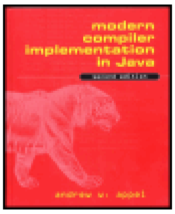
Modern Compiler Implementation in Java, Second Edition



by Andrew W. Appel and Jens Palsberg ISBN:052182060x

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Back Cover

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The first part of the book, Fundamentals of Compilation, is suitable for a one-semester first course in compiler design. The second part, Advanced Topics, which includes the compilation of object-oriented and functional languages, garbage collection, loop optimization, SSA form, instruction scheduling, and optimization for cache-memory hierarchies, can be used for a second-semester or graduate course.

This new edition has been rewritten extensively to include more discussion of Java and object-oriented programming concepts, such as visitor patterns. A unique feature in the newly redesigned compiler project in Java for a subset of Java itself. The project includes both front-end and back-end phases, so that students can build a complete working compiler in one semester.

About the Authors

Andrew W. Appel is Professor of Computer Science at Princeton University. He has done research and published papers on compilers, functional programming languages, runtime systems and garbage collection, type systems, and computer security; he is also the author of the book *Compiling with Continuations*. He is a designer and founder of the Standard ML of New Jersey project. In 1998, Appel was elected a Fellow of the Association for Computing Machinery for “significant research contributions in the area of programming languages and compilers” and for his work as editor-in-chief (1993-7) of the *ACM Transactions on Programming Languages and Systems*, the leading journal in the field of compilers and programming languages.

Hens Palsberg is Associate Professor of Computer Science at Purdue University. His research interests are programming languages, compilers, software engineering, and information security. He has authored more than 50 technical papers in these areas and a book with Michael Schwartzbach, *Object-Oriented Type Systems*. In 1998, he received the National Science Foundation Faculty Early Career Development Award, and in 1999, the Purdue

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Preface

This book is intended as a textbook for a one- or two-semester course in compilers. Students will see the theory behind different components of a compiler, the programming techniques used to put the theory into practice, and the interfaces used to modularize the compiler. To make the interfaces and programming examples clear and concrete, we have written them in Java. Another edition of this book is available that uses the ML language.

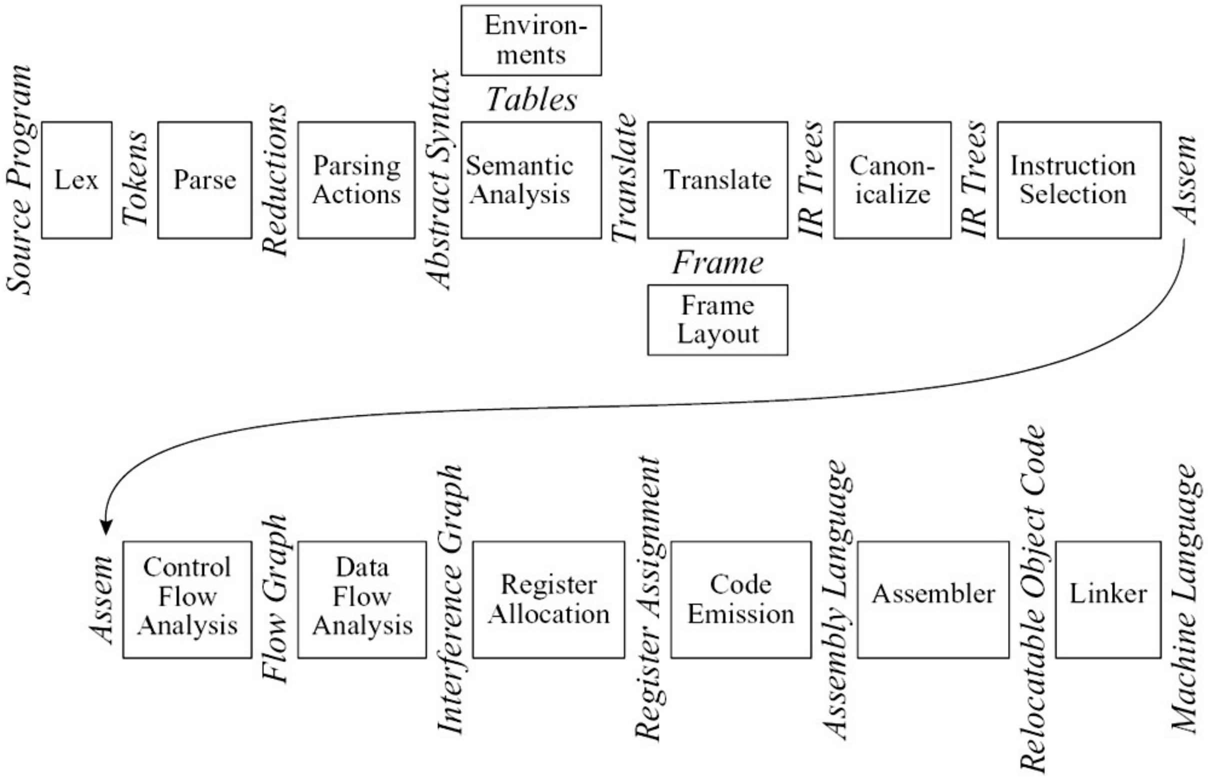
Implementation project The "student project compiler" that we have out-lined is reasonably simple, but is organized to demonstrate some important techniques that are now in common use: abstract syntax trees to avoid tangling syntax and semantics, separation of instruction selection from register allocation, copy propagation to give flexibility to earlier phases of the compiler, and containment of target-machine dependencies. Unlike many "student compilers" found in other textbooks, this one has a simple but sophisticated back end, allowing good register allocation to be done after instruction selection.

This second edition of the book has a redesigned project compiler: It uses a subset of Java, called MiniJava, as the source language for the compiler project, it explains the use of the parser generators JavaCC and SableCC, and it promotes programming with the Visitor pattern. Students using this edition can implement a compiler for a language they're familiar with, using standard tools, in a more object-oriented style.

Each chapter in Part I has a programming exercise corresponding to one module of a compiler. Software useful for the exercises can be found at
http://uk.cambridge.org/resources/052182060X (outside North America);
http://us.cambridge.org/titles/052182060X.html (within North America).

Exercises Each chapter has pencil-and-paper exercises; those marked with a star are more challenging, two-star problems are difficult but solvable, and the occasional three-star exercises are not known to have a solution.

Course sequence The figure shows how the chapters depend on each other.



- A one-semester course could cover all of Part I (Chapters 1-12), with students implementing the project compiler (perhaps working in groups); in addition, lectures could cover selected topics from Part II.
- An advanced or graduate course could cover Part II, as well as additional topics from the current literature. Many of the Part II chapters can stand independently from Part I, so that an advanced course could be taught to students who have used a different book for their first course.
- In a two-quarter sequence, the first quarter could cover Chapters 1-8, and the second quarter could cover Chapters 9-12 and some chapters from Part II.

Acknowledgments Many people have provided constructive criticism or helped us in other ways on this book. Vidyut Samanta helped tremendously with both the text and the software for the new edition of the book. We would also like to thank Leonor Abraido-Fandino, Scott Ananian, Nils Andersen, Stephen Bailey, Joao Cangussu, Maia Ginsburg, Max Hailperin, David Hanson, Jeffrey Hsu, David MacQueen, Torben Mogensen, Doug Morgan, Robert Netzer, Elma Lee Noah, Mikael Petterson, Benjamin Pierce, Todd Proebsting, Anne Rogers, Barbara Ryder, Amr Sabry, Mooly Sagiv, Zhong Shao, Mary Lou Soffa, Andrew Tolmach, Kwangkeun Yi, and Kenneth Zadeck.

Part One: Fundamentals of Compilation

Chapter List

Chapter 1: Introduction

A **compiler** was originally a program that "compiled" subroutines [a link-loader]. When in 1954 the combination "algebraic compiler" came into use, or rather into misuse, the meaning of the term had already shifted into the present one.

Bauer and Eickel [1975]

OVERVIEW

This book describes techniques, data structures, and algorithms for translating programming languages into executable code. A modern compiler is often organized into many phases, each operating on a different abstract "language." The chapters of this book follow the organization of a compiler, each covering a successive phase.

To illustrate the issues in compiling real programming languages, we show how to compile MiniJava, a simple but nontrivial subset of Java. Programming exercises in each chapter call for the implementation of the corresponding phase; a student who implements all the phases described in Part I of the book will have a working compiler. MiniJava is easily extended to support class extension or higher-order functions, and exercises in Part II show how to do this. Other chapters in Part II cover advanced techniques in program optimization. Appendix A describes the MiniJava language.

The interfaces between modules of the compiler are almost as important as the algorithms inside the modules. To describe the interfaces concretely, it is useful to write them down in a real programming language. This book uses Java - a simple object-oriented language. Java is *safe,* in that programs cannot circumvent the type system to violate abstractions; and it has garbage collection, which greatly simplifies the management of dynamic storage allocation. Both of these properties are useful in writing compilers (and almost any kind of software).

This is not a textbook on Java programming. Students using this book who do not know Java already should pick it up as they go along, using a Java programming book as a reference. Java is a small enough language, with simple enough concepts, that this should not be difficult for students with good programming skills in other languages.

1.1 MODULES AND INTERFACES

Any large software system is much easier to understand and implement if the designer takes care with the fundamental abstractions and interfaces. Figure 1.1 shows the phases in a typical compiler. Each phase is implemented as one or more software modules.

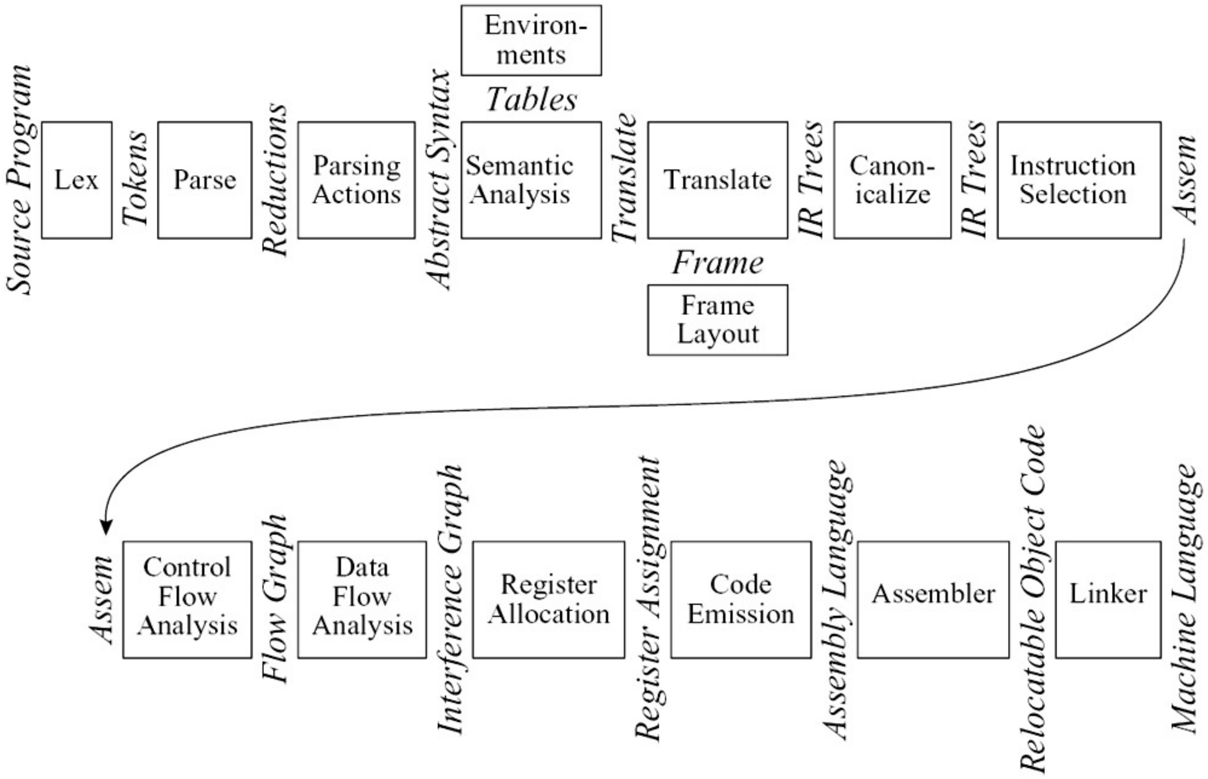


Figure 1.1: Phases of a compiler, and interfaces between them.

Breaking the compiler into this many pieces allows for reuse of the components. For example, to change the target machine for which the compiler produces machine language, it suffices to replace just the Frame Layout and Instruction Selection modules. To change the source language being compiled, only the modules up through Translate need to be changed. The compiler can be attached to a language-oriented syntax editor at the *Abstract Syntax* interface.

The learning experience of coming to the right abstraction by several iterations of *think-implement-redesign* is one that should not be missed. However, the student trying to finish a compiler project in one semester does not have this luxury. Therefore, we present in this book the outline of a project where the abstractions and interfaces are carefully thought out, and are as elegant and general as we are able to make them.

Some of the interfaces, such as *Abstract Syntax, IR Trees,* and *Assem*, take the form of data structures: For example, the Parsing Actions phase builds an *Abstract Syntax* data structure and passes it to the Semantic Analysis phase. Other interfaces are abstract data types; the *Translate* interface is a set of functions that the Semantic Analysis phase can call, and the *Tokens* interface takes the form of a function that the Parser calls to get the next token of the input program.

DESCRIPTION OF THE PHASES

Each chapter of Part I of this book describes one compiler phase, as shown in Table 1.2

Table 1.2: Description of compiler phases.

Chapter	Phase	Description
2	Lex	Break the source file into individual words, or *tokens*.

Table 1.2: Description of compiler phases.

Chapter	Phase	Description
3	Parse	Analyze the phrase structure of the program.
4	Semantic Actions	Build a piece of *abstract syntax tree* corresponding to each phrase.
5	Semantic Analysis	Determine what each phrase means, relate uses of variables to their definitions, check types of expressions, request translation of each phrase.
6	Frame Layout	Place variables, function-parameters, etc. into activation records (stack frames) in a machine-dependent way.
7	Translate	Produce *intermediate representation trees* (IR trees), a notation that is not tied to any particular source language or target-machine architecture.
8	Canonicalize	Hoist side effects out of expressions, and clean up conditional branches, for the convenience of the next phases.
9	Instruction Selection	Group the IR-tree nodes into clumps that correspond to the actions of target-machine instructions.
10	Control Flow Analysis	Analyze the sequence of instructions into a *control flow graph* that shows all the possible flows of control the program might follow when it executes.
10	Dataflow Analysis	Gather information about the flow of information through variables of the program; for example, *liveness analysis* calculates the places where each program variable holds a still-needed value (is *live*).
11	Register Allocation	Choose a register to hold each of the variables and temporary values used by the program; variables not live at the same time can share the same register.
12	Code Emission	Replace the temporary names in each machine instruction with machine registers.

This modularization is typical of many real compilers. But some compilers combine Parse, Semantic Analysis, Translate, and Canonicalize into one phase; others put Instruction Selection much later than we have done, and combine it with Code Emission. Simple compilers omit the Control Flow Analysis, Data Flow Analysis, and Register Allocation phases.

We have designed the compiler in this book to be as simple as possible, but no simpler. In particular, in those places where corners are cut to simplify the implementation, the structure of the compiler allows for the addition of more optimization or fancier semantics without violence to the existing interfaces.

1.2 TOOLS AND SOFTWARE

Two of the most useful abstractions used in modern compilers are *contextfree grammars*, for parsing, and *regular expressions*, for lexical analysis. To make the best use of these abstractions it is helpful to have special tools, such as *Yacc* (which converts a grammar into a parsing program) and *Lex* (which converts a declarative specification into a lexical-analysis program). Fortunately, such tools are available for Java, and the project described in this book makes use of them.

The programming projects in this book can be compiled using any Java compiler. The parser generators *JavaCC* and *SableCC* are freely available on the Internet; for information see the World Wide Web page

http://uk.cambridge.org/resources/052182060X (outside North America);

http://us.cambridge.org/titles/052182060X.html (within North America).

Source code for some modules of the MiniJava compiler, skeleton source code and support code for some of the programming exercises, example MiniJava programs, and other useful files are also available from the same Web address. The programming exercises in this book refer to this directory as `$MINIJAVA/` when referring to specific subdirectories and files contained therein.

1.3 DATA STRUCTURES FOR TREE LANGUAGES

Many of the important data structures used in a compiler are *intermediate representations* of the program being compiled. Often these representations take the form of trees, with several node types, each of which has different attributes. Such trees can occur at many of the phase-interfaces shown in Figure 1.1.

Tree representations can be described with grammars, just like programming languages. To introduce the concepts, we will show a simple programming language with statements and expressions, but no loops or if-statements (this is called a language of *straight-line programs*).

The syntax for this language is given in Grammar 1.3.

GRAMMAR 1.3: A straight-line programming language.

Stm → *Stm*; *Stm*	(CompoundStm)
Stm → `id :=` *Exp*	(AssignStm)
Stm → `print` (*ExpList*)	(PrintStm)

Exp → id	(IdExp)
Exp → num	(NumExp)
Exp → *Exp Binop Exp*	(OpExp)
Exp → (*Stm*, *Exp*)	(EseqExp)
ExpList → *Exp*, *ExpList*	(PairExpList)
ExpList → *Exp*	(LastExpList)
Binop →+	(Plus)
Binop →−	(Minus)
Binop →×	(Times)
Binop → /	(Div)

The informal semantics of the language is as follows. Each *Stm* is a statement, each *Exp* is an expression. s_1; s_2 executes statement s_1, then statement s_2. i :=e evaluates the expression e, then "stores" the result in variable i. `print`(e_1, e_2,…, e_n) displays the values of all the expressions, evaluated left to right, separated by spaces, terminated by a newline.

An *identifier expression*, suchas i, yields the current contents of the variable i. A *number* evaluates to the named integer. An *operator expression* e_1 op e_2 evaluates e_1, then e_2, then applies the given binary operator. And an *expression sequence* (s, e) behaves like the C-language "comma" operator, evaluating the statement s for side effects before evaluating (and returning the result of) the expression e.

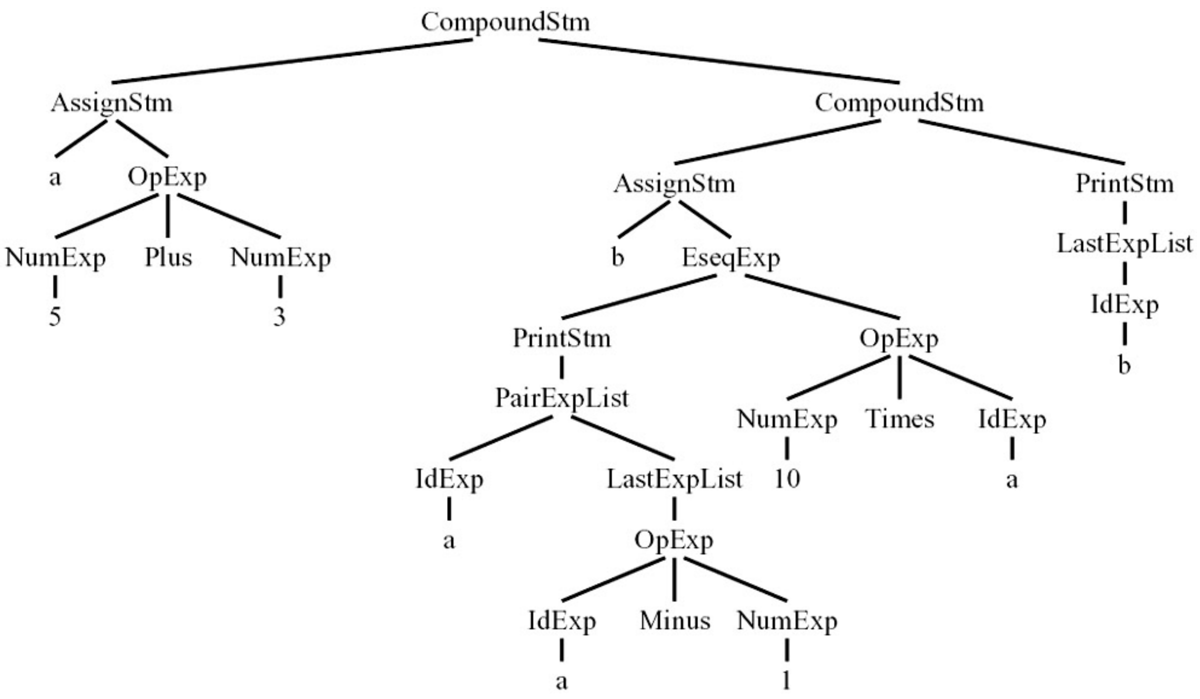
For example, executing this program

```
a := 5+3; b := (print(a, a-1), 10*a); print(b)
```

prints

```
8 7
80
```

How should this program be represented inside a compiler? One representation is *source code*, the characters that the programmer writes. But that is not so easy to manipulate. More convenient is a tree data structure, with one node for each statement (Stm) and expression (Exp). Figure 1.4 shows a tree representation of the program; the nodes are labeled by the production labels of Grammar 1.3, and each node has as many children as the corresponding grammar production has right-hand-side symbols.



```
a := 5 + 3 ; b := ( print ( a , a - 1 ) , 10 * a ) ; print ( b )
```

Figure 1.4: Tree representation of a straight-line program.

We can translate the grammar directly into data structure definitions, as shown in Program 1.5. Each grammar symbol corresponds to an `abstract class` in the data structures:

Grammar	**class**
Stm	Stm
Exp	Exp
ExpList	ExpList
id	String
num	int

PROGRAM 1.5: Representation of straight-line programs.

```
public abstract class Stm {}

public class CompoundStm extends Stm {
   public Stm stm1, stm2;
   public CompoundStm(Stm s1, Stm s2) {stm1=s1; stm2=s2;}}

public class AssignStm extends Stm {
   public String id; public Exp exp;
   public AssignStm(String i, Exp e) {id=i; exp=e;}}

public class PrintStm extends Stm {
   public ExpList exps;
   public PrintStm(ExpList e) {exps=e;}}

public abstract class Exp {}

public class IdExp extends Exp {
   public String id;
   public IdExp(String i) {id=i;}}

public class NumExp extends Exp {
   public int num;
   public NumExp(int n) {num=n;}}

public class OpExp extends Exp {
   public Exp left, right; public int oper;
   final public static int Plus=1, Minus=2, Times=3, Div=4;
   public OpExp(Exp l, int o, Exp r) {left=l; oper=o; right=r;}}

public class EseqExp extends Exp {
   public Stm stm; public Exp exp;
   public EseqExp(Stm s, Exp e) {stm=s; exp=e;}}

public abstract class ExpList {}

public class PairExpList extends ExpList {
   public Exp head; public ExpList tail;
   public PairExpList(Exp h, ExpList t) {head=h; tail=t;}}

public class LastExpList extends ExpList {
   public Exp head;
   public LastExpList(Exp h) {head=h;}}
```

For each grammar rule, there is one *constructor* that belongs to the class for its left-hand-side symbol. We simply *extend* the abstract class with a "concrete" class for each grammar rule. The constructor (class) names are indicated on the right-hand side of Grammar 1.3.

Each grammar rule has right-hand-side components that must be represented in the data structures. The CompoundStm has two Stm's on the right-hand side; the AssignStm has an identifier and an expression; and so on.

These become *fields* of the subclasses in the Java data structure. Thus, CompoundStm has two fields (also called *instance variables*) called `stm1` and `stm2`; AssignStm has fields `id` and `exp`.

For Binop we do something simpler. Although we could make a Binop class - with subclasses for Plus, Minus, Times, Div - this is overkill because none of the subclasses would need any fields. Instead we make an "enumeration" type (in Java, actually an integer) of constants (`final int` variables) local to the OpExp class.

Programming style We will follow several conventions for representing tree data structures in Java:

1. Trees are described by a grammar.
2. A tree is described by one or more abstract classes, each corresponding to a symbol in the grammar.
3. Each abstract class is *extended* by one or more subclasses, one for each grammar rule.
4. For each nontrivial symbol in the right-hand side of a rule, there will be one field in the corresponding class. (A trivial symbol is a punctuation symbol such as the semicolon in CompoundStm.)
5. Every class will have a constructor function that initializes all the fields.
6. Data structures are initialized when they are created (by the constructor functions), and are never modified after that (until they are eventually discarded).

Modularity principles for Java programs A compiler can be a big program; careful attention to modules and interfaces prevents chaos. We will use these principles in writing a compiler in Java:

1. Each phase or module of the compiler belongs in its own package.
2. "Import on demand" declarations will not be used. If a Java file begins with
3. `import A.F.*; import A.G.*; import B.*; import C.*;`

   then the human reader *will have to look outside this file* to tell which package defines the `X` that is used in the expression `X.put()`.

4. "Single-type import" declarations are a better solution. If the module begins
5. `import A.F.W; import A.G.X; import B.Y; import C.Z;`

   then you can tell *without looking outside this file* that `X` comes from `A.G`.

6. Java is naturally a multithreaded system. We would like to support multiple simultaneous compiler threads and compile two different programs simultaneously, one in each compiler thread. Therefore, static variables must be avoided unless they are `final` (constant). We never want two compiler threads to be updating the same (static) instance of a variable.

PROGRAM STRAIGHT-LINE PROGRAM INTERPRETER

Implement a simple program analyzer and interpreter for the straight-line programming language. This exercise serves as an introduction to *environments* (symbol tables mapping variable names to information about the variables); to *abstract syntax* (data structures representing the phrase structure of programs); to *recursion over tree data structures*, useful in many parts of a compiler; and to a *functional style* of programming without assignment statements.

It also serves as a "warm-up" exercise in Java programming. Programmers experienced in other languages but new to Java should be able to do this exercise, but will need supplementary material (such as textbooks) on Java.

Programs to be interpreted are already parsed into abstract syntax, as described by the data types in Program 1.5.

However, we do not wish to worry about parsing the language, so we write this program by applying data constructors:

```
Stm prog =
new CompoundStm(new AssignStm("a",
                     new OpExp(new NumExp(5),
                               OpExp.Plus, new NumExp(3))),
new CompoundStm(new AssignStm("b",
   new EseqExp(new PrintStm(new PairExpList(new IdExp("a"),
             new LastExpList(new OpExp(new IdExp("a"),
                           OpExp.Minus,new NumExp(1))))),

        new OpExp(new NumExp(10), OpExp.Times,
                  new IdExp("a")))),
new PrintStm(new LastExpList(new IdExp("b")))));
```

Files with the data type declarations for the trees, and this sample program, are available in the directory `$MINIJAVA/chap1`.

