I) $(3x^2+2xy)\,dx + (x^2+\cos y)\,dy = 0 \Rightarrow M\,dx + N\,dy = 0$

$M = 3x^2+2xy$ $\quad N = x^2+\cos y$

$\frac{\partial M}{\partial y} = 2x$ $\quad \frac{\partial N}{\partial x} = 2x$ $\quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ECD exacta

$F(x,y) = C$ $\quad M = \frac{\partial F}{\partial x} = F_x \rightarrow \frac{\partial M}{\partial y} = \frac{\partial F_x}{\partial y} = F_{xy}$

$dF = 0$ $\quad N = \frac{\partial F}{\partial y} = F_y$ $\quad \frac{\partial F}{\partial x} = \frac{\partial F_y}{\partial x} = F_{yx}$ Iguales

$\underbrace{\frac{\partial F}{\partial x}}_{M}\,dx + \underbrace{\frac{\partial F}{\partial y}}_{N}\,dy = 0$

$\frac{\partial F}{\partial x} = 3x^2+2xy$

$\int \partial F = \int (3x^2+2xy)\,\partial x$

$F = \frac{\cancel{6}x^3}{\cancel{3}} + \frac{\cancel{2}x^2y}{\cancel{2}} + C(y)$

$F = 2x^3 + x^2y + C(y)$ $\quad \frac{\partial F}{\partial y} = 0 + x^2 + C'(y)$

$\frac{\partial F}{\partial y} = x^2 + C'(y) = x^2 + \cos y$

$\cancel{x^2} + C'(y) = \cancel{x^2} + \cos(y)$ $\quad F(x,y) = C$

Particular

$C'(y) = \cos(y)$ $\quad 2x^3 + x^2y + \text{Sen}(y) + C = C_0$ $\quad y(0) = \pi/2$ (x, y)

$C(y) = \int \cos(y)\,dy$ $\quad 2x^3 + x^2y + \text{Sen}(y) = K$

$C(y) = \text{Sen}(y) + C$ $\quad 2(0^3) + (0^2)(\pi/2) + \text{Sen}(\pi/2) = K$

$K = 1$

$f(x,y) = 2x^3 + x^2y + \text{Sen}(y) = 1$ Sol general $\quad x = [0,3]$ $\quad h = \frac{x_f - x_0}{n}$

$2x^3 + x^2y = 1 - \text{Sen}(y)$ $\quad y(0) = \pi/2$ $\quad n = \frac{x_f - x_0}{h}$

$x^2(2x+y) = 1 - \text{Sen}(y)$

h	n
0.1	30
0.05	60
0.01	300

50

$\frac{dy}{dx} = x^2 + y^2$ $\quad \frac{dy}{dx} = x^2 + y^2$ $\quad \frac{dy}{dx} - x^2 = y^2$

$dy = (x^2 + y^2)\,dx$ $\quad \frac{dx}{dy}\,(+x^2)\frac{dy}{dx}$ $\quad dy + (x^2)dx = dy\,y$

$dy + (-x^2)\,dx = (y^2)dx$

$\frac{dy}{dx} = x^2 + y^2$ $\quad y(0) = 0$ $\quad h = 0{,}1 \,,\, 0{,}05 - 0{,}01$

$y(1) = 0$

$n = \frac{1-0}{0{,}1} = 10$ $\quad n = \frac{1-0}{0{,}05} = 20$ $\quad n = \frac{1-0}{0{,}01} = 100$

2) Un tanque contiene 200L de agua que se han disuelto 30g de sal y le entran 4L/min de solución con 1g de sal por litro, está bien mezclado y de él liquido con un flujo de 2L/min. Calcule la cantidad $A(t)$ de gramos de sal que hay en el tanque en cualquier momento t.

b) $A(t)$ = Cantidad de sal $\quad$ entrada $= \frac{4L}{min} \rightarrow \frac{1g}{L}$ $\quad t$ = tiempo

