# Analysis of individual earns

## Group 29

## 1 Introduction

**Dataset 29** were collected from the US 1994 Census database. This dataset contain data on individuals regarding their income level, and various socioeconomic factors. You will have access to the following variables:

- Age The age of the individual in years
- Education Highest education level obtained by the individual
- Marital Status The marital status of the individual
- Occupation The occupation of the individual
- Sex The sex of the individual
- Hours Pw Number of hours worked per week by the individual
- Nationality The nationality of birth of the individual
- Income A factor variable with two levels: >50k if the individual earns more than \$50k per year, or <=50k if the individual earns less than or equal to \$50k per year

#### 1.1 Task

Imagine you have been tasked by the government to identify which features based on the census data impact the income an individual makes: - Which factors influence whether an individual earns more than \$50k per year?

#### You should:

- 1. Conduct an analysis to answer your question using a Generalised Linear Model (GLM)
- 2. Summarise your results in the form of a presentation

## 2 Exploratory Data Analysis

Summary statistics of **Income** are presented in the following for each column separately. Observe the correlation of the numeric of data:

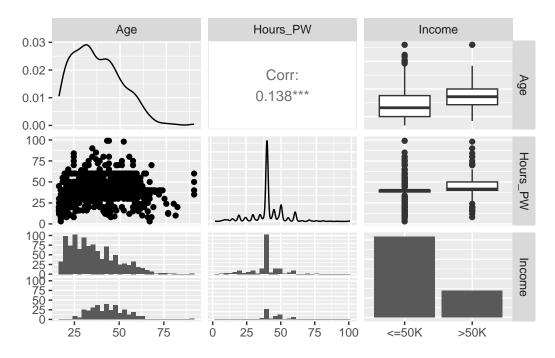


Figure 1: Numeric Data Correlation

## 2.1 Data levels

[1]	"10th"	"11th"	"12th"	"1st-4th"	"5th-6th"
[6]	"7th-8th"	"9th"	"Assoc-acdm"	"Assoc-voc"	"Bachelors"
[11]	"Doctorate"	"HS-grad"	"Masters"	"Prof-school"	"Some-college"
[1]	"Divorced"	"Marrie	ed-civ-spouse"	"Married-spo	use-absent"
[4]	"Never-married"	"Separa	ated"	"Widowed"	

```
[1] "Adm-clerical"
                         "Craft-repair"
                                              "Exec-managerial"
 [4] "Farming-fishing"
                         "Handlers-cleaners" "Machine-op-inspct"
 [7] "Other-service"
                         "Priv-house-serv"
                                              "Prof-specialty"
[10] "Protective-serv"
                         "Sales"
                                              "Tech-support"
[13] "Transport-moving"
[1] "Female" "Male"
 [1] "Cambodia"
                                   "Canada"
 [3] "China"
                                   "Columbia"
 [5] "Cuba"
                                   "Dominican-Republic"
 [7] "El-Salvador"
                                   "England"
 [9] "France"
                                   "Germany"
[11] "Greece"
                                   "Guatemala"
                                   "India"
[13] "Haiti"
[15] "Iran"
                                   "Ireland"
                                   "Jamaica"
[17] "Italy"
[19] "Japan"
                                   "Laos"
[21] "Mexico"
                                   "Outlying-US(Guam-USVI-etc)"
                                   "Poland"
[23] "Philippines"
[25] "Portugal"
                                   "Puerto-Rico"
[27] "Scotland"
                                   "South"
[29] "Taiwan"
                                   "Trinadad&Tobago"
[31] "United-States"
                                  "Vietnam"
Chi-square test for Education vs Income:
    Pearson's Chi-squared test
data: contingency_table
X-squared = 193.07, df = 14, p-value < 2.2e-16
Chi-square test for Marital_Status vs Income :
    Pearson's Chi-squared test
data: contingency_table
X-squared = 263.24, df = 5, p-value < 2.2e-16
```

Chi-square test for Occupation vs Income:

```
Pearson's Chi-squared test
```

```
data: contingency_table
X-squared = 167.57, df = 12, p-value < 2.2e-16</pre>
```

Chi-square test for Sex vs Income :

Pearson's Chi-squared test with Yates' continuity correction

```
data: contingency_table
X-squared = 57.101, df = 1, p-value = 4.139e-14
```

Chi-square test for Nationality vs Income :

Pearson's Chi-squared test

```
data: contingency_table
X-squared = 28.476, df = 31, p-value = 0.5965
```

#### 2.2 Income Distribution

Income distribution:

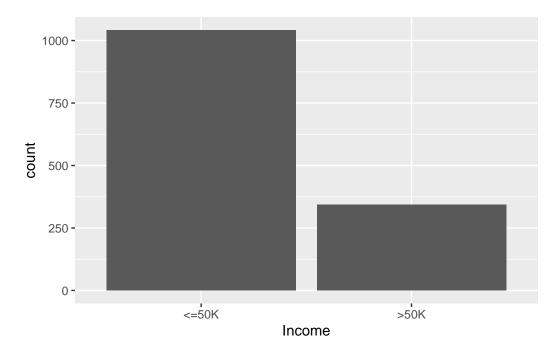


Figure 2: Income Distribution.

The income distribution analysis reveals that 75.2% of individuals earn <=50K, while only 24.8% earn >50K. This indicates a significant class imbalance, where the majority of the dataset consists of low-income individuals.

This imbalance can impact predictive modeling, as machine learning algorithms may become biased towards the dominant class (<=50K), leading to poor classification performance for high-income individuals. If a classification model is trained without addressing this issue, it may struggle to accurately predict the >50K category.

## 2.3 Occupation by Income

Income Adm	-clerical	Craft-	repair	Exe	c-manage	erial F	arming-fis	hing	
<=50K 13	.5% (141)	14.8%	(154)		9.3%	(97)	3.6		(37)
>50K 7	.0% (24)	13.1%	(45)		23.0%	(79)	1.5%	(5)	
Handlers-c	Leaners M	achine-	op-ins	pct (	Other-se	ervice	Priv-house	-serv	
5.	3% (55)	•	7.9% (	82)	14.1%	(147)	0.4		(4)
1.	7% (6)	4	4.1% (	14)	0.6%	(2)	0.0		(0)
Prof-speci	alty Prot	ective-	serv		Sales 7	Cech-su	pport Trans	sport-r	noving
8.8%	(92)	2.0%	(21) 1	2.1%	(126)	2.3%	(24)	5.9	(61)
25.3%	(87)	2.9%	(10) 1	2.5%	(43)	3.8%	(13)	4.7	(16)

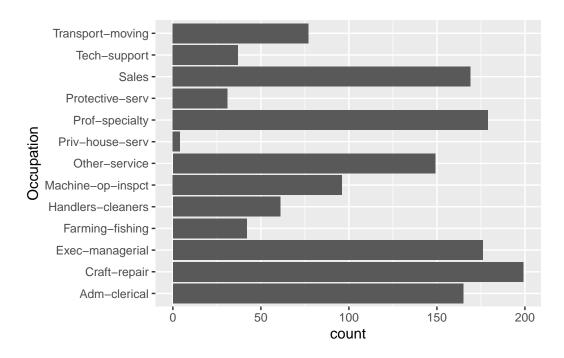


Figure 3: Occupation Distribution.

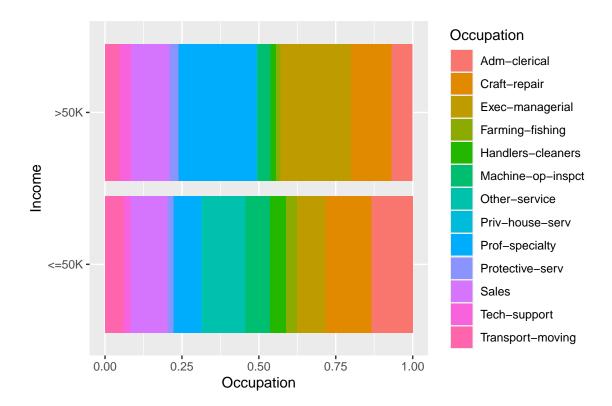


Figure 4: Income by Occupation.

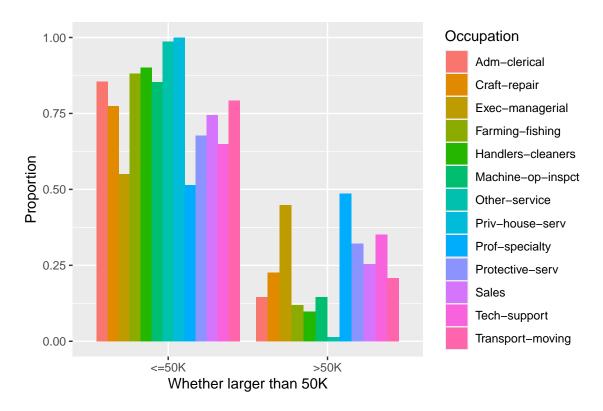


Figure 5: Income by Occupation.

The analysis of occupation by income shows significant variation across different job categories.

Administrative/Clerical roles: Approximately 85.6% of individuals in this category earn <=50K, while only 14.4% earn >50K, indicating that these jobs are predominantly lower-income. Craft and repair jobs: 76.4% of workers earn <=50K, with 23.6% earning >50K, suggesting a slightly better income distribution compared to clerical jobs. Executive/Managerial roles: This category has the highest proportion of high-income earners, with 44.9% earning >50K, showing that leadership positions significantly increase the likelihood of higher income. Farming/Fishing occupations: 88.1% of individuals earn <=50K, and only 11.9% earn >50K, reinforcing that agricultural jobs tend to be lower-paying. Handlers/Cleaners: The lowest income distribution, with 90.3% earning <=50K and only 9.7% earning >50K, indicating very limited access to higher salaries. This distribution highlights that occupational type is a key factor in determining income, with executive roles offering the highest proportion of high-income earners, while farming, cleaning, and clerical jobs predominantly fall into the lower-income category. These findings suggest that education, experience, and industry type play crucial roles in income disparities.

## 2.4 Education by Income

Income 10th 12th 1st-4th 5th-6th 7th-8th 9th 11th <=50K 3.6% (37) 3.5% (36) 1.7% (18) 1.2% (12) 1.3% (14) 2.4% (25) 2.3(24) >50K 0.9% (3) 0.9% (3) 0.0% (0) 0.0% (0) 0.0% (0) 0.6% (2) 0.9% (3) Bachelors Doctorate HS-grad Assoc-acdm Assoc-voc Masters Prof-school 3.9% (41) 4.8% (50) 13.4% (139) 0.2% (2) 34.6% (360) 3.0% (31) 0.3% (3) 2.9% (10) 4.1% (14) 24.7% (85) 4.4% (15) 21.8% (75) 13.4% (46) 4.7(16)Some-college 23.9 (249)20.9% (72)

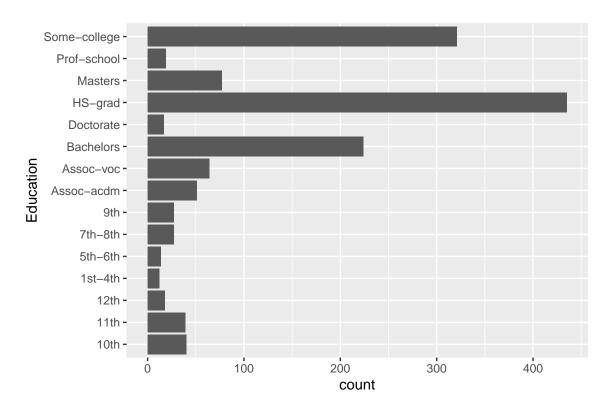


Figure 6: Education Distribution.

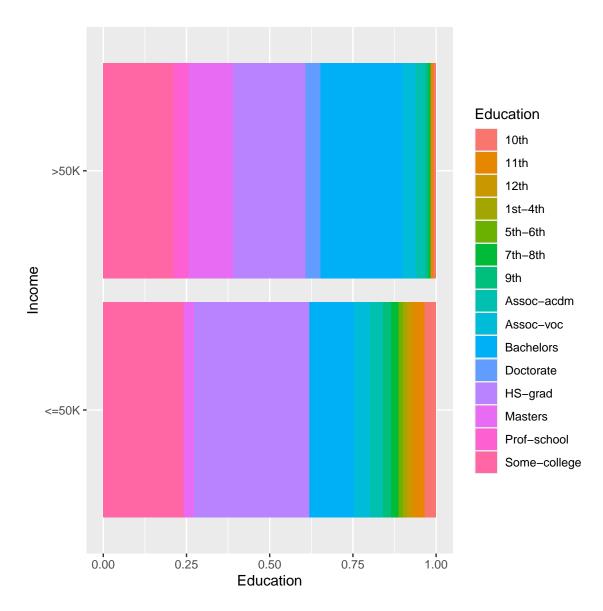


Figure 7: Income by Education.

