

Isomorphism in Union-Closed Sets

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In 1979, Péter Frankl proposed a famous conjecture about finite union-closed families. He stated that in every such family, there exists an element that appears in at least half of the sets. Despite significant efforts, the problem has remained unsolved for more than four decades.

Union-Closed Conjecture (Frankl, 1979)

Let $\mathcal{K} \subseteq 2^{[n]}$ be a union-closed family of sets. Then there exists an element $i \in \bigcup \mathcal{K}$ such that: $|\mathcal{K}| \leq 2|\mathcal{K}^i|$, where

$$\mathcal{K}^i = \{A \in \mathcal{K} \mid i \in A\}.$$

Example

Example 1: Consider the family of sets:

$$\mathcal{K} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

The union-closed property holds, and element a (or b) appears in at least half the sets.

Example 2: Consider the family of sets:

$$\mathcal{K} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

The union-closed property holds, and element a (or b) appears in at least half the sets.

Thank You!

Any Questions?

- Email: m.moghadas11235@gmail.com
- Paper available on ArXiv.