Tarea 1 Control - Brian Sebastian Caceres Pinzon 1803245

Ejercicio 1

Modelo Matemático.

$$\dot{x} = v$$
$$\dot{v} = \frac{1}{m}[-\frac{cI_L^2}{x} + mg]$$
$$\dot{I}_L = \frac{1}{L}[V(t) - I_L R]$$

Puntos de equilibrio.

$$\ddot{x} = \dot{I}_L = 0$$
$$v(t) - I_L = 0$$
$$-\frac{cI_L^2}{x} + mg = 0$$

Constantes:

$$R = 10\Omega$$
$$c = 2$$
$$m = 0.2kg$$

Puntos de operación.

$$\bar{x} = 0.05m$$
$$I_L = \sqrt{\frac{mg\bar{x}}{c}}$$
$$\bar{I}_L = \sqrt{\frac{9.8 * 0.2 * 0.05}{2}} = 0.22A$$
$$\bar{v}(t) = \bar{I}_L R = 0.22 * 10 = 2.2V$$

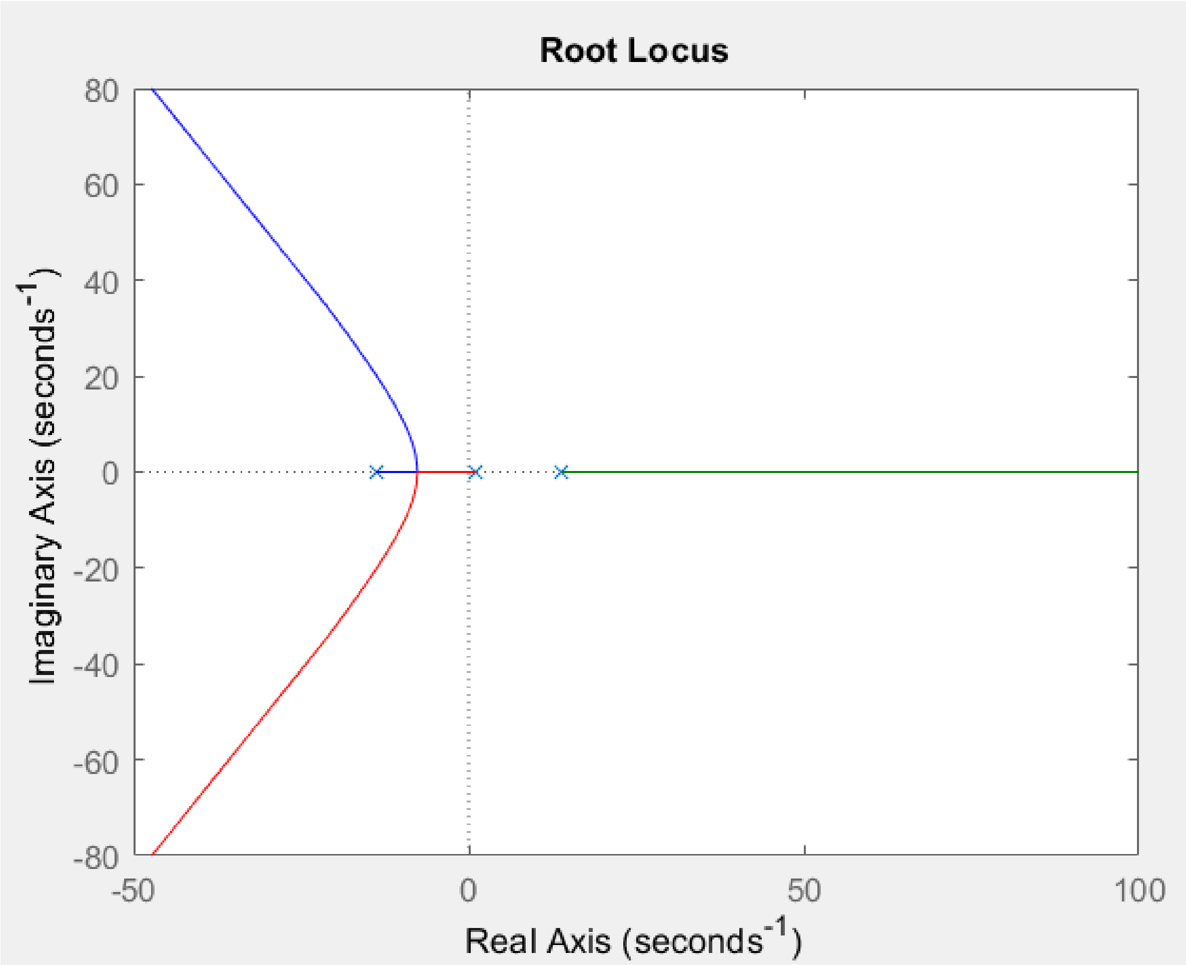
Linealizando.