Writing interpreters without side effects (that is, assignment statements that update variables and data structures) is a good introduction to *denotational semantics* and *attribute grammars*, which are methods for describing what programming languages do. It's often a useful technique in writing compilers, too; compilers are also in the business of saying what programming languages do.

Therefore, in implementing these programs, never assign a new value to any variable or object field except when it is initialized. For local variables, use the initializing form of declaration (for example, `int i=j+3;`)andfor each class, make a constructor function (like the `CompoundStm` constructor in Program 1.5).

1. Write a Java function `int maxargs(Stm s)` that tells the maximum number of arguments of any `print` statement within any subexpression of a given statement. For example, `maxargs(prog)` is 2.
2. Write a Java function `void interp(Stm s)` that "interprets" a program in this language. To write in a "functional programming" style - in which you never use an assignment statement - initialize each local variable as you declare it.

Your functions that examine each `Exp` will have to use `instanceof` to determine which subclass the expression belongs to and then cast to the proper subclass. Or you can add methods to the `Exp` and `Stm` classes to avoid the use of `instanceof`.

For part 1, remember that print statements can contain expressions that contain other print statements.

For part 2, make two mutually recursive functions `interpStm` and `interpExp`. Represent a "table", mapping identifiers to the integer values assigned to them, as a list of `id` × `int` pairs.

```
class Table {
  String id; int value; Table tail;
  Table(String i, int v, Table t) {id=i; value=v; tail=t;}
}
```

Then `interpStm` is declared as

```
Table interpStm(Stm s, Table t)
```

taking a table t_1 as argument and producing the new table t_2 that's just like t_1 except that some identifiers map to different integers as a result of the statement.

For example, the table t_1 that maps a to 3 and maps c to 4, which we write $\{a \mapsto 3, c \mapsto 4\}$ in mathematical notation, could be represented as the linked list .

Now, let the table t_2 be just like t_1, except that it maps c to 7 instead of 4. Mathematically, we could write,

t_2 = update (t_1, c, 7),

where the update function returns a new table $\{a \mapsto 3, c \mapsto 7\}$.

On the computer, we could implement t_2 by putting a new cell at the head of the linked list: , as long as we assume that the *first* occurrence of *c* in the list takes precedence over any later occurrence.

Therefore, the `update` function is easy to implement; and the corresponding `lookup` function

```
int lookup(Table t, String key)
```

just searches down the linked list. Of course, in an object-oriented style, `int lookup(String key)` should be a method of the `Table` class.

Interpreting expressions is more complicated than interpreting statements, because expressions return integer values *and* have side effects. We wish to simulate the straight-line programming language's assignment statements without doing any side effects in the interpreter itself. (The `print` statements will be accomplished by interpreter side effects, however.) The solution is to declare `interpExp` as

```
class IntAndTable {int i; Table t;
    IntAndTable(int ii, Table tt) {i=ii; t=tt;}
   }
IntAndTable interpExp(Exp e, Table t) …
```

The result of interpreting an expression e_1 with table t_1 is an integer value *i* and a new table t_2. When interpreting an expression with two subexpressions (such as an `OpExp`), the table t_2 resulting from the first subexpression can be used in processing the second subexpression.

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        new OpExp(new NumExp(10), OpExp.Times,
                  new IdExp("a")))),
new PrintStm(new LastExpList(new IdExp("b")))));
```

Files with the data type declarations for the trees, and this sample program, are available in the directory `$MINIJAVA/chap1`.

Writing interpreters without side effects (that is, assignment statements that update variables and data structures) is a good introduction to *denotational semantics* and *attribute grammars*, which are methods for describing what programming languages do. It's often a useful technique in writing compilers, too; compilers are also in the business of saying what programming languages do.

Therefore, in implementing these programs, never assign a new value to any variable or object field except when it is initialized. For local variables, use the initializing form of declaration (for example, `int i=j+3;`)andfor each class, make a constructor function (like the `CompoundStm` constructor in Program 1.5).

1. Write a Java function `int maxargs(Stm s)` that tells the maximum number of arguments of any `print` statement within any subexpression of a given statement. For example, `maxargs(prog)` is 2.
2. Write a Java function `void interp(Stm s)` that "interprets" a program in this language. To write in a "functional programming" style - in which you never use an assignment statement - initialize each local variable as you declare it.

Your functions that examine each `Exp` will have to use `instanceof` to determine which subclass the expression belongs to and then cast to the proper subclass. Or you can add methods to the `Exp` and `Stm` classes to avoid the use of `instanceof`.

For part 1, remember that print statements can contain expressions that contain other print statements.

For part 2, make two mutually recursive functions `interpStm` and `interpExp`. Represent a "table", mapping identifiers to the integer values assigned to them, as a list of `id` × `int` pairs.

```
class Table {
  String id; int value; Table tail;
  Table(String i, int v, Table t) {id=i; value=v; tail=t;}
}
```

Then `interpStm` is declared as

```
Table interpStm(Stm s, Table t)
```

taking a table t_1 as argument and producing the new table t_2 that's just like t_1 except that some identifiers map to different integers as a result of the statement.

For example, the table t_1 that maps a to 3 and maps c to 4, which we write $\{a \mapsto 3, c \mapsto 4\}$ in mathematical notation, could be represented as the linked list .

Now, let the table t_2 be just like t_1, except that it maps c to 7 instead of 4. Mathematically, we could write,

t_2 = update (t_1, c, 7),

where the update function returns a new table $\{a \mapsto 3, c \mapsto 7\}$.

On the computer, we could implement t_2 by putting a new cell at the head of the linked list: , as long as we assume that the *first* occurrence of c in the list takes precedence over any later occurrence.

Therefore, the `update` function is easy to implement; and the corresponding `lookup` function

```
int lookup(Table t, String key)
```

just searches down the linked list. Of course, in an object-oriented style, `int lookup(String key)` should be a method of the `Table` class.

Interpreting expressions is more complicated than interpreting statements, because expressions return integer values *and* have side effects. We wish to simulate the straight-line programming language's assignment statements without doing any side effects in the interpreter itself. (The `print` statements will be accomplished by interpreter side effects, however.) The solution is to declare `interpExp` as

```
class IntAndTable {int i; Table t;
    IntAndTable(int ii, Table tt) {i=ii; t=tt;}
   }
IntAndTable interpExp(Exp e, Table t) …
```

The result of interpreting an expression e_1 with table t_1 is an integer value i and a new table t_2. When interpreting an expression with two subexpressions (such as an `OpExp`), the table t_2 resulting from the first subexpression can be used in processing the second subexpression.

Chapter 2: Lexical Analysis

lex-i-cal: of or relating to words or the vocabulary of a language as distinguished from its grammar and construction

Webster's Dictionary

OVERVIEW

To translate a program from one language into another, a compiler must first pull it apart and understand its structure and meaning, then put it together in a different way. The front end of the compiler performs analysis; the back end does synthesis.

The analysis is usually broken up into

Lexical analysis: breaking the input into individual words or "tokens";

Syntax analysis: parsing the phrase structure of the program; and

Semantic analysis: calculating the program's meaning.

The lexical analyzer takes a stream of characters and produces a stream of names, keywords, and punctuation marks; it discards white space and comments between the tokens. It would unduly complicate the parser to have to account for possible white space and comments at every possible point; this is the main reason for separating lexical analysis from parsing.

Lexical analysis is not very complicated, but we will attack it with highpowered formalisms and tools, because similar formalisms will be useful in the study of parsing and similar tools have many applications in areas other than compilation.

2.1 LEXICAL TOKENS

A lexical token is a sequence of characters that can be treated as a unit in the grammar of a programming language. A programming language classifies lexical tokens into a finite set of token types. For example, some of the token types of a typical programming language are

Type	Examples
ID	`foo n14 last`
NUM	`73 0 00 515 082`
REAL	`66.1 .5 10. 1e67 5.5e-10`
IF	`if`
COMMA	`,`
NOTEQ	`!=`
LPAREN	`(`
RPAREN	`)`

Punctuation tokens such as IF, VOID, RETURN constructed from alphabetic characters are called *reserved words* and, in most languages, cannot be used as identifiers.

Examples of nontokens are

comment	`/* try again */`
preprocessor directive	`#include<stdio.h>`
preprocessor directive	`#define NUMS 5, 6`
macro	`NUMS`
blanks, tabs, and newlines	

In languages weak enough to require a macro preprocessor, the preprocessor operates on the source character stream, producing another character stream that is then fed to the lexical analyzer. It is also possible to integrate macro processing with lexical analysis.

Given a program such as

```
float match0(char *s) /* find a zero */
{if (!strncmp(s, "0.0", 3))
  return 0.;
}
```

the lexical analyzer will return the stream

```
FLOAT   ID(match0)   LPAREN   CHAR   STAR   ID(s)   RPAREN
LBRACE   IF   LPAREN   BANG   ID(strncmp)   LPAREN   ID(s)
COMMA   STRING(0.0)   COMMA   NUM(3)   RPAREN   RPAREN
RETURN   REAL(0.0)   SEMI   RBRACE   EOF
```

where the token-type of each token is reported; some of the tokens, such as identifiers and literals, have *semantic values* attached to them, giving auxiliary information in addition to the token-type.

How should the lexical rules of a programming language be described? In what language should a lexical analyzer be written?

We can describe the lexical tokens of a language in English; here is a description of identifiers in C or Java:

An identifier is a sequence of letters and digits; the first character must be a letter. The underscore _ counts as a letter. Upper- and lowercase letters are different. If the input stream has been parsed into tokens up to a given character, the next token is taken to include the longest string of characters that could possibly constitute a token. Blanks, tabs, newlines, and comments are ignored except as they serve to separate tokens. Some white space is required to separate otherwise adjacent identifiers, keywords, and constants.

And any reasonable programming language serves to implement an ad hoc lexer. But we will specify lexical tokens using the formal language of *regular expressions*, implement lexers using *deterministic finite automata*, and use mathematics to connect the two. This will lead to simpler and more readable lexical analyzers.

2.2 REGULAR EXPRESSIONS

Let us say that a *language* is a set of *strings*; a string is a finite sequence of *symbols*. The symbols themselves are taken from a finite *alphabet*.

The Pascal language is the set of all strings that constitute legal Pascal programs; the language of primes is the set of all decimal-digit strings that represent prime numbers; and the language of C reserved words is the set of all alphabetic strings that cannot be used as identifiers in the C programming language. The first two of these languages are infinite sets; the last is a finite set. In all of these cases, the alphabet is the ASCII character set.

When we speak of languages in this way, we will not assign any meaning to the strings; we will just be attempting to classify each string as in the language or not.

To specify some of these (possibly infinite) languages with finite descriptions, we will use the notation of *regular expressions*. Each regular expression stands for a set of strings.

Symbol: For each symbol **a** in the alphabet of the language, the regular expression **a** denotes the language containing just the string `a`.

Alternation: Given two regular expressions *M* and *N*, the alternation operator written as a vertical bar   makes a new regular expression *M*   *N*. A string is in the language of *M*   *N* if it is in the language of *M* or in the language of *N*. Thus, the language of **a**   **b** contains the two strings `a` and `b`.

Concatenation: Given two regular expressions *M* and *N*, the concatenation operator · makes a new regular expression *M* · *N*. A string is in the language of *M* · *N* if it is the concatenation of any two strings α and β such that α is in the language of *M* and β is in the language of *N*. Thus, the regular expression (**a**   **b**) · **a** defines the language containing the two strings `aa` and `ba`.

Epsilon: The regular expression ϵ represents a language whose only string is the empty string. Thus, (*a* · *b*)   ϵ represents the language `{"", "ab"}`.

Repetition: Given a regular expression *M*, its Kleene closure is *M**. A string is in *M** if it is the concatenation of zero or more strings, all of which are in *M*. Thus, ((**a**   **b**) · **a**)* represents the infinite set `{"", "aa", "ba", "aaaa", "baaa", "aaba", "baba", "aaaaaa", …}`.

Using symbols, alternation, concatenation, epsilon, and Kleene closure we can specify the set of ASCII characters corresponding to the lexical tokens of a programming language. First, consider some examples:

(**0** | **1**)* · **0**	Binary numbers that are multiples of two.
b*(**abb***)*(**a**|ϵ)	Strings of `a`'s and `b`'s with no consecutive `a`'s.
(**a**|**b**)***aa**(**a**|**b**)*	Strings of `a`'s and `b`'s containing consecutive `a`'s.

In writing regular expressions, we will sometimes omit the concatenation symbol or the epsilon, and we will assume that Kleene closure "binds tighter" than concatenation, and concatenation binds tighter than alternation; so that **ab** | **c** means (**a** · **b**) | **c**, and (**a** |) means (**a** | ϵ).

Let us introduce some more abbreviations: [**abcd**] means (**a** | **b** | **c** | **d**), [**b**-**g**] means [**bcdefg**], [**b**-**gM**-**Qkr**] means [**bcdefgMNOPQkr**], *M*? means (*M* | ϵ), and M^{+} means (*M*·*M**). These extensions are convenient, but none extend the descriptive power of regular expressions: Any set of strings that can be described with these abbreviations could also be described by just the basic set of operators. All the operators are summarized in Figure 2.1.

a	An ordinary character stands for itself.
ϵ	The empty string. Another way to write the empty string.
$M \mid N$	Alternation, choosing from *M* or *N*.
$M \cdot N$	Concatenation, an *M* followed by an *N*.
MN	Another way to write concatenation.
*M**	Repetition (zero or more times).
M^{+}	Repetition, one or more times.
M?	Optional, zero or one occurrence of *M*.
[a − zA − Z]	Character set alternation.
·	A period stands for any single character except newline.
`"a.+*"`	Quotation, a string in quotes stands for itself literally.

Figure 2.1: Regular expression notation.

Using this language, we can specify the lexical tokens of a programming language (Figure 2.2).

```
if                                IF
[a-z][a-z0-9]*                    ID
[0-9]+                            NUM
([0-9]+"."[0-9]*)|([0-9]*"."[0-9]+)  REAL
("--"[a-z]*"\n")|(" "|"\n"|"\t")+   no token, just white space
.                                 error
```

Figure 2.2: Regular expressions for some tokens.

The fifth line of the description recognizes comments or white space but does not report back to the parser. Instead, the white space is discarded and the lexer resumed. The comments for this lexer begin with two dashes, contain only alphabetic characters, and end with newline.

Finally, a lexical specification should be *complete*, always matching some initial substring of the input; we can always achieve this by having a rule that matches any single character (and in this case, prints an "illegal character" error message and continues).

These rules are a bit ambiguous. For example, does `if8` match as a single identifier or as the two tokens `if` and `8`? Does the string `if 89` begin with an identifier or a reserved word? There are two important disambiguation rules used by Lex, JavaCC, SableCC, and other similar lexical-analyzer generators:

Longest match: The longest initial substring of the input that can match any regular expression is taken as the next token.

Rule priority: For a *particular* longest initial substring, the first regular expression that can match determines its token-type. This means that the order of writing down the regular-expression rules has significance.

Thus, `if8` matches as an identifier by the longest-match rule, and `if` matches as a reserved word by rule-priority.

2.3 FINITE AUTOMATA

Regular expressions are convenient for specifying lexical tokens, but we need a formalism that can be implemented as a computer program. For this we can use finite automata (N.B. the singular of automata is automaton). A finite automaton has a finite set of *states*; *edges* lead from one state to another, and each edge is labeled with a *symbol*. One state is the *start* state, and certain of the states are distinguished as *final* states.

Figure 2.3 shows some finite automata. We number the states just for convenience in discussion. The start state is numbered 1 in each case. An edge labeled with several characters is shorthand for many parallel edges; so in the ID machine there are really 26 edges each leading from state 1 to 2, each labeled by a different letter.

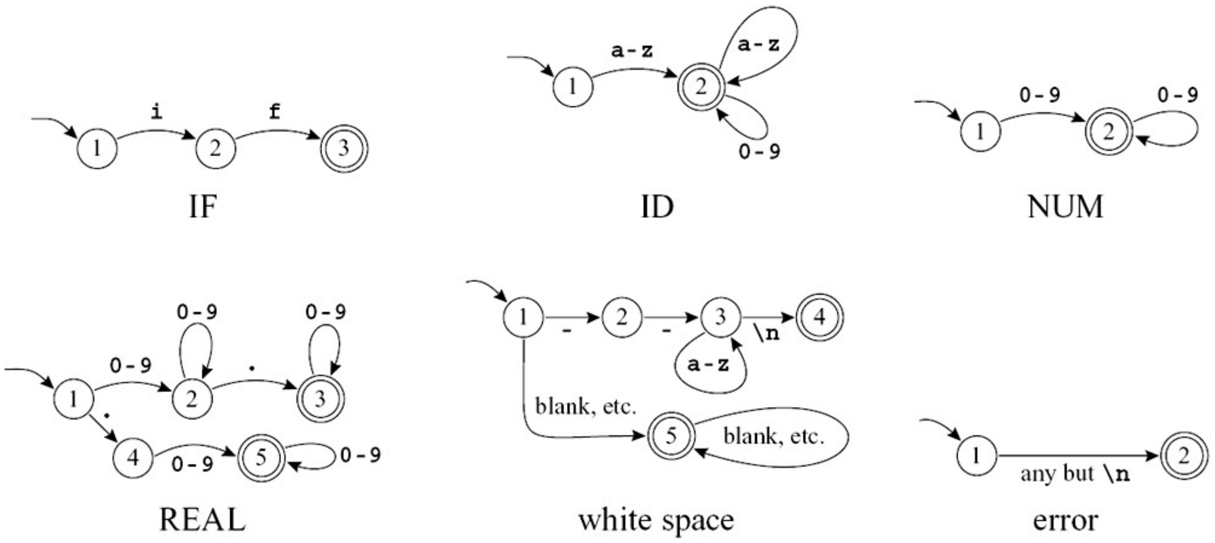


Figure 2.3: Finite automata for lexical tokens. The states are indicated by circles; final states are indicated by double circles. The start state has an arrow coming in from nowhere. An edge labeled with several characters is shorthand for many parallel edges.

In a *deterministic* finite automaton (DFA), no two edges leaving from the same state are labeled with the same symbol. A DFA *accepts* or *rejects* a string as follows. Starting in the start state, for each character in the input string the automaton follows exactly one edge to get to the next state. The edge must be labeled with the input character. After making *n* transitions for an *n*-character string, if the automaton is in a final state, then it accepts the string. If it is not in a final state, or if at some point there was no appropriately labeled edge to follow, it rejects. The *language* recognized by an automaton is the set of strings that it accepts.

For example, it is clear that any string in the language recognized by automaton ID must begin with a letter. Any single letter leads to state 2, which is final; so a single-letter string is accepted. From state 2, any letter or digit leads back to state 2, so a letter followed by any number of letters and digits is also accepted.

In fact, the machines shown in Figure 2.3 accept the same languages as the regular expressions of Figure 2.2.

These are six separate automata; how can they be combined into a single machine that can serve as a lexical analyzer? We will study formal ways of doing this in the next section, but here we will just do it ad hoc: Figure 2.4 shows such a machine. Each final state must be labeled with the token-type that it accepts. State 2 in this machine has aspects of state 2 of the IF machine and state 2 of the ID machine; since the latter is final, then the combined state must be final. State 3 is like state 3 of the IF machine and state 2 of the ID machine; because these are both final we use *rule priority* to disambiguate - we label state 3 with IF because we want this token to be recognized as a reserved word, not an identifier.

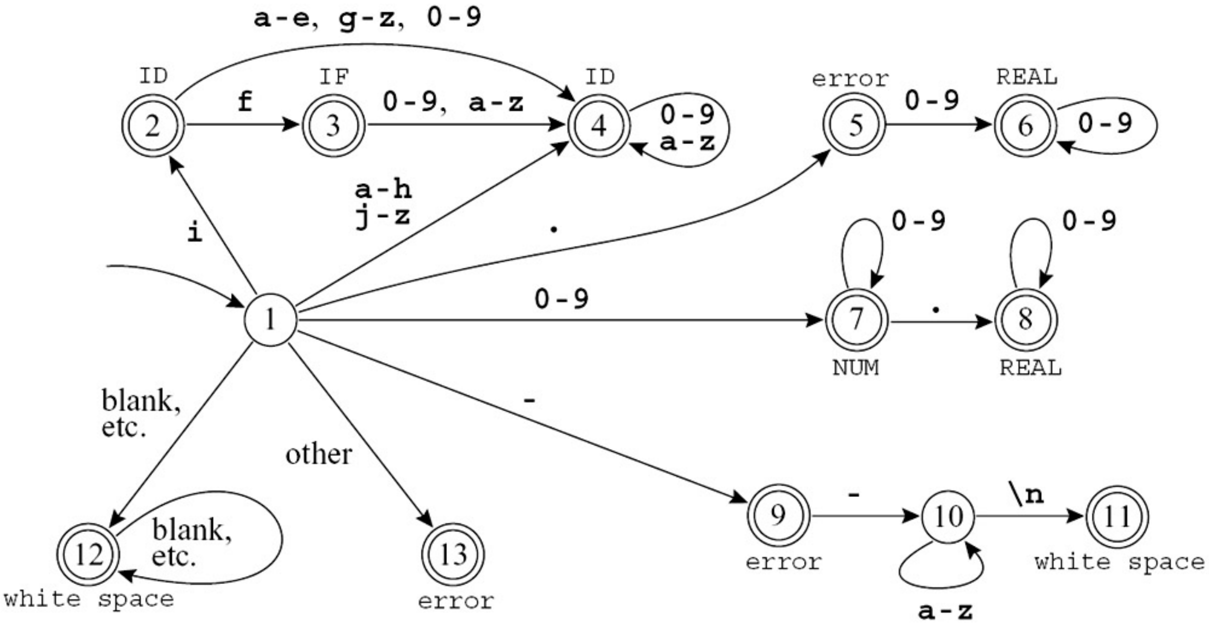


Figure 2.4: Combined finite automaton.

We can encode this machine as a transition matrix: a two-dimensional array (a vector of vectors), subscripted by state number and input character. There will be a "dead" state (state 0) that loops to itself on all characters; we use this to encode the absence of an edge.

