

Modelado de LaGrange:

$$\begin{aligned}
 T &= \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) \\
 x &= l\sin(\theta) \\
 y &= l\cos(\theta) + A\sin(\omega t) \\
 \dot{x} &= \sin(\theta)\dot{l} + l\sin(\theta)\dot{\theta} \\
 \dot{y} &= \cos(\theta)\dot{l} - l\sin(\theta)\dot{\theta} + A\omega\sin(\omega t)
 \end{aligned}$$

Energía cinética:

$$T = \frac{1}{2}m[\dot{l}^2 + l^2\dot{\theta}^2 + (A\omega\sin(\omega t))^2 + 2A\omega\cos(\omega t)[\cos(\theta)\dot{l} - l\sin(\theta)\dot{\theta}]]$$

Energía potencial:

$$\begin{aligned}
 V_g &= mgy \\
 V_m &= \frac{1}{2}K[l - l_0]^2 \\
 V &= V_m + V_g = \frac{1}{2}K[l - l_0]^2 + mg[l\cos(\theta) + A\sin(\omega t)]
 \end{aligned}$$

Lagrangiano:

$$\frac{1}{2}m[\dot{l}^2 + l^2\dot{\theta}^2 + (A\omega\sin(\omega t))^2 + 2A\omega\cos(\omega t)[\cos(\theta)\dot{l} - l\sin(\theta)\dot{\theta}]] - \frac{1}{2}K[l - l_0]^2 - mg[l\cos(\theta) + A\sin(\omega t)]$$

Ecuación 1:

$$\begin{aligned}
 \frac{\partial L}{\partial \theta} &= -mA\omega\cos(\omega t)[\sin(\theta)\dot{l} + l\cos(\theta)\dot{\theta}] + mgl\sin\theta \\
 \frac{\partial L}{\partial \dot{\theta}} &= ml^2\dot{\theta} - mA\omega\cos(\omega t)l\sin(\theta) \\
 \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}}\right] &= ml^2\ddot{\theta} + 2ml\dot{l}\dot{\theta} + mA\omega^2\sin(\omega t)l\sin(\theta) - mA\omega\cos(\omega t)[\dot{l}\sin\theta + l\cos\theta\dot{\theta}] \\
 l\ddot{\theta} + 2\dot{l}\dot{\theta} + [A\omega^2\sin(\omega t) - g]\sin(\theta) &= FL
 \end{aligned}$$

Ecuación 2:

$$\begin{aligned}
 \frac{\partial L}{\partial l} &= m\dot{\theta}^2 - mA\omega\cos(\omega t)\sin(\theta)\dot{\theta} - K[l - l_0] - mg\cos(\theta) \\
 \frac{\partial L}{\partial \dot{l}} &= m\dot{l} + mA\omega\cos(\omega t)\cos(\theta) \\
 \frac{d}{dt}\left[\frac{\partial L}{\partial \dot{l}}\right] &= m\ddot{l} - mA\omega^2\sin(\omega t)\cos(\theta) - mA\omega\cos(\omega t)\sin(\theta)\dot{\theta} \\
 \ddot{l} - l\dot{\theta}^2 + \frac{K}{m}[l - l_0] + [g - A\omega^2\sin(\omega t)]\cos\theta &= 0
 \end{aligned}$$

Ecuaciones:

$$\begin{aligned}
 l\ddot{\theta} + 2\dot{l}\dot{\theta} + [A\omega^2\sin(\omega t) - g]\sin(\theta) &= Fl \\
 \ddot{l} - l\dot{\theta}^2 + \frac{K}{m}[l - l_0] + [g - A\omega^2\sin(\omega t)]\cos\theta &= 0
 \end{aligned}$$

