Modelo

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \left(\frac{-1}{\frac{Jb}{R^2} + M} \right) Mg \sin(u)$$

Constantes

$M = 0{,}02\ kg$

$g = -9{,}81\ m/s^2$

$Jb = 1 \times 10^{-6}\ kg\,m^2$

$R = 0{,}01 [m]$

$J = 0{,}02\ kg\,m^2$

linealizando por Jacobiano.

$$\begin{bmatrix} \dot{x}_1\delta \\ \dot{x}_2\delta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1\delta \\ x_2\delta \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{327 \cos(u)}{50} \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$$

Ptos equilibrio

$\bar{x}_1 = 0{,}1\,m$

$\dot{x}_1 = 0$

$\dot{x}_2 = 0 \rightarrow 0 = \frac{327}{50} \sin(u) \rightarrow \bar{u} = 0$

$\bar{x}_3 = 0$

Ackerman Brian Careres Pinzón 1803245

$C = [\hat{B} \quad \hat{A}\hat{B} \quad \hat{A}^2\hat{B}]$

Hallamos $\hat{A}$ y $\hat{B}$

$$\hat{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 6{,}54 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 6{,}54 \\ 0 \end{bmatrix}$$

Hallamos la matriz de Controlabilidad

$$C = \begin{bmatrix} 0 & 6{,}54 & 0 \\ 6{,}54 & 0 & 0 \\ 0 & 0 & -6{,}54 \end{bmatrix}$$

Hallamos φA con el polinomio deseado

$$\varphi A = \hat{A}^3 + 9{,}6\hat{A}^2 + 14{,}106\,\hat{A} + 10{,}45 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\varphi A = \begin{bmatrix} 10{,}45 & 14{,}1060 & 0 \\ 0 & 10{,}45 & 0 \\ -14{,}1060 & -9{,}6 & 10{,}45 \end{bmatrix}$$

Aplicando el método de Ackerman

$$K = [0 \ 0 \ 1] \cdot C^{-1} \cdot \varphi A.$$

$$K = [0 0 1] \begin{bmatrix} 0 & 6{,}54 & 0 \\ 6{,}54 & 0 & 0 \\ 0 & 0 & -6{,}54 \end{bmatrix} \begin{bmatrix} 10{,}45 & 14{,}1060 & 0 \\ 0 & 10{,}45 & 0 \\ -14{,}1060 & -9{,}6 & 10{,}45 \end{bmatrix}$$

Se obtienen las K

$$K = \begin{bmatrix} 2{,}1569 \\ 1{,}4679 \\ -1{,}5979 \end{bmatrix} \begin{matrix} \leftarrow K_1 \\ \leftarrow K_2 \\ \leftarrow K_i \end{matrix}$$

Observador

Matriz de observabilidad

$$O = \begin{bmatrix} C \\ CA \end{bmatrix} \Rightarrow \det(O) = 1 \quad \text{es observable}$$

$\xi = 0{,}5$ $\qquad O = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$t_s = 0{,}025$ segundos

$$\omega_n = \frac{4}{(0{,}5)(0{,}025)} = 320 \left[\frac{rad}{s}\right]$$

Polinomio deseado $= s^2 + 2\xi\omega_n s + \omega_n^2$

Orden 2 $\rightarrow$ 2 estados a observar

Polinomio deseado $= s^2 + 320s + 102400$

$$\varphi_{obs} = A^2 + 320A + 102400 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$L = \varphi_{obs} \cdot O^{-1} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

matlab

$$L = \begin{bmatrix} 320 \\ 102400 \end{bmatrix} \begin{matrix} \leftarrow L_1 \\ \leftarrow L_2 \end{matrix}$$