Cellular Automata Traffic Model Simulation and Analysis based on Fundamental Diagrams

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1. Abstract

In order to simulate traffic flow and examine the relationship between density, flow, and velocity, this study produced four single-plane cellular automata models: the One-space model, the Two-space model, the Multi-vehicle model, and the Nagel-Schreckenberg (NaSch) model. This article then examined how different models contribute to differences in these measurements.

2. Introduction

Vehicle movement on highways and transit networks has been analyzed and predicted using traffic models. Among different traffic models, Cellular automata (CA) proposed by VON NEUMANN (Schwartz, von Neumann and Burks, 1967), which is a simplified system for the simulation of complex transportation systems, has been famous due to its ability to reproduce features of Flow-density measurements. CA is a system dxiscretized in space and time. For the CA model for traffic study, roads are sliced into cells, containing a vehicle (with velocity) or empty. Then the system simulates through a constant of time steps (Rohde, 2005).

In the CA model, the length of a cell is 7.5m, given by the minimum space headway between vehicles in a jam. A unit of time step Δt is between 0.6 and 1.2s, which is an average of driver reaction time (Čulík, Kalašová and Štefancová, 2022). According to Wu and Brilon (2014), the velocity in the CA model ranges from 0 to $v_{max} = 6\ cells/\Delta t$, which is equivalent to $162km/h$ for $\Delta t = 1s$.

Cells are named after their location on the road. For instance, cell 1 is the beginning of the road, the next cell is cell 2, and the last cell of the road is cell L (L denotes the total number of cells in the road). The traffic system is set on a circle (close system). A vehicle restarts its journey at the beginning when it reaches the end of the route.

In this project, we established a CA model for a simple traffic study. Then, we reform the model by adding a different kind of vehicle. Lastly, based on these, a Nagel-Schreckenberg model is constructed to compare the features of the models.

This article is organized as follows: At the beginning we describe the four models (Sect. 3) and the measurement process (Sect. 4) . In Section 5 we discuss and contrast the fundamental diagrams and analysis of the four models. In the concluding Section 6 we summarize of results and compare them with real traffic.

3. Model Setup

3.1. One-space model

In this CA model, each cell could be occupied or unoccupied by a vehicle. Vehicles are initially arranged in the cells at random.

For update steps, this model assumes that every road cell is updated instantly for update steps. Each vehicle is updated based on the road for the previous time step. The movement for a vehicle in cell i is defined as:

If the next cell (cell $i+1$) is occupied, the vehicle in cell i does not move. Otherwise, the vehicle in cell i goes to the cell $i+1$.

The movement with the rules defined are shown in Fig.1, where each black cell represents an occupied cell.

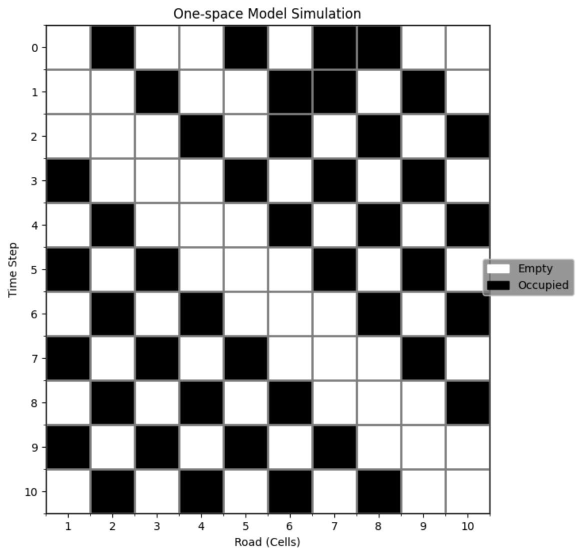

Fig.1 –Simulation for 4 vehicles in a 10-cell road using One-space Model

3.2. Two-space model

The goal of this model is to investigate how various movement logics alter the system. All the presumptions and setups are the same as in Model 1. The only modification is that vehicles now need two spaces to move (therefore we call them two-space vehicles), which means they can only go 1 cell further if the following 2 cells along the direction are unoccupied. Fig.2 shows how vehicles move with the rules mentioned.

