantum network nodes. The entanglement facilitates teleportation of qubits between the nodes. A resulting application is a large-scale quantum network such as a quantum internet. It involves large distances which unfortunately introduces noise and imperfections into the quantum communication channels. This renders the distribution and maintenance of entanglement between nodes impossible. To solve this, researchers proposed using quantum repeaters. In the quantum repeater protocol, an extensive quantum channel gets fragmented into multiple small segments. Quantum repeaters from each part then distribute entangled states between adjacent nodes. Quantum entanglement switching is performed down the chain, eventually entangling the sender and receiver stations, forming a full large-scale quantum channel. Numerous components and protocols go into making a practical fully functioning quantum repeater. To understand the quantum repeater infrastructure, we simulate it on the simulation platforms; Qiskit and NetSquid and already available quantum computers from IBM. Further research is needed in the quantum purification protocol used to ensure entangled qubits maintain a high fidelity that passes the channel operation threshold. We study purification protocols to identify the best purification strategy for quantum repeaters. We perform optimization schemes to determine the extents of effects from various purification protocols. This research presents an understanding of the implementation, execution and working of quantum repeaters and their corresponding protocols. The results from this research should provide a mirror for future approaches to implementing practical quantum repeaters and challenges existing and those bound to arise.

DECLARATION

I declare that this research is my original work and has not been submitted elsewhere for research. Where other people's work or my own work has been used, this has properly been acknowledged and referenced in accordance with the University of Nairobi's requirements.

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List of Abbreviations

QR	Quantum Repeater
HEP	Heralded Entanglement Purification
HEG	Heralded Entanglement Generation
QEC	Quantum Error Correction
BSM	Bell State Measurement
NISQ	Noisy Intermediate-Scale Quantum
EAAS	Entanglement As A Service

CHAPTER 1

INTRODUCTION

1.1 Research Background

Quantum communication leverages the principles of quantum mechanics to transmit qubits between remote locations. The qubits encode information that is to be transmitted. The qubits themselves can be encoded by various physical entities such as the spin of an atom or the polarization state of a photon (Ruihong and Ying, 2019), but to name a few.

At the heart of quantum communication is the principle of quantum entanglement. Quantum entanglement gives rise to the phenomena of quantum teleportation as a new paradigm protocol for communication (Bennett et al., 1993). In the quantum teleportation protocol, the two endpoints that are, the receiver and sender of information must be entangled to make a complete quantum communication channel which can teleport either qubits or entangled qubits.

Currently, the quantum communication channels make use of either optical fibres or free space, both of which are affected by noise during transmission. When using an optical fibre channel, the photon intensity is attenuated exponentially with transmission distance (Ruihong and Ying, 2019). This limits long-distance communication channels. To counter this long-distance limitation, one has to use quantum repeaters (Briegel et al., 1998).

To use quantum repeaters, the communication channels get fragmented into small segments composed of nodes or relay stations, where each segment has a quantum repeater which plays its role in extending entanglement to its adjacent nodes. In this research project the terms nodes, relay stations and stations are used interchangeably and imply the same thing unless otherwise stated. The length of the segments is chosen such that it is less than the attenuation length of the channel (Das et al., 2021). Entanglement is usually established between every two adjacent nodes in the network. The Quantum entanglement switching protocol is then used between entangled stations to extend the entanglement along the entire link. For instance, a photon from one of the entangled

pairs is teleported from one station to the next, essentially teleporting an entanglement from one node to the next (Gisin and Thew, 2007). Eventually, this forms a large-scale quantum link from the sender to the receiver station (Ruihong and Ying, 2019). Each station has a quantum memory to store the entangled state before being used in the entanglement switching protocol. A quantum entanglement purification protocol is used as well at each station to increase the fidelity of entanglement between the stations (Bennett et al., 1996). The loss of fidelity arises from noise or imperfections in the communication channel.

This research implements the whole infrastructure of a quantum repeater using quantum circuits and executes it on quantum computers to examine the numerous components and protocols that go into making a fully functioning quantum repeater that can be deployed in the real world away from experimental labs.

