(1) a) $(3x^2+2xy)\,dx + (x^2+\cos y)\,dy = 0$; $y(0) = \frac{\pi}{2}$ $h=0,1$

$$dy = -\frac{3x^2+2xy}{x^2+\cos y}\,dx$$

$$\frac{dy}{dx} = -\frac{3x^2+2xy}{x^2+\cos y}$$

$M(x,y)\,dx + N(x,y)\,dy = 0$

$M(x,y) = 3x^2+2xy$ $\quad N(x,y) = x^2+\cos y$

$M_y = 2x$ $\quad N_x = 2x$

$F(x,y) = \int M\,dx$

$= \int (3x^2+2xy)\,dx$

$= x^3 + x^2y + g(y)$

$F_y = x^2 + g'(y)$

$x^2 + \cos y = x^2 + g'(y)$

$g'(y) = \cos y$

$g(y) = \operatorname{sen}(y) + C$

$C_1 - C = K$

$F(x,y) = x^3 + x^2y + \operatorname{sen}(y) + C = C_1$

$F(0, \frac{\pi}{2}) = (0)^3 + (0)^2\frac{\pi}{2} + \operatorname{sen}(\frac{\pi}{2}) + C$

$C_1 - C = 1$

$F(x,y) = K$

$x^3 + x^2y + \operatorname{sen}(y) = -1$

$3^3 + 3^2y + \operatorname{sen}(y) = 1$

$y = -2.8577$

5.0

Taller metodos

200 litros agua
30g sal
in 4 L/min
1g/L
2 L/min

A(t) = ? g sal en tanque

a) $\frac{dx}{dt} = V_1C_1 - V_2C_2$

$C_2 = \frac{x}{Q+(V_1-V_2)t}$

$\frac{dx}{dt} = V_1C_1 - V_2 \frac{x}{Q+(V_1-V_2)t}$

$\frac{dx}{dt} + \frac{2}{200+(4-2)t} x = 4(1)$

$\frac{dx}{dt} + \frac{2}{2(100+t)} x = 4$

$\frac{dx}{dt} + \underbrace{\frac{x}{(100+t)}}_{P(t)} = 4$

$F_I = e^{\int \frac{1}{100+t} dt}$ $u = 100+t$, $du = 1$

$e^{\int \frac{du}{u}} = e^{\ln u} = 100+t$

$(100+t)\frac{dx}{dt} + (100+t)\frac{x}{(100+t)} = (100+t)(4)$

$(100+t)\frac{dx}{dt} + x = (100+t)4$

$\int 100+t \frac{dx}{dt} dt + \int x dt$

$(100+t)x + xt$

$(100+t+t)x \rightarrow (100+2t)x$

$$\frac{d}{dt}[(100+2t)x] = (100+t)\,4$$

$$\int d(100+2t)x = 4\int(100+t)\,dt$$

$$(100+2t)x = 4\left(100t + \frac{t^2}{2} + C\right)$$

$$x(t) = \frac{4\left(100t + \frac{t^2}{2} + C\right)}{100+2t}$$

$$C = 110$$

$$x(t) = \frac{400t + 2t^2 + 440}{100+2t}$$

$$x(2) = \frac{400(2) + 2(2)^2 + 440}{100 + 2(2)} = 12$$

$$x(5) = \frac{400(5) + 2(5)^2 + 440}{100 + 2(5)} = 22{,}6$$

(b)

$$\frac{dV}{dt} = R_{in} - R_{out}$$

$$R_{in} = (0{,}06\,\%)\cdot(2000\,ft^3/min) = 1{,}2\,ft^3/min$$

$$R_{out} = \left(\frac{V(t)}{8000}\right)(2000\,ft^3/min) = \frac{V(t)}{4}\,ft^3/min$$

$$\frac{dV}{dt} = 1{,}2 - \frac{V(t)}{4} \rightarrow \frac{dV}{dt} + \frac{1}{4}V = 1{,}2 \rightarrow V' + \frac{1}{4}V = 1{,}2$$

$$V(0) = 0{,}2\,\% \times 8000\,ft^3 = 16\,ft^3$$

$$V' = 1{,}2 - \frac{1}{4}V$$

(4) $\frac{dx_1}{dt} = -0,1 x_1 x_2$; $x_1(0) = 10$

$\frac{dx_2}{dt} = -x_1$; $x_2(0) = 15$

$\frac{dx}{dt} = f(x_1, x_2, t)$ | $\frac{dy_2}{dt} = g(x_1, x_2, t)$

$k_1 = hf(x_1, x_2, t)$ | $l_1 = hg(x_1, x_2, t)$

$k_2 = hf(x_1 + \frac{k_1}{2}, x_2 + \frac{l_1}{2}, t + \frac{h}{2})$ | $l_2 = hg(x_1 + \frac{k_1}{2}, x_2 + \frac{l_1}{2}, t + \frac{h}{2})$

$k_3 = hf(x_1 + \frac{k_2}{2}, x_2 + \frac{l_2}{2}, t + \frac{h}{2})$ | $l_3 = hg(x_1 + \frac{k_2}{2}, x_2 + \frac{l_2}{2}, t + \frac{h}{2})$