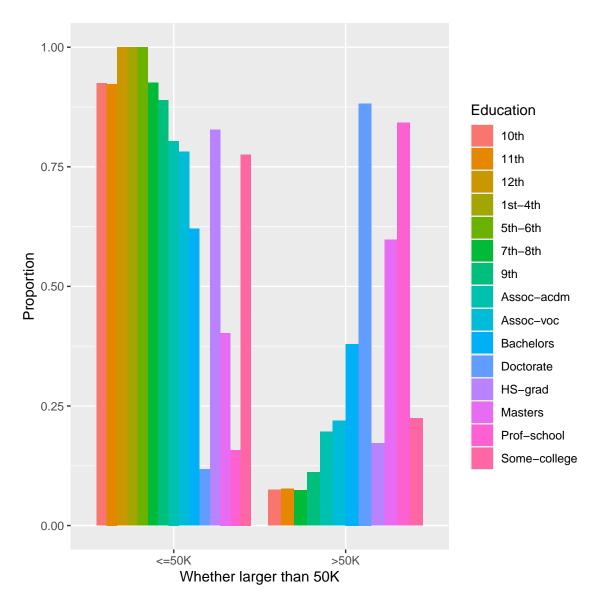


Figure 8: Income by Education.

The distribution of income across different education levels shows a clear correlation between higher education and increased income potential:

Lower education levels (1st-4th, 5th-6th, 12th grade):

100% of individuals in these categories earn <=50K, with no individuals earning >50K. This indicates that individuals with minimal education have almost no access to high-income jobs. 10th and 11th grade education:

93.8% of individuals with a 10th-grade education earn <=50K, while only 6.3% earn >50K. 92.9% of individuals with an 11th-grade education earn <=50K, with 7.1% in the high-income group. Although slightly better than primary education, these groups still have a very low likelihood of earning above 50K.

Key Takeaways: Education level is a strong predictor of income. Individuals without a high school diploma have almost no chance of reaching the high-income category. These findings highlight the importance of higher education in increasing earning potential and access to well-paying jobs.

### 2.5 Martial\_Status by Income

Table 1: Martial Status Distribution.

Income	Divorced	Married-civ-spouse	Married-spouse-absent	Never-married	Separated	Widowed
<=50K	15.1% (157)	34.5% (359)	1.8% (19)	42.3% (440)	3.9% (41)	2.4% (25)
>50K	7.3% (25)	84.3% (290)	0.3% (1)	7.3% (25)	0.3% (1)	0.6% (2)

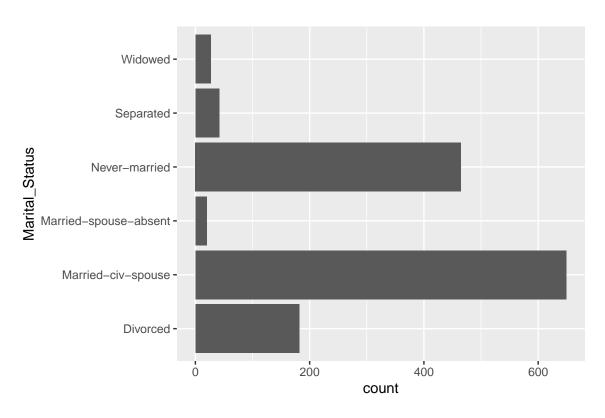


Figure 9: Marital\_Status Distribution.

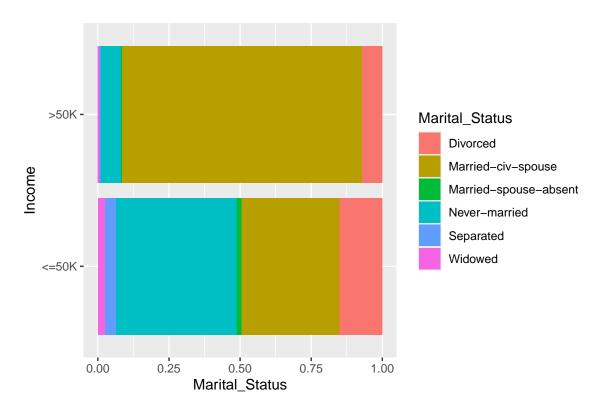


Figure 10: Income by Marital\_Status.

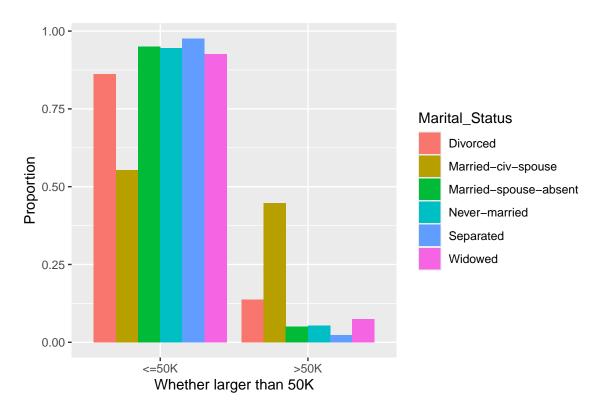


Figure 11: Income by Marital Status.

The distribution of income by marital status reveals a strong correlation between marriage and income level:

#### Married individuals (civil spouse):

44.3% earn >50K, the highest proportion among all marital statuses. This suggests that marriage (especially in stable relationships) is associated with higher earnings, possibly due to dual-income households or increased financial stability. Divorced individuals:

85.8% earn <=50K, while 14.2% earn >50K. Although lower than married individuals, this group has a higher proportion of high-income earners than single or separated individuals. Never-married and separated individuals:

95.5% of never-married individuals and 97.9% of separated individuals earn <=50K, with only a very small fraction entering the >50K category. This indicates that single individuals are more likely to belong to lower-income groups, possibly due to younger age demographics and lower accumulated financial resources. Key Takeaways: Being married is strongly associated with higher income levels. Single and separated individuals are less likely to earn above 50K, which may reflect differences in career stability, accumulated wealth, or household income. This analysis suggests that marital status can play a significant role in economic well-being.

# 2.6 Sex by Income

Table 2: Sex Distribution.

Income	Female	Male
<=50K	37.2% (387)	62.8% (654)
>50K	15.1% (52)	84.9% (292)

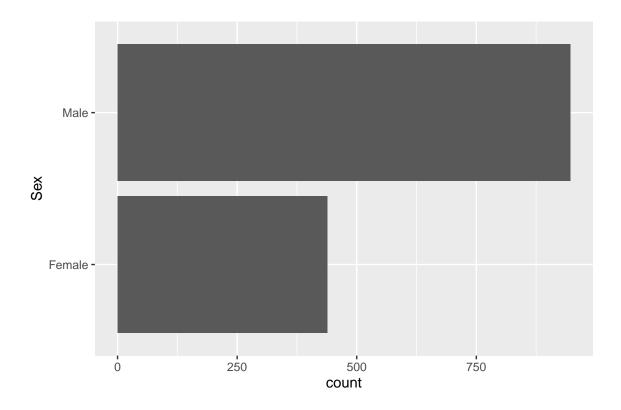


Figure 12: Sex Distribution.

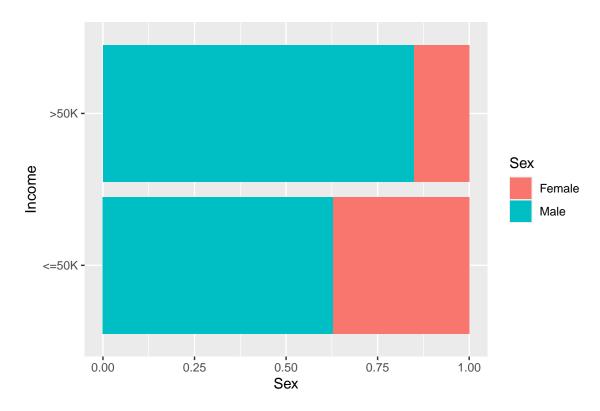


Figure 13: Income by Sex.

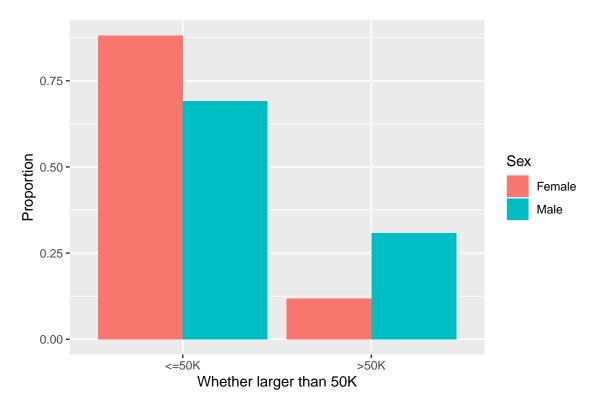


Figure 14: Income by Sex.

The distribution of income by gender reveals a significant disparity between males and females:

#### Females:

88.2% earn <=50K, while only 11.8% earn >50K. This indicates that women are much less likely to be in the high-income category. Males:

69.6% earn <=50K, whereas 30.4% earn >50K. The proportion of males earning above 50K is significantly higher than that of females. Key Takeaways: Men are significantly more likely to earn >50K compared to women. The income gap suggests possible gender disparities in wages, career advancement, or occupational roles. These findings highlight the need to investigate underlying causes, such as differences in industries, job roles, or social factors affecting earnings.

#### 2.7 Nationality by Income

```
>50K 0.3% (1) 0.9% (3) 0.3% (1) 0.0% (0) 0.3% (1)
                                                            0.0
                                                                    (0)
El-Salvador England France Germany Greece Guatemala Haiti
                                                                    India
  0.5% (5) 0.2% (2) 0.1% (1) 0.4% (4) 0.2% (2) 0.5% (5) 0.1% (1) 0.3 (3)
  0.0% (0) 0.0% (0) 0.0% (0) 0.9% (3) 0.3% (1) 0.0% (0) 0.0% (0) 0.0 (0)
    Iran Ireland
                  Italy Jamaica
                                     Japan
                                               Laos
                                                       Mexico
0.1% (1) 0.1% (1) 0.2% (2) 0.2% (2) 0.2% (2) 0.2% (2) 2.8
                                                                    (29)
0.0\% (0) 0.3\% (1) 0.0\% (0) 0.0\% (0) 0.3\% (1) 0.0\% (0) 0.3\% (1)
Outlying-US(Guam-USVI-etc) Philippines Poland Portugal Puerto-Rico Scotland
                 0.1% (1)
                            0.7% (7) 0.3% (3) 0.1% (1)
                                                          0.5% (5) 0.1(1)
                 0.0% (0)
                            0.6% (2) 0.3% (1) 0.3% (1)
                                                          0.6% (2) 0.0(0)
          Taiwan Trinadad&Tobago United-States Vietnam
0.3% (3) 0.1% (1)
                       0.1% (1)
                                  89.5% (932) 0.5
                                                                     (5)
0.0% (0) 0.6% (2)
                    0.0% (0)
                                  93.9% (323) 0.0
                                                                     (0)
```

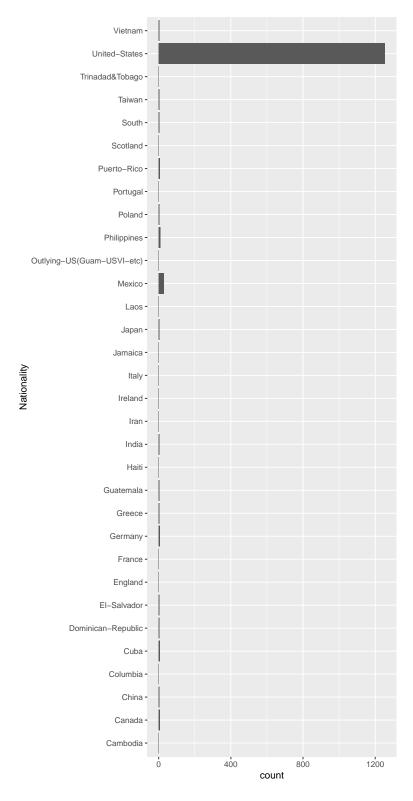


Figure 15: Nationality Distribution.

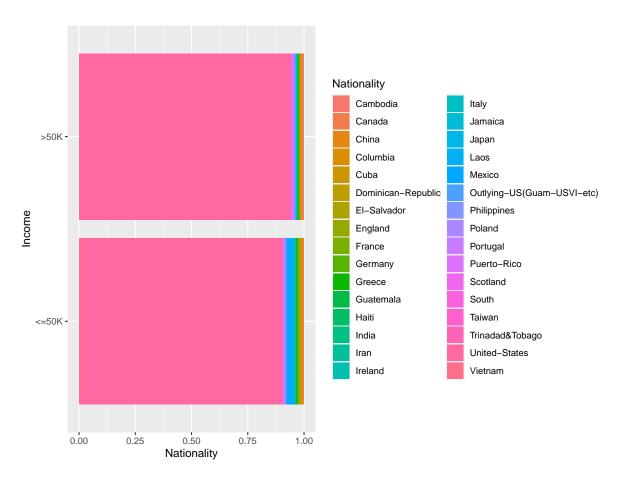


Figure 16: Income by Nationality.

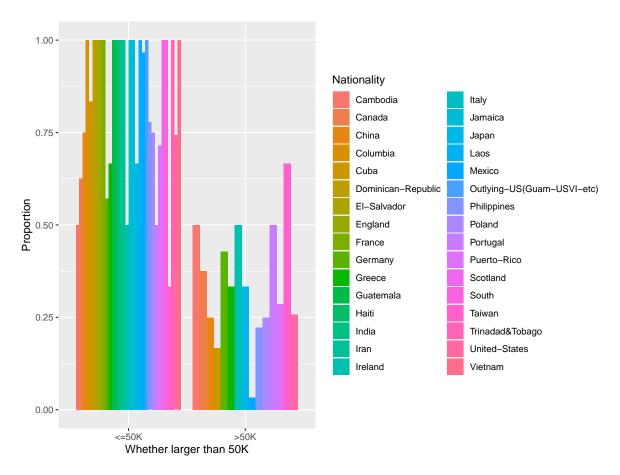


Figure 17: Income by Nationality.

