Taller: Integrales y Ecuaciones Diferenciales Ordinarias

1.

I. $(3x^2+2xy)\,dx + (x^2+\cos y)\,dy = 0; \quad y(0) = \frac{\pi}{2}$

$(x^2+\cos y)\,dy = -(3x^2+2xy)\,dx$

$$\frac{dy}{dx} = \frac{-(3x^2+2xy)}{(x^2+\cos y)}$$

2.

$$\frac{dA(t)}{dt} = S_e - S_s \; ; \qquad A(0) = 30g$$

$$S_e = 4L/_{min} \cdot 1g/_L = 4g/_{min}$$

$$S_s = 2L/_{min} \cdot \frac{A(t)}{(4-2)t + 200} = \frac{A(t)}{t+100} \, g/_{min}$$

$$\frac{dA(t)}{dt} = 4 - \frac{A(t)}{t+100}$$

$$\frac{dA(t)}{dt} + \frac{A(t)}{t+100} = 4 \; ; \qquad \mu(t) = e^{\int \frac{dt}{t+100}} = t+100 \longrightarrow \text{Factor Integrante}$$

$$\underbrace{(t+100)\frac{dA(t)}{dt} + A(t)}\; = 4(t+100)$$

$$\frac{d(A(t)(t+100))}{dt} = 4(t+100) \; ; \qquad \frac{d(A(t)(t+100))}{dt} = (t+100)\frac{dA(t)}{dt} + A(t)$$

$$\int d(A(t)(t+100)) = \int 4(t+100)\,dt$$

$$A(t)(t+100) = 2t^2 + 400t + C$$

$$A(t) = \frac{2t^2 + 400t + C}{t+100}$$

Para $t=0$:

$$30 = \frac{C}{100}$$

$$C = 300$$

$$\boxed{A(t) = \frac{2t^2 + 400t + 300}{t+100}} \; ⓐ$$

Para $t_b = 2$:

$$12 = \frac{8 + 800 + C}{102}$$

$$12(102) - 8 - 800 = C$$

$$C = 416$$

$$\boxed{A(t_b) = \frac{2t_b^2 + 400t_b + 416}{t_b + 100}} \; ⓑ$$

Para $t_b = 5$:

$$A(5) = \frac{2(25) + 400(5) + 416}{5+100}$$

$$\boxed{A(5) = 23.48571429\,g} \; ⓑ$$

3.

$\frac{dv}{dt} = A_e - A_s$; $v(0) = \left(\frac{0.2\%}{100\%}\right) \cdot 8000 ft^3 = 16 ft^3$

$A_e = \frac{0.06\%}{100}\left(2000 ft^3/min\right) = 1.2 ft^3/min$

$A_s = \frac{v(t)}{8000}\left(2000 ft^3/min\right) = \frac{v(t)}{4} ft^3/min$

$\frac{dv}{dt} = 1.2 - \frac{v(t)}{4}$

$\frac{dv}{dt} + \frac{v(t)}{4} = 1.2$; $\mu(t) = e^{\int \frac{dt}{4}} = e^{t/4} \longrightarrow$ Factor Integrante

$e^{t/4}\frac{dv}{dt} + \frac{e^{t/4}}{4} v(t) = 1.2 e^{t/4}$

$\frac{d(v(e^{t/4}))}{dt} = 1.2 e^{t/4}$

$\int d(v(e^{t/4})) = \int 1.2 e^{t/4} dt$

$v(e^{t/4}) = 4.8 e^{t/4} + C$

$v(t) = 4.8 + \frac{C}{e^{t/4}}$

Para $t = 0$:

$16 = 4.8 + \frac{C}{1}$

$C = 11.2$

$\boxed{v(t) = 4.8 + \frac{11.2}{e^{t/4}}}$

Para $t = 2$:

$v(2) = 4.8 + \frac{11.2}{e^{1/2}}$

$v(2) = 11.59314339 ft^3$

$\boxed{\% CO_2 = \frac{v(2)}{8000} \cdot 100\% = 0.144914292\%}$

5.