$V(t)$ = Volumen del fluido

Salida $= \frac{2L}{min}$ $\quad A = g/min$

$A(0) = 30g$

$V(0) = 200L$

$\frac{\Delta A}{\Delta t}$ = entrada − salida $\quad \frac{dA(t)}{dt} = 4 - \frac{A(t)}{100+t}$

Cantidad de sal en la entrada en un tiempo t $= \frac{4\cancel{L}}{min} \cdot \frac{1g}{\cancel{L}} = \frac{4g}{min}$

Cantidad de sal en la salida en un tiempo t $= \left(\frac{2L}{min}\right)\left(\frac{A(t)}{200L + (4L/min - 2L/min)t}\right) = \frac{2A(t)}{200 + (2t)} = \frac{A(t)}{100+t}$

$$\frac{dA(t)}{dt} = 4 - \frac{A(t)}{100+t}$$ es de la forma $\frac{dy}{dx}$ donde $A(t) = y$ y $x = t$

$$\frac{dy}{dx} = 4 - \frac{y}{100+x}$$

$A(2) = 12g \to y(2) = 12g \to$

x	y
2	12

$A(5) = ?$

$$\frac{dy}{dx} = \frac{400+4x-y}{100+x}$$

$$(100+x)\,dy = (4x-y+400)\,dx$$

$$\frac{dy}{dx} + \frac{1}{100+x}y = 4 \to y' + \frac{1}{100+x}y = 4$$

$Q(x) = 4$

$P(x) = \frac{1}{100+x}$

$$y'(x) + P(x)\cdot y = Q(x)$$

$$y\mu = \int q\mu\,dx \qquad \mu = e^{\int P(x)dx} = e^{\int \frac{1}{100+x}dx} = e^{\int \frac{1}{u}du} = e^{\ln u} = u$$

$\mu = 100+x$

$u = 100+x$
$du = dx$

$$y(100+x) = \int 4(100+x)\,dx \to y(100+x) = 4\int 100+x\,dx$$

$$\to y = \frac{4(100x + x^2(\tfrac{1}{2}) + C)}{100+x} = \frac{400x + 2x^2 + 4C}{100+x} = \frac{400x + 2x^2 + K}{100+x}$$

$$A(t) = \frac{400t + 2t^2 + K}{100+t} \qquad A(2) = 12g$$

$$12 = \frac{400(2) + 2(2)^2 + K}{100+2}$$

$$1224 = 800 + 8 + K \qquad \therefore \quad A(t) = 2t^2 + 2$$

$$K = 416$$

$$A(t) = \frac{2t(200+t) + 416}{t+100}$$

$$A(5) = \frac{2(5)(200+5) + 416}{5+100} = 23{,}48571429\ g$$

(3) Air Containing 0,06% Carbon dioxide is pumped into a room whose Volumen is 8000 ft^3. the air is pumped in at rate 2000 ft^3/min, and the circulated air is pumped out at the same rate. If there is an initial concentraition of 0.2 carbon dioxide in the room, determine the subsequent amount in the room at time t. what is the concentration of carbon dioxide at 2 minutes?

$$C(0) = 0{,}2$$

(1)

Entrada $\frac{0{,}06\% \; CO_2}{100}$ $2000 \; ft^3/min$

Salida $2000 \; ft^3/min$

Entrada $120 \frac{ft^3}{min} \; CO_2$

$$\text{Salida} = 2000 \; ft^3/min \left(\frac{C(t)}{8000 + (2000 - 2000)t} \right)$$

$$\frac{dv}{dt} = R_{in} - R_{out} \longrightarrow \frac{v(t)}{4} = \frac{v(t)}{8000} \, 2000 \frac{ft^3}{min} =$$

$$\downarrow$$

$$(0{,}06\%)\left(\frac{2000\,ft^3}{min}\right)$$

$$\frac{dv}{dt} = 1{,}2 - \frac{v(t)}{4} \quad \therefore \quad \frac{dv}{dt} + \frac{1}{4}v(t) = 1{,}2$$

