# Lecture 2: Atomic Data Types/Homogeneous vectors

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R can be summarized in three principles (John M. Chambers, 2016)

- Everything that exists in R is an object.
- Everything that happens in R is a function call.
- Interfaces to other languages are a part of R.

# 1 R Objects

- An object in R is (internally) represented as a pair: (symbol, value).
- A **symbol** is assigned a **value** by the use of an arrow pointing to the left (<-).
- There are less favored ways:
  - A simple equality sign (=).
  - Using the **assign()** function.

## 1.1 Examples

• Clean up the global environment i.e. remove all objects from the current R environment.

Recommended!

```
rm(list=ls())
ls()
```

character(0)

• preferred way to assign variables

```
x <- 5.0
x
```

[1] 5

• alternative 1: mainly used to assign default function arguments

```
y = 5.0
y
```

[1] 5

```
mysamplevariance <- function(x, av=0){

n <- length(x)
   if(n>1){
      return(1.0/(n-1)*sum((x-av)^2))
   }
   else{
      stop("ERROR:: Dividing by zero (n==1) || (n==0) ")
   }
}

x <- rnorm(10)
mysamplevariance(x)</pre>
```

```
[1] 1.471668
```

```
mysamplevariance(x,mean(x))
```

[1] 1.459569

```
var(x)
[1] 1.459569

• alternative 2: even less used
assign("z", 5.0)
z
[1] 5

• functions are objects
f <- mean
f
function (x, ...)
UseMethod("mean")
<bytecode: 0x56201f4c2f60>
<environment: namespace:base>
val <- f(1:10)
val</pre>
```

[1] 5.5

"Nothing exists except atoms and empty space; everything else is opinion". (Democritos)

# 2 Atomic Data Types

## 2.1 The core/atomic data types

- R has the following 6 **atomic** data types:
  - logical (i.e. boolean)
  - integer
  - double
  - character (i.e. string)
  - complex
  - raw (i.e. byte)

The latter 2 types (i.e. complex and especially raw) are less common.

The **typeof()** function determines the **INTERNAL** storage/type of an R object.

### 2.1.1 Examples

• boolean/logical values: either TRUE or FALSE

```
x1 <- TRUE
x1
```

[1] TRUE

typeof(x1)

- [1] "logical"
  - integer values  $(\in \mathbb{Z})$ :

```
x2 <- 3L
```

x2
[1] 3

typeof(x2)

- [1] "integer"
  - double (precision) values:

```
x3 <- 3.14
x3
```

```
[1] 3.14
```

typeof(x3)

[1] "double"

• character values/strings

```
x4 <- "Hello world"
x4</pre>
```

[1] "Hello world"

typeof(x4)

- [1] "character"
  - complex values ( $\in \mathbb{C}$ ):

```
x5 <- 2.0 + 3i
x5
```

[1] 2+3i

typeof(x5)

[1] "complex"

## 2.2 Operations on atomic data types

```
• logical operators: ==, !=, &&, ||, !
   • numerical operators: +, -, *, /, ^, ** (same as the caret), but also:
        – integer division: \%/\%
        - modulo operation: %%
        − Note: matrix multiplication will be performed using %*%
   • character/string manipulation:
        - nchar():
       - paste():
       - cat():
       - sprintf():
       - substr():
       - strsplit():
        - Note: Specialized R libraries were developed to manipulate strings e.g. stringr
   • explicit cast/conversion: https://data-flair.training/blogs/r-string-manipulation/
        as.{logical, integer, double, complex, character}()
   • explicit test of the type of a variable:
        is.{logical, integer, double, complex, character}()
2.2.1
       Examples
   • Logical operators:
x <-3
y <-7
(x <= 3) && (y == 7)
[1] TRUE
! (y<7)
[1] TRUE
   • Mathematical operations
2**4
[1] 16
7%%4
[1] 3
7/4
[1] 1.75
7%/%4
[1] 1
```