The income distribution across different nationalities varies significantly:

#### Cambodia:

50% earn <=50K, and 50% earn >50K. This is the most balanced distribution among the nationalities observed. Canada:

62.5% earn <=50K, while 37.5% earn >50K. A relatively high proportion of individuals from Canada belong to the high-income group. China:

75% earn  $\leq 50$ K, and 25% earn  $\geq 50$ K. While some individuals reach the high-income group, the majority remain below 50K. Colombia:

100% earn <=50K, with no individuals in the >50K category. This suggests that people from this nationality are more likely to be in lower-income jobs. Cuba:

83.3% earn <=50K, while 16.7% earn >50K. The majority still earn less than 50K, but a small proportion reaches higher income levels. Key Takeaways: Nationality appears to be a factor influencing income distribution. Some nationalities, such as Canada and Cambodia, have higher proportions of high-income individuals, while others, like Colombia and Cuba, have a stronger concentration in the low-income group. These differences could be attributed to factors such as job opportunities, skill levels, or immigration status affecting income potential.

However, in the dataset, there are 1255 samples with American nationality, accounting for 90.7% of the total. Due to this highly imbalanced distribution, nationality is not suitable as an explanatory variable.

## 2.8 Age by Income

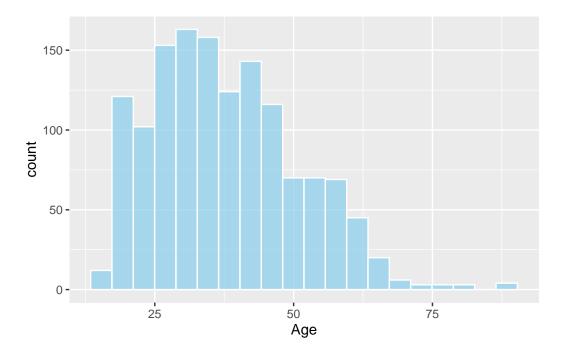


Figure 18: Age Distribution.

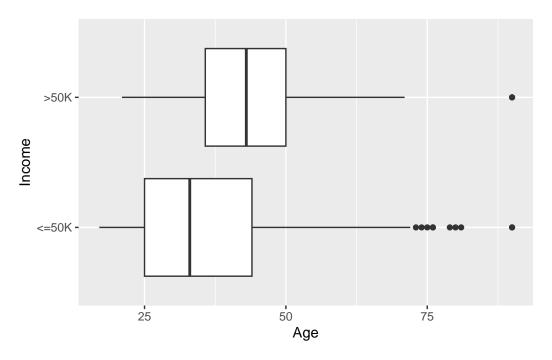


Figure 19: Income by Age.

The analysis of age distribution and its relationship to income reveals the following trends:

### Age Distribution:

The dataset contains individuals aged 17 to 90 years old. The age distribution appears roughly normal, with most individuals concentrated in their 30s and 40s. Income and Age Statistics:

Low-income group (<=50K): Average age: 36.1 years 25th percentile: 25 years 50th percentile (median): 33 years 75th percentile: 45 years High-income group (>50K): Average age: 43.9 years 25th percentile: 35 years 50th percentile (median): 43 years 75th percentile: 51 years Key Takeaways: Older individuals are more likely to be in the high-income group. The median age of high-income earners (43 years) is significantly higher than that of low-income earners (33 years). This suggests that experience, seniority, and career progression contribute to higher earnings over time. Younger individuals tend to belong to the low-income group. This is likely because they are in the early stages of their careers, earning entry-level salaries.

# 2.9 Hours\_pw by Income

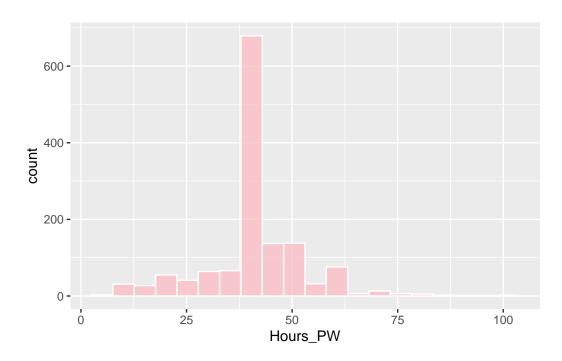


Figure 20: Hours\_PW Distribution.

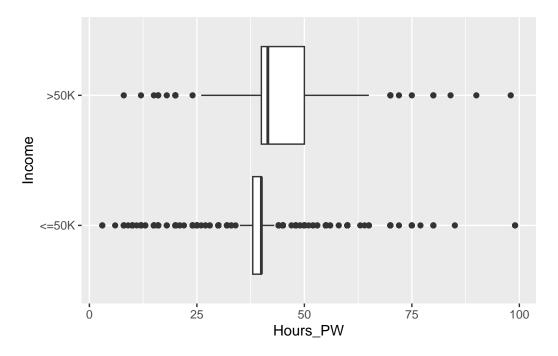


Figure 21: Income by Hours\_PW.

The analysis of weekly work hours and its relationship to income reveals key differences between low and high earners:

#### Work Hours Distribution:

The dataset contains a variety of work hours, with most individuals working around 40 hours per week. The distribution appears right-skewed, meaning some individuals work significantly longer hours. Work Hours Statistics by Income:

Low-income group (<=50K): The most common work hours are 40 hours per week, with 546 individuals working this amount. There are 58 unique work hour values, indicating a wide range of working schedules. High-income group (>50K): The most common work hours are also 40 hours per week, but only 142 individuals in this category work this amount. There are 39 unique work hour values, suggesting slightly less variability in working hours compared to low-income earners. Key Takeaways: Most individuals work 40 hours per week, regardless of income level. High-income earners tend to work more stable hours, while low-income earners exhibit greater variation in work schedules. Longer working hours do not necessarily guarantee higher income, suggesting that job type and skill level are more influential factors in determining earnings.

## 3 Model selection

fitting full model.

Implementation: ROI | Solver: lpsolve Separation: TRUE Existence of maximum likelihood estimates (Intercept) Age 0 0 Education11th Education12th -Inf Education1st-4th Education5th-6th -Inf Education7th-8th Education9th 0 EducationAssoc-acdm EducationAssoc-voc EducationBachelors EducationDoctorate EducationHS-grad EducationMasters EducationProf-school EducationSome-college SexMale Hours\_PW 0 0 Marital\_StatusMarried-civ-spouse Marital\_StatusMarried-spouse-absent Marital\_StatusNever-married Marital\_StatusSeparated OccupationCraft-repair Marital\_StatusWidowed OccupationExec-managerial OccupationFarming-fishing OccupationHandlers-cleaners OccupationMachine-op-inspct OccupationOther-service OccupationPriv-house-serv -Inf OccupationProtective-serv OccupationProf-specialty OccupationSales OccupationTech-support OccupationTransport-moving NationalityCanada

```
NationalityColumbia
                     NationalityChina
                                                                         -Inf
                      NationalityCuba
                                              NationalityDominican-Republic
                                     0
                                                                         -Inf
               NationalityEl-Salvador
                                                          NationalityEngland
                                                                         -Inf
                    NationalityFrance
                                                          NationalityGermany
                                  -Inf
                                                        NationalityGuatemala
                    NationalityGreece
                                     0
                                                                         -Inf
                     NationalityHaiti
                                                            NationalityIndia
                                  -Inf
                                                                         -Inf
                      NationalityIran
                                                          NationalityIreland
                     NationalityItaly
                                                          NationalityJamaica
                                  -Inf
                                                                         -Inf
                     NationalityJapan
                                                             NationalityLaos
                                     0
                                                                         -Inf
                    NationalityMexico NationalityOutlying-US(Guam-USVI-etc)
               NationalityPhilippines
                                                           NationalityPoland
                  NationalityPortugal
                                                     NationalityPuerto-Rico
                                     0
                  NationalityScotland
                                                            NationalitySouth
                                  -Inf
                                                                         -Inf
                    NationalityTaiwan
                                                 NationalityTrinadad&Tobago
             NationalityUnited-States
                                                          NationalityVietnam
                                                                         -Inf
0: finite value, Inf: infinity, -Inf: -infinity
Call:
glm(formula = Income ~ Age + Education + Sex + Hours_PW + Marital_Status +
    Occupation + Nationality, family = binomial(link = "logit"),
    data = data)
Coefficients:
                                         Estimate Std. Error z value Pr(>|z|)
                                      -4.710e+00 2.198e+00 -2.142 0.032163
(Intercept)
```

0

Age	3.840e-02	7.626e-03	5.036 4.76e-07
Education11th	1.105e+00	9.856e-01	1.121 0.262436
Education12th	-1.448e+01	1.244e+03	-0.012 0.990718
Education1st-4th	-2.981e+01	1.875e+03	-0.016 0.987315
Education5th-6th	-1.524e+01	1.475e+03	-0.010 0.991753
Education7th-8th	-1.686e-01	1.027e+00	-0.164 0.869615
Education9th	2.560e-01	9.462e-01	0.271 0.786769
EducationAssoc-acdm	1.612e+00	7.895e-01	2.042 0.041190
EducationAssoc-voc	8.442e-01	7.523e-01	1.122 0.261769
EducationBachelors	2.034e+00	7.010e-01	2.902 0.003707
EducationDoctorate	4.593e+00	1.098e+00	4.183 2.88e-05
EducationHS-grad	1.042e+00	6.809e-01	1.531 0.125784
EducationMasters	2.290e+00	7.394e-01	3.097 0.001952
EducationProf-school	3.103e+00	9.911e-01	3.131 0.001741
EducationSome-college	1.450e+00	6.857e-01	2.115 0.034466
SexMale	1.914e-01	2.490e-01	0.769 0.442085
Hours_PW	2.978e-02	7.914e-03	3.762 0.000168
Marital_StatusMarried-civ-spouse	1.936e+00	2.892e-01	6.695 2.15e-11
Marital_StatusMarried-spouse-absent	2.740e-01	1.158e+00	0.237 0.813013
Marital_StatusNever-married	-5.338e-01	3.586e-01	-1.489 0.136554
Marital_StatusSeparated	-1.594e+00	1.068e+00	-1.493 0.135565
Marital_StatusWidowed	-8.949e-01	8.426e-01	-1.062 0.288201
OccupationCraft-repair	-9.979e-02	3.548e-01	-0.281 0.778491
OccupationExec-managerial	3.437e-01	3.422e-01	1.004 0.315155
OccupationFarming-fishing	-1.698e+00	6.081e-01	-2.792 0.005240
OccupationHandlers-cleaners	-1.772e-01	5.794e-01	-0.306 0.759728
OccupationMachine-op-inspct	-5.569e-01	4.414e-01	-1.261 0.207134
OccupationOther-service	-2.477e+00	7.982e-01	-3.103 0.001913
OccupationPriv-house-serv	-1.633e+01	2.648e+03	-0.006 0.995080
OccupationProf-specialty	6.458e-01	3.595e-01	1.796 0.072471
OccupationProtective-serv	-2.791e-02	5.369e-01	-0.052 0.958538
OccupationSales	-5.323e-02	3.641e-01	-0.146 0.883781
OccupationTech-support	1.037e+00	5.441e-01	1.905 0.056736
OccupationTransport-moving	-3.092e-01	4.362e-01	-0.709 0.478456
NationalityCanada	-1.789e+00	2.313e+00	-0.773 0.439233
NationalityChina	-2.918e+00	3.035e+00	-0.961 0.336342
NationalityColumbia	-1.680e+01	6.523e+03	-0.003 0.997945
NationalityCuba	-1.802e+00	2.365e+00	-0.762 0.445958
NationalityDominican-Republic	-1.660e+01	2.909e+03	-0.006 0.995448
NationalityEl-Salvador	-1.838e+01	2.495e+03	-0.007 0.994122
NationalityEngland	-1.812e+01	4.607e+03	-0.004 0.996862
NationalityFrance	-1.804e+01	6.523e+03	-0.003 0.997793
NationalityGermany	-1.414e+00	2.206e+00	-0.641 0.521713

