

$$\dot{x}_1 = x_2$$

$$\ddot{x}_2 = \left(\frac{-1}{\frac{J_b}{R^2} + M} \right) m g \sin \alpha$$

$$M = 0.02$$

$$g = -9.81$$

$$J_b = 1 \times 10^{-6}$$

$$R = 0.01$$

$$\alpha = 0.02$$

Jacobiano

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-m g \cos(\alpha)}{M + \frac{J_b}{R^2}} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Linearization

$$\bar{x}_1 = 0.1 \text{ [m]}$$

$$\dot{\bar{x}}_2 = \dot{\bar{x}}_1 = 0$$

$$0 = \left(\frac{-1}{\frac{J_b}{R^2} + M} \right) m g \sin \alpha$$

$$u = 0$$

$$G = \frac{327 \cos(\alpha)}{305^2}$$

Espacio de estados

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0.654 \end{bmatrix}$$

matriz de controlabilidad

$$P_c = \begin{bmatrix} B & AB \end{bmatrix}$$

$$P_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det(P_c) = 1 \rightarrow \text{es controlable}$$

matriz de observabilidad

$$P_o = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$$P_o = \begin{bmatrix} 0 & 0.654 \\ 0.654 & 0 \end{bmatrix}$$

$$\det(P_o) = -42.777 \rightarrow \text{es observable}$$

servo sistema

$$P_c = \det \left[sI - \begin{bmatrix} A-BK & BK_i \\ -C & 0 \end{bmatrix} \right]$$

$$K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$$

$$P_c = \begin{bmatrix} -k_1 & -k_2 & k_1 \\ 1 & 0 & 0 \\ 0 & -\frac{327}{50} & 0 \end{bmatrix}$$

$$P_c = s^3 + k_1 s^2 + k_2 s + \frac{327 k_1}{50}$$

$$PD = s^3 + 6.857s^2 + 7.465s + 3.339$$

se iguala $PD = PC$

$$6.857 = k_1$$

$$k_1 = 6.857$$

$$7.465 = k_2$$

$$k_2 = 7.465$$

$$3.339 = \frac{327}{50} k_i$$

$$k_i = 0.8164$$

Observador de estados

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$

$$PCO = \det [sI - (A - LC)]$$

$$PCO = \det \left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -327L_1/50 \\ 1 & -327L_2/50 \end{bmatrix} \right]$$

$$PCO = s^2 + \frac{327}{50} L_2 s + \frac{327L_1}{50}$$

$$Pdo = s^2 + 320s + 102400$$

se iguala $Pdo = PCO$

$$\frac{327}{50} L_2 = 320$$

$$L_1 = 15657$$

$$L_2 = 48.92$$

$$\frac{327L_1}{50} = 102400$$