$$V = \frac{\int \mu\, Q(t)}{\mu} = \frac{\int e^{t/4}(1{,}2)\,dt}{e^{t/4}}$$

$$\mu = e^{\int \frac{1}{4}\,dt} \qquad = \frac{4{,}8e^{t/4} + K}{e^{t/4}}$$

$$\mu = e^{t/4}$$

$$V = 4{,}8 + Ke^{-t/4}$$

$$V(0) = 0{,}2\% \cdot 8000ft^3 = 16ft^3$$

$$16 = 4{,}8 + Ke^{0}$$

$$16 = 4{,}8 + K$$

$$11{,}2 = K \qquad \therefore \quad v(t) = 4{,}8 + 11{,}2e^{-t/4}$$

$$v(2) = 4{,}8 + 11{,}2\,e^{-2/4}$$

$$v(2) = 11{,}59314339$$

5) La ecuación diferencial que representa la caida de voltaje en un circuito LR está dado por

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}=E(t) \qquad I=\frac{dq}{dt}$$

$$20\frac{d^2q}{dt^2}+2\frac{dq}{dt}=120 \quad \therefore \quad 20m^2+2m=0$$

$m^2+1m=0$

$m=0$

$m(10m+1)=0$

$m=-1/10$

$y_h=c_1e^{0t}+c_2e^{-t/10}$

$y_h=c_1+c_2e^{-t/10}$

$120 \therefore y_p=At$

$y_p'=A$

$y''=0$

$0+2A=120$

$A=60$

$y_p=60t$

$q(0)=1$

$q(t)=c_2e^{-t/10}+60t+c_1$

$1=c_2+c_1$

$I(0)=0$

$I(t)=q'(t)=c_2e^{-t/10}\left(-\frac{1}{10}\right)+60$

$I(0)=0=-\frac{c_2e^0}{10}+60$

$0=-\frac{c_2}{10}+60$

$\frac{c_2}{10}=60$

$c_2=600 \quad \therefore \quad 1=600+c_1$

$c_1=-599$

$q(t)=600e^{-t/10}+60t-599$

$I(t)=600e^{-t/10}\left(-\frac{1}{10}\right)+60$

$I(t)=-60e^{-t/10}+60$

$q(2)=12,23845185$

$I(2)=10,87615482$

$I(2)=11,23278724 \rightarrow$ excel

$$E=\left|\frac{10,87615482-11,23278724}{10,87615482}\right|$$

$E=0,03279030418$ absoluto

$E=3,27903\%$

$\cancel{q'(2)}=12,2$

$q'(2)=10,87615482$

$q'(2)=10,87615545$ excel

$$E=\left|\frac{10,87615482-10,87615545}{10,87615482}\right|$$

$E=5,792488\times10^{-8}$

$E=5,792488\times10^{-6}\%$

6) Determine el numero de particiones n, por el metodo de trapecios

$$\int_{0.1}^{2} f(x)\,dx$$

Si se sabe que $f'(x) = -\frac{5}{4}\ln(2x^2+1)$ y se quiere aproximar con un error menor a 10^{-2}, Hallar el valor de la integral haciendo uso del número de particiones hallado

$$f'(x) = -\frac{5}{4}\ln(2x^2+1)$$

$$f(x) = \int -\frac{5}{4}\ln(2x^2+1)\,dx = -\frac{5}{4}\int \ln(2x^2+1)\,dx$$

$$U = 2x^2+1$$
$$du = 4x\,dx$$

$$\cancel{|f''(x)| < \left|\frac{K(b-a)^3}{12n^2}\right|}$$

$$|f''(x)| < M$$

$$ET \leq \frac{(b-a)^3}{12n^2}M$$

$$f''(x) = -\frac{5}{4}\left(\frac{1}{2x^2+1}\cdot 4x\right)$$

$$f''(x) = \frac{-5x}{2x^2+1} \leq M$$

$$f'''(x) = \frac{(-5)(2x^2+1)-(-5x)(4x)}{(2x^2+1)^2}$$

$$= \frac{-10x^2-5+20x^2}{(2x^2+1)^2}$$

$$= \frac{10x^2-5}{(2x^2+1)^2}$$

$$= \frac{5(2x^2-1)}{(2x^2+1)^2} = 0$$

$$= 5(2x^2-1) = 0$$

$$2x^2-1 = 0$$

$$x^2 = 1/2$$

$$x = \pm\sqrt{1/2} \rightarrow \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$|x| = \frac{\sqrt{2}}{2}$$

