Estimating the Life-Cycle Model of Labor Supply of Males in Kyrgyzstan

by

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Abstract

The aim of this study is to present and estimate MaCurdy's empirical model of the labor supply over the life-cycle using the sample of prime-age males in Kyrgyzstan. The data set used in the analysis is taken from Life in Kyrgyzstan panel survey of households and individuals. The sample includes 60 males who were continuously employed, married, and who were 25–46 years old during the sample period which covers years 2010–2012. 2SLS estimation technique was applied to estimate the labor supply function and the specification for the individual effects. The former provided the estimate of the intertemporal substitution elasticity, the latter determined the parameters necessary to describe the labor supply response to parametric wage changes. Results suggest that, contrary to the existing literature and MaCurdy's own findings,the estimated coefficient for the intertemporal elasticity is negative, however statistically insignificant. Uncompensated and compensated elasticities are estimated to be negative and close to zero. The lack of predictive power of the proposed instruments is likely to be the main reason for the obtained results. This issue can be addressed in future research.

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Chapter 1

Introduction

The aim of this study is to present and estimate the empirical model of labor supply over the life-cycle, which was described in the classic paper of MaCurdy (1981), and to characterize the labor supply behavior of prime-age males in Kyrgyzstan.

Most empirical research on labor supply deals with the static model that analyzes the behavior of an individual who chooses whether to work and how many hours to work in a single time period (Borjas, 2005, p.64). Borjas also argues that the main drawback of this one-period setting is that the model ignores the fact that consumption and leisure decisions are made by the individual in each period over his entire life. "There is a great deal of evidence" suggesting that the average individual has a certain profile of life cycle path of wages and working hours (Borjas, 2005, p.64). Hundreds of studies have estimated Mincer (1974) earnings function (Heckman et al., 2003) and concluded that the lifetime wage paths are concave: wages are low when the individual enters the market, they reach the peak at about age 50 and remain stable or decrease thereafter. The individual's supply of labor exhibits a similar pattern (Borjas, 2005, p.64). Thus, the dynamics of wage and hours of work suggest that the individual's preferences towards consumption and leisure change over time. Therefore, the estimates of the labor supply elasticity reported in studies (some of them are summarized in Borjas (2005, pp.45-46)) that assume one-period decision-making do not allow to differentiate between factors determining individuals' dynamic behavior and factors determining differences in consumption and hours of work across consumers. To address this issue, the life-cycle models of individual decision-making were elaborated (Ghez and Becker, 1975; Heckman, 1974; Heckman and Macurdy, 1980; Lucas and Rapping, 1969; Mincer, 1962). The life-cycle model of labor supply developed by MaCurdy (1981) assumes decision-making in multiple periods, in which individuals maximize the lifetime utility subject to constraints. The model extends Friedman's (1957) permanent income theory to a framework with time varying relative price of consumption and leisure. This intertemporal setting allows distinguishing between the labor supply effects coming from the across-workers differences and those associated with movements along the life-cycle wage path.[1]

From microeconomic perspective, it is already a convention that the estimates of the labor supply elasticity are quite small (Keane and Rogerson, 2012). Ghez and Becker (1975) reported one of the earliest sets of estimates of the intertemporal substitution elasticity in their seminal paper. The coefficients obtained in their study range from -0.068 to 0.44. Smith (1975) reports the intertemporal elasticity of 0.32 in the analysis of cross-sectional data aggregated by age cohorts. Similar results are found in three classic papers by: MaCurdy (1981), Browning et al. (1985), and Altonji (1986) with the estimated elasticities of 0.10–0.23, 0.09, and 0.31 respectively. Since the intertemporal elasticity (or changes in labor supply coming from movements along the age-earnings profile) is the upper bound for compensated and/or uncompensated elasticities (or changes in labor supply coming from changes of the entire age-earnings profile), the latter are even smaller (Keane and Rogerson, 2012; MaCurdy, 1981).

There is not much research done in this field in Kyrgyzstan. This study aims to make the first step by estimating the life-cycle model of labor supply from the sample of married males following MaCurdy's methodology. The analysis makes use of the country's most recent longitudinal data set for the period 2010–2012. The rest of the paper is organized as follows: chapter 2 presents the life-cycle model of consumption and leisure, chapter 3 describes the empirical specifications for the presented life-cycle model, chapter 4 focuses on the interpretation of different elasticities, chapter 5 provides the empirical analysis, and chapter 6 concludes.

[1] The analysis is performed under the assumption that individuals live in the world of perfect certainty. MaCurdy says, however, that with some minor modifications the model provides accurate description of consumers' dynamic behavior under uncertainty as well (1981, p.1063).

Chapter 2

A Model of the Individual's Choice of Consumption and Leisure over the Life-Cycle

This chapter presents the model described in MaCurdy's (1981) influential paper. The model assumes that the utility of the individual, in period t, U_t, is given by the amount of consumption of market-purchased good, C_t, and the number of hours spent in nonmarket activities, L_t, and is concave: $U_t = U(C_t, L_t)$. The individual is assumed to operate in an environment of perfect certainty. The monotonically increasing lifetime utility function of the individual, G, assuming that a lifetime consists of $T + 1$ periods, is then given by:

$$G = \sum_{t=0}^{T} \frac{1}{(1+\rho)^t} U(C_t, L_t), \tag{2.1}$$

where ρ is the individual's rate of time preference. The worker then chooses C_t and L_t to maximize the lifetime preference function, G, subject to a wealth constraint:

$$A_0 + \sum_{t=0}^{T} R_t H_t W_t = \sum_{t=0}^{T} R_t C_t, \tag{2.2}$$

where A_0 is the initial assets of the individual, $H_t \equiv N - L_t$ is the number of hours of work in period t with N being the total number of hours in the period, W_t is the real wage rate in period t assumed to be exogenously given, $R_t \equiv 1/[(1+r_1)(1+r_2)\cdots(1+r_t)]$ is the discount rate that converts the value of the real income in period t into the corresponding value in period 0 with $R_0 = 1$, and r_t is the real interest rate in period t, at which the individual can freely borrow or lend.

