Introduction to R*

Lecture 5: Environments, Running R , Libraries and some probability distributions

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Last updated: 11/18/2022 @ 15:46:03

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R comes with the most important probability distributions installed.
For the theoretical underpinnings, see e.g. (Casella & Berger, 2002).

Probability distributions can be grosso modo classified into:

- discrete distributions
- continuous distributions

4.1 Discrete distributions

Let $\mathbb{P}(X = k; \{\xi\}))$ be a discrete probability mass function when the random variable $X = k$ and which depends on the parameter set $\{\xi\}$.
Let `keyword` be the (variable) name of the corresponding distribution. Then,

- **d**`keyword`$(k, \ldots)$: calculates the probability $\mathbb{P}(X = k)$
- **p**`keyword`$(k, \ldots)$: calculates the cumulative probability function (CDF) at k:
  $$F(X = k; \{\xi\}) := \sum_{j=0}^{k} \mathbb{P}(X = j)$$
- **q**`keyword`$(p, \ldots)$: calculates the value of k where $p = F(k; \{\xi\})$ or $k = \lceil F^{-1}(p; \{\xi\}) \rceil$
- **r**`keyword`$(n, \ldots)$: generates a vector of n random values sampled from the distribution `keyword`.

Some common discrete probability distributions (implemented in R) are displayed in Table 1.

keyword	Name	$\mathbb{P}(X = k; \{\xi\})$	Parameter set $(\{\xi\})$
binom	Binomial	$\binom{n}{k} p^k (1-p)^{n-k}$	$0 \leq p \leq 1$
nbinom	Negative Binomial	$\binom{k+r-1}{k} (1-p)^k p^r$	$0 \leq p \leq 1$; $r > 0$
hyper	Hypergeometric	$\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$	$N \in \{0, 1, 2, \ldots\}$; $K, n \in \{0, 1, \ldots, N\}$
pois	Poisson	$\frac{\lambda^k e^{-\lambda}}{k!}$	$0 < \lambda < \infty$

Table 1: A few common discrete probability distributions.

4.1.1 Examples

- Let's consider the following distribution: binom$(p = 0.3, n = 5)$.

```
n <- 5
p <- 0.3
# Alternative code for the binom distribution
mybinom <- function(n,p){
    v <- vector(mode="double", length=(n+1))
    for(k in 0:n){
      v[k+1] <- choose(n,k)*p^k*(1-p)^(n-k)
    }
    return(v)
}
pvec <- mybinom(n,p)
```

  - Value of the PMF at $k = \{0, 1, \ldots, 5\}$:

```
for(k in 0:n){
    cat(sprintf("  P(X=%d):%8.6f and should be %8.6f\n",
                k, dbinom(k, size=n, p), pvec[k+1]))
}
```

```
  P(X=0):0.168070 and should be 0.168070
  P(X=1):0.360150 and should be 0.360150
  P(X=2):0.308700 and should be 0.308700
  P(X=3):0.132300 and should be 0.132300
  P(X=4):0.028350 and should be 0.028350
  P(X=5):0.002430 and should be 0.002430
```

  - Value of the CDF at $k = \{0, 1, \ldots, 5\}$:

```
for(k in 0:n){
    cat(sprintf("  F(X=%d):%8.6f and should be %8.6f\n",
    k, pbinom(k,size=n,p), sum(pvec[1:(k+1)]) ))
}
```

```
  F(X=0):0.168070 and should be 0.168070
```

```
  F(X=1):0.528220 and should be 0.528220
  F(X=2):0.836920 and should be 0.836920
  F(X=3):0.969220 and should be 0.969220
  F(X=4):0.997570 and should be 0.997570
  F(X=5):1.000000 and should be 1.000000
```

– The quantile function:

```
pvec <- c(0.0, 0.25, 0.50, 0.75, 1.00)
for(item in pvec){
    cat(sprintf("  P:%4.2f => k=%d\n",
    item, qbinom(item,size=n, prob=p)))
}
```

```
  P:0.00 => k=0
  P:0.25 => k=1
  P:0.50 => k=1
  P:0.75 => k=2
  P:1.00 => k=5
```

