4,5

1) I. $(3x^2+2xy)\,dx+(x^2+\cos(y))\,dy=0$

$$\Rightarrow \frac{dy}{dx}=-\frac{3x^2+2xy}{x^2+\cos y}$$

$y(0)=\pi/2$ para $h=0.1,\ 0.05,\ 0.01$ $\quad y(3)=?$

2) $V_{in}=4\,L/min$ → Velocidad de entrada

$d_{in}=1\,(g)$ → Concentración que entra.

$V_{out}=2\,L/min$ → Velocidad de salida.

$y(t)$ = Cantidad de litros en el tiempo

$$y(t)=y_0+(V_{in}-V_{out})\,t$$

$x(t)$ = Cantidad de gramos en el tiempo

$$\frac{dx}{dt}=V_{in}d_{in}-V_{out}d_{out}$$

$$\frac{dx}{dt}=V_{in}d_{in}-\frac{V_{out}\,x}{y_0+(V_{in}-V_{out})t}$$

$$\frac{dx}{dt}+\underbrace{\frac{V_2\,x}{y_0(V_{in}-V_{out})}}_{P(x)}=\underbrace{V_{in}d_{in}}_{Q(x)}$$

$$F_I=e^{\int P(t)\,dt}$$

$$\frac{dx}{dt}=200-170e^{\left(\frac{-t}{100}\right)}$$

3) $R_{in} = (0{,}06)(2000\,ft^3/min) = 1{,}2\,ft^3/min$ 11,59

R_{out} = Concentration of CO_2

$\Rightarrow R_{in} = \frac{V_t}{8000}(2000\,ft^3/min) = \frac{V_t}{4}\,ft^3/min$

$\frac{dv}{dt} = 1{,}2 - \frac{v(t)}{4} \rightarrow \frac{dv}{dt} + \frac{1}{4}v = 1{,}2$

$e^{\int \frac{1}{4} dt} = e^{\frac{t}{4}}$

$\frac{d}{dt} = 1{,}2e^{t/4}$ $\quad y(t) = 2 \times 10^{-3} \times 8000 = 16\,ft$

$v(t) = 4{,}8 + 11{,}2\,e^{-\frac{t}{4}}$

5. $L \frac{d^2q}{dt^2} + R\frac{dq}{dt} = E(t)$

inductor = 20H, $R = 2\,\Omega$, $E = 120\,V$

q: carga del circuito

$I = \frac{dq}{dt}$

$q(0) = 1C$

$I(0) = 0A$

$q'' + 0{,}1\,q' = 6$

$q'' + 0{,}1\,q' - 6 = 0$

$m_1 = 2{,}4 \quad m_2 = -0{,}71$

$q(t) = C_1 e^{2{,}4t} + C_2 e^{-0{,}71t}$

$1 = C_1 e^0 + C_2 e^0$

$1 = C_1 + C_2$

$I(t) = 2{,}4\,C_1 e^{2{,}4t} - 0{,}71\,C_2 e^{-0{,}71t}$

$0 = 2{,}4\,C_1 - 0{,}71\,C_2$

$C_1 = 0{,}5102 \quad C_2 = 0{,}4898$

$\Rightarrow Q(t) = 0{,}5102\,e^{2{,}4t} + 0{,}4898\,e^{-0{,}71t}$

$\Rightarrow I(t) = 1{,}2245\,(e^{2{,}4t} - e^{-0{,}71t})$

$$\left.\begin{aligned} Q(2) &= 61{,}99\ C \\ I(2) &= 148{,}78\ A \end{aligned}\right\} a$$

$h = 0{,}25$ para $I(2)$

6) $\int_{0.1}^{2} f(x)\,dx$; $f(x) = -\frac{5}{4}\ln(2x^2+1)$ $\epsilon = 10^{-2}$

$$f''(x) = \frac{-5x}{2x^2+1}$$

$$f'''(x) = \frac{-5(-2x^2+1)}{(2x^2+1)^2} = 0 \Rightarrow \left.\begin{matrix} x = 0.7071 \\ x = -0.7071 \end{matrix}\right\}$$ $|f''(x)|$ con ambos x valor $= K$

$$|f''(0.7071)| = \left|\frac{-5(0.7071)}{2(0.7071)^2+1}\right| = 1.7678 = K$$

$$\epsilon_T \leq \left|\frac{K(b-a)^3}{12n^2}\right|$$

$$n^2 \leq \left|\frac{1.7678(2-0.1)^3}{12(10^{-2})}\right|$$

$$n = 10.05 \approx 11$$

7) $f(x) = e^{\operatorname{sen}(x)}$ 4 longitud curva $= n$

$$L = \int_0^6 \sqrt{1 + [f'(x)]^2}\,dx \Rightarrow f'(x) = e^{\operatorname{sen}(x)}\cos(x)$$

$$L = \int_0^6 \sqrt{1 + (e^{\operatorname{sen}(x)}\cos(x))^2} = 7.8637 \approx 8$$

longitud curva

$$4(8) = n = 32$$

a) $i(t) = (60-t)^2 + (60-t)\,\text{sen}(\sqrt{t})$

$i'(t) = -2(60-t) - \text{sen}(\sqrt{t}) + \dfrac{\cos(\sqrt{t})(60-t)}{2\sqrt{t}}$