$k_4 = hf(x_1 + k_3, x_2 + l_3, t + h)$ | $l_4 = hg(x_1 + k_3, x_2 + l_3, t + h)$

$x(t+h) = x(t) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ | $x_2(t+h) = x_2(t) + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$

Ganarian las fuerzas convencionales

1) $A \times 5$

2) $\text{inv}(A) \times B^t$

3)

4) $\frac{\text{adj}(A)}{\det(A)} \times \text{inv}(B)$

Scribe

(5) a)

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = E(t) \qquad q(0) = 1C$$

$$q'' + \frac{R}{L}q' = \frac{E(t)}{L} \rightarrow q'' + \frac{1}{10}q' = 6 \rightarrow q'' + \frac{1}{10}q' - 6 = 0$$

$$q'' + \frac{1}{10}q' - 6 = 0$$

$$m_1 = \frac{12}{5} \qquad q(t) = C_1 e^{\frac{12}{5}t} + C_2 e^{-5/2 \cdot t}$$

$$m_2 = -\frac{5}{2} \qquad 1 = C_1 e^0 + C_2 e^0$$

$$1 = C_1 + C_2$$

$$\frac{dq(t)}{dt} = I(t) \qquad I(t) = C_1 \frac{12}{5} e^{\frac{12}{5}t} - \frac{5}{2}C_2 e^{-5/2 \cdot t} \qquad I(0) = 0A$$

$$0 = \frac{12}{5}C_1 - \frac{5}{2}C_2 \qquad C_1 = \frac{25}{49} \qquad C_2 = \frac{24}{49}$$

$$q(t) = \frac{25}{49}e^{\frac{12}{5}\cdot t} + \frac{24}{49}e^{-\frac{5}{2}\cdot t}$$

$$I(t) = \frac{60}{49}\left(e^{12/5 \cdot t} - e^{-5/2 \cdot t}\right)$$

$$I(2) = 148{,}78\ A$$

$$q(2) = 61{,}99\ C$$

b)

$$I' = -\frac{1}{10} I + 6$$

$$I' + \frac{1}{10} I - 6 = 0$$

$$q' = 6t - \frac{1}{10} q$$

$$q'' + \frac{1}{10} q' - 6 = 0$$

$$\int q'' \, dt + \frac{1}{10} \int q' \, dt = 6 \int dt$$

$$q' + \frac{1}{10} q = 6t$$

6) $\int_{0,1}^{2} F(x)\,dx$

$F(x) = \int -\frac{5}{4}\ln(2x^2+1)\,dx$

$F(x) = -\frac{5}{4}\left(x\ln(2x^2+1) + \sqrt{2}\arctan(\sqrt{2}x) - 2x\right)$

$\int_{0,1}^{2} F x\,dx = -1,66\ldots$

$F'(x) = -\frac{5}{4}\ln(2x^2+1)$

$F''(x) = -\frac{5}{4}\frac{(4x)}{2x^2+1}$

$F''(x) = -\frac{5x}{2x^2+1}$

$F'''(x) = -\frac{5(2x^2+1)+5x(4x)}{(2x^2+1)^2}$

$F'''(x) = 0$

$0 = -10x^2 - 5 + 20x^2$

$0 = 10x^2 - 5$

$x = \sqrt{5/10} = 0,707$

$|F''(0,707)| = 1,76 \rightarrow k$

$E_T = \frac{K(b-a)^3}{12n^2}$

$n = \sqrt{\frac{K(b-a)^3}{12E_T}}$

$n = \sqrt{\frac{1,76(2-0,1)^3}{12(1\times10^{-2})}} = 10$

$\Delta x = \frac{2-0,1}{10} = 0,19$

x_0	=	0,1	$F(x0) = -13,98$
x_1	=	0,29	$F(x1) = -38,75$
x_2	=	0,48	$F(x2) = -69,43$
x_3	=	0,67	$F(x3) = -75,68$
x_4	=	0,86	$F(x4) = -88,22$
x_5	=	1,05	$F(x5) = -97,97$
x_6	=	1,24	$F(x6) = -105,6$
x_7	=	1,43	$F(x7) = -111,9$
x_8	=	1,67	$F(x8) = -117$
x_9	=	1,81	$F(x9) = -121,4$
x_{10}	=	2	$F(x10) = -125,1$

$\frac{0,19}{2}(-13,92 + 2(-815,95) - 125,1) = \int_{0,1}^{2} F(x)\,dx$

$\int_{0,1}^{2} F x\,dx = -1,66\ldots$

(7)