– Sampling random numbers from the distribution:

```
tot <- 15
vec <- rbinom(tot,size=n, prob=p)
print(vec)
```

```
 [1] 2 1 1 1 0 1 1 2 3 1 2 2 2 3 2
```

4.2 Continuous distributions

Let $f(x; \{\xi\})$ be a continuous probability density function (pdf), which depends on the variable x and the parameter set $\{\xi\}$.
Let `keyword` be the (variable) name of the corresponding distribution. Then,

- **d**`keyword`$(x, \ldots)$: calculates the value of the pdf at x, i.e. $f(x; \{\xi\})$
- **p**`keyword`$(x, \ldots)$: calculates the cumulative probability function (cdf) at x: $F(x; \{\xi\}) := \int_{-\infty}^{x} f(t; \{\xi\})\, dt$
- **q**`keyword`$(p, \ldots)$: calculates the value of x where $p = F(x; \{\xi\})$ or $x = F^{-1}(p; \{\xi\})$
- **r**`keyword`$(n, \ldots)$: generates a vector of n random values sampled from the distribution `keyword`.

Some common continuous probability distributions (implemented in `R`) are displayed in Table 2.

`keyword`	Name	$f(x;\{\xi\})$	`Dom`(x)	Parameter set ($\{\xi\}$)
`unif`	Uniform	$\frac{1}{(b-a)}$	$a \le x \le b$	a, b
`norm`	Normal	$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$-\infty < x < \infty$	$-\infty < \mu < \infty$, $\sigma > 0$
`cauchy`	Cauchy	$\frac{1}{\pi\sigma} \frac{1}{1+\left(\frac{x-\theta}{\sigma}\right)^2}$	$-\infty < x < \infty$	$-\infty < \theta < \infty$, $\sigma > 0$
`t`	Student's t	$\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})} \frac{1}{\nu\pi} \frac{1}{\left(1+\frac{x^2}{\nu}\right)^{(\nu+1)/2}}$	$-\infty < x < \infty$	$\nu = 1, 2, \ldots$
`chisq`	Chi-squared	$\frac{1}{\Gamma(\nu/2)2^{(\nu/2)}} x^{(\nu/2)-1} e^{-\frac{x}{2}}$	$0 \le x < \infty$	$\nu = 1, 2, \ldots$
`f`	F	$\frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \frac{x^{(\nu_1-2)/2}}{\left(1+\left(\frac{\nu_1}{\nu_2}\right)x\right)^{(\nu_1+\nu_2)/2}}$	$0 \le x < \infty$	$\nu_1, \nu_2 = 1, 2, \ldots$
`exp`	Exponential	$\lambda e^{-\lambda x}$	$0 \le x < \infty$	$\lambda > 0$

Table 2: A few common continuous probability distributions.

where $\Gamma(x)$ stands for the gamma function which has the following mathematical form:

$$\Gamma(x) := \int_0^\infty t^{x-1} e^{-x} dt$$

4.2.1 Examples

- Let's consider the following distribution: $N(\mu = 5.0, \sigma^2 = 4.0)$.
  Therefore, `distro:norm`

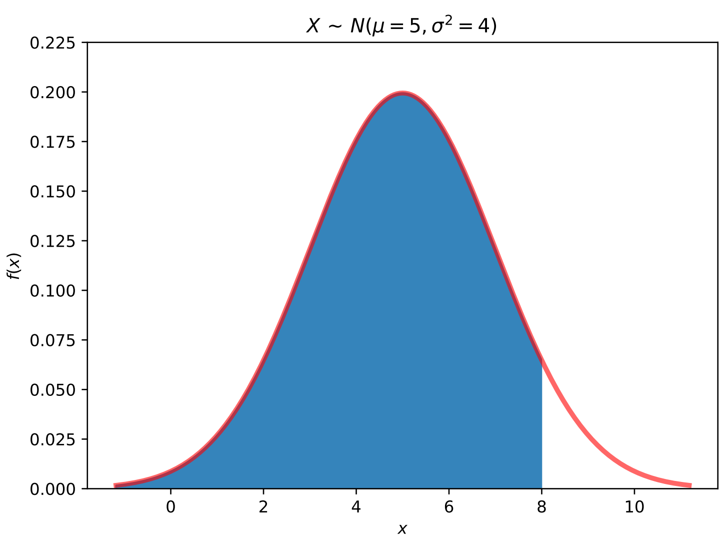

Figure 1: Plot of the normal distribution (red). The area under the curve (blue) represents the cumulative probability at $x = 8.0$.

```r
x <- 8.0
mu <- 5.0
sigma <- 2.0
```

