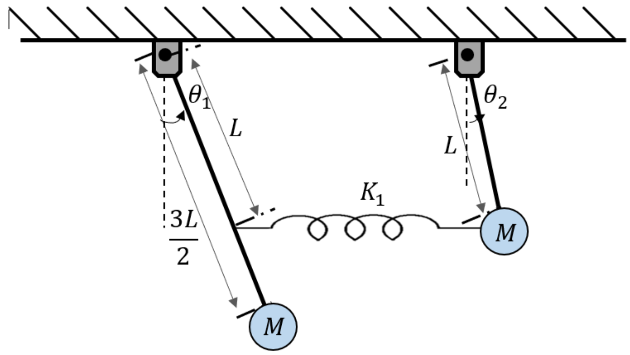
Solución Parcial Control

Punto uno



coordenadas generalizadas θ_1 y θ_2

Energía cinética

$$T = \frac{1}{2}M\left(\frac{3L}{2}\right)^2 \dot{\theta}_1^2 + \frac{1}{2}ML^2\dot{\theta}_2^2$$

Energía Potencial

$$U = Mg\frac{3L}{2}(1 - cos(\theta_1)) + MgL(1 - cos(\theta_2)) + \frac{1}{2}kL^2(sen(\theta_1) - sen(\theta_2))^2$$

Lagrangiano

$$L = \frac{1}{2}M\left(\frac{3L}{2}\right)^2 \dot{\theta}_1^2 + \frac{1}{2}ML^2\dot{\theta}_2^2 - Mg\frac{3L}{2}(1 - cos(\theta_1)) - MgL(1 - cos(\theta_2)) - \frac{1}{2}kL^2(sen(\theta_1) - sen(\theta_2))^2$$

$$L = \frac{1}{2}M\left(\frac{3L}{2}\right)^2 \dot{\theta}_1^2 + \frac{1}{2}ML^2\dot{\theta}_2^2 + Mg\frac{3L}{2}cos(\theta_1) + MgLcos(\theta_2) - \frac{1}{2}kL^2(sen(\theta_1) - sen(\theta_2))^2 - Mg\frac{3L}{2} - MgL$$

Ecuaciones de movimiento Euler Lagrange

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0$$

Primer término del lagrangiano primera coordenada generalizada

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = M\left(\frac{3L}{2}\right)^2 \ddot{\theta}_1$$

Segundo término lagrangiano

$$\frac{\partial L}{\partial \theta_1}$$

$$\frac{\partial L}{\partial \theta_1} = -Mg\frac{3L}{2}sen(\theta_1) - \; kL^2(sen(\theta_1)cos(\theta_1) - sen(\theta_2)cos(\theta_1))$$

La ecuación del movimiento 1

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0$$

$$M\left(\frac{3L}{2}\right)^2 \ddot{\theta}_1 + Mg\frac{3L}{2}sen(\theta_1) + \; kL^2cos(\theta_1)(sen(\theta_1) - sen(\theta_2)) = 0$$

Primer término del lagrangiano para la segunda coordenada generalizada

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = ML^2\ddot{\theta}_2$$

$$T = \frac{1}{2}M\left(\frac{3L}{2}\right)^2 \dot{\theta}_1^2 + \frac{1}{2}ML^2\dot{\theta}_2^2 + Mg\frac{3L}{2}cos(\theta_1) + MgLcos(\theta_2) - \frac{1}{2}kL^2(sen(\theta_1) - sen(\theta_2))^2 - Mg\frac{3L}{2} - MgL$$

Segundo término lagrangiano

$$\frac{\partial L}{\partial \theta_2}$$

$$\frac{\partial L}{\partial \theta_2} = -MgLsen(\theta_2) + kL^2(cos(\theta_2)sen(\theta_1) - cos(\theta_2)sen(\theta_2))$$

La ecuación del movimiento 2

$$ML^2\ddot{\theta}_2 + MgLsen(\theta_2) - kL^2cos(\theta_2)(sen(\theta_1) - sen(\theta_2)) = 0$$

Modelo

$$M\left(\frac{3L}{2}\right)^2 \ddot{\theta}_1 + Mg\frac{3L}{2}sen(\theta_1) + \; kL^2cos(\theta_1)(sen(\theta_1) - sen(\theta_2)) = 0$$

$$ML^2\ddot{\theta}_2 + MgLsen(\theta_2) - kL^2cos(\theta_2)(sen(\theta_1) - sen(\theta_2)) = 0$$

Definir Variables de estado θ_1 , $\dot{\theta}_1 = \omega_1$, θ_2 , $\dot{\theta}_2 = \omega_2$

$$\dot{\theta}_1 = \omega_1$$

$$\dot{\omega}_1 = \frac{1}{M\left(\frac{3L}{2}\right)^2}\left(-Mg\frac{3L}{2}sen(\theta_1) - \ kL^2(cos(\theta_1)sen(\theta_1) - cos(\theta_1)sen(\theta_2))\right)$$

$$\dot{\theta}_2 = \omega_2$$

$$\dot{\omega}_2 = \frac{1}{ML^2}\left(-MgLsen(\theta_2) + kL^2(cos(\theta_2)sen(\theta_1) - cos(\theta_2)sen(\theta_2))\right)$$

Puntos de operación $\theta_1 = \overline{\theta_1}$, $\theta_2 = \overline{\theta_2}$

Linealización

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{\partial f_2}{\partial \theta_1} & 0 & \frac{\partial f_2}{\partial \theta_2} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\partial f_4}{\partial \theta_1} & 0 & \frac{\partial f_4}{\partial \theta_2} & 0 \end{bmatrix}$$