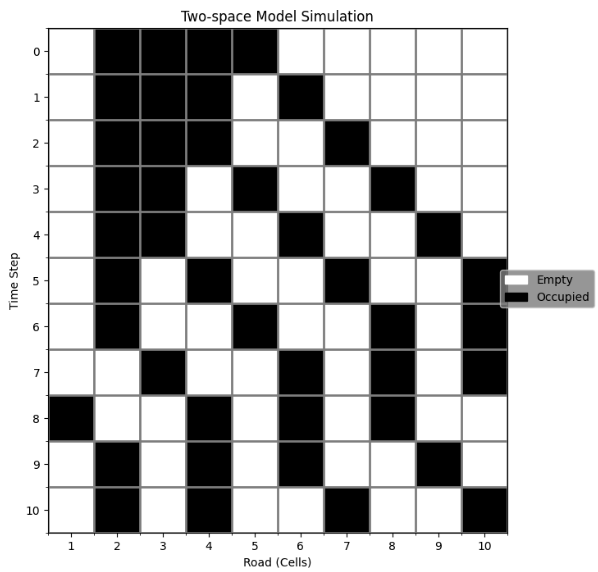

Fig.2 –Simulation for 4 vehicles in a 10-cell road using Two-space Model

3.3. Multi-vehicle model

This model's objective is to demonstrate how the allocation of the different vehicles might optimize road utilization. The vehicles in this model are defined into two groups, cars and buses, which follow the movement rule of the One-space model and the Two-space model, respectively. Fig.3 shows how cars and buses move with the rules mentioned, where a red cell and a blue cell represent a car and a bus respectively.

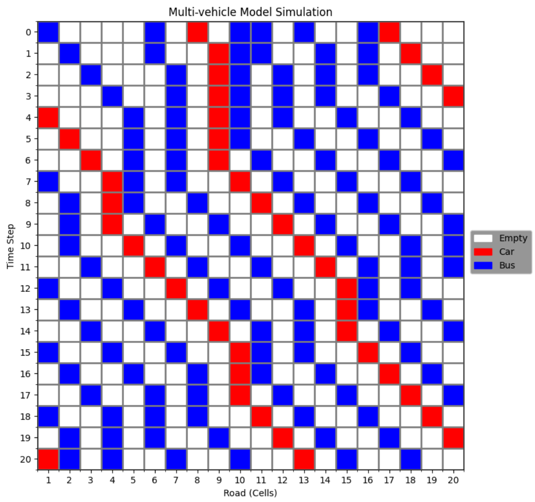

Fig.3 – Simulation for 2 cars and 6 buses in a 20-cell road using Multi-vehicle Model

Vehicles are expected to carry a certain number of passengers. A car can accommodate up to 5 people, while a bus has a maximum capacity of 30 people. Based on the settings for passengers, the model can now investigate how transportation allocation impacts the road given a number of passengers.

3.4. Nagel-Schreckenberg (NaSch) Model

Nagel and Schreckenberg (1992) have introduced a CA model for simulations of highway traffic. The NaSch model in this project is simplified and based on their model. The model is defined on a one-dimensional array with L cells, each cell may either be occupied by a vehicle or be empty. Each vehicle has an integer velocity with values between 0 and $v_{max} =$

6 $cells/\Delta t$. The following four rules update a vehicle with velocity v:

1. Acceleration: If the distance to the next car ahead is larger than $v+1$ and $v < v_{max}$, then the velocity is increased by 1
2. Deceleration: If the distance to the next car ahead is j and $j \leq v$, the vehicle reduces its velocity to $j-1$
3. Randomization: With probability p, the velocity is decreased by 1
4. Car motion: Each vehicle goes v cell ahead

Fig.4 shows how vehicles move following the rules in the NaSch model.

