Quantum Computing Approaches for the Quadratic Knapsack Problem (QKP)

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June 9, 2025

Abstract

Brief summary of the QKP, motivation for using quantum computing approaches.

1 Introduction

- Definition of the Quadratic Knapsack Problem (QKP)
- Talk about QUBOs.
- Motivation to explore quantum algorithms

2 Literature Review

Chronologically organized presentation of key works relevant to QKP and quantum computation.

2.1 Quantum-Inspired Evolutionary Algorithms (QIEA)

These algorithms are a classical heuristic that simulates concepts of quantum computing, but doesn't require quantum hardware to execute. QEIA is **inspired** by quantic mechanics, but can be performed **classically**.

2.1.1 *A Novel Quantum Evolutionary Algorithm for Quadratic Knapsack Problem, (2009)* [1]

- **Approach:** QKP has a graph-theoretic interpretation, is a generalization of the *Clique* problem. Algorithm starts with the *greedy* solution. The first Q-gate has a greater tendency for exploration, the other two Q-gates, looking to converge towards the best solution. Observations are made to checking the constraint.

- **Contribution:** Algorithm uses concepts of superposition and quantum measurement (not really interesting).

2.1.2 *An Angle-expressed Quantum Evolutionary Algorithm for Quadratic Knapsack Problem, (2019)* [2]

- **Approach:** The authors don't start in uniform random values, instead, define an "initial value density" for each item $d_i = \frac{p_i}{w_i}$. Q-Gate of rotation Bit, the current state versus the best previous global solution are compared. Observations are made to checking the constraint. Implementation a H_ε-Gate prevents states from being located.

- **Contribution:** The idea of starting with a preferential state.

2.2 Quantum Approximate Optimization Algorithm (QAOA)

QAOA is a variational quantum algorithm used to solve combinatorial optimization problems. It works by encoding the problem into a quantum circuit and using a classical optimizer to find the optimal parameters that yield the best solution. QAOA can solve binary optimization problems like QUBOs.

2.2.1 *Translating Constraints into QUBOs for the Quadratic Knapsack Problem, (2023)* [3]

- **Approach:** They present six different QUBO formulations of the QKP, all of these formulations use a different technique to include the weight constraint into the objective function. Is introduced penalty values for the constraint. The results are obtained using the D-Wave implementation as a simulated annealing. The best performance is obtained by a formulation that uses no auxiliary variables for modelling the inequality constraint.

- **Contribution:** The authors present different and useful QUBOs formulations to implement in Quantum Annealing, but all require more than n qubits to transform the problem into QUBO.

2.2.2 *Solving Quadratic Knapsack Problem with Biased Quantum State Optimization Algorithm, (2024)* [4]

- **Approach:** They implement a QAOA algorithm, and use gradient and gradient-free optimizers with initial parameters (β, γ). They proposed a novel initial state that allow to execute in a low-depth circuits, and without using more qubits as is proposed by [3] in the six QUBOs formulations. Furthermore, don't need to include the penalty method such [2]. This article is the first to apply QAOA in solving the quadratic knapsack problem. The results are obtained using the qiskit-aer simulator.

- **Contribution:** The authors propose a new quantum state that improves the QAOA performance and does not require additional variables to transform the problem into QUBO.

3 Summary of Existing Methods

Strengths and weaknesses. Identified gaps or open problems.

4 Proposed Method

5 Experimental Plan / Future Work

- Simulation tools (Qiskit, D-Wave Ocean, PennyLane)
- Benchmarks or datasets

References

[1] A. Narayan, C. Patvardhan, A novel quantum evolutionary algorithm for quadratic knapsack problem, in: 2009 IEEE International Conference on Systems, Man and Cybernetics, IEEE, 2009, pp. 1388–1392.

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[3] T. Bontekoe, F. Phillipson, W. v. d. Schoot, Translating constraints into qubos for the quadratic knapsack problem, in: International Conference on Computational Science, Springer, 2023, pp. 90–107.

[4] H. P. N. Ha, V. H. Nguyen, A. S. Ta, Solving quadratic knapsack problem with biased quantum state optimization algorithm, in: M. Sevaux, A.-L. Olteanu, E. G. Pardo, A. Sifaleras, S. Makboul (Eds.), Metaheuristics, Springer Nature Switzerland, Cham, 2024, pp. 268–280.