```
int edges[][] = { /* ...012...-...e f g h i j... */
/* state 0 */    {0,0,...0,0,0...0...0,0,0,0,0,0...},
/* state 1 */    {0,0,...7,7,7...9...4,4,4,4,2,4...},
/* state 2 */    {0,0,...4,4,4...0...4,3,4,4,4,4...},
/* state 3 */    {0,0,...4,4,4...0...4,4,4,4,4,4...},
/* state 4 */    {0,0,...4,4,4...0...4,4,4,4,4,4...},
/* state 5 */    {0,0,...6,6,6...0...0,0,0,0,0,0...},
/* state 6 */    {0,0,...6,6,6...0...0,0,0,0,0,0...},
/* state 7 */    {0,0,...7,7,7...0...0,0,0,0,0,0...},
/* state 8 */    {0,0,...8,8,8...0...0,0,0,0,0,0...},
  et cetera
}
```

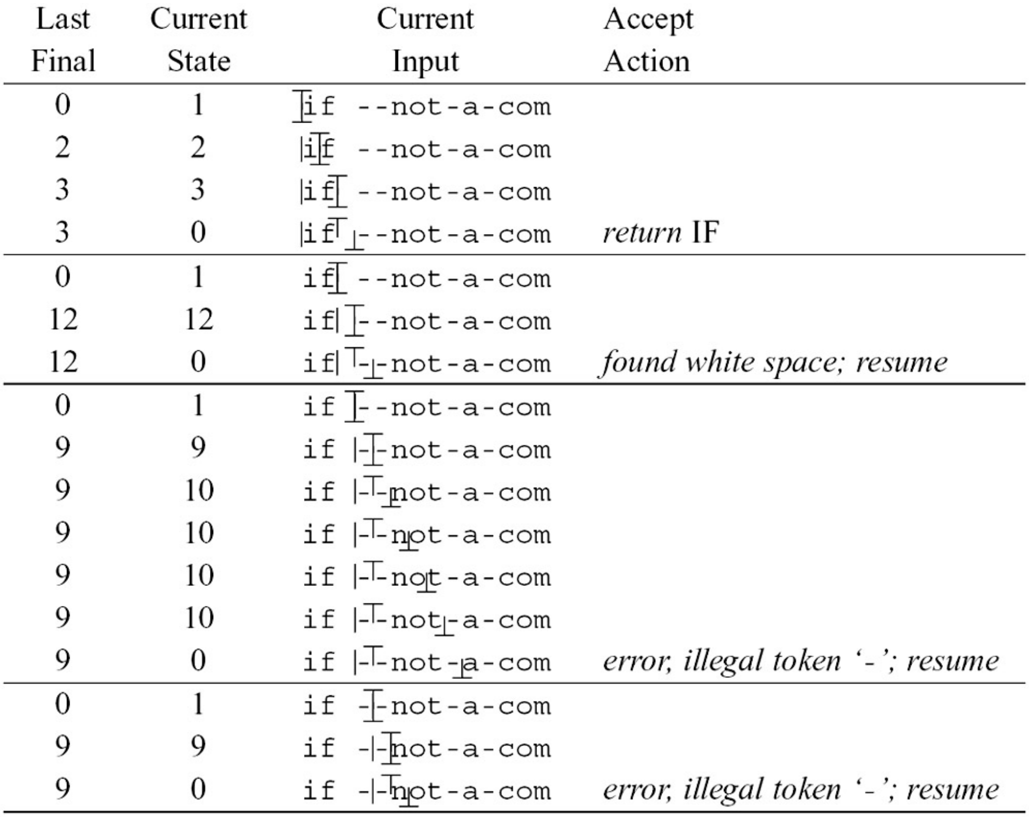
There must also be a "finality" array, mapping state numbers to actions - final state 2 maps to action ID, and so on.

RECOGNIZING THE LONGEST MATCH

It is easy to see how to use this table to recognize whether to accept or reject a string, but the job of a lexical analyzer is to find the longest match, the longest initial substring of the input that is a valid token. While interpreting transitions, the lexer must keep track of the longest match seen so far, and the position of that match.

Keeping track of the longest match just means remembering the last time the automaton was in a final state with two variables, `Last-Final` (the state number of the most recent final state encountered) and `Input-Position-at-Last-Final`. Every time a final state is entered, the lexer updates these variables; when a *dead* state (a nonfinal state with no output transitions) is reached, the variables tell what token was matched, and where it ended.

Figure 2.5 shows the operation of a lexical analyzer that recognizes longest matches; note that the current input position may be far beyond the most recent position at which the recognizer was in a final state.



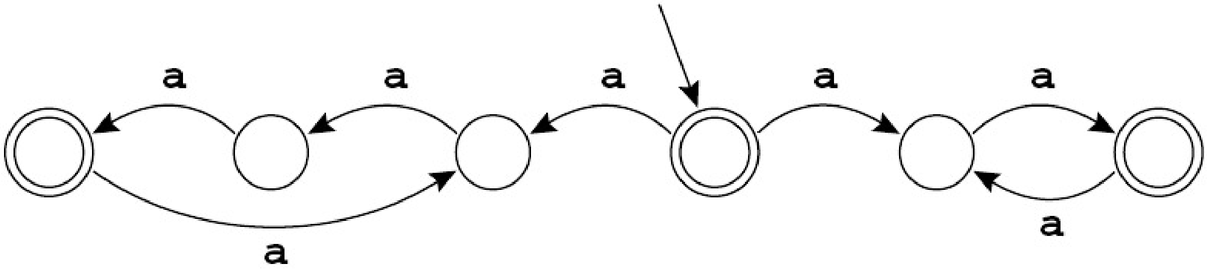
Last Final	Current State	Current Input	Accept Action
0	1	⊤⊥if --not-a-com	
2	2	|i⊤⊥f --not-a-com	
3	3	|if⊤⊥ --not-a-com	
3	0	|if⊤ ⊥--not-a-com	*return* IF
0	1	if⊤⊥ --not-a-com	
12	12	if| ⊤⊥--not-a-com	
12	0	if| ⊤-⊥-not-a-com	*found white space; resume*
0	1	if ⊤⊥--not-a-com	
9	9	if |-⊤⊥-not-a-com	
9	10	if |-⊤-⊥not-a-com	
9	10	if |-⊤-n⊥ot-a-com	
9	10	if |-⊤-no⊥t-a-com	
9	10	if |-⊤-not⊥-a-com	
9	0	if |-⊤-not-⊥a-com	*error, illegal token '-'; resume*
0	1	if -⊤⊥-not-a-com	
9	9	if -|-⊤⊥not-a-com	
9	0	if -|-⊤n⊥ot-a-com	*error, illegal token '-'; resume*

Figure 2.5: The automaton of Figure 2.4 recognizes several tokens. The symbol | indicates the input position at each successive call to the lexical analyzer, the symbol ⊥ indicates the current position of the automaton, and ⊤ indicates the most recent position in which the recognizer was in a final state.

2.4 NONDETERMINISTIC FINITE AUTOMATA

A nondeterministic finite automaton (NFA) is one that has a choice of edges - labeled with the same symbol - to follow out of a state. Or it may have special edges labeled with ϵ (the Greek letter epsilon) that can be followed without eating any symbol from the input.

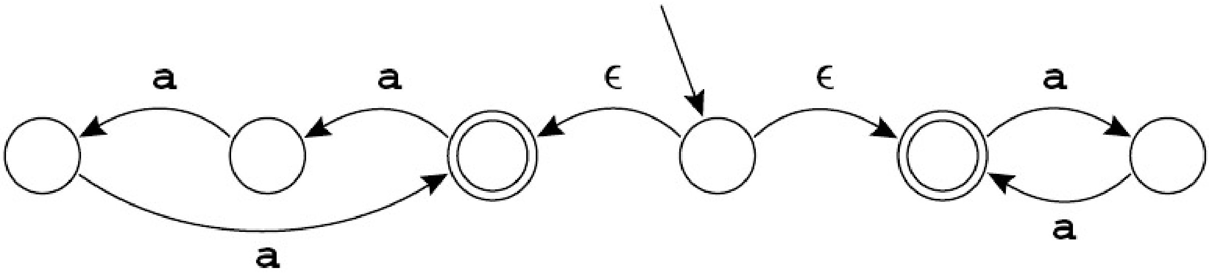
Here is an example of an NFA:



In the start state, on input character a, the automaton can move either right or left. If left is chosen, then strings of a's whose length is a multiple of three will be accepted. If right is chosen, then even-length strings will be accepted. Thus, the language recognized by this NFA is the set of all strings of a's whose length is a multiple of two or three.

On the first transition, this machine must choose which way to go. It is required to accept the string if there is *any* choice of paths that will lead to acceptance. Thus, it must "guess", and must always guess correctly.

Edges labeled with ϵ may be taken without using up a symbol from the input. Here is another NFA that accepts the same language:

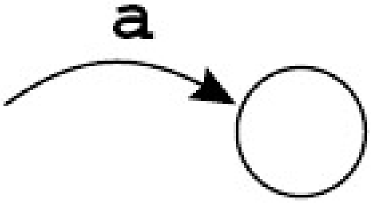


Again, the machine must choose which ϵ-edge to take. If there is a state with some ϵ-edges and some edges labeled by symbols, the machine can choose to eat an input symbol (and follow the corresponding symbol-labeled edge), or to follow an ϵ-edge instead.

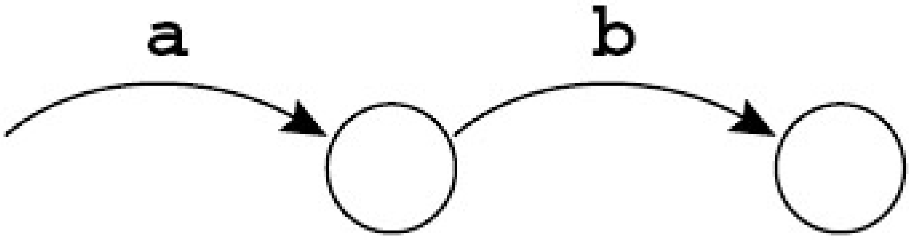
CONVERTING A REGULAR EXPRESSION TO AN NFA

Nondeterministic automata are a useful notion because it is easy to convert a (static, declarative) regular expression to a (simulatable, quasi-executable) NFA.

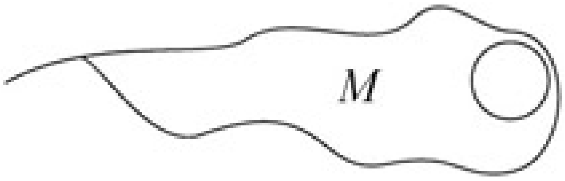
The conversion algorithm turns each regular expression into an NFA with a *tail* (start edge) and a *head* (ending state). For example, the single-symbol regular expression **a** converts to the NFA



The regular expression **ab**, made by combining **a** with **b** using concatenation, is made by combining the two NFAs, hooking the head of **a** to the tail of **b**. The resulting machine has a tail labeled by **a** and a head into which the **b** edge flows.



In general, any regular expression *M* will have some NFA with a tail and head:



We can define the translation of regular expressions to NFAs by induction. Either an expression is primitive (a single symbol or ϵ) or it is made from smaller expressions. Similarly, the NFA will be primitive or made from smaller NFAs.

Figure 2.6 shows the rules for translating regular expressions to nondeterministic automata. We illustrate the algorithm on some of the expressions in Figure 2.2 - for the tokens IF, ID, NUM, and **error**. Each expression is translated to an NFA, the "head" state of each NFA is marked final with a different token type, and the tails of all the expressions are joined to a new start node. The result - after some merging of equivalent NFA states - is shown in Figure 2.7.

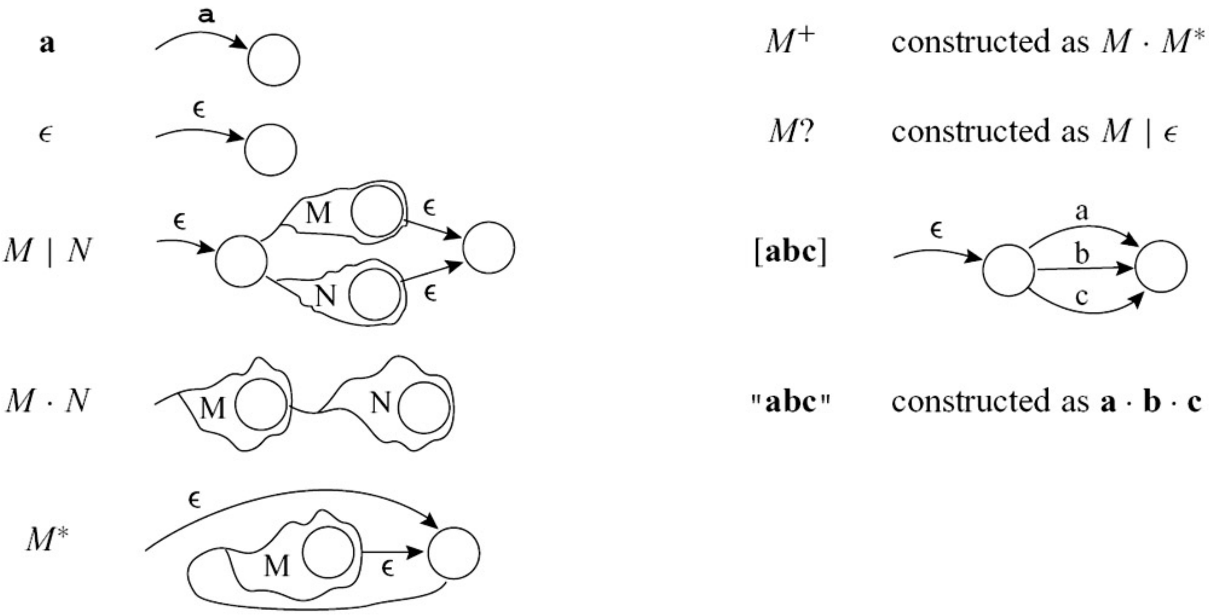


Figure 2.6: Translation of regular expressions to NFAs.

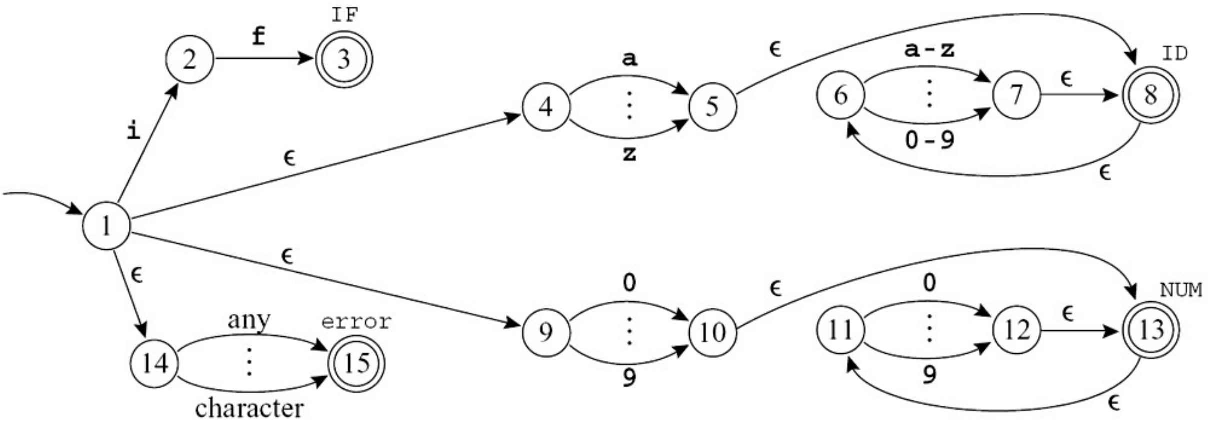


Figure 2.7: Four regular expressions translated to an NFA.

CONVERTING AN NFA TO A DFA

As we saw in Section 2.3, implementing deterministic finite automata (DFAs) as computer programs is easy. But implementing NFAs is a bit harder, since most computers don't have good "guessing" hardware.

We can avoid the need to guess by trying every possibility at once. Let us simulate the NFA of Figure 2.7 on the string `in`. We start in state 1. Now, instead of guessing which ϵ-transition to take, we just say that at this point the NFA might take any of them, so it is in one of the states $\{1, 4, 9, 14\}$; that is, we compute the ϵ-*closure* of $\{1\}$. Clearly, there are no other states reachable without eating the first character of the input.

Now, we make the transition on the character `i`. From state 1 we can reach 2, from 4 we reach 5, from 9 we go nowhere, and from 14 we reach 15. So we have the set f2, 5, 15g. But again we must compute the ϵ-closure: From 5 there is an ϵ-transition to 8, and from 8 to 6. So the NFA must be in one of the states $\{2, 5, 6, 8, 15\}$.

On the character `n`, we get from state 6 to 7, from 2 to nowhere, from 5 to nowhere, from 8 to nowhere, and from 15 to nowhere. So we have the set $\{7\}$; its ϵ-closure is $\{6, 7, 8\}$.

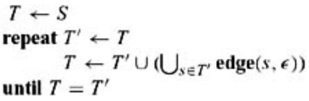
Now we are at the end of the string `in`; is the NFA in a final state? One of the states in our possible-states set is 8, which is final. Thus, `in` is an ID token.

We formally define ϵ-closure as follows. Let **edge**(s, c) be the set of all NFA states reachable by following a single edge with label c from state s.

For a set of states S, **closure**(S) is the set of states that can be reached from a state in S without consuming any of the input, that is, by going only through ϵ-edges. Mathematically, we can express the idea of going through ϵ-edges by saying that **closure**(S) is the smallest set T such that

$$T = S \cup \left(\bigcup_{s \in T} \mathbf{edge}(s, \epsilon) \right).$$

We can calculate T by iteration:



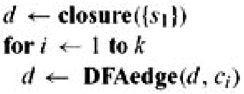
```
$T \leftarrow S$
repeat $T' \leftarrow T$
        $T \leftarrow T' \cup (\bigcup_{s \in T'} \mathbf{edge}(s, \epsilon))$
until $T = T'$
```

Why does this algorithm work? *T* can only grow in each iteration, so the final *T* must include *S*. If $T = T'$ after an iteration step, then *T* must also include . Finally, the algorithm must terminate, because there are only a finite number of distinct states in the NFA.

Now, when simulating an NFA as described above, suppose we are in a set $d = \{s_i; s_k; s_l\}$ of NFA states s_i ; s_k; s_l. By starting in *d* and eating the input symbol *c*, we reach a new set of NFA states; we'll call this set **DFAedge**(*d; c*):

$$\mathbf{DFAedge}(d, c) = \mathbf{closure}(\bigcup_{s \in d} \mathbf{edge}(s, c))$$

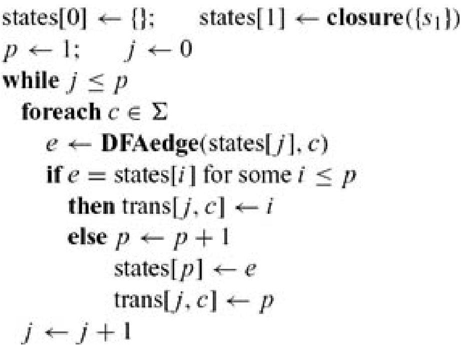
Using **DFAedge**, we can write the NFA simulation algorithm more formally. If the start state of the NFA is s_1, and the input string is $c_1, \ldots, c_k$, then the algorithm is



```
$d \leftarrow \mathbf{closure}(\{s_1\})$
for $i \leftarrow 1$ to $k$
  $d \leftarrow \mathbf{DFAedge}(d, c_i)$
```

Manipulating sets of states is expensive - too costly to want to do on every character in the source program that is being lexically analyzed. But it is possible to do all the sets-of-states calculations in advance. We make a DFA from the NFA, such that each set of NFA states corresponds to one DFA state. Since the NFA has a finite number *n* of states, the DFA will also have a finite number (at most 2^n) of states.

DFA construction is easy once we have **closure** and **DFAedge** algorithms. The DFA start state d_1 is just **closure**(s_1), as in the NFA simulation algorithm. Abstractly, there is an edge from d_i to d_j labeled with *c* if d_j = **DFAedge**(d_i, *c*). We let **Σ** be the alphabet.



```
states[0] $\leftarrow$ {};      states[1] $\leftarrow$ closure({$s_1$})
$p \leftarrow 1$;     $j \leftarrow 0$
while $j \leq p$
  foreach $c \in \Sigma$
    $e \leftarrow$ DFAedge(states[$j$], $c$)
    if $e$ = states[$i$] for some $i \leq p$
      then trans[$j, c$] $\leftarrow i$
      else $p \leftarrow p + 1$
           states[$p$] $\leftarrow e$
           trans[$j, c$] $\leftarrow p$
  $j \leftarrow j + 1$
```

The algorithm does not visit unreachable states of the DFA. This is extremely important, because in principle the DFA has 2^n states, but in practice we usually find that only about *n* of them are reachable from the start state. It is important to avoid an exponential blowup in the size of the DFA interpreter's transition tables, which will form part of the working compiler.

A state *d* is *final* in the DFA if any NFA state in states[*d*] is final in the NFA. Labeling a state *final* is not enough; we must also say what token is recognized; and perhaps several members of states[*d*] are final in the NFA. In this case we label *d* with the token-type that occurred first in the list of regular expressions that constitute the lexical specification. This is how *rule priority* is implemented.

After the DFA is constructed, the "states" array may be discarded, and the "trans" array is used for lexical analysis.

Applying the DFA construction algorithm to the NFA of Figure 2.7 gives the automaton in Figure 2.8.

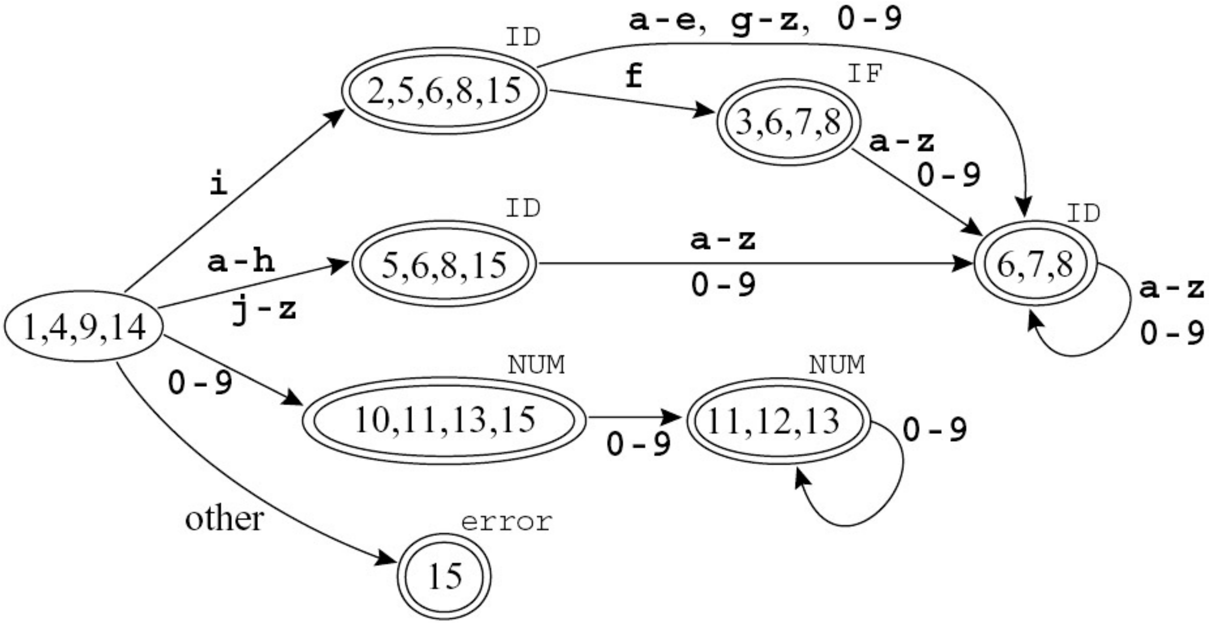
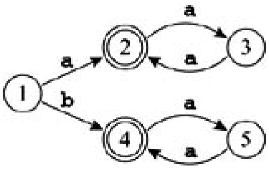


Figure 2.8: NFA converted to DFA.

This automaton is suboptimal. That is, it is not the smallest one that recognizes the same language. In general, we say that two states s_1 and s_2 are equivalent when the machine starting in s_1 accepts a string σ if and only if starting in s_2 it accepts σ. This is certainly true of the states labeled **5, 6, 8, 15** and **6, 7, 8** in Figure 2.8, and of the states labeled **10, 11, 13, 15** and **11, 12, 13**. In an automaton with two equivalent states s_1 and s_2, we can make all of s_2's incoming edges point to s_1 instead and delete s_2.

How can we find equivalent states? Certainly, s_1 and s_2 are equivalent if they are both final or both nonfinal and, for any symbol c, trans[s_1, c] = trans[s_2, c]; **10, 11, 13, 15** and **11, 12, 13** satisfy this criterion. But this condition is not sufficiently general; consider the automaton



Here, states 2 and 4 are equivalent, but trans[2, a] ≠ trans[4, a].

After constructing a DFA it is useful to apply an algorithm to minimize it by finding equivalent states; see Exercise 2.6.

2.5 LEXICAL-ANALYZER GENERATORS

DFA construction is a mechanical task easily performed by computer, so it makes sense to have an automatic *lexical-analyzer generator* to translate regular expressions into a DFA.

JavaCC and SableCC generate lexical analyzers and parsers written in Java. The lexical analyzers are generated from *lexical specifications*; and, as explained in the next chapter, the parsers are generated from grammars.

For both JavaCC and SableCC, the lexical specification and the grammar are contained in the same file.

JAVACC

The tokens described in Figure 2.2 are specified in JavaCC as shown in Program 2.9. A JavaCC specification starts with an optional list of options followed by a Java compilation unit enclosed between `PARSER_BEGIN(name)` and `PARSER_END(name)`. The same name must follow `PARSER_BEGIN` and `PARSER_END`; it will be the name of the generated parser (`MyParser` in Program 2.9). The enclosed compilation unit must contain a class declaration of the same name as the generated parser.

PROGRAM 2.9: JavaCC specification of the tokens from Figure 2.2.

```
PARSER_BEGIN(MyParser)
   class MyParser {}
PARSER_END(MyParser)
/* For the regular expressions on the right, the token on the left will be
returned:/*
TOKEN : {
    < IF: "if" >
  | < #DIGIT: ["0"-"9"] >
  | < ID: ["a"-"z"] (["a"-"z"]|<DIGIT>) >
  | < NUM: (<DIGIT>)+ >
  | < REAL: ( (<DIGIT>)+ "." (<DIGIT>)* ) |
          ( (<DIGIT>)* "." (<DIGIT>)+ )>
}
/* The regular expressions here will be skipped during lexical analysis: */
SKIP : {
     <"--" (["a"-"z"])* ("\n" | "\r" | "\r\n")>
  |""
  | "\t"
  | "\n"
}
/* If we have a substring that does not match any of the regular
expressions in TOKEN or SKIP,
   JavaCC will automatically throw an error. */
void Start() :
{}
{ ( <IF> | <ID> | <NUM> | <REAL> )* }
```

Next is a list of grammar productions of the following kinds: a *regular-expression production* defines a token, a *token-manager declaration* can be used by the generated lexical analyzer, and two other kinds are used to define the grammar from which the parser is generated.

A lexical specification uses regular-expression productions; there are four kinds: `TOKEN`, `SKIP`, `MORE`, and `SPECIAL_TOKEN`. We will only need `TOKEN` and `SKIP` for the compiler project in this book. The kind `TOKEN` is used to specify that the matched string should be transformed into a token that should be communicated to the parser. The kind `SKIP` is used to specify that the matched string should be thrown away.

In Program 2.9, the specifications of ID, NUM, and REAL use the abbreviation DIGIT. The definition of DIGIT is preceeded by # to indicate that it can be used only in the definition of other tokens.

The last part of Program 2.9 begins with void Start. Itisa *production* which, in this case, allows the generated lexer to recognize any of the four defined tokens in any order. The next chapter will explain productions in detail.

SABLECC

The tokens described in Figure 2.2 are specified in SableCC as shown in Program 2.10. A SableCC specification file has six sections (all optional):

1. Package declaration: specifies the root package for all classes generated by SableCC.
2. Helper declarations: a list of abbreviations.
3. State declarations: support the state feature of, for example, GNU FLEX; when the lexer is in some state, only the tokens associated with that state are recognized. States can be used for many purposes, including the detection of a beginning-of-line state, with the purpose of recognizing tokens only if they appear at the beginning of a line. For the compiler described in this book, states are not needed.
4. Token declarations: each one is used to specify that the matched string should be transformed into a token that should be communicated to the parser.
5. Ignored tokens: each one is used to specify that the matched string should be thrown away.
6. Productions: are used to define the grammar from which the parser is generated.

PROGRAM 2.10: SableCC specification of the tokens from Figure 2.2.