$$f''\left(\frac{\sqrt{2}}{2}\right) = \frac{-5\left(\frac{\sqrt{2}}{2}\right)}{2\left(\frac{\sqrt{2}}{2}\right)^2+1} \leq M = \frac{5\sqrt{2}}{4} = 1,767$$

$$= \frac{-\frac{5\sqrt{2}}{2}}{\frac{2}{1}} \qquad \frac{-5\left(-\frac{\sqrt{2}}{2}\right)}{2\left(-\frac{\sqrt{2}}{2}\right)+1}$$

$$= -\frac{5\sqrt{2}}{4} \qquad = \frac{5\sqrt{2}}{4}$$

$$10^{-2} \leq \left|\frac{(2-0{,}1)^3}{12n^2} \cdot \frac{5\sqrt{2}}{4}\right|$$

$$10^{-2} \leq \frac{(6{,}859)\,5\sqrt{2}}{48n^2}$$

$$n^2 \leq \frac{5\sqrt{2}\,(6{,}859)}{(48)(10^{-2})}$$

$$n \leq \sqrt{\frac{5\sqrt{2}\,(6{,}859)}{(48)(10^{-2})}}$$

$$n \leq 10{,}052 \to n = 11$$

$$-\frac{5}{4}\int_{0,1}^{2} \ln(2x^2+1)\,dx$$

$u = \ln(2x^2+1)$ $\quad v = x$

$du = \frac{4x}{2x^2+1}\,dx$ $\quad dv = 1$

$$-\frac{5}{4}\left(x\ln(2x^2+1) - 4\int_{0,1}^{2} \frac{x^2}{2x^2+1}\,dx\right)$$

$$4\int_{0,1}^{2} \frac{x^2}{2x^2+1}\,dx = \cancel{4}\int_{0,1}^{2} \frac{u^2}{2\sqrt{2}(u^2+1)}\,du$$

$\cancel{u = x}$ $\quad x = \frac{1}{\sqrt{2}}u$

$\cancel{du = dx}$

$$= \frac{2}{\sqrt{2}}\int_{0,1}^{2} \frac{u^2}{u^2+1}\,du = \frac{2}{\sqrt{2}}\left(\int_{0,1}^{2} -\frac{1}{u^2+1}\,du + \int_{0,1}^{2} du\right) = \frac{2}{\sqrt{2}}\left(-\arctan(u) + u\right)$$

$$\frac{u^2}{u^2+1} = \frac{u^2-(u^2+1)}{u^2+1} + 1 = -\frac{1}{u^2+1} + 1$$

$$= \frac{2}{\sqrt{2}}\left(-\tan^{-1}(\sqrt{2}x) + \sqrt{2}x\right) = +\sqrt{2}\left(-\tan^{-1}(\sqrt{2}x) + \sqrt{2}x\right)$$

$$= -\frac{5}{4}\left(x\ln(2x^2+1) - \sqrt{2}\left(-\tan^{-1}(\sqrt{2}x) + \sqrt{2}x\right)\right)$$

$$-\frac{5}{4}\int_{0,1}^{2} x\ln(2x^2+1) - \sqrt{2}\left(-\tan^{-1}(\sqrt{2}x) + \sqrt{2}x\right)$$

$n = 11$ $\quad \Delta x = \frac{2-0{,}1}{11}$

$\Delta x = \frac{19}{110}$

$$\frac{\Delta x}{2}\left[f(x_0) + 2\sum_{i=0}^{n-1} f(x_i) + f(x_{n-1})\right]$$