NationalityGreece	-1.153e+00	3.135e+00	-0.368 0.713110
NationalityGuatemala	-1.548e+01	2.162e+03	-0.007 0.994285
NationalityHaiti	-1.707e+01	6.523e+03	-0.003 0.997912
${\tt NationalityIndia}$	-2.097e+01	3.539e+03	-0.006 0.995273
NationalityIran	-1.884e+01	6.523e+03	-0.003 0.997696
NationalityIreland	6.005e-01	2.728e+00	0.220 0.825742
NationalityItaly	-1.874e+01	4.184e+03	-0.004 0.996426
NationalityJamaica	-1.688e+01	4.609e+03	-0.004 0.997078
NationalityJapan	-1.502e+00	2.608e+00	-0.576 0.564588
NationalityLaos	-1.838e+01	4.187e+03	-0.004 0.996497
NationalityMexico	-2.297e+00	2.282e+00	-1.006 0.314240
NationalityOutlying-US(Guam-USVI-etc)	-1.694e+01	6.523e+03	-0.003 0.997928
NationalityPhilippines	-2.274e+00	2.375e+00	-0.957 0.338412
NationalityPoland	-2.433e+00	2.397e+00	-1.015 0.310036
NationalityPortugal	1.404e+01	1.326e+03	0.011 0.991550
NationalityPuerto-Rico	4.044e-01	2.361e+00	0.171 0.863991
NationalityScotland	-1.343e+01	6.523e+03	-0.002 0.998357
NationalitySouth	-1.776e+01	3.246e+03	-0.005 0.995635
NationalityTaiwan	-3.320e+00	2.486e+00	-1.336 0.181695
NationalityTrinadad&Tobago	-1.855e+01	6.523e+03	-0.003 0.997730
NationalityUnited-States	-1.815e+00	2.002e+00	-0.906 0.364805
NationalityVietnam	-1.958e+01	2.253e+03	-0.009 0.993067
·			
(Intercept)	*		
(Intercept) Age	* ***		
<del>-</del>			
Age			
Age Education11th			
Age Education11th Education12th			
Age Education11th Education12th Education1st-4th			
Age Education11th Education12th Education1st-4th Education5th-6th			
Age Education11th Education12th Education1st-4th Education5th-6th Education7th-8th			
Age Education11th Education12th Education1st-4th Education5th-6th Education7th-8th Education9th	***		
Age Education11th Education12th Education1st-4th Education5th-6th Education7th-8th Education9th EducationAssoc-acdm	***		
Age Education11th Education12th Education1st-4th Education5th-6th Education7th-8th Education9th EducationAssoc-acdm EducationAssoc-voc	***		
Age Education11th Education12th Education1st-4th Education5th-6th Education7th-8th Education9th EducationAssoc-acdm EducationAssoc-voc EducationBachelors	***  *  *		
Age Education11th Education12th Education1st-4th Education5th-6th Education7th-8th Education9th EducationAssoc-acdm EducationAssoc-voc EducationBachelors EducationDoctorate	***  *  *		
Age Education11th Education12th Education1st-4th Education5th-6th Education7th-8th Education9th EducationAssoc-acdm EducationAssoc-voc EducationBachelors EducationDoctorate EducationHS-grad	***  *  **  **		
Age Education11th Education12th Education1st-4th Education5th-6th Education7th-8th Education9th EducationAssoc-acdm EducationBachelors EducationBoctorate EducationHS-grad EducationMasters EducationProf-school	***  *  *  **  **  **		
Age Education11th Education12th Education1st-4th Education5th-6th Education7th-8th Education9th EducationAssoc-acdm EducationAssoc-voc EducationBachelors EducationDoctorate EducationHS-grad EducationMasters	***  *  *  **  **  **		
Age Education11th Education12th Education1st-4th Education5th-6th Education7th-8th Education9th EducationAssoc-acdm EducationAssoc-voc EducationBachelors EducationDoctorate EducationHS-grad EducationMasters EducationProf-school EducationSome-college	***  *  *  **  **  **		
Education11th Education12th Education1st-4th Education5th-6th Education9th EducationAssoc-acdm EducationAssoc-voc EducationBachelors EducationDoctorate EducationHS-grad EducationMasters EducationProf-school EducationSome-college SexMale Hours_PW	***  *  *  **  **  **  **  **		
Age Education11th Education12th Education1st-4th Education5th-6th Education9th EducationAssoc-acdm EducationAssoc-voc EducationBachelors EducationDoctorate EducationHS-grad EducationMasters EducationProf-school EducationSome-college SexMale	***  *  *  *  *  *  *  *  *  *  *  *  *		

Marital\_StatusNever-married

Marital\_StatusSeparated

Marital\_StatusWidowed

OccupationCraft-repair

OccupationExec-managerial

OccupationFarming-fishing

OccupationHandlers-cleaners

OccupationMachine-op-inspct

OccupationOther-service

OccupationPriv-house-serv

OccupationProf-specialty

OccupationProtective-serv

OccupationSales

OccupationTech-support

OccupationTransport-moving

NationalityCanada

NationalityChina

NationalityColumbia

NationalityCuba

NationalityDominican-Republic

NationalityEl-Salvador

NationalityEngland

NationalityFrance

NationalityGermany

NationalityGreece

NationalityGuatemala

NationalityHaiti

NationalityIndia

NationalityIran

NationalityIreland

NationalityItaly

NationalityJamaica

NationalityJapan

NationalityLaos

NationalityMexico

NationalityOutlying-US(Guam-USVI-etc)

NationalityPhilippines

NationalityPoland

NationalityPortugal

NationalityPuerto-Rico

 ${\tt NationalityScotland}$ 

NationalitySouth

NationalityTaiwan

NationalityTrinadad&Tobago NationalityUnited-States NationalityVietnam

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 1552.71 on 1384 degrees of freedom Residual deviance: 974.31 on 1319 degrees of freedom

AIC: 1106.3

Number of Fisher Scoring iterations: 17

#### Call:

#### Coefficients:

	Estimate	Std. Error	z value	Pr(> z )
(Intercept)	-6.433e+00	9.215e-01	-6.982	2.92e-12 ***
Age	3.793e-02	7.413e-03	5.117	3.10e-07 ***
Education11th	1.288e+00	9.495e-01	1.357	0.17493
Education12th	-1.362e+01	7.644e+02	-0.018	0.98578
Education1st-4th	-1.459e+01	9.286e+02	-0.016	0.98747
Education5th-6th	-1.447e+01	9.224e+02	-0.016	0.98748
Education7th-8th	-1.669e-01	1.019e+00	-0.164	0.86992
Education9th	3.944e-01	9.297e-01	0.424	0.67141
EducationAssoc-acdm	1.560e+00	7.818e-01	1.996	0.04596 *
EducationAssoc-voc	8.264e-01	7.447e-01	1.110	0.26717
EducationBachelors	1.943e+00	6.912e-01	2.811	0.00494 **
EducationDoctorate	4.400e+00	1.094e+00	4.022	5.77e-05 ***
EducationHS-grad	1.043e+00	6.714e-01	1.554	0.12023
EducationMasters	2.264e+00	7.285e-01	3.107	0.00189 **
EducationProf-school	3.036e+00	9.789e-01	3.102	0.00192 **
EducationSome-college	1.422e+00	6.767e-01	2.102	0.03557 *
Hours_PW	3.046e-02	7.687e-03	3.963	7.40e-05 ***
Marital_StatusMarried-civ-spouse	1.941e+00	2.657e-01	7.306	2.76e-13 ***
Marital_StatusMarried-spouse-absent	-4.485e-02	1.120e+00	-0.040	0.96806
Marital_StatusNever-married	-5.497e-01	3.490e-01	-1.575	0.11531
Marital_StatusSeparated	-1.611e+00	1.066e+00	-1.511	0.13083

```
Marital_StatusWidowed
                                 -9.765e-01 8.396e-01 -1.163 0.24481
                                -6.118e-02 3.259e-01 -0.188 0.85109
OccupationCraft-repair
OccupationExec-managerial
                                 4.468e-01 3.265e-01 1.368 0.17124
OccupationFarming-fishing
                                 -1.621e+00 5.942e-01 -2.728 0.00638 **
OccupationHandlers-cleaners
                                -9.311e-02 5.548e-01 -0.168 0.86672
OccupationMachine-op-inspct
                                 -5.220e-01 4.147e-01 -1.259 0.20812
OccupationOther-service
                                 -2.445e+00 7.761e-01 -3.150 0.00163 **
                                 -1.542e+01 1.606e+03 -0.010 0.99234
OccupationPriv-house-serv
OccupationProf-specialty
                                 7.109e-01 3.459e-01 2.055 0.03985 *
OccupationProtective-serv
                                8.969e-02 5.165e-01 0.174 0.86213
                                 3.198e-02 3.426e-01 0.093 0.92563
OccupationSales
OccupationTech-support
                                1.079e+00 5.209e-01 2.072 0.03827 *
OccupationTransport-moving
                               -2.654e-01 4.154e-01 -0.639 0.52289
```

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 1552.7 on 1384 degrees of freedom Residual deviance: 998.6 on 1351 degrees of freedom

AIC: 1066.6

Number of Fisher Scoring iterations: 16

Implementation: ROI | Solver: lpsolve

Separation: TRUE

Existence of maximum likelihood estimates

EducationProf-school

(Intercept)	Age
0	0
Education11th	Education12th
0	-Inf
Education1st-4th	Education5th-6th
-Inf	-Inf
Education7th-8th	Education9th
0	0
EducationAssoc-acdm	EducationAssoc-voc
0	0
EducationBachelors	EducationDoctorate
0	0
EducationHS-grad	EducationMasters
0	0

EducationSome-college

```
0
                                                                        0
                            Hours PW
                                        Marital_StatusMarried-civ-spouse
                                   0
Marital_StatusMarried-spouse-absent
                                             Marital_StatusNever-married
                                                                        0
            Marital StatusSeparated
                                                   Marital StatusWidowed
                                                                        0
             OccupationCraft-repair
                                               OccupationExec-managerial
          OccupationFarming-fishing
                                             OccupationHandlers-cleaners
                                                                        0
        OccupationMachine-op-inspct
                                                 OccupationOther-service
                                                OccupationProf-specialty
          OccupationPriv-house-serv
          OccupationProtective-serv
                                                          OccupationSales
             OccupationTech-support
                                              OccupationTransport-moving
                                                                        0
```

0: finite value, Inf: infinity, -Inf: -infinity

The model selection process involves fitting a Generalized Linear Model (GLM) with a binomial logistic regression to predict whether an individual's income falls into the >50K or <=50K category.

- 1. Full Model Fitting The initial model (full\_model) includes all available predictor variables: Age, Education, Sex, Work Hours per Week (Hours\_PW), Marital Status, Occupation, and Nationality. A separation detection test (detect\_separation\_full\_model) is applied to check if certain variables lead to perfect separation, which may cause convergence issues in the model. The summary of the full model provides insights into which variables are statistically significant.
- 2. Model Optimization using Stepwise AIC Stepwise Akaike Information Criterion (AIC) selection is applied (stepAIC\_model), which iteratively removes the least significant predictors to find the best-performing model. The final optimized model retains only the most relevant predictors: Age, Education, Work Hours per Week, Marital Status, and Occupation. A separation detection test (detect\_separation\_stepAIC\_model) is performed again to check whether the optimized model exhibits perfect separation issues.
- 3. Key Takeaways: The full model includes all predictors but may contain unnecessary variables that do not contribute significantly. Stepwise AIC selection helps refine the model by retaining only the most informative variables, reducing overfitting and improving interpretability. The final optimized model suggests that Age, Education, Work Hours, Marital Status, and Occupation are the strongest predictors of income. Nationality and Sex are removed in the AIC-selected model, indicating they may not have a

significant impact on predicting income in this dataset. The model suffers from perfect separation issues, with the explanatory variables Education and Occupation exhibiting perfect separation.

## 3.1 Option choose of model

To resolve the perfect separation issue, merge the categories of some explanatory variables.

Table 3: Martial\_Status Distribution.

Income	Married	Unmarried
<=50K	36.3% (378)	63.7% (663)
> 50 K	84.6% (291)	15.4%(53)

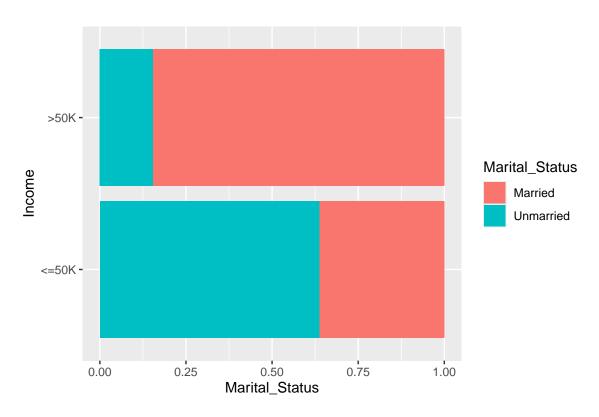


Figure 22: Income by Marital\_Status.

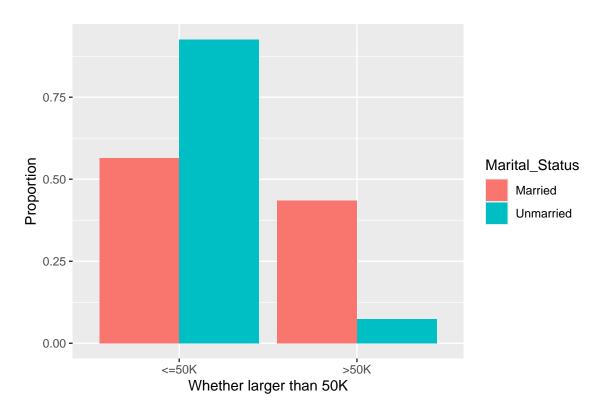


Figure 23: Income by Marital\_Status.

Table 4: Education Distribution.

Income	High_Education	Low_Education
<=50K	84.1% (875)	15.9% (166)
>50K	96.8% (333)	3.2% (11)

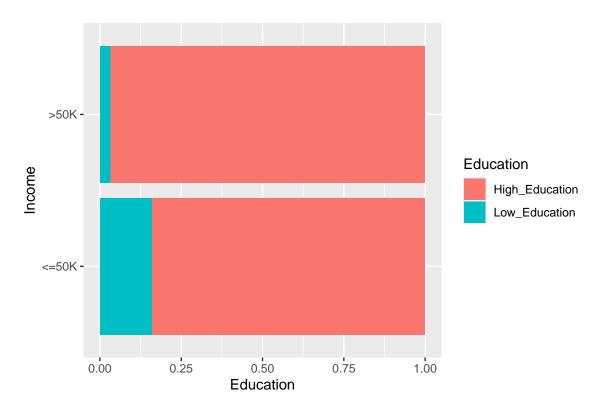


Figure 24: Income by Education.