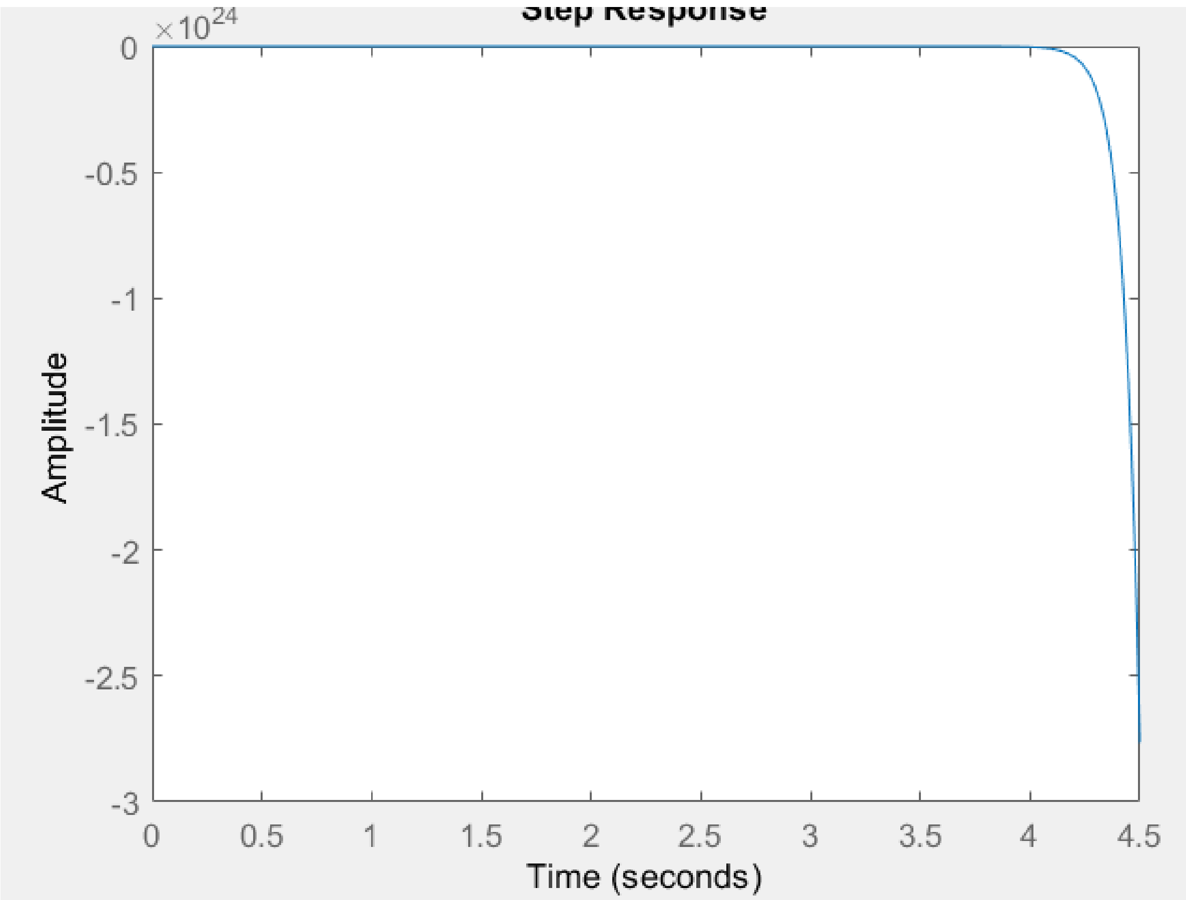
$$\dot{v}\delta = -\frac{2cI_L\delta}{mx} + \frac{cI_L^2}{mx^2\delta}$$
$$\dot{x}\delta = v$$
$$\dot{I}_L = \frac{v(t)\delta}{L} - \frac{RI_L\delta}{L}$$

Función de transferencia si la salida es x.

$$ft = -\frac{44}{5s^3 - 5s^2 - 968s + 968}$$

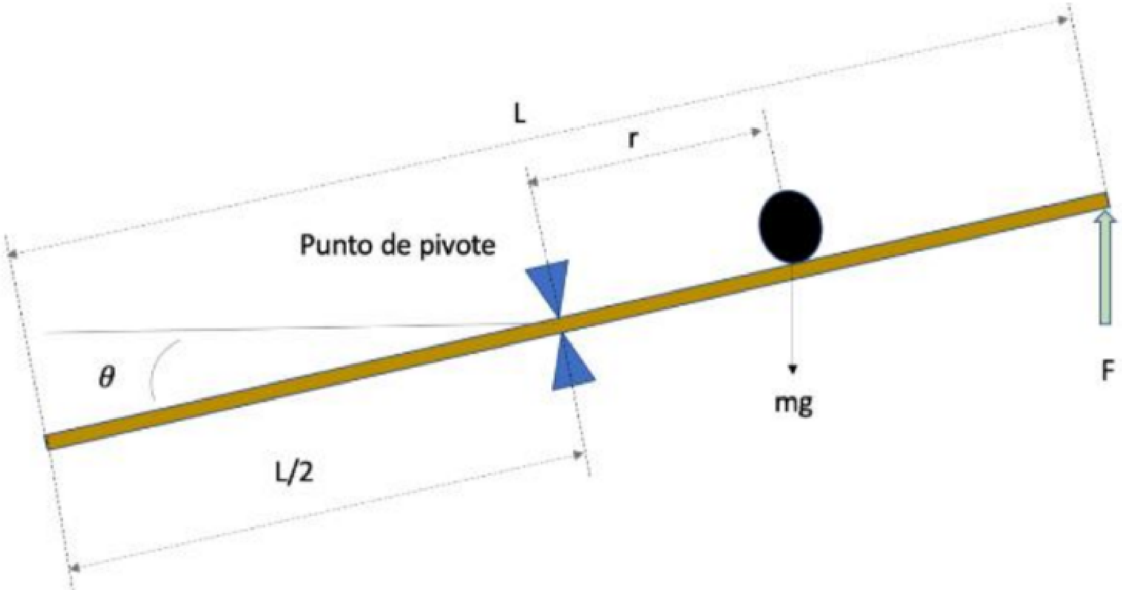
Por medio del criterio de la ubicación de los polos, se observa que el sistema es inestable y también al escalón.