$x_i = a + i\Delta x$

7) Una joven llamada Emma Madera de Gallo que le encanta las caminatas ecologicas contrata un tour para caminar por una montaño en cercanías de la ciudad, ella comienza por el punto A y luego de 6Km llega al punto B, ella se siente agotada de las piernas y se pregunta cuanta longitud ha caminado por el borde de la montaña, para esto acude a su amigo que estudia ingenieria y le dice que se puede calcular haciendo uso de la longitud de curva, ayude a Emma a calcular cuanto ha caminado por el borde de la montaña, haciendo uso de la regla de simpson con un numero de particiones igual a 4 veces la longitud lineal de A hasta B

$$f(x) = e^{sen(x)}$$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$f(x) = e^{sen(x)}$$

$$f'(x) = e^{sen(x)} \cdot cos(x)$$

$$L = \int_0^6 \sqrt{1 + \left(cos(x) e^{sen(x)}\right)^2}\, dx$$

$$L = \int_0^6 \sqrt{1 + cos^2(x) e^{2sen(x)}}\, dx = 7,86370054$$

$$L = \int_0^6 \sqrt{1 + cos(x) e^{sen(x)}}\, dx \simeq 7,86460601$$

$$E = 9,66542401 \times 10^{-5}$$

Simpson

$$\Delta x = \frac{b-a}{n} \qquad x_i = a + i\Delta x$$

$$E_s \leq \left| \frac{K(b-a)^2}{180n^4} \right|$$

$$n = (6-0)4$$

$n = 24$ particiones

$$\Delta x = \frac{6-0}{24} = 0,25$$

8) Si un capacitor se mantiene no cargado, el voltaje alrededor de este se presenta como una funcion dada por:

$$V(t) = \frac{1}{C} \int_0^t i(s)\, ds$$

Se sabe que la corriente está dado por:

$$i(t) = (60-t^2)^2 + (60-t)\, sen\sqrt{t}$$

$$i(t) = (60-t)^2 + (60-t)\,\text{Sen}(\sqrt{t})$$

$$i'(t) = 2(60-t)(-1) + (-1)\,\text{Sen}\sqrt{t} + (60-t)\cos\sqrt{t} \cdot \frac{1}{2\sqrt{t}}$$

$$i'(t) = -120 + 2t - \text{Sen}\sqrt{t} + \frac{(60-t)\cos\sqrt{t}}{2\sqrt{t}}$$

$$i''(t) = 2 - \cos\sqrt{t} \cdot \frac{1}{2\sqrt{t}} + \frac{\left[(-1)\cos\sqrt{t} + (60-t)(-\text{Sen}\sqrt{t}) \cdot \frac{1}{2\sqrt{t}}\right] 2\sqrt{t} - \left[(60-t)\cos\sqrt{t} \cdot 2\left(\frac{1}{2\sqrt{t}}\right)\right]}{(2\sqrt{t})^2}$$

$$i''(t) = 2 - \frac{\cos\sqrt{t}}{2\sqrt{t}} + \frac{\left(-\cos\sqrt{t} - \frac{(60-t)(\text{Sen}\sqrt{t})}{2\sqrt{t}}\right) 2\sqrt{t} - \left(\frac{(60-t)\cos\sqrt{t}}{\sqrt{t}}\right)}{4t}$$

$$i''(t) = 4\sqrt{t} - \cos\sqrt{\ }$$

$$i''(t) = 2 - \frac{\cos\sqrt{t}}{2\sqrt{t}} + \frac{-2\sqrt{t}\cos\sqrt{t} + (t-60)\,\text{Sen}\sqrt{t} - \frac{(60-t)\cos\sqrt{t}}{\sqrt{t}}}{4t}$$

$$i''(t) = 2 - \frac{\cos\sqrt{t}}{2\sqrt{t}} + \frac{\frac{-2t\cos\sqrt{t} + \sqrt{t}(t-60)\,\text{Sen}\sqrt{t} - (60-t)\cos\sqrt{t}}{\sqrt{t}}}{\frac{4t}{1}}$$

