Sensores y Laboratorio 2019-I

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Filtro de Kalman

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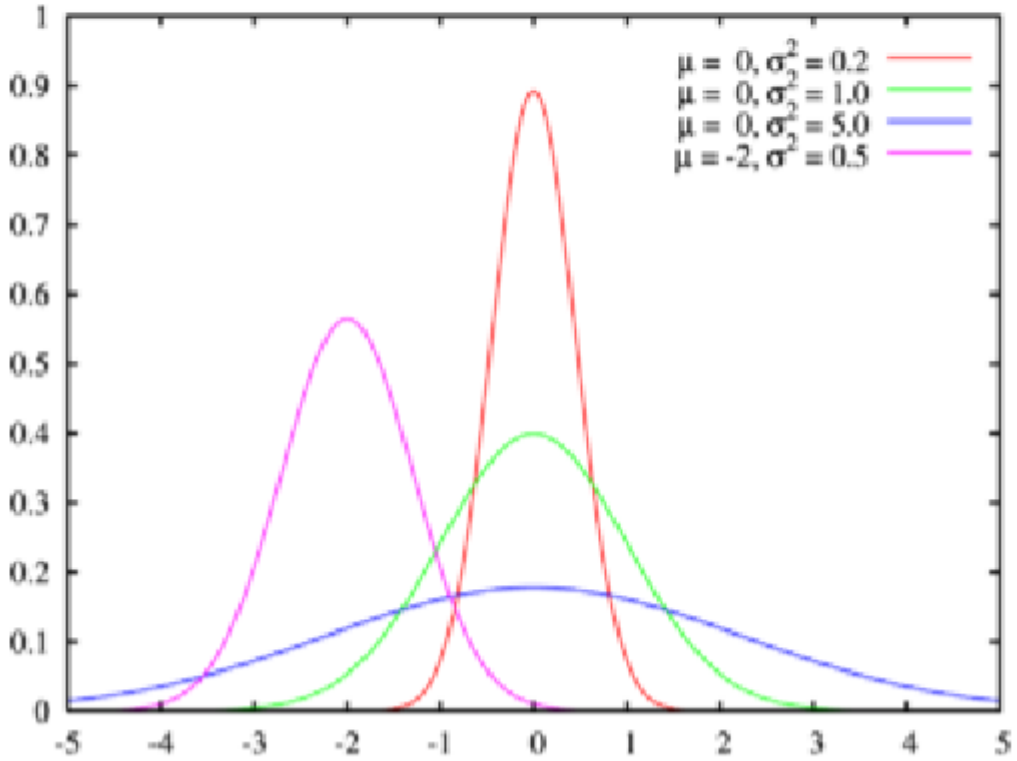
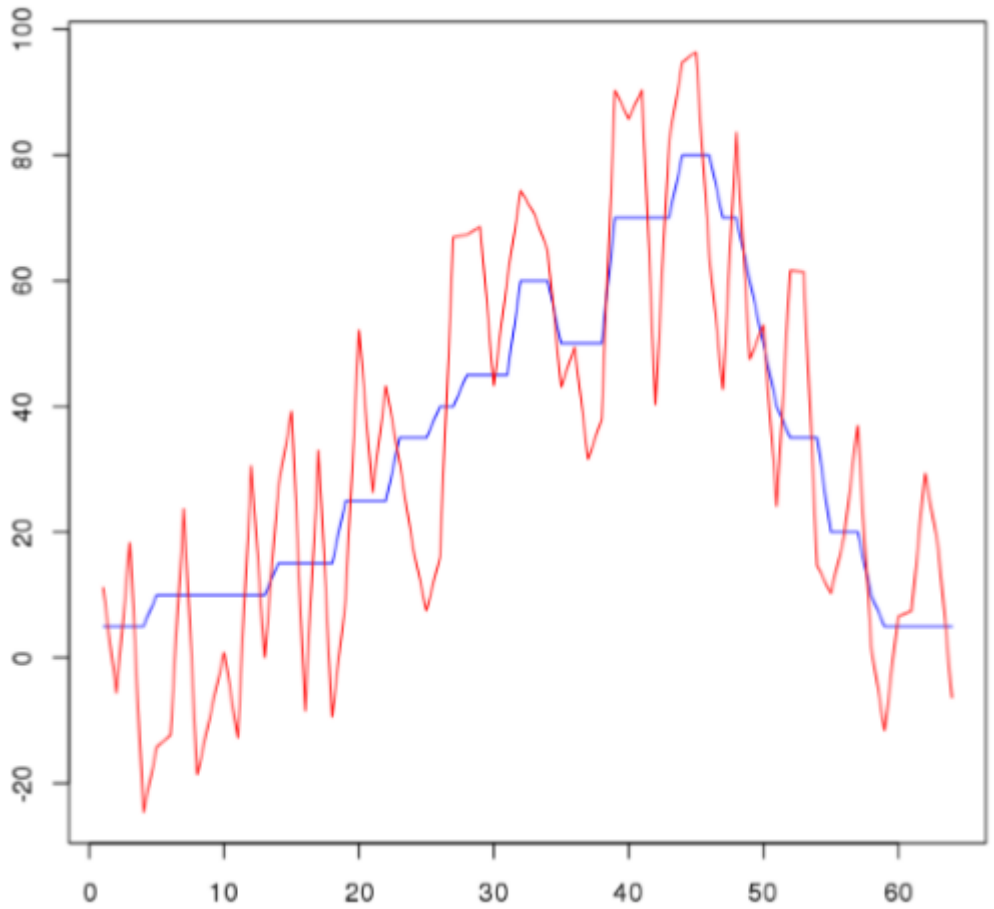
Rudolf E. Kalman (1960)

