Notas de clase semana 7

Andrés Castro

1. Simulación control retroalimentación de estados (regulación).
2. Servosistema tipo 1 para entrada escalón o sistemas de seguimiento.
3. Observador de orden completo.
4. Retroalimentación de estados por matriz de T y Ackerman. (place(A,B,p))
5. Servosistema tipo 2 y 3 para entrada rampa y parábola.
6. Ejercicios.

Ganancias levitador magnético.

$$\begin{bmatrix} K1 \\ K2 \\ K3 \end{bmatrix} = \begin{bmatrix} -32 \\ -2.41 \\ -0.4 \end{bmatrix}$$

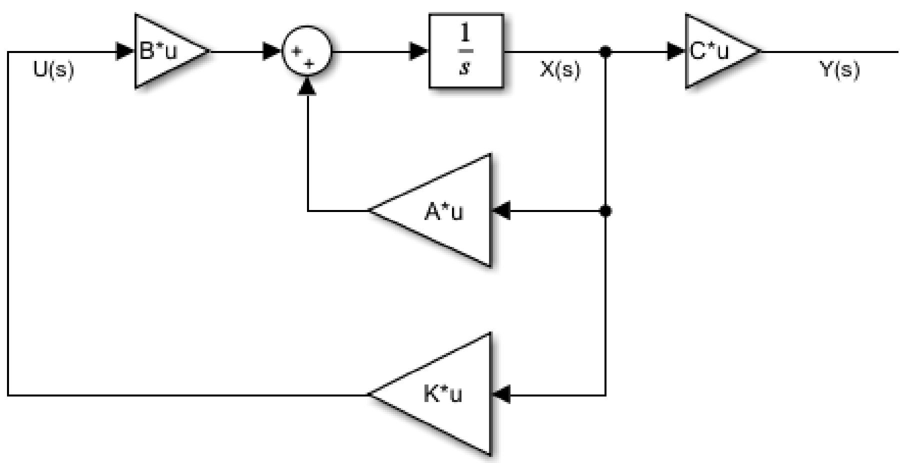


Figura 1: Simulación con el modelo lineal

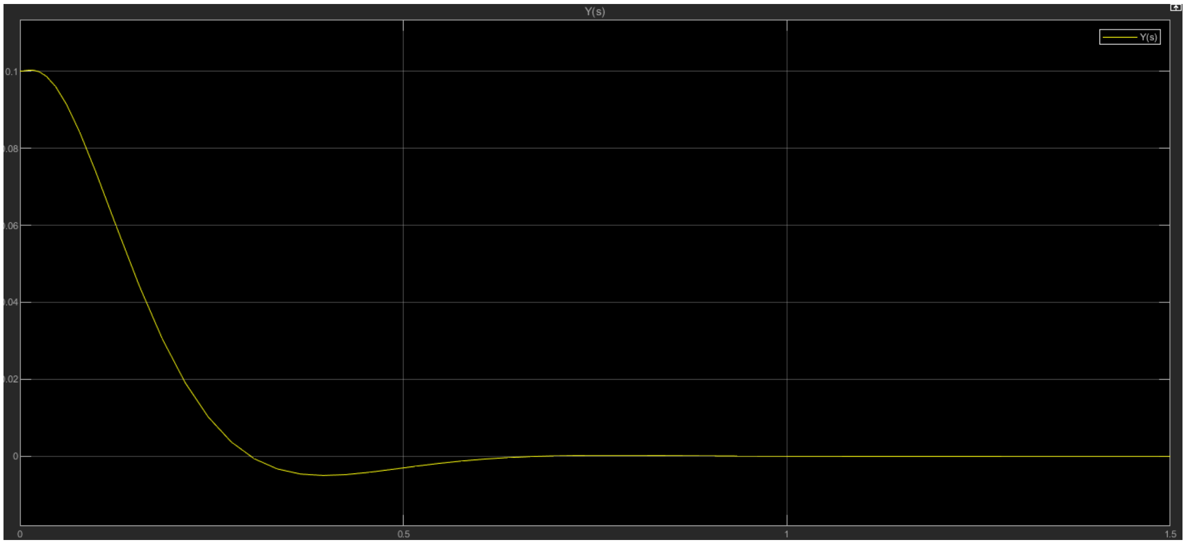


Figura 2. Salida posición X1.

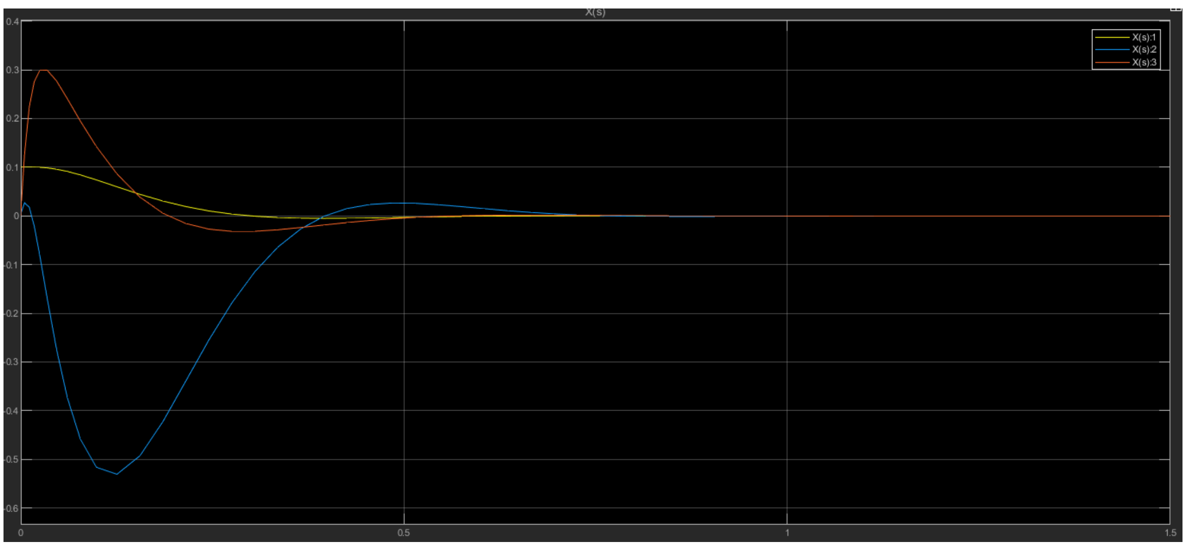


Figura 3. Los tres estados estados.