1.2 Statement of the Problem

Quantum repeaters are necessary for the future quantum communication technologies such as quantum internet (Briegel et al., 1998; Gisin and Thew, 2007). They will extend the range of transmission links to inter-continental global scale, powering the future of global quantum network. Individual components and necessary ingredients that make up quantum repeaters have been tested and demonstrated severally with different technologies and approaches (Gisin and Thew, 2010). However, implementing them from physics principles and beyond the lab to the real world is a huge technological challenge. A lot of research is needed and ongoing not only into the individual components but the whole full-scale architecture of a quantum repeater as well. One such key component under active research is entanglement purification. It is a necessary protocol in first generation or near-term quantum repeaters (Muralidharan et al., 2016). It plays an essential role in ensuring entangled states maintain high fidelity throughout the channel. However, there exists uncertainty as to the right purification strategy to be applied and at what stages it should be applied (Kozlowski et al., 2020). Circuit optimizations to the purification protocols are also necessary to achieve better efficiencies, shorter circuits, fewer purification rounds and less resources cost in

terms of energy and time (Muralidharan et al., 2016).

If we are to realise practical quantum repeaters for future quantum communication technologies, we need to understand how quantum repeaters can be implemented successfully. This research seeks to design and implement a full-scale circuit of a quantum repeater and examine its components. Different purification strategies will be implemented and results analysed to identify the optimum strategy. Performance analysis will be conducted on the quantum repeater under optimization schemes and different purification protocols and the effects studied to bring out any useful correlations.

1.3 Research Objectives

1.3.1 Main Objective

To design and implement a modern quantum repeater infrastructure for future quantum communication technologies and teleportation applications.

1.3.2 Specific Objectives

The specific objectives emanating from the review of the literature are to:

- i. Design and implement a complete quantum circuit of a quantum repeater whose executions emulate its expected workings and behaviour in a real world quantum network.
- ii. Explore various purification strategies to find the optimum strategy.
- iii. Perform optimizations on the purification circuits and studying the effects of various purification protocols on the overall purification optimization scheme.

1.4 Justification and Significance of the Study

Developments in quantum communication will revolutionize the entire communication infrastructure as we know it. A key infrastructure for the future of communication is the quantum internet

or network whose backbone will be quantum repeaters. This exciting technological idea calls for a lot of research into quantum repeaters - to understand how to implement them and integrate them into our current communication protocols.

Luckily enough, our current quantum computers, Noisy Intermediate-Scale Quantum (NISQ) computers can emulate the noisy environment that would be found in a communication channel (Das et al., 2021). This makes them suitable for use in emulating the behaviour of quantum communication channels and the technology that goes into establishing it. Identifying an optimum purification strategy for near-term quantum repeaters means better efficiency of operation of future practical quantum repeaters.

Research into quantum repeaters will help ease the technological challenge of finally implementing and deploying them into the real world. This makes it highly important to examine quantum repeater protocols to examine their performance considering their practical implementation using simulation platforms (Liao et al., 2022).

CHAPTER 2

LITERATURE REVIEW

2.1 Quantum Communication

Quantum communication involves transferring quantum states between remote locations, where the sender is traditionally known as Alice and the receiver Bob (Gisin and Thew, 2007). Alice should be able to send her quantum state $|\psi\rangle_A$, to Bob without loss of its quantum properties. The transmission of a single qubit or an EPR pair (Bennett et al., 1993) is achieved using the phenomenon of quantum teleportation (Bennett et al., 1993), when the two parties share an entangled Bell state, say $|\Phi^+\rangle$. Bell states are an integral element in quantum communication and are applied in many protocols.

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \quad (2.1)$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad |\Phi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \quad (2.2)$$

2.2 Quantum Teleportation

Quantum teleportation is a manifestation of quantum non-locality (Gisin and Thew, 2007). The quantum teleportation protocol between Alice and Bob requires that they are both connected by a classical and quantum channel. The quantum channel is established by sharing an entangled Bell state between them (Bennett et al., 1993). Local quantum operations will be performed on their respective qubits and a classical channel used to transmit Bell measurement results to Bob who uses them to reconstruct the state to be teleported on his side. Essentially, Alice transmits two classical bits to Bob in order to teleport one qubit to Bob.

The teleportation of a single photon through a quantum optical channel follows the procedure as below (Gisin and Thew, 2007):

- Distribution of entanglement. Entangled photons pairs are generated and sent through optical fibers to Alice and Bob in distant locations. This establishes the quantum teleportation channel.
- Bell state measurement (BSM). Alice performs a BSM between the photon from the entangled pair and the photon encoded with the quantum state to be teleported.
- BSM results. Alice transmits the results of the BSM through a classical channel to Bob.
- Unitary operation. Bob performs unitary operations on his photon from the entangled pair based on the result of the BSM sent to him by Alice.
- Teleportation. Instantly, Bob's photon now has the exact quantum state that was initially encoded into one of Alice's photon.

