software, they are instead first analyzed by the spacecraft orbit determination software. Range bias, which present the systematic error in the geometric position of the planet as seen from the Earth, can be estimated while computing the orbit of the spacecraft. These rang bias impose strong constraints on the orbits of the planet, as well as on other solar system parameters. In consequence, such data not only allow the construction of an accurate planetary ephemeris, they also contribute significantly to our knowledge of parameters such as asteroid masses. A detailed description of such analysis using MGS, and MESSENGER spacecraft radiometric data is discussed in Chapters 3, and 5, respectively.

2.3 Radiometric data

The radiometric data which are produced by the NASA DSN Multimission Navigation (MM-NAV) Radio Metric Data Conditioning Team (RMDCT) is called Orbit Data File (ODF)[1]. These ODF are used to determine the spacecraft trajectories, gravity field affecting them, and radio propagation conditions. Each ODF is in standard JPL binary format and consists of many 36-byte logical records, which falls into 7 primary groups. In this work, we have developed an independent software to extract the contents of these ODFs. This software reads the binary ODF and writes the contents in specific format, called GINS format. GINS is the orbit determination software, independently developed at the CNES (see Section 2.5).

2.3.1 ODF contents

The ODF contains several groups of informations. An ODF usually contains most groups, but may not have all. The format of such groups are given in Kwok (2000). The brief description of the contents of these groups is given below.

2.3.1.1 Group 1

- This group is usually a first group among the several records. It identifies the spacecraft ID, the file creation time, the hardware, and the software associated with the ODF. This group also provides the information about the reference date and time for ODF time-tags. Currently the ODF data time-tags are referenced to Earth Mean Equatorial equinox of 1950 (EME-50).

[1] http://geo.pds.nasa.gov/

2.3.1.2 Group 2

- This group is usually a second group among the several groups records. It contains the string character that some time used to identify the contents of the data record, such as, TIMETAG, OBSRVBL, FREQ, ANCILLARY-DATA.

2.3.1.3 Group 3

- This is the third group that usually contains majority of the data included in the ODF. According to the data categories, the description of this group is given below.

2.3.1.3.1 Time-tags

- **Observable time:** First in this category is the Doppler and range observable time TT measured at the receiving station. Observable time TT corresponds to the time at the midpoint of the count interval, T_c. The integer and the fractional part of this time-tag (TT) is given separately in ODF. The integer part is measured from 0 hours UTC on 1 January 1950, whereas the fractional part is given in milliseconds.
- **Count interval:** Doppler observables are derived from the change in the Doppler cycle count. The time period on which these counts are accumulated is called count interval or compression time T_c. Typically count times have a duration of tens of seconds to a few thousand of seconds. For example, count time could be between 1-10 s when the spacecraft is near a planet or roughly 1000 s for interplanetary cruise.
- **Station delay:** This gives the information corresponding to the downlink and uplink delay at the receiving and at the transmitting station respectively. It is given in nanosecond in the ODF.

2.3.1.3.2 Format IDs

- **Spacecraft ID:** It identities the spacecraft ID which corresponds to ODF data. For example: 94 for MGS
- **Data type ID:** As mentioned before, the radiometric data could be one-, two-, and three-way Doppler and two-way range. The ODF provides a specific ID associated with these data set. For example: 11, 12, and 13 integers give in ODF correspond to one-, two-, and three-way Doppler respectively, whereas 37 stands for two-way range.

- **Station ID:** This is an integer that gives the receiving and transmitting stations ID that are associated with the time period covered by the ODF. The transmitting station ID is set to zero, if the date type is one-way Doppler.

- **Band ID:** It identifies the uplink (at transmitting station), downlink (at receiving station), and exciter band (at receiving station) ID. The ID of these bands are set to 1, 2, and 3 for S, X, and Ka band.

- **Date Validity ID:** It is the quality indicator of the data. It set to zero for a good quality of data and set to one for a bad data.

2.3.1.3.3 Observables

- **Reference frequency:** It is the frequency measured at the reception time t_3 at the receiving station in UTC (see Section 2.4.1). This frequency can be constant or ramped. However, the given reference frequency in the ODF could be a reference oscillator frequency f_q, or a transmitter frequency f_T, or a Doppler reference frequency f_{REF}. The computed values of Doppler observables are directly affected by the f_{REF}. Hence, the computation of the f_{REF} from the reference oscillator frequency f_q, or from the transmitter frequency f_T is discussed below.