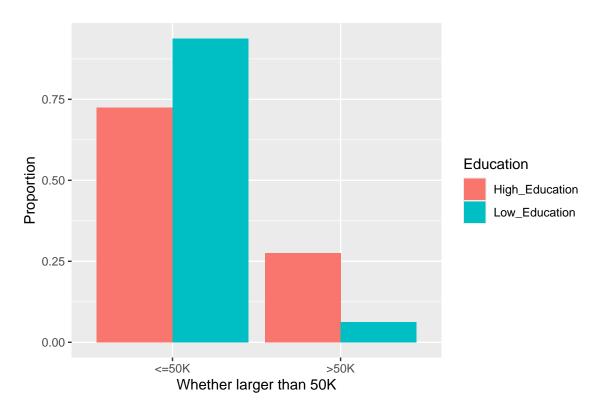


Figure 25: Income by Education.

To address the complete separation issue, some explanatory variables have been merged into broader categories. This helps ensure that each category has sufficient data points, improving the stability of the logistic regression model.

# 1. Education Level Merging Low Education:

Groups together individuals with education levels from 1st grade to 12th grade (e.g., 1st-4th, 5th-6th, 7th-8th, 9th, 10th, 11th, 12th). These individuals generally have no formal higher education and are more likely to be in low-income jobs. High Education:

Includes individuals with some form of post-secondary education (e.g., Associate degree, Bachelor's, Master's, Doctorate, HS-grad, Prof-school, Some-college). These individuals have better earning potential and are more likely to be in high-income categories.

# 2. Marital Status Merging Unmarried:

Combines Separated, Widowed, Never-married, and Divorced into a single category. These individuals tend to have lower financial stability compared to married individuals. Married:

Combines Married-spouse-absent and Married-civ-spouse into a single category. Married individuals generally have more financial stability, often benefiting from dual-income households.

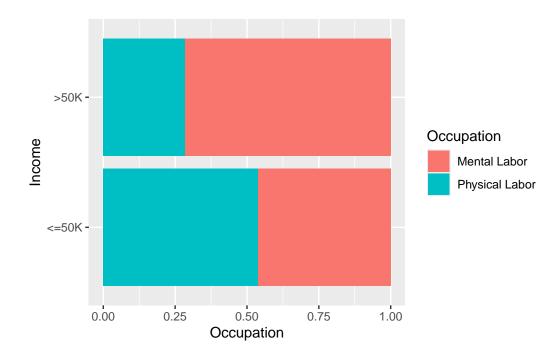
# 3. Visualizing the Effects of Merging

- Marital Status vs. Income:
   The distribution of income across the newly grouped Married and Unmarried categories is displayed.
- Education vs. Income:

  The newly created Low Education and High Education categories allow for a clearer comparison of income distribution between different education levels. Key

Takeaways: Merging categories helps reduce model complexity and prevent complete separation issues. Individuals with higher education levels are more likely to earn  $>50 \mathrm{K}$  compared to those in the Low Education category. Married individuals show a higher proportion of high-income earners, reinforcing previous findings about marital status and financial stability. These transformations improve the model's ability to generalize and make more accurate predictions.

# 3.2 Combine occupation types (physical labor and mental labor)



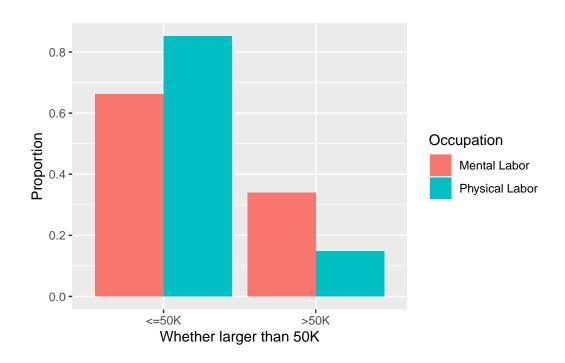


Table 5: Occupation Distribution (physical labor and mental labor).

Income	Mental Labor	Physical Labor
<=50K	46.1% (480)	53.9% (561)
>50K	71.5% (246)	28.5% (98)

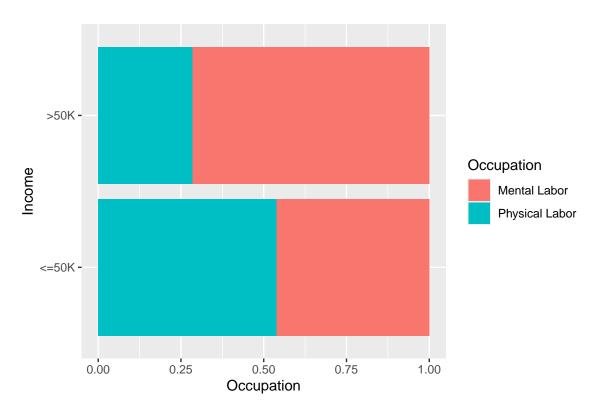


Figure 26: Income by Occupation

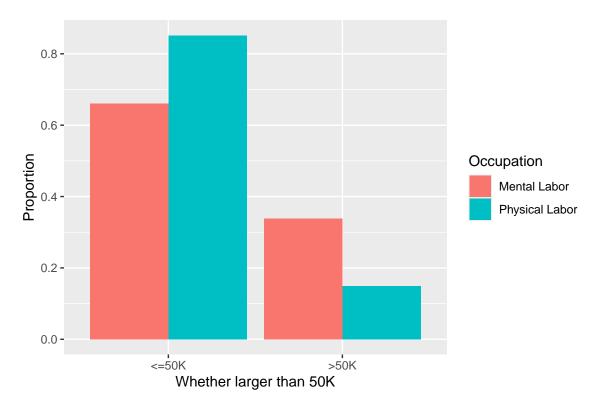


Figure 27: Income by Occupation

To simplify occupational categories and address class imbalance, occupations have been grouped into two broader categories:

# Mental Labor (White-collar jobs):

Includes occupations such as Administrative clerical, Executive managerial, Professional specialty, Sales, and Technical support. These jobs typically involve cognitive work, decision-making, and problem-solving.

## Physical Labor (Blue-collar jobs):

Includes Craft repair, Farming and fishing, Handlers and cleaners, Machine operators, Transport workers, Other services, Private household services, and Protective services. These jobs are more labor-intensive and require physical exertion.

# Analysis of Income Distribution after Merging:

Mental labor jobs have a higher proportion of individuals earning >50K, reinforcing the idea that cognitive and managerial roles tend to offer better salaries. Physical labor jobs predominantly fall in the <=50K category, suggesting that manual labor occupations generally provide lower wages. The proportion of high-income earners in mental labor jobs is significantly higher

than in physical labor jobs, highlighting the economic advantage of cognitive and executive occupations.

Key Takeaways: Merging occupations into Physical vs. Mental labor simplifies the analysis while preserving meaningful insights. Mental labor positions are more likely to be associated with higher salaries. Physical laborers predominantly fall into the low-income category, likely due to industry pay standards and job requirements. These findings emphasize the importance of education and skill specialization in securing higher-paying jobs.

# 3.3 Combine occupation types (by PRC Job Classification List)

ref:https://zchweb.oss-cn-beijing.aliyuncs.com/contract/temp/2021122116541363304.pdf

Table 6: Occupation Distribution (PRC Job Classification List).

Income	1	2	3	4	5	6
<=50K	9.3% (97)	11.1% (116)	13.5% (141) $7.0% (24)$	33.9% (353)	3.6% (37)	28.5% (297)
>50K	23.0% (79)	29.1% (100)		17.7% (61)	1.5% (5)	21.8% (75)

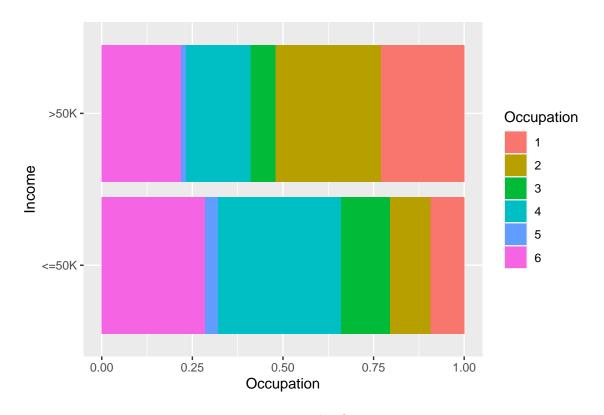


Figure 28: Income by Occupation

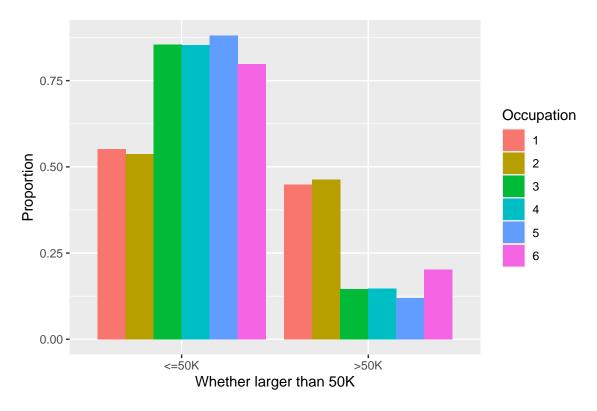


Figure 29: Income by Occupation

To further refine the occupation categories, we classify jobs according to the PRC Job Classification List. This classification system groups occupations based on skill levels and job nature, which allows for a more structured income comparison.

# Income Distribution Analysis After Merging:

Category 1 (Senior Management) has the highest proportion of individuals earning >50K, reinforcing the idea that executive roles are highly paid. Category 2 (Specialists & Technical Support) also has a notable presence in the high-income group, indicating that specialized skills lead to better salaries. Category 4 (Sales & Service) and Category 5 (Agriculture & Fishing) have the lowest share of high-income earners, highlighting the financial struggles in these job sectors.

Key Takeaways: The PRC Job Classification List provides a structured way to analyze income disparities across different occupational groups. Management and technical jobs tend to have higher salaries, while sales, service, and agricultural jobs have a greater share of low-income earners. Grouping occupations in this way allows for more precise policy recommendations and workforce planning.

# 3.4 Combine occupation types (by International Standard Classification of Occupations (ISCO-08))

ref:https://ilostat.ilo.org/methods/concepts-and-definitions/classification-occupation/

Table 7: Occupation Distribution (ISCO-08).

Income	1	2	3	4	5	6	7	8	9
<=50K >50K	9.3% (97) 23.0% (79)	8.8% (92) 25.3% (87)	2.3% (24) $3.8% (13)$	13.5% (141) 7.0% (24)	28.6% (298) 16.0% (55)	3.6% (37) 1.5% (5)	14.8% (154) 13.1% (45)	13.7% (143) 8.7% (30)	5.3% (55) 1.7% (6)

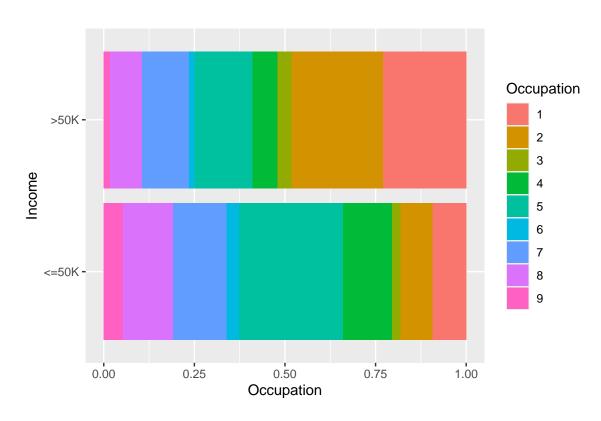


Figure 30: Income by Occupation

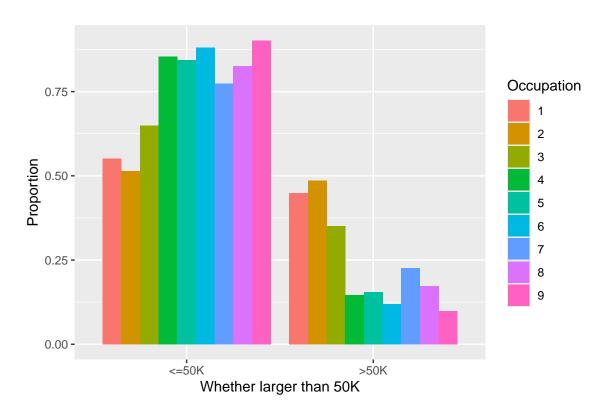


Figure 31: Income by Occupation

```
Call:
```

```
glm(formula = Income ~ Age + Education + Sex + Hours_PW + Marital_Status +
    Occupation, family = binomial(link = "logit"), data = data.new.1)
```

### Coefficients:

```
Estimate Std. Error z value Pr(>|z|)
(Intercept)
                        -2.966040
                                    0.445192 -6.662 2.69e-11 ***
Age
                         0.037715
                                    0.006356
                                              5.934 2.96e-09 ***
EducationLow_Education
                        -1.441793
                                    0.349529 -4.125 3.71e-05 ***
SexMale
                                    0.204248
                         0.585571
                                               2.867 0.00414 **
Hours_PW
                         0.030770
                                    0.006887
                                               4.468 7.89e-06 ***
                                    0.187060 -10.640 < 2e-16 ***
Marital_StatusUnmarried -1.990257
OccupationPhysical Labor -1.320743
                                    0.163077 -8.099 5.55e-16 ***
```

