

2. Gradient Descent for Multiple Variables

August 6, 2021

1 Cost function and Gradient Descent with Multiple Features

From the previous section we derived a matrix expression of the hypothesis function for multiple features. Given $h_{\theta}(x) = \Theta^T \times X$ where Θ is given as $\begin{pmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{pmatrix}$

(the parameters of the features) and X is given as $\begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{pmatrix}$

Now our cost function can be given in terms of the feature vector Θ as below

$$J(\Theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^i) - y^i)^2 \quad (1)$$

Substituting this into the gradient descent algorithm, we get

$$\begin{aligned} \{\theta_j &= \theta_j - \alpha \times \frac{\partial}{\partial \theta_j} J(\Theta) \\ &= \theta_j - \frac{\alpha}{2m} \sum_{i=1}^m \frac{\partial}{\partial \theta_j} (h_{\theta}(x^i) - y^i)^2 \\ &= \theta_j - \frac{\alpha}{2m} \sum_{i=1}^m \frac{\partial}{\partial \theta_j} (h_{\theta}(x^i) - y^i)^2 \\ &= \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^i) - y^i) x_j^i; \text{ for } j = 1, \dots, n \end{aligned} \quad (2)$$

because we defined the feature x_0 in the previous section and the vector X contains it, we can arrive at the general definition of the gradient descent function defined in equation 2