• String operations

```
s <- "Hello"
nchar(s)
[1] 5
news <- paste(s,"World")</pre>
[1] "Hello World"
sprintf("My new string:%20s\n", news)
[1] "My new string:
                        Hello World\n"
city <- "Witwatersrand"</pre>
substr(city,4,8)
[1] "water"
  • Conversion and testing of types
s <- "Hello World"
is.character(s)
[1] TRUE
s1 <- "-500"
is.character(s1)
[1] TRUE
s2 <- as.double(s1)
is.character(s2)
[1] FALSE
is.double(s2)
[1] TRUE
s3 <- as.complex(s2)
[1] -500+0i
sqrt(s3)
[1] 0+22.36068i
```

### 2.2.2 Exercise

- - Calculate  $log_2(10)$  using R's log() function
  - Perform the inverse operation and check that you get 10 back
- Let z = 3 + 4i
  - Use R's Re(), Im() functions to extract the real and imaginary parts of z.
  - Calculate the modulus of z using R's Mod() function and check whether you the same answer using √R(z)² + S(z)².
    Calculate the argument of z using R's Arg() function and check
  - Calculate the argument of z using R's Arg() function and check whether you have the same answer using  $\arctan\left(\frac{\Im(z)}{\Re(z)}\right)$ .

## 3 Atomic vectors

- An atomic vector is a data structure containing elements of only one atomic data type.
   Therefore, an atomic vector is homogeneous.
- Atomic vectors are stored in a **linear** fashion.
- R does NOT have scalars:
  - An atomic vector of length 1 plays the role of a scalar.
  - Vectors of **length 0** also exist (and they have some use!).
- A **list** is a vector not necessarily of the atomic type.

A list is also known as a **recursive/generic** vector (vide infra).

### 3.1 Creation of atomic vectors

Atomic vectors can be created in a multiple ways:

- Use of the **vector()** function.
- Use of the **c()** function (**c** stands for concatenate).
- Use of the column operator:
- Use of the **seq()** and **rep()** functions.

The length of a vector can be retrieved using the length() function.

## 3.1.1 Examples

```
• use of the vector() function:

x <- vector() # Empty vector (Default:'logical')
x

logical(0)
length(x)

[1] 0
typeof(x)

[1] "logical"

x <- vector(mode="complex", length=4)
x

[1] 0+0i 0+0i 0+0i 0+0i
length(x)

[1] 4
x

[1] 0+0i 0+0i 0+0i 0+0i
x[1] <- 4
x

[1] 4+0i 0+0i 0+0i 0+0i
0+0i
0+0i
0+0i
0+0i
```

```
• use of the c() function:
x1 \leftarrow c(3, 2, 5.2, 7)
[1] 3.0 2.0 5.2 7.0
x2 <- c(8, 12, 13)
[1] 8 12 13
x3 \leftarrow c(x2, x1)
x3
[1] 8.0 12.0 13.0 3.0 2.0 5.2 7.0
x4 <- c(FALSE, TRUE, FALSE)
[1] FALSE TRUE FALSE
x5 <- c("Hello", "Salt", "Lake", "City")
x5
[1] "Hello" "Salt" "Lake" "City"
  • use of the column operator:
y1 <- 1:10
y1
[1] 1 2 3 4 5 6 7 8 9 10
y2 <- 5:-5
у2
[1] 5 4 3 2 1 0 -1 -2 -3 -4 -5
y3 <- 2.3:10
уЗ
[1] 2.3 3.3 4.3 5.3 6.3 7.3 8.3 9.3
y4 <- 2.0*7:1
y4
[1] 14 12 10 8 6 4 2
y5 <- 1:7-1
у5
[1] 0 1 2 3 4 5 6
  • seq() and rep() functions
z1 <- seq(from=1, to=15, by=3)</pre>
[1] 1 4 7 10 13
```

```
z2 <- seq(from=-2,to=5,length=4)
z2

[1] -2.0000000  0.3333333  2.6666667  5.0000000

z3 <- rep(c(3,2,4), time=2)
z3

[1] 3 2 4 3 2 4

z4 <- rep(c(3,2,4), each=3)
z4

[1] 3 3 3 2 2 2 4 4 4

z5 <- rep(c(1,7), each=2, time=3)
z5

[1] 1 1 7 7 1 1 7 7 1 1 7 7

length(z5)

[1] 12</pre>
```

## 3.2 Operations on vectors: element-wise

- All operations on vectors in R happen element by element (cfr. NumPy).
- Vector Recycling:

If 2 vectors of **different** lengths are involved in an operation, the **shortest vector** will be repeated until all elements of the longest vector are matched. A message will be sent to the stdout.