The conditions for an optimum are the wealth constraint (2.2), and the following:

$$\frac{\partial U_t}{\partial C_t} = R_t(1+\rho)^t\lambda, \qquad t = 0,\ldots,T, \tag{2.3}$$

$$\frac{\partial U_t}{\partial L_t} \geq R_t(1+\rho)^t\lambda W_t, \quad t = 0,\ldots,T, \tag{2.4}$$

where λ is the Lagrange multiplier associated with the wealth constraint. It represents the marginal utility of the initial wealth. Condition (2.3) determines the individual's optimal choice of consumption. It states that the individual chooses consumption so that its marginal utility equals the marginal utility of the initial wealth adjusted for a discount factor which is determined by the interest rate, r_t, and individual rate of time preference, ρ. Condition (2.4) characterizes the individual's optimal choice of labor supply. If it is equality, then the individual supplies positive number of hours to the market (i.e. $H_t > 0$). If it is strict inequality, then the individual spends all time on leisure (i.e. $H_t = 0$).

Using implicit function theorem, equations (2.3) and (2.4) can be solved for consumption and labor supply as functions of the form:

$$C_t = C[R_t(1+\rho)^t\lambda, W_t], \quad t = 0,\ldots,T, \tag{2.5}$$

$$H_t = H[R_t(1+\rho)^t\lambda, W_t], \quad t = 0,\ldots,T. \tag{2.6}$$

MaCurdy refers to the expressions (2.5) and (2.6) as "λ constant" consumption and labor supply functions, because they represent the demand functions for consumption and hours of work holding constant the marginal utility of wealth. Concavity of U_t and normality of goods C_t and L_t imply[1]:

$$\frac{\partial C_t}{\partial \lambda} < 0 \quad \frac{\partial C_t}{\partial (R_t(1+\rho)^t)} < 0, \tag{2.7}$$

$$\frac{\partial H_t}{\partial \lambda} \geq 0 \quad \frac{\partial H_t}{\partial (R_t(1+\rho)^t)} \geq 0 \quad \frac{\partial H_t}{\partial W_t} \geq 0. \tag{2.8}$$

Substituting (2.5) and (2.6) into (2.2) yields:

$$A_0 + \sum_{t=0}^{T} R_t W_t H\left[R_t(1+\rho)^t\lambda, W_t\right] = \sum_{t=0}^{T} R_t C\left[R_t(1+\rho)^t\lambda, W_t\right], \tag{2.9}$$

which implicitly defines the optimal value of λ. λ can then be expressed as a function of

[1] See the proof in Heckman (1974, p.191)

initial assets, lifetime wages, the rate of time preference and interest rates:

$$\lambda = \lambda(W_t, A_0, \rho, r_t), \quad t = 0, \ldots, T.$$

It can be shown that the following expressions are true for λ[2]:

$$\frac{\partial \lambda}{\partial A_0} < 0 \;\; \frac{\partial \lambda}{\partial W_t} \leq 0, \quad t = 0, \ldots, T, \tag{2.10}$$

which show that the marginal utility of initial wealth decreases with both initial assets and wages. Defined by (2.9), λ is a permanent effect that affects the individual's choice of consumption and leisure. It is a "sufficient statistic" for all historic and future information that one needs to choose C_t and L_t optimally, because in an environment of perfect certainty each individual knows his wage-time path from the initial period, therefore, λ does not have to be revised. Thus, λ constant functions completely describe the individual's behavior over the life time, which allows to analyze the labor supply effects of different levels of λ based on differences in lifetime wage profiles, initial assets, individual rates of time preference, and tastes between individuals (or for otherwise identical individuals) (MaCurdy, 1981, p.1063).

[2] See the proof in Heckman (1974, p.192)

Chapter 3

An Empirical Model of Labor Supply over the Life-Cycle

This chapter presents an empirical model based on the theoretical model described above. As in the previous chapter, the notation is consistent with that of MaCurdy (1981). The model is a two-step procedure: the first step provides the specification for estimating the intertemporal labor supply elasticity; the second step describes the framework for estimating effects of parametric wage changes.

3.1 Labor Supply Response to Movement along a Life-Cycle Wage Path

The following functional form is assumed for individual i's utility at time t:

$$U_{it} = U_i(C_{it}, L_{it}) = \Upsilon_{1it}(C_{it})^{\omega_1} - \Upsilon_{2it}(H_{it})^{\omega_2}, \tag{3.1}$$

where $\omega_2 > 1 > \omega_1 > 0$ are constant parameters that are the same for all individuals, and Υ_{1it} and Υ_{2it} are positive time-varying determinants of tastes for consumption and work respectively.