It should be noted that BSM is a difficult process with only partial BSM being realized (Gisin and Thew, 2010). Using linear optics allows distinguishing 2 out of 4 Bell states and provides maximum fidelity of 1 (Herbst et al., 2015).

2.3 Quantum Repeaters

Quantum repeaters are based on quantum relays. Quantum relays teleport entangled states from one node to another in a quantum channel. They are however limited to intermediate distances. Augmenting quantum relays with a few useful components results in a powerful and more elegant device - the quantum repeater. The main components of quantum repeaters are:

- Quantum entanglement switching for swapping entangled states between adjacent nodes.
- Quantum entanglement purification for enhancing fidelity of the entangled states.
- Quantum memory for storing quantum states for efficient on demand retrieval.

These components have a few limitations arising from imperfections of the source of entangled particles, the quantum operations involved and the interconnecting communication channels (Herbst

et al., 2015).

Herbst et al., managed to use a quantum repeater to teleport an entangled state, a photon, between the Canary islands of La Palma to Tenerife, a distance of about 143 km (Herbst et al., 2015). The entanglement swapping experiment used two polarization-entangled photon pairs generated in two identical spontaneous parametric down-conversion (SPDC) sources using a non-linear crystal, β -barium borate (BBO) (Herbst et al., 2015).

2.4 Quantum Memory

Quantum memories offer on demand reversible storage and buffering of qubits across a quantum network without losing the qubit's quantum properties. They can also store entangled states. Quantum memories ought to preserve entanglement just as well as they preserve qubits

The best quantum memory currently is a simple fiber optic loop (Gisin and Thew, 2007). Different approaches are used in the building and realization of quantum memories. Most approaches such as polarization of atom-photon systems and atomic ensembles focus on storing single qubits of single modes. However, other approaches such as rare-earth offers possibility of storing multiple qubits and modes.

2.5 Quantum Repeater Purification Protocols

Purification has to be done to keep the states at high fidelity. The three popular purification protocols

- Bennett's protocol (Bennett et al., 1996). Both Alice and Bob apply Controlled-NOT operations between the two qubits of the Bell pair, then measuring in computational basis and transmitting the results over a classical channel between the nodes (Das et al., 2021). Purification is a success if the measurements agree and the resulting state is kept.
- Deutsch's protocol (Deutsch et al., 1996). The states get represented as Bloch vectors in a Bloch sphere. Alice performs a rotational $R_x(\pi/2)$ on her qubits while Bob performs the

inverse rotation ($-\pi/2$). The rest of the process then proceeds as in Bennett's protocol.

- Multi-qubit entanglement purification. The purification protocol is executed on multiple qubits simultaneously (Das et al., 2021).

Performance aspects considered for purification protocols are; fidelity of the purified Bell pair, success probability and circuit length (Krastanov et al., 2019). All protocols work towards obtaining shorter circuits, achieving higher success rates and better final fidelities (Krastanov et al., 2019).

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Research Approach

This research project takes a theoretical and computational approach. The architectural design of the quantum repeater is modelled based on the use of quantum optics as opposed to earth to satellite links. The conceptual implementation is however, the same.

A convenient approach to understanding the quantum repeater infrastructure is emulating it on simulation platforms such as NetSquid and currently available quantum computers such as those from IBM. The environment exposed to superconducting qubits in the IBM quantum computers can ideally emulate the same environment quantum repeaters will be exposed to when in real-world operation (Das et al., 2021).

3.2 Design and Implementation Approach

Each execution stage and protocol of the quantum repeater gets translated into a modular quantum circuit that can get independently executed on a simulation platform such as IBM Qiskit and NetSquid. The modularity of the code will help test out different purification strategies, protocols and components of the quantum repeater for better analysis. The quantum circuits will first be executed on a native simulation using IBM Qiskit's QASM simulator before being finally executed on actual real IBM quantum computers.

Performance analysis will get done based on the fidelity of the purified Bell pair, the bandwidth of the channel and the stability of the quantum repeater (Krastanov et al., 2019). Optimisation schemes will be applied to the entanglement purification circuits to analyse their performance. Attention will get paid to the limitation imposed by working with finite resources.

