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Rank of the Word FOCUS

Date: 06/27/2003 at 01:41:26
From: Irfan Nasir
Subject: Permutation and combination

Dear Dr. Math

The letters "CFOSU" are arranged in dictionary order. What is the rank of the word "FOCUS" in this order?

Thank you a lot in anticipation.

Date: 07/10/2003 at 17:16:59
From: Doctor Achilles
Subject: Re: Permutation and combination

Hi Irfan,

Thanks for writing to Dr. Math.

Let's use a code to solve this problem. We have the following letters:

C

F

O

S

U

And we will take all the permutations of these letters and arrange them in alphabetical order.

I think it will be helpful to think of these letters as numbers. The first letter, alphabetically, in this list is C, so let's replace "C" with the number 1.

The second letter is F, so let's replace it with 2.

And so on.

Using this code, the word "FOCUS" will be the sequence:

2 3 1 5 4

The first permutation in the sequence "alphabetically" is:

1 2 3 4 5

Then

1 2 3 5 4

Then

1 2 4 3 5

And it goes on like that.

The permutation we want is, again:

2 3 1 5 4

Which starts with a 2, so it comes AFTER every permutation that starts with a 1. How many permutations start with a 1? The answer to that is the number of ways the digits:

  2 3 4 5

can be arranged. A quick look at our FAQ on permutations

   Permutations and Combinations
   http://www.mathforum.com/dr.math/faq/faq.comb.perm.html

tells us that that number of combinations is going to be 4! (4*3*2*1) or 24.

So there are 24 permutations that begin with 1. That means that the 25th permutation is:

  2 1 3 4 5

Now we've gotten the first digit in place. The next digit we want is 3. So we are going to have to work through all of the permutations that start with "2 1". How many are there? 3! (3*2*1) or 6.

So the 31st permutation is:

  2 3 1 4 5

And the 32nd permutation is:

  2 3 1 5 4

Which is the one we wanted.

So the steps to solving this are:

1) Assign a number to each letter using alphabetical order.

2) Determine what number sequence corresponds to the word you want.

3) Determine how many permutations it takes to get the first letter into place.

4) Now that the first letter is set, focus on the remaining letters and repeat step 3 until you're done.

I hope this helps. If you have other questions or you'd like to talk about this some more, please write back.

- Doctor Achilles, The Math Forum
  http://mathforum.org/dr.math/

Associated Topics:
High School Permutations and Combinations