Since this model is designed to investigate males' labor supply behavior, it is reasonable to assume that the individual's optimal choice of labor supply is the interior solution to condition (2.4) (that is, assuming equality so that $H_{it} > 0$). With this assumption, the λ constant labor supply function takes the following form:

$$\ln H_{it} = \frac{1}{\omega_2 - 1}\left[\ln \lambda_i - \ln \Upsilon_{2it} - \ln \omega_2 + \ln\left(R_t(1+\rho)^t\right) + \ln W_{it}\right]. \tag{3.2}$$

In order to ensure that tastes towards consumption and work are distributed randomly across individuals, Υ_{2it} is defined as:

$$\Upsilon_{2it} = \sigma_i - u^*_{it}, \tag{3.3}$$

where σ_i represents unobserved time-invariant individual-specific factors, and u^*_{it} is a zero-mean disturbance. If we plug (3.3) into (3.2), the labor supply function turns to:

$$\ln H_{it} = F_i + \delta \sum_{k=0}^{t} (\rho - r_k) + \delta \ln W_{it} + u_{it}, \tag{3.4}$$

where $F_i = \frac{1}{\omega_2 - 1}(\ln \lambda_i - \sigma_i - \ln \omega_2)$, $\delta = \frac{1}{\omega_2 - 1}$, $u_{it} = \delta u^*_{it}$, $r_0 = \rho$, and log linear approximations $\ln(1 + r_t) \approx r_t$ and $\ln(1 + \rho) \approx \rho$ were used. Assuming constant real interest rate, r, labor supply function given by (3.4) reduces to:

$$\ln H_{it} = F_i + bt + \delta \ln W_{it} + u_{it}, \tag{3.5}$$

where $b = \delta(\rho - r)$.

Equations (3.4) and (3.5) are two alternative specifications for the estimation of the intertemporal labor supply elasticity, δ. Since the intercept term, F_i, is correlated with W_{it} through λ_i, estimating (3.4) or (3.5) would lead to a biased estimate of δ. To avoid this bias, F_i is treated as a fixed effect. According to proposition (2.8), δ is expected to be positive. MaCurdy refers to δ as an "intertemporal substitution elasticity", since there is no income effect associated with a change in the wage rate at each period of the lifetime, because the lifetime wage path is known to the individual since the initial period by perfect certainty assumption and there are no other sources of a wage change than the movement along the path.

Estimating labor supply equations such as (3.4) or (3.5) is considerably simplified by the fact that all the necessary variables are observed within the sample. Since λ_i is contained in F_i, its estimated effect on the supply of labor reflects all the historic and future information that individual i needs to choose consumption and leisure in each period t. Thus, there is no need to formulate the individual's future plans and take into account future (that is, out-of-sample) wage rates when estimating the labor supply response at any period t, which is a significant simplification of the estimation process (MaCurdy, 1981, p.1066).

3.2 Labor Supply Response to a Parametric Wage Change

Equations (3.4) and (3.5) produce the estimate of the intertemporal substitution elasticity. However, this is not enough to describe all aspects of labor supply behavior of individuals. It is necessary to characterize labor supply responses to individual-specific factors, such as lifetime wage profile or initial wealth. These factors are contained in λ and, thus, influence labor supply through F_i. The value of λ is determined by equation (2.9), which does not provide a closed-form expression for λ.

To estimate individual effects, F_i is assumed to be approximated by the following function:

$$F_i = Z_i\psi + \sum_{t=0}^{K} \gamma_t \ln W_{it} + A_{i0}\theta + a_i, \tag{3.6}$$

where Z_i is a vector of observed factors, a_i is an error term, and ψ, γ_t, and θ are the parameters assumed to be constant among individuals. Specification (3.6) implicitly assumes that lifetime consists of $K + 1$ periods. According to expressions given by (2.10), parameters γ_t's and θ are expected to be negative.

Contrary to labor supply equations (3.4) and (3.5), we do not observe the variables that are present in the specification for individual effects (3.6). The issue is especially important with the explanatory variables – lifetime wage rates and initial assets – because the estimate for the dependent variable, F_i, can be obtained from estimating labor supply equation.

To address this issue, certain assumptions regarding the lifetime wage and income paths are required. In particular, the lifetime wage path is assumed to follow a quadratic profile in age:

$$\ln W_{it} = \pi_{0i} + \pi_{1i}\, age_{it} + \pi_{2i}\, age_{it}^2 + e_{it}, \tag{3.7}$$

where π_{0i}, π_{1i}, and π_{2i} are linear functions defined as:

$$\pi_{ji} = M_i g_j, \quad j = 0, 1, 2, \tag{3.8}$$

where M_i is a vector containing observable age-invariant exogenous determinants of wages such as education, $g_j, j = 0, 1, 2$, are parameter vectors, and e_{it} is an error term.

Similarly, the lifetime property income path is assumed to follow a quadratic profile in age:

$$Y_{it} = \alpha_{0i} + \alpha_{1i}\, age_{it} + \alpha_{2i}\, age_{it}^2 + v_{it}, \tag{3.9}$$

where $Y_{it} = A_{it} r$ is the income flow generated from the investment of A_{it} assets at the interest rate of r and α_{0i}, α_{1i}, and α_{2i} are linear functions defined as:

$$\alpha_{ji} = S_i q_j, \quad j = 0, 1, 2, \tag{3.10}$$

where S_i is a vector of observable age-invariant factors, $q_j, j = 0, 1, 2$, are vectors of parameters, and v_{it} is an error term; α_{0i} can then be used in place of the initial wealth, A_{0i}, in equation (3.6).

Substituting equation (3.7) and the expression $\alpha_{0i} = A_{0i} r$ into (3.6) yields:

$$F_i = Z_i \psi + \pi_{0i} \bar{\gamma_0} + \pi_{1i} \bar{\gamma_1} + \pi_{2i} \bar{\gamma_2} + \alpha_{0i} \bar{\theta} + \eta_i, \tag{3.11}$$

where

$$\bar{\gamma_j} = \sum_{t=0}^{K} age^j \gamma_t, \quad j = 0, 1, 2, \quad \bar{\theta} = \frac{\theta}{r}.$$

Estimating equation (3.11) provides the full set of parameter estimates needed to describe labor supply responses to parametric wage changes. The estimates of $\bar{\gamma_0}$, $\bar{\gamma_1}$, and $\bar{\gamma_2}$ together with the estimate of intertemporal substitution elasticity, δ, given by (3.4) and/or (3.5), determine the way labor supply reacts to either change in the slope of the lifetime wage profile or a shift of the entire profile. The estimate of $\bar{\theta}$ allows to describe how labor supply is affected by changes in initial property income. Thus, the empirical strategy undertaken in this paper completely describes an individual's life-cycle labor suppy behavior.