CHAPTER 4

THEORETICAL INPUT

4.1 Quantum Repeater

We follow along the sequence of events, protocols and operations executed by a quantum repeater. In this case, a three-segmented channel is considered, running from station A to B to C and finally D. The goal is to implement a full quantum communication link from A to D via the sub-stations B and C.

4.1.1 Entanglement Generation

The circuit implementation that prepares and generates an entangled pair takes in two qubits as input and performs Hadamard and Controlled-NOT unitary gate operations on them. Each EPR pair gets distributed to adjacent nodes. One pair, $|\Phi^+\rangle_{AB}$ gets to entangles A and B while the other pair, $|\Phi^+\rangle_{CD}$ gets to entangles C and D.

4.1.2 Quantum Entanglement Distribution

The first distribution is that of the EPR pairs $|\Phi^+\rangle_{AB}$ and $|\Phi^+\rangle_{CD}$ to their respective nodes, each node taking one of the qubits from a pair. The distribution stage that involves the quantum repeater requires the distribution of entanglement along the transmission line from sender to receiver. This entanglement distribution relies on quantum memories, entanglement purification protocols and entanglement swapping protocols to distribute entanglement between nodes from the start of the communication link to the end.

4.1.3 Quantum Memory

The entangled state $|\Phi^+\rangle_{AB}$ is momentarily stored in a quantum memory and only retrieved when it is needed to perform entanglement distribution between nodes B and C to get the entangled state $|\Phi^+\rangle_{BC}$.

4.1.4 Quantum Entanglement Purification

The three common purification protocols: Bennett's protocol (Bennett et al., 1996), Deutsch's protocol (Deutsch et al., 1996) and multi-qubit purification protocol (Das et al., 2021), will be used and experimented with since each has its complexity of implementation. They also provide varying fidelity levels and produce varying degrees of overhead during circuit operation.

4.1.5 Quantum Entanglement Swapping

The two Bell pairs form a combined 4-qubit quantum state $|\psi\rangle_{ABCD} = |\Phi^+\rangle_{AB} \otimes |\Phi^+\rangle_{CD}$. BSM measurement is performed between the qubits in B and C. Depending on the results of the measurement, an appropriate Pauli correction operation I, Z, X, Y gets performed on the qubit in D (Das et al., 2021). The result is the projection of qubits in A and D into the state $|\Phi^+\rangle_{AD}$ and the entanglement between nodes B and C in the state $|\Phi^+\rangle_{BC}$. Teleportation can now occur directly from node A to D because the entanglement distributed to D from A maintains a complete quantum communication link not limited by spatial separation.

CHAPTER 5

RESULTS AND DISCUSSION

5.1 Experimental Conventions

Here, we detail the various conventions used in this research work as regards the experimental setup and quantum circuits as will be seen in the figures displayed.

Foremost, the sender of information is always denoted as Alice while the recipient is always Bob.

Other intermediary players are denoted either as Charlie or Telamon, unless otherwise stated.

All quantum gates are used and denoted as provided by the Qiskit framework. Any custom gates or instructions are labelled appropriately.

5.2 Building Quantum Repeater Components

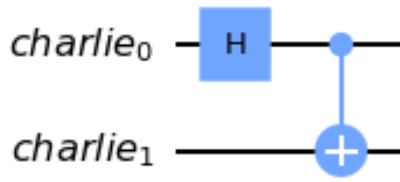
First, the various individual and conceptual components making up the quantum repeater are designed and implemented and studies before being all compiled to the final complete quantum repeater design.

5.2.1 Bell States

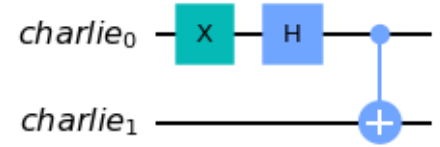
The prototypical entangled states of a two-qubit quantum system are the maximally entangled qubits, usually denoted as: $|\Phi^\pm\rangle$ and $|\Psi^\pm\rangle$. The Hadamard and Controlled-NOT gates are used to generate all the Bell pairs:

5.2.2 Quantum Teleportation

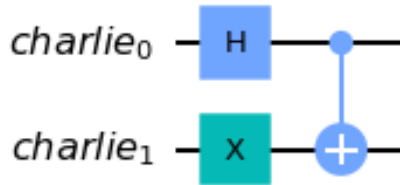
Next, the teleportation protocol was studied. The protocol was simulated natively using Qiskit's QASM Simulator and later on a real IBM quantum computer.