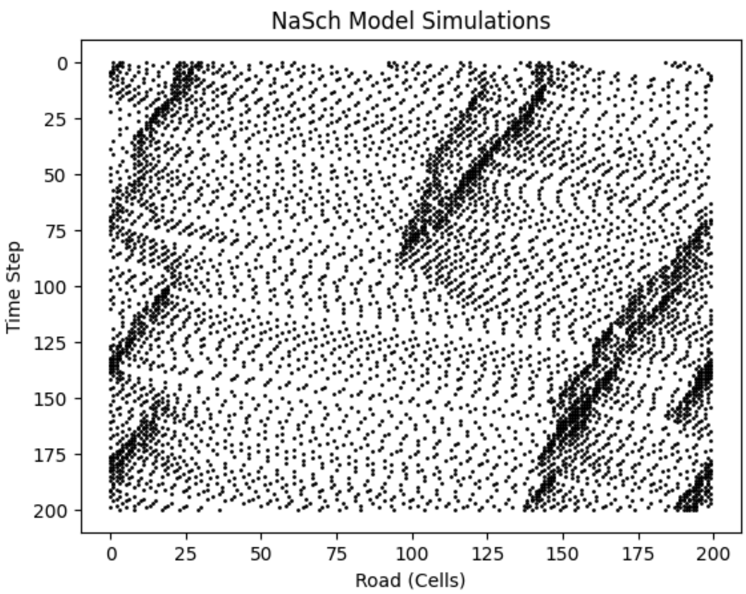

Fig.4 – Simulation for 30 vehicles in a 200-cell road for NaSch model, each black dot represents a vehicle

4. Data Measurement

4.1. Definitions of System and Sample Data

This project will focus on the analysis of density, flow and velocity. Nagel and Schreckenberg (1992) defined constant system density

$$p = \frac{N}{L}$$

N : Number of vehicles

L : Number of cells

However, it is impossible to calculate the total number of all vehicles in reality. To simulate real conditions, sample density $\bar{p}$ is measured on a fixed cell i over a time period T starting from time step t_0

$$\bar{p}\,(i, t_0, T) = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} n_i(t)$$

where $n_i(t)$ is defined as

$$n_i(t) = \begin{cases} 1\ if\ cell\ i\ is\ occupied\ at\ the\ time\ step\ t \\ 0\ otherwise \end{cases}$$

According to Prof L.H. Immers and S. Logghe (2002), system flow during the time period $t_0 + 1$ to $t_0 + T$ is defined as

$$q(t_0, T) = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} \frac{\sum_{i=1}^{L} v_i}{L}$$

Sample Flow $\bar{q}$ between i and $i + 1$ is defined by:

$$\bar{q}(i, t_0, T) = \frac{1}{T} \sum_{t=t_0+1}^{t_0+T} m_i(t)$$

where $m_i(t)$ is defined by:

$$m_i(t) = \begin{cases} 1\ if\ a\ vehicle\ motion\ is\ detected\ between\ cell\ i\ and\ i+1\ at\ the\ time\ step\ t \\ 0\ otherwise \end{cases}$$

Average velocity v for the system over the period T is:

$$v(t_0, T) = \frac{1}{T} \sum_{t=t_0+1}^{t_o+T} \frac{\sum_{i=1}^{L} v_i}{N}$$

To mimic real conditions, the average velocity given by sampling in cell i over a period T is calculated as:

$$\bar{v}(i, t_0, T) = \frac{\sum_{t=t_0+1}^{t_o+T} s_i(t)}{\sum_{t=t_0+1}^{t_0+T} g_i(t)}$$

$s_i(t)$: the sum of the velocity of vehicles passing through, arriving or starting at cell i at time step t

$g_i(t)$: the total number of vehicles passing through cell i at time step t

4.2. Measurement Method

Each data had a measurement cell randomly assigned by the system for sample measurement. The measurement then adheres to the guidelines listed below.

In order to measure sample density, the density counter will count 1 for that time step if the measurement cell is occupied (count 0 otherwise). The mean is eventually determined for the final result.

Regarding sample flow, the measurement cell was checked to see if there was a movement on the measurement cell. The system counts the number for each time step and calculates the mean.

To measure sample velocity, if a vehicle passes through, arrives, or begins to move at the measurement cell. The vehicle's velocity is noted. Finally, the mean is calculated.

5. Analysis and Discussion

In this section, the fundamental diagram, which means plots including density, flow and velocity) and analysis based on different models will be covered (Immers and Logghe, 2002)). All the fundamental diagrams are simulated under 400 time steps and a 1200-cell road.