Chapter 4

Interpretation of Substitution Elasticities

To proceed with estimations it is necessary to distinguish between the types of labor supply elasticities and their interpretation. As it was mentioned in chapter 3, page 7, the parameter $\delta > 0$ measures the labor supply response to a wage change coming from the movement along the lifetime wage path. The parameters estimated by equation (3.11) help to determine the hours of work response to parametric wage changes, that is, the changes coming from the differences across individuals, or the so-called compensated and uncompensated substitution elasticities. The description here follows closely the notation of MaCurdy.

Consider a setting of two individuals with concave lifetime wage paths II and III respectively such that wage path III is higher than wage path II at some period t' and identical during the rest of the lifetime. Denote the absolute value of this wage difference between individuals II and III in period t' by Δ. The wage difference generates two effects for the supply of labor of individual III. First, he decreases his value of F by $\gamma_{t'} \times \Delta$, as suggested by equation (3.6). Because F contains λ, expression (2.8) implies a decrease in the supply of hours of work. Thus, at all periods except at period t', the amount of hours supplied to the market by individual III will be lower by some constant compared to individual II. Second, according to the labor supply equation, individual III's supply of labor will increase by $\delta \times \Delta$ at period t'. Hence, the total effect of the wage difference is $(\delta + \gamma_{t'}) \times \Delta$, which is positive if the estimated intertemporal substitution is large enough. If this is true, individual III will work more than individual II in period t'.

The effects $(\delta + \gamma_{t'})$ and $\gamma_{t'}$ discussed above stand for cross- and own-uncompensated substitution elasticities respectively. These elasticities reflect the labor supply response to a parametric wage change. Since intertemporal substitution elasticity is greater than uncompensated elasticities, wage changes due to the movement along the lifetime wage profile are expected to have a greater effect on the supply of labor than parametric wage changes.

Now suppose that we add individual I to the previous two-individual setting. Individual I faces a lifetime wage path that is lower than that of individual II by some constant over the lifetime. The parametric wage change in this situation is the shift from wage path I to wage path II. This shift is equivalent to the increase of the intercept π_0 from equation (3.7) by the amount $\Delta\pi_0$. Such a wage change introduces two effects on the supply of labor in each period. The first effect is that individual I has to decrease his value of F by $\bar{\gamma_0} \times \Delta\pi_0$, as suggested by equation (3.11). This is followed by a lower supply of labor in all periods. The second effect is associated with an increase in the number of hours worked by the amount $\delta \times \Delta\pi_0$, according to the labor supply equation. The total effect is, thus, $(\delta + \bar{\gamma_0}) \times \Delta\pi_0$, which can be either positive or negative depending on the corresponding values of δ and $\bar{\gamma_0}$.

Having described the intertemporal and uncompensated substitution elasticities, MaCurdy stresses the importance of distinguishing compensated substitution elasticities from the above two. He shows that the own- and cross-compensated elasticities can be written as $\delta + \gamma_t - E_t\theta$ and $\gamma_t - E_t\theta$ respectively, where $E_t = H_t W_t$ is real earnings in period t. In sum, all three measures are related and become even exactly the same once there are no income effects. Provided that leisure is a normal good, intertemporal and uncompensated substitution elasticities correspond to an upper and a lower bound for compensated elasticity, or $\delta > \delta + \gamma_t - E_t\theta > \delta + \gamma_t$.

Chapter 5

Empirical Analysis

The first section of this chapter describes the data set, sampling procedure and the variables used in the empirical analysis. The second section presents the results and discussion.

5.1 Data and Variables

The estimations are based on the sample from the "Life in Kyrgyzstan" panel survey of households and individuals under the research project prepared by German Institute for Economic Research DIW Berlin and contributors (2016). The sample is composed of 60 prime-age males for the years 2010–2012[1] resulting in a total of 180 observations. To be included in the sample individuals had to satisfy several criteria: be continuously employed during the sample period and receive money for the work on a regular basis; be continuously married for the sample period; be 25–46 years old at the time the survey took place; be the head of the household. The variable weekly hours of work reported for the last week was used as a regressand and the wage rate, obtained by dividing the monthly earnings by the number of hours worked for the last week and 4, was used as a regressor in the labor supply equation. Most individuals report wages in Kyrgyz National currency – soms. For those who reported in dollars, the wage was converted into soms by the average exchange rate for the corresponding year.[2]

The wage variable deserves special attention in the analysis. As suggested by Borjas (1980), defining the wage rate as the result of the division of wages by the number of hours worked for the corresponding period (be that a week, a month, or a year) creates a spurious correlation between dependent and independent variables, which leads to a downward bias in the estimate of the labor supply elasticity due to measurement errors in hours worked.

[1] The data for the year 2013 became available, however, due to a substantial amount of errors and missing values, it is not used in this work.

[2] According to the National Bank's of Kyrgyz Republic data (2016), the average USD/KGS exchange rate for the years 2010–2012 is about 46.4 KG soms per 1 US dollar.