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

```
Null deviance: 1552.7 on 1384 degrees of freedom
Residual deviance: 1109.9 on 1378 degrees of freedom
```

AIC: 1123.9

Number of Fisher Scoring iterations: 6

Implementation: ROI | Solver: lpsolve

Separation: FALSE

Existence of maximum likelihood estimates

(Intercept) Age EducationLow\_Education
0 0 0

SexMale Marital\_StatusUnmarried OccupationPhysical Labor
0 0 0

Hours\_PW

O: finite value, Inf: infinity, -Inf: -infinity

## Call:

glm(formula = Income ~ Age + Education + Sex + Hours\_PW + Marital\_Status +
 Occupation, family = binomial(link = "logit"), data = data.new.2)

# Coefficients:

Estimate Std. Error z value Pr(>|z|)(Intercept) -3.252612 0.510402 -6.373 1.86e-10 \*\*\* Age EducationLow\_Education -1.436365 0.346423 -4.146 3.38e-05 \*\*\* 0.488605 0.215606 2.266 SexMale 0.0234 \* Hours\_PW 0.035850 0.007234 4.956 7.19e-07 \*\*\* Marital\_StatusUnmarried -2.093813 0.195001 -10.737 < 2e-16 \*\*\* Occupation2 0.707103 0.253029 2.795 0.0052 \*\* Occupation3 -0.486564 0.319354 - 1.5240.1276 Occupation4 0.550179 -4.751 2.03e-06 \*\*\* Occupation5 -2.613814 0.237695 -4.436 9.15e-06 \*\*\* Occupation6 -1.054506

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 1552.7 on 1384 degrees of freedom

```
Residual deviance: 1079.5 on 1374 degrees of freedom
```

AIC: 1101.5

Number of Fisher Scoring iterations: 6

```
Implementation: ROI | Solver: lpsolve
```

Separation: FALSE

Existence of maximum likelihood estimates

(Intercept)	Age	${\tt EducationLow\_Education}$
0	0	0
SexMale	Marital_StatusUnmarried	Occupation2
0	0	0
Occupation3	Occupation4	Occupation5
0	0	0
${\tt Occupation 6}$	Hours_PW	

0: finite value, Inf: infinity, -Inf: -infinity

# Call:

# Coefficients:

ODELLICIENTS.					
	Estimate	Std. Error	z value	Pr(> z )	
(Intercept)	-3.262143	0.514533	-6.340	2.30e-10	***
Age	0.041171	0.006597	6.241	4.35e-10	***
EducationLow_Education	-1.431424	0.347718	-4.117	3.84e-05	***
SexMale	0.476856	0.216949	2.198	0.027948	*
Hours_PW	0.036079	0.007286	4.952	7.35e-07	***
${\tt Marital\_StatusUnmarried}$	-2.094087	0.195332	-10.721	< 2e-16	***
Occupation2	0.730937	0.261676	2.793	0.005217	**
Occupation3	0.562765	0.487845	1.154	0.248675	
Occupation4	-0.489309	0.319379	-1.532	0.125506	
Occupation5	-0.999851	0.250967	-3.984	6.78e-05	***
Occupation6	-2.614898	0.550520	-4.750	2.04e-06	***
Occupation7	-0.931695	0.268166	-3.474	0.000512	***
Occupation8	-1.212247	0.291251	-4.162	3.15e-05	***
Occupation9	-0.951597	0.516129	-1.844	0.065224	

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 1552.7 on 1384 degrees of freedom Residual deviance: 1078.4 on 1371 degrees of freedom

AIC: 1106.4

Number of Fisher Scoring iterations: 6

Implementation: ROI | Solver: lpsolve

Separation: FALSE

Existence of maximum likelihood estimates

EducationLow_Education	Age	(Intercept)
0	0	0
Occupation2	Marital_StatusUnmarried	SexMale
0	0	0
Occupation5	Occupation4	Occupation3
0	0	0
Occupation8	Occupation7	Occupation6
0	0	0
	Hours_PW	Occupation9
	0	0

0: finite value, Inf: infinity, -Inf: -infinity

The International Standard Classification of Occupations (ISCO-08) is used to categorize occupations into structured groups based on job function and skill level. This classification allows for a globally standardized approach to analyzing income distribution by profession.

Income Distribution Analysis After Merging:

Category 1 (Managers) and Category 2 (Professionals) have the highest share of individuals earning >50K, emphasizing that managerial and specialized roles offer better earnings. Category 5 (Service & Sales) and Category 6 (Agricultural & Fishery) exhibit the lowest proportion of high-income earners, indicating the financial constraints faced by these workers. Category 7 (Craft Workers) and Category 8 (Machine Operators) have an intermediate income distribution, suggesting that skilled manual labor provides moderate earnings.

Model Selection and Performance Comparison:

The stepwise AIC model selection process was applied separately to datasets using different occupational classification methods (Mental vs. Physical Labor, PRC Job Classification, and ISCO-08). All models showed improvement in fitting compared to the full model, but the best performance was observed with the ISCO-08 classification. Nationality was removed in the AIC-selected models, reinforcing the earlier observation that nationality has little impact on income prediction.

Key Takeaways: ISCO-08 classification provides a structured way to assess income distribution across occupations. Managers and professionals dominate the high-income group, while service, sales, and agriculture workers struggle with lower wages. Machine operators and craft workers occupy a middle ground, earning more than service workers but less than professionals. The final model using ISCO-08 classification achieves a better balance between accuracy and interpretability, making it a robust choice for workforce and economic analysis.

# 4 Model Check

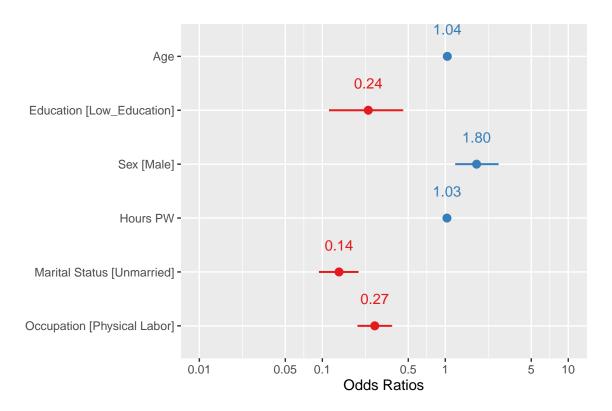


Figure 32: Model Plot for stepAIC\_model\_new.1

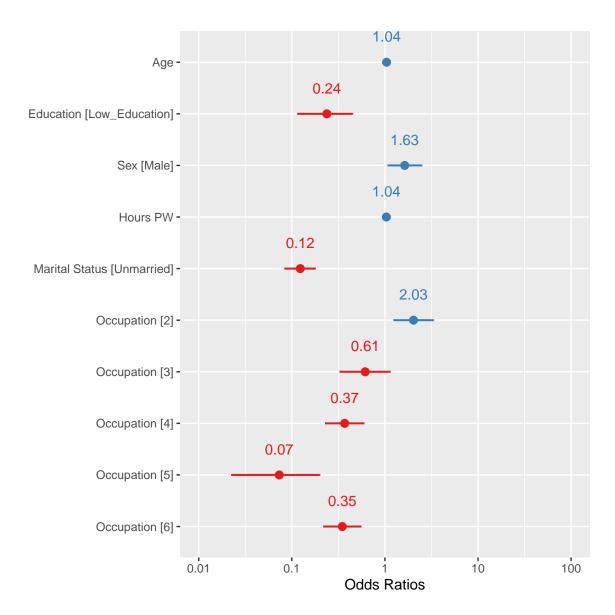


Figure 33: Model Plot for stepAIC $\_$ model $\_$ new.2

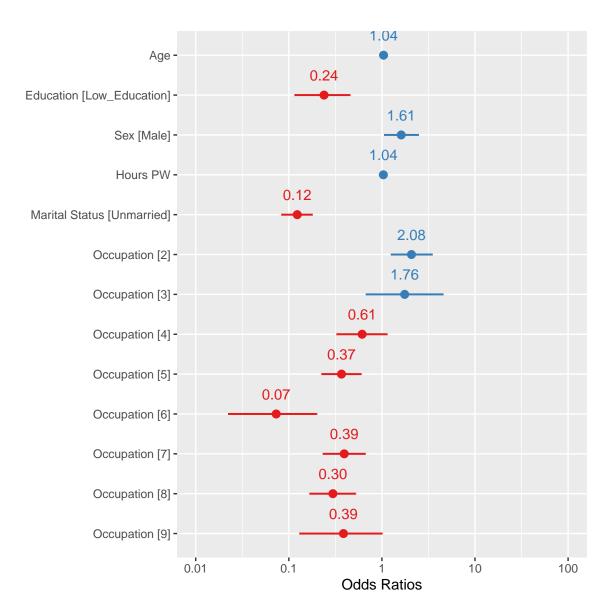


Figure 34: Model Plot for stepAIC\_model

Analysis of Deviance Table

Model: binomial, link: logit

Response: Income

Terms added sequentially (first to last)

# Df Deviance Resid. Df Resid. Dev Pr(>Chi)

NULL			1384	1552.7	
Age	1	90.112	1383	1462.6 < 2.2e-16 ***	c
Education	1	53.385	1382	1409.2 2.742e-13 ***	c
Sex	1	62.449	1381	1346.8 2.734e-15 ***	c
Hours_PW	1	36.563	1380	1310.2 1.478e-09 ***	c
Marital_Status	1	129.706	1379	1180.5 < 2.2e-16 ***	:
Occupation	1	70.560	1378	1109.9 < 2.2e-16 ***	:

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Analysis of Deviance Table

Model: binomial, link: logit

Response: Income

Terms added sequentially (first to last)

# Df Deviance Resid. Df Resid. Dev Pr(>Chi)

NULL			1384	1552.7	
Age	1	90.112	1383	1462.6 < 2.2e-16	***
Education	1	53.385	1382	1409.2 2.742e-13	***
Sex	1	62.449	1381	1346.8 2.734e-15	***
Hours_PW	1	36.563	1380	1310.2 1.478e-09	***
Marital_Status	1	129.706	1379	1180.5 < 2.2e-16	***
Occupation	5	101.032	1374	1079.5 < 2.2e-16	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Analysis of Deviance Table

Model: binomial, link: logit

Response: Income

Terms added sequentially (first to last)

Df Deviance Resid. Df Resid. Dev Pr(>Chi)

NULL 1384 1552.7

Age 1 90.112 1383 1462.6 < 2.2e-16 \*\*\*

```
Education
                    53.385
                                 1382
                                          1409.2 2.742e-13 ***
                1
Sex
                1
                    62.449
                                 1381
                                          1346.8 2.734e-15 ***
Hours_PW
                    36.563
                                 1380
                                          1310.2 1.478e-09 ***
                1
Marital_Status
                1
                   129.706
                                1379
                                          1180.5 < 2.2e-16 ***
Occupation
                   102.097
                                          1078.4 < 2.2e-16 ***
                                 1371
                0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Signif. codes:
```

Area under the curve: 0.882

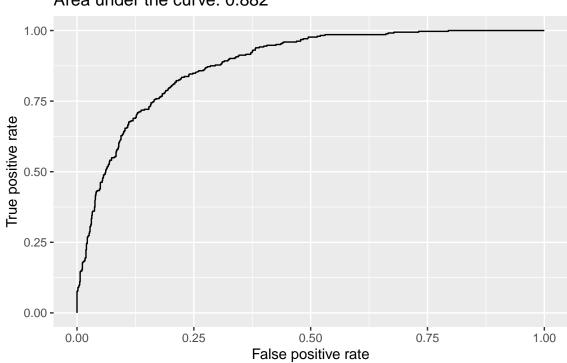


Figure 35: ROC curve for stepAIC\_model

# Area under the curve: 0.851 1.00 0.75 0.50 0.00 0.25 0.50 False positive rate

Figure 36: ROC curve for stepAIC\_model\_new.1

# Area under the curve: 0.86 1.00 0.75 0.25 0.00 0.25 0.50 False positive rate

Figure 37: ROC curve for stepAIC\_model\_new.2

# Area under the curve: 0.861

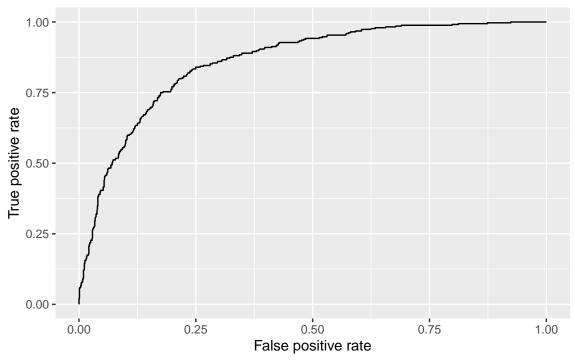


Figure 38: ROC curve for stepAIC\_model\_new.3

To ensure the reliability and effectiveness of the logistic regression models, multiple validation techniques are applied, including coefficient visualization, ROC curve analysis, and ANOVA testing.