#### 3.2.1 Examples

```
x <- -3:3

x

[1] -3 -2 -1 0 1 2 3

y <- 1:7

y

[1] 1 2 3 4 5 6 7

xy <- x*y

xy

[1] -3 -4 -3 0 5 12 21

xpy <- x^y
xpy

[1] -3 4 -1 0 1 64 2187
```

```
x <- 0:10
y <- 1:2
length(x)

[1] 11
length(y)

[1] 2
x

[1] 0 1 2 3 4 5 6 7 8 9 10
y

[1] 1 2
x+y

Warning in x + y: longer object length is not a multiple of shorter object length

[1] 1 3 3 5 5 7 7 9 9 11 11</pre>
```

## 3.3 Retrieving elements of vectors

- Indexing: starts at 1 (not 0 like C/C++, Python, Java, ....) see also: Edsger Dijkstra: Why numbering should start at zero
- Use of vector with indices to extract values.
- Advanced features:
  - use of boolean values to extract values.
  - the membership operator: %in%.
  - the deselect/omit operator: -
  - which(): returns the indices for which the condition is true.
  - **any**()/**all**() functions.
    - \* any(): TRUE if at least 1 value is true
    - \* all(): TRUE if all values are true

## 3.3.1 Examples

[1] 2

• Use of a simple index:

```
x <- seq(2,100,by=15)
x[4]
[1] 47
x[1]</pre>
```

• Select several indices at once using vectors:

```
[1] 2 17 32 47 62 77 92
x[3:5]
[1] 32 47 62
x[c(1,3,5,7)]
[1] 2 32 62 92
x[seq(1,7,by=2)]
[1] 2 32 62 92
  • Extraction via booleans (i.e. retain only those values that are equal to TRUE):
X
[1] 2 17 32 47 62 77 92
[1] FALSE FALSE FALSE TRUE TRUE TRUE
x[x>45]
[1] 47 62 77 92
  • Use of the %in% operator:
[1] 2 17 32 47 62 77 92
10 %in% x
[1] FALSE
62 %in% x
[1] TRUE
c(32,33,43) %in% x
[1] TRUE FALSE FALSE
!(c(32,33,43) \%in\% x)
[1] FALSE TRUE TRUE
  • Negate/filter out the elements with negative indices:
[1] 2 17 32 47 62 77 92
```

```
x[-c(2,4,6)]

[1] 2 32 62 92

z <- x[-1] - x[-length(x)]

z

[1] 15 15 15 15 15 15
```

• The which() function returns only those indices of which the condition/expression is true.

```
[1] -1.2358762 -1.5636180 -0.8614797 -0.6143570 -1.7241800 2.3487254
[7] 2.5304723 1.4193973 -0.2664692 2.3185802

which(vecnum>1.0)

[1] 6 7 8 10

• Use of the any()/all() functions.

y <- seq(0,100,by=10)
x

[1] 2 17 32 47 62 77 92

y

[1] 0 10 20 30 40 50 60 70 80 90 100

any(x<y)

Warning in x < y: longer object length is not a multiple of shorter object length
[1] TRUE

all(x[6:7]>y[2:3])
```

#### 3.4 Hash tables

[1] TRUE

# Sample 10 numbers from N(0,1)

vecnum <- rnorm(n=10)</pre>

vecnum

A **hash table** is a data structure which implements an associative array or dictionary. It is an abstract data which maps data to keys.