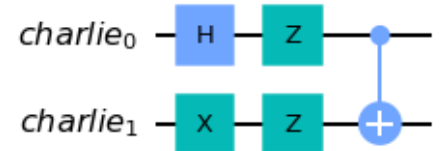
(a) $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$



(b) $|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$



(c) $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$



(d) $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

Figure 5.1: The four Bell pairs

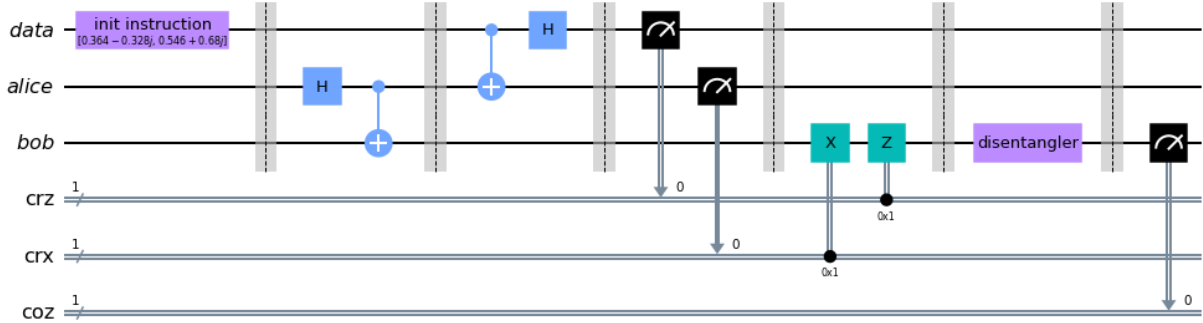


Figure 5.2: Quantum circuit for the teleportation protocol executed on the QASM Simulator

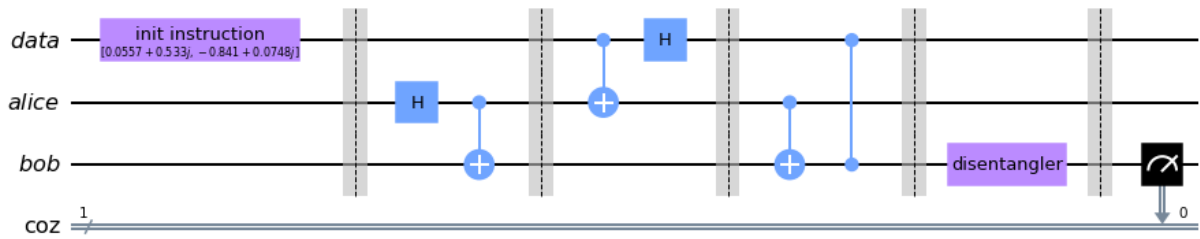


Figure 5.3: Quantum circuit for the teleportation protocol executed on a real IBM quantum computer.