```
Helpers
    digit = ['0'..'9'];
Tokens
    if = 'if';
    id = ['a'..'z'](['a'..'z'] | (digit))*;
    number = digit+;
    real = ((digit)+ '.' (digit)*) |
           ((digit)* '.' (digit)+);
    whitespace = (' ' | '\t' | '\n')+;
    comments = ('--' ['a'..'z']* '\n');
Ignored Tokens
    whitespace,
    comments;
```

PROGRAM LEXICAL ANALYSIS

Write the lexical-analysis part of a JavaCC or SableCC specification for MiniJava. Appendix A describes the syntax of MiniJava. The directory

```
$MINIJAVA/chap2/javacc
```

contains a test-scaffolding file Main.java that calls the lexer generated by javacc. It also contains a README file that explains how to invoke javacc. Similar files for sablecc can be found in $MINIJAVA/chap2/sablecc.

FURTHER READING

Lex was the first lexical-analyzer generator based on regular expressions [Lesk 1975]; it is still widely used.

Computing ϵ-closure can be done more efficiently by keeping a queue or stack of states whose edges have not yet been checked for ϵ-transitions [Aho et al. 1986]. Regular expressions can be converted directly to DFAs without going through NFAs [McNaughton and Yamada 1960; Aho et al. 1986].

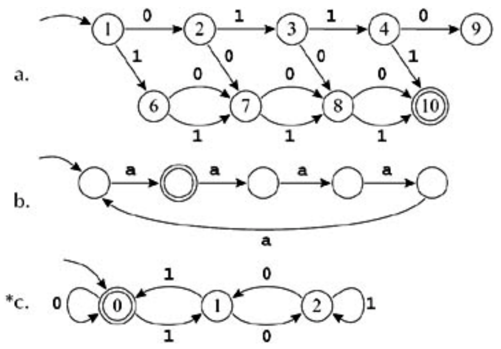
DFA transition tables can be very large and sparse. If represented as a simple two-dimensional matrix (*states* × *symbols*), they take far too much memory. In practice, tables are compressed; this reduces the amount of memory required, but increases the time required to look up the next state [Aho et al. 1986].

Lexical analyzers, whether automatically generated or handwritten, must manage their input efficiently. Of course, input is buffered, so that a large batch of characters is obtained at once; then the lexer can process one character at a time in the buffer. The lexer must check, for each character, whether the end of the buffer is reached. By putting a *sentinel* - a character that cannot be part of any token - at the end of the buffer, it is possible for the lexer to check for end-of-buffer only once per token, instead of once per character [Aho et al. 1986]. Gray [1988] uses a scheme that requires only one check per line, rather than one per token, but cannot cope with tokens that contain end-of-line characters. Bumbulis and Cowan [1993] check only once around each cycle in the DFA; this reduces the number of checks (from once per character) when there are long paths in the DFA.

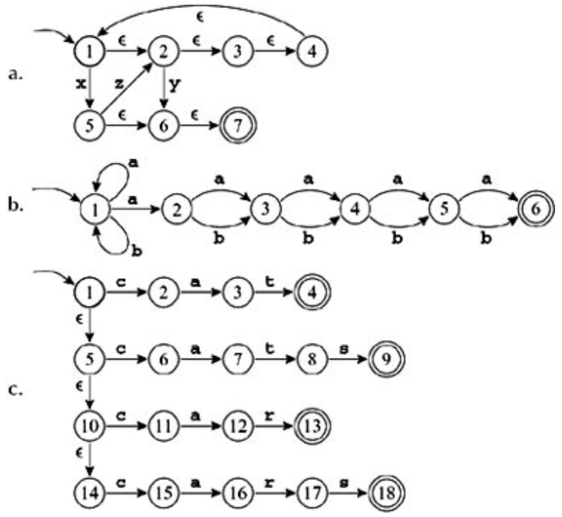
Automatically generated lexical analyzers are often criticized for being slow. In principle, the operation of a finite automaton is very simple and should be efficient, but interpreting from transition tables adds overhead. Gray [1988] shows that DFAs translated directly into executable code (implementing states as case statements) can run as fast as hand-coded lexers. The Flex "fast lexical-analyzer generator" [Paxson 1995] is significantly faster than Lex.

EXERCISES

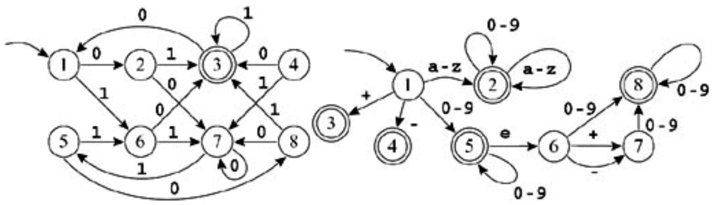
- **2.1** Write regular expressions for each of the following.
    a. Strings over the alphabet {*a*, *b*, *c*} where the first *a* precedes the first *b*.
    b. Strings over the alphabet {*a*, *b*, *c*} with an even number of *a*'s.
    c. Binary numbers that are multiples of four.
    d. Binary numbers that are greater than 101001.
    e. Strings over the alphabet {*a*, *b*, *c*} that don't contain the contiguous substring *baa*.
    f. The language of nonnegative integer constants in C, where numbers beginning with 0 are *octal* constants and other numbers are decimal constants.
    g. Binary numbers *n* such that there exists an integer solution of $a^n+b^n = c^n$.
- **2.2** For each of the following, explain why you're not surprised that there is no regular expression defining it.
    a. Strings of *a*'s and *b*'s where there are more *a*'s than *b*'s.
    b. Strings of *a*'s and *b*'s that are palindromes (the same forward as backward).
    c. Syntactically correct Java programs.
- **2.3** Explain in informal English what each of these finite-state automata recognizes.



- **2.4** Convert these regular expressions to nondeterministic finite automata.
  a. (**if**|**then**|**else**)
  b. **a**((**b**|**a***c)**x**)*j**x***a**
- **2.5** Convert these NFAs to deterministic finite automata.

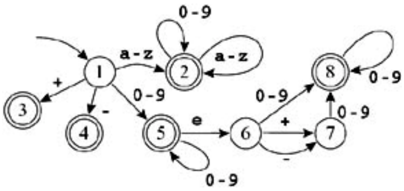


- **2.6** Find two equivalent states in the following automaton, and merge them to produce a smaller automaton that recognizes the same language. Repeat until there are no longer equivalent states.



Actually, the general algorithm for minimizing finite automata works in reverse. First, find all pairs of inequivalent ststes. States *X, Y* are inequivalent if *X* is final and *Y* is not or (by iteration) if $X \xrightarrow{a} X'$and $X \xrightarrow{a} X'$ (Y naar Y') and *X*′, *Y*′ are inequivalent. After this iteration ceases to find new pairs of inequivalent states, then *X; Y* are equivalent if they are not inequivalent. See Hopcroft and Ullman [1979], Theorem 3.10.

- ***2.7** Any DFA that accepts at least one string can be converted to a regular expression. Convert the DFA of Exercise 2.3c to a regular expression. **Hint:** First, pretend state 1 is the start state. Then write a regular expression for excursions to state 2 and back, and a similar one for excursions to state 0 and back. Or look in Hopcroft and Ullman [1979], Theorem 2.4, for the algorithm.
- ***2.8** Suppose this DFA were used by Lex to find tokens in an input file.



a. How many characters past the end of a token might Lex have to examine before matching the token?
b. Given your answer *k* to part (a), show an input file containing at least two tokens such that *the first call* to Lex will examine *k* characters *past the end of the first token* before returning *the first* token. If the answer to part (a) is zero, then show an input file containing at least two tokens, and indicate the endpoint of each token.

- **2.9** An interpreted DFA-based lexical analyzer uses two tables,

  `edges` indexed by state and input symbol, yielding a state number, and `final` indexed by state, returning 0 or an action-number.

  Starting with this lexical specification,

  ```
  (aba)+    (action 1);
  (a(b*)a)  (action 2);
  (a|b)     (action 3);
  ```

  generate the `edges` and `final` tables for a lexical analyzer.

  Then show each step of the lexer on the string `abaabbaba`. Be sure to show the values of the important internal variables of the recognizer. There will be repeated calls to the lexer to get successive tokens.

- ****2.10** Lex has a *lookahead operator* `/` so that the regular expression `abc/def` matches `abc` only when followed by `def` (but `def` is not part of the matched string, and will be part of the next token(s)). Aho et al. [1986] describe, and Lex [Lesk 1975] uses, an incorrect algorithm for implementing lookahead (it fails on `(a|ab)/ba` with input `aba`, matching `ab` where it should match `a`). Flex [Paxson 1995] uses a better mechanism that works correctly for `(a|ab)/ba` but fails (with a warning message) on `zx*/xy*`.

Design a better lookahead mechanism.

Chapter 3: Parsing

syn-tax: the way in which words are put together to form phrases, clauses, or sentences.

Webster's Dictionary

OVERVIEW

The abbreviation mechanism discussed in the previous chapter, whereby a symbol stands for some regular expression, is convenient enough that it is tempting to use it in interesting ways:

$$\begin{aligned} \mathit{digits} &= [0-9]+ \\ \mathit{sum} &= (\mathit{digits}\ \text{``+''})^{*}\ \mathit{digits} \end{aligned}$$

These regular expressions define sums of the form `28+301+9`.

But now consider

$$\begin{aligned} \mathit{digits} &= [0-9]+ \\ \mathit{sum} &= \mathit{expr}\ \text{``+''}\ \mathit{expr} \\ \mathit{expr} &= \text{``(''}\ \mathit{sum}\ \text{``)''}\ \mid \mathit{digits} \end{aligned}$$

This is meant to define expressions of the form:

```
(109+23)
61
(1+(250+3))
```

in which all the parentheses are balanced. But it is impossible for a finite automaton to recognize balanced parentheses (because a machine with *N* states cannot remember a parenthesis-nesting depth greater than *N*), so clearly *sum* and *expr* cannot be regular expressions.

So how does a lexical analyzer implement regular-expression abbreviations such as `digits`? The answer is that the right-hand-side (`[0-9]+`) is simply substituted for `digits` wherever it appears in regular expressions, *before* translation to a finite automaton.

This is not possible for the *sum*-and-*expr* language; we can first substitute *sum* into *expr*, yielding

$$\mathit{expr} = \text{``(''}\ \mathit{expr}\ \text{``+''}\ \mathit{expr}\ \text{``)''}\ \mid \mathit{digits}$$

but now an attempt to substitute *expr* into itself leads to

$$\mathit{expr} = \text{``(''}\ (\ \text{``(''}\ \mathit{expr}\ \text{``+''}\ \mathit{expr}\ \text{``)''}\ \mid \mathit{digits}\ )\ \text{``+''}\ \mathit{expr}\ \text{``)''}\ \mid \mathit{digits}$$

and the right-hand side now has just as many occurrences of *expr* as it did before - in fact, it has more!

Thus, the notion of abbreviation does not add expressive power to the language of regular expressions - there are no additional languages that can be defined - unless the abbreviations are recursive (or mutually recursive, as are *sum* and *expr*).

The additional expressive power gained by recursion is just what we need for parsing. Also, once we have abbreviations with recursion, we do not need alternation except at the top level of expressions, because the definition

$$expr = ab(c \mid d)e$$

can always be rewritten using an auxiliary definition as

$$\begin{aligned} aux &= c \mid d \\ expr &= a\ b\ aux\ e \end{aligned}$$

In fact, instead of using the alternation mark at all, we can just write several allowable expansions for the same symbol:

$$\begin{aligned} aux &= c \\ aux &= d \\ expr &= a\ b\ aux\ e \end{aligned}$$

The Kleene closure is not necessary, since we can rewrite it so that

$$expr = (a\ b\ c)*$$

becomes

$$\begin{aligned} expr &= (a\ b\ c)\ expr \\ expr &= \epsilon \end{aligned}$$

What we have left is a very simple notation, called *context-free grammars*. Just as regular expressions can be used to define lexical structure in a static, declarative way, grammars define syntactic structure declaratively. But we will need something more powerful than finite automata to parse languages described by grammars.

In fact, grammars can also be used to describe the structure of lexical tokens, although regular expressions are adequate - and more concise - for that purpose.

3.1 CONTEXT-FREE GRAMMARS

As before, we say that a *language* is a set of *strings*; each string is a finite sequence of *symbols* taken from a finite *alphabet*. For parsing, the strings are source programs, the symbols are lexical tokens, and the alphabet is the set of token-types returned by the lexical analyzer.

A context-free grammar describes a language. A grammar has a set of *productions* of the form

symbol → *symbol symbol … symbol*

where there are zero or more symbols on the right-hand side. Each symbol is either *terminal*, meaning that it is a token from the alphabet of strings in the language, or *nonterminal*, meaning that it appears on the left-hand side of some production. No token can ever appear on the left-hand side of a production. Finally, one of the nonterminals is distinguished as the *start symbol* of the grammar.

Grammar 3.1 is an example of a grammar for straight-line programs. The start symbol is *S* (when the start symbol is not written explicitly it is conventional to assume that the left-hand nonterminal in the first production is the start symbol). The terminal symbols are

```
id print num, + ( ) := ;
```

GRAMMAR 3.1: A syntax for straight-line programs.

1. *S* → *S*; *S*
2. *S* → id := *E*
3. *S* → print (*L*)
4. *E* → id
5. *E* → num
6. *E* → *E* + *E*
7. *E* → (*S*, *E*)
8. *L* → *E*
9. *L* → *L*, *E*

and the nonterminals are *S*, *E*, and *L*. One sentence in the language of this grammar is

```
id := num; id := id + (id := num + num, id)
```

where the source text (before lexical analysis) might have been

```
a : = 7;
b : = c + (d : = 5 + 6, d)
```

The token-types (terminal symbols) are `id`, `num`, `:=`, and so on; the names (`a,b,c,d`) and numbers (7, 5, 6) are *semantic values* associated with some of the tokens.

DERIVATIONS

To show that this sentence is in the language of the grammar, we can perform a *derivation*: Start with the start symbol, then repeatedly replace any nonterminal by one of its right-hand sides, as shown in Derivation 3.2.

DERIVATION 3.2

- $\underline{S}$
- *S* ; $\underline{S}$
- $\underline{S}$; id := *E*
- id := $\underline{E}$; id := *E*
- id := num ; id := $\underline{E}$
- id := num ; id := $\underline{E}$ + *E*
- id := num ; id := $\underline{E}$ + (*S*, *E*)
- id := num ; id := id + ($\underline{S}$, *E*)
- id := num ; id := id + (id := $\underline{E}$, *E*)
- id := num ; id := id + (id := *E* + *E*, $\underline{E}$)
- id := num ; id := id + (id := $\underline{E}$ + *E*, id)
- id := num ; id := id + (id := num + $\underline{E}$, id)
- id := num ; id := id + (id := num + num, id)

There are many different derivations of the same sentence. A *leftmost* derivation is one in which the leftmost nonterminal symbol is always the one expanded; in a *rightmost* derivation, the rightmost nonterminal is always the next to be expanded.

Derivation 3.2 is neither leftmost nor rightmost; a leftmost derivation for this sentence would begin,

- $\underline{S}$
- $\underline{S}$; S
- id := $\underline{E}$; S
- id := num ; $\underline{S}$
- id := num ; id := $\underline{E}$
- id := num ; id := $\underline{E}$ + E
- ⋮

PARSE TREES

A *parse tree* is made by connecting each symbol in a derivation to the one from which it was derived, as shown in Figure 3.3. Two different derivations can have the same parse tree.

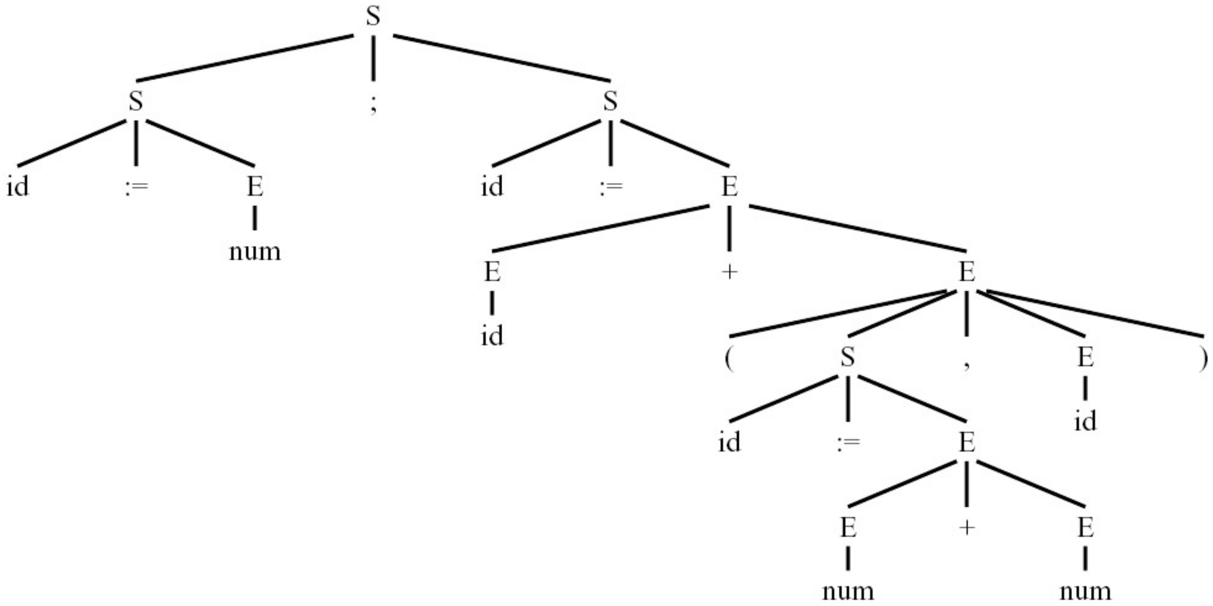


Figure 3.3: Parse tree.

AMBIGUOUS GRAMMARS

A grammar is *ambiguous* if it can derive a sentence with two different parse trees. Grammar 3.1 is ambiguous, since the sentence `id := id+id+id` has two parse trees (Figure 3.4).

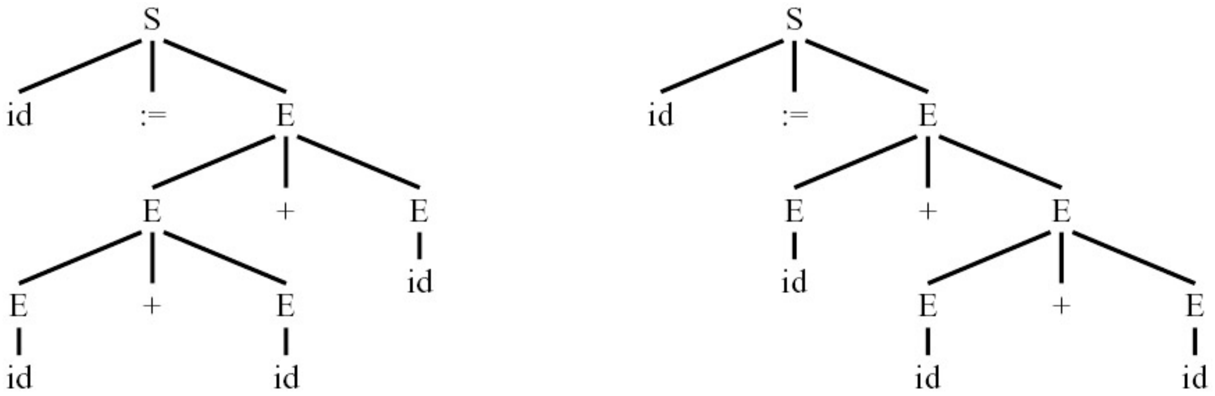


Figure 3.4: Two parse trees for the same sentence using Grammar 3.1.

Grammar 3.5 is also ambiguous; Figure 3.6 shows two parse trees for the sentence `1-2-3`, and Figure 3.7 shows two trees for `1+2*3`. Clearly, if we use parse trees to interpret the meaning of the expressions, the two parse trees for `1-2-3` mean different things: $(1 - 2) - 3$ D-4 versus $1 - (2 - 3)$ D 2. Similarly, $(1 + 2) \times 3$ is not the same as $1 + (2 \times 3)$. And indeed, compilers do use parse trees to derive meaning.

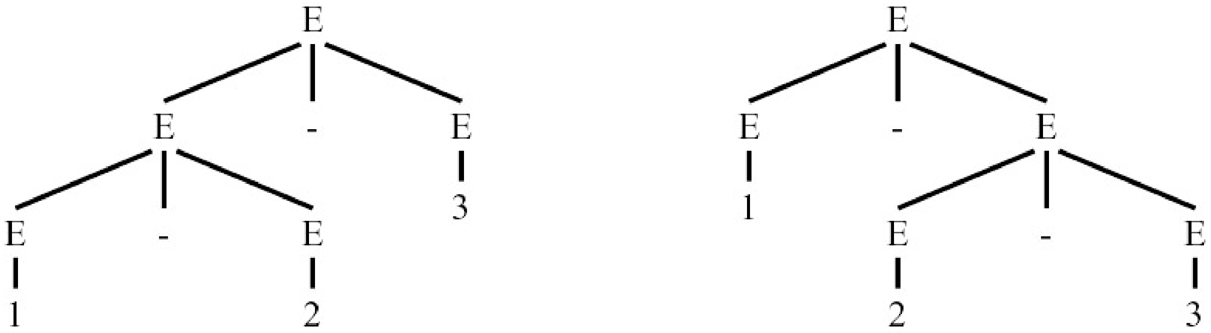


Figure 3.6: Two parse trees for the sentence `1-2-3` in Grammar 3.5.

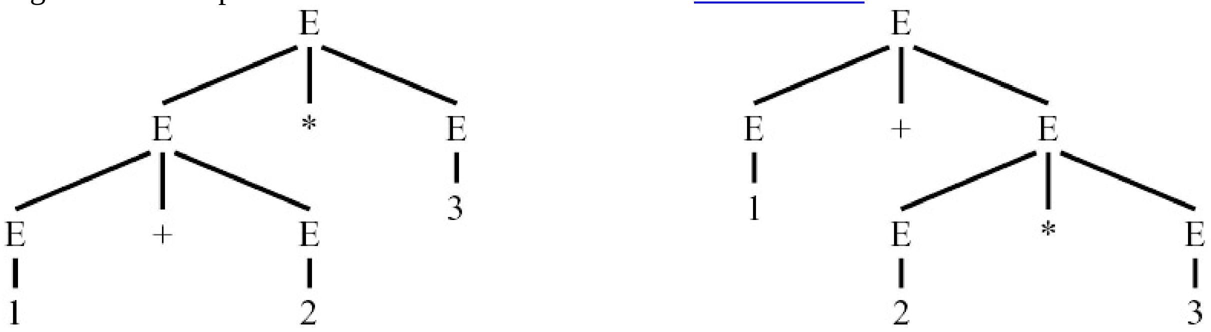


Figure 3.7: Two parse trees for the sentence `1+2*3` in Grammar 3.5.

GRAMMAR 3.5

- *E* → id
- *E* → num
- *E* → *E* * *E*
- *E* → *E* / *E*
- *E* → *E* + *E*
- *E* → *E* − *E*
- *E* → (*E*)

GRAMMAR 3.8

- $E \to E + T$
- $E \to E - T$
- $E \to T$
- $T \to T * F$
- $T \to T / F$
- $T \to F$
- $F \to$ id
- $F \to$ num
- $F \to (E)$

Therefore, ambiguous grammars are problematic for compiling: In general, we would prefer to have unambiguous grammars. Fortunately, we can often transform ambiguous grammars to unambiguous grammars.

Let us find an unambiguous grammar that accepts the same language as Grammar 3.5. First, we would like to say that * *binds tighter* than +, or has *higher precedence*. Second, we want to say that each operator *associates to the left*, so that we get $(1 - 2) - 3$ instead of $1 - (2 - 3)$. We do this by introducing new nonterminal symbols to get Grammar 3.8.

The symbols *E*, *T*, and *F* stand for *expression*, *term*, and *factor*; conventionally, factors are things you multiply and terms are things you add.

This grammar accepts the same set of sentences as the ambiguous grammar, but now each sentence has exactly one parse tree. Grammar 3.8 can never produce parse trees of the form shown in Figure 3.9 (see Exercise 3.17).

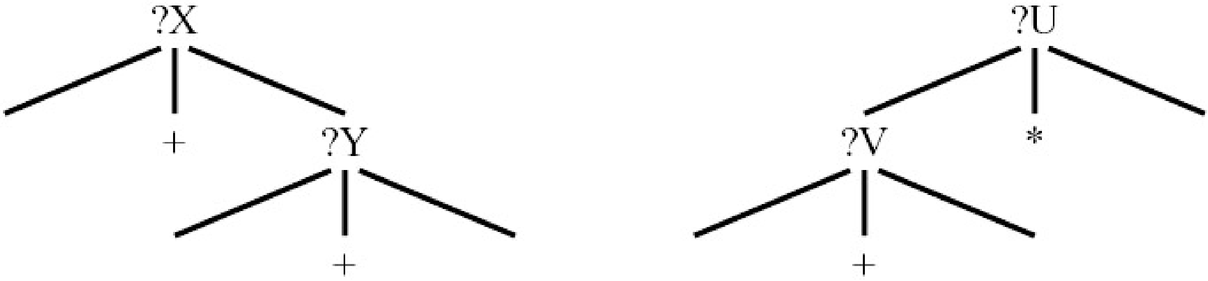


Figure 3.9: Parse trees that Grammar 3.8 will never produce.

Had we wanted to make * associate to the right, we could have written its production as $T \to F * T$.

We can usually eliminate ambiguity by transforming the grammar. Though there are some languages (sets of strings) that have ambiguous grammars but no unambiguous grammar, such languages may be problematic as *programming* languages because the syntactic ambiguity may lead to problems in writing and understanding programs.

END-OF-FILE MARKER

Parsers must read not only terminal symbols such as +, −, num, and so on, but also the end-of-file marker. We will use $ to represent end of file.

Suppose S is the start symbol of a grammar. To indicate that \$ must come after a complete S-phrase, we augment the grammar with a new start symbol S' and a new production $S' \rightarrow S\$$.

In Grammar 3.8, E is the start symbol, so an augmented grammar is Grammar 3.10.

GRAMMAR 3.10

- $S \rightarrow E\ \$$
-
- $E \rightarrow E + T$
- $E \rightarrow E - T$
- $E \rightarrow T$
-
- $T \rightarrow T * F$
- $T \rightarrow T \,/\, F$
- $T \rightarrow F$
-
- $F \rightarrow$ id
- $F \rightarrow$ num
- $F \rightarrow (E)$

3.2 PREDICTIVE PARSING

Some grammars are easy to parse using a simple algorithm known as *recursive descent*. In essence, each grammar production turns into one clause of a recursive function. We illustrate this by writing a recursive-descent parser for Grammar 3.11.

GRAMMAR 3.11

- $S \rightarrow$ if E then S else S
- $S \rightarrow$ begin $S\ L$
- $S \rightarrow$ print E
-
- $L \rightarrow$ end
- $L \rightarrow$; $S\ L$
-
- $E \rightarrow$ num = num

A recursive-descent parser for this language has one function for each nonterminal and one clause for each production.