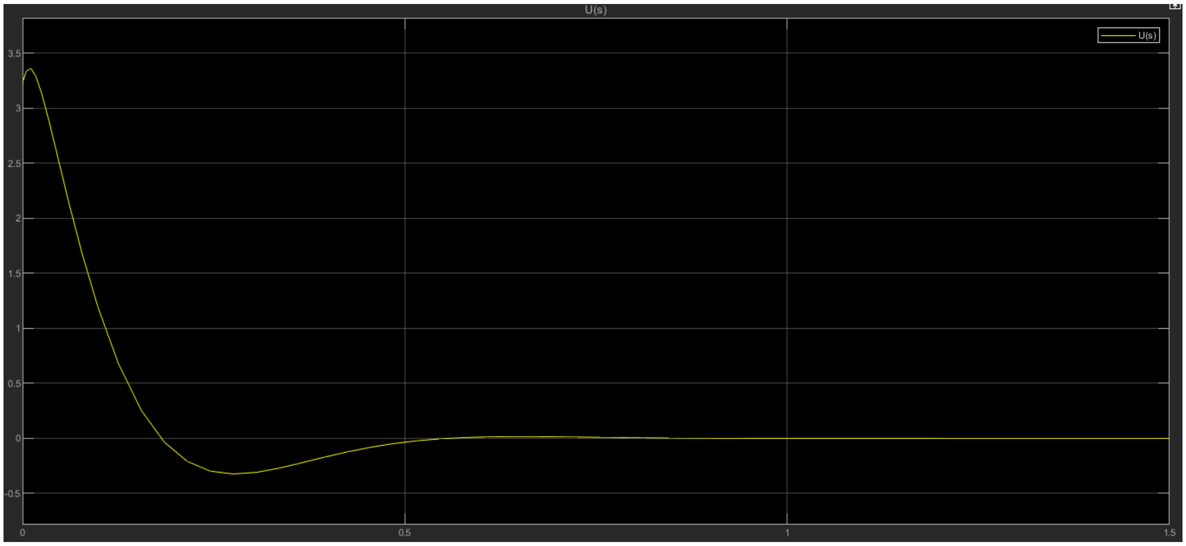


Figura 4. Señal de control.

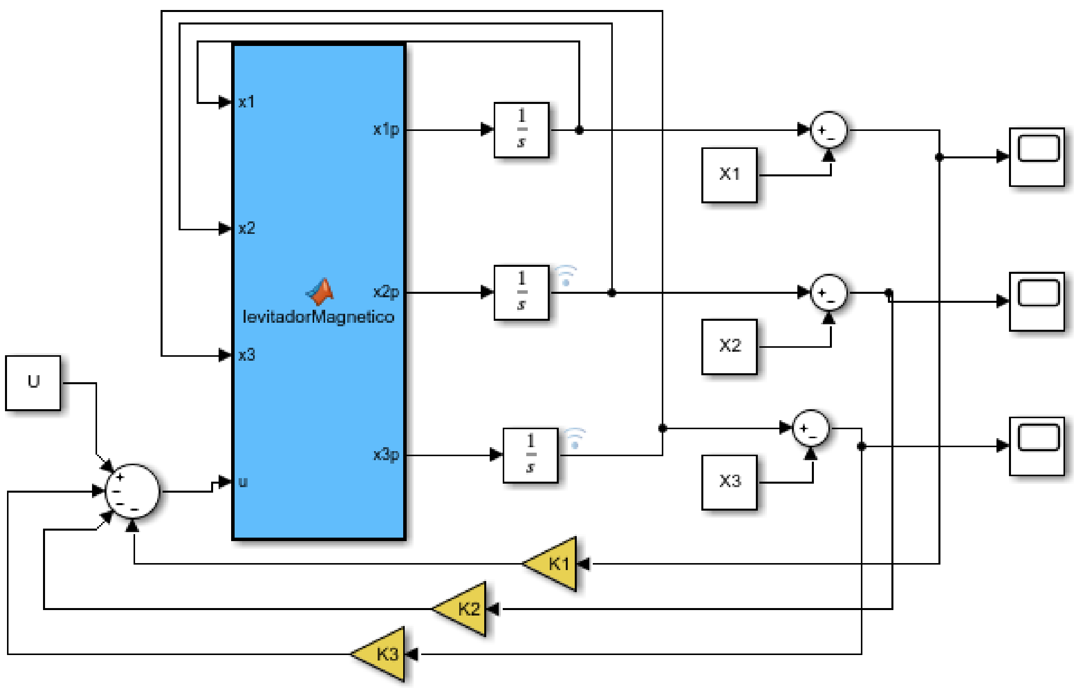


Figura 5. Simulación modelo No lineal.