- There are several ways to create one:
  - Map names to an existing vector
  - Add names when creating the vector
- To remove the map, map the names to NULL

#### 3.4.1 Examples

• Creation of 2 independent vectors

```
capitals <- c("Albany", "Providence", "Hartford", "Boston", "Montpelier", "Concord", "Augusta")</pre>
states <- c("NY", "RI", "CT", "MA", "VT", "NH", "ME")
capitals
[1] "Albany"
                  "Providence" "Hartford"
                                                            "Montpelier"
                                              "Boston"
[6] "Concord"
                  "Augusta"
states
[1] "NY" "RI" "CT" "MA" "VT" "NH" "ME"
capitals[3]
[1] "Hartford"
  • Create the hashtable/dictionary
# Method 1
names(capitals) <- states</pre>
capitals
                        RI
                                      CT
                                                    MA
                                                                  VT
                                                                               NH
    "Albany" "Providence"
                                              "Boston" "Montpelier"
                              "Hartford"
                                                                        "Concord"
   "Augusta"
capitals["MA"]
      MA
"Boston"
names(capitals)
[1] "NY" "RI" "CT" "MA" "VT" "NH" "ME"
# Method 2
phonecode <- c("801"="SLC", "206"="Seattle", "307"="Wyoming")</pre>
phonecode
                 206
                           307
      801
    "SLC" "Seattle" "Wyoming"
phonecode["801"]
  801
"SLC"
```

• Dissociate the 2 vectors

```
names(capitals) <- NULL
capitals</pre>
```

- [1] "Albany" "Providence" "Hartford" "Boston" "Montpelier"
- [6] "Concord" "Augusta"

## 3.5 NA (Not Available values)

- NA: stands for 'Not Available'/Missing values
- has length of 1.
- is.na(): test all elements of a vector for NA values.
- some functions e.g. mean() return NA when an instance of NA is present.

#### 3.5.1 Examples

• Check of the NA availability

```
x <- c(NA, 1, 2, NA) is.na(x)
```

- [1] TRUE FALSE FALSE TRUE
  - Functions on a vector containing NA

mean(x)

[1] NA

mean(x, na.rm=TRUE)

[1] 1.5

#### 3.6 Alia

### Still to be developed!

- boolean: Vector operators vs. unique value
- && vs. &.
- || vs. |.
- xor()

#### 3.7 Exercises

- Use the **seq()** function to generate the following sequence: 6 13 20 27 34 41 48
- R has the its own inversion function, rev(), e.g,:

```
x <- seq(from=2, to=33, by=3)
x</pre>
```

```
[1] 2 5 8 11 14 17 20 23 26 29 32
```

```
y <- rev(x)
y</pre>
```

[1] 32 29 26 23 20 17 14 11 8 5 2

Invert the vector x without invoking the **rev()** function.

- Create the following R vector using  $\mathbf{only}$  the  $\mathbf{seq()}$  and  $\mathbf{rep()}$  functions:
  - -8 -8 -8 -8 0 8 8 8 16 16 16 16 16
- Create the following vector: -512 -216 -64 -8 0 8 64 216 512 1000
- Generate a random vector of integers using the following code:

```
x <- sample(x=0:1000, size=100, replace=TRUE)
```

- Invoke the above code to generate the vector  ${\tt x}$
- Find if there are any integers in the vector **x** which can be divided by 4 and 6
- Find those numbers and their corresponding indices in the vector **x**.

"It is my experience that proofs involving matrices can be shortened by 50% if one throws the matrices out" (Emil Artin)

## 4 Matrices & Arrays

Matrices and arrays are homogeneous atomic vectors with an extra attribute: dimension

By default, the elements are stored in a **column-major** fashion. (cfr. **Fortran**). However, we can store the elements in **row-major** order (cfr. **C**) as well.

#### 4.1 Creation of matrices

Matrices can be created in different ways:

- use of the **matrix()** function
- use of rbind()/cbind()
- set the attributes() of a vector
- special functions like e.g. diag()

#### 4.1.1 Examples

• use of the **matrix()** function:

The matrix() function creates a matrix based on a vector.

By default, the elements are stored in a column-major fashion.

