

① I. $\frac{dy}{dx} = -\frac{(3x^2 + 2xy)}{(x^2 + \cos y)} = -\frac{3x^2 + 2xy}{x^2 + \cos y}$

② a) $\frac{dA(t)}{dt} = \left(\frac{12}{L}\right) \left(\frac{4L}{\text{min}}\right) - \left(\frac{A(t)}{200 + (-2)t}\right) \left(\frac{2L}{\text{min}}\right)$
 $\frac{dA(t)}{dt} = 4 - \frac{A(t)}{100+t} \Rightarrow A'(t)(100+t) = 400 + 4t - A(t)$

b) $A(t) = C_1(t+100) + 4(t+100) \ln(t+100) \Rightarrow C_1 = -18,38221419$
 $A(5) = 24,5297$

③ $\frac{dv}{dt} = 1,2 - \frac{v(t)}{4} \Rightarrow \frac{dv}{dt} + \frac{1}{4}v(t) = 1,2 \Rightarrow v'(t) + \frac{1}{4}v(t) = 1,2$

$$v = 4,8 + C e^{-t/4}$$

$$v(0) = 0,2\% \cdot 8000 \text{ ft}^3 = \frac{0,2}{100} \cdot 8000 = 16 \text{ ft}^3$$

$$v(0) = 16$$

$$16 = 4,8 + C \Rightarrow C = 11,2$$

$$v(t) = 4,8 + 11,2 e^{-t/4}$$

$$v(2) = 11,59$$

$$6) f''(x) = -\frac{5x}{2x^2-1}$$

$$f'''(x) = \frac{5(2x^2-1)}{(2x^2+1)^2} \Rightarrow f'''(x) = 0 \quad \frac{5(2x^2-1)}{(2x^2+1)^2} = 0$$

$$2x^2 - 1 = 0$$

$$x = |\sqrt{1/2}| = 0,70710$$

$$10^{-2} \leq \left| \frac{12,125}{12n^2} \right| \Rightarrow n^2 \leq \frac{12,125}{12 \cdot 10^{-2}} \Rightarrow n \leq 10,052$$

$$n = 11$$

$$\Delta x = \frac{b-a}{n} \approx 0,1727 \quad f''(\sqrt{1/2}) = |-1,7678| \Rightarrow K = 1,7678$$

$$\int f'(x) = -\frac{5}{4}(x \ln(2x^2+1) - \sqrt{2}(-\tan^{-1}(\sqrt{2}x) + \sqrt{2}x)) + C$$

$$7) n = 6(4) = 24$$

$$f(x) = e^{\sin(x)} \quad f'(x) = -\cos x e^{\sin(x)}$$

$$g(x) = \sqrt{1+y'} = \sqrt{1 - \cos x} e^{\sin(x)}$$

$$8) a) f'''(x_0) = \frac{f(x_0-3h) - 8f(x_0-2h) + 13f(x_0-h) - 13f(x_0+h) + 8f(x_0+2h) - f(x_0+3h)}{8h^3}$$

$$= f(x_0+3h) \quad \text{Excel}$$

$$b) f''(x) = \frac{-3x \cos(\sqrt{x}) + x\sqrt{x} \sin(\sqrt{x}) - 60\sqrt{x} \sin \sqrt{x} - 60 \cos \sqrt{x}}{4x\sqrt{x}} + 2$$

$$f''(12,4251787380887) = 0,873 \Rightarrow K = 0,873$$

Excel

$$9) f'''(t) = -4(2t \ln(t) + t) = 0$$

$$t = 0 \quad 2 \ln(t) + 1 \Rightarrow t = 1/\sqrt{e}$$

$$f''(t) = -4t^2 \ln(t) + 1 \Rightarrow f''(t) = 1,7358 \Rightarrow K = 1,7358$$

$$\epsilon_T = 2 \cdot 10^{-4} \Rightarrow n^2 \leq \left| \frac{(b-a)^3 K}{12 \epsilon_T} \right| \Rightarrow n \leq 19,24 \Rightarrow n = 20$$