5.1. Analysis based on One-space model

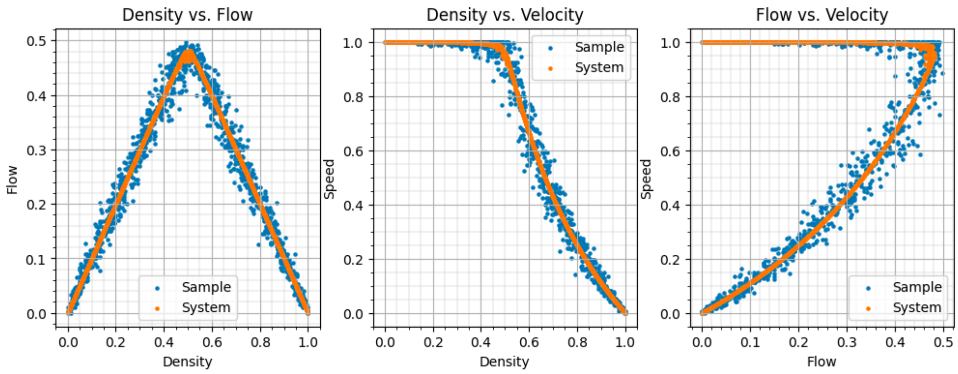

Fig.5 – Fundamental diagram for One-space model

This diagram (Fig.5) demonstrates that flow reaches its maximum at a density roughly 0.5 and diminishes at higher or lower densities. The outcome is comparable to the Gazis and Herman Model generated by Fadhloun, Rakha and Loulizi (2016)

The density vs. velocity plot indicates that average velocity falls with increasing density. When the density is less than or equal to 0.4, the average velocity stay close to 1.0. However, once the density exceeds 0.4, the average velocity drops. Using Numpy's Polyfit function, we determine that the maximum density for an average velocity of 0.9 is 0.51.

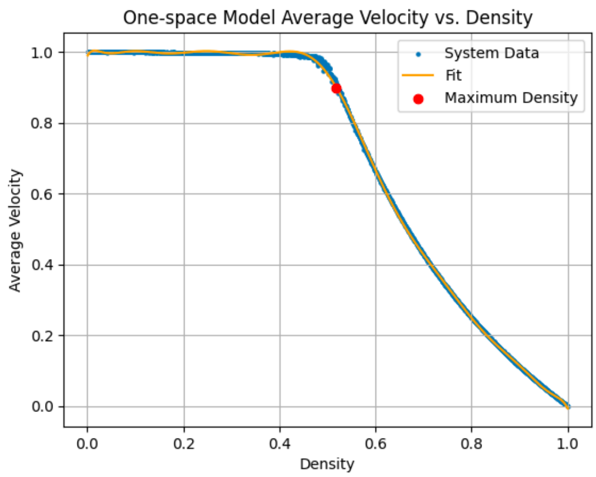

Fig.6 –Velocity vs. density graph for One-space model

From Fig.6, it is significant that, most of the time, there is a positive correlation between flow and average velocity. However, if the speed exceeds a particular value (nearly 1.0), speed growth will cause a rapid decline in flow. This is because significantly large velocities are contributed by low density, and low density leads to low flow.