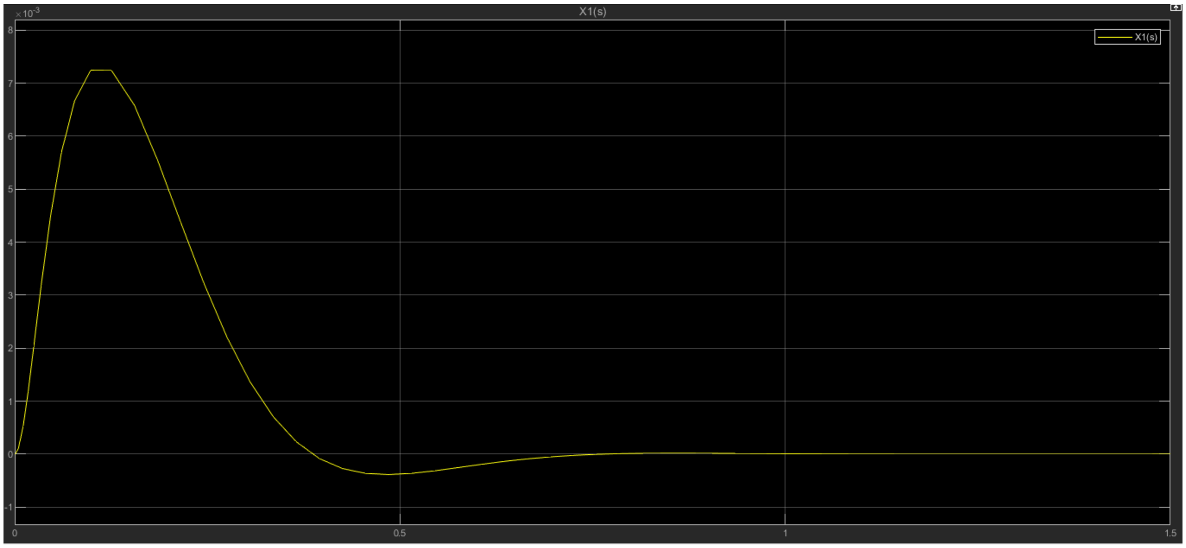


Figura 5. Simulación con modelo NO lineal.

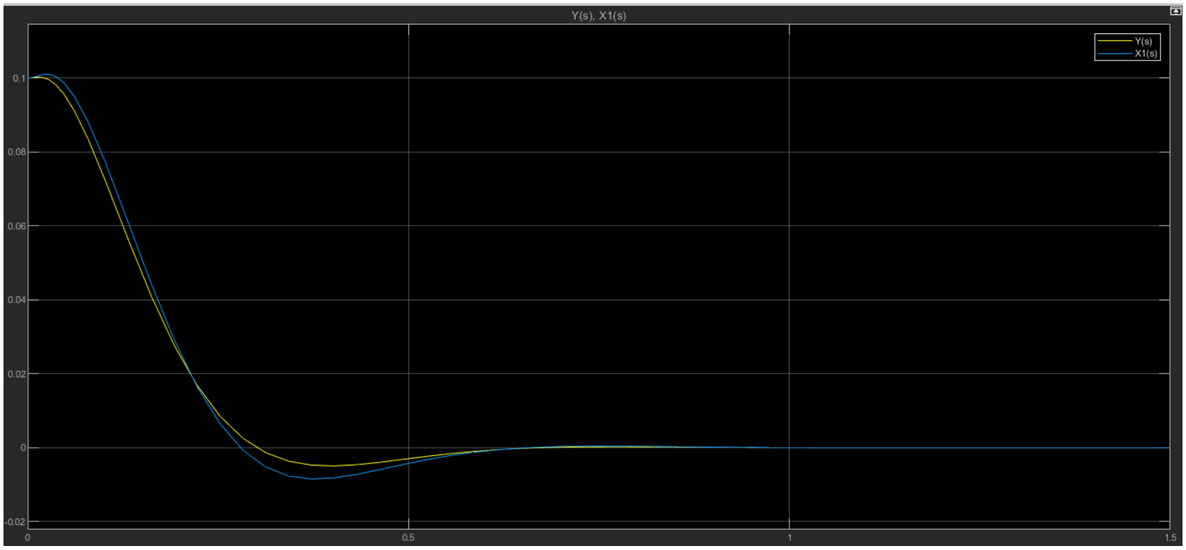
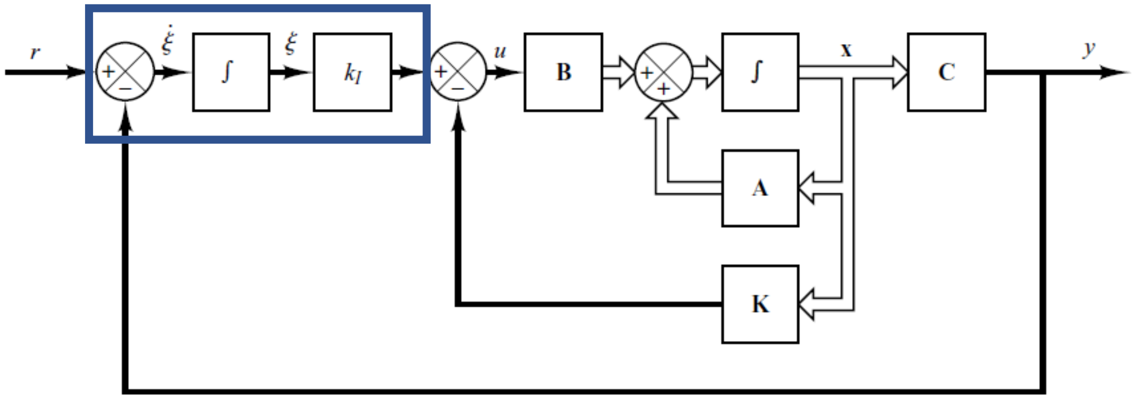


Figura 7. Comparación de los dos modelos.

Servosistema tipo 1 (para entrada escalón) sistemas de seguimiento.
Obtener el espacio de estados del siguiente sistema.