$i''(t) = \dfrac{(-3t-60)\cos(\sqrt{t}) + \sqrt{t}\,\text{sen}(\sqrt{t})(x-60)}{4t^{3/2}} + 2$

$i'''(t) = \dfrac{\sqrt{t}\cos(\sqrt{t})(-57t+t^2+180) + \text{sen}(\sqrt{t})(3t^2+180t)}{8t^3} + 0$

b) $K \geq |i''(t)| = K \geq |i''(12.976)| = K \geq 2.8767$

$$E_T \leq \left| \frac{K(b-a)^3}{12n^2} \right|$$

$$n \leq \sqrt{\left| \frac{K(b-a)^3}{12E_T} \right|}$$

$$n \leq \sqrt{\frac{2.8767(16)^3}{12(0.01)}}$$

$$n \leq 313.355 \approx 314$$

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1. $\displaystyle\int_{0,3}^{1,1} \underbrace{\left(-\tfrac{1}{3}t^4 \ln(t) + \tfrac{7}{36}t^4 + \tfrac{1}{2}t^2\right)}_{f(x)} dt$

Error $= 2 \times 10^{-4}$ $\quad a = 0,3 \quad b = 1,1 \quad \Delta x = \dfrac{b-a}{n}$

$\int = 0,290882$

$$E_T \leq \left| \frac{k(1,1-0,3)^3}{12n^2} \right|$$

$$f'(x) = \frac{-12x^3 \ln(x) + 4x^3 + 9x}{9}$$

$$f''(x) = -4x^2 \ln(x) + 1$$

$$f'''(x) = \underbrace{-4(2x\ln(x) + x) = 0}_{\text{max y mínimo}}$$

$$x = 0,60653$$

$$f''(x) = -4(0,60653)^2 \ln(0,60653) + 1 = 1,735758$$

$$2 \times 10^{-4} \leq \frac{1,735758(1,1-0,3)^3}{12n^2}$$

$$n^2 \leq \frac{1,735758(1,1-0,3)^3}{12 \cdot 2 \times 10^{-4}}$$

$$n = \sqrt{[illegible]} = 21,91438 \approx 22$$

max y mínima

$x = 0,60653$

$f''(x) = -4(0,60653)^2 \ln(0,60653) + 1 = 1,7357758$

$$2 \times 10^{-4} \leq \frac{1,7357758\,(1,1-0,3)^3}{12\, n^2}$$

$$n^2 \leq \frac{1,7357758\,(1,1-0,3)^3}{12 \cdot 2 \times 10^{-4}}$$

$$n = \sqrt{462,8200} = 21,51438 \approx 22$$

$$\Delta x = \frac{1,1-0,3}{22}$$

$$\int_{0,3}^{1,1} \left(-1/2\, t^4 \ln(t) + 3/32\, t^4 + 1/2\, t^4\right) dt =$$

		$f(x)$			
x_0	0,3	-0,0498	x_{11}	0,7	-0,3202
x_1	0,3363	-0,0627	x_{12}	0,7363	-0,3592
x_2	0,3727	-0,0795	x_{13}	0,7727	-0,4015
x_3	0,4090	-0,0974	x_{14}	0,8090	-0,4464
x_4	0,4454	-0,1174	x_{15}	0,8454	-0,4932
x_5	0,4818	-0,1396	x_{16}	0,8818	-0,5317
x_6	0,5181	-0,1640	x_{17}	0,9181	-0,5794
x_7	0,5545	-0,1907	x_{18}	0,9545	-0,6218
x_8	0,5909	-0,2166	x_{19}	0,9909	-0,6817
x_9	0,6272	-0,2504	x_{20}	1,0272	-0,7344
x_{10}	0,6636	-0,2844	x_{21}	1,0636	-0,8882
			x_{22}	1,1	-0,9434

$$= \frac{0,03636\,[23,9263093]}{3}$$

$= 0,2880376$

error = 0,0071.

10

$$H = \int_{t_1}^{t_2} Q(t)\, c(t)\, dt$$

$$\int_{2}^{8} (9 + 4\cos^2(0{,}4t))(5e^{-0{,}5t} + 2e^{0{,}15t})$$

$n = 6$ (promedio de tiempo) $= 6 \cdot 5 = 30$

$$\Delta x = \frac{8-2}{30} = 0{,}2.$$