```
final int IF=1, THEN=2, ELSE=3, BEGIN=4, END=5, PRINT=6,
          SEMI=7, NUM=8, EQ=9;

int tok = getToken();

void advance() {tok=getToken();}
void eat(int t) {if (tok==t) advance(); else error();}

void S() {switch(tok) {
        case IF: eat(IF); E(); eat(THEN); S();
                 eat(ELSE); S(); break;
        case BEGIN: eat(BEGIN); S(); L(); break;
        case PRINT: eat(PRINT); E(); break;
        default: error();
       }}
void L() {switch(tok) {
        case END: eat(END); break;
        case SEMI: eat(SEMI); S(); L(); break;
        default: error();
```

```
         }}
void E() { eat(NUM); eat(EQ); eat(NUM); }
```

With suitable definitions of `error` and `getToken`, this program will parse very nicely.

Emboldened by success with this simple method, let us try it with Grammar 3.10:

```
void S() {  E(); eat(EOF); }
void E() {switch (tok) {
          case ?: E(); eat(PLUS); T(); break;
          case ?: E(); eat(MINUS); T(); break;
          case ?: T(); break;
          default: error();
         }}
void T() {switch (tok) {
          case ?: T(); eat(TIMES); F(); break;
          case ?: T(); eat(DIV); F(); break;
          case ?: F(); break;
          default: error();
         }}
```

There is a *conflict* here: The E function has no way to know which clause to use. Consider the strings `(1*2-3)+4` and `(1*2-3)`. In the former case, the initial call to E should use the $E \rightarrow E + T$ production, but the latter case should use $E \rightarrow T$.

Recursive-descent, or *predictive*, parsing works only on grammars where the *first terminal symbol* of each subexpression provides enough information to choose which production to use. To understand this better, we will formalize the notion of FIRST sets, and then derive conflict-free recursive-descent parsers using a simple algorithm.

Just as lexical analyzers can be constructed from regular expressions, there are parser-generator tools that build predictive parsers. But if we are going to use a tool, then we might as well use one based on the more powerful LR(1) parsing algorithm, which will be described in Section 3.3.

Sometimes it's inconvenient or impossible to use a parser-generator tool. The advantage of predictive parsing is that the algorithm is simple enough that we can use it to construct parsers by hand - we don't need automatic tools.

FIRST AND FOLLOW SETS

Given a string γ of terminal and nonterminal symbols, FIRST(γ) is the set of all terminal symbols that can begin any string derived from γ. For example, let $\gamma = T * F$. Any string of terminal symbols derived from γ must start with `id`, `num`, or `(`. Thus, FIRST($T * F$) = {`id`, `num`, `(`}.

If two different productions $X \rightarrow \gamma_1$ and $X \rightarrow \gamma_2$ have the same lefthand-side symbol (X) and their right-hand sides have overlapping FIRST sets, then the grammar cannot be parsed using predictive parsing. If some terminal symbol I is in FIRST(γ_1) and also in FIRST(γ_2), then the X function in a recursive-descent parser will not know what to do if the input token is I.

The computation of FIRST sets looks very simple: If γ = *X Y Z*, it seems as if *Y* and *Z* can be ignored, and FIRST(*X*) is the only thing that matters. But consider Grammar 3.12. Because *Y* can produce the empty string - and therefore *X* can produce the empty string - we find that FIRST(*X Y Z*) must include FIRST(*Z*). Therefore, in computing FIRST sets, we must keep track of which symbols can produce the empty string; we say such symbols are *nullable*. And we must keep track of what might follow a nullable symbol.

GRAMMAR 3.12

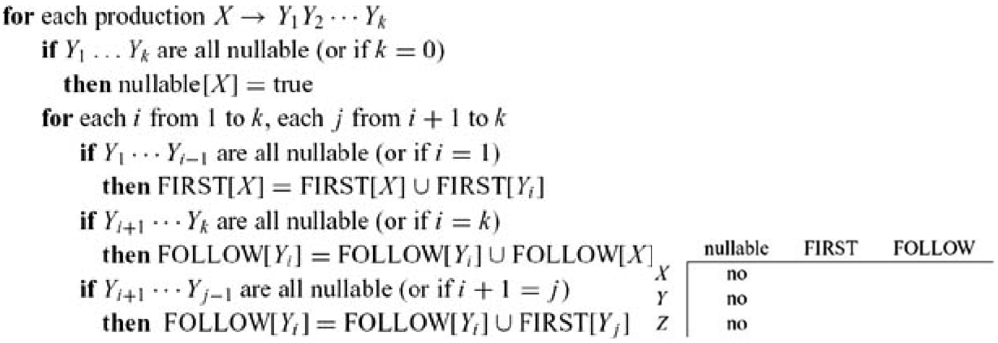
- $Z \rightarrow d$
- $Z \rightarrow X\ Y\ Z$
- $Y \rightarrow$
- $Y \rightarrow c$
- $X \rightarrow Y$
- $X \rightarrow a$

With respect to a particular grammar, given a string γ of terminals and nonterminals,

- nullable(*X*) is true if *X* can derive the empty string.
- FIRST(γ) is the set of terminals that can begin strings derived from γ.
- FOLLOW(*X*) is the set of terminals that can immediately follow *X*. That is, $t \in$ FOLLOW(*X*) if there is any derivation containing *Xt*. This can occur if the derivation contains *X Y Zt* where *Y* and *Z* both derive ϵ.

A precise definition of FIRST, FOLLOW, and nullable is that they are the smallest sets for which these properties hold:

For each terminal symbol *Z*, FIRST[*Z*] = {*Z*}.



```
for each production $X \rightarrow Y_1 Y_2 \cdots Y_k$
  if $Y_1 \ldots Y_k$ are all nullable (or if $k = 0$)
    then nullable[$X$] = true
  for each $i$ from 1 to $k$, each $j$ from $i + 1$ to $k$
    if $Y_1 \cdots Y_{i-1}$ are all nullable (or if $i = 1$)
      then FIRST[$X$] = FIRST[$X$] $\cup$ FIRST[$Y_i$]
    if $Y_{i+1} \cdots Y_k$ are all nullable (or if $i = k$)
      then FOLLOW[$Y_i$] = FOLLOW[$Y_i$] $\cup$ FOLLOW[$X$]
    if $Y_{i+1} \cdots Y_{j-1}$ are all nullable (or if $i + 1 = j$)
      then FOLLOW[$Y_i$] = FOLLOW[$Y_i$] $\cup$ FIRST[$Y_j$]
```

	nullable	FIRST	FOLLOW
X	no		
Y	no		
Z	no		

Algorithm 3.13 for computing FIRST, FOLLOW, and nullable just follows from these facts; we simply replace each equation with an assignment statement, and iterate.

ALGORITHM 3.13: Iterative computation of *FIRST*, *FOLLOW*, and *nullable*.

Algorithm to compute FIRST, FOLLOW, *and* nullable.

Initialize FIRST and FOLLOW to all empty sets, and nullable to all false.

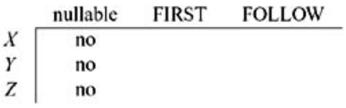
```
for each terminal symbol Z
   FIRST[Z] ← {Z}
repeat
```

```
  for each production $X \rightarrow Y_1 Y_2 \ldots Y_k$
        if $Y_1 \ldots Y_k$ are all nullable (or if $k = 0$)
               then nullable[$X$] ← true
       for each $i$ from 1 to $k$, each $j$ from $i + 1$ to $k$
              if $Y_1 \ldots Y_{i-1}$ are all nullable (or if $i = 1$)
                   then FIRST[$X$] ← FIRST[$X$] ∪ FIRST[$Y_i$ ]
              if $Y_{i+1} \ldots Y_k$ are all nullable (or if $i = k$)
                   then FOLLOW[$Y_i$] ← FOLLOW[$Y_i$] ∪ FOLLOW[$X$]
             if $Y_{i+1} \ldots Y_{j-1}$ are all nullable (or if $i + 1 = j$)
                   then FOLLOW[$Y_i$] ← FOLLOW[$Y_i$] ∪ FIRST[$Y_j$]
until FIRST, FOLLOW, and nullable did not change in this iteration.
```

Of course, to make this algorithm efficient it helps to examine the productions in the right order; see Section 17.4. Also, the three relations need not be computed simultaneously; nullable can be computed by itself, then FIRST, then FOLLOW.

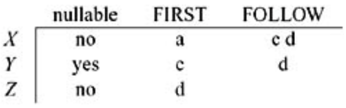
This is not the first time that a group of equations on sets has become the algorithm for calculating those sets; recall the algorithm on page 28 for computing ϵ-closure. Nor will it be the last time; the technique of iteration to a fixed point is applicable in dataflow analysis for optimization, in the back end of a compiler.

We can apply this algorithm to Grammar 3.12. Initially, we have:



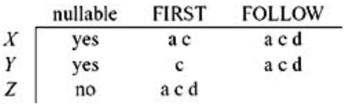
	nullable	FIRST	FOLLOW
X	no		
Y	no		
Z	no		

In the first iteration, we find that $a \in$ FIRST[X], Y is nullable, $c \in$ FIRST[Y], $d \in$ FIRST[Z], $d \in$ FOLLOW[X], $c \in$ FOLLOW[X], $d \in$ FOLLOW[Y]. Thus:



	nullable	FIRST	FOLLOW
X	no	a	c d
Y	yes	c	d
Z	no	d	

In the second iteration, we find that X is nullable, $c \in$ FIRST[X], $\{a; c\} \subseteq$ FIRST[Z], $\{a, c, d\} \subseteq$ FOLLOW[X], $\{a, c, d\} \subseteq$ FOLLOW[Y]. Thus:



	nullable	FIRST	FOLLOW
X	yes	a c	a c d
Y	yes	c	a c d
Z	no	a c d	

The third iteration finds no new information, and the algorithm terminates.

It is useful to generalize the FIRST relation to strings of symbols:

FIRST($X\gamma$) = FIRST[X]   if not nullable[X]

FIRST($X\gamma$) = FIRST[X] ∪ FIRST(γ)   if nullable[X]

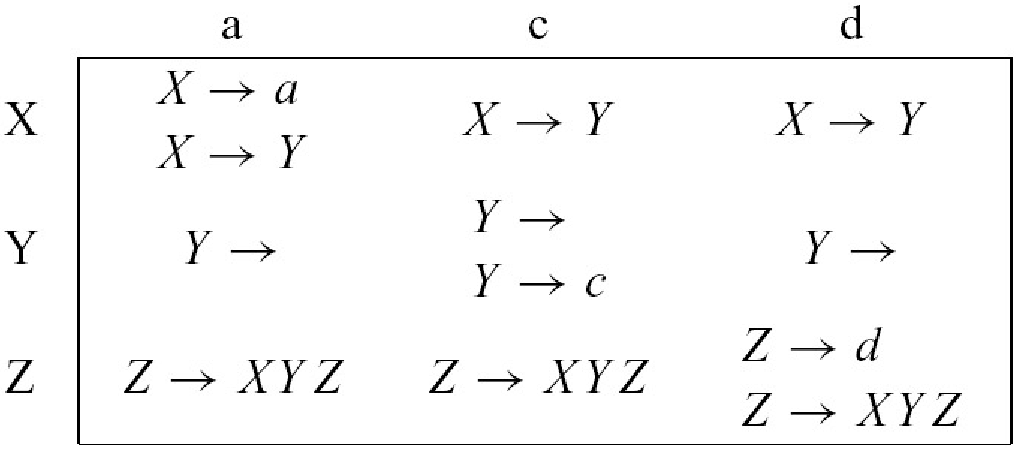
and similarly, we say that a string γ is nullable if each symbol in γ is nullable.

CONSTRUCTING A PREDICTIVE PARSER

Consider a recursive-descent parser. The parsing function for some nonterminal *X* has a clause for each *X* production; it must choose one of these clauses based on the next token *T* of the input. If we can choose the right production for each (*X*, *T*), then we can write the recursive-descent parser. All the information we need can be encoded as a two-dimensional table of productions, indexed by nonterminals *X* and terminals *T*. Thisiscalleda *predictive parsing* table.

To construct this table, enter production $X \to \gamma$ in row *X*, column *T* of the table for each $T \in$ FIRST(γ). Also, if γ is nullable, enter the production in row *X*, column *T* for each $T \in$ FOLLOW[*X*].

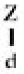
Figure 3.14 shows the predictive parser for Grammar 3.12. But some of the entries contain more than one production! The presence of duplicate entries means that predictive parsing will not work on Grammar 3.12.

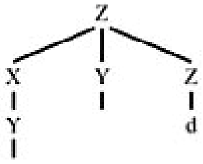


	a	c	d
X	$X \to a$ $X \to Y$	$X \to Y$	$X \to Y$
Y	$Y \to$	$Y \to$ $Y \to c$	$Y \to$
Z	$Z \to XYZ$	$Z \to XYZ$	$Z \to d$ $Z \to XYZ$

Figure 3.14: Predictive parsing table for Grammar 3.12.

If we examine the grammar more closely, we find that it is ambiguous. The sentence *d* has many parse trees, including:





An ambiguous grammar will always lead to duplicate entries in a predictive parsing table. If we need to use the language of Grammar 3.12 as a programming language, we will need to find an unambiguous grammar.

Grammars whose predictive parsing tables contain no duplicate entries are called LL(1). This stands for *left-to-right parse, leftmost-derivation, 1-symbol lookahead*. Clearly a recursive-descent (predictive) parser examines the input left-to-right in one pass (some parsing algorithms do not, but these are generally not useful for compilers). The order in which a predictive parser expands nonterminals into right-hand sides (that is, the recursive-descent parser calls functions corresponding to nonterminals) is just the order in which a leftmost derivation expands nonterminals. And a recursive-descent parser does its job just by looking at the next token of the input, never looking more than one token ahead.

We can generalize the notion of FIRST sets to describe the first *k* tokens of a string, and to make an LL(*k*) parsing table whose rows are the nonterminals and columns are every sequence of *k* terminals. This is rarely done (because the tables are so large), but sometimes when you write a recursive-descent parser by hand you need to look more than one token ahead.

Grammars parsable with LL(2) parsing tables are called LL(2) grammars, and similarly for LL(3), etc. Every LL(1) grammar is an LL(2) grammar, and so on. No ambiguous grammar is LL(*k*)forany *k*.

ELIMINATING LEFT RECURSION

Suppose we want to build a predictive parser for Grammar 3.10. The two productions

$$\begin{aligned} &E \rightarrow E + T \\ &E \rightarrow T \end{aligned}$$

are certain to cause duplicate entries in the LL(1) parsing table, since any token in FIRST(*T*) will also be in FIRST(*E* + *T*). The problem is that *E* appears as the first right-hand-side symbol in an *E*-production; this is called *left recursion*. Grammars with left recursion cannot be LL(1).

To eliminate left recursion, we will rewrite using right recursion. We introduce a new nonterminal *E*′, and write

$$E \rightarrow T\ E'$$

$$\begin{aligned} &E' \rightarrow +\ T\ E' \\ &E' \rightarrow \end{aligned}$$

This derives the same set of strings (on *T* and +) as the original two productions, but now there is no left recursion.

In general, whenever we have productions *X* → *X*γ and *X* → α, where α does not start with *X*, we know that this derives strings of the form αγ*, an α followed by zero or more γ. So we can rewrite the regular expression using right recursion:

$$\begin{pmatrix} X \rightarrow X\,\gamma_1 \\ X \rightarrow X\,\gamma_2 \\ X \rightarrow \alpha_1 \\ X \rightarrow \alpha_2 \end{pmatrix} \Longrightarrow \begin{pmatrix} X \rightarrow \alpha_1\,X' \\ X \rightarrow \alpha_2\,X' \\ X' \rightarrow \gamma_1\,X' \\ X' \rightarrow \gamma_2\,X' \\ X' \rightarrow \end{pmatrix}$$

Applying this transformation to Grammar 3.10, we obtain Grammar 3.15.

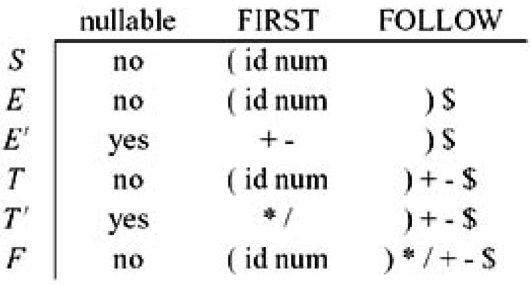
GRAMMAR 3.15

- *S* → *E* $
- *E* → *T E*′
- *E*′ → + *T E*′
- *E*′ →− *T E*′
- *E*′ →
- *T* → *F T*′

- $T' \rightarrow * F T'$
- $T' \rightarrow / F T'$
- $T' \rightarrow$
- $F \rightarrow$ id
- $F \rightarrow$ num
- $F \rightarrow (E)$

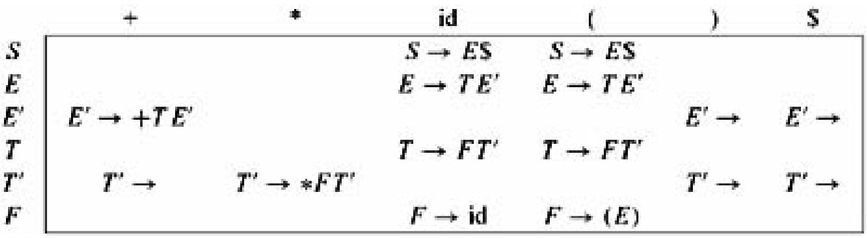
To build a predictive parser, first we compute nullable, FIRST, and FOLLOW (Table 3.16). The predictive parser for Grammar 3.15 is shown in Table 3.17.

Table 3.16: Nullable, FIRST, and FOLLOW for Grammar 3.15.



	nullable	FIRST	FOLLOW
S	no	(id num	
E	no	(id num	) $
E'	yes	+ -	) $
T	no	(id num	) + - $
T'	yes	* /	) + - $
F	no	(id num	) * / + - $

Table 3.17: Predictive parsing table for Grammar 3.15. We omit the columns for `num`, `/`, and -, as they are similar to others in the table.



	+	*	id	(	)	$
S			$S \rightarrow E\$$	$S \rightarrow E\$$		
E			$E \rightarrow TE'$	$E \rightarrow TE'$		
E'	$E' \rightarrow +TE'$				$E' \rightarrow$	$E' \rightarrow$
T			$T \rightarrow FT'$	$T \rightarrow FT'$		
T'	$T' \rightarrow$	$T' \rightarrow *FT'$			$T' \rightarrow$	$T' \rightarrow$
F			$F \rightarrow$ id	$F \rightarrow (E)$		

LEFT FACTORING

We have seen that left recursion interferes with predictive parsing, and that it can be eliminated. A similar problem occurs when two productions for the same nonterminal start with the same symbols. For example:

$$S \rightarrow \text{if } E \text{ then } S \text{ else } S$$
$$S \rightarrow \text{if } E \text{ then } S$$

In such a case, we can *left factor* the grammar - that is, take the allowable endings (*else S* and ϵ) and make a new nonterminal X to stand for them:

$$S \rightarrow \text{if } E \text{ then } S\ X$$
$$X \rightarrow$$
$$X \rightarrow \text{else } S$$

The resulting productions will not pose a problem for a predictive parser. Although the grammar is still ambiguous - the parsing table has two entries for the same slot - we can resolve the ambiguity by using the *else S* action.

ERROR RECOVERY

Armed with a predictive parsing table, it is easy to write a recursive-descent parser. Here is a representative fragment of a parser for Grammar 3.15:

```
void T() {switch (tok) {
        case ID:
        case NUM:
        case LPAREN: F(); Tprime(); break;
        default: error!
      }}
void Tprime() {switch (tok) {
        case PLUS: break;
        case TIMES: eat(TIMES); F(); Tprime(); break;
        case EOF: break;
        case RPAREN: break;
        default: error!
      }}
```

A blank entry in row *T*, column *x* of the LL(1) parsing table indicates that the parsing function `T()` does not expect to see token *x* - this will be a syntax error. How should *error* be handled? It is safe just to raise an exception and quit parsing, but this is not very friendly to the user. It is better to print an error message and recover from the error, so that other syntax errors can be found in the same compilation.

A syntax error occurs when the string of input tokens is not a sentence in the language. Error recovery is a way of finding some sentence similar to that string of tokens. This can proceed by deleting, replacing, or inserting tokens.

For example, error recovery for `T` could proceed by inserting a `num` token. It's not necessary to adjust the actual input; it suffices to pretend that the `num` was there, print a message, and return normally.

```
void T() {switch (tok) {
        case ID:
        case NUM:
        case LPAREN: F(); Tprime(); break;
        default: print("expected id, num, or left-paren");
      }}
```

It's a bit dangerous to do error recovery by insertion, because if the error cascades to produce another error, the process might loop infinitely. Error recovery by deletion is safer, because the loop must eventually terminate when end-of-file is reached.

Simple recovery by deletion works by skipping tokens until a token in the FOLLOW set is reached. For example, error recovery for T' could work like this:

```
int Tprime_follow [] = {PLUS, RPAREN, EOF};

void Tprime() { switch (tok) {
        case PLUS: break;
        case TIMES: eat(TIMES); F(); Tprime(); break;
        case RPAREN: break;
        case EOF: break;
        default: print("expected +, *, right-paren,
                        or end-of-file");
                 skipto(Tprime_follow);
      }}
```

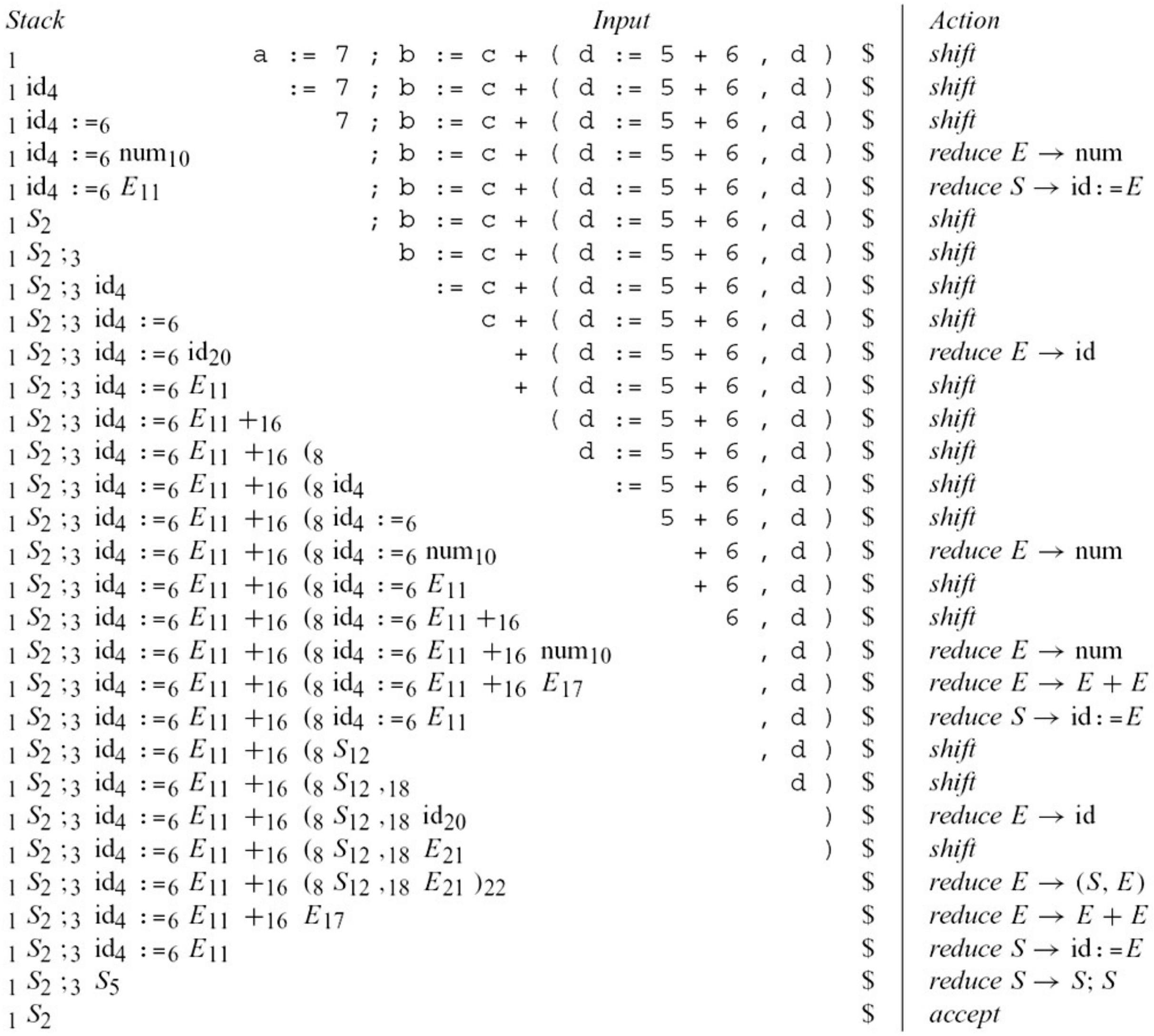
A recursive-descent parser's error-recovery mechanisms must be adjusted (sometimes by trial and error) to avoid a long cascade of error-repair messages resulting from a single token out of place.

3.3 LR PARSING

The weakness of LL(*k*) parsing techniques is that they must *predict* which production to use, having seen only the first *k* tokens of the right-hand side. A more powerful technique, LR(*k*) parsing, is able to postpone the decision until it has seen input tokens corresponding to the entire right-hand side of the production in question (and *k* more input tokens beyond).

LR(*k*) stands for *left-to-right parse, rightmost-derivation, k-token lookahead.* The use of a rightmost derivation seems odd; how is that compatible with a left-to-right parse? Figure 3.18 illustrates an LR parse of the program