  (i) When the given frequency in the ODF is f_q, then it is needed to first compute the transmitter frequency, which is given by (Moyer, 2003):

$$\mathrm{f_T(t) = T_3 \times f_q(t) + T_4} \tag{2.1}$$

  where T3 and T4 are the transmitter-band dependent constants as given in Table 2.1. From Eq. 2.1, one can compute the transmitter frequency at the receiving station and at the transmitting station by replacing the time t to t_3 and t_1 respectively. Thus, the f_{REF} at reception time can calculated by multiplying the spacecraft transponder ratio with f_T:

$$\mathrm{f_{REF}(t_3) = M_{2_R} \times f_T(t_3)} \tag{2.2}$$

Table 2.1: Constants dependent upon transmitter or exciter band

Band	Transmitter Band			
	T_1	T_2	T_3	T_4 (Hz)
S	240	221	96	0
X	240	749	32	6.5×10^9
Ku	142	153	1000	-7.0×10^9
Ka	14	15	1000	1.0×10^{10}

Table 2.2: Spacecraft transponder ratio M_2 (M_{2_R})

Uplink (Exciter) band	Downlink band S	Downlink band X	Downlink band Ka
S	$\frac{240}{221}$	$\frac{880}{221}$	$\frac{3344}{221}$
X	$\frac{240}{749}$	$\frac{880}{749}$	$\frac{3344}{749}$
Ka	$\frac{240}{3599}$	$\frac{880}{3599}$	$\frac{3344}{3599}$

where M_{2_R} is the spacecraft transponder ratio (see Table 2.2). It is the function of the exciter band at the transmitting station and of the downlink band at the receiving station. Whereas, M_2 given in Table 2.2 is the function of uplink band at the transmitting station and the downlink band at the receiving station. Hence, the corresponding frequency $f(t_1)$ at the transmission time t_1 can be calculated by,

$$\mathrm{f(t_1) = M_2 \times f_T(t_1)} \tag{2.3}$$

In Eqs. 2.2 and 2.3, $f_T(t_3)$ and $f_T(t_1)$ can be calculated from the Eq. 2.1. If the given value of f_q in the ODF is ramped then f_q is calculated through ramp-table (see Group 4).

(ii) When the transmitter frequency at reception time $f_T(t_3)$ is given in the ODF, then Eq. 2.2 can be used to compute the f_{REF}. The given $f_T(t_3)$ could be the constant or ramped. The ramped $f_T(t_3)$ can be calculated from the ramped table. However, when the spacecraft is the transmitter (one-way Doppler), then f_T is given by (Moyer, 2003):

$$\mathrm{f_T(t) = C_2 \times f_{S/C}} \tag{2.4}$$

where C_2 is the downlink frequency multiplier. Table 2.3 shows the standard DSN values of the C_2 for S, X, and Ka downlink bands for the data point. $f_{S/C}$ is the spacecraft transmitter frequency which is given by (Moyer, 2003):

$$\mathrm{f_{S/C} = f_{T_0} + \Delta f_{T_0} + f_{T_1}(t - t_0) + f_{T_2}(t - t_0)^2} \tag{2.5}$$

where f_{T_0} is the nominal value of $f_{S/C}$ and give in ODF. Δf_{T_0}, f_{T_1}, and f_{T_2} are the solve-for quadratic coefficients used to represent the departure of $f_{S/C}$. The quadratic coefficients are specified by time block with start time t_0.

(iii) Finally, the given frequency in the ODF could be a constant value of f_{REF}. This value is usually constant for a given pass.