The use of the flag byrow=TRUE will store the data in a row-major fashion.

```
A <- matrix(data=1:10, nrow=2) # Column-major (like Fortran)
     [,1] [,2] [,3] [,4] [,5]
                       7
[1,]
        1
             3
                  5
[2,]
                  6
B <- matrix(data=c(2,3,893,0.17), nrow=2, ncol=2)
     [,1]
            [,2]
[1,]
        2 893.00
[2,]
            0.17
```

```
C <- matrix(data=1:10, nrow=2, byrow=TRUE) # Row-major (like C, C++)
C

[,1] [,2] [,3] [,4] [,5]
[1,] 1 2 3 4 5
[2,] 6 7 8 9 10
```

• use of the **rbind()/cbind()** functions:

```
rbind(): Bind several vectors (as rows) into a matrix.
cbind(): Bind several vectors (as columns) into a matrix.
```

```
A <- rbind(1:10,11:20)
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
[1,]
             2
                   3
                        4
                              5
                                   6
                                         7
                                              8
                                                   9
                                                         10
[2,]
       11
             12
                  13
                       14
                             15
                                  16
                                        17
                                                   19
                                                         20
typeof(A)
[1] "integer"
class(A)
[1] "matrix" "array"
```

```
B \leftarrow cbind(1:5,6:10,11:15)
      [,1] [,2] [,3]
[1,]
         1
               6
                    11
[2,]
         2
               7
                    12
[3,]
         3
               8
                    13
[4,]
                    14
[5,]
         5
              10
                    15
class(B)
```

- [1] "matrix" "array"
  - $\bullet \;$  modifying the  $\operatorname{\mathbf{dim}}$  attribute.

The **fundamental** difference between an R vector and matrix is the presence (in the case of matrices) of a non NULL dim attribute.

We change easily convert a vector into a matrix by setting the dimensions of the vector:

- through the dim() function.
- through the attr() function.

The inverse can be done as well by setting the dim attribute of matrix to NULL.

#### 4.1.2 Examples

```
A <- 1:10
typeof(A)

[1] "integer"
class(A)

[1] "integer"
dim(A)
```

NULL

```
# Matrix
B <- matrix(1:10,nrow=2,ncol=5,byrow=TRUE)</pre>
typeof(B)
[1] "integer"
class(B)
[1] "matrix" "array"
dim(B)
[1] 2 5
# Vector
A <- 1:10
[1] 1 2 3 4 5 6 7 8 9 10
dim(A)
NULL
typeof(A)
[1] "integer"
class(A)
[1] "integer"
\# OPTION I: Using the dim function transform a vector into a matrix
dim(A) \leftarrow c(2,5)
     [,1] [,2] [,3] [,4] [,5]
[1,] 1 3 5 7 9
[2,]
       2 4 6 8 10
dim(A)
[1] 2 5
typeof(A)
[1] "integer"
class(A)
[1] "matrix" "array"
```

```
\# Converting the matrix back to a vector
dim(A) <- NULL</pre>
dim(A)
NULL
typeof(A)
[1] "integer"
class(A)
[1] "integer"
# Option II: More general way
# Convert vector into a matrix
A <- 1:8
[1] 1 2 3 4 5 6 7 8
class(A)
[1] "integer"
attr(A,'dim') <- c(2,4)
     [,1] [,2] [,3] [,4]
[1,]
       1 3 5
[2,]
        2
              4
                   6
class(A)
[1] "matrix" "array"
# Convert matrix into a vector.
attr(A, 'dim') <- NULL</pre>
[1] 1 2 3 4 5 6 7 8
class(A)
[1] "integer"
4.2
      Operations on matrices
  • Operations like *,/, + happpen element-wise.
   • There are also more specialized functions:
       - the mean over rows and columns (rowMeans(), colMeans())
       - linear algebra functions (\%*\%, \mathbf{t}(), ...)
```