$$i''(t) = 2 - \frac{\cos\sqrt{t}}{2\sqrt{t}} + \frac{\sqrt{t}(t-60)\,\text{Sen}\sqrt{t} + (t-60)\cos\sqrt{t} - 2t\cos\sqrt{t}}{4t\sqrt{t}}$$

$$i''(t) = 2 - \frac{2t\cos\sqrt{t} + [\sqrt{t}\,\text{Sen}\sqrt{t} + \cos\sqrt{t}](t-60) - 2t\cos\sqrt{t}}{4t\sqrt{t}}$$

$$i''(t) = \frac{-4t\cos\sqrt{t} + t\sqrt{t}\,\text{Sen}\sqrt{t} + t\cos\sqrt{t} - 60\sqrt{t}\,\text{Sen}\sqrt{t} - 60\cos\sqrt{t}}{4t\sqrt{t}} + 2$$

$$i''(t) = \frac{t\sqrt{t}\,\text{Sen}\sqrt{t} - 60\sqrt{t}\,\text{Sen}\sqrt{t} - 60\cos\sqrt{t} - 3t\cos\sqrt{t}}{4t\sqrt{t}} + 2$$

$$i'''(t) = \frac{t\sqrt{t}\,\text{sen}\sqrt{t} - 60\sqrt{t}\,\text{sen}\sqrt{t} - 60\cos\sqrt{t} - 3t\cos\sqrt{t}}{4t\sqrt{t}} + 2$$

$$i''''(t) = \frac{\left[\sqrt{t}\,\text{sen}\sqrt{t} + t\left(\frac{1}{2\sqrt{t}}\right)\text{sen}\sqrt{t} + t\sqrt{t}\cos\sqrt{t}\left(\frac{1}{2\sqrt{t}}\right) - 60\left(\frac{1}{3\sqrt{t}}\right)\text{sen}\sqrt{t} - 60\sqrt{t}\cos\sqrt{t}\left(\frac{1}{2\sqrt{t}}\right) - 60(-\text{sen}\sqrt{t})\left(\frac{1}{2\sqrt{t}}\right) - 3\cos\sqrt{t} - 3t\left(-\text{sen}\sqrt{t}\cdot\frac{1}{2\sqrt{t}}\right)\right]4t\sqrt{t} - \left(4\sqrt{t} + \frac{4t}{2\sqrt{t}}\right)\left(t\sqrt{t}\,\text{sen}\sqrt{t} - 60\sqrt{t}\,\text{sen}\sqrt{t} - 60\cos\sqrt{t} - 3t\cos\sqrt{t}\right)}{16t^3}$$

$$i''''(t) \frac{\left(4t^2\text{sen}\sqrt{t} + \frac{4t^2\text{sen}\sqrt{t}}{2} + \frac{4t^2\sqrt{t}\cos\sqrt{t}}{2} - \frac{240t\,\text{sen}\sqrt{t}}{2} - \frac{240t\sqrt{t}\cos\sqrt{t}}{2} + \frac{240t\,\text{sen}\sqrt{t}}{2} - 12t\sqrt{t}\cos\sqrt{t} + \frac{12t^2\text{sen}\sqrt{t}}{2}\right) - \left(4t^2\text{sen}\sqrt{t} - 240t\,\text{sen}\sqrt{t} - 240\sqrt{t}\cos\sqrt{t} - 12t\sqrt{t}\cos\sqrt{t} + \frac{4t^2\text{sen}\sqrt{t}}{2} - \frac{240t\,\text{sen}\sqrt{t}}{2} - \frac{240t\cos\sqrt{t}}{2\sqrt{t}} - \frac{12t^2\cos\sqrt{t}}{2\sqrt{t}}\right)}{16t^3}$$