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} = E(t) \qquad I = \frac{dq}{dt} \qquad \frac{dI}{dt} = \frac{d^2q}{dt^2}$$

20 H
2 Ω
120 V
$q(0) = 1C$
$I(0) = 0A$

$$20\frac{d^2q}{dt^2} + 2\frac{dq}{dt} = 120$$

$$20\frac{di}{dt} + 2i = 120 \Rightarrow \frac{di}{dt} + \frac{1}{10}i = 6$$

$$P(t) = \frac{1}{10}$$

$$u(t) = e^{\int \frac{1}{10} dt} = e^{1/10t}$$

$$e^{1/10t}\frac{di}{dt} + \frac{1}{10}i\,e^{1/10t} = 6e^{1/10t}$$

$$\frac{d}{dt}\left(e^{1/10t}i\right) = 6e^{1/10t} \Rightarrow \int \frac{d}{dt}\left(e^{1/10t}i\right)dt = 6\int e^{1/10t}\,dt$$

$$\Rightarrow e^{1/10t}i = 60e^{1/10t} + C$$

$$i(t) = \frac{60e^{1/10t} + C}{e^{1/10t}} = 60 + Ce^{-1/10t}$$

$$i(0) = 60 + Ce^{-1/10(0)} \Rightarrow 60 + C = 0 \Rightarrow C = -60$$

$$I(t) = 60 - 60e^{-1/10t} \quad ; \quad I(2) = 60 - 60e^{-1/10(2)} = 10.87615162\ A$$

$$\int i(t)dt = q(t) \Rightarrow \int 60 - 60e^{-1/10t}dt = q(t) \Rightarrow \int 60(1 - e^{-1/10t})dt = q(t)$$

$$\Rightarrow 60\int 1 - e^{-1/10t}\,dt = 60\left(t + 10e^{-t/10}\right) \Rightarrow 60t + 600e^{-t/10} + C = q(t)$$

$$q(0) = 60(1) + 600e^{-1/10} + C \Rightarrow 60 + 600e^{-1/10} + C = 1$$

$$C = 1 - 60 - 600e^{-1/10} = -601.902 \Rightarrow q(2) = 60(2) + 600e^{-(2)/10} - 601.902$$

$$q(2) = 9.336451847\ C$$

6) $\int_{0,1}^{2} f(x)\,dx$

$$f(x) = -\frac{5}{4}\left(x(\ln(2x^2+1)-2) + \sqrt{2}\tan^{-1}(\sqrt{2}x)\right)$$

$$f'(x) = -\frac{5}{4}\left(\ln(2x^2+1)\right)$$

$$f''(x) = \frac{-5x}{2x^2+1} = \frac{-5(0,707)}{2(0,707)^2+1} = -1,767766953$$

$$f'''(x) = \frac{5(2x^2-1)}{(2x^2+1)^2}$$

$(2x^2+1) \neq 0$ $\qquad$ $2x^2 = 1$ $\qquad$ $x = \sqrt{1/2}$

$2x^2-1 = 0$ $\qquad$ $x^2 = 1/2$ $\qquad$ $x = 0,707$

$$E_r \leq \left| \frac{K(b-a)^3}{12n^2} \right| \Rightarrow 10^{-3} \leq \frac{(1,767766953)(1,9)^3}{12n^2}$$

$$n \leq \sqrt{\frac{(1,767766953)(6,859)}{12\times 10^{-3}}}$$

$n \leq 31,787$

$n \simeq 32$

7) $A = 0$

$B = 6$

$$f(x) = e^{\sin(x)}$$

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$f'(x) = e^{\sin(x)} \cos(x)$$

$$L = \int_0^6 \sqrt{1 + (e^{\sin(x)} \cos(x))^2}\, dx$$

$$L \approx 7{,}8637 \approx 8$$

$$n = 24 = 6 \cdot 4$$

$$f''(x) = -e^{\sin(x)}(\sin(x) - \cos^2(x))$$

$$f'''(x) = -e^{\sin(x)} \cos(x)(3\sin(x) - \cos^2(x) + 1)$$

$$= -e^{\sin(x)} \cos(x)(3\sin(x) + \sin^2 x)$$

$$f^{(4)}(x) = e^{\sin(x)}(3\sin^2 x + (1 - 6\cos^2(x))\sin(x) + \cos^4(x) - 4\cos^2(x))$$

$$f^{(5)}(x) = e^{\sin(x)} \cos(x)(15\sin^2(x) + (15 - 10\cos^2(x))\sin(x) + \cos^4(x) - 10\cos^2(x) + 1)$$