```
a:=7;
b:=c+(d:=5+6,d)
```



Stack	*Input*	*Action*
$_1$	a := 7 ; b := c + (d := 5 + 6 , d) \$	*shift*
$_1\ \text{id}_4$	:= 7 ; b := c + (d := 5 + 6 , d) \$	*shift*
$_1\ \text{id}_4\ :=_6$	7 ; b := c + (d := 5 + 6 , d) \$	*shift*
$_1\ \text{id}_4\ :=_6\ \text{num}_{10}$	; b := c + (d := 5 + 6 , d) \$	*reduce* $E \rightarrow \text{num}$
$_1\ \text{id}_4\ :=_6\ E_{11}$	; b := c + (d := 5 + 6 , d) \$	*reduce* $S \rightarrow \text{id}:=E$
$_1\ S_2$	; b := c + (d := 5 + 6 , d) \$	*shift*
$_1\ S_2\ ;_3$	b := c + (d := 5 + 6 , d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4$	:= c + (d := 5 + 6 , d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6$	c + (d := 5 + 6 , d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ \text{id}_{20}$	+ (d := 5 + 6 , d) \$	*reduce* $E \rightarrow \text{id}$
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}$	+ (d := 5 + 6 , d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}$	(d := 5 + 6 , d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8$	d := 5 + 6 , d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ \text{id}_4$	:= 5 + 6 , d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ \text{id}_4\ :=_6$	5 + 6 , d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ \text{id}_4\ :=_6\ \text{num}_{10}$	+ 6 , d) \$	*reduce* $E \rightarrow \text{num}$
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ \text{id}_4\ :=_6\ E_{11}$	+ 6 , d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ \text{id}_4\ :=_6\ E_{11}\ +_{16}$	6 , d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ \text{num}_{10}$	, d) \$	*reduce* $E \rightarrow \text{num}$
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ E_{17}$	, d) \$	*reduce* $E \rightarrow E + E$
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ \text{id}_4\ :=_6\ E_{11}$	, d) \$	*reduce* $S \rightarrow \text{id}:=E$
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ S_{12}$	, d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ S_{12}\ ,_{18}$	d) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ S_{12}\ ,_{18}\ \text{id}_{20}$	) \$	*reduce* $E \rightarrow \text{id}$
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ S_{12}\ ,_{18}\ E_{21}$	) \$	*shift*
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ (_8\ S_{12}\ ,_{18}\ E_{21}\ )_{22}$	\$	*reduce* $E \rightarrow (S, E)$
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}\ +_{16}\ E_{17}$	\$	*reduce* $E \rightarrow E + E$
$_1\ S_2\ ;_3\ \text{id}_4\ :=_6\ E_{11}$	\$	*reduce* $S \rightarrow \text{id}:=E$
$_1\ S_2\ ;_3\ S_5$	\$	*reduce* $S \rightarrow S; S$
$_1\ S_2$	\$	*accept*

Figure 3.18: Shift-reduce parse of a sentence. Numeric subscripts in the *Stack* are DFA state numbers; see Table 3.19.

using Grammar 3.1, augmented with a new start production $S' \rightarrow S\$$.

The parser has a *stack* and an *input*. The first *k* tokens of the input are the *lookahead*. Based on the contents of the stack and the lookahead, the parser performs two kinds of actions:

Shift: Move the first input token to the top of the stack.

Reduce: Choose a grammar rule $X \rightarrow A\ B\ C$; pop *C*, *B*, *A* from the top of the stack; push *X* onto the stack.

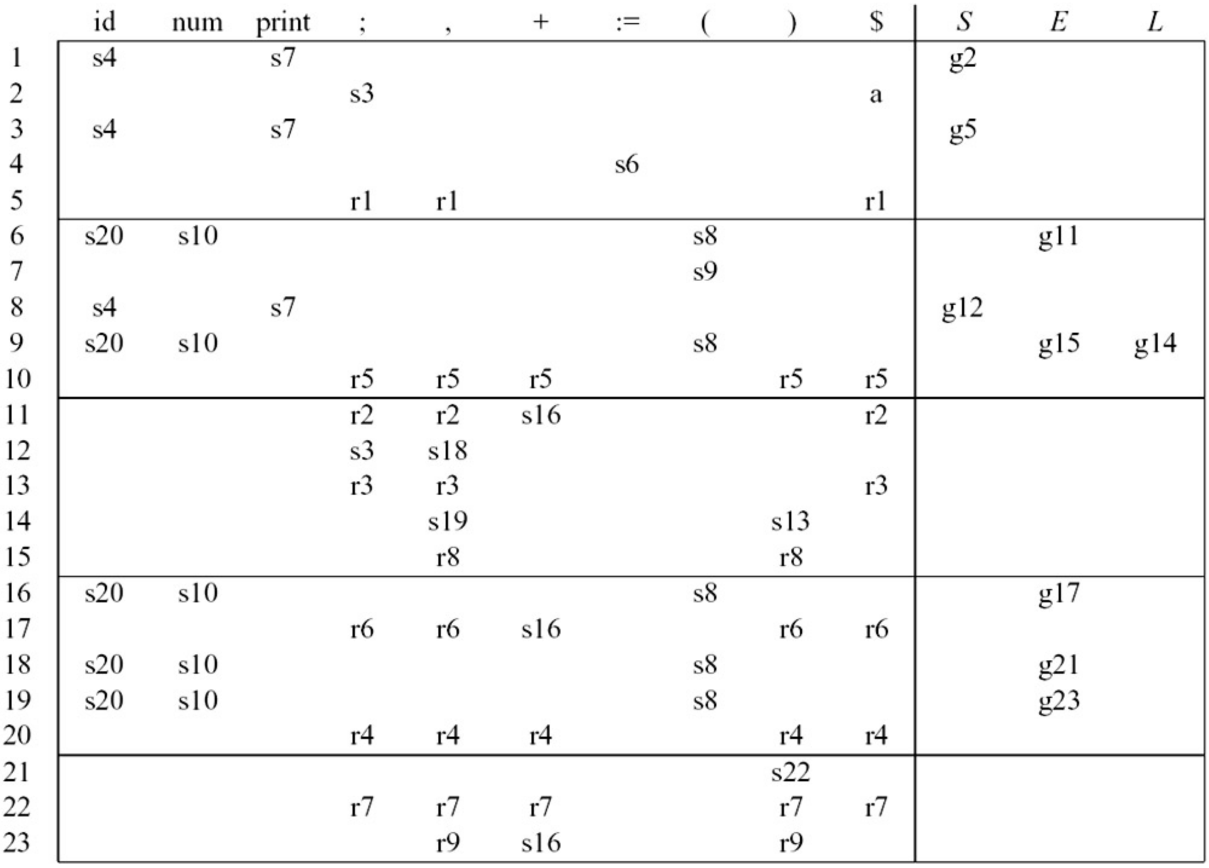
Initially, the stack is empty and the parser is at the beginning of the input. The action of shifting the end-of-file marker $ is called *accepting* and causes the parser to stop successfully.

In Figure 3.18, the stack and input are shown after every step, along with an indication of which action has just been performed. The concatenation of stack and input is always one line of a rightmost derivation; in fact, Figure 3.18 shows the rightmost derivation of the input string, upside-down.

LR PARSING ENGINE

How does the LR parser know when to shift and when to reduce? By using a deterministic finite automaton! The DFA is not applied to the input - finite automata are too weak to parse context-free grammars - but to the stack. The edges of the DFA are labeled by the symbols (terminals and nonterminals) that can appear on the stack. Table 3.19 is the transition table for Grammar 3.1.

Table 3.19: LR parsing table for Grammar 3.1.



	id	num	print	;	,	+	:=	(	)	$	*S*	*E*	*L*
1	s4		s7								g2		
2				s3						a			
3	s4		s7								g5		
4							s6						
5				r1	r1					r1			
6	s20	s10						s8				g11	
7								s9					
8	s4		s7								g12		
9	s20	s10						s8				g15	g14
10				r5	r5	r5			r5	r5			
11				r2	r2	s16				r2			
12				s3	s18								
13				r3	r3					r3			
14					s19				s13				
15					r8				r8				
16	s20	s10						s8				g17	
17				r6	r6	s16			r6	r6			
18	s20	s10						s8				g21	
19	s20	s10						s8				g23	
20				r4	r4	r4			r4	r4			
21									s22				
22				r7	r7	r7			r7	r7			
23					r9	s16			r9				

The elements in the transition table are labeled with four kinds of actions:

s*n* Shift into state *n*;

g*n* Goto state *n*;

r*k* Reduce by rule *k*;

a Accept;
Error (denoted by a blank entry in the table).

To use this table in parsing, treat the shift and goto actions as edges of a DFA, and scan the stack. For example, if the stack is id := *E*, then the DFA goes from state 1 to 4 to 6 to 11. If the next input token is a semicolon, then the ";" column in state 11 says to reduce by rule 2.

The second rule of the grammar is *S* → id:=*E*, so the top three tokens are popped from the stack and *S* is pushed.

The action for "+" in state 11 is to shift; so if the next token had been + instead, it would have been eaten from the input and pushed on the stack.

Rather than rescan the stack for each token, the parser can remember instead the state reached for each stack element. Then the parsing algorithm is

Look up top stack state, and input symbol, to get action; If action is

Shift(n): Advance input one token; push n on stack.

Reduce(k): Pop stack as many times as the number of symbols on the right-hand side of rule k;
Let X be the left-hand-side symbol of rule k;
In the state now on top of stack, look up X to get "goto n";
Push n on top of stack.

Accept: Stop parsing, report success.

Error: Stop parsing, report failure.

LR(0) PARSER GENERATION

An LR(k) parser uses the contents of its stack and the next k tokens of the input to decide which action to take. Table 3.19 shows the use of one symbol of lookahead. For $k = 2$, the table has columns for every two-token sequence and so on; in practice, $k > 1$ is not used for compilation. This is partly because the tables would be huge, but more because most reasonable programming languages can be described by *LR*(1) grammars.

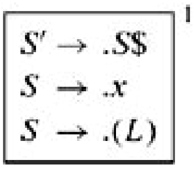
LR(0) grammars are those that can be parsed looking only at the stack, making shift/reduce decisions without any lookahead. Though this class of grammars is too weak to be very useful, the algorithm for constructing LR(0) parsing tables is a good introduction to the LR(1) parser construction algorithm.

We will use Grammar 3.20 to illustrate LR(0) parser generation. Consider what the parser for this grammar will be doing. Initially, it will have an empty stack, and the input will be a complete S-sentence followed by $; that is, the right-hand side of the S' rule will be on the input. We indicate this as $S' \rightarrow .S\$$ where the dot indicates the current position of the parser.

GRAMMAR 3.20

0. $S' \rightarrow S\$$
1. $S \rightarrow (L)$
2. $S \rightarrow x$
3. $L \rightarrow S$
4. $L \rightarrow L, S$

In this state, where the input begins with S, that means that it begins with any possible right-hand side of an S-production; we indicate that by

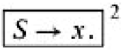


$S' \to .S\$$
$S \to .x$
$S \to .(L)$

1

Call this state 1. A grammar rule, combined with the dot that indicates a position in its right-hand side, is called an *item* (specifically, an *LR(0) item*). A state is just a set of items.

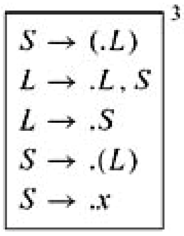
Shift actions In state 1, consider what happens if we shift an *x*. Wethen know that the end of the stack has an *x*; we indicate that by shifting the dot past the *x* in the $S \to x$ production. The rules $S' \to .S\$$ and $S \to .(L)$ are irrelevant to this action, so we ignore them; we end up in state 2:



$S \to x.$

2

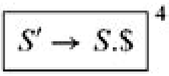
Or in state 1 consider shifting a left parenthesis. Moving the dot past the parenthesis in the third item yields $S \to (.L)$, where we know that there must be a left parenthesis on top of the stack, and the input begins with some string derived by *L*, followed by a right parenthesis. What tokens can begin the input now? We find out by including all *L*-productions in the set of items. But now, in one of those *L*-items, the dot is just before an *S*, so we need to include all the *S*-productions:



$S \to (.L)$
$L \to .L, S$
$L \to .S$
$S \to .(L)$
$S \to .x$

3

Goto actions In state 1, consider the effect of parsing past some string of tokens derived by the *S* nonterminal. This will happen when an *x* or left parenthesis is shifted, followed (eventually) by a reduction of an *S*-production. All the right-hand-side symbols of that production will be popped, and the parser will execute the goto action for *S* in state 1. The effect of this can be simulated by moving the dot past the *S* in the first item of state 1, yielding state 4:



$S' \to S.\$$

4

Reduce actions In state 2 we find the dot at the end of an item. This means that on top of the stack there must be a complete right-hand side of the corresponding production ($S \to x$), ready to reduce. In such a state the parser could perform a reduce action.

The basic operations we have been performing on states are **closure**(*I*) and **goto**(*I*, *X*), where *I* is a set of items and *X* is a grammar symbol (terminal or nonterminal). **Closure** adds more items to a set of items when there is a dot to the left of a nonterminal; **goto** moves the dot past the symbol *X* in all items.

```
Closure(I) =                        Goto(I, X) =
   repeat                            set J to the empty set
     for any item A → α.Xβ in I   for any item A → α:Xβ in I
```

```
      for any production X → γ
         I ← I ∩ {X → .γ}
until I does not change.
return I
```

```
add A → αX.β to J
return Closure(J)
```

Now here is the algorithm for LR(0) parser construction. First, augment the grammar with an auxiliary start production $S' \rightarrow S\$$. Let T be the set of states seen so far, and E the set of (shift or goto) edges found so far.

```
Initialize T to {Closure({S′ → :S$})}
Initialize E to empty.
repeat
      for each state I in T
          for each item A → α.Xβ in I
             let J be Goto(I, X)
             T ← T ∪ {J}
             E ← E ∪ {I *** J}
until E and T did not change in this iteration
```

However, for the symbol \$ we do not compute **Goto**(*I;* \$); instead we will make an **accept** action.

For Grammar 3.20 this is illustrated in Figure 3.21.

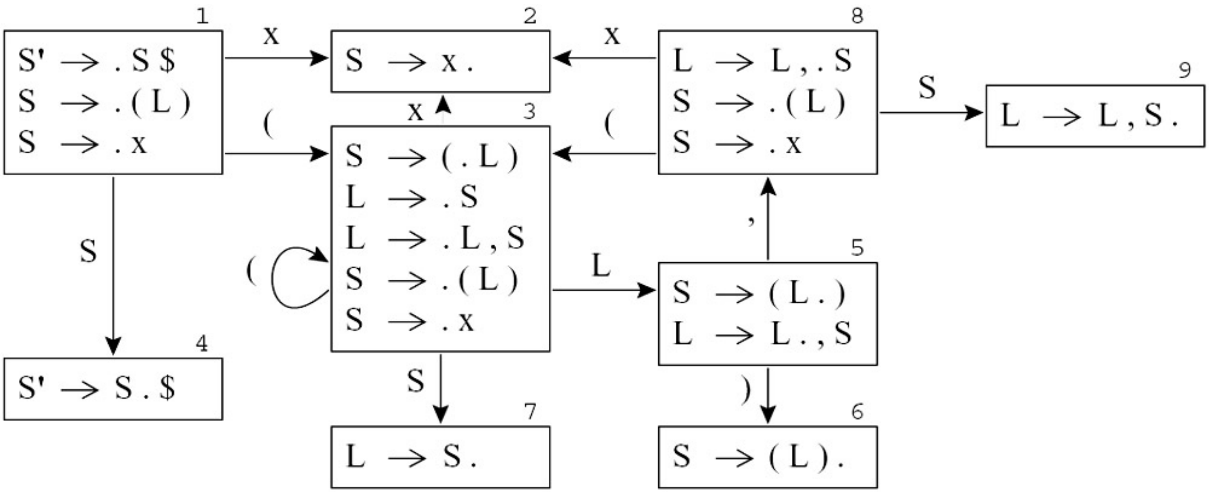


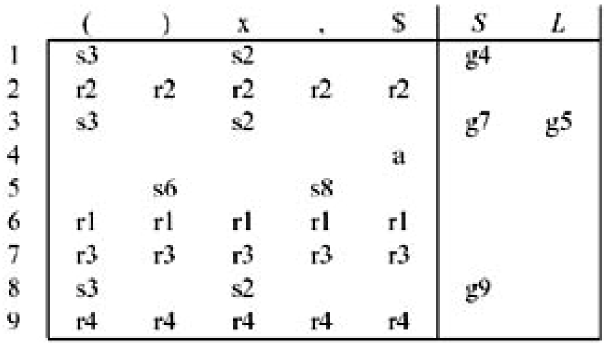
Figure 3.21: LR(0) states for Grammar 3.20.

Now we can compute set R of LR(0) reduce actions:

```
R ← {}
for each state I in T
    for each item A → α. in I
       R ← R ∪ {(I, A → α)}
```

We can now construct a parsing table for this grammar (Table 3.22). For each edge $I \xrightarrow{X} J$ where X is a terminal, we put the action *shift* J at position (I, X) of the table; if X is a nonterminal, we put *goto* J at position (I, X). For each state I containing an item $S' \rightarrow S.\$$ we put an *accept* action at (I, \$). Finally, for a state containing an item $A \rightarrow \gamma.$ (production n with the dot at the end), we put a *reduce* n action at (I, Y) for every token Y.

Table 3.22: LR(0) parsing table for Grammar 3.20.



	(	)	x	,	$	S	L
1	s3		s2			g4	
2	r2	r2	r2	r2	r2		
3	s3		s2			g7	g5
4					a		
5		s6		s8			
6	r1	r1	r1	r1	r1		
7	r3	r3	r3	r3	r3		
8	s3		s2			g9	
9	r4	r4	r4	r4	r4		

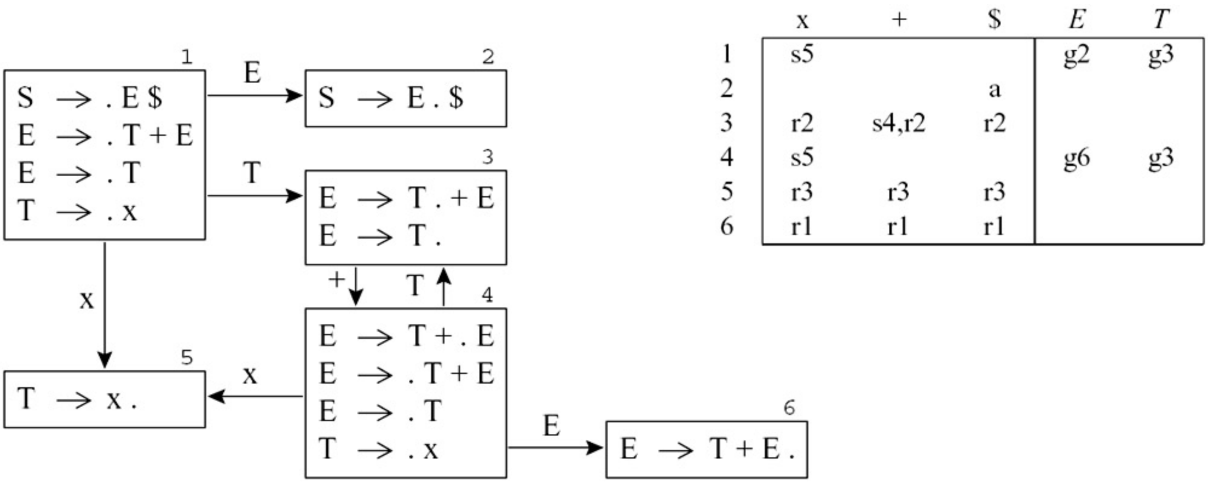
In principle, since LR(0) needs no lookahead, we just need a single action for each state: A state will shift or reduce, but not both. In practice, since we need to know what state to shift into, we have rows headed by state numbers and columns headed by grammar symbols.

SLR PARSER GENERATION

Let us attempt to build an LR(0) parsing table for Grammar 3.23. The LR(0) states and parsing table are shown in Figure 3.24.

GRAMMAR 3.23

1. $S \rightarrow E\ \$$
2. $E \rightarrow T + E$
3. $E \rightarrow T$
4. $T \rightarrow x$



	x	+	$	E	T
1	s5			g2	g3
2			a		
3	r2	s4,r2	r2		
4	s5			g6	g3
5	r3	r3	r3		
6	r1	r1	r1		

Figure 3.24: LR(0) states and parsing table for Grammar 3.23.

In state 3, on symbol +, there is a duplicate entry: The parser must shift into state 4 and also reduce by production 2. This is a conflict and indicates that the grammar is not LR(0) - it cannot be parsed by an LR(0) parser. We will need a more powerful parsing algorithm.

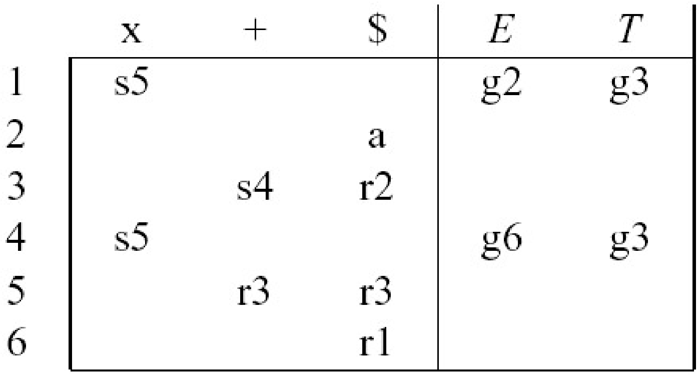
A simple way of constructing better-than-LR(0) parsers is called SLR, which stands for simple LR. Parser construction for SLR is almost identical to that for LR(0), except that we put reduce actions into the table only where indicated by the FOLLOW set.

Here is the algorithm for putting reduce actions into an SLR table:

```
R ← {}
for each state I in T
    for each item A → α. in I
       for each token X in FOLLOW(A)
          R ← R ∪ {(I, X, A → α)}
```

The action (*I*, *X*, *A* → α) indicates that in state *I*, on lookahead symbol *X*, the parser will reduce by rule *A* → α.

Thus, for Grammar 3.23 we use the same LR(0) state diagram (Figure 3.24), but we put fewer reduce actions into the SLR table, as shown in Figure 3.25.



	x	+	$	*E*	*T*
1	s5			g2	g3
2			a		
3		s4	r2		
4	s5			g6	g3
5		r3	r3		
6			r1		

Figure 3.25: SLR parsing table for Grammar 3.23.

The SLR class of grammars is precisely those grammars whose SLR parsing table contains no conflicts (duplicate entries). Grammar 3.23 belongs to this class, as do many useful programming-language grammars.

LR(1) ITEMS; LR(1) PARSING TABLE

Even more powerful than SLR is the LR(1) parsing algorithm. Most programming languages whose syntax is describable by a context-free grammar have an LR(1) grammar.

The algorithm for constructing an LR(1) parsing table is similar to that for LR(0), but the notion of an *item* is more sophisticated. An LR(1) item consists of a *grammar production*, a *right-hand-side position* (represented by the dot), and a *lookahead symbol*. The idea is that an item (*A* → α.β, *x*) indicates that the sequence α is on top of the stack, and at the head of the input is a string derivable from β*x*.

An LR(1) state is a set of LR(1) items, and there are **Closure** and **Goto** operations for LR(1) that incorporate the lookahead:

```
Closure(I) =
 repeat
  for any item (A → α.Xβ, z) in I
     for any production X → γ
         for any w ∈ FIRST(βz)
           I ← I ∪ {(X → .γ, w)}
 until I does not change
```

```
Goto(I, X) =
             J ← {}
       for any item (A → α.Xβ, z) in I
           add (A → αX.β, z) to J
      return Closure(J).
```