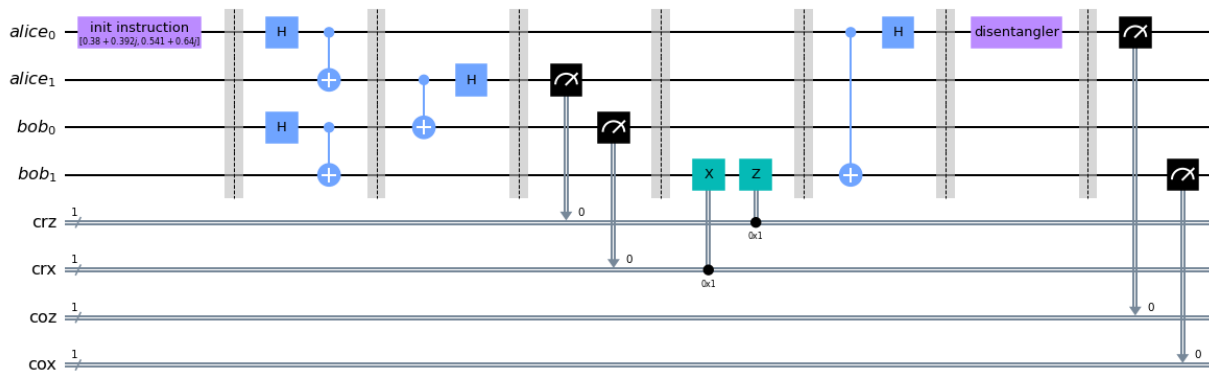


Figure 5.4: Quantum circuit for quantum entanglement swapping protocol

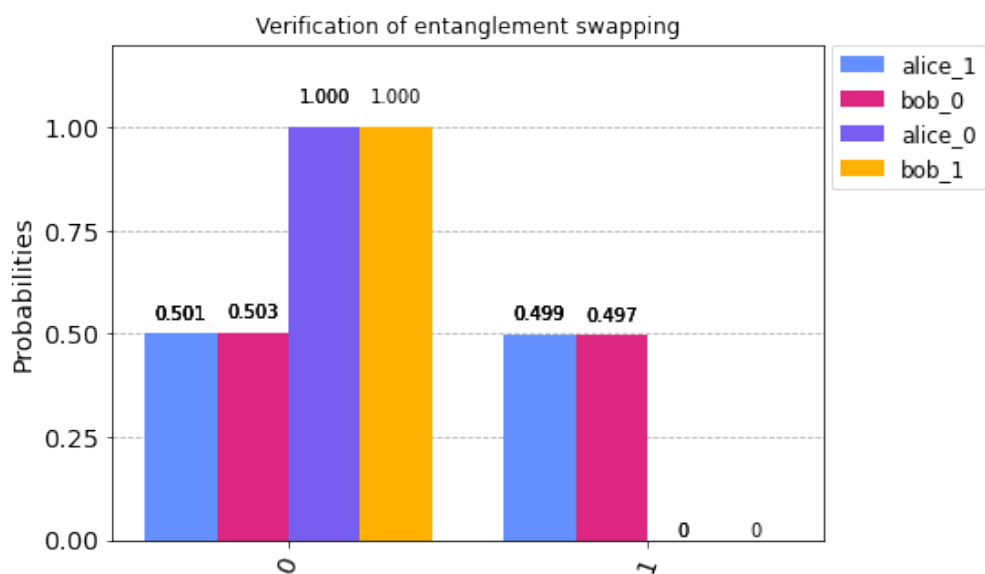


Figure 5.5: Results for the verification of quantum entanglement swapping

5.2.3 Quantum Entanglement Switching

5.2.4 Quantum Purification

5.3 Complete Quantum Repeater Architecture

5.4 Purification Strategies

5.5 Effects of Purification Protocols on Purification Optimization schemes

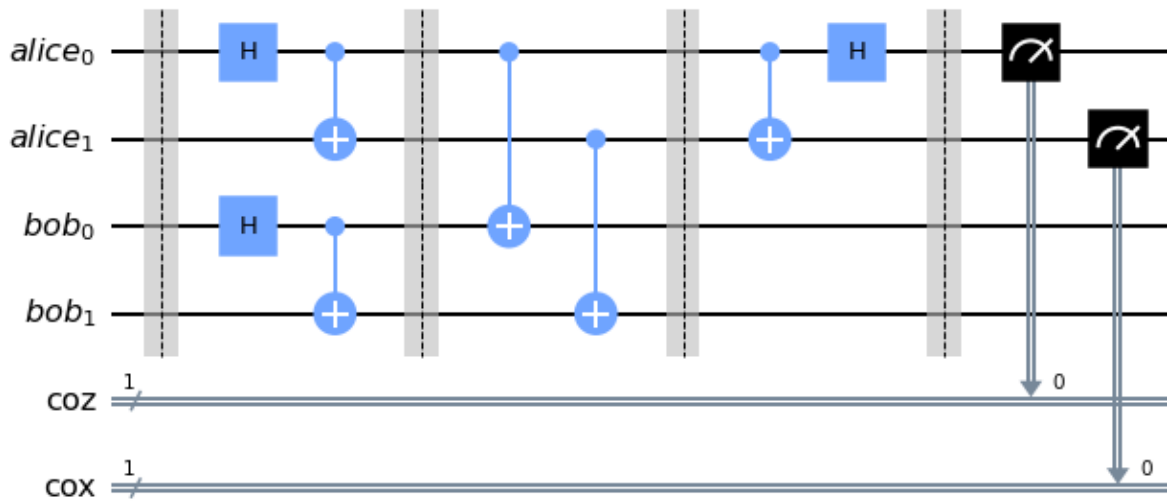