Borjas refers to this bias as a "division bias". He then proposes two techniques to mitigate this bias. In terms of the above notation, let us define the wage rate as:

$$W = E/H, \tag{5.1}$$

where E is earnings for some period, and H is the number of hours worked for the corresponding period. The first technique is associated with instrumenting the wage rate defined by (5.1) by the fitted values obtained from estimating the earnings function:

$$\ln W = f(X), \tag{5.2}$$

where X is a vector of wage determinants. Theoretically, the wage rates predicted by (5.2) are not subject to measurement errors in hours of work, and should, thus, avoid the spurious correlation.

The second method proposed by Borjas involves the use of another measure of earnings, which is not related to the measure of labor supply. In his study he discusses the implications of having two alternative measures of labor supply – usual hours of work, H_U, and hours worked for the last week, H_W. This allows to obtain usual wage rate, E/H_U , and wage rate for the last week, E/H_W. Although, obviously, the former is spuriously correlated with usual hours of work, the same is not true for the latter. Thus, Borjas estimates the"cross-divide" labor supply equations of the form:

$$\ln H_U = \alpha + \beta \ln(E/H_W) + \xi, \tag{5.3}$$

or, alternatively:

$$\ln H_W = \alpha + \beta \ln(E/H_U) + \xi. \tag{5.4}$$

In his paper, Borjas provides a substantial evidence that proposed techniques greatly improve the estimates of the labor supply elasticity[3]. He also shows that the second method is more efficient and leads to a more positive coefficient. However, due to the absence of the alternative measure of labor supply in the data set, the first method of correction for division bias is used in this work. See Table A1 in appendices for the description of the rest of the variables used in this research.

[3] For more details see Borjas (1980)

5.2 Results

The results of estimating labor supply equations (3.4) and (3.5), and specification for individual effects (3.11) are presented in this section. The first part provides the estimates of intertemporal substitution elasticity, whereas the second part reports the estimates of parameters $\bar{\gamma_0}$, $\bar{\gamma_1}$, $\bar{\gamma_2}$, and $\bar{\theta}$. Combining the results, the uncompensated elasticities, and, thus, the upper and lower bounds of compensated elasticities can be computed and the life-cycle behavior of the individual can be described. As before, the notation is similar to that of MaCurdy.

5.2.1 Estimates of Labor Supply Functions

The labor supply equation (3.4) is estimated by First-Difference estimation technique. Denoting the difference operator by D and applying it to equation (3.4) yields:

$$D \ln H_{it} = \delta(\rho - r_t) + \delta D \ln W_{it} + \epsilon_{it}, \quad t = 2, 3, \ldots, \tau, \tag{5.5}$$

where τ is the number of sample periods (which is 3 in this work), or, in vector notation:

$$D \ln H_i = \beta + D \ln W_i \delta + \epsilon_i, \tag{5.6}$$

where the dependent variable, intercept, explanatory variable, and error term are all $(\tau-1)\times 1$ dimensional vectors with $D \ln H_i' = [D \ln H_{i\tau}, \ldots, D \ln H_{i2}]$, $\beta' = [\delta(\rho-r_\tau), \ldots, \delta(\rho-r_2)]$, $D \ln W_i' = [D \ln W_{i\tau}, \ldots, D \ln W_{i2}]$, $\epsilon_i' = [\epsilon_{i\tau}, \ldots, \epsilon_{i2}]$.

Equation (5.6) is estimated by Two Stage Least Squares estimation procedure. The endogenous variable entering this equation is $D \ln W_i$. The variables used to instrument wage rate in the first stage are *education, educations squared, age, age squared*, interactions between education and age variables, and year dummies.

In his study, MaCurdy also specifies an alternative equation that relates changes in hours of work to changes in real earnings, E, as follows (in vector notation):

$$D \ln H_i = \kappa + D \ln E_i \frac{\delta}{1+\delta} + \epsilon_i \frac{1}{1+\delta}, \tag{5.7}$$

where, as in equation (5.6), all variables are $(\tau - 1) \times 1$ dimensional vectors with $D \ln E_i' = [D \ln(H_{i\tau} W_{i\tau}), \ldots, D \ln(H_{i2} W_{i2})]$, and $\kappa' = \left[\frac{\delta(\rho-r_\tau)}{1+\delta}, \cdots, \frac{\delta(\rho-r_2)}{1+\delta}\right]$. The estimate of the intertemporal substitution elasticity can be recovered from the estimated coefficient on $D \ln E_i$.

Results of estimating equations (5.6) and (5.7) are shown in Table 5.1. First and third columns report the estimates for a without-year-dummies specification of (5.6) and (5.7), while columns 2 and 4 report the estimates of δ with year dummies included as explanatory

Table 5.1: **Estimates of the Intertemporal Substitution Elasticity**

	(1) $D \ln H$	(2) $D \ln H$	(3) $D \ln H$	(4) $D \ln H$
$D \ln W$	−0.276*	−0.176	–	–
	(0.163)	(0.166)	–	–
year 2010	–	−0.19*	–	−0.15
		(0.11)		(0.146)
year 2011	–	−0.008	–	0.03
		(0.08)		(0.085)
$D \ln E$	–	–	−0.03	−0.056
			(0.20)	(0.19)
Intercept	0.12**	–	0.066	–
	(0.06)		(0.07)	
1st stage F-statistic	1.36	1.36	0.95	0.95
Observations	120	120	120	120

Notes: Standard errors are in parentheses.
*** - significant at 0.01, ** - significant at 0.05, * - significant at 0.1.
The coefficients and standard errors in the $D \ln E$ row are for δ. Denote the coefficient on $D \ln E$ as $\hat{\phi}$. $\hat{\delta}$ is then computed according to the relationship $\hat{\delta} = \hat{\phi}/(1 - \hat{\phi})$. Standard error is computed according to: $s.e.(\hat{\delta}) = [(1 + \hat{\delta})^2 \times s.e.(\hat{\phi})]/\hat{\phi}$.

variables. The first setting allows to estimate the value of δ when the interest rate r_t is constant (that is, equation (3.5)), while the second specification allows r_t to vary over the sample period. If the interest rate is constant over time and explanatory variable is the change in the wage rate, the estimated elasticity is negative and scarcely significant. When the interest rate is allowed to vary, the explanatory power of wage rate falls by almost a half as more variation is explained by year effects. The same is partly true when the change in the wage rate is replaced by the change in earnings as the explanatory variable. When the interest rate is time-varying, again, more variation in hours of work is due to year effects, however, the labor supply response remains at roughly the same level. This implies that, in comparison with wage rate, real earnings is more stable to the inclusion of other relevant variables. In all four cases the labor supply response to movement along the lifetime wage profile is negative.