```
 return I
```

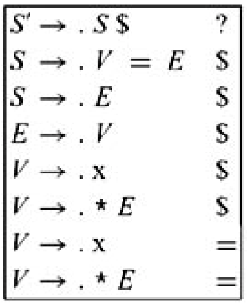
The start state is the closure of the item ($S' \rightarrow .S$ \$, ?), where the lookahead symbol `?` will not matter, because the end-of-file marker will never be shifted.

The reduce actions are chosen by this algorithm:

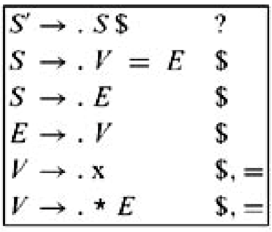
```
R ← {}
for each state I in T
    for each item (A → α., z) in I
       R ← R ∪{(I, z, A → α)}
```

The action (I, z, $A \rightarrow \alpha$) indicates that in state I, on lookahead symbol z, the parser will reduce by rule $A \rightarrow \alpha$.

Grammar 3.26 is not SLR (see Exercise 3.9), but it is in the class of LR(1) grammars. Figure 3.27 shows the LR(1) states for this grammar; in the figure, where there are several items with the same production but different lookahead, as at left below, we have abbreviated as at right:



$S' \rightarrow .S\$$	?
$S \rightarrow .V = E$	\$
$S \rightarrow .E$	\$
$E \rightarrow .V$	\$
$V \rightarrow .x$	\$
$V \rightarrow .*E$	\$
$V \rightarrow .x$	=
$V \rightarrow .*E$	=



$S' \rightarrow .S\$$	?
$S \rightarrow .V = E$	\$
$S \rightarrow .E$	\$
$E \rightarrow .V$	\$
$V \rightarrow .x$	\$, =
$V \rightarrow .*E$	\$, =

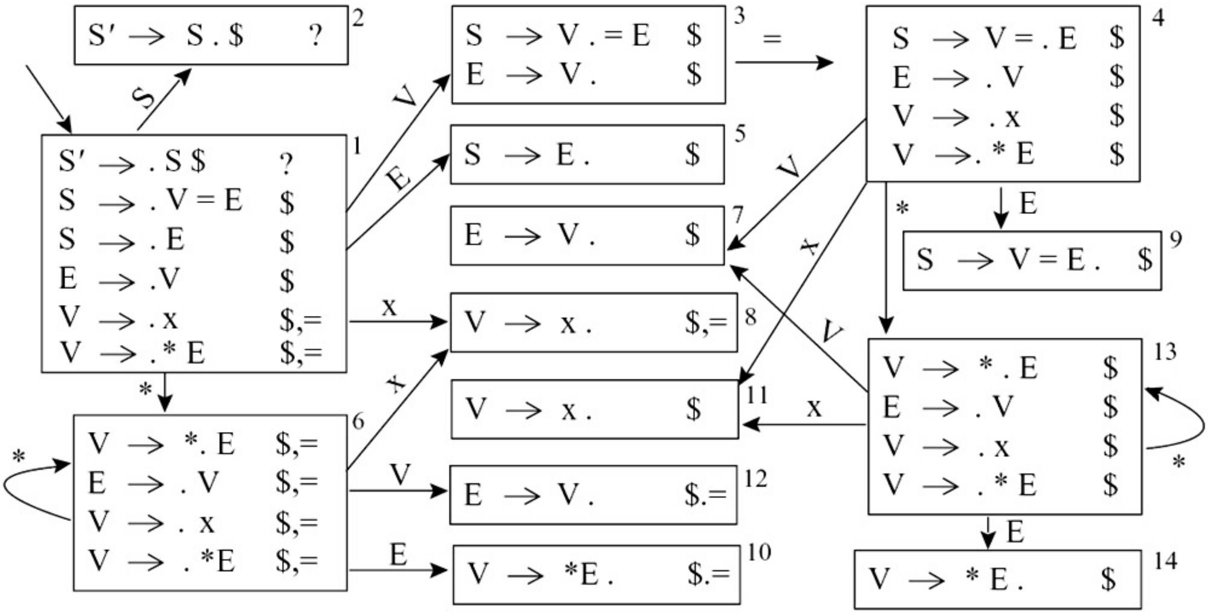


Figure 3.27: LR(1) states for Grammar 3.26.

GRAMMAR 3.26: A grammar capturing the essence of expressions, variables, and pointer-dereference (by the `*`) operator in the C language.

0. $S' \rightarrow S\,\$$

1. $S \rightarrow V = E$

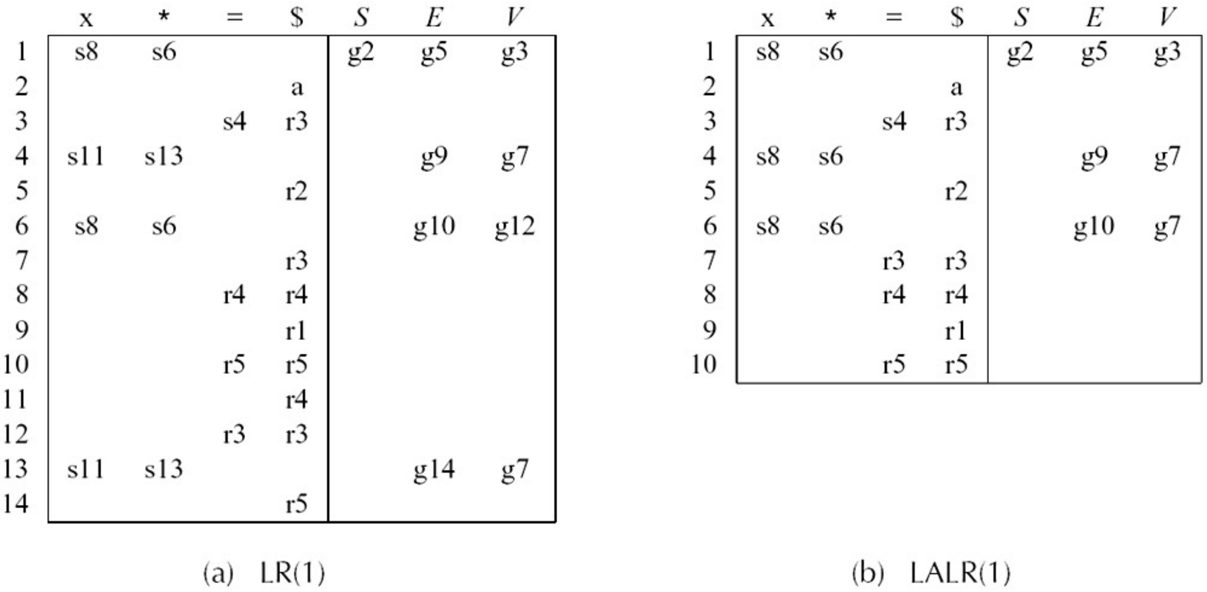
2. $S \rightarrow E$

3. $E \rightarrow V$

4. *V* → x   5. *V* → * *E*

The LR(1) parsing table derived from this state graph is Table 3.28a. Wherever the dot is at the end of a production (as in state 3 of Figure 3.27, where it is at the end of production *E* → *V*), then there is a *reduce* action for that production in the LR(1) table, in the row corresponding to the state number and the column corresponding to the lookahead of the item (in this case, the lookahead is $). Whenever the dot is to the left of a terminal symbol or nonterminal, there is a corresponding shift or goto action in the LR(1) parsing table, just as there would be in an LR(0) table.

Table 3.28: LR(1) and LALR(1) parsing tables for Grammar 3.26.



	x	*	=	$	*S*	*E*	*V*
1	s8	s6			g2	g5	g3
2				a			
3			s4	r3			
4	s11	s13				g9	g7
5				r2			
6	s8	s6				g10	g12
7				r3			
8			r4	r4			
9				r1			
10			r5	r5			
11				r4			
12			r3	r3			
13	s11	s13				g14	g7
14				r5			

(a) LR(1)

	x	*	=	$	*S*	*E*	*V*
1	s8	s6			g2	g5	g3
2				a			
3			s4	r3			
4	s8	s6				g9	g7
5				r2			
6	s8	s6				g10	g7
7			r3	r3			
8			r4	r4			
9				r1			
10			r5	r5			

(b) LALR(1)

LALR(1) PARSING TABLES

LR(1) parsing tables can be very large, with many states. A smaller table can be made by merging any two states whose items are identical except for lookahead sets. The result parser is called an LALR(1) parser, for *lookahead LR(1).*

For example, the items in states 6 and 13 of the LR(1) parser for Grammar 3.26 (Figure 3.27) are identical if the lookahead sets are ignored. Also, states 7 and 12 are identical except for lookahead, as are states 8 and 11 and states 10 and 14. Merging these pairs of states gives the LALR(1) parsing table shown in Table 3.28b.

For some grammars, the LALR(1) table contains reduce-reduce conflicts where the LR(1) table has none, but in practice the difference matters little. What does matter is that the LALR(1) parsing table requires less memory to represent than the LR(1) table, since there can be many fewer states.

HIERARCHY OF GRAMMAR CLASSES

A grammar is said to be LALR(1) if its LALR(1) parsing table contains no conflicts. All SLR grammars are LALR(1), but not vice versa. Figure 3.29 shows the relationship between several classes of grammars.

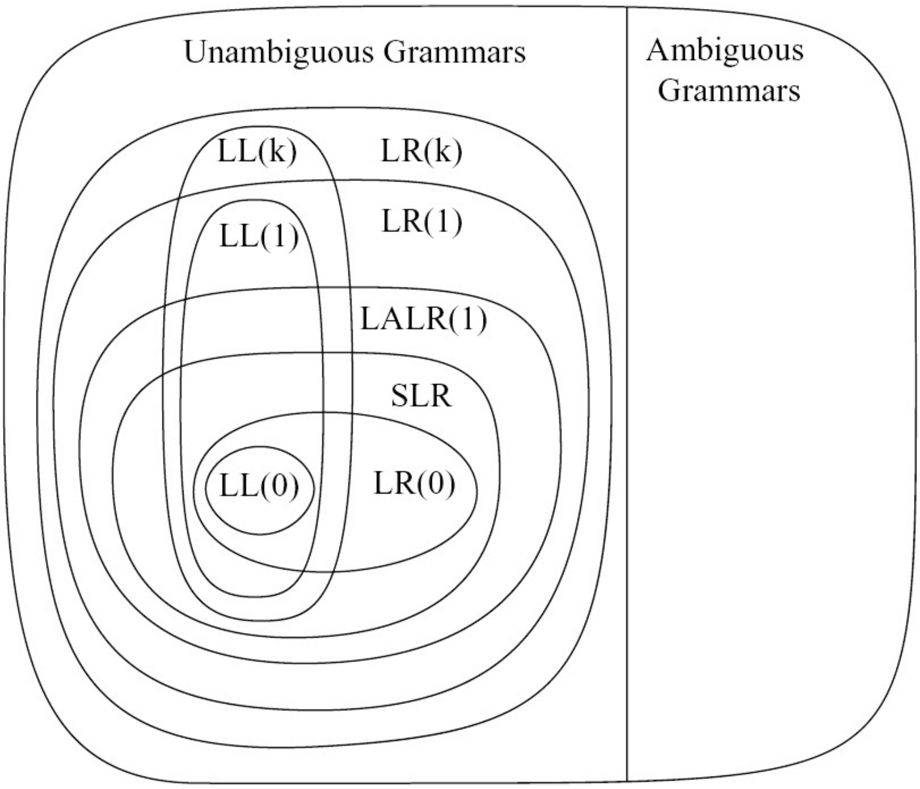


Figure 3.29: A hierarchy of grammar classes.

Any reasonable programming language has a LALR(1) grammar, and there are many parser-generator tools available for LALR(1) grammars. For this reason, LALR(1) has become a standard for programming languages and for automatic parser generators.

LR PARSING OF AMBIGUOUS GRAMMARS

Many programming languages have grammar rules such as

- $S \to$ if E then S else S
- $S \to$ if E then S
- $S \to$ other

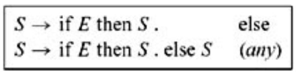
which allow programs such as

```
if a then if b then s1 else s2
```

Such a program could be understood in two ways:

```
(1)       if a then { if b then s1 else s2 }
(2)       if a then { if b then s1 } else s2
```

In most programming languages, an `else` must match the most recent possible `then`, so interpretation *(1)* is correct. In the LR parsing table there will be a shift-reduce conflict:



$S \rightarrow$ if E then S .	else
$S \rightarrow$ if E then S . else S	(*any*)

Shifting corresponds to interpretation *(1)* and reducing to interpretation *(2)*.

The ambiguity can be eliminated by introducing auxiliary nonterminals *M* (for *matched statement*)and *U* (for *unmatched statement*):

- $S \rightarrow M$
- $S \rightarrow U$
- $M \rightarrow$ if E then M else M
- $M \rightarrow$ other
- $U \rightarrow$ if E then S
- $U \rightarrow$ if E then M else U

But instead of rewriting the grammar, we can leave the grammar unchanged and tolerate the shift-reduce conflict. In constructing the parsing table this conflict should be resolved by shifting, since we prefer interpretation *(1)*.

It is often possible to use ambiguous grammars by resolving shift-reduce conflicts in favor of shifting or reducing, as appropriate. But it is best to use this technique sparingly, and only in cases (such as the *dangling-else* described here, and operator-precedence to be described on page 74) that are well understood. Most shift-reduce conflicts, and probably all reduce-reduce conflicts, should not be resolved by fiddling with the parsing table. They are symptoms of an ill-specified grammar, and they should be resolved by eliminating ambiguities.

3.4 USING PARSER GENERATORS

The task of constructing a parser is simple enough to be automated. In the previous chapter we described the lexical-analyzer aspects of JavaCC and SableCC. Here we will discuss the parser-generator aspects of these tools. Documentation for JavaCC and SableCC are available via this book's Web site.

JAVACC

JavaCC is an LL(k) parser generator. Productions are of the form:

```
void Assignment() : {} { Identifier() "=" Expression() ";" }
```

where the left-hand side is `Assignment()`; the right-hand side is enclosed between the last two curly brackets; `Assignment()`, `Identifier()`, and `Expression()` are nonterminal symbols; and `"="` and `";"` are terminal symbols.

Grammar 3.30 can be represented as a JavaCC grammar as shown in Grammar 3.31. Notice that if we had written the production for `StmList()` in the style of Grammar 3.30, that is,

```
void StmList() :
{}
{ Stm()
| StmList( ) ";" Stm()
}
```

GRAMMAR 3.30

1. $P \rightarrow L$
2. $S \rightarrow$ id := id
3. $S \rightarrow$ while id do S
4. $S \rightarrow$ begin L end
5. $S \rightarrow$ if id then S
6. $S \rightarrow$ if id then S else S
7. $L \rightarrow S$
8. $L \rightarrow L$; S

GRAMMAR 3.31: JavaCC version of Grammar 3.30.

```
PARSER_BEGIN(MyParser)
   public class MyParser {}
PARSER_END(MyParser)

SKIP :
{ " " | "\t" | "\n" }

TOKEN :
{ < WHILE: "while" >
| < BEGIN: "begin" >
| < END: "end" >
| < DO: "do" >
| < IF: "if" >
| < THEN: "then" >
| < ELSE: "else" >
| < SEMI: ";" >
| < ASSIGN: "=" >
| < ID: ["a"-"z"](["a"-"z"] | ["0"-"9"])* >
}

void Prog() :
{}
{ StmList() <EOF> }

void StmList() :
{}
{ Stm() StmListPrime() }

void StmListPrime() :
{}
{ ( ";" Stm() StmListPrime() )? }

void Stm() :
{}
{ <ID> "=" <ID>
| "while" <ID> "do" Stm()
| "begin" StmList() "end"
| LOOKAHEAD(5) /* we need to lookahead till we see "else" */
"if" <ID> "then" Stm()
| "if" <ID> "then" Stm() "else" Stm()
}
```

then the grammar would be left recursive. In that case, JavaCC would give the following error:

```
Left recursion detected: "StmList... --> StmList..."
```

We used the techniques mentioned earlier to remove the left recursion and arrive at Grammar 3.31.

SABLECC

SableCC is an LALR(1) parser generator. Productions are of the form:

```
assignment = identifier assign expression semicolon ;
```

where the left-hand side is `assignment`; the right-hand side is enclosed between `=` and `;`; `assignment`, `identifier`, and `expression` are nonterminal symbols; and `assign` and `semicolon` are terminal symbols that are defined in an earlier part of the syntax specification.

Grammar 3.30 can be represented as a SableCC grammar as shown in Grammar 3.32. When there is more than one alternative, SableCC requires a name for each alternative. A name is given to an alternative in the grammar by prefixing the alternative with an identifier between curly brackets. Also, if the same grammar symbol appears twice in the same alternative of a production, SableCC requires a name for at least one of the two elements. Element names are specified by prefixing the element with an identifier between square brackets followed by a colon.

GRAMMAR 3.32: SableCC version of Grammar 3.30.

```
Tokens
    while = 'while';
    begin = 'begin';
    end = 'end';
    do = 'do';
    if = 'if';
    then = 'then';
    else = 'else';
    semi = ';';
    assign = '=';
    whitespace = (' ' | '\t' | '\n')+;
    id = ['a'..'z'](['a'..'z'] | ['0'..'9'])*;
Ignored Tokens
    whitespace;
Productions
    prog = stmlist;

    stm =  {assign} [left]:id assign [right]:id |
           {while} while id do stm |
           {begin} begin stmlist end |
           {if_then} if id then stm |
           {if_then_else} if id then [true_stm]:stm else [false_stm]:stm;

    stmlist = {stmt} stm |
              {stmtlist} stmlist semi stm;
```

SableCC reports shift-reduce and reduce-reduce conflicts. A shift-reduce conflict is a choice between shifting and reducing; a reduce-reduce conflict is a choice of reducing by two different rules.

SableCC will report that the Grammar 3.32 has a shift-reduce conflict. The conflict can be examined by reading the detailed error message SableCC produces, as shown in Figure 3.33.

```
shift/reduce conflict in state [stack: TIf TId TThen PStm *] on TElse in {
        [ PStm = TIf TId TThen PStm * TElse PStm ] (shift),
        [ PStm = TIf TId TThen PStm * ] followed by TElse (reduce)
}
```

Figure 3.33: SableCC shift-reduce error message for Grammar 3.32.

SableCC prefixes productions with an uppercase ‘P' and tokens with an uppercase ‘T', and replaces the first letter with an uppercase when it makes the objects for the tokens and productions. This is what you see on the stack in the error message in Figure 3.33. So on the stack we have tokens for `if`, `id`, `then`, and a production that matches a `stm`, and now we have an `else` token. Clearly this reveals that the conflict is caused by the familiar dangling `else.`

In order to resolve this conflict we need to rewrite the grammar, removing the ambiguity as in Grammar 3.34.

GRAMMAR 3.34: SableCC productions of Grammar 3.32 with conflicts resolved.

```
Productions
    prog = stmlist;

    stm = {stm_without_trailing_substm}
              stm_without_trailing_substm |
          {while} while id do stm |
          {if_then} if id then stm |
          {if_then_else} if id then stm_no_short_if
                         else [false_stm]:stm;

    stm_no_short_if = {stm_without_trailing_substm}
                          stm_without_trailing_substm |
                         {while_no_short_if}
                          while id do stm_no_short_if |
                      {if_then_else_no_short_if}
                         if id then [true_stm]:stm_no_short_if
                               else [fals_stm]:stm_no_short_if;

    stm_without_trailing_substm = {assign} [left]:id assign [right]:id |
                                     {begin} begin stmlist end ;
    stmlist = {stmt} stm | {stmtlist} stmlist semi stm;
```

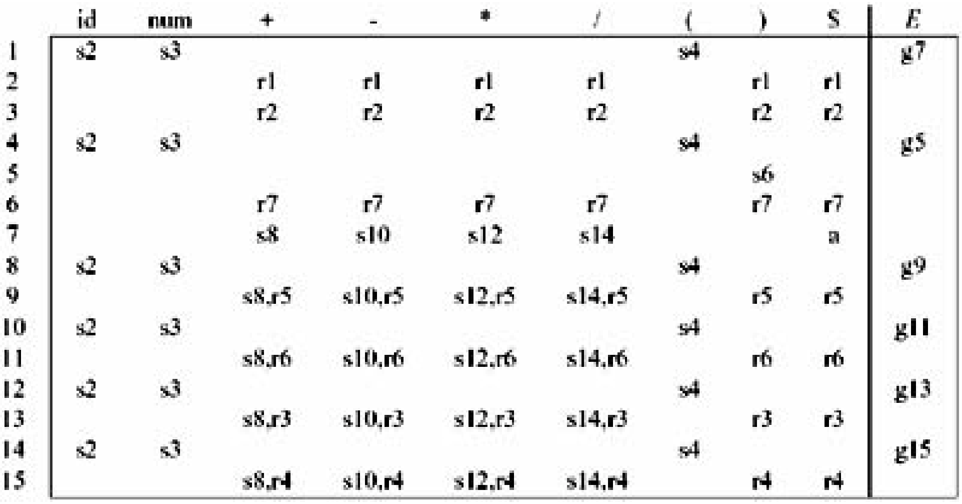
PRECEDENCE DIRECTIVES

No ambiguous grammar is LR(*k*) for any *k*; the LR(*k*) parsing table of an ambiguous grammar will always have conflicts. However, ambiguous grammars can still be useful if we can find ways to resolve the conflicts.

For example, Grammar 3.5 is highly ambiguous. In using this grammar to describe a programming language, we intend it to be parsed so that * and = bind more tightly than + and −, and that each operator associates to the left. We can express this by rewriting the unambiguous Grammar 3.8.

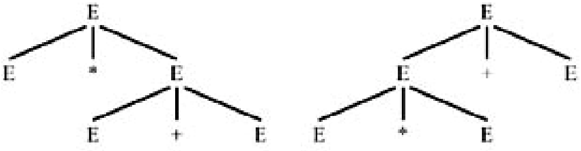
But we can avoid introducing the *T* and *F* symbols and their associated "trivial" reductions $E \rightarrow T$ and $T \rightarrow F$. Instead, let us start by building the LR(1) parsing table for Grammar 3.5, as shown in Table 3.35. We find many conflicts. For example, in state 13 with lookahead + we find a conflict between *shift into state 8* and *reduce by rule 3*. Two of the items in state 13 are

Table 3.35: LR parsing table for Grammar 3.5.



	id	num	+	-	*	/	(	)	$	E
1	s2	s3					s4			g7
2			r1	r1	r1	r1		r1	r1	
3			r2	r2	r2	r2		r2	r2	
4	s2	s3					s4			g5
5								s6		
6			r7	r7	r7	r7		r7	r7	
7			s8	s10	s12	s14			a	
8	s2	s3					s4			g9
9			s8,r5	s10,r5	s12,r5	s14,r5		r5	r5	
10	s2	s3					s4			g11
11			s8,r6	s10,r6	s12,r6	s14,r6		r6	r6	
12	s2	s3					s4			g13
13			s8,r3	s10,r3	s12,r3	s14,r3		r3	r3	
14	s2	s3					s4			g15
15			s8,r4	s10,r4	s12,r4	s14,r4		r4	r4	

In this state the top of the stack is … *E* * *E*. Shifting will lead to a stack … *E* * *E*+ and eventually … *E* * *E* + *E* with a reduction of *E* + *E* to *E*. Reducing now will lead to the stack … *E* and then the + will be shifted. The parse trees obtained by shifting and reducing are



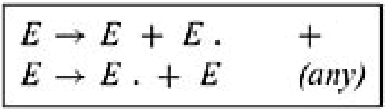
Shift

Reduce

If we wish * to bind tighter than +, we should reduce instead of shift. So we fill the (13, +) entry in the table with r3 and discard the s8 action.

Conversely, in state 9 on lookahead *, we should shift instead of reduce, so we resolve the conflict by filling the (9, *) entry with s12.

The case for state 9, lookahead + is



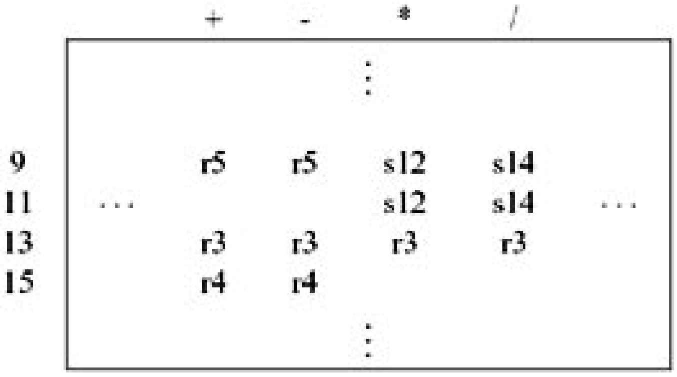
$E \rightarrow E + E\,.$	$+$
$E \rightarrow E\,.+ E$	*(any)*

Shifting will make the operator right-associative; reducing will make it leftassociative. Since we want left associativity, we fill (9, +) with r5.

Consider the expression $a - b - c$. In most programming languages, this associates to the left, as if written $(a - b) - c$. But suppose we believe that this expression is inherently confusing, and we want to force the programmer to put in explicit parentheses, either $(a - b) - c$ or $a - (b - c)$. Then we say that the minus operator is *nonassociative*, and we would fill the $(11, -)$ entry with an error entry.

The result of all these decisions is a parsing table with all conflicts resolved (Table 3.36).

Table 3.36: Conflicts of Table 3.35 resolved.



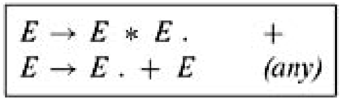
		+	-	*	/	
				⋮		
9		r5	r5	s12	s14	
11	. . .			s12	s14	. . .
13		r3	r3	r3	r3	
15		r4	r4			
				⋮		

Yacc has *precedence directives* to indicate the resolution of this class of shift-reduce conflicts. (Unfortunately, SableCC does not have precedence directives.) A series of declarations such as

```
precedence nonassoc EQ, NEQ;
precedence left PLUS, MINUS;
precedence left TIMES, DIV;
precedence right EXP;
```

indicates that + and - are left-associative and bind equally tightly; that * and / are left-associative and bind more tightly than +; that ʎ is right-associative and binds most tightly; and that = and ≠ are nonassociative, and bind more weakly than +.

In examining a shift-reduce conflict such as



$E \to E * E\,.$	+
$E \to E\,.\,+ E$	*(any)*

there is the choice of shifting a *token* and reducing by a *rule*. Should the rule or the token be given higher priority? The precedence declarations (`precedence left`, etc.) give priorities to the tokens; the priority of a rule is given by the last token occurring on the right-hand side of that rule. Thus the choice here is between a rule with priority * and a token with priority +; the rule has higher priority, so the conflict is resolved in favor of reducing.

When the rule and token have equal priority, then a `left` precedence favors reducing, `right` favors shifting, and `nonassoc` yields an error action.

Instead of using the default "rule has precedence of its last token", we can assign a specific precedence to a rule using the `%prec` directive. This is commonly used to solve the "unary minus" problem. In most programming languages a unary minus binds tighter than any binary operator, so $-6 * 8$ is parsed as $(-6) * 8$, not $-(6 * 8)$. Grammar 3.37 shows an example.

GRAMMAR 3.37: Yacc grammar with precedence directives.

```
%{ declarations of yylex and yyerror %}
%token INT PLUS MINUS TIMES UMINUS
%start exp

%left PLUS MINUS
%left TIMES
%left UMINUS
%%

exp : INT
    | exp PLUS exp
    | exp MINUS exp
    | exp TIMES exp
    | MINUS exp    %prec UMINUS
```

The token UMINUS is never returned by the lexer; it's just a placeholder in the chain of `precedence` declarations. The directive `%prec UMINUS` gives the rule `exp::= MINUS exp` the highest precedence, so reducing by this rule takes precedence over shifting any operator, even a minus sign.

Precedence rules are helpful in resolving conflicts, but they should not be abused. If you have trouble explaining the effect of a clever use of precedence rules, perhaps instead you should rewrite the grammar to be unambiguous.

SYNTAX VERSUS SEMANTICS

Consider a programming language with *arithmetic expressions* such as $x + y$ and *boolean expressions* such as $x + y = z$ or $a\&(b = c)$. Arithmetic operators bind tighter than the boolean operators; there are arithmetic variables and boolean variables; and a boolean expression cannot be added to an arithmetic expression. Grammar 3.38 gives a syntax for this language.

GRAMMAR 3.38: Yacc grammar with precedence directives.