ROC Curve Analysis & AUC Comparison The ROC curve (Receiver Operating Characteristic curve) is plotted for each model to assess its classification performance. AUC (Area Under the Curve) values are calculated: A higher AUC (closer to 1) indicates a better-performing model. A lower AUC (closer to 0.5) suggests poor classification performance. The AUC values of different models are compared, helping to determine which occupational classification method improves predictive accuracy.

ANOVA Model Comparison ANOVA (Analysis of Variance) tests compare model fits: A significant p-value (< 0.05) indicates that additional predictors improve model performance. If models have similar p-values, a simpler model may be preferred to avoid overfitting. The results help in deciding whether stepAIC\_model\_new.1, stepAIC\_model\_new.2, or stepAIC\_model\_new.3 should be used for the final analysis.

Key Takeaways: The ROC curve and AUC values help determine the best occupational classification method in terms of prediction accuracy. Model coefficients provide insights into

the most influential factors affecting income. ANOVA helps verify whether additional variables significantly improve prediction performance. The final model selection should balance accuracy and interpretability, preferring models with a high AUC while avoiding excessive complexity.

## 4.1 Consider the interaction effects

```
Call:
glm(formula = Income ~ Age + Education + Sex + Hours PW + Marital_Status +
    Occupation + Age: Marital_Status + Education: Sex + Sex: Marital_Status +
    Marital_Status:Occupation, family = binomial(link = "logit"),
    data = data.new.1)
Coefficients:
                                                  Estimate Std. Error z value
                                                 -2.189254 0.495876 -4.415
(Intercept)
Age
                                                  0.031676 0.008176 3.874
EducationLow_Education
                                                -15.313498 481.667910 -0.032
SexMale
                                                 Hours PW
                                                  0.032659
                                                            0.007021 4.651
Marital_StatusUnmarried
                                                 -4.514939
                                                            0.707999 -6.377
OccupationPhysical Labor
                                                 -1.420146
                                                             0.180064 -7.887
Age:Marital_StatusUnmarried
                                                  0.030029
                                                             0.013723 2.188
EducationLow_Education:SexMale
                                                 14.012202 481.668031 0.029
SexMale:Marital_StatusUnmarried
                                                  1.546335
                                                             0.444445
                                                                       3.479
Marital_StatusUnmarried:OccupationPhysical Labor
                                                  0.633460
                                                             0.383444 1.652
                                                Pr(>|z|)
(Intercept)
                                                1.01e-05 ***
                                                0.000107 ***
Age
                                                0.974637
EducationLow_Education
SexMale
                                                0.823862
Hours_PW
                                                3.30e-06 ***
Marital_StatusUnmarried
                                                1.81e-10 ***
OccupationPhysical Labor
                                                3.10e-15 ***
Age:Marital_StatusUnmarried
                                                0.028653 *
EducationLow_Education:SexMale
                                                0.976792
SexMale:Marital_StatusUnmarried
                                                0.000503 ***
Marital_StatusUnmarried:OccupationPhysical Labor 0.098529 .
___
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 1552.7 on 1384 degrees of freedom Residual deviance: 1084.2 on 1374 degrees of freedom

AIC: 1106.2

Number of Fisher Scoring iterations: 16

### Call:

glm(formula = Income ~ Age + Education + Sex + Hours\_PW + Marital\_Status + Occupation + Age: Marital\_Status + Sex: Marital\_Status, family = binomial(link = "logit"), data = data.new.1)

## Coefficients:

CCCITICIONOD.					
	Estimate	Std. Error	z value	Pr(> z )	
(Intercept)	-2.270129	0.491722	-4.617	3.90e-06	***
Age	0.031818	0.008089	3.933	8.38e-05	***
EducationLow_Education	-1.462369	0.346401	-4.222	2.43e-05	***
SexMale	-0.037620	0.264656	-0.142	0.8870	
Hours_PW	0.032681	0.007023	4.654	3.26e-06	***
Marital_StatusUnmarried	-4.369683	0.705494	-6.194	5.87e-10	***
OccupationPhysical Labor	-1.292511	0.160702	-8.043	8.77e-16	***
Age:Marital_StatusUnmarried	0.028900	0.013623	2.121	0.0339	*
SexMale:Marital_StatusUnmarried	1.692187	0.431822	3.919	8.90e-05	***

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 1552.7 on 1384 degrees of freedom Residual deviance: 1090.2 on 1376 degrees of freedom

AIC: 1108.2

Number of Fisher Scoring iterations: 6

Implementation: ROI | Solver: lpsolve

Separation: FALSE

Existence of maximum likelihood estimates

(Intercept) Age SexMale

EducationLow\_Education

```
0 Hours_PW Marital_StatusUnmarried
0 0

OccupationPhysical Labor Age:Marital_StatusUnmarried
0 0

SexMale:Marital_StatusUnmarried
0

0: finite value, Inf: infinity, -Inf: -infinity
```

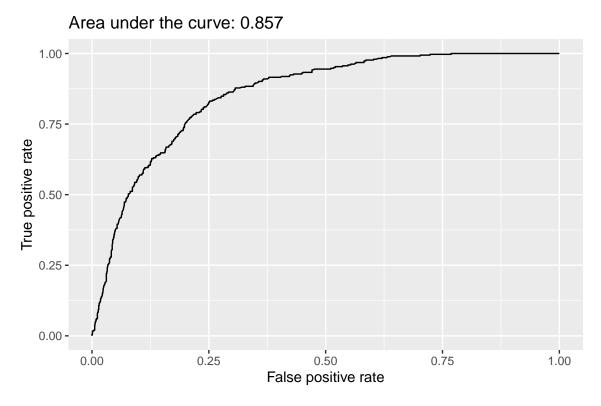


Figure 39: ROC curve for model\_new.1.interaction

```
Call:
glm(formula = Income ~ Age + Education + Sex + Hours_PW + Marital_Status +
    Occupation + Age:Marital_Status + Education:Sex + Education:Hours_PW +
    Sex:Marital_Status + Sex:Occupation + Marital_Status:Occupation,
    family = binomial(link = "logit"), data = data.new.2)
```

## Coefficients:

Estimate Std. Error z value Pr(>|z|)

```
Age
                                   3.958e-02 8.521e-03
                                                          4.645 3.41e-06 ***
EducationLow_Education
                                  -1.704e+01 4.614e+02 -0.037 0.97054
                                  -7.620e-01 5.285e-01 -1.442 0.14933
SexMale
Hours PW
                                   3.742e-02 7.619e-03
                                                          4.911 9.04e-07 ***
Marital_StatusUnmarried
                                  -5.198e+00 9.421e-01 -5.517 3.44e-08 ***
Occupation2
                                    5.189e-01 6.916e-01
                                                          0.750 0.45305
Occupation3
                                   -1.388e+00 6.260e-01 -2.218 0.02656 *
                                  -2.057e+00 7.192e-01 -2.860 0.00423 **
Occupation4
Occupation5
                                    1.500e+01 1.069e+03
                                                          0.014 0.98880
Occupation6
                                              7.775e-01 -1.839 0.06588 .
                                   -1.430e+00
Age:Marital_StatusUnmarried
                                   2.268e-02 1.443e-02
                                                         1.572 0.11594
                                                          0.030 0.97608
EducationLow_Education:SexMale
                                    1.384e+01 4.614e+02
EducationLow_Education:Hours_PW
                                   4.163e-02 2.911e-02 1.430 0.15264
                                   1.647e+00 5.299e-01
                                                          3.107 0.00189 **
SexMale:Marital_StatusUnmarried
SexMale:Occupation2
                                                          0.383 0.70137
                                   2.714e-01 7.078e-01
SexMale:Occupation3
                                   1.591e+00 7.707e-01 2.064 0.03899 *
                                   1.022e+00 7.444e-01
                                                          1.373 0.16987
SexMale:Occupation4
SexMale:Occupation5
                                  -1.790e+01 1.069e+03 -0.017 0.98664
SexMale:Occupation6
                                   2.372e-01 7.994e-01
                                                          0.297 0.76666
Marital_StatusUnmarried:Occupation2 6.281e-01 6.740e-01
                                                          0.932 0.35144
Marital_StatusUnmarried:Occupation3 -3.286e-01 9.822e-01 -0.335 0.73795
Marital_StatusUnmarried:Occupation4 1.450e+00
                                              6.810e-01
                                                          2.129 0.03329 *
Marital_StatusUnmarried:Occupation5 -1.209e+01 1.069e+03 -0.011 0.99097
Marital_StatusUnmarried:Occupation6 1.684e+00 6.627e-01
                                                          2.541 0.01104 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1552.7 on 1384 degrees of freedom
Residual deviance: 1030.1 on 1360 degrees of freedom
AIC: 1080.1
Number of Fisher Scoring iterations: 16
```

-2.073e+00 6.763e-01 -3.065 0.00218 \*\*

(Intercept)

glm(formula = Income ~ Age + Education + Sex + Hours\_PW + Marital\_Status +

data = data.new.2)

Occupation + Sex:Marital\_Status + Sex:Occupation, family = binomial(link = "logit"),

## Coefficients:

	Estimate	Std. Error	z value	Pr(> z )	
(Intercept)	-2.703368	0.599795	-4.507	6.57e-06	***
Age	0.045978	0.006855	6.708	1.98e-11	***
EducationLow_Education	-1.513441	0.348787	-4.339	1.43e-05	***
SexMale	-0.708754	0.489645	-1.447	0.14776	
Hours_PW	0.040908	0.007421	5.513	3.53e-08	***
Marital_StatusUnmarried	-3.585524	0.449124	-7.983	1.42e-15	***
Occupation2	0.781470	0.570534	1.370	0.17078	
Occupation3	-1.266576	0.563022	-2.250	0.02447	*
Occupation4	-1.558491	0.662737	-2.352	0.01869	*
Occupation5	2.604163	1.505891	1.729	0.08375	
Occupation6	-1.190624	0.729529	-1.632	0.10267	
SexMale:Marital_StatusUnmarried	1.956976	0.501180	3.905	9.43e-05	***
SexMale:Occupation2	0.032557	0.638509	0.051	0.95933	
SexMale:Occupation3	1.269024	0.704213	1.802	0.07154	
SexMale:Occupation4	0.747952	0.713819	1.048	0.29472	
SexMale:Occupation5	-5.390792	1.625243	-3.317	0.00091	***
SexMale:Occupation6	0.283107	0.768359	0.368	0.71253	

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 1552.7 on 1384 degrees of freedom Residual deviance: 1050.7 on 1368 degrees of freedom

AIC: 1084.7

Number of Fisher Scoring iterations: 6

Implementation: ROI | Solver: lpsolve

Separation: FALSE

Existence of maximum likelihood estimates

 (Intercept)
 Age

 0
 0

 EducationLow\_Education
 SexMale

 0
 0

 Hours\_PW
 Marital\_StatusUnmarried

 0
 0

 Occupation2
 Occupation3

Occupation4 Occupation5

0: finite value, Inf: infinity, -Inf: -infinity

# Area under the curve: 0.868

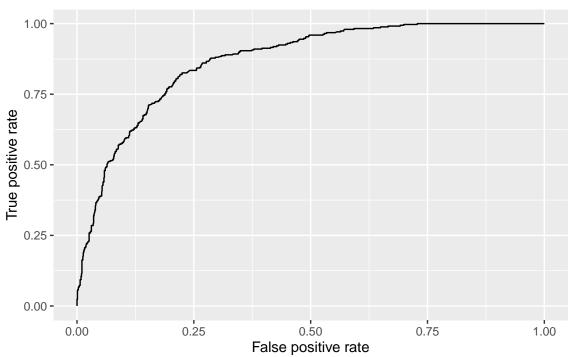


Figure 40: ROC curve for model\_new.2.interaction

# Call:

```
glm(formula = Income ~ Age + Education + Sex + Hours_PW + Marital_Status +
    Occupation + Age:Occupation + Education:Sex + Education:Marital_Status +
    Sex:Marital_Status, family = binomial(link = "logit"), data = data.new.3)
```

# Coefficients:

```
Estimate Std. Error z value
(Intercept)
                                                -3.518053
                                                            0.856736 -4.106
                                                 0.058001
                                                            0.017461
                                                                       3.322
Age
EducationLow_Education
                                               -15.965073 510.132302 -0.031
SexMale
                                                -0.339964
                                                            0.294255 - 1.155
Hours PW
                                                            0.007539
                                                 0.041293
                                                                      5.478
Marital StatusUnmarried
                                                -3.377304
                                                            0.397452 - 8.497
                                                                       1.676
Occupation2
                                                 1.729139
                                                            1.031589
Occupation3
                                                -2.075271
                                                            2.716100 -0.764
Occupation4
                                                -0.462935
                                                            1.188057 -0.390
Occupation5
                                                 0.510018
                                                            0.950263
                                                                       0.537
Occupation6
                                                 1.765975
                                                            2.241941
                                                                       0.788
                                                -0.117523
                                                            1.030062 -0.114
Occupation7
Occupation8
                                                -1.747957
                                                            1.218622 - 1.434
Occupation9
                                                -5.786074
                                                            2.476922 -2.336
Age:Occupation2
                                                -0.022482
                                                            0.024172 -0.930
Age:Occupation3
                                                 0.073706
                                                            0.065199
                                                                      1.130
Age:Occupation4
                                                -0.001889
                                                            0.027509 -0.069
Age:Occupation5
                                                            0.021804 -1.546
                                                -0.033699
Age:Occupation6
                                                -0.096113
                                                            0.051625 - 1.862
Age:Occupation7
                                                -0.016641
                                                            0.023975 - 0.694
Age:Occupation8
                                                 0.013741
                                                            0.027381
                                                                       0.502
Age:Occupation9
                                                 0.123809
                                                            0.056975
                                                                       2.173
EducationLow_Education:SexMale
                                                14.141329 510.132309
                                                                       0.028
EducationLow_Education:Marital_StatusUnmarried
                                                 1.354707
                                                            0.750523
                                                                       1.805
SexMale:Marital_StatusUnmarried
                                                 1.663952
                                                            0.456993
                                                                       3.641
                                               Pr(>|z|)
(Intercept)
                                               4.02e-05 ***
Age
                                               0.000895 ***
EducationLow_Education
                                               0.975034
SexMale
                                               0.247951
Hours_PW
                                               4.31e-08 ***
Marital_StatusUnmarried
                                                < 2e-16 ***
Occupation2
                                               0.093701 .
Occupation3
                                               0.444830
                                               0.696790
Occupation4
Occupation5
                                               0.591466
Occupation6
                                               0.430873
Occupation7
                                               0.909164
Occupation8
                                               0.151466
                                               0.019492 *
Occupation9
Age:Occupation2
                                               0.352339
Age:Occupation3
                                               0.258277
```