- **Doppler observable:** Doppler observables are derived from the change in the Doppler cycle count $N(t_3)$, which accumulates during the compression time T_c at the receiving station. These observables in the ODF are defined as follows:

$$\mathrm{Observable} = \left(\frac{\mathrm{B}}{|\mathrm{B}|}\right) \times \left[\left(\frac{\mathrm{N_j - N_i}}{\mathrm{t_j - t_i}}\right) - |\mathrm{F_b \times K + B}|\right] \tag{2.6}$$

Table 2.3: Downlink frequency multiplier C_2

Multiplier	Downlink Band		
	S	X	Ka
C_2	1	$\frac{880}{230}$	$\frac{3344}{240}$

where:
B = Bias placed on receiver
N_i = Doppler count at time t_i
N_j = Doppler count at time t_j
t_i = start time of interval
t_j = end time of interval
F_b = frequency bias
K = 1 for S-band receivers
= 11/3 for X-band receivers
= 176/27 for Ku-band receivers
= 209/15 for Ka-band receivers

$$F_b = (X_1/X_2) \times (X_3 \times f_R + X_4) - f_{s/c} + R_3 \qquad \text{for } 1-\text{way Doppler} \tag{2.7}$$

$$F_b = (X_1/X_2) \times (X_3 \times f_{q_R} + X_4) - (T_1/T_2) \times (T_3 \times f_{q_T} + T_4) \qquad \text{for } 2/3-\text{way Doppler} \tag{2.8}$$

$$T_c = t_j - t_i \qquad \text{compression time} \tag{2.9}$$

where:
f_{q_R} = Receiver oscillator frequency at time t_3
$f_{s/c}$ = Spacecraft (beacon) frequency
f_{q_T} = Transmitter oscillator frequency at time t_1
R_3 = 0 for all receiving bands
T_1 to T_4 = Transmitters band (Table 2.1)
X_1 to X_4 = Exciter band (same value as transmitter band, Table 2.1)

Figure 2.5 shows an example of two and three way Doppler observables extracted from the MGS ODF.

- **Range observable:** Range observables are obtained from the ranging machine at the receiving station. These range observables are measured in range units (see Section 2.4.3) and defined in ODF as follows:

$$\text{Observable} = R - C + Z - S \tag{2.10}$$

where:
R = range measurement
C = station delay calibration
Z = Z-height correction
S = spacecraft delay

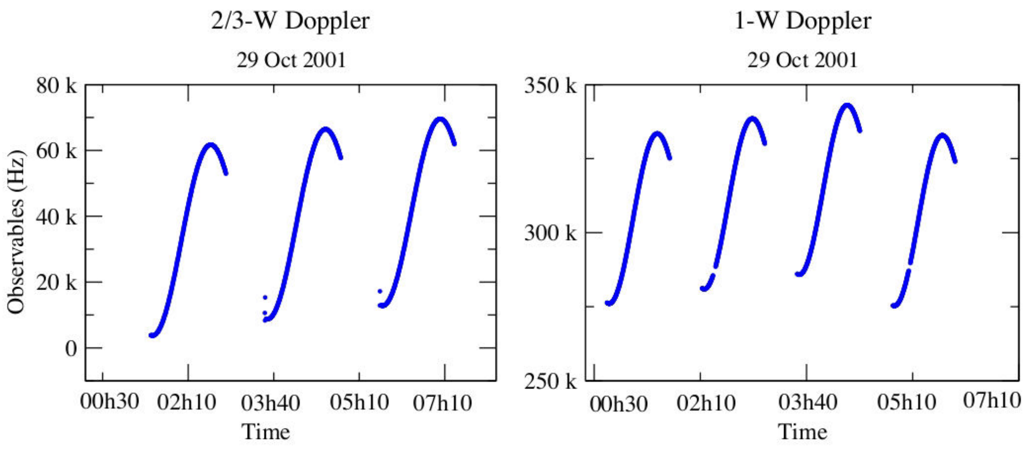

Figure 2.5: One- ,Two-, and three-way Doppler observables of the MGS spacecraft.

