Modelado de LaGrange:

$$T=rac{1}{2}mv^2=rac{1}{2}m(\dot{x}^2+\dot{y}^2) \ x=lsin(heta) \ y=lcos(heta)+Asin(\omega t) \ \dot{x}=sin(heta)\dot{l}+lsin(heta)\dot{ heta} \ \dot{y}=cos(heta)\dot{l}-lsin(heta)\dot{ heta}+A\omega sin(\omega t)$$

Energía cinética:

$$T=rac{1}{2}m[{\dot l}^{\,2}+l^{2}{\dot heta}^{2}+(A\omega sin(\omega t))^{2}+2A\omega cos(\omega t)[cos(heta){\dot l}\,-lsin(heta){\dot heta}]]$$

Energía potencial:

$$V_g=mgy$$
 $V_m=rac{1}{2}K[l-l_0]^2$ $V=V_m+V_g=rac{1}{2}K[l-l_0]^2+mg[lcos(heta)+Asin(\omega t)]$

Lagrangiano:

$$\frac{1}{2}m[\dot{l}^{2}+l^{2}\dot{\theta}^{2}+(A\omega sin(\omega t))^{2}+2A\omega cos(\omega t)[cos(\theta)\dot{l}-lsin(\theta)\dot{\theta}]]-\frac{1}{2}K[l-l_{0}]^{2}-mg[lcos(\theta)+Asin(\omega t)]$$

Ecuación 1:

$$egin{aligned} rac{\partial L}{\partial heta} &= -mA\omega cos(\omega t)[sin(heta)\,\dot{l} + lcos(heta)\dot{ heta}] + mglsin heta \ &rac{\partial L}{\partial \dot{ heta}} &= ml^2\,\dot{ heta} - mA\omega cos(\omega t)lsin(heta) \ &rac{d}{dt}[rac{\partial L}{\partial \dot{ heta}} &= ml^2\,\ddot{ heta} + 2ml\dot{l}\,\dot{ heta} + mA\omega^2 sin(\omega t)lsin(heta) - mA\omega cos(\omega t)[\dot{l}\,sin heta + lcos heta\dot{ heta}]] \ &l\ddot{ heta} + 2\dot{l}\,\dot{ heta} + [A\omega^2 sin(wt) - g]sin(heta) &= FL \end{aligned}$$

Ecuación 2:

$$\begin{split} \frac{\partial L}{\partial l} &= ml\dot{\theta}^2 - mA\omega cos(\omega t)sin(\theta)\dot{\theta} - K[l-l_o] - mgcos(\theta) \\ &\frac{\partial L}{\partial \dot{l}} = m\dot{l} + mA\omega cos(\omega t)cos(\theta) \\ \\ \frac{d}{dt}[\frac{\partial L}{\partial \dot{l}}] &= m\ddot{l} - mA\omega^2sin(\omega t)cos(\theta) - mA\omega cos(\omega t)sin(\theta)\dot{\theta} \\ \\ \ddot{l} &= l\dot{\theta}^2 + \frac{K}{m}[l-l_o] + [g - A\omega^2sin(\omega t)]cos\theta = 0 \end{split}$$

Ecuaciones:

$$egin{aligned} l\ddot{ heta} + 2\dot{l}\,\dot{ heta} + [A\omega^2 sin(wt) - g]sin(heta) &= Fl \ \ddot{l} - l\dot{ heta}^2 + rac{K}{m}[l - l_o] + [g - A\omega^2 sin(\omega t)]cos heta &= 0 \end{aligned}$$