6 −2
−120 −12 +12

$$i''''(t) = \frac{\left(4t^2\text{sen}\sqrt{t} + 2t^2\text{sen}\sqrt{t} + 2t^2\sqrt{t}\cos\sqrt{t} - 120t\,\text{sen}\sqrt{t} - 120t\sqrt{t}\cos\sqrt{t} + 120t\,\text{sen}\sqrt{t} - 12t\sqrt{t}\cos\sqrt{t} + 6t^2\text{sen}\sqrt{t}\right) - 4t^2\text{sen}\sqrt{t} + 240t\,\text{sen}\sqrt{t} + 240\sqrt{t}\cos\sqrt{t} + 12t\sqrt{t}\cos\sqrt{t} - 2t^2\text{sen}\sqrt{t} + 120t\,\text{sen}\sqrt{t} + \frac{120t\cos\sqrt{t}}{\sqrt{t}} + \frac{6t^2\cos\sqrt{t}}{\sqrt{t}}}{16t^3}$$

$$i''''(t) = \frac{6t^2\text{sen}\sqrt{t} + 2t^2\sqrt{t}\cos\sqrt{t} + 360t\,\text{sen}\sqrt{t} - 120t\sqrt{t}\cos\sqrt{t} + 240\sqrt{t}\cos\sqrt{t} + \frac{120\sqrt{t}\,t\cos\sqrt{t}}{t} + \frac{6t^2\sqrt{t}\cos\sqrt{t}}{t}}{16t^3}$$

$$i'''' = \frac{\frac{6t^3\text{sen}\sqrt{t} + 2t^3\sqrt{t}\cos\sqrt{t} + 360t^2\text{sen}\sqrt{t} - 120t^2\sqrt{t}\cos\sqrt{t} + 240t\sqrt{t}\cos\sqrt{t} + 120\sqrt{t}\,t\cos\sqrt{t} + 6t^2\sqrt{t}\cos\sqrt{t}}{t}}{16t^3}$$

$$i'''' = \frac{6t^3\text{sen}\sqrt{t} + 2t^3\sqrt{t}\cos\sqrt{t} + 360t^2\text{sen}\sqrt{t} - 120t^2\sqrt{t}\cos\sqrt{t} + 240t\sqrt{t}\cos\sqrt{t} + 120t\sqrt{t}\cos\sqrt{t} + 6t^2\sqrt{t}\cos\sqrt{t}}{16t^4}$$

$$i'''' = \frac{\overset{3}{\cancel{6}}t^2\text{sen}\sqrt{t} + \overset{1}{\cancel{2}}t^2\sqrt{t}\cos\sqrt{t} + \overset{180}{\cancel{360}}t\,\text{sen}\sqrt{t} - \overset{60}{\cancel{120}}t\sqrt{t}\cos\sqrt{t} + \overset{120}{\cancel{240}}\sqrt{t}\cos\sqrt{t} + \overset{60}{\cancel{120}}\sqrt{t}\cos\sqrt{t} + \overset{3}{\cancel{6}}t\sqrt{t}\cos\sqrt{t}}{\overset{8}{\cancel{16}}t^3}$$

$$i'''' = \frac{3t^2\text{sen}\sqrt{t} + 180t\,\text{sen}\sqrt{t} + t^2\sqrt{t}\cos\sqrt{t} - 57t\sqrt{t}\cos\sqrt{t} + 180\sqrt{t}\cos\sqrt{t}}{8t^3} \quad [12{,}5 \quad 13{,}5]$$

Regla falso

$i'''(0) = 12{,}9753098839825$

b) $t = 16$ $\quad b = 16$, $a = 0$ $\qquad E_r \leq \left| \frac{(b-a)^3 M}{12n^2} \right|$ $\quad f''(x) \leq M$, $i''(t) \leq M$

$$i''(t) = \frac{t\sqrt{t}\,\mathrm{Sen}\sqrt{t} - 60\sqrt{t}\,\mathrm{Sen}\sqrt{t} - 60\cos\sqrt{t} - 3t\cos\sqrt{t}}{4t\sqrt{t}} + 2$$