Root Locus
80
60
40
20
0
-20
-40
-60
-80
-50
0
50
100
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})


$\times 10^{24}$
0
-0.5
-1
-1.5
-2
-2.5
-3
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Amplitude
Time (seconds)

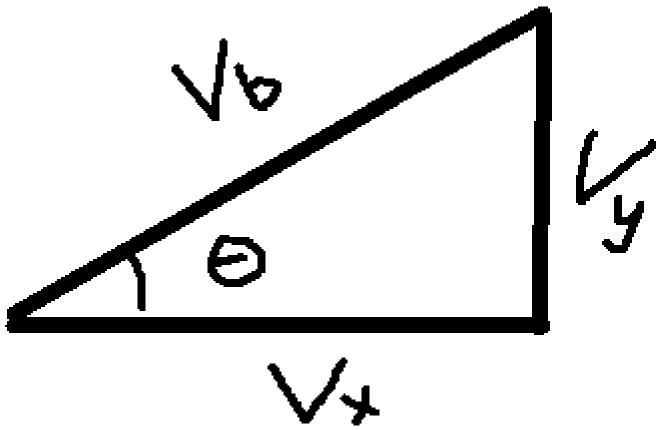
Ejercicio 2

- usando el enfoque de Euler-Lagrange verificar el modelo matemático del siguiente sistema (bola sobre riel).
- Obtener el modelo lineal y la función de transferencia.



$$T = T_{viga} + T_{bola}$$
$$T_{viga} = T_{rot} = \frac{1}{2}Iw^2$$
$$T_{bola} = T_{tras} + T_{rot} = \frac{1}{2}mv_b^2 + \frac{1}{2}Iw^2$$

Calculo de componentes para hallar vb



$$x = rcos(\theta) \rightarrow \dot{x} = \dot{r}cos(\theta) - r\dot{\theta}sen(\theta)$$
$$y = rsen(\theta) \rightarrow \dot{y} = \dot{r}sen(\theta) + r\dot{\theta}cos(\theta)$$
$$v_b^2 = \dot{x}^2 + \dot{y}^2$$

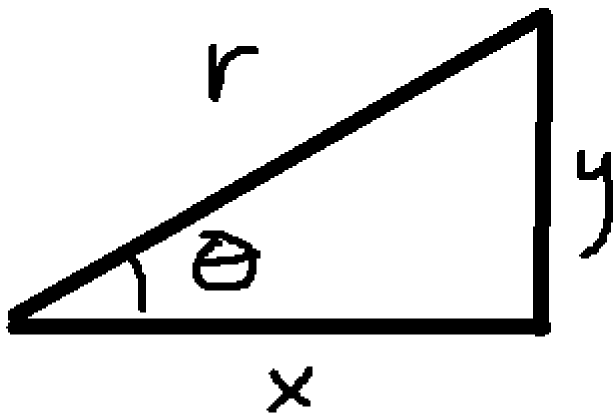
Se obtiene Vb y w para la energía cinética en la bola

$$v_b^2 = (\dot{r}cos(\theta) - r\dot{\theta}sen(\theta))^2 + (\dot{r}sen(\theta) + r\dot{\theta}cos(\theta))^2$$
$$v_b^2 = \dot{r}^2cos^2(\theta) + r^2\dot{\theta}^2sen^2(\theta) + \dot{r}^2sen^2(\theta) + r^2\dot{\theta}^2cos^2(\theta)$$
$$v_b^2 = \dot{r}^2(cos^2(\theta) + sen^2(\theta)) + r^2\dot{\theta}^2(sen^2(\theta) + cos^2(\theta))$$
$$v_b^2 = \dot{r}^2 + r^2\dot{\theta}^2$$
$$\theta R = r \rightarrow \dot{\theta}R = \dot{r} \rightarrow wR = \dot{r} \rightarrow w = \frac{\dot{r}}{R}$$

Energía cinética total del sistema

$$T_{viga} = \frac{1}{2} J_v \dot{\theta}^2$$
$$T_{bola} = \frac{1}{2} m(\dot{r}^2 + r^2 \dot{\theta}^2) + \frac{J_b \dot{r}^2}{2R^2}$$
$$T = \frac{1}{2} J_v \dot{\theta}^2 + \frac{m\dot{r}^2}{2} + \frac{mr^2\dot{\theta}^2}{2} + \frac{J_b \dot{r}^2}{2R^2}$$

Energía Potencial



$$U = U_{viga} + U_{bola}$$
$$U_{viga} = 0$$
$$U_{bola} = mgrsen(\theta)$$
$$U = mgrsen(\theta)$$