ξ *Salida del integrador, Variable de estado del sistema escalar*

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$$

$$\boldsymbol{u} = -\boldsymbol{Kx} + K_I\xi$$

$$\dot{\xi} = r - y$$

$$\dot{\xi} = r - \boldsymbol{Cx}$$

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{B}(-\boldsymbol{Kx} + K_I\xi)$$

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} - \boldsymbol{BKx} + \boldsymbol{B}\,K_I\xi$$

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} - \boldsymbol{BK})\boldsymbol{x} + \boldsymbol{B}\,K_I\xi$$

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} - \boldsymbol{BK} & \boldsymbol{B}\,K_I \\ -\boldsymbol{C} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \xi \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ 1 \end{bmatrix} r$$

Polinomio característico en lazo cerrado

$$\det\left(sI - \begin{bmatrix} \boldsymbol{A} - \boldsymbol{BK} & \boldsymbol{B}\,K_I \\ -\boldsymbol{C} & 0 \end{bmatrix}\right) = PD$$

Ejemplo

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -3 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$\boldsymbol{A-BK}$$

$$\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -3 & -5\end{bmatrix}-\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}\begin{bmatrix}K1 & K2 & K3\end{bmatrix}$$

$$A-BK=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -3 & -5\end{bmatrix}-\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ K1 & K2 & K3\end{bmatrix}$$

$$A-BK=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2-K1 & -3-K2 & -5-K3\end{bmatrix}$$

$$\begin{bmatrix}\boldsymbol{A-BK} & \boldsymbol{B}\,K_I\\ -\boldsymbol{C} & 0\end{bmatrix}$$

$$\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -2-K1 & -3-K2 & -5-K3 & K_I\\ -1 & 0 & 0 & 0\end{bmatrix}$$

$$sI-\begin{bmatrix}\boldsymbol{A-BK} & \boldsymbol{B}\,K_I\\ -\boldsymbol{C} & 0\end{bmatrix}$$

$$\begin{bmatrix}s & 0 & 0 & 0\\ 0 & s & 0 & 0\\ 0 & 0 & s & 0\\ 0 & 0 & 0 & s\end{bmatrix}-\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -2-K1 & -3-K2 & -5-K3 & K_I\\ 1 & 0 & 0 & 0\end{bmatrix}$$

Polinomio característico en lazo cerrado

$$\det\left(\begin{bmatrix}s & 0 & 0 & 0\\ 0 & s & 0 & 0\\ 0 & 0 & s & 0\\ 0 & 0 & 0 & s\end{bmatrix}-\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -2-K1 & -3-K2 & -5-K3 & K_I\\ 1 & 0 & 0 & 0\end{bmatrix}\right)$$

$$s^4+(K3+5)s^3+(K2+3)s^2+(K1+2)s+KI=PD$$

$$\zeta = 0.9$$

$$T_2 = 7seg$$

$$\beta = 10$$

$$PD = (s^2 + 2\zeta\omega_n s + {\omega_n}^2)(s + \beta\zeta\omega_n)(s + \beta\zeta\omega_n)$$