Figure 5.6: Quantum circuit for Bennett's purification protocol

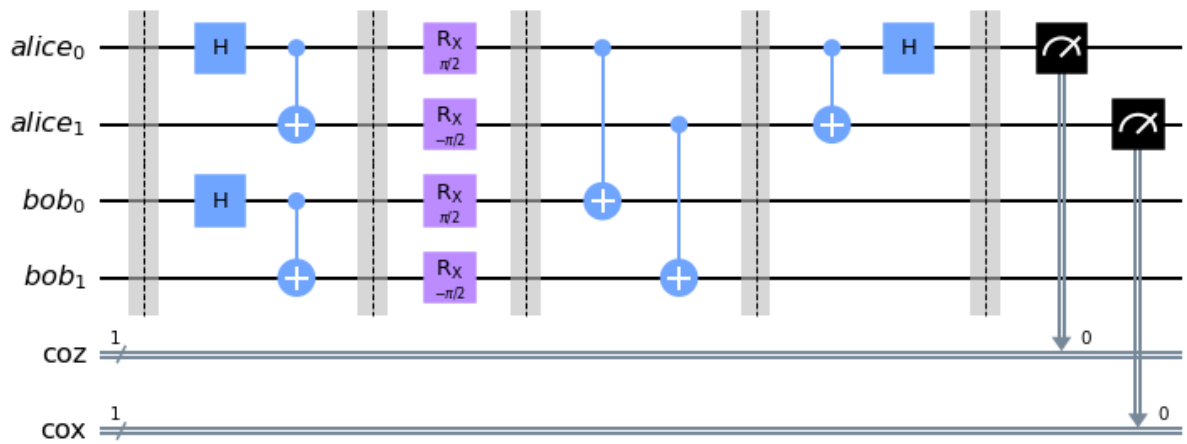


Figure 5.7: Quantum circuit for Deutsch's purification protocol

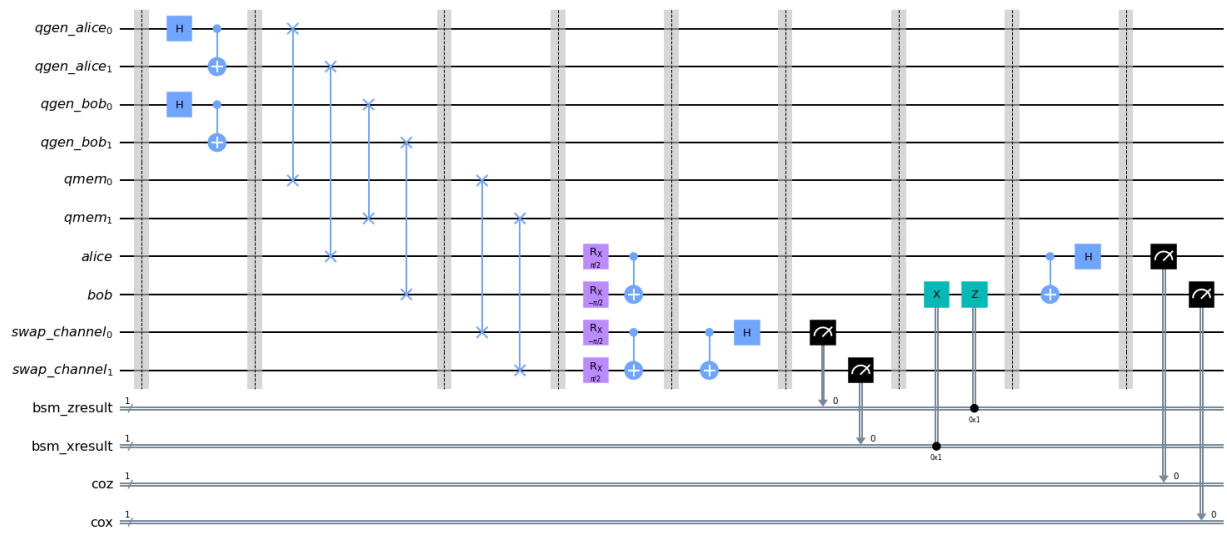


Figure 5.8: Quantum circuit for the full complete quantum repeater, implementing Deutsch's purification protocol just before the swapping protocol stage.

CHAPTER 6

CONCLUSION AND WAY FORWARD

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