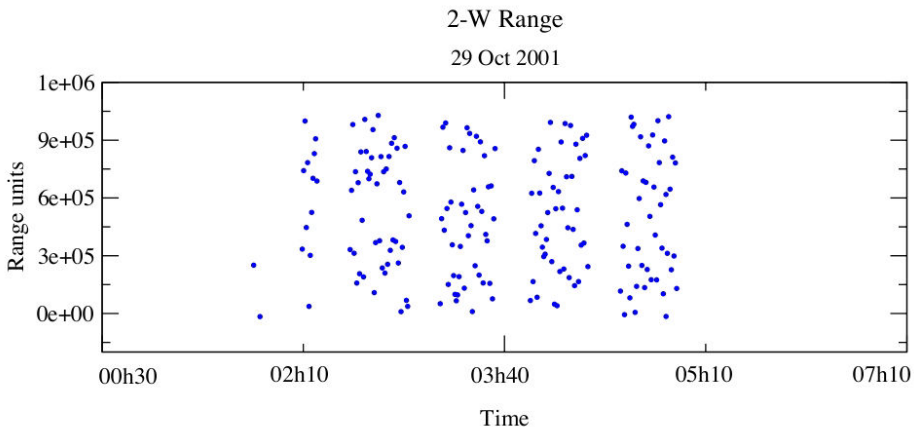

Figure 2.6: Two way range observables of the MGS spacecraft.

Figure 2.6 shows an example of two way range observables extracted from the MGS′s ODF.

2.3.1.4 Group 4

- Ramp groups are usually the fourth of several groups of record in ODF. This group contains the information about the tuning of the receiver or transmitter on the Earth station. Ramping is a technique to achieve better quality communication with spacecraft when its velocity varies with respect to ground stations and it has been implemented at the DSN. There is usually one ramp group for each DSN station. The contents of this group and the procedure to calculates the ramped transmitter frequency $f_T(t)$ is described below.

2.3.1.4.1 Ramp tables

As mentioned in Group 3, the reference frequency given in ODF can be a constant or ramped. When the given frequency is ramped then the reference frequency is computed through the ramp table. The ramp table contains the start UTC time t_o, end UTC time t_f, the values of ramped frequency f_o at the start time t_o, the constant time derivative of frequency (ramp rate) $\dot{f}$, and the tracking station. The ramp table can be specified as the reference oscillator frequency $f_q(t)$ or the transmitter frequency $f_T(t)$. However, Eq. 2.1 can be used to convert reference oscillator frequency $f_q(t)$ into the transmitter frequency $f_T(t)$. The ramped frequency can be then calculated by:

$$\mathrm{f_T(t) = f_o + \dot{f}(t - t_o)} \tag{2.11}$$

where t is the interpolation time. For Doppler observables, the ramp table for the receiving station gives the ramped transmitter frequency $f_T(t)$ as a function of time. This ramped frequency or a constant value of $f_T(t)$ at the receiving station can be then used to calculate the Doppler reference frequency $f_{REF}(t_3)$ at receiving station using Eq. 2.2.
The Figure 2.7 shows an example of the ramped frequency f_o and the ramp rate $\dot{f}$ plotted over the start time t_o of the ramp table. These ramp informations are extracted from the MGS ODF.