$$PD = s^4 + 12.6s^3 + 46.1s^2 + 41.9s + 13.2$$

$$(K3 + 5) = 12.6$$

$$(K2 + 3) = 46.1$$

$$(K1 + 2) = 41.9$$

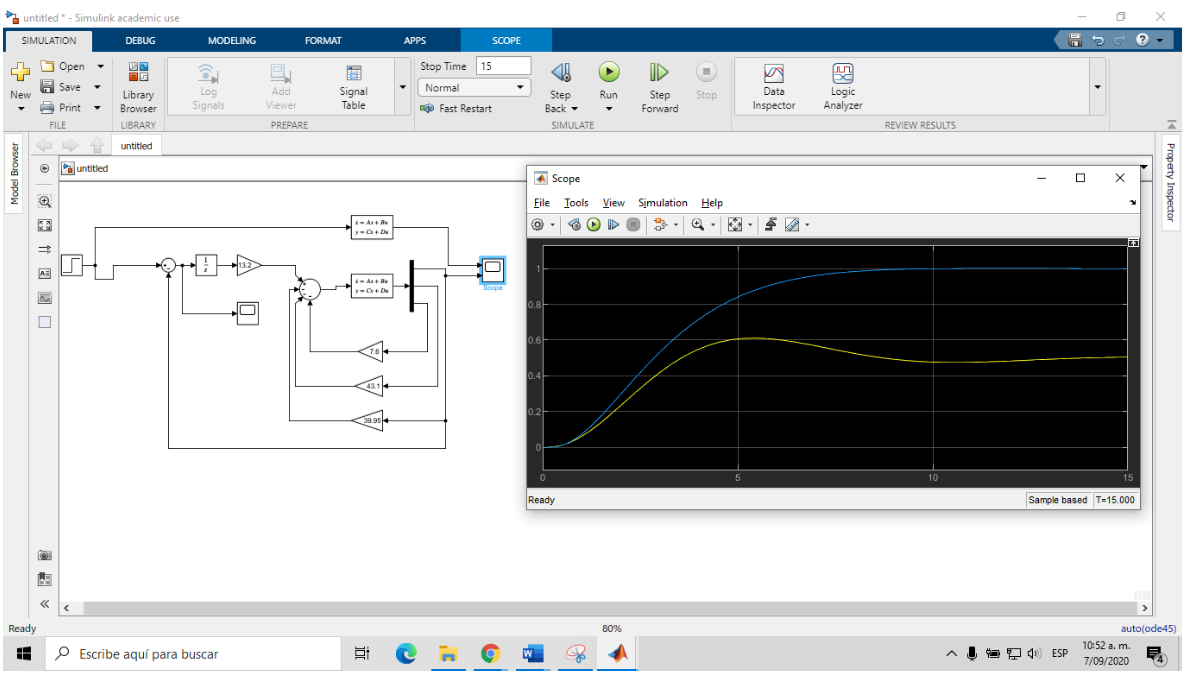
$$KI = 13.2$$

$$K3 = 7.6$$

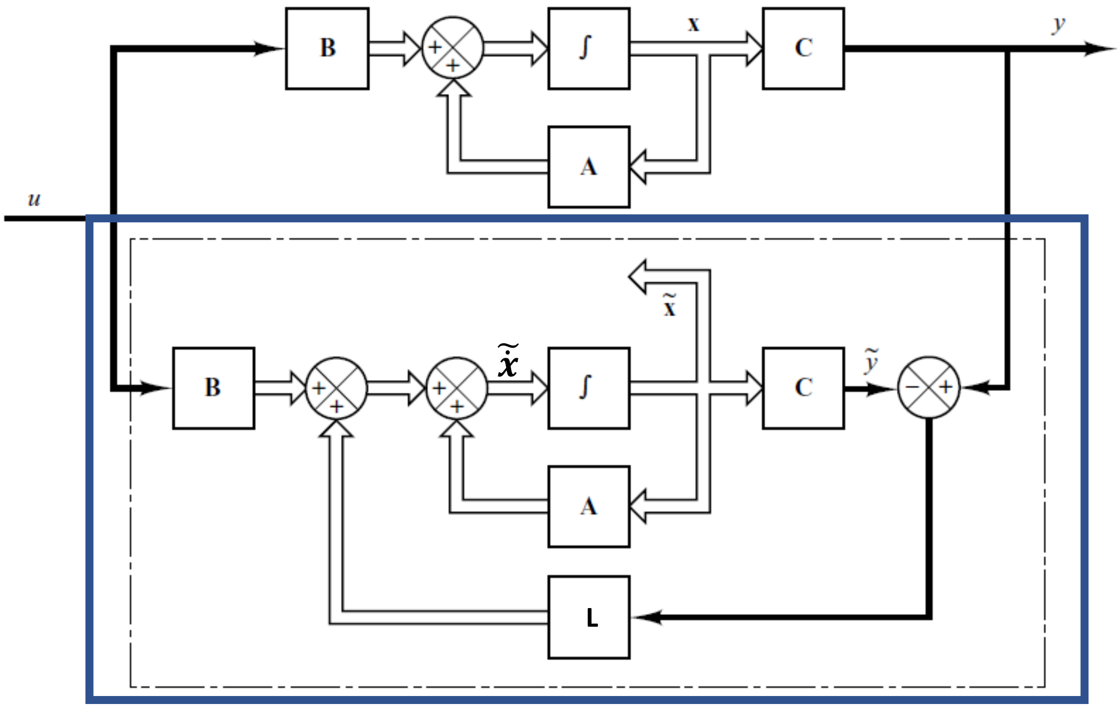
$$K2 = 43.1$$

$$K1 = 39.95$$

$$KI = 13.2$$



Observador de orden completo.



$$\tilde{\boldsymbol{x}}\ estados\ estimado$$

$$\boldsymbol{L}\ Vector\ de\ ganancias\ del\ observador$$

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u$$

$$y = \boldsymbol{C}\boldsymbol{x}$$

$$\dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}\tilde{\boldsymbol{x}} + \boldsymbol{B}u + \boldsymbol{L}(y - \tilde{y})$$

$$\tilde{y} = \boldsymbol{C}\tilde{\boldsymbol{x}}$$

Obtener el espacio de estados del error

$$\dot{\boldsymbol{x}} - \dot{\tilde{\boldsymbol{x}}} = \dot{\boldsymbol{e}}$$

$$\dot{\boldsymbol{x}} - \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u - (\boldsymbol{A}\tilde{\boldsymbol{x}} + \boldsymbol{B}u + \boldsymbol{L}(y - \tilde{y}))$$

$$\dot{\boldsymbol{x}} - \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u - (\boldsymbol{A}\tilde{\boldsymbol{x}} + \boldsymbol{B}u + \boldsymbol{L}(\boldsymbol{c}x - \boldsymbol{c}\tilde{x}))$$

$$\dot{\boldsymbol{x}} - \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u - (\boldsymbol{A}\tilde{\boldsymbol{x}} + \boldsymbol{B}u + \boldsymbol{L}\boldsymbol{c}x - \boldsymbol{L}\boldsymbol{c}\tilde{x})$$

$$\dot{\boldsymbol{x}} - \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u - \boldsymbol{A}\tilde{\boldsymbol{x}} - \boldsymbol{B}u - \boldsymbol{L}\boldsymbol{c}x + \boldsymbol{L}\boldsymbol{c}\tilde{x}$$

$$\dot{\boldsymbol{x}} - \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{Ax} \cancel{+ \boldsymbol{Bu}} - \boldsymbol{A\tilde{x}} \cancel{- \boldsymbol{Bu}} - \boldsymbol{Lcx} + \boldsymbol{Lc\tilde{x}}$$

$$\dot{\boldsymbol{x}} - \dot{\tilde{\boldsymbol{x}}} = \boldsymbol{Ax} - \boldsymbol{A\tilde{x}} - \boldsymbol{Lcx} + \boldsymbol{Lc\tilde{x}}$$

$$\dot{\boldsymbol{x}} - \dot{\tilde{\boldsymbol{x}}} = (\boldsymbol{A} - \boldsymbol{Lc})\boldsymbol{x} - (\boldsymbol{A} - \boldsymbol{Lc})\boldsymbol{\tilde{x}}$$

$$\dot{\boldsymbol{x}} - \dot{\tilde{\boldsymbol{x}}} = (\boldsymbol{A} - \boldsymbol{Lc})(\boldsymbol{x} - \boldsymbol{\tilde{x}})$$

$$\dot{\boldsymbol{e}} = (\boldsymbol{A} - \boldsymbol{Lc})(\boldsymbol{e})$$

$$\boldsymbol{det}(\boldsymbol{sI} - (\boldsymbol{A} - \boldsymbol{Lc})) = \boldsymbol{PDo}$$