Lagrangiano

$$L = T - U$$
$$L = \frac{1}{2} J_v \dot{\theta}^2 + \frac{m\dot{r}^2}{2} + \frac{mr^2\dot{\theta}^2}{2} + \frac{J_b \dot{r}^2}{2R^2} - mgrsen(\theta)$$

Ecuaciones de Lagrange

$$\frac{d}{dt}(\frac{dL}{d\dot{\theta}}) - \frac{dL}{d\theta} + \frac{dD}{d\dot{\theta}} = \tau$$
$$\frac{d}{dt}(\frac{dL}{d\dot{r}}) - \frac{dL}{dr} = 0$$
$$\frac{dL}{d\dot{r}} = m\dot{r} \to \frac{d}{dt}(\frac{dL}{d\dot{r}}) = m\ddot{r} + \frac{J_b \ddot{r}}{R^2}$$
$$\frac{dL}{dr} = mr\dot{\theta}^2 - mgsen(theta)$$
$$\frac{dL}{d\dot{\theta}} = J_V \dot{\theta} + mr^2\dot{\theta} \to \frac{d}{dt}(\frac{dL}{d\dot{\theta}}) = J_V \ddot{\theta} + mr^2 \ddot{\theta}$$
$$\frac{dL}{d\dot{\theta}} = -mgrcos(\theta)$$

Ecuaciones de movimiento

$$\ddot{r}(m + \frac{J_b}{R^2}) - mr\dot{\theta}^2 + mgsen(\theta) = 0$$
$$\ddot{\theta}(J_v + J_b + mr^2) + mgrcos(\theta) + 2mr\dot{r}\dot{\theta} = \tau$$

Linealización

Puntos de Equilibrio

$$\ddot{r} = \dot{r} = \ddot{\theta} = \dot{\theta} = 0$$
$$\ddot{r}(m + \frac{J_b}{R^2}) - mr\dot{\theta}^2 + mgsen(\theta) = 0 \to 0 = 0$$
$$\ddot{\theta}(J_v + J_b + mr^2) + mgrcos(\theta) + 2mr\dot{r}\dot{\theta} = \tau \to \tau = mgr\cos(\theta)$$

Puntos de operación

$$\bar{\theta} = \bar{w} = \bar{\dot{r}} = 0$$
$$r = 0.03[m]$$
$$\tau = mg\bar{r}\cos(\theta) \to \tau = 0.012054$$

Jacobiano

Cambio de variables

$$\dot{\theta} = w$$
$$\dot{r} = v_r$$

Parámetros

$$m = 0.041[kg]$$
$$J_V = 0.473[Kgm^2]$$
$$J_b = 3.69e - 6[Kgm^2]$$
$$r = 0.5[m]$$
$$L = 1[m]$$

Espacio de estados linealizado

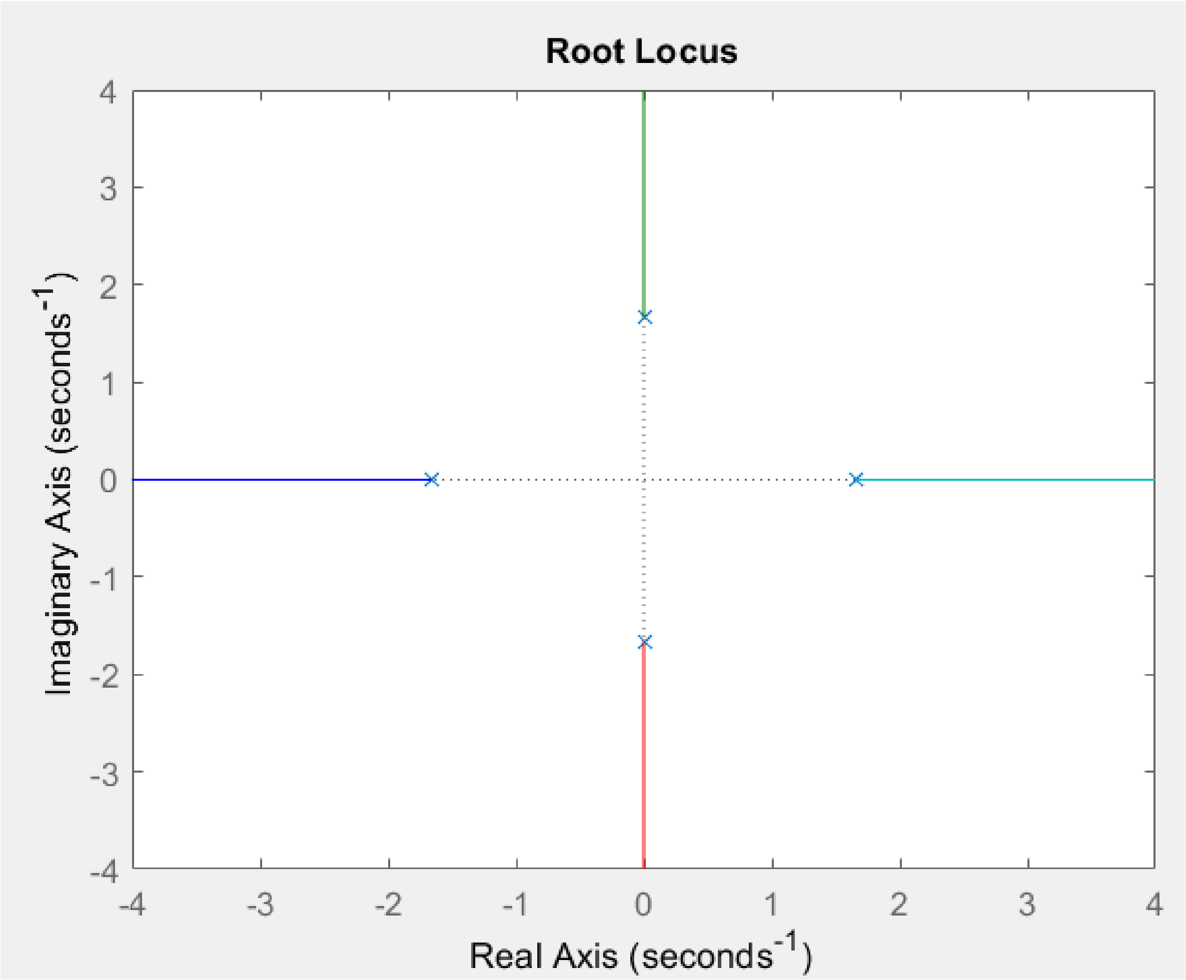
$$\begin{bmatrix} \dot{\theta} \\ \dot{w} \\ \dot{r} \\ \dot{v}_r \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -0.00027 & 0 \\ 0 & 0 & 0 & 1 \\ -9.79 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ w \\ r \\ v_r \end{bmatrix} + \begin{bmatrix} 0 \\ 2.11 \\ 0 \\ 0 \end{bmatrix} \tau$$
$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ w \\ r \\ v_r \end{bmatrix}$$