A negative estimated intertemporal substitution elasticity is a result that contradicts to the theoretical prediction and most existing literature on the life-cycle labor supply as shown in chapter 1. This finding implies the presence of an income effect which outweighs the substitution effect from moving along the age-earnings profile, while the model's assumption of perfect certainty guarantees that this income effect is zero (see chapter 2). The main reason of such disagreement between theory and empirics is the weak performance of the instruments, as suggested by the values of F-statistic from the 1st stage regressions (see Table A2 for the full 1st stage results). This fact is verified by inspecting how the value of δ changes as we move from columns 1 and 2 to columns 3 and 4 of table 5.1. It is apparent that the value of $\hat{\delta}$ tends to get closer towards 0 and become less negative when the explanatory variable is the change in real earnings, $D \ln E$. This is a clear sign of an existing division bias as described in section 5.1. In this situation the estimates of the intertemporal substitution from the earnings labor supply function are certainly preferable. Keeping in mind wide confidence intervals of estimated coefficients, it is reasonable to conclude that, "according to the results, intertemporal substitution elasticity for prime-age males in Kyrgyzstan is essentially zero".

5.2.2 Estimates of the Effects of Parametric Wage Change

Estimating equation (3.11) is complicated by the fact that we do not observe any of the variables that enter the equation. To address this issue, for each variable MaCurdy constructs an observable one with the mean equal to the corresponding unobserved factor and then estimates the equation using the constructed variables.

According to MaCurdy, the observable subsitutes for the parameters π_{2i}, π_{1i}, and π_{0i} of lifetime wage growth specification (3.7) are defined as[4]:

$$\tilde{\pi}_{2i} = \frac{1}{\tau - 2} \sum_{t=1}^{\tau-2} \frac{1}{t} \left[\frac{D_{t+1} \ln W_{i,t+2}}{t+1} - D_1 \ln W_{i2} \right], \tag{5.8}$$

$$\tilde{\pi}_{1i} = \frac{1}{\tau - 1} \sum_{t=1}^{\tau-1} \left[\frac{D_t \ln W_{i,t+1}}{t} - \tilde{\pi}_{2i} (2 \times age_{i,t+1} - t) \right], \tag{5.9}$$

$$\tilde{\pi}_{0i} = \frac{1}{\tau} \sum_{t=1}^{\tau} \left[\ln W_{it} - \tilde{\pi}_{1i} \times age_{it} - \tilde{\pi}_{2i} (age_{it})^2 \right], \tag{5.10}$$

where, as before, τ is the number of sample periods. The variables $\tilde{\pi}_{2i}$, $\tilde{\pi}_{1i}$, and $\tilde{\pi}_{0i}$ are constructed one after another in the order they appear above.

The observed values for the parameters of lifetime income path specification (3.9) are obtained similarly, except that there is now Y_{it} instead of $\ln W_{it}$. The implied definition for the initial property income, α_{0i}, is therefore:

$$\tilde{\alpha}_{0i} = \frac{1}{\tau} \sum_{t=1}^{\tau} \left[\ln H_{it} - \hat{b} \times age_{it} - \hat{\delta} (age_{it})^2 \right]. \tag{5.11}$$

The problem of not observing the dependent variable of equation (3.11) is addressed by

[4] For more details on the strategy that leads to such definitions see MaCurdy (1981, pp.1080-1081).

constructing a measurable variable using the estimates of the labor supply equation (3.5). Specifically, individual fixed effects can be estimated by:

$$\tilde{F}_i = \frac{1}{\tau} \sum_{t=1}^{\tau} (\ln H_{it} - \hat{b} \times age_{it} - \hat{\delta} \ln W_{it}). \tag{5.12}$$

Replacing the parameters and a dependent variable of the equation (3.11) with their constructed counterparts, one obtains a system of simultaneous equations:

$$\tilde{\pi}_{ji} = M_i g_j + \eta_j, \; j = 0, 1, 2, \tag{5.13}$$

$$\tilde{\alpha}_{0i} = S_i q_0 + \eta_3, \tag{5.14}$$

$$\tilde{F}_i = \psi + \tilde{\pi}_{0i}\bar{\gamma}_0 + \tilde{\pi}_{1i}\bar{\gamma}_1 + \tilde{\pi}_{2i}\bar{\gamma}_2 + \tilde{\alpha}_{0i}\bar{\theta} + \eta_4, \tag{5.15}$$

where the vectors of exogenous factors M_i and S_i and parameters g_0, g_1, g_2, and q_0 are defined by (3.8) and (3.10), and η_0, η_1, η_2, and η_4 are error terms. The variables $\tilde{F}_i$, $\tilde{\pi}_{0i}$, $\tilde{\pi}_{1i}$, $\tilde{\pi}_{2i}$, and $\tilde{\alpha}_{0i}$ enter the system endogenously. Two Stage Least Squares estimation procedure is used to estimate the coefficients $\bar{\gamma}_0$, $\bar{\gamma}_1$, $\bar{\gamma}_2$, and $\bar{\theta}$. In this study, the elements of vectors M_i and S_i are *education*, *education squared*, urban dummy, and a set of position and industry dummies (see Table A1 for the description).