$$i''(i'''(0)) = 2{,}876683400376 35 = M$$

$$10^{-2} \leq \frac{16^3 M}{12n^2}$$

$$n \leq \sqrt{\frac{(16)^3 M}{(12)(10^{-2})}} \qquad n \leq 313{,}354102$$

$n = 314$

4) Haga uso del método de trapecios para determinar el número n de particiones necesarias para el cálculo de la siguiente integral con aproximación menor a 2×10^{-4}

$$\int_{0.3}^{1.1}\left(-\frac{1}{3}t^4\ln(t)+\frac{7}{36}t^4+\frac{1}{2}t^2\right)dt$$

$$n\le\sqrt{\frac{(1.1-0.3)^3M}{(12)(10^{-4})}}$$

$$|f''(x)|\le M$$

$$f(t)=-\frac{1}{3}t^4\ln(t)+\frac{7}{36}t^4+\frac{1}{2}t^2$$

$$f'(t)=-\frac{1}{3}(4t^3)\ln(t)+\left(-\frac{1}{3}t^4\left(\frac{1}{t}\right)\right)+\left(\frac{7}{36}\right)(4t^3)+\frac{1}{2}(2t)$$

$$f'(t)=-\frac{4}{3}t^3\ln(t)-\frac{t^3}{3}+\frac{28}{36}t^3+t$$

$$f'(t)=\frac{-4t^3\ln(t)}{3}-\frac{1}{3}t^3+\frac{7}{9}t^3+t$$

$$f'(t)=\frac{-12t^3\ln(t)-3t^3+7t^3+9t}{9}$$

$$f'(t)=\frac{-12t^3\ln(t)+4t^3+9t}{9}$$

$$f''(t)=\frac{1}{9}\left(-12(3t^2)\ln(t)+(-12)(t^3)\left(\frac{1}{t}\right)+4(3t^2)+9\right)$$

$$f''(t)=\frac{1}{9}\left(-36t^2\ln(t)-\cancel{12t^2}+\cancel{12t^2}+9\right)$$

$$\boxed{f''(t)=-4t^2\ln(t)+1}$$

$$f'''(t)=-4(2t)\ln(t)+(-4)(t^2)\left(\frac{1}{t}\right)+0$$

$$f'''(t)=-8t\ln(t)-4t$$

$$f'''(t)=-4(2t\ln(t)+t)$$

$$0=-4(2t\ln(t)+t)$$

$$0=2t\ln(t)+t$$

$$2\cancel{t}\ln(t)=-\cancel{t}$$

$$\ln(t)=-1/2$$

$$e^{\ln(t)}=e^{-1/2}$$

$$t=e^{-1/2}$$

$$f''(e^{-1/2})=-4(e^{-1/2})^2\ln(e^{-1/2})+1$$

$$\therefore f''(e^{-1/2})=+4(e^{-1})(+1/2)+1$$

$$f''(e^{-1/2})=\frac{2}{e}+1=M$$

$$n\le\sqrt{\frac{(0,8)^3\left(\frac{2}{e}+1\right)}{(12)(10^{-4})}}$$

$$n\le 27,21=28$$

$$\Delta x=\frac{0,8}{28}=\frac{1}{35}$$

Integral = 0,29061

Integral = 0,290961998

valor real = 0,297761 7933

ERROR = 0,028

10) La Cantidad de masa transportada por una tubería sobre un periodo de tiempo puede ser calculada por

$$M = \int_{t_1}^{t_2} Q(t)\,c(t)\,dt$$

M = masa (mg) t_1 = tiempo inicial t_2 = tiempo final $Q(t)$ = tasa de flujo $\frac{m^3}{min}$ y $c(t)$ = concentración $\frac{mg}{m^3}$

la representación temporal de las variaciones de flujo y concentración están dadas por

$Q(t) = 9 + 4\cos^2(0{,}4t)$ $t_1 = 2\,min$ $t_{prom} = 5\,min$

$c(t) = 5e^{-0{,}5t} + 2e^{0{,}15t}$ $t_2 = 8\,min$

$n = 30$

$\Delta x = \frac{8-2}{30} = \frac{1}{5}$ $M = 322{,}3483453731\,89$ mg