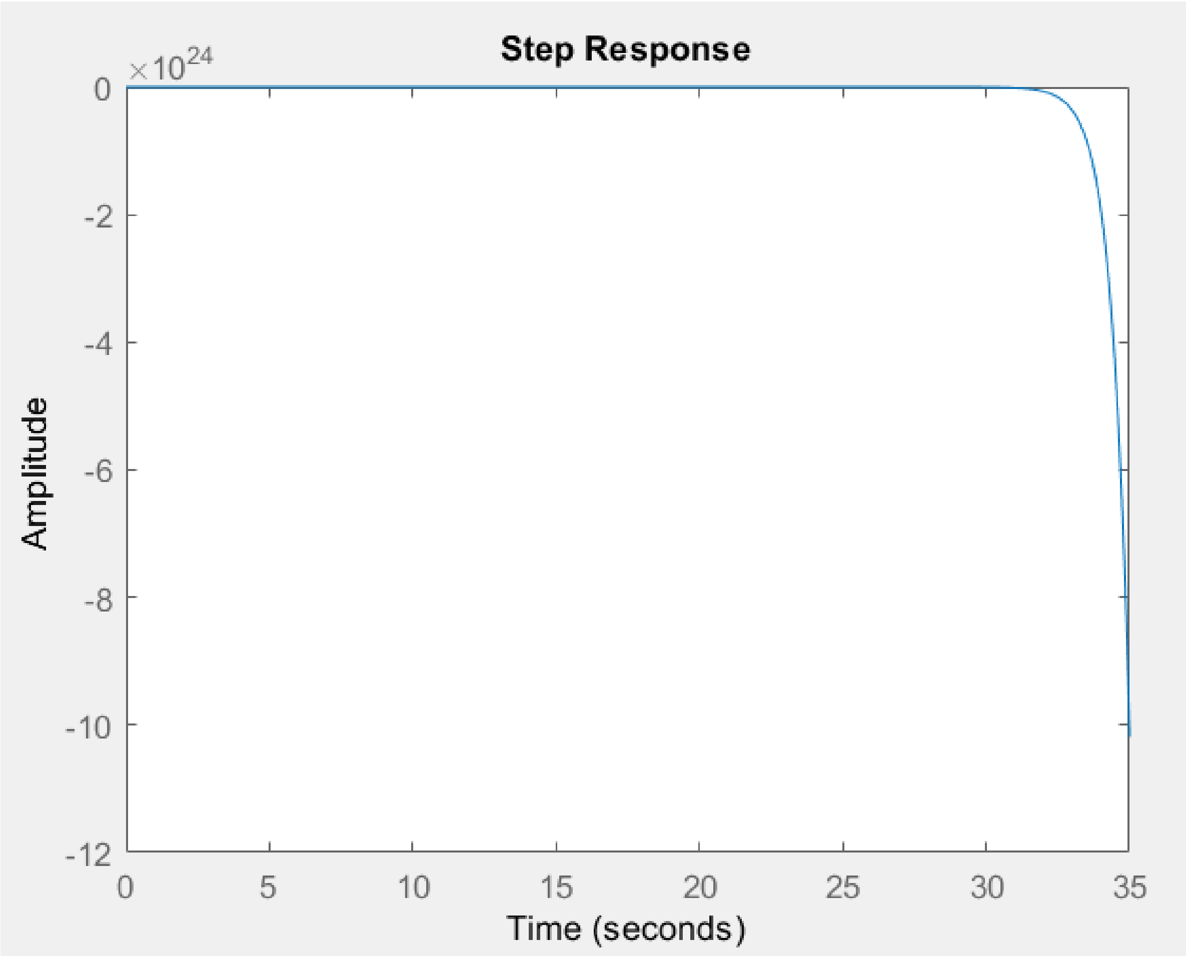
Función de transferencia**

$$ft = \frac{-18.83}{s^4 + (2.22e - 16)s^3 + (5.329e - 15)s^2 - (5.329e - 15)s - 7.567}$$