- Value of the PDF at x:

  ```r
  cat(sprintf("The density at %f is %12.10f\n", x, dnorm(x,mean=mu, sd=sigma)))
  ```

  ```
  The density at 8.000000 is 0.0647587978
  ```

- Value of the CDF at x:

  ```r
  prob <- pnorm(x,mean=mu,sd=sigma)
  cat(sprintf("The Cumulative Probability at %f is %12.10f", x, prob))
  ```

  ```
  The Cumulative Probability at 8.000000 is 0.9331927987
  ```

- The quantile function:

  ```r
  cat(sprintf("The point where the Cumulative Probability is %12.10f: %8.4f",
              prob, qnorm(prob, mean=mu, sd=sigma)))
  ```

  ```
  The point where the Cumulative Probability is 0.9331927987:   8.0000
  ```

- Sampling random numbers from the distribution:

  ```r
  vec <- rnorm(n=10, mean=mu, sd=sigma)
  print(vec)
  ```

  ```
   [1] 6.474410 2.043741 6.225198 4.247351 5.462166 5.915501 5.401859 5.530572
   [9] 4.589846 1.723328
  ```

4.3 Exercises:

1. Generate vectors with 10^4, 10^5, 10^6, 10^7 random numbers from the $\chi^2(\nu = 5)$ distribution. Calculate the mean, as well as the variance for each of those vectors. (**mean()**, **var()**)
   Note: If $X \sim \chi^2(\nu) \Rightarrow \mathbb{E}[X] = \nu$ and $\mathbb{V}[X] = 2\nu$

2. A brewer from the far-away lands of Hatu wants to follow the land's alcohol ordinance (i.e. a maximum of 5% ethanol per volume). In order to make sure that he doesn't violate the regulation he sent a batch of independent samples to a certified lab. The lab results are to be found in the file `data/beer.csv`.

   He further assumes that the beer samples are normally distributed i.e. $N(\mu, \sigma^2)$ where μ is known but σ^2 is unknown.

   His plan is to perform a simple one-sided hypothesis test:

   $$H_0 : \mu_0 = 5.0$$
   $$H_1 : \mu \neq \mu_0$$

   Let $c_{1-\alpha}$ be the (critical) point that separates the acceptance region ($\mathcal{A}$) with $\mathbb{P}(\mathcal{A}) = 1 - \alpha$ from the rejection region ($\mathcal{R}$) with $\mathbb{P}(\mathcal{R}) = \alpha$. Therefore,

$$\begin{aligned}\alpha &= \mathbb{P}(\overline{X} > c_{1-\alpha}|\mu = \mu_0)\\ &= \mathbb{P}\Big(\frac{\overline{X} - \mu_0}{s/\sqrt{n}} > \frac{c_{1-\alpha} - \mu_0}{s/\sqrt{n}}\Big)\\ &= \mathbb{P}\Big(\frac{\overline{X} - \mu_0}{s/\sqrt{n}} > t_{1-\alpha}(\nu = n-1)\Big)\end{aligned}$$

where t stands for `Student's t` distribution, s^2 is the sample variance, n the number of measurements.

- Read the lab results from the file `data/beer.csv`.
- Calculate the numerical value (τ) of the test-statistic T, given by:

$$T = \frac{\overline{X} - \mu_0}{s/\sqrt{n}}$$

- Determine $t_{0.95}(\nu = n-1)$, i.e. the critical point such that $\mathbb{P}(\mathcal{R}) = 0.05$.
- Decide whether the brewer should reject H_0 (i.e. reject if $\tau > t_{0.95}(\nu = n-1)$).
- What is the probability of the area under the curve for $t \in [\tau, +\infty)$?

3. Check this link if you are interested in the origin of `Student's t` distribution.

Bibliography

Casella G. & Berger R.L. (2002). Statistical Inference. Duxbury Advanced Series in Statistics and Decision Sciences. Thomson Learning.