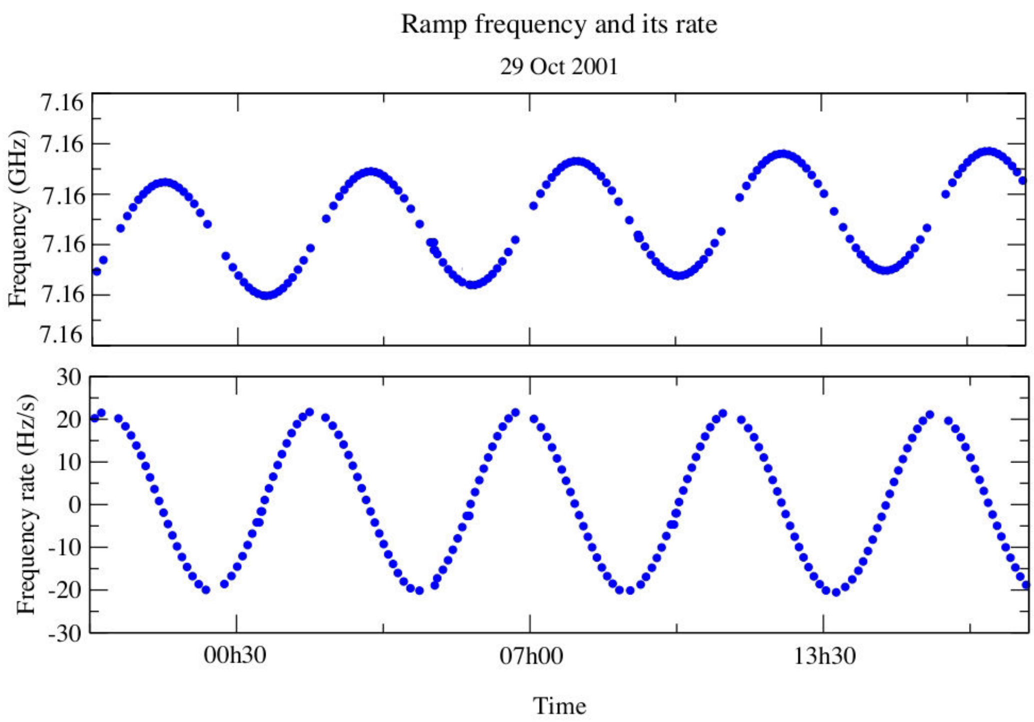

Figure 2.7: Ramped frequency f_o and frequency rate $\dot{f}$ measured for MGS spacecraft.

2.3.1.5 Group 5

- This is a clock offset group. It is usually the fifth of several groups of record in ODF. It contains information on clock offsets at DSN stations contributing to the ODF. This group may be omitted from the ODF and used only with VLBI data. It contains the start and end time of the clock offset which is measured from 0 hours UTC on 1 January 1950. It also includes the DSN station ID and the correspond clock offset given in nanoseconds. The informations of this group are generally not useful for the radioscience studies.

2.3.1.6 Group 6

- This group is usually not include in the ODF and omitted all the time.

2.3.1.7 Group 7

- It is a data summary group which contains summary information on contents of the ODF, such as, the first and last date of the data sample, total number of samples, used transmitting and receiving stations, band ID, and the type of data available in the ODF. This group is optional and may be omitted from the ODF.

2.4 Observation Model

For given spacecraft radiometric data obtained by the DSN are described in Section 2.3. These data record for each data point contains ID information which is necessary to unambiguously identify the data point and the observed value of the observable (see Group 3 of Section 2.3.1). In order to better understand these radiometric data and to estimate the precise orbit of the spacecraft, it is then necessary to compute the observables.

The computation of the observables requires the time and frequency information of the transmitted frequencies at the transmitter. The various time scales and their transformations used for these computation are described in Section 2.4.1. Using the time scale transformations, the reception time t_3 and the transmission time t_1 can be then derived from the light-time delay described in the Section 2.4.2. Using these informations it is then possible to compute the Doppler and range observables as described in Section 2.4.3.

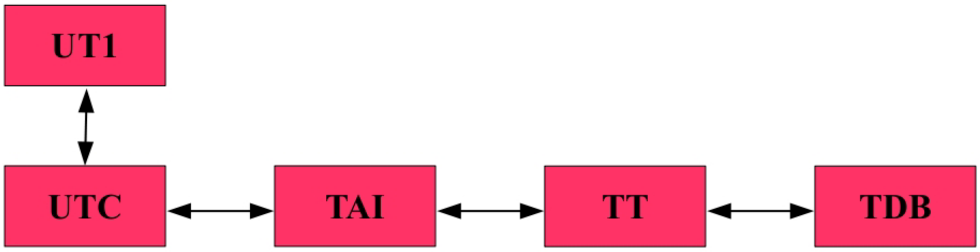

Figure 2.8: Transformation between the time scales.

2.4.1 Time scales

As described in Group 3 of Section 2.3.1, the given time in the ODF is measured in UTC from 0 hours, 1 January 1950. However, the orbital computations of the celestial body and the artificial satellite are described in TDB. Therefore, it is necessary to transform the given UTC time into TDB. The transformation between these time scales is give in Figure 2.8.

2.4.1.1 Universal Time (UT or UT1)

UT1 (or UT) is the modern equivalent of mean solar time. It is defined through the relationship with the Earth rotation angle (formerly through sidereal time), which is the Greenwich hour angle of the mean equinox of date, measured in the true equator of date. Owing the Earth rotation rate which is slightly irregular for geophysical reasons and is gradually decreasing, the UT1 is not uniform. Hence, this makes Universal Time (UT1) unsuitable for use as a time scale in physics applications.