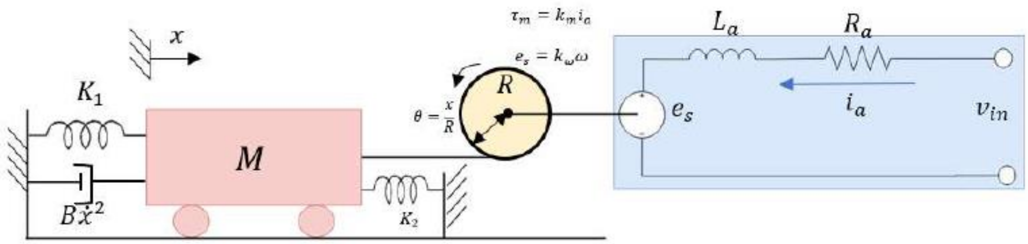
$$\frac{\partial f_2}{\partial \theta_1} = \frac{1}{M\left(\frac{3L}{2}\right)^2}\left(-Mg\frac{3L}{2}cos(\overline{\theta}_1) - \ kL^2(cos(2\overline{\theta_1}\ ) + sen(\overline{\theta_1}\ )sen(\overline{\theta}_2))\right)$$

$$\frac{\partial f_2}{\partial \theta_2} = \frac{1}{M\left(\frac{3L}{2}\right)^2}\left(-cos(\overline{\theta}_1)cos(\overline{\theta}_2))\right)$$

$$\frac{\partial f_4}{\partial \theta_1} = \frac{1}{ML^2}\left(kL^2(cos(\overline{\theta}_1)cos(\overline{\theta}_1))\right)$$

$$\frac{\partial f_4}{\partial \theta_2} = \frac{1}{ML^2}\left(-MgLcos(\overline{\theta}_2) + kL^2(-sen(\overline{\theta}_2)sen(\overline{\theta}_1) - cos(2\overline{\theta}_2))\right)$$

Punto dos



$$\dot{x} = v$$
$$\dot{v} = \frac{1}{M}\left(-(K1 + k2)x - Bv^2 + k_m\frac{i_a}{R}\right)$$
$$\frac{di_a}{dt} = \frac{1}{L_a}\left(-R_a i_a - k_\omega\frac{v}{R} + Vin\right)$$

$$\dot{x} = v$$
$$\dot{v} = (-2x - v^2 + i_a)$$
$$\frac{di_a}{dt} = (-i_a - v + Vin)$$

$$\dot{x} = v$$
$$\dot{v} = \frac{1}{M}\left(-(K1 + k2)x - Bv^2 + k_m\frac{i_a}{R}\right)$$
$$\frac{di_a}{dt} = \frac{1}{L_a}\left(-R_a i_a - k_\omega\frac{v}{R} + Vin\right)$$

Puntos de operación

$$0 = \bar{V}$$
$$2\bar{X} = (\bar{I}_a)$$
$$\bar{I}_a = (\bar{V}_{in})$$

Modelo Lineal

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{-(K1+k2)}{M} & -\dfrac{B}{M}\overline{V} & \dfrac{k_m}{RM} \\ 0 & -\dfrac{k_\omega}{RL_a} & -\dfrac{R_a}{L_a} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{L_a} \end{bmatrix}$$

Si M=1, K1=1, k2=1, B=1, $k_m = 1$, $k_\omega = 1$, R=1, L=1

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & -2\overline{V} & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Función de transferencia

$$G(s) = C(sI - A)^{-1}B$$

$$\frac{X(s)}{Vin(s)} = \frac{1}{s^3 + s^2 + 3s + 2}$$

PIDD²

$$G_c(s) = \frac{K_p s + K_I + K_D s^2 + K_{D2} s^3}{s}$$

Función de transferencia en lazo abierto

$$G(s) = \frac{1}{s^3 + s^2 + 3s + 2} \frac{K_p s + K_I + K_D s^2 + K_{D2} s^3}{s}$$

$$Kp = \lim_{s \to 0} G(s) = \infty$$

$$ess = \frac{1}{Kp} = 0$$

Función de transferencia en lazo cerrado

$$M(s) = \frac{\frac{1}{s^3+s^2+3s+2} * \frac{K_p s + K_I + K_D s^2 + K_{D2} s^3}{s}}{1 + \frac{1}{s^3+s^2+3s+2} * \frac{K_p s + K_I + K_D s^2 + K_{D2} s^3}{s}}$$

$$\frac{K_p s + K_I + K_D s^2 + K_{D2} s^3}{s(s^3+s^2+3s+2) + K_p s + K_I + K_D s^2 + K_{D2} s^3}$$

Polinomio característico en lazo cerrado

$$s(s^3+s^2+3s+2) + K_p s + K_I + K_D s^2 + K_{D2} s^3$$

$$s^4 + s^3 + 3s^2 + 2s + K_p s + K_I + K_D s^2 + K_{D2} s^3$$

$$s^4 + s^3 + K_{D2} s^3 + 3s^2 + K_D s^2 + 2s + K_p s + K_I$$

$$s^4 + (1 + K_{D2})s^3 + (3 + K_D)s^2 + \left(2 + K_p\right)s + K_I$$

$$\zeta = 0.59$$

$$Tss = 0.5s$$

Polinomio deseado

$$PD = (s^2 + 2\zeta\omega_n s + {\omega_n}^2)(s + 10\zeta\omega_n)(s + 10\zeta\omega_n)$$

Coeficientes del polinio deseado

1.0000   11.6667   40.1361   37.5129   17.4967

$$1 + K_{D2} = 11.6667 \implies K_{D2} = 10.6667$$

$$3 + K_D = 40.1361 \implies K_D = 37.1361$$

$$2 + K_p = 37.5129 \implies K_p = 39.5129$$

$$K_I = 17.4967$$