$$x = 5{,}0791068$$

$$f^{(4)}(10,995576) = 0,748272$$

$$E_s \leq \left| \frac{(0,748272)\cdot(6-0)^5}{180\,(24)^4} \right|$$

$$E_s \leq 97,4 \times 10^{-6}$$

8) $V_{(t)} = 1/c \int_0^t i(t)\,dt$

$$V_{(t)} = \int_0^t i(t)\,dt$$

$$i(t) = (60-t)^2 + (60-t)\sin\sqrt{t}$$

$$i'(t) = -2(60-x) + \frac{\cos\sqrt{x}\,(60-x)}{2\sqrt{x}} - \sin\sqrt{x}$$

$$i''(t) = \frac{-\cos\sqrt{x}}{\sqrt{x}} - \frac{\sin\sqrt{x}\,(60-x)}{4x} - \frac{\cos\sqrt{x}\,(60-x)}{4x^{3/2}} + 2$$

$$i'''(t) = \frac{\cos\sqrt{x}\,x^4 + 3\sin\sqrt{x}\,x^{7/2} - 57\cos\sqrt{x}\,x^3 + 180\sin\sqrt{x}\,x^{5/2} + 180\cos\sqrt{x}\,x^2}{8x^{9/2}}$$

Metodo de Newton-Raphson Modificado

$$f_2'(x) = \frac{\cos(\sqrt{x})x^4 + 3\sin(\sqrt{x})x^{9/2} - 57\cos(\sqrt{x})x^3 + 180\sin(\sqrt{x})x^{5/2} + 180\cos(\sqrt{x})x^2}{8x^{11/2}}$$

$$f_2'''(x) = \frac{-\sin(\sqrt{x})x^{9/2} + 2\cos\sqrt{x}\,x^4 + 515\sin\sqrt{x}\,x^{7/2} + 351\cos\sqrt{x}\,x^3 - 900\sin\sqrt{x}\,x^{5/2} - 900\cos\sqrt{x}\,x^2}{16x^{11/2}}$$

$$f_2''(x) = \frac{-\cos\sqrt{x}\,x^5 - 45\cos\sqrt{x}\,x^4 + 555\sin\sqrt{x}\,x^{9/2} + 2655\cos\sqrt{x}\,x^3 - 6300\sin\sqrt{x}\,x^{5/2} - 6300\cos\sqrt{x}\,x^2}{32x^{13/2}}$$

$t = 72,9753 \quad i''(72,9753) = 2,876 \approx 3 = K$

$$E_T \leq \left| \frac{K(b-a)^3}{12n^2} \right|$$

$$n \leq \sqrt{\frac{K(76-0)^3}{72 \cdot 0,07}}$$

$n = 320$

9. $\displaystyle\int_{0.3}^{1.1}\left(-\frac{1}{3}t^4\ln(t)+\frac{7}{36}t^4+\frac{1}{2}t^2\right)dt$ $\quad |F''(x)| \leq K$

$E_T \leq \left|\dfrac{K(b-a)^3}{12n^2}\right| \qquad F(x) = -\frac{1}{3}t^4\ln(t)+\frac{7}{36}t^4+\frac{1}{2}t^2$

$F'(x) = -\frac{4}{3}t^3\ln(t) - \frac{1}{3}t^4\cdot\frac{1}{t} + \frac{7}{9}t^3 + t$

$= -\frac{4}{3}t^3\ln(t) - \frac{1}{3}t^3 + \frac{7}{9}t^3 + t = -\frac{4}{3}t^3\ln(t) + \frac{4}{9}t^3 + t$

$F''(x) = -4t^2\ln(t) - \frac{4}{3}t^3\frac{1}{t} + \frac{4}{3}t^2 + 1$

$= \underline{-4t^2\ln(t) + 1}$

Para hallar un máximo o mínimo $F'''(x)$

$\Rightarrow F'''(x) = -8t\ln(t) - 4t^2\frac{1}{t} = -8t\ln(t) - 4t$

$= -4t(2\ln(t)+1)$

$-4t(2\ln(t)+1) = 0 \qquad -4t = 0 \quad ó \quad 2\ln(t)+1 = 0$

$t = 0 \qquad e^{\ln(t)} = e^{-\frac{1}{2}}$

Evaluar el valor en $F''(x)$

$t = 0.6065$

$\Rightarrow F''(0.6065) = 1.7357 = K \rightarrow$ cota superior

$n = \sqrt{\dfrac{K(b-a)^3}{12|E_T|}} = \sqrt{\dfrac{1.7357(4/5)^3}{12 \times 2\times10^{-4}}} = 20$ particiones