2.4.1.2 Coordinated Universal Time (UTC)

Coordinated Universal Time (UTC) is the basis of civilian time which is the standard time for 0° longitude along the Greenwich meridian. Since January 1, 1972, UTC is given in unit of SI seconds and has been derived from the International Atomic Time (TAI). UTC is close to UT1 and maintained within 0.90 second of the observed UT1 by adding a positive or negative leap second to UTC. Figure 2.9 shows the time history of the $\triangle$UT1 since 1962, which can be defined as the time scale difference between UT1 and UTC:

$$\triangle \mathrm{UT1} = \mathrm{UT1} - \mathrm{UTC} \tag{2.12}$$

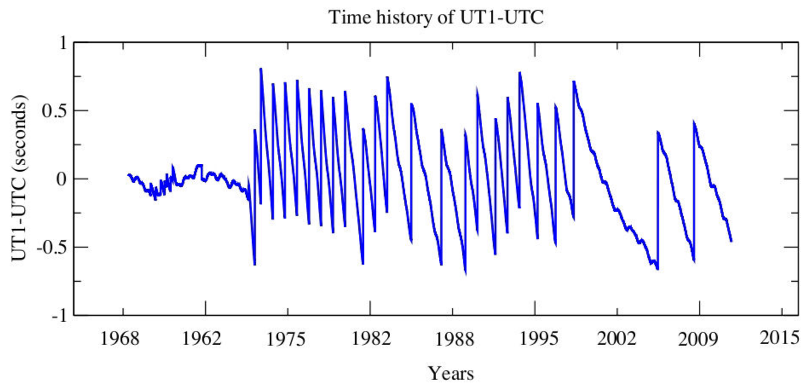

Figure 2.9: Time history of the △UT1 since 1962. These values of △UT1 are extracted from EOP file.

The △UT1 can be extracted from the Earth Orientation Parameters (EOP)[2] file and at any given time, △UT1 can be obtained by interpolating this file.

2.4.1.3 International Atomic Time (TAI)

The International Atomic Time (TAI) is measured in the unit of SI second and defined the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom (Moyer, 2003). TAI is a laboratory time scale, independent of astronomical phenomena apart from having been synchronized to solar time. TAI is obtained from a worldwide system of synchronized atomic clocks. It is calculated as a weighted average of times obtained from the individual clocks, and corrections are applied for known effects.

TAI is ahead of UTC by an integer number of seconds. The Figure 2.10 shows the time history of the difference between TAI and UTC time scales △TAI since 1973. The value of the △TAI can be extracted from the International Earth Rotation and Reference Systems Service (IERS)[3] and it given by

$$\triangle \mathrm{TAI} = \mathrm{TAI} - \mathrm{UTC} \tag{2.13}$$

[2] http://www.iers.org/IERS/EN/DataProducts/EarthOrientationData
[3] http://www.iers.org/

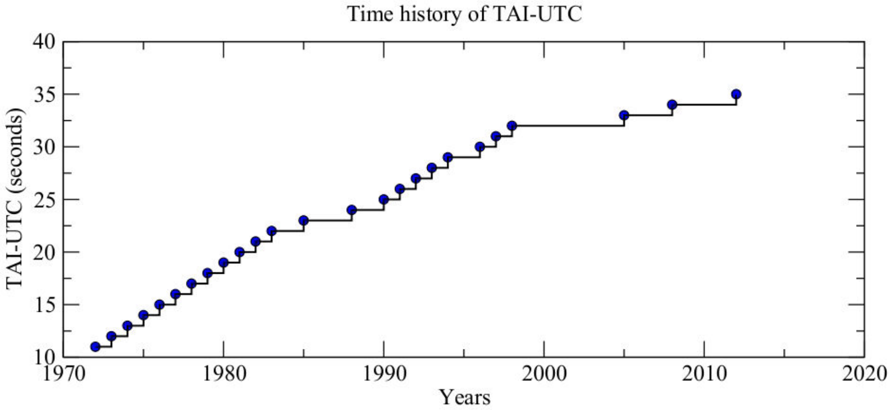

Figure 2.10: Time history of the $\triangle$TAI since 1973.

2.4.1.4 Terrestrial Time (TT)

TT is the theoretical time scale for clocks at sea-level. In a modern astronomical time standard, it defined by the IAU as a measurement time for astronomical observations made from the surface of the Earth. TT runs parallel to the atomic timescale TAI and it is ahead of TAI by a certain number of seconds which is given as

$$\mathrm{TT} - \mathrm{TAI} = 32.184\mathrm{s} \tag{2.14}$$

From Figure 2.8 and from Eqs. 2.13 and 2.14, one can transform the time scale from UTC to TT or from TT to UTC.