$\boldsymbol{PDo}$ *Debe ser mas rápido que el sistema controlado*

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\boldsymbol{A} - \boldsymbol{Lc}$$

$$\boldsymbol{L} = \begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -5 \end{bmatrix} - \begin{bmatrix} L1 \\ L2 \\ L3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -5 \end{bmatrix} - \begin{bmatrix} L1 & 0 & 0 \\ L2 & 0 & 0 \\ L3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -L1 & 1 & 0 \\ -L2 & 0 & 1 \\ -2 - L3 & -3 & -5 \end{bmatrix}$$

$$\boldsymbol{det}(\boldsymbol{sI} - (\boldsymbol{A} - \boldsymbol{Lc}))$$

$$\det\left(sI - \begin{bmatrix} -L1 & 1 & 0 \\ -L2 & 0 & 1 \\ -2-L3 & -3 & -5 \end{bmatrix}\right)$$

$$\det\left(\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -L1 & 1 & 0 \\ -L2 & 0 & 1 \\ -2-L3 & -3 & -5 \end{bmatrix}\right)$$

$$\det\left(\begin{bmatrix} s+L1 & -1 & 0 \\ L2 & s & -1 \\ 2+L3 & 3 & s+5 \end{bmatrix}\right)$$

$$s^3 + (L1+5)s^2 + (5L1+L2+3)s + 3L1 + 5L2 + L3 + 2 = PDo$$

$$\zeta = 0.7$$

$$T_s = 0.7seg$$

$$\beta = 10$$

$$PDo$$

$$(s^2 + 2\zeta\omega_n s + {\omega_n}^2)(s + \beta\zeta\omega_n)$$

$$PDo = s^3 + 68.6s^2 + 720\,s + 3810$$

$$L1 + 5 = 68.6$$

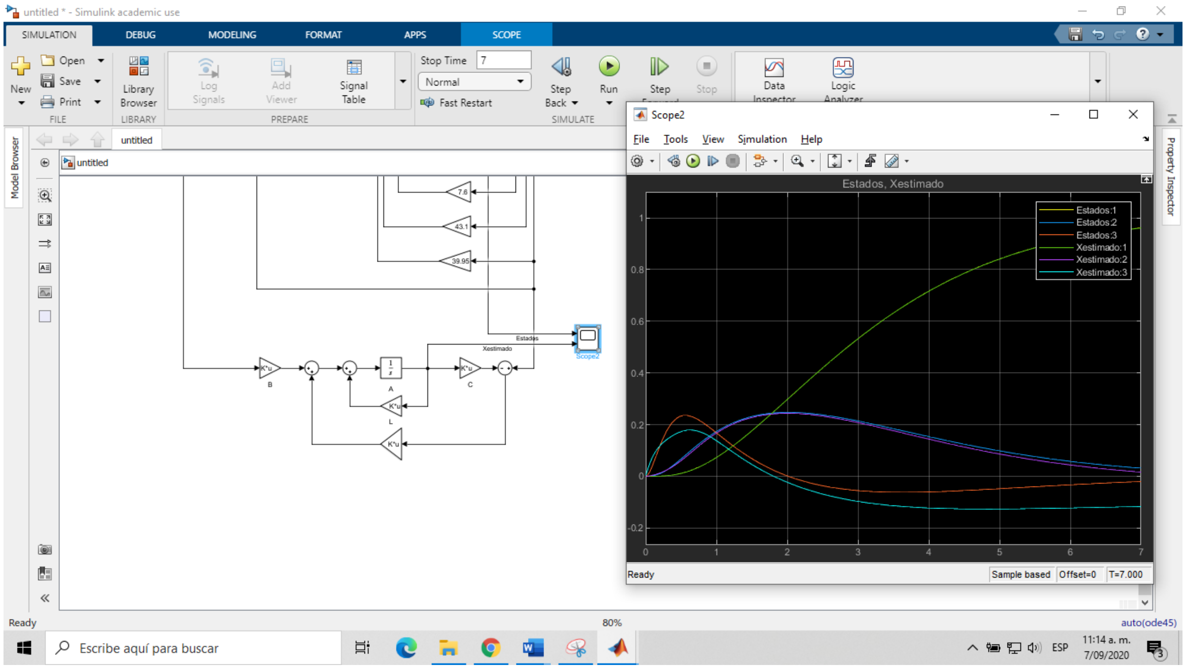
$$(5L1 + L2 + 3) = 720$$

$$3L1 + 5L2 + L3 + 2 = 3810$$

L1=63.6

L2=399

L3=1622.2



Ejercicio realizar el servosistema de tipo 1 y el observador para el levitador magnético.

Fórmula de Akerman.