Filtro de Kalman

Distribución Normal



Filtro de Kalman



Filtro de Kalman

Modelo matemático continuo conocido

$$\dot{x} = Ax + Bu$$

Filtro de Kalman

Modelo matemático discreto conocido

$$\mathbf{x}(k+1) = \mathbf{F}(k)\mathbf{x}(k) + \mathbf{G}(k)\mathbf{u}(k) + \mathbf{v}(k)$$

$$k = 0,1,\ldots$$

Filtro de Kalman

Predicción de Kalman

$$\hat{\mathbf{x}}_{t|t-1} = \mathbf{F}_t \hat{\mathbf{x}}_{t-1|t-1}$$

$$\mathbf{P}_{t|t-1} = \mathbf{F}_t \mathbf{P}_{t-1|t-1} \mathbf{F}_t^T + \mathbf{Q}_t$$

Filtro de Kalman

$$\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{H}_t^T\left(\mathbf{H}_t\mathbf{P}_{t|t-1}\mathbf{H}_t^T + \mathbf{R}_t\right)^{-1}$$

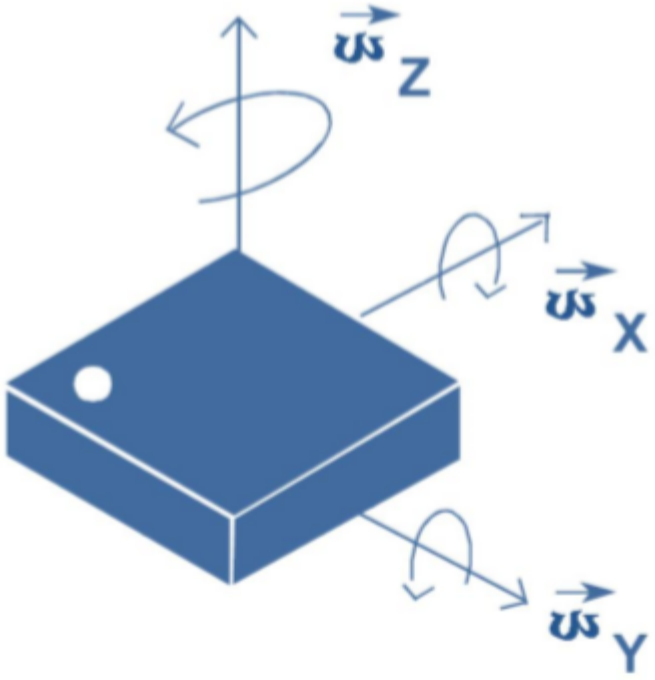
$$\hat{\mathbf{x}}_{t|t} = \hat{\mathbf{x}}_{t|t-1} + \mathbf{K}_t\left(\mathbf{y}_t - \mathbf{H}_t\hat{\mathbf{x}}_{t|t-1}\right)$$

$$\mathbf{P}_{t|t} = (\mathbf{I} - \mathbf{K}_t\mathbf{H}_t)\mathbf{P}_{t|t-1}$$

Filtro de Kalman

Sistema de rotacional con Giróscopo

$$\dot{x} = Ax$$



$$x = \begin{bmatrix} \boldsymbol{\theta} \\ \boldsymbol{\omega} \end{bmatrix}$$

$$\begin{bmatrix} \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta} \\ \boldsymbol{\omega} \end{bmatrix}$$

Filtro de Kalman

Pasar el sistema dinámico a discreto,

$$e^{A\tau} = I + \sum_{i=1}^{\infty} \frac{A^i \tau^i}{i!}$$

Filtro de Kalman

Modelamiento del ruido,

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & q_f \end{bmatrix}$$

Filtro de Kalman

$$\mathbf{Q}(\Delta t) = \int_0^{\Delta t} e^{\mathbf{A}\tau}\mathbf{Q}e^{\mathbf{A}^\top\tau}d\tau$$

Filtro de Kalman

$$\mathbf{Q}(\Delta t) = \int_0^{\Delta t} \begin{pmatrix} \tau^2 q_f & \tau q_f \\ \tau q_f & q_f \end{pmatrix} d\tau$$

Filtro de Kalman

$$Q(\Delta t) = \begin{pmatrix} \frac{\tau^3}{3} q_f & \frac{\tau^2}{2} q_f \\ \frac{\tau^2}{2} q_f & \tau q_f \end{pmatrix}$$

Filtro de Kalman

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$