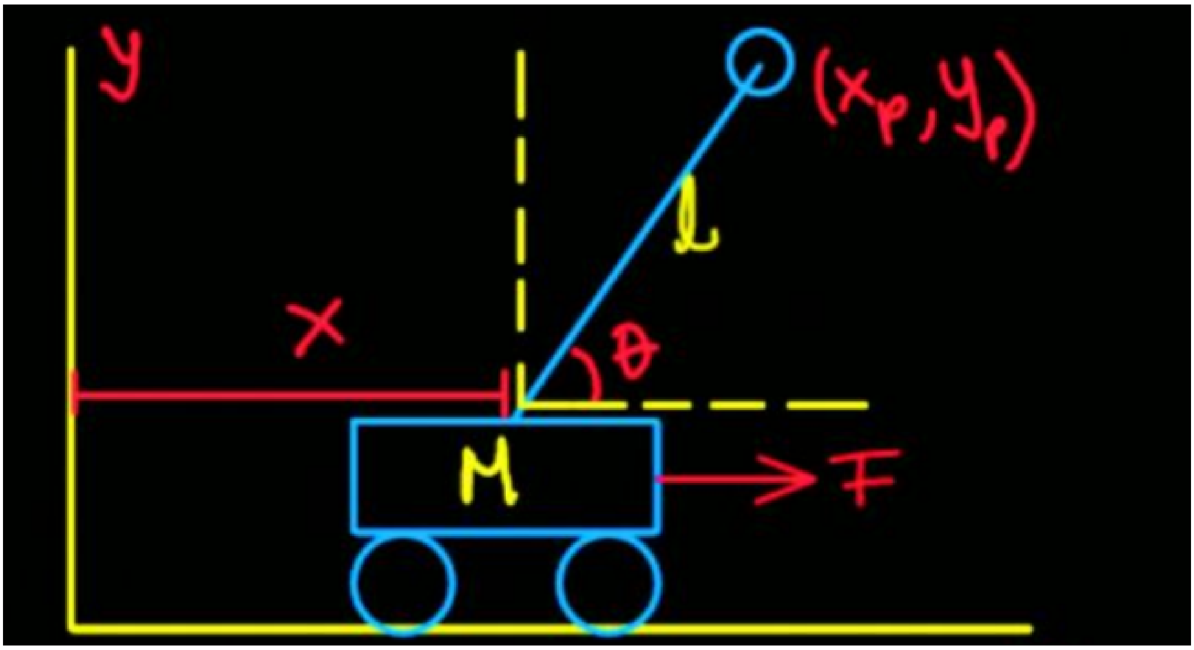
Estabilidad del sistema

Se observa que el sistema es inestable. Por medio del criterio de ubicación de polos y a la respuesta al escalón.


Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4


Step Response
$\times 10^{24}$
Amplitude
Time (seconds)
0
-2
-4
-6
-8
-10
-12
0
5
10
15
20
25
30
35

Ejercicio 3



Ecuaciones de Movimiento

$$1.\ddot{X}(M+m)-\ddot{\theta}(mLcos(\theta))+mL\dot{\theta}^2 sen)\theta+b\dot{X}=F(t)$$
$$2.\ddot{\theta}(mL^2+I)-\ddot{X}(mLcos(\theta))+\dot{\theta}C-mgLsen(\theta)=0$$

Dejando en cada ecuación solo una máxima derivada

$$w_p=-\frac{CMw+Cmw-FLmcos(\theta)-Lgm^2 sin(\theta)+L^2m^2w^2cos(\theta)sin(\theta)-LMgmsen(\theta)+Lbmvcos(\theta)}{-L^2m^2cos(\theta)^2+L^2m^2+ML^2m+Im+IM}$$
$$V_P=-\frac{Ibv-FI-FL^2m+L^2bmv+L^3m^2w^2sin(\theta)+ILmw^2sen(\theta)+CLmwcos(\theta)-L^2gm^2cos(\theta)sin(\theta)}{-L^2m^2cos(\theta)^2+L^2m^2+ML^2m+Im+IM}$$

Reemplazando los valores iniciales

$$Movil$$
$$M=0.48b=3.83$$
$$Péndulo$$
$$m=0.16L=0.25I=0.0043C=0.00218g=9.8$$
$$x_p=v$$
$$v_p=\frac{25F}{6}-\frac{383v}{64}-\frac{w^2sen(\theta)}{16}+\frac{cos(\theta)(500cos(\theta)sin(\theta)w^2+436w-78400sen(\theta)-12500Fcos(\theta)+47875vcos(\theta))}{(16)(500cos(\theta)^2-2860)}$$
$$\theta_p=w$$
$$w_p=\frac{500cos(\theta)sin(\theta)w^2+436w-78400sin(\theta)-12500Fcos(\theta)+47875vcos(\theta)}{500cos(\theta)^2-2860}$$