Existe una fórmula que me permite obtener las ganancias del controlador y se conoce como la fórmula de Akerman.

$$K = [0\ 0 \ldots 0\ 1]{P_c}^{-1}\phi(A)$$

Formula de Ackerman

$$P_c = [B\ AB\ A^2B \ldots\ A^{n-1}B]$$

Matriz de controlabilidad

$$\phi(A) = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_n$$

Polinomio deseado evaluado en A

$$\phi(A) = A^n + \alpha_{n-1}A^{n-1} + \cdots + \alpha_n I$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$P_c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ 1 & -5 & 22 \end{bmatrix}$$

$$PD = s^3 + 14.4s^2 + 82.1s + 172.8$$

$$\phi(A) = A^3 + 14.4A^2 + 82.1A + 172.8I$$

$$\phi(A) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -5 \end{bmatrix}^3 + 14.4 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -5 \end{bmatrix}^2 + 82.1 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -5 \end{bmatrix} + 172.8 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\phi(A) = \begin{bmatrix} 170.8 & 79.1 & 9.4 \\ -18.8 & 142.6 & 32.1 \\ -64.2 & -115.1 & -17.9 \end{bmatrix}$$

$${P_c}^{-1} = \begin{bmatrix} 3 & 5 & 1 \\ 5 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$K = [0\ 0 \ldots 0\ 1]{P_c}^{-1}\phi(A)$$

$$K = [0\;0\;1]\begin{bmatrix}3 & 5 & 1\\5 & 1 & 0\\1 & 0 & 0\end{bmatrix}\begin{bmatrix}170.8 & 79.1 & 9.4\\-18.8 & 142.6 & 32.1\\-64.2 & -115.1 & -17.9\end{bmatrix}$$

$$[K1 \quad K2 \quad K3] = [170.8 \quad 79.1 \quad 9.4]$$

Matriz de transformación T.

$$K = [\alpha_n - a_n \quad \alpha_{n-1} - a_{n-1} \quad \dots \quad \alpha_1 - a_1]T^{-1}$$

$$pd = s^n + \alpha_{n-1}s^{n-1} + \dots + \alpha_n$$

Polinomio deseado

$$s^n + a_{n-1}s^{n-1} + \dots + a_n = |sI - A|$$

Polinomio característico del sistema

$$T = P_c * W$$

$$W = \begin{bmatrix}a_{n-1} & a_{n-2} \cdots & a_1 & 1\\a_{n-2} & a_{n-3} \cdots & 1 & 0\\\vdots & \vdots & \vdots & \vdots\\a_1 & 1 & 0 & 0\end{bmatrix}$$

Ejemplo

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\-2 & -3 & -5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}u$$

$$y = [1 \quad 0 \quad 0]\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$$s^3 + 5s^2 + 3s + 2 = |sI - A|$$

$$T = \begin{bmatrix}0 & 0 & 1\\0 & 1 & -5\\1 & -5 & 22\end{bmatrix} * \begin{bmatrix}3 & 5 & 1\\5 & 1 & 0\\1 & 0 & 0\end{bmatrix}$$

$$T^{-1} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$T^{-1} * A * T = La\ forma\ canónica\ controlable\ de\ A$$

$$T^{-1}B = forma\ canónica\ controlable\ de\ B$$

$$s^3 + 14.4s^2 + 82.1s + 172.8$$

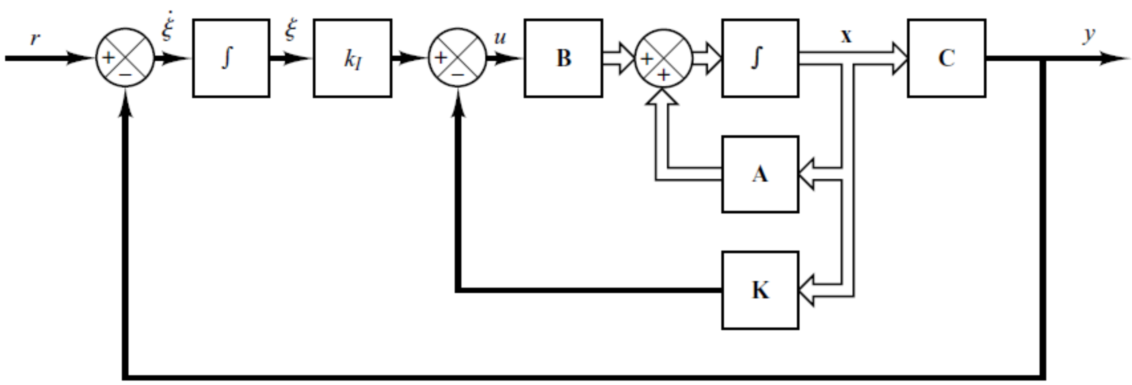
$$s^3 + 5s^2 + 3s + 2$$

$$K = [\alpha_n - a_n \quad \alpha_{n-1} - a_{n-1} \quad \dots \quad \alpha_1 - a_1]T^{-1}$$

$$K = [172.8 - 2 \quad 82.1 - 3 \quad 14.4 - 5]$$

$$[K1 \quad K2 \quad K3] = [170.8 \quad 79.1 \quad 9.4]$$

Para servo sistemas



$$\xi \ Salida\ del\ integrador, Variable\ de\ estado\ del\ sistema\ escalar$$

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$$

$$\boldsymbol{u} = -\boldsymbol{Kx} + K_I \xi$$

$$\dot{\xi} = r - y$$

$$\dot{\xi} = r - \boldsymbol{Cx}$$

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$$

$$\dot{\xi} = r - \boldsymbol{Cx}$$

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{C} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \xi \end{bmatrix} + \begin{bmatrix} \boldsymbol{B} \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$\hat{A} = \begin{bmatrix} \boldsymbol{A} & 0 \\ -\boldsymbol{C} & 0 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} \boldsymbol{B} \\ 0 \end{bmatrix}$$

$$K = [0\ 0 \dots 0\ 1]\widehat{P_c}^{-1}\phi(\hat{A})$$

$$\widehat{P_c} = \begin{bmatrix} 0 & 0 & 1 & -5 \\ 0 & 1 & -5 & 22 \\ 1 & -5 & 22 & -97 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\phi(\hat{A}) = \hat{A}^4 + 12.6\hat{A}^3 + 46.1\hat{A}^2 + 41.9\hat{A} + 13.2I$$

$$[K1 \quad K2 \quad K3 \quad KI] = [\, 39.9 \quad 43.1 \quad 7.6 \quad -13.2]$$

Mediante transformación T

$$\left|sI - \hat{A}\right|$$

$$T = \hat{P}_C * W$$

$$K = [\alpha_n - a_n \quad \alpha_{n-1} - a_{n-1} \quad \dots \quad \alpha_1 - a_1]T^{-1}$$