Table 5.2 presents the results of estimating the system of simultaneous equations given by (5.13)–(5.15). The first row reports the estimates, where the dependent variable, $\tilde{F}_i$, is obtained using the coefficients of $\hat{b}$ and $\hat{\delta}$ from the "hours–wage rate" labor supply specification, whereas the second row provides the estimates that correspond to "hours–earning" equation. It can be inferred from the table that all $\bar{\gamma}$'s obtained from both specifications are negative in accordance with the theoretical prediction (2.10). The estimates of $\bar{\theta}$ are scanty and statistically insignificant. In the original study, MaCurdy obtains very similar coefficients.

Table 5.2: Estimates of the Structural Equation for Individual Fixed Effects

$\hat{b}$	$\hat{\delta}$	$\bar{\gamma}_0$	$\bar{\gamma}_1$	$\bar{\gamma}_2$	$\bar{\theta}$	Intercept
0.12	−0.276	−0.0005	−0.05*	−2.28	2.33e-08	−0.75
		(0.0003)	(0.026)	(1.62)	(1.62e-08)	(0.5)
0.066	−0.03	−0.0003	−0.03*	−1.51	1.2e-08	0.84***
		(0.0002)	(0.02)	(1.04)	(1.04e-08)	(0.32)

Notes: Standard errors are in parentheses.
*** - significant at 0.01, ** - significant at 0.05, * - significant at 0.1.
The variable $\tilde{F}_i$ is estimated based on the values of $\hat{b}$ and $\hat{\delta}$ as given by equation (5.12).

Since the estimate of δ reported in the first row of Table 5.2 is subject to a significant downward bias, as discussed in subsection 5.2.1, it will not make any sense to describe the life-cycle behavior of individuals based on the $\bar{\gamma}$'s and $\bar{\theta}$ associated with this value of $\hat{\delta}$ and, thus, $\tilde{F}_i$. The estimates from the second row are more appropriate for that purpose.

The effect of initial permanent income, which is reflected by the value of $\bar{\theta}$, seems to bring little value in describing the labor-supply behavior of prime-age males in Kyrgyzstan. Indeed, an increase in the initial wealth's worth by the amount of one million soms[5] leads to an increase in the hours of work by 0.012 percent. In this work, the measure of nonwage income includes all the household income excluding the wage of the household head.

To obtain the estimate of cross- and own-uncompensated substitution elasticities, one needs to combine the estimates of $\bar{\gamma_0}$ and δ. If one divides the estimated value of $\bar{\gamma_0}$ by the average working life (MaCurdy assumes it to last about 40 years), then one gets the average cross-uncompensated elasticity, which would be very close to zero keeping in mind how small is $\bar{\gamma_0}$. If one then adds to it the value of $\hat{\delta}$, s/he gets the estimate of the "average own-uncompensated elasticity" which would be approximately equal to -0.03. Since the average own-compensated elasticity is bounded by the intertemporal substitution elasticity (from above) and the average own-uncompensated elasticity (from below), its value should very close to -0.03 as well, which means that an increase in individual's wage rate of 10% in period s is expected to decrease that individual's supply of labor by about 0.3% in period s and by practically 0% in all other periods.

Labor supply responses to shifts in lifetime wage profiles can also be analyzed with the coefficients from Table 5.2. A uniform 10% increase in wage rate involves a change in the individual's number of working hours by the amount $(\bar{\gamma_0} + \hat{\delta}) \times 10\% = (-0.0003 - 0.03) \times 10\% = -0.303$ during the whole sample period. The change of slope of the wage profile involves the responses of labor supply of $(\bar{\gamma_1} + \hat{\delta} \times age)$ multiplied by the amount of the percentage change in the slope coefficient π_1 or a $(\bar{\gamma_2} + \hat{\delta} \times age^2)$ multiplied by the percentage change in π_2 if the wage profile changes due to the quadratic term. However, due to the practically zero value of $\hat{\delta}$, the responses of labor supply are not meaningful.

[5] Which is approximately 21,500 USD if the average exchange rate for the period 2010–2012 is used.

Chapter 6

Conclusion

This paper presents and estimates the empirical model of labor supply over the life-cycle first described by MaCurdy (1981). The model divides the analysis of the individual's labor supply behavior into two steps, where in the first step it provides the framework for estimating the labor supply responses to wage changes from movements along the lifetime wage profile, while in the second step the focus is on determining the parameters that are necessary to explain the responses of the number of hours worked to parametric wage changes.

The role of different substitution elasticities is analyzed and tested empirically. Unfortunately, their theoretical importance was not completely verified in this research. The wage rate variable obtained by the division of earnings by the hours of work did not show itself as a strong predictor of hours of work due to a downward bias caused by a spurious correlation with the dependent variable. The use of earnings instead of wage rates in MaCurdy's alternative specification improved the estimate of the intertemporal substitution, which, however, remained insignificant. The fact that the effect of a change in the wage rate coming from the movement along the life-cycle path is "essentially zero" affected the estimates of the compensated and uncompensated elasticities making them close to zero as well. This can be improved in future research when more powerful instruments may be available.

Despite the relative weakness of the instruments used in this analysis, the estimation of the structural equation for individual effects still provided some empirical evidence for the theoretical implications of the model and agreed with the original work's several findings. Finally, this study makes use of the most recent data set and provides a contribution in this field of labor economics research in Kyrgyzstan.