2.4.1.5 Barycentric Dynamical Time (TDB)

TT and Geocentric Coordinate Time (TCG) are the geocenter time scales to be used in the vicinity of the Earth, while Barycentric Coordinate Time (TCB) and TDB are the solar system barycentric time scales to be used for planetary ephemerides or interplanetary spacecraft navigation. Transformation between these time scales are plotted in Figure 2.11.

The geocentric coordinate time, TCG, is appropriate for theoretical studies of geocentric ephemerides and differ from the TT by a constant rate with linear transformation (McCarthy and Petit, 2004):

$$\mathrm{TCG} - \mathrm{TT} = \mathrm{L_G} \times (\mathrm{JD} - \mathrm{T_0}) \times 86400 \tag{2.15}$$

where L_G = 6.969290134×10^{10}, T_0 = 2443144.5003725, and JD is TAI measured in Julian days. T_0 is JD at 1977 January 1, 00h 00m 00s TAI. The time-scale used in the ephemerides of planetary spacecraft, as well as that of solar system bodies, is the barycentric dynamical time,

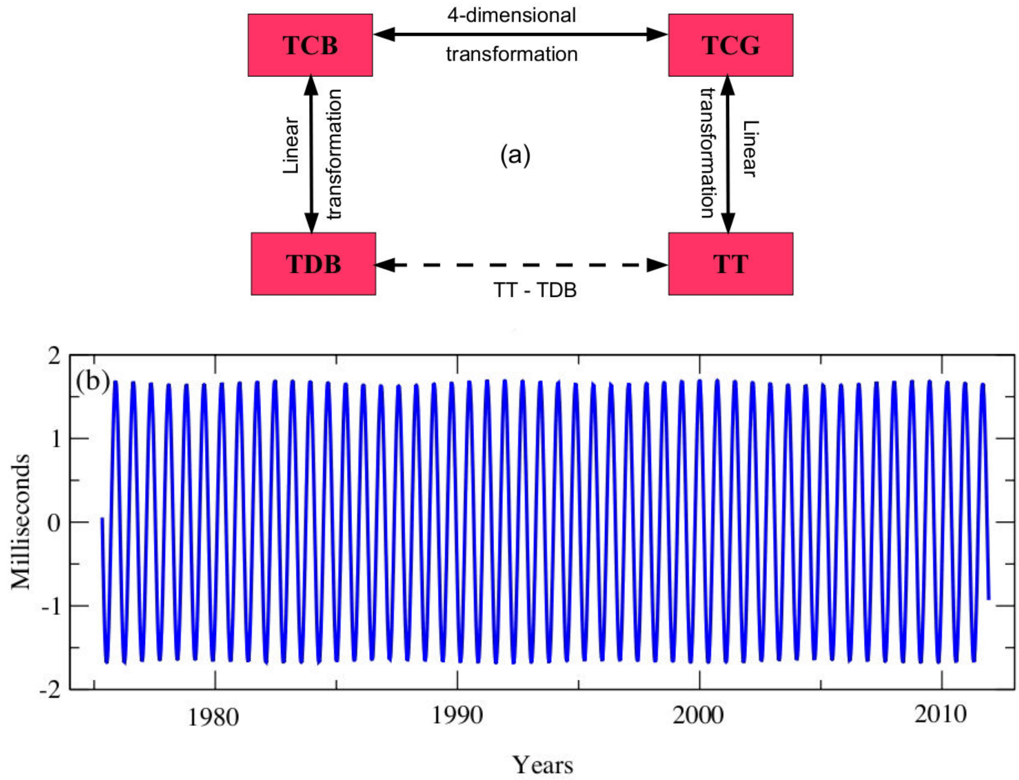

Figure 2.11: Panels: (a) transformations of various coordinate time scales, (b) time history of TT - TDB.

