Isomorphism in Union-Closed Sets

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Outline

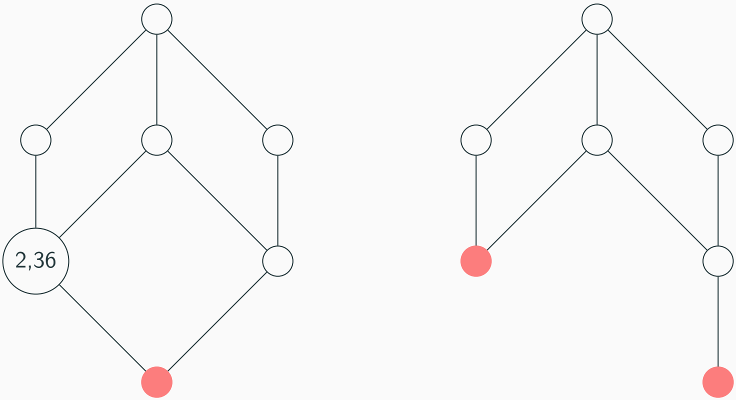

Figure 1: Two lattice diagrams. The red nodes belong to $\mathcal{K}^{\perp}$.

Introduction

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- Definition of Union-Closed Families of Sets
- Péter Frankl's Union-Closed Set Conjecture
- Our Focus: Structural Properties of Isomorphisms

Definitions and Theorems

Definitions

Union-Closed Family: A collection $\mathcal{K} \subseteq 2^{[n]}$ is union-closed if for all $A, B \in \mathcal{K}$, we have $A \cup B \in \mathcal{K}$.

Isomorphism: A bijection $h : \mathcal{K}_1 \to \mathcal{K}_2$ such that:

$$h(A \cup B) = h(A) \cup h(B) \quad \forall A, B \in \mathcal{K}_1.$$

Main Theorem

Theorem: For every isomorphism $h : \mathcal{K}_1 \to \mathcal{K}_2$, there exists a corresponding hyperisomorphism $H : \bigcup \mathcal{K}_1 \to \bigcup \mathcal{K}_2$ such that:

$$h(A) = \{H(a) \mid a \in A\}, \quad \forall A \in \mathcal{K}_1.$$

Conclusion

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- Structural preservation under isomorphisms
- Connection to the Union-Closed Set Conjecture
- Future work: applications of hyperisomorphisms

Thank You!

Questions?