$f(x) = e^{\operatorname{sen}(x)}$

$L = \int_a^b \sqrt{(1 + (y')^2}\, dx$

$f'(x) = e^{\operatorname{sen}(x)} \cos(x)$

$a = 0 \qquad b = 6$

$L = \int_0^6 \sqrt{1 + (e^{\operatorname{sen}(x)} \cos(x))^2}\, dx$

$n = 24$

$L = \int_0^6 \sqrt{1 + e^{2\operatorname{sen}(x)} \cos^2(x)}\, dx$

$x_i = a + i\Delta x$

$\Delta x = \frac{b-a}{24} = \frac{1}{4}$

$f(x) = \sqrt{1 + e^{2\operatorname{sen}(x)} \cos^2(x)}$

$\sqrt{t} + t \rightarrow dv + t^2 \rightarrow du + 2v$

$\sqrt{t} = v$ $\qquad \frac{1}{2\sqrt{t}} \cdot 2\sqrt{t}$

(B) $i(t) = (60-t)^2 + (60-t)\,\text{sen}(\sqrt{t})$

$i''(t) = -2(60-t) + \frac{(60-x)\cos\sqrt{x}}{2\sqrt{x}} - \text{sen}(\sqrt{x})$

$i''(x) = 2 - \frac{\cos\sqrt{x}}{2\sqrt{x}} + \left[\frac{-\left[\frac{(60-x)\,\text{sen}\sqrt{x}}{2\sqrt{x}} - \cos\sqrt{x}\right] 2\sqrt{x} - \frac{1}{\sqrt{x}}(60-x)\cos\sqrt{x}}{(4x)^2} \right]$

$= 2 - \frac{\cos\sqrt{x}}{2\sqrt{x}} + \left[\frac{-(60-x)\,\text{sen}\sqrt{x} - 2\sqrt{x}\cos\sqrt{x} - (60\sqrt{x} - x\sqrt{x})\cos\sqrt{x}}{2x\sqrt{x}} \right]$

$= 2 + \frac{-3x\cos\sqrt{x} + x\sqrt{x}\,\text{sen}(\sqrt{x}) - 60\sqrt{x}\,\text{sen}(\sqrt{x}) - 60\cos\sqrt{x}}{4x\sqrt{x}}$

$i'''(t) = \frac{x^2\sqrt{x}\cos\sqrt{x} + 3x^2\,\text{sen}\sqrt{x} + 180x\,\text{sen}\sqrt{x} - 57x\sqrt{x}\cos\sqrt{x} + 180\sqrt{x}\cos\sqrt{x}}{8x^3}$

$0 = i'''(t)$

$\Rightarrow t = 12.9753$

$i''(12.9753) = 2,8766834 \rightarrow K$

$E_T \leq \left| \frac{K(b-a)^3}{12n^2} \right|$

$0,01 \leq \frac{K(16-0)^3}{12n^2}$

$n \geq \sqrt{\frac{K(16-0)^3}{12 \cdot 0,01}}$

(9) $\int_{0,3}^{1,1} \left(\frac{1}{3}t^4 \ln(t) + \frac{7}{36}t^4 + \frac{1}{2}t^2\right) dt$

$\int_{0,3}^{1,1} F(t)\,dt = 0,29088$

$F(t) = -\frac{1}{3}t^4 \ln(t) + \frac{7}{36}t^4 + \frac{1}{2}t^2$

$F'(t) = -\frac{4}{3}t^3 \ln t - \frac{t^4}{3}\cdot\frac{1}{t} + \frac{7}{9}t^3 + t$

$F'(t) = -\frac{4}{3}t^3 \ln(t) + \frac{4}{9}t^3 + t$

$F''(t) = -4t^2 \ln(t) - \frac{4}{3}t^2 + \frac{4}{3}t^2 + 1$

$F''(t) = -4t^2 \ln(t) + 1$

$F'''(t) = -8t \ln(t) - 4t$

$F'''(t) = 0$

$0 = -8t(\ln(t)) - 4t$

$t = 0,6065$

$F''(0,6065) = 1,7357 \rightarrow K$

$$n = \sqrt{\frac{K(b-a)^3}{12\,ET}}$$

$$n = \sqrt{\frac{1,7357\,(1,1-0,3)^3}{12\,(2\times10^{-4})}} = 19,24 \approx 20$$

$\Delta x = \frac{1,1-0,3}{20} = 0,04$

Fx0	0,049	Fx10	0,32
Fx1	0,065	Fx11	0,36
Fx2	0,083	Fx12	0,40
Fx3	0,103	Fx13	0,45
Fx4	0,126	Fx14	0,50
Fx5	0,15	Fx15	0,55
Fx6	0,18	Fx16	0,609
Fx7	0,21	Fx17	0,665
Fx8	0,244	Fx18	0,723
Fx9	0,28	Fx19	0,782
		Fx20	0,8431

$\int_{0,3}^{1,1} = \frac{0,04}{2}\left[0,049 + 2(7,09) + 0,8431\right] = 0,2998$

$$M = \int_{t_1}^{t_2} Q(t)\, C(t)\, dt$$

$t_1 = 2$
$t_2 = 8$

$Q(t) = 9 + 4\cos^2(0{,}4t)$

$t_{transporte} = 6$ min

$C(t) = 5e^{-0{,}5t} + 2e^{0{,}5t}$

$n = \frac{t_{transp}}{2} \quad 6 = 18$

$\Delta x = \frac{b-a}{n} = \frac{6}{18} = \frac{1}{3}$