5.2.Analysis based on Two-space model

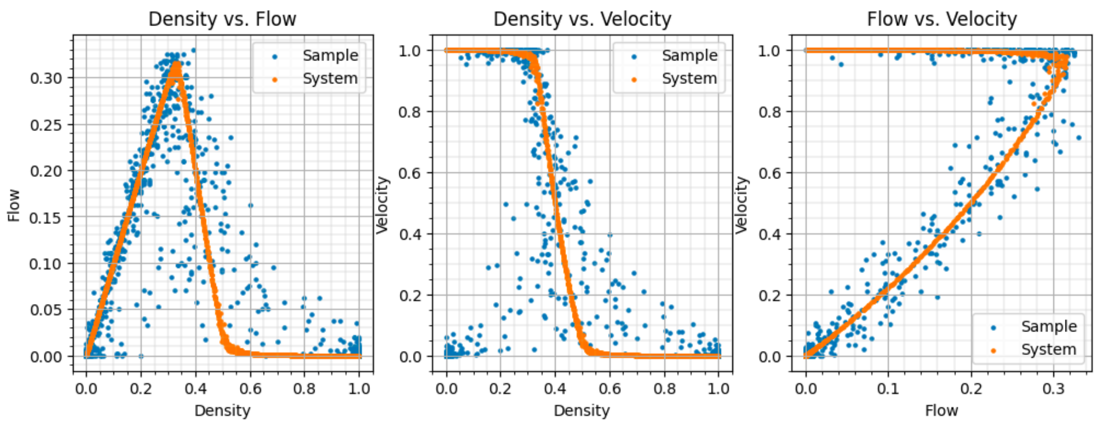

Fig.7 – Fundamental diagram for Two-space model

The plots (Fig.7) show that, at about 0.3 density, the most significant flow is represented. This result shows a smaller maximum flow and corresponding density compared to the one-space model, which means two-space vehicles require more space than one-space vehicles (vehicles in the One-space model).

The trend of the flow vs. average velocity graph resembles that of the One-space model. In contrast to the One-space model, the maximum flow for the maximum speed is substantially smaller.

The graph (Fig.8) shows that a larger density will cause a smaller velocity in this model. When the density is lower than 0.32, the system can maintain an average velocity higher than 0.9. However, after that, growth in density will cause a rapid decrease in average velocity. When density is larger than 0.55, the average velocity is lower than 0.01, which means almost no vehicle movements.

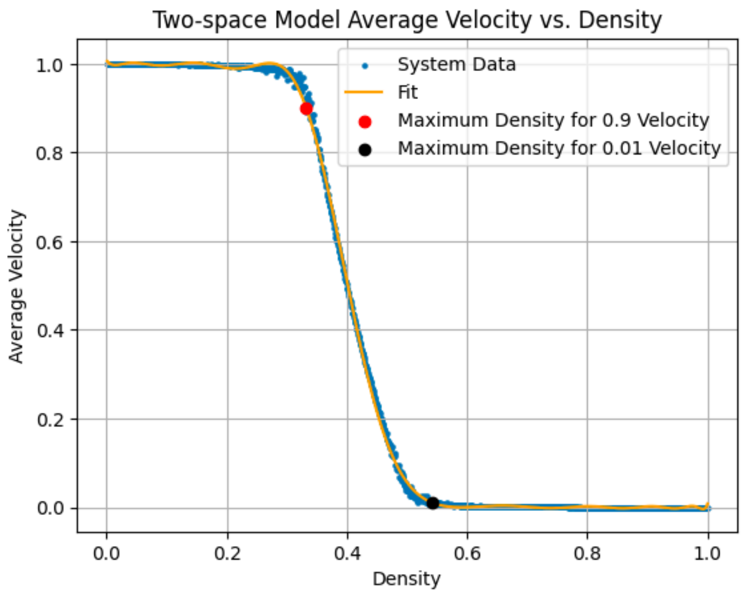

Fig.8 – Velocity vs. density graph for Two-space model

Note that the sample data differs significantly from system data due to errors produced in sampling density when density is high and velocity is less than 1.0.

The sample density bias is negatively correlated with average velocity. When the average velocity is about 1.0, the sample density is comparable to the system density. When the average velocity falls below 1.0, the corresponding density represents a more significant variance.

When the average velocity is 0, all vehicles are immobile, which results in the measurement cell continuously counting 1 or 0 in every time step (depending on whether the initial cell is occupied). The outcome is highly biased data.

The error follows the same general pattern as the bias in the result of Nagel and Schreckenberg (1992), which states that higher-density data exhibit higher variance. In the meantime, it shows that the model needs to be more realistic, as it is impossible for the average velocity to be 0.