Para las ganancias del observador

$$L = \phi(A) * P_o^{-1} * \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

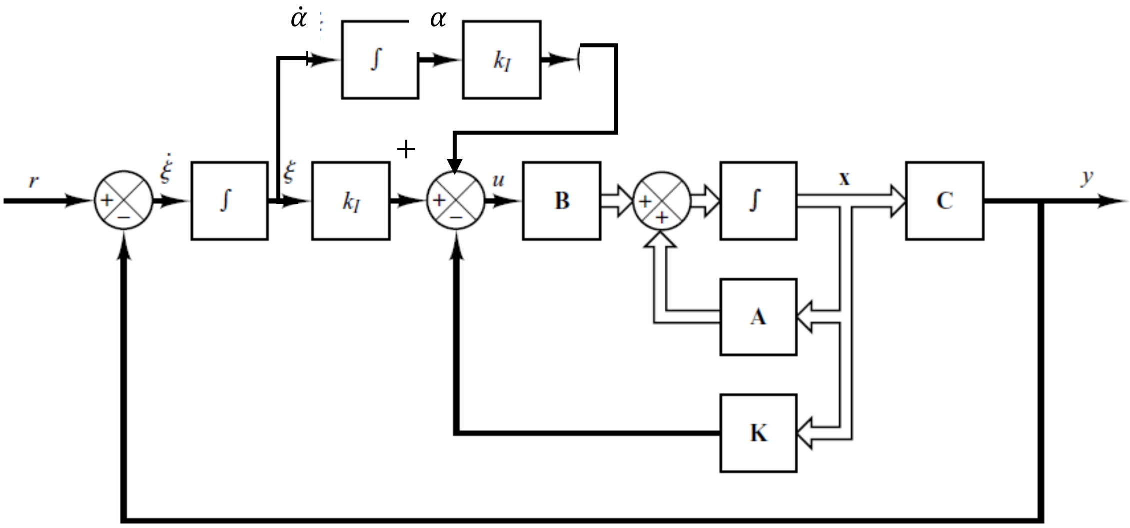
Por matriz de transformación

$$L = Q \begin{bmatrix} \alpha_n - a_n \\ \alpha_{n-1} - a_{n-1} \\ \vdots \\ \alpha_1 - a_1 \end{bmatrix}$$

$$Q = (W * P_O{}^*)^{-1}$$

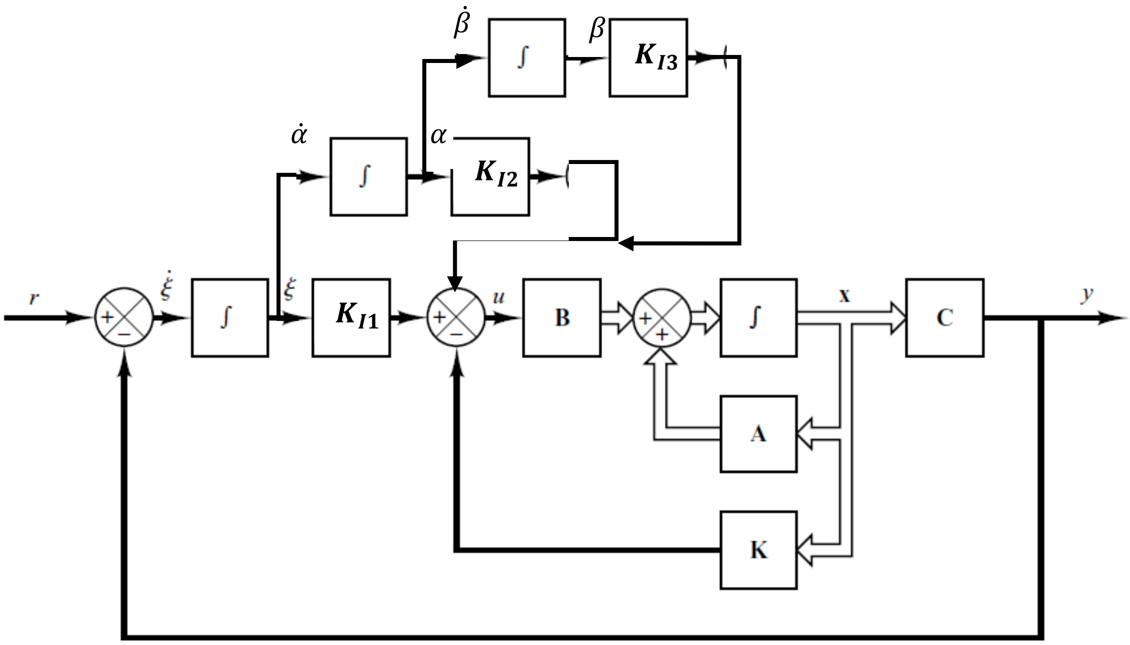
$$P_O{}^* \ \ es\ la\ matriz\ transpuesta\ conjugada$$

Servosistema tipo 2



$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\xi} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} (\boldsymbol{A}-\boldsymbol{BK}) & \boldsymbol{B}\,K_{I1} & \boldsymbol{B}K_{I2} \\ -\boldsymbol{C} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \xi \\ \alpha \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{1} \\ \boldsymbol{0} \end{bmatrix} r$$

Servosistema tipo 3



$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\xi} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} (\boldsymbol{A}-\boldsymbol{BK}) & \boldsymbol{B}\,K_{I1} & \boldsymbol{B}K_{I2} & \boldsymbol{B}K_{I3} \\ -\boldsymbol{C} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \xi \\ \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} r$$