## 4.2.1 Examples

[1,] 1 3 5 [2,] 2 4 6

• Operations (by **default: element-by-element**): A <- matrix(1:10, nrow=2) B <- matrix( seq(10, 100, by=10), nrow=2)</pre> [,1] [,2] [,3] [,4] [,5] [1,] 1 3 5 7 9 [2,] 2 4 6 8 10 [,1] [,2] [,3] [,4] [,5] [1,] 10 30 50 70 90 [2,] 20 40 60 80 100 A\*B[,1] [,2] [,3] [,4] [,5] [1,] 10 90 250 490 810 [2,] 40 160 360 640 1000 C <- matrix(rep(2,10), nrow=2)</pre> C\*\*A[,1] [,2] [,3] [,4] [,5] [1,] 2 8 32 128 512 [2,] 4 16 64 256 1024 • Calculate row and column means : # Means of rows and columns [,1] [,2] [,3] [,4] [,5] [1,] 1 3 5 7 9 [2,] 2 4 6 8 10 rowMeans(A) [1] 5 6 colMeans(A) [1] 1.5 3.5 5.5 7.5 9.5 • Matrix multiplication (%\*%) : A <- matrix(1:6, nrow=2) [,1] [,2] [,3]

```
B <- matrix(seq(10,120,by=10), nrow=3)</pre>
     [,1] [,2] [,3] [,4]
[1,]
       10
             40
                   70 100
[2,]
       20
                   80
                      110
[3,]
       30
             60
                   90 120
C <- A%*%B
     [,1] [,2] [,3] [,4]
[1,] 220 490 760 1030
[2,] 280 640 1000 1360
dim(C)
[1] 2 4
   • Linear algebra routines
Some of the more common ones in R:
   • solve(): invert a square matrix
   • diag()
       - extracts the diagonal of a matrix when a matrix is provided.
       - creates a diagonal matrix when a vector is provided.
   • eigen(): calculates the eigenvalues and eigenvectors of a matrix
   • det() : calculates the determinant of a matrix.
   • t(): calculates the transpose<sup>1</sup> of a matrix.
# Invert matrix A
A \leftarrow matrix(c(1, 3, 2, 4), ncol = 2, byrow = T)
Ainv <- solve(A)
Ainv %*% A
     [,1] [,2]
[1,]
        1
              0
[2,]
        0
# Create a diagonal matrix
C \leftarrow diag(c(1,4,7))
     [,1] [,2] [,3]
[1,]
              0
                    0
         1
[2,]
                    0
              0
                    7
[3,]
# Extract the diagonal elements
```

D <- matrix(1:8,nrow=4)</pre>

<sup>&</sup>lt;sup>1</sup>Can also be used for dataframes (see later)

```
[1,]
[2,]
        2
[3,]
        3
             7
[4,]
diag(D)
[1] 1 6
# Calculate eigenvalues and eigenvectors of A
r <- eigen(A)
eigen() decomposition
$values
[1] 5.3722813 -0.3722813
$vectors
           [,1]
                      [,2]
[1,] -0.5657675 -0.9093767
[2,] -0.8245648 0.4159736
# Eigenvalues
r$values
[1] 5.3722813 -0.3722813
# Matrix with eigenvectors
r$vectors
           [,1]
                      [,2]
[1,] -0.5657675 -0.9093767
[2,] -0.8245648 0.4159736
# Diagonal Matrix (Similarity Transformation)
solve(r$vectors) %*% A %*% r$vectors
              [,1]
                         [,2]
[1,] 5.372281e+00 0.0000000
[2,] -3.330669e-16 -0.3722813
Note that under the hood R calls BLAS and LAPACK.
# Find the version used of BLAS and LAPACK
La_library()
[1] "/usr/lib/x86_64-linux-gnu/lapack/liblapack.so.3.7.1"
extSoftVersion()["BLAS"]
                                              BLAS
"/usr/lib/x86_64-linux-gnu/blas/libblas.so.3.7.1"
```

## 4.3 Hash Tables

[,1] [,2]

#### Other types **5**

- Attributes
- Special types:
  - FactorsDateTime
- NA, NaN, NULL

## Other topics on Data structures

- List
- Dataframe & Tibble
- IO (read.csv, read.file)
- Names
- Subsetting, [[]] vs. []

# Conditionals & Loops

- if, else, else if switch and elseif
- $\bullet$  for
- while
- repeat
- return()

## **Environments**

- search(), attach, detach
- library

## **Functions**

- lexical scoping
- simple functions
- args(), formals()
- default arg, ...
- lazy evaluation
- closure
- anonymous functions
- $\bullet\,\,$  make your own operators
- loop functions: {l,s,m}apply, split

# Capita selecta

• profiling, debugging