5.3.Analysis based on Multi-vehicle model

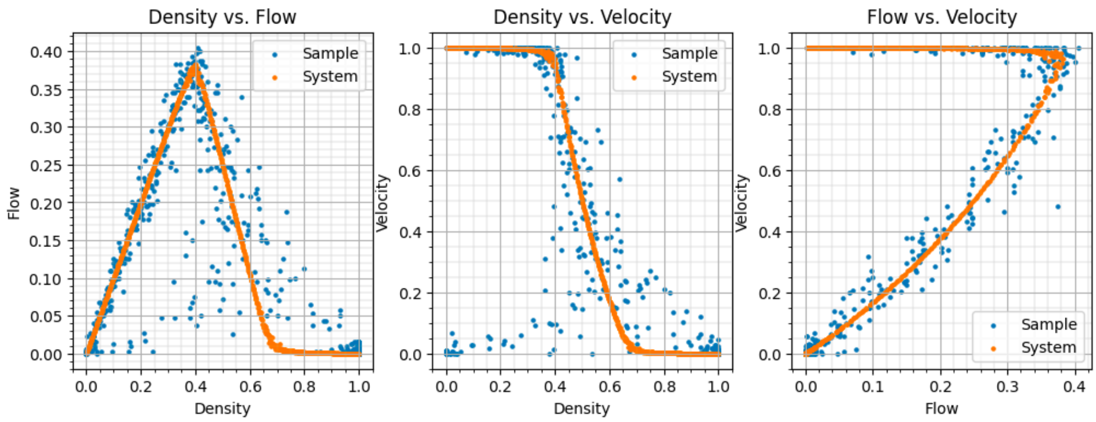

Fig.9 – Fundamental diagram for Multi-vehicle model

In this analysis (Fig.9), the number of cars and buses is equal. The correlations between density, flow and velocity show a similar trend to the Two-space model's result, but they result in higher maximum flow and a higher corresponding density.

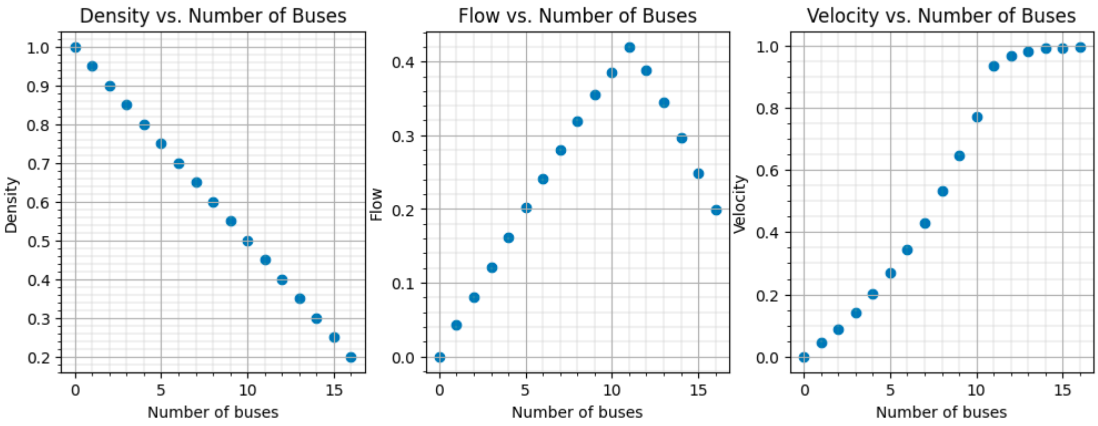

Fig.10 – Relation between number of buses and density, flow, velocity in Multi-vehicle model. The simulation is run under 500 passengers, a 100-cell road and 100 time steps.

The graphs (Fig.10) illustrate how the allocation of different vehicles impacts the road. Where there are more buses on the road, the density if lower and velocity is higher. However, the flow reaches a maximum when the number of buses is 11. Therefore, a combination of cars and buses has the best performance on flow compared to just cars or buses.

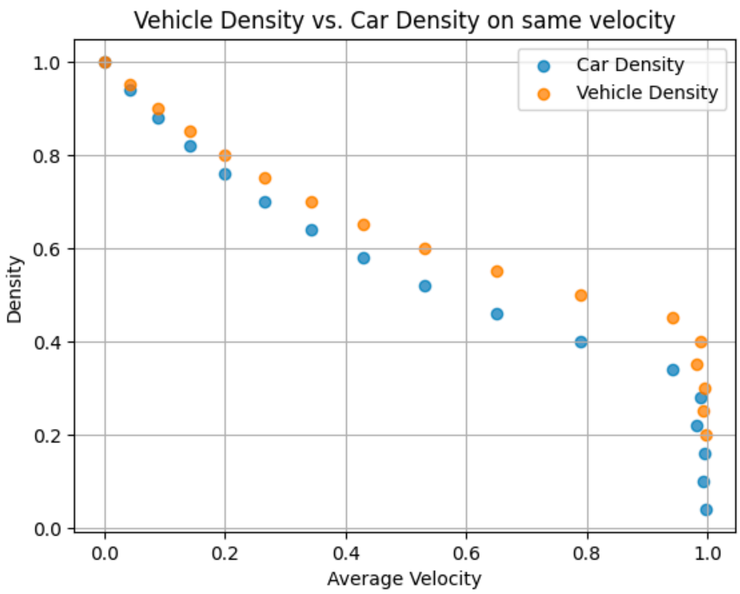

Fig.11 – Comparison between vehicle density and car density based on the same velocity