TDB, a scaled version of the barycentric coordinate time, TCB, (the time coordinate of the IAU space-time metric BCRS) (Klioner, 2008). The TDB stays close to TT (< 2ms, see panel *b* of Figure 2.11) on the average by suppressing a drift in TCB due to the combined effect of the terrestrial observer orbital speed and the gravitational potential from the Sun and planets by applying a linear transformation (McCarthy and Petit, 2004):

$$\mathrm{TCB} - \mathrm{TDB} = \mathrm{L_B}(\mathrm{JD} - \mathrm{T_0}) \times 86400 - \mathrm{TDB_0} \tag{2.16}$$

where $\mathrm{T_0}$ = 2443144.5003725, L_B = 1.550519768×10^{-8}, $\mathrm{TDB_0}$ = -6.55×10^5 s, and JD is the TCB Julian date which is $\mathrm{T_0}$ for the event 1977 January 1, 00h 00m 00s TAI.

The barycentric coordinate time, TCB, is appropriate for applications where the observer is imagined to be stationary in the solar system so that the gravitational potential of the solar system vanishes at their location and is at rest relative to the solar system barycenter (Klioner, 2008). The transformation from TCG to TCB thus takes account of the orbital speed of the geocenter and the gravitational potential from the Sun and planets. The difference between TCG and TCB involves a full 4-dimensional GR transformation (McCarthy and Petit, 2004):

$$\mathrm{TCB} - \mathrm{TCG} = c^{-2}\left\{\int_{t_0}^{t}\left[\frac{v_e^2}{2} + U_{ext}\,(\vec{x}_e)\right]dt + \vec{v}_e.(\vec{x} - \vec{x}_e)\right\} + O(c^{-4}) \tag{2.17}$$

where $\vec{x}_e$ and $\vec{v}_e$ are the barycentric position and velocity of the geocenter, the $\vec{x}$ is the barycentric position of the observer and U_{ext} is the Newtonian potential of all of the solar system bodies apart from the Earth, evaluated at the geocenter. In this formula, t is TCB and t_0 is chosen to be consistent with 1977 January 1, 00h 00m 00s TAI. The neglected terms, $O(c^{-4})$, are of order 10^{-16} in rate for terrestrial observers. U_{ext} $(\vec{x}_e)$ and $\vec{v}_e$ are all ephemeris-dependent, and so the resulting TCB belongs to that particular ephemeris, and the term $\vec{v}_e.(\vec{x} - \vec{x}_e)$ is zero at the geocenter.

The all above set of Equations 2.15-2.17 are precisely modeled in INPOP. The numerical integration has been performed to obtain a realization of Equation 2.17 with a nanosecond accuracy (Fienga et al., 2009). The difference between TT-TDB therefore can be extracted at any time from the INPOP planetary ephemeris using the tool called calceph[4]. The spacecraft orbit determination software GINS (see Section 2.5), integrates the equations of motion in the specific coordinate time called, ephemeris time (ET). In GINS, this time is also referred to as TDB, as defined by Moyer (2003). As discrepancies between TT and TDB or ET are smaller than 2 ms (see panel *b* of Figure 2.11), the transformation between the time scales defined either in INPOP or GINS are analogous and show consistency between both software.

2.4.2 Light time solution

The light time solution is used to compute the one-way or round-trip light time of the signal propagating between the tracking station on the Earth and the spacecraft. In order to compute the Doppler and range observables, the first step is to obtain the light time solution. This solution can be modeled by computing the positions and velocities of the transmitter at the transmitting time $t_1(TDB)$, spacecraft at the bouncing time $t_2(TDB)$ (for round-trip) or transmitting time $t_2(TDB)$ (for one-way), and receiver at the receiving time $t_3(TDB)$.

For round-trip light time, spacecraft observations involve two tracking stations, a transmitter, and a receiver which may not be at the same location. Therefore, two light time solutions must be computed, one for up-leg of the signal (transmitter to spacecraft) and one for down-leg (spacecraft to receiver). However, one-way light time requires only single solution because the signal is transmitting by the spacecraft to the receiver. These solutions can be obtained in the Solar system barycenter space-time reference frame for a spacecraft located anywhere in the Solar system.

Since spacecraft observations are usually given at receiver time UTC (see section 2.3.1), the computation sequence therefore works backward in time: given the receiver time $t_3(UTC)$,

[4] http://www.imcce.fr/inpop/calceph/