```
Age:Occupation4
                                                0.945240
Age:Occupation5
                                                0.122222
Age:Occupation6
                                                0.062640 .
Age:Occupation7
                                                0.487613
Age:Occupation8
                                                0.615767
Age:Occupation9
                                                0.029776 *
EducationLow_Education:SexMale
                                                0.977885
{\tt EducationLow\_Education:Marital\_StatusUnmarried~0.071072~.}
SexMale:Marital_StatusUnmarried
                                                0.000271 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1552.7 on 1384 degrees of freedom
Residual deviance: 1036.0 on 1360 degrees of freedom
AIC: 1086
Number of Fisher Scoring iterations: 16
```

# Call:

glm(formula = Income ~ Age + Sex + Education + Hours\_PW + Marital\_Status +
 Occupation + Marital\_Status:Occupation, family = binomial(link = "logit"),
 data = data.new.3)

# Coefficients:

occirioronob.					
	Estimate	Std. Error	z value	Pr(> z )	
(Intercept)	-3.304748	0.529508	-6.241	4.34e-10	***
Age	0.044727	0.006817	6.561	5.34e-11	***
SexMale	0.445923	0.222749	2.002	0.04529	*
EducationLow_Education	-1.446628	0.346257	-4.178	2.94e-05	***
Hours_PW	0.037683	0.007366	5.116	3.12e-07	***
Marital_StatusUnmarried	-2.915217	0.531143	-5.489	4.05e-08	***
Occupation2	0.722076	0.336026	2.149	0.03164	*
Occupation3	0.697480	0.692564	1.007	0.31389	
Occupation4	-0.208668	0.380242	-0.549	0.58316	
Occupation5	-1.264080	0.288458	-4.382	1.17e-05	***
Occupation6	-3.060971	0.609949	-5.018	5.21e-07	***
Occupation7	-1.186279	0.297107	-3.993	6.53e-05	***
Occupation8	-1.623163	0.330524	-4.911	9.07e-07	***
Occupation9	-1.035350	0.586947	-1.764	0.07774	

```
Marital_StatusUnmarried:Occupation2 0.525588
                                                0.648670
                                                           0.810 0.41779
Marital_StatusUnmarried:Occupation3
                                     0.213914
                                                 1.136972
                                                           0.188 0.85076
Marital_StatusUnmarried:Occupation4 -0.879872
                                                0.941130
                                                          -0.935 0.34983
Marital_StatusUnmarried:Occupation5
                                     1.301111
                                                0.658313
                                                            1.976 0.04811 *
Marital StatusUnmarried:Occupation6
                                     3.306006
                                                 1.320659
                                                            2.503 0.01230 *
Marital_StatusUnmarried:Occupation7
                                     1.645927
                                                0.717551
                                                            2.294 0.02180 *
Marital StatusUnmarried:Occupation8
                                     2.161565
                                                0.714046
                                                            3.027
                                                                  0.00247 **
Marital_StatusUnmarried:Occupation9
                                     0.676033
                                                 1.272549
                                                           0.531 0.59525
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 1552.7
                           on 1384
                                    degrees of freedom
Residual deviance: 1054.8
                           on 1363
                                    degrees of freedom
AIC: 1098.8
Number of Fisher Scoring iterations: 6
Implementation: ROI | Solver: lpsolve
Separation: FALSE
Existence of maximum likelihood estimates
                        (Intercept)
                                                                     Age
                            SexMale
                                                 EducationLow_Education
                                                Marital_StatusUnmarried
                           Hours PW
                                  0
                                                                       0
                        Occupation2
                                                            Occupation3
                                                                       0
                        Occupation4
                                                             Occupation5
                                  0
                                                                       0
                        Occupation6
                                                             Occupation7
                                  0
                                                                       0
                                                             Occupation9
                        Occupation8
Marital_StatusUnmarried:Occupation2 Marital_StatusUnmarried:Occupation3
Marital_StatusUnmarried:Occupation4 Marital_StatusUnmarried:Occupation5
Marital_StatusUnmarried:Occupation6 Marital_StatusUnmarried:Occupation7
```

0

Marital\_StatusUnmarried:Occupation8 Marital\_StatusUnmarried:Occupation9 0 0 0: finite value, Inf: infinity, -Inf: -infinity

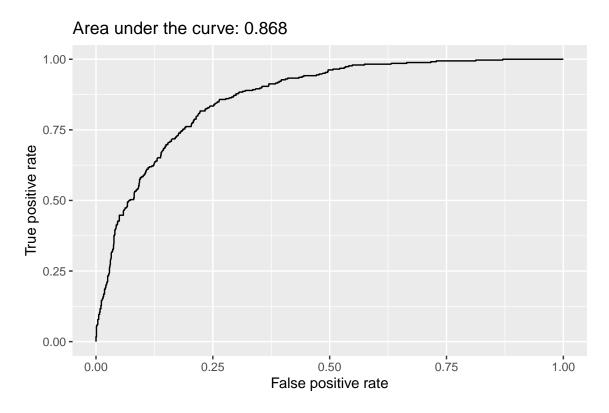


Figure 41: ROC curve for model\_new.3.interaction

To enhance predictive accuracy and account for potential interdependencies, interaction terms between variables are introduced into the logistic regression model. This allows us to evaluate how combinations of factors influence income classification ( $\leq$ 50K vs.  $\geq$ 50K).

Key Interaction Effects Identified Model using Physical vs. Mental Labor (model\_new.1.interaction): Significant interactions found: Age  $\times$  Marital Status Sex  $\times$  Marital Status This suggests that age influences the impact of marital status on income, and gender differences in income depend on marital status. Model using PRC Job Classification (model\_new.2.interaction):

Additional significant interactions: Sex  $\times$  Marital Status Sex  $\times$  Occupation Marital Status  $\times$  Occupation This highlights that gender effects on income vary across job categories, and marital status plays a role in occupational income disparities. Model using ISCO-08 Job Classification (model\_new.3.interaction):

The most significant interaction: Marital Status × Occupation This suggests that marital status interacts with occupation type to influence income, possibly due to differences in career stability or dual-income households.

Model Performance Assessment ROC curve analysis: The AUC values are compared for interaction models, showing whether incorporating interaction effects improves classification performance. Detect separation tests: Ensures that models do not suffer from convergence issues caused by perfect separation. Stepwise AIC optimization: Helps reduce overfitting by removing redundant interaction terms.

Key Takeaways: Including interaction terms improves model interpretability and accuracy. Marital Status and Occupation consistently interact, meaning these two factors jointly affect income potential. Gender interacts with both Marital Status and Occupation, indicating income disparities linked to societal roles. ROC analysis suggests that models with interactions perform better than those without, validating the importance of capturing interdependencies between variables.

# 5 Model Assumption

The Hosmer-Lemeshow test p-values for all seven models are greater than 0.05, indicating a good model fit. The predicted probabilities do not show a significant difference from the actual data distribution, confirming that the models are suitable for use.

```
Hosmer and Lemeshow goodness of fit (GOF) test

data: stepAIC_model$y, fitted(stepAIC_model)
X-squared = 3.5824, df = 8, p-value = 0.8927

Hosmer and Lemeshow goodness of fit (GOF) test

data: stepAIC_model_new.1$y, fitted(stepAIC_model_new.1)
X-squared = 7.7147, df = 8, p-value = 0.4618

Hosmer and Lemeshow goodness of fit (GOF) test

data: stepAIC_model_new.2$y, fitted(stepAIC_model_new.2)
X-squared = 6.6156, df = 8, p-value = 0.5786
```

Hosmer and Lemeshow goodness of fit (GOF) test

data: stepAIC\_model\_new.3\$y, fitted(stepAIC\_model\_new.3)
X-squared = 10.221, df = 8, p-value = 0.2499

Hosmer and Lemeshow goodness of fit (GOF) test

data: model\_new.1.interaction\$y, fitted(model\_new.1.interaction)
X-squared = 9.4961, df = 8, p-value = 0.3022

Hosmer and Lemeshow goodness of fit (GOF) test

data: model\_new.2.interaction\$y, fitted(model\_new.2.interaction)
X-squared = 9.1216, df = 8, p-value = 0.3321

Hosmer and Lemeshow goodness of fit (GOF) test

data: model\_new.3.interaction\$y, fitted(model\_new.3.interaction)
X-squared = 6.8584, df = 8, p-value = 0.552

When evaluating the prediction accuracy of the seven models, all models achieved an accuracy greater than 0.8, with a difference of no more than 0.02 between them. This indicates that the models can effectively distinguish between high-income (>50K) and low-income (<=50K) groups.

Actual Predicted <=50K >50K <=50K 958 154

>50K

[1] "Accuracy1: 0.829"

83 190

Actual
Predicted <=50K >50K
<=50K 942 165
>50K 99 179

# [1] "Accuracy2: 0.809"

Actual

Predicted <=50K >50K <=50K 961 169 >50K 80 175

[1] "Accuracy3: 0.82"

Actual

Predicted <=50K >50K <=50K 965 169 >50K 76 175

[1] "Accuracy4: 0.823"

Actual

Predicted <=50K >50K <=50K 938 150 >50K 103 194

[1] "Accuracy5: 0.817"

Actual

Predicted <=50K >50K <=50K 958 161 >50K 83 183

[1] "Accuracy6: 0.824"

Actual

Predicted <=50K >50K <=50K 957 171 >50K 84 173

[1] "Accuracy7: 0.816"

The AIC of model\_new.2.interaction is the smallest, indicating the best model fit. Additionally, model\_new.2.interaction achieves the highest AUC of 0.868, demonstrating the strongest ability to classify high-income (>50K) and low-income (<=50K) groups.

```
df AIC stepAIC_model 34 1066.601 stepAIC_model_new.1 7 1123.930 stepAIC_model_new.2 11 1101.458 stepAIC_model_new.3 14 1106.393 model_new.1.interaction 9 1108.156 model_new.2.interaction 17 1084.720 model_new.3.interaction 22 1098.772
```

The X-axis  $(0 \to 1)$  represents the predicted probability, where 0 indicates low income, 1 indicates high income, and 0.5 represents model uncertainty. The Y-axis (residuals) represents the prediction error. A residual value close to 0 indicates that the predicted result is close to the actual value, with a small error, while a residual value far from 0 indicates a larger discrepancy between the predicted and actual values.

At points where the X-axis is close to 0 and 1, residual values are larger, indicating higher prediction errors in these regions. However, this is a normal phenomenon in logistic regression. When the X-axis is near 0.5, the residual values are smaller, suggesting that the model fits better in the middle probability range.

# **Residuals vs Fitted Values**

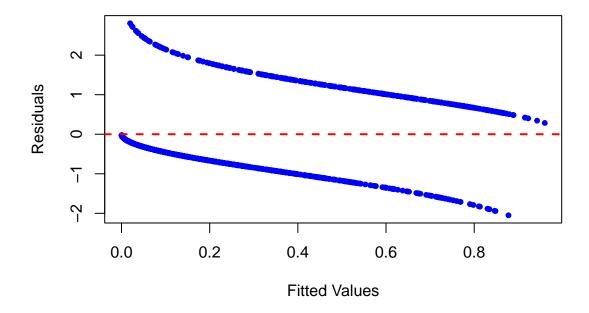


Figure 42: Residuals vs Fitted Values.

The residuals follow a normal distribution in the middle region but deviate from the diagonal line at both ends, especially on the right side. However, slight tail deviations are common in logistic regression.

# **QQ Plot of Standardized Residuals**

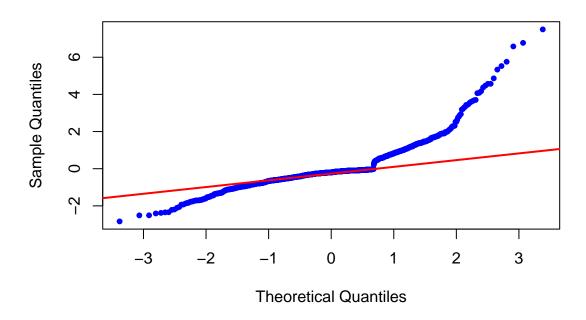


Figure 43: QQ Plot of Standardized Residuals.