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Appendices

Table A1: Description of the Variables

Variable	Description	
H	The number of hours worked for the last week at the time the survey took place	
E	The earnings of an individual for a period of time (usually, a month)	
W	The wage rate obtained by dividing earnings by hours worked last week (and usually by 4, since the majority of individuals receive monthly wages)	
age	The age of an individual	
age^2	The square of an individual's age	
education	Years spent studying	
$education^2$	The square of years spent studying	
urban	1 - if a household is situated in an urban area 0 - if a household is situated in a rural area	
i307	Set of dummy variables coded as follows:	
	1 - legislator, senior official, manager	6 - skilled agricultural or fishery worker
	2 - professional	7 - craft and related trades
	3 - technician, associated professional	8 - plant or machine operator or assembler
	4 - clerk	9 - unskilled worker
	5 - service worker, shop or market sales worker	10 - armed forces
i308	Set of dummy variables coded as follows:	
	1 - agriculture and fishing	9 - finance
	2 - mining	10 - real estate, renting and business activities
	3 - manufacturing	11 - public administration
	4 - energy and water	12 - education
	5 - construction	13 - health and social work
	6 - trade and repair	14 - utilities, social and personal services
	7 - hotels and restaurants	15 - private households with employed persons
	8 - transport and communications	16 - extra-territorial organizations

Table A2: First Stage Results for Labor Supply Equations

	(1) $D \ln W$	(2) $D \ln E$
D education	134.8	172.5
	(106.7)	(107.2)
D education^2	−4.6	−6.15
	(4)	(4.025)
D age	51.8	63.4
	(38.2)	(38.34)
D age^2	−0.71	−0.847
	(0.52)	(0.523)
D (*education* $\times$ *age*)	−7.51	−9.41
	(5.9)	(5.93)
D (*education*^{2} $\times$ *age*)	0.26	0.34
	(0.22)	(0.22)
D (*education* $\times$ *age*^{2})	0.103	0.126
	(0.08)	(0.081)
D (*education*^{2} $\times$ *age*^{2})	−0.004	−0.0045
	(0.003)	(0.003)
D year2011	−0.09	−0.00085
	(0.08)	(0.085)
Intercept	−0.002	0.156
	(0.19)	(0.19)
Observations	120	120
R-squared	0.100	0.072
F-statistic	1.360	0.95

Notes: Standard errors are in parentheses.
*** - significant at 0.01, ** - significant at 0.05, * - significant at 0.1.
D denotes difference operator.

Table A3: First Stage Results of the Structural Equation for Individual Effects

	(1) $\tilde{\pi}_0$	(2) $\tilde{\pi}_1$	(3) $\tilde{\pi}_2$	(4) $\tilde{\alpha}_0$
education	7,413***	−196.6***	2.753***	6.432e+06
	(2,06)	(50.6)	(0.661)	(5.031e+06)
education^2	−272.1***	7.114***	−0.0996***	−256.478
	(77.4)	(1.9)	(0.0249)	−189.056
i3072	3,24	−88.4	1.12	−1.34e+07
	(3,72)	(91.2)	(1.19)	(9.1e+06)
i3073	−3,14	70	−1.01	−1.36e+07
	(3,96)	(97.2)	(1.27)	(9.67e+06)
i3074	3,37	−86	0.82	−1.23e+07
	(3,93)	(96.5)	(1.26)	(9.59e+06)
i3075	1,74	−49.55	0.65	−1.61e+07*
	(3,63)	(89.1)	(1.17)	(8.87e+06)
i3076	774.8	−27.99	0.23	−1.66e+07*
	(3,56)	(87.35)	(1.14)	(8.7e+06)
i3077	−1,07	14.33	−0.28	−1.8e+07**
	(3,53)	(86.65)	(1.13)	(8.62e+06)
i3078	523.4	12.59	−0.26	−2.36e+07**
	(3,91)	(96.12)	(1.26)	(9.56e+06)
i3079	−777.0	16.61	−0.320	−1.667e+07*
	(3,562)	(87.47)	(1.144)	(8.701e+06)
i3081	−854.7	22.69	−0.245	1.099e+07*
	(2,421)	(59.46)	(0.778)	(5.914e+06)
i3082	−760.4	14.36	−0.118	2.155e+06
	(2,796)	(68.66)	(0.898)	(6.830e+06)
i3084	−3,192	75.78	−0.816	−9.696e+06
	(2,727)	(66.98)	(0.876)	(6.663e+06)
i3085	−441.7	10.19	−0.0909	1.265e+07**
	(2,423)	(59.50)	(0.778)	(5.918e+06)
i3086	182.6	−6.064	0.0476	9.276e+06
	(2,358)	(57.92)	(0.757)	(5.761e+06)
i3088	−223.4	2.550	−0.0204	958.385
	(2,314)	(56.83)	(0.743)	(5.652e+06)
i30811	1,332	−24.40	0.369	1.893e+07**
	(3,163)	(77.68)	(1.016)	(7.727e+06)
i30814	−2,898	71.77	−0.75	−2.519e+06
	(2,837)	(69.66)	(0.911)	(6.929e+06)
urban	−1,903	61.06*	−0.780*	1.218e+07***
	(1,313)	(32.24)	(0.42)	(3.207e+06)
Intercept	−48,524***	1,262***	−17.67***	−3.371e+07
	(12,483)	(306.6)	(4.01)	(3.050e+07)
Observations	60	60	60	60
R-squared	0.465	0.499	0.500	0.590
F-statistic	1.83	2.09	2.11	3.03

Notes: Standard errors are in parentheses.
*** - significant at 0.01, ** - significant at 0.05, * - significant at 0.1.
Variables *i3071, i30710, i3083, i3087, i3089, i30810, i30812, i30813*, and *i30815* were omitted due to collinearity.