Linealizando mediante jacobiano

$$\begin{bmatrix} \partial\dot{x} \\ \partial\dot{v} \\ \partial\dot{\theta} \\ \partial\dot{w} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_1 & b_1 & c_1 \\ 0 & 0 & 0 & 1 \\ 0 & a_2 & b_2 & c_2 \end{bmatrix} \begin{bmatrix} \partial x \\ \partial v \\ \partial\theta \\ \partial w \end{bmatrix} + \begin{bmatrix} 0 \\ d_1 \\ 0 \\ d_2 \end{bmatrix} F$$

$$a_1 = \frac{47875cos(\theta)^2}{16\partial_3} - \frac{383}{64}$$

$$b_1 = \frac{125cos(\theta)^2 sen(\theta)\partial_1}{2\partial_3^2} - \frac{sen(\theta)\partial_1}{16\partial_3} - \frac{cos(\theta)\partial_2}{16\partial_3} - \frac{w^2 cos(\theta)}{16}$$

$$c_1 = \frac{cos(\theta)\partial_4}{16\partial_3} - \frac{wsen(\theta)}{8}$$

$$d_1 = \frac{25}{16} - \frac{3125cos(\theta)^2}{(4)(500cos(\theta)^2 - 2860)}$$

$$a_2 = \frac{47875cos(\theta)}{\partial_3}$$

$$b_2 = \frac{1000cos(\theta)sen(\theta)\partial_1}{\partial_3^2} - \frac{\partial_2}{\partial_3}$$

$$c_2 = \frac{\partial_4}{\partial_3}$$

$$d_2 = -\frac{12500cos(\theta)}{500cos(\theta)^2 - 2860}$$

Reemplazando punto de operación.

$$\theta = w = 0$$
$$\partial_1 = -12500F + 47875v$$
$$\partial_2 = 78400$$
$$\partial_3 = -2360$$
$$\partial_4 = 436$$

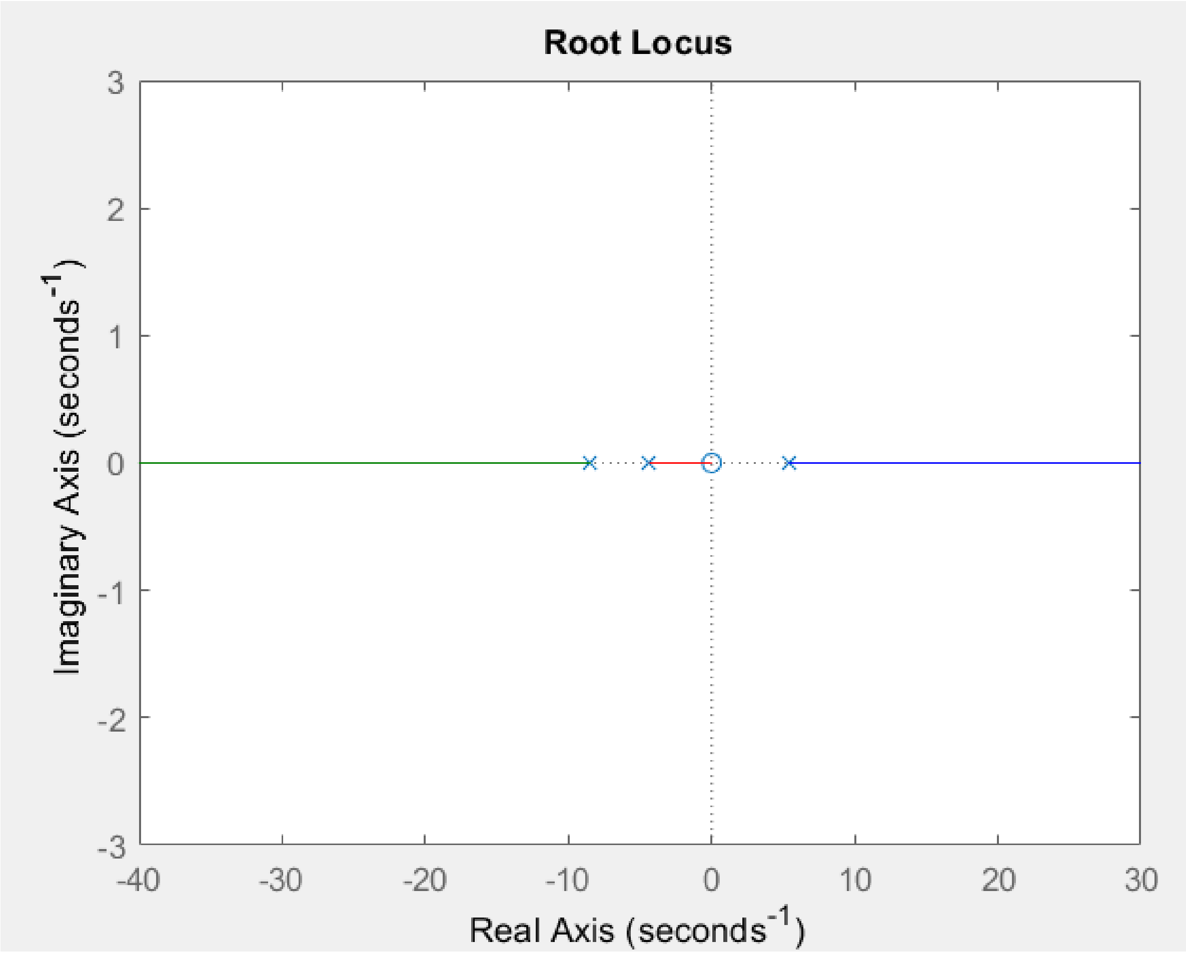
$$\begin{bmatrix} \partial\dot{x} \\ \partial\dot{v} \\ \partial\dot{\theta} \\ \partial\dot{w} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -7.2523 & 2.0763 & -0.0115 \\ 0 & 0 & 0 & 1 \\ 0 & -20.2860 & 33.2203 & -0.1847 \end{bmatrix} \begin{bmatrix} \partial x \\ \partial v \\ \partial\theta \\ \partial w \end{bmatrix} + \begin{bmatrix} 0 \\ 1.8935 \\ 0 \\ 5.2966 \end{bmatrix} F$$