The plot (Fig.11) shows that the difference between vehicle and car density grows with average velocity. This validates the previous finding that more buses lead to faster velocity.

5.3.Analysis based on the NaSch model

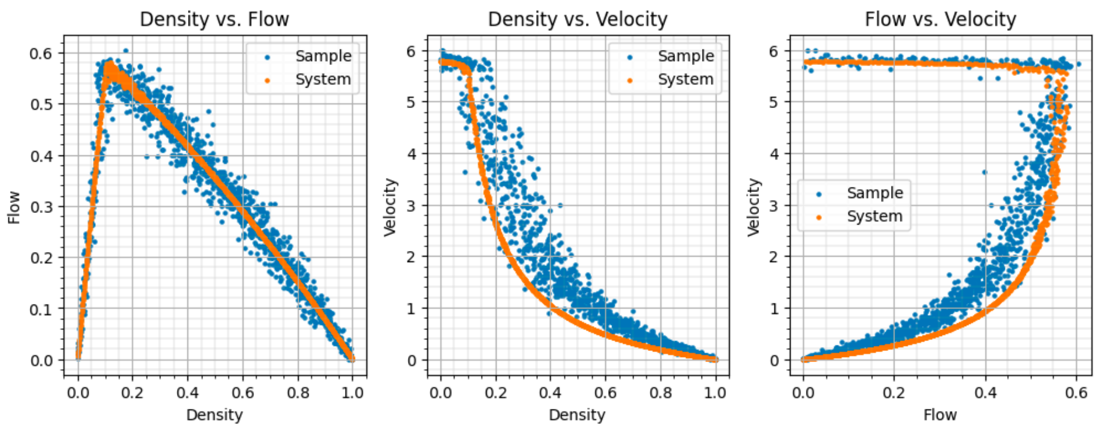

Fig.12 – Fundamental diagram for the NaSch model

This graph (Fig.12) shows similarity with One-space model in how the three measurements affect each other. It also shows the same trend with Nagel and Schreckenberg's results.

When density is about 0.1, flow and average velocity reach a maximum. A density lower than 0.1 causes higher average velocity but a lower flow. A density higher than 0.1 causes a reduction in both flow and speed.

Note that the data of sample average velocity is mostly larger than the theorem average velocity by using the sampling method mentioned. This is due to low sampling rate and frequency as the model has only one velocity counting ceil in a road of 200 cells.

6. Conclusions

This article presents four CA models for single-plane traffic simulations. It has been shown that density, flow and velocity follow a specific pattern. There always exists density, which contributes to maximum flow and velocity close to its maximum value. Moreover, the multi-vehicle model demonstrates that, given a constant passenger load, a mix of various vehicles can maintain the highest flow.

The NaSch model produces a triangular fundamental diagram similar to that of Juan Carlos Muñoz and Daganzo (2002) and rejects the curve model created by Gazis (1992). As the NaSch model displays a similar trend to the real flow-density relationship mentioned by Al-Obaedi and Yousif (2010), it is more persuasive that the NaSch model is closer to reality. Additionally, all models mentioned in this article show that the flow vs density graph ought to resemble a triangle. When considering the graph of flow vs. velocity and density vs. velocity, the NaSch model's performance is comparable to Romanowsaka's result (2021).

CA models have limitations as position, velocity, acceleration and time are not discrete but linear in real cases. It is also unable to simulate large scenes due to the restricted spacing capacity (Jun et al., 2019). In addition, more factors such as weather, humidity, vehicle mechanism, and driver's concentration should be taken into account. More rules, like Velocity-dependent-randomization (Tian, Zhu and Jiang, 2003), Modified-comfortable-driving (Barlovic et al., 1998). Could be added to the model to address this.

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