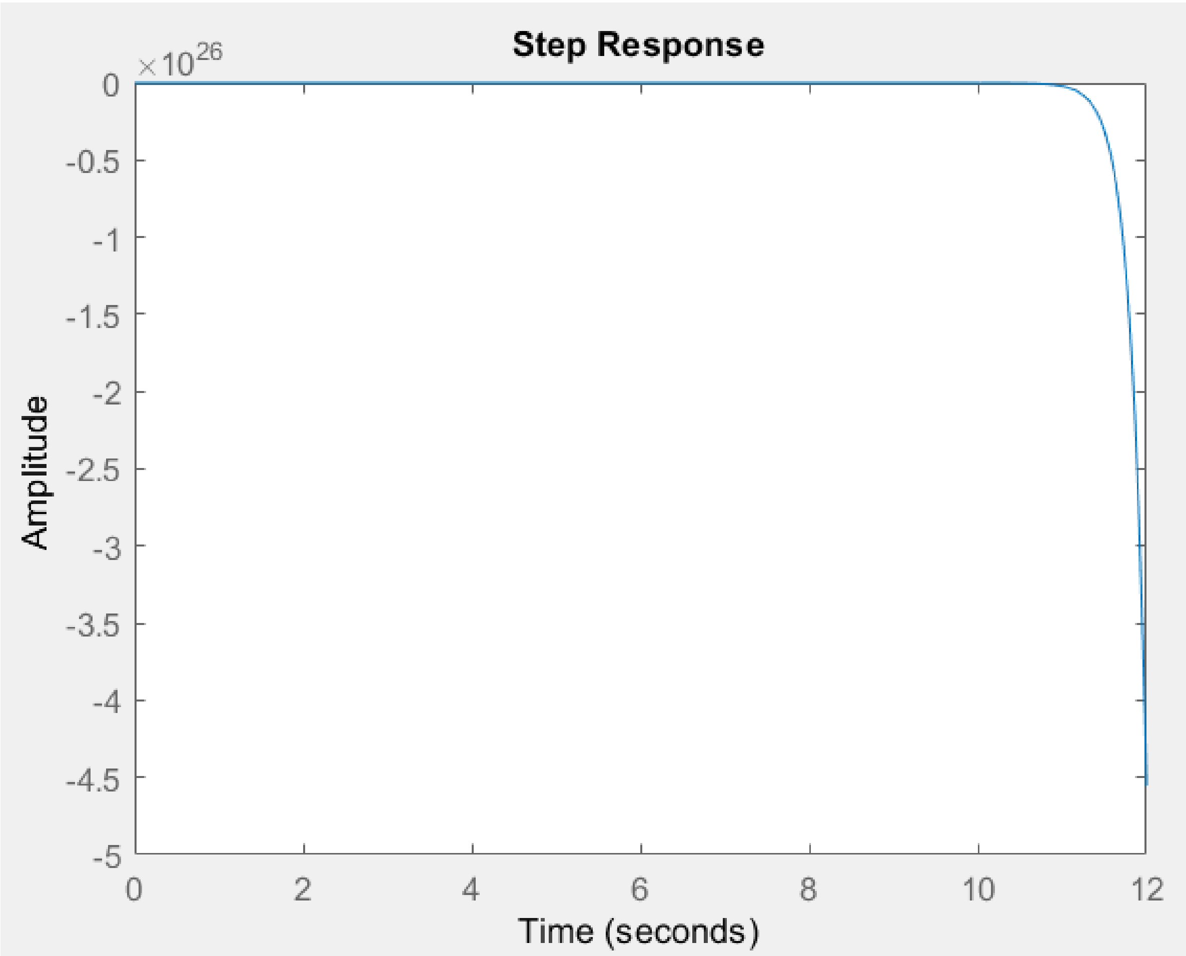
$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \partial x \\ \partial v \\ \partial\theta \\ \partial w \end{bmatrix}$$

$$Ft(s) = \frac{-5.2966s - 0.00099118}{s^3 + 7.437s^2 - 32.1141s - 198.8038}$$

Estabilidad del sistema

Se observa que el sistema es inestable. Por medio del criterio de los polos y a la respuesta al escalon.


Root Locus
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
3
2
1
0
-1
-2
-3
-40
-30
-20
-10
0
10
20
30


Step Response
×10^26
Amplitude
Time (seconds)
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
0
2
4
6
8
10
12