```
%token ID ASSIGN PLUS MINUS AND EQUAL
%start stm
%left OR
%left AND
%left PLUS
%%

stm : ID ASSIGN ae
    | ID ASSIGN be

be  : be OR be
    | be AND be
    | ae EQUAL ae
    | ID
```

```
ae  : ae PLUS ae
    | ID
```

The grammar has a reduce-reduce conflict. How should we rewrite the grammar to eliminate this conflict?

Here the problem is that when the parser sees an identifier such as *a,* it has no way of knowing whether this is an arithmetic variable or a boolean variable - syntactically they look identical. The solution is to defer this analysis until the "semantic" phase of the compiler; it's not a problem that can be handled naturally with context-free grammars. A more appropriate grammar is

- $S \rightarrow \text{id} : E$

- $E \rightarrow \text{id}$
- $E \rightarrow E \,\&\, E$
- $E \rightarrow E = E$
- $E \rightarrow E + E$

Now the expression $a + 5\&b$ is syntactically legal, and a later phase of the compiler will have to reject it and print a semantic error message.

3.5 ERROR RECOVERY

LR(k) parsing tables contain shift, reduce, accept, and error actions. On page 58 we claimed that when an LR parser encounters an error action it stops parsing and reports failure. This behavior would be unkind to the programmer, who would like to have *all* the errors in her program reported, not just the first error.

RECOVERY USING THE ERROR SYMBOL

Local error recovery mechanisms work by adjusting the parse stack and the input *at the point where the error was detected* in a way that will allow parsing to resume. One local recovery mechanism - found in many versions of the Yacc parser generator - uses a special *error* symbol to control the recovery process. Wherever the special *error* symbol appears in a grammar rule, a sequence of erroneous input tokens can be matched.

For example, in a Yacc grammar we might have productions such as

- $exp \rightarrow ID$
- $exp \rightarrow exp + ext$
- $exp \rightarrow (\ exps\ )$
- $exps \rightarrow exp$
- $exps \rightarrow exps\ ;\ exp$

Informally, we can specify that if a syntax error is encountered in the middle of an expression, the parser should skip to the next semicolon or right parenthesis (these are called *synchronizing tokens*) and resume parsing. We do this by adding error-recovery productions such as

- *exp* → (*error*)
- *exps* → *error* ; *exp*

What does the parser generator do with the *error* symbol? In parser generation, *error* is considered a terminal symbol, and shift actions are entered in the parsing table for it as if it were an ordinary token.

When the LR parser reaches an error state, it takes the following actions:

1. Pop the stack (if necessary) until a state is reached in which the action for the *error* token is *shift*.
2. Shift the *error* token.
3. Discard input symbols (if necessary) until a lookahead is reached that has a nonerror action in the current state.
4. Resume normal parsing.

In the two *error* productions illustrated above, we have taken care to follow the *error* symbol with an appropriate synchronizing token - in this case, a right parenthesis or semicolon. Thus, the "nonerror action" taken in step 3 will always *shift*. If instead we used the production *exp* → *error*, the "nonerror action" would be *reduce*, and (in an SLR or LALR parser) it is possible that the original (erroneous) lookahead symbol would cause another error after the reduce action, without having advanced the input. Therefore, grammar rules that contain *error* not followed by a token should be used only when there is no good alternative.

Caution One can attach *semantic actions* to Yacc grammar rules; whenever a rule is reduced, its semantic action is executed. Chapter 4 explains the use of semantic actions. Popping states from the stack can lead to seemingly "impossible" semantic actions, especially if the actions contain side effects. Consider this grammar fragment:

```
statements: statements exp SEMICOLON
          | statements error SEMICOLON
          | /* empty */

exp : increment exp decrement
    |ID

increment: LPAREN    {: nest=nest+1; :}
decrement: RPAREN    {: nest=nest-1; :}
```

"Obviously" it is true that whenever a semicolon is reached, the value of `nest` is zero, because it is incremented and decremented in a balanced way according to the grammar of expressions. But if a syntax error is found after some left parentheses have been parsed, then states will be popped from the stack without "completing" them, leading to a nonzero value of `nest`. The best solution to this problem is to have side-effect-free semantic actions that build abstract syntax trees, as described in Chapter 4.

Unfortunately, neither JavaCC nor SableCC support the *error*-symbol errorrecovery method, nor the kind of global error repair described below.

GLOBAL ERROR REPAIR

Global error repair finds the smallest set of insertions and deletions that would turn the source string into a syntactically correct string, *even if the insertions and deletions are not at a point where an LL or LR parser would first report an error*.

Burke-Fisher error repair We will describe a limited but useful form of global error repair, which tries every possible single-token insertion, deletion, or replacement at every point that occurs no earlier than *K* tokens before the point where the parser reported the error. Thus, with *K* = 15, if the parsing engine gets stuck at the 100th token of the input, then it will try every possible repair between the 85th and 100th tokens.

The correction that allows the parser to parse furthest past the original reported error is taken as the best error repair. Thus, if a single-token substitution of `var` for `type` at the 98th token allows the parsing engine to proceed past the 104th token without getting stuck, this repair is a successful one. Generally, if a repair carries the parser *R* = 4 tokens beyond where it originally got stuck, this is "good enough."

The advantage of this technique is that the LL(*k*) or LR(*k*) (or LALR, etc.) grammar is not modified at all (no *error* productions), nor are the parsing tables modified. Only the parsing engine, which interprets the parsing tables, is modified.

The parsing engine must be able to back up *K* tokens and reparse. To do this, it needs to remember what the parse stack looked like *K* tokens ago. Therefore, the algorithm maintains *two* parse stacks: the *current* stack and the *old* stack. A queue of *K* tokens is kept; as each new token is shifted, it is pushed on the *current* stack and also put onto the tail of the queue; simultaneously, the head of the queue is removed and shifted onto the *old* stack. With each *shift* onto the old or current stack, the appropriate reduce actions are also performed. Figure 3.39 illustrates the two stacks and queue.

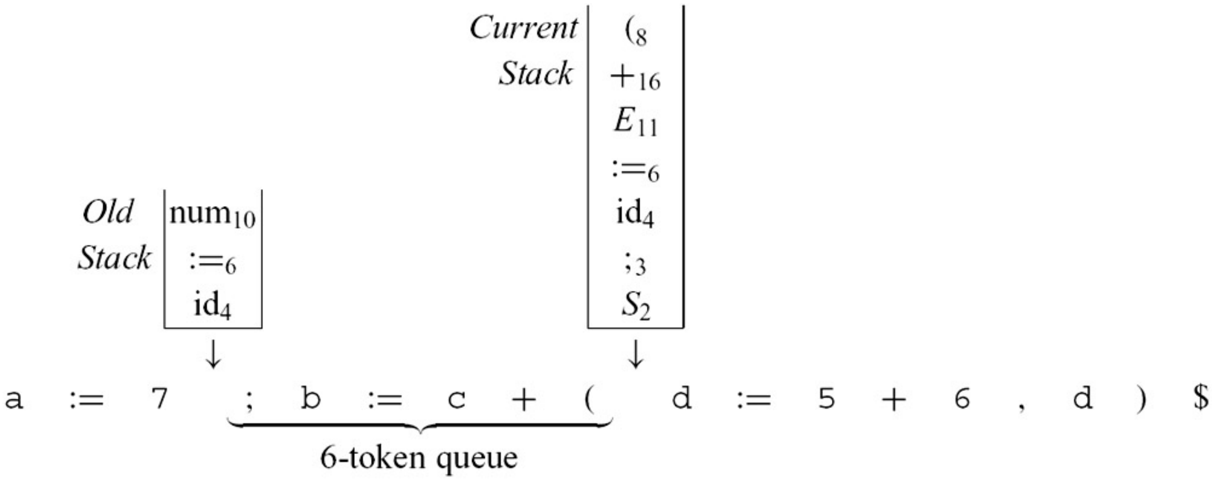


Figure 3.39: Burke-Fisher parsing, with an error-repair queue. Figure 3.18 shows the complete parse of this string according to Table 3.19.

Now suppose a syntax error is detected at the *current* token. For each possible insertion, deletion, or substitution of a token at any position of the queue, the Burke-Fisher error repairer makes that change to within (a copy of) the queue, then attempts to reparse from the *old* stack. The success of a modification is in how many tokens past the *current* token can be parsed; generally, if three or four new tokens can be parsed, this is considered a completely successful repair.

In a language with N kinds of tokens, there are $K + K \cdot N + K \cdot N$ possible deletions, insertions, and substitutions within the K -token window. Trying this many repairs is not very costly, especially considering that it happens only when a syntax error is discovered, not during ordinary parsing.

Semantic actions Shift and reduce actions are tried repeatedly and discarded during the search for the best error repair. Parser generators usually perform programmer-specified semantic actions along with each reduce action, but the programmer does not expect that these actions will be performed repeatedly and discarded - they may have side effects. Therefore, a Burke-Fisher parser does not execute any of the semantic actions as reductions are performed on the *current* stack, but waits until the same reductions are performed (permanently) on the *old* stack.

This means that the lexical analyzer may be up to $K + R$ tokens ahead of the point to which semantic actions have been performed. If semantic actions affect lexical analysis - as they do in C, compiling the `typedef` feature - this can be a problem with the Burke-Fisher approach. For languages with a pure context-free grammar approach to syntax, the delay of semantic actions poses no problem.

Semantic values for insertions In repairing an error by insertion, the parser needs to provide a semantic value for each token it inserts, so that semantic actions can be performed as if the token had come from the lexical analyzer. For punctuation tokens no value is necessary, but when tokens such as numbers or identifiers must be inserted, where can the value come from? The ML-Yacc parser generator, which uses Burke-Fischer error correction, has a `%value` directive, allowing the programmer to specify what value should be used when inserting each kind of token:

```
%value ID ("bogus")
%value INT (1)
%value STRING ("")
```

Programmer-specified substitutions Some common kinds of errors cannot be repaired by the insertion or deletion of a single token, and sometimes a particular single-token insertion or substitution is very commonly required and should be tried first. Therefore, in an ML-Yacc grammar specification the programmer can use the `%change` directive to suggest error corrections to be tried first, before the default "delete or insert each possible token" repairs.

```
%change      EQ -> ASSIGN | ASSIGN -> EQ
       |    SEMICOLON ELSE -> ELSE  | -> IN INT END
```

Here the programmer is suggesting that users often write "`; else`"where they mean "`else`" and so on. These particular error corrections are often useful in parsing the ML programming language.

The insertion of `in 0 end` is a particularly important kind of correction, known as a *scope closer*. Programs commonly have extra left parentheses or right parentheses, or extra left or right brackets, and so on. In ML, another kind of nesting construct is `let` … `in` … `end`. If the programmer forgets to close a scope that was opened by a left parenthesis, then the automatic singletoken insertion heuristic can close this scope where necessary. But to close a `let` scope requires the insertion of three tokens, which will not be done automatically unless the compiler-writer has suggested "change *nothing* to `in 0 end`" as illustrated in the `%change` command above.

PROGRAM PARSING

Use JavaCC or SableCC to implement a parser for the MiniJava language. Do it by extending the specification from the corresponding exercise in the previous chapter. Appendix A describes the syntax of MiniJava.

FURTHER READING

Conway [1963] describes a predictive (recursive-descent) parser, with a notion of FIRST sets and left-factoring. LL(*k*) parsing theory was formalized by Lewis and Stearns [1968].

LR(*k*) parsing was developed by Knuth [1965]; the SLR and LALR techniques by DeRemer [1971]; LALR(1) parsing was popularized by the development and distribution of Yacc [Johnson 1975] (which was not the first parser generator, or "compiler-compiler", as can be seen from the title of the cited paper).

Figure 3.29 summarizes many theorems on subset relations between grammar classes. Heilbrunner [1981] shows proofs of several of these theorems, including LL(*k*) ⊂ LR(*k*) and LL(1) 6 ⊊ LALR(1) (see Exercise 3.14). Backhouse [1979] is a good introduction to theoretical aspects of LL and LR parsing.

Aho et al. [1975] showed how deterministic LL or LR parsing engines can handle ambiguous grammars, with ambiguities resolved by precedence directives (as described in Section 3.4).

Burke and Fisher [1987] invented the error-repair tactic that keeps a *K* token queue and two parse stacks.

EXERCISES

- **3.1** Translate each of these regular expressions into a context-free grammar.
  a. $((xy{*}x)\quad(yx{*}y))?$
  b. ((0   1)$^+$"."(0   1)*)   ((0   1)*"."(0   1)$^+$)
- ***3.2** Write a grammar for English sentences using the words
- `time, arrow, banana, flies, like, a, an, the, fruit`

  and the semicolon. Be sure to include all the senses (noun, verb, etc.) of each word. Then show that this grammar is ambiguous by exhibiting more than one parse tree for "time flies like an arrow; fruit flies like a banana."

- **3.3** Write an unambiguous grammar for each of the following languages. **Hint:** One way of verifying that a grammar is unambiguous is to run it through Yacc and get no conflicts.
  - a. Palindromes over the alphabet {*a*, *b*} (strings that are the same backward and forward).
  - b. Strings that match the regular expression *a***b** and have more *a*'s than *b*'s.
  - c. Balanced parentheses and square brackets. Example: `([[](()[()][])])`
  - *d. Balanced parentheses and brackets, where a closing bracket also closes any outstanding open parentheses (up to the previous open bracket). Example: `[([](()[(][])]`. **Hint:** First, make the language of balanced parentheses and brackets, where extra open parentheses are allowed; then make sure this nonterminal must appear within brackets.

- e. All subsets and permutations (without repetition) of the keywords `public final static synchronized transient`. (Then comment on how best to handle this situation in a real compiler.)
- f. Statement blocks in Pascal or ML where the semicolons separate the statements:
- `( statement ; ( statement ; statement ) ; statement )`
- g. Statement blocks in C where the semicolons terminatethe statements:
- `{ expression; { expression; expression; } expression; }`

- **3.4** Write a grammar that accepts the same language as Grammar 3.1, but that is suitable for LL(1) parsing. That is, eliminate the ambiguity, eliminate the left recursion, and (if necessary) left-factor.
- **3.5** Find nullable, FIRST, and FOLLOW sets for this grammar; then construct the LL(1) parsing table.

  0. $S' \to S\,\$$
  1. $S \to$
  2. $S \to XS$
  3. $B \to$ `\ begin { WORD }`
  4. $E \to$ `\ end { WORD }`
  5. $X \to BSE$
  6. $X \to \{\, S \,\}$
  7. $X \to$ `WORD`
  8. $X \to$ `begin`
  9. $X \to$ `end`
  10. $X \to$ `\ WORD`

- **3.6**

  . Calculate nullable, FIRST, and FOLLOW for this grammar:

  - $S \to u\ B\ D\ z$
  - $B \to B\ v$
  - $B \to w$
  - $D \to E\ F$
  - $E \to y$
  - $E \to$
  - $F \to x$
  - $F \to$

  a. Construct the LL(1) parsing table.
  b. Give evidence that this grammar is not LL(1).
  c. Modify the grammar **as little as possible** to make an LL(1) grammar that accepts the same language.

- ***3.7**

  . Left-factor this grammar.

1. $S \to G\,\$$
2. $G \to P$
3. $G \to PG$
4. $P \to \mathrm{id} : R$
5. $R \to$

6. $R \rightarrow$ id R

   a. Show that the resulting grammar is LL(2). You can do this by constructing FIRST sets (etc.) containing two-symbol strings; but it is simpler to construct an LL(1) parsing table and then argue convincingly that any conflicts can be resolved by looking ahead one more symbol.
   b. Show how the `tok` variable and `advance` function should be altered for recursive-descent parsing with two-symbol lookahead.
   c. Use the grammar class hierarchy (Figure 3.29) to show that the (leftfactored) grammar is LR(2).
   d. Prove that no string has two parse trees according to this (left-factored) grammar.

- **3.8** Make up a tiny grammar containing left recursion, and use it to demonstrate that left recursion is not a problem for LR parsing. Then show a small example comparing growth of the LR parse stack with left recursion versus right recursion.
- **3.9** Diagram the LR(0) states for Grammar 3.26, build the SLR parsing table, and identify the conflicts.
- **3.10** Diagram the LR(1) states for the grammar of Exercise 3.7 (without left-factoring), and construct the LR(1) parsing table. Indicate clearly any conflicts.
- **3.11** Construct the LR(0) states for this grammar, and then determine whether it is an SLR grammar.

  0. $S \rightarrow B$ \$

  1. $B \rightarrow$ id P

  2. $B \rightarrow$ id *(E]

  3. $P \rightarrow$

  4. $P \rightarrow$ (E)

  5. $E \rightarrow B$

  6. $E \rightarrow B, E$

- **3.12**

  . Build the LR(0) DFA for this grammar:

0. $S \rightarrow E$ \$

1. $E \rightarrow$ id

2. $E \rightarrow$ id (E)

3. $E \rightarrow E$ + id

   a. Is this an LR(0) grammar? Give evidence.
   b. Is this an SLR grammar? Give evidence.
   c. Is this an LR(1) grammar? Give evidence.

- **3.13** Show that this grammar is LALR(1) but not SLR:

  0. $S \rightarrow X$ \$

  1. $X \rightarrow Ma$

  2. $X \rightarrow bMc$

  3. $X \rightarrow dc$

  4. $X \rightarrow bda$

  5. $M \rightarrow d$

- **3.14** Show that this grammar is LL(1) but not LALR(1):

  0. $S \rightarrow$ (X

  1. $S \rightarrow E$]

2. $S \rightarrow F)$
3. $X \rightarrow E)$
4. $X \rightarrow F]$
5. $E \rightarrow A$
6. $F \rightarrow A$
7. $A \rightarrow$

- ***3.15** Feed this grammar to Yacc; from the output description file, construct the LALR(1) parsing table for this grammar, with duplicate entries where there are conflicts. For each conflict, show whether shifting or reducing should be chosen so that the different kinds of expressions have "conventional" precedence. Then show the Yacc-style precedence directives that resolve the conflicts this way.

0. $S \rightarrow E$ \$
1. $E \rightarrow$ while E do E
2. $E \rightarrow$ id := E
3. $E \rightarrow E + E$
4. $E \rightarrow$ id

- ***3.16** Explain how to resolve the conflicts in this grammar, using precedence directives, or grammar transformations, or both. Use Yacc or SableCC as a tool in your investigations, if you like.

0. $E \rightarrow$ id
1. $E \rightarrow EBE$
2. $B \rightarrow +$
3. $B \rightarrow -$
4. $B \rightarrow \times$
5. $B \rightarrow /$

- ***3.17** Prove that Grammar 3.8 cannot generate parse trees of the form shown in Figure 3.9. **Hint:** What nonterminals could possibly be where the ?X is shown? What does that tell us about what could be where the ?Y is shown?

Chapter 4: Abstract Syntax

ab-stract: disassociated from any specific instance

Webster's Dictionary

OVERVIEW

A compiler must do more than recognize whether a sentence belongs to the language of a grammar - it must do something useful with that sentence. The *semantic actions* of a parser can do useful things with the phrases that are parsed.

In a recursive-descent parser, semantic action code is interspersed with the control flow of the parsing actions. In a parser specified in JavaCC, semantic actions are fragments of Java program code attached to grammar productions. SableCC, on the other hand, automatically generates syntax trees as it parses.

4.1 SEMANTIC ACTIONS

Each terminal and nonterminal may be associated with its own type of semantic value. For example, in a simple calculator using Grammar 3.37, the type associated with `exp` and `INT` might be `int`; the other tokens would not need to carry a value. The type associated with a token must, of course, match the type that the lexer returns with that token.

For a rule *A* → *B C D*, the semantic action must return a value whose type is the one associated with the nonterminal *A*. But it can build this value from the values associated with the matched terminals and nonterminals *B*, *C*, *D*.

RECURSIVE DESCENT

In a recursive-descent parser, the semantic actions are the values returned by parsing functions, or the side effects of those functions, or both. For each terminal and nonterminal symbol, we associate a *type* (from the implementation language of the compiler) of *semantic values* representing phrases derived from that symbol.

Program 4.1 is a recursive-descent interpreter for part of Grammar 3.15. The tokens `ID` and `NUM` must now carry values of type `string` and `int`, respectively. We will assume there is a lookup table mapping identifiers to integers. The type associated with *E; T; F;* etc., is `int`, and the semantic actions are easy to implement.

PROGRAM 4.1: Recursive-descent interpreter for part of Grammar 3.15.

```
class Token {int kind; Object val;
             Token(int k, Object v) {kind=k; val=v;}
            }
final int EOF=0, ID=1, NUM=2, PLUS=3, MINUS=4, ...

int lookup(String id) { ... }

int F_follow[] = { PLUS, TIMES, RPAREN, EOF };

int F() {switch (tok.kind) {
         case ID:  int i=lookup((String)(tok.val)); advance(); return i;
         case NUM: int i=((Integer)(tok.val)).intValue();
                   advance(); return i;
         case LPAREN: eat(LPAREN);
                      int i = E();
                      eatOrSkipTo(RPAREN, F_follow);
                      return i;
         case EOF:
         default:   print("expected ID, NUM, or left-paren");
                    skipto(F_follow); return 0;
         }}

int T_follow[] = { PLUS, RPAREN, EOF };

int T() {switch (tok.kind) {
         case ID:
         case NUM:
         case LPAREN: return Tprime(F());
         default: print("expected ID, NUM, or left-paren");
                  skipto(T_follow);
                  return 0;
         }}

int Tprime(int a) {switch (tok.kind) {
        case TIMES: eat(TIMES); return Tprime(a*F());
        case PLUS:
        case RPAREN:
        case EOF: return a;
        default: ...
       }}

void eatOrSkipTo(int expected, int[] stop) {
   if (tok.kind==expected)
        eat(expected);
   else {print(...); skipto(stop);}
}
```

The semantic action for an artificial symbol such as T' (introduced in the elimination of left recursion) is a bit tricky. Had the production been $T \rightarrow T * F$, then the semantic action would have been

```
int a = T(); eat(TIMES); int b=F(); return a*b;
```

With the rearrangement of the grammar, the production $T' \rightarrow *FT'$ is missing the left operand of the *. One solution is for T to pass the left operand as an argument to T', as shown in Program 4.1.

AUTOMATICALLY GENERATED PARSERS

A parser specification for JavaCC consists of a set of grammar rules, each annotated with a semantic action that is a Java statement. Whenever the generated parser reduces by a rule, it will execute the corresponding semantic action fragment.

Program 4.2 shows how this works for a variant of Grammar 3.15. Every `INTEGER_CONSTANT` terminal and every nonterminal (except `Start`) carries a value. To access this value, give the terminal or nonterminal a name in the grammar rule (such as `i` in Program 4.2), and access this name as a variable in the semantic action.

PROGRAM 4.2: JavaCC version of a variant of Grammar 3.15.

```
void Start() :
{ int i; }
{ i=Exp() <EOF> { System.out.println(i); }
}
int Exp() :
{ int a,i; }
{ a=Term()
  ( "+" i=Term() { a=a+i; }
  | "-" i=Term() { a=a-i; }
  )*
  { return a; }
}
int Term() :
{ int a,i; }
{ a=Factor()
  ( "*" i=Factor() { a=a*i; }
  | "/" i=Factor() { a=a/i; }
  )*
  { return a; }
}
int Factor() :
{ Token t; int i; }
{ t=<IDENTIFIER>      { return lookup(t.image); }
| t=<INTEGER_LITERAL> { return Integer.parseInt(t.image); }
| "(" i=Exp() ")"     { return i; }
}
```

SableCC, unlike JavaCC, has no way to attach action code to productions. However, SableCC automatically generates syntax tree classes, and a parser generated by SableCC will build syntax trees using those classes. For JavaCC, there are several companion tools, including JJTree and JTB (the Java Tree Builder), which, like SableCC, generate syntax tree classes and insert action code into the grammar for building syntax trees.

4.2 ABSTRACT PARSE TREES

It is possible to write an entire compiler that fits within the semantic action phrases of a JavaCC or SableCC parser. However, such a compiler is difficult to read and maintain, and this approach constrains the compiler to analyze the program in exactly the order it is parsed.

To improve modularity, it is better to separate issues of syntax (parsing) from issues of semantics (type-checking and translation to machine code). One way to do this is for the parser to produce a *parse tree* - a data structure that later phases of the compiler can traverse. Technically, a parse tree has exactly one leaf for each token of the input and one internal node for each grammar rule reduced during the parse.

Such a parse tree, which we will call a *concrete parse tree*, representing the *concrete syntax* of the source language, may be inconvenient to use directly. Many of the punctuation tokens are redundant and convey no information - they are useful in the input string, but once the parse tree is built, the structure of the tree conveys the structuring information more conveniently.

Furthermore, the structure of the parse tree may depend too much on the grammar! The grammar transformations shown in Chapter 3 - factoring, elimination of left recursion, elimination of ambiguity - involve the introduction of extra nonterminal symbols and extra grammar productions for technical purposes. These details should be confined to the parsing phase and should not clutter the semantic analysis.

An *abstract syntax* makes a clean interface between the parser and the later phases of a compiler (or, in fact, for the later phases of other kinds of program-analysis tools such as dependency analyzers). The abstract syntax tree conveys the phrase structure of the source program, with all parsing issues resolved but without any semantic interpretation.

Many early compilers did not use an abstract syntax data structure because early computers did not have enough memory to represent an entire compilation unit's syntax tree. Modern computers rarely have this problem. And many modern programming languages (ML, Modula-3, Java) allow forward reference to identifiers defined later in the same module; using an abstract syntax tree makes compilation easier for these languages. It may be that Pascal and C require clumsy *forward* declarations because their designers wanted to avoid an extra compiler pass on the machines of the 1970s.

Grammar 4.3 shows an abstract syntax of the expression language is Grammar 3.15. This grammar is completely impractical for parsing: The grammar is quite ambiguous, since precedence of the operators is not specified.

GRAMMAR 4.3: Abstract syntax of expressions.

- $E \rightarrow E + E$
- $E \rightarrow E - E$
- $E \rightarrow E * E$
- $E \rightarrow E / E$
- $E \rightarrow$ id
- $E \rightarrow$ num

However, Grammar 4.3 is not meant for parsing. The parser uses the *concrete syntax* to build a parse tree for the *abstract syntax*. The semantic analysis phase takes this *abstract syntax tree*; it is not bothered by the ambiguity of the grammar, since it already has the parse tree!

The compiler will need to represent and manipulate abstract syntax trees as data structures. In Java, these data structures are organized according to the principles outlined in Section 1.3: an abstract class for each nonterminal, a subclass for each production, and so on. In fact, the classes of Program 4.5 are abstract syntax classes for Grammar 4.3. An alternate arrangement, with all the different binary operators grouped into an `OpExp` class, is also possible.

Let us write an interpreter for the expression language in Grammar 3.15 by first building syntax trees and then interpreting those trees. Program 4.4 is a JavaCC grammar with semantic actions that produce syntax trees. Each class of syntax-tree nodes contains an `eval` function; when called, such a function will return the value of the represented expression.

PROGRAM 4.4: Building syntax trees for expressions.

```
Exp Start() :
  { Exp e; }
  { e=Exp() { return e; }
  }
Exp Exp() :
  { Exp e1,e2; }
  { e1=Term()
      ( "+" e2=Term() { e1=new PlusExp(e1,e2); }
      | "-" e2=Term() { e1=new MinusExp(e1,e2); }
      )*
    { return e1; }
  }
Exp Term() :
  { Exp e1,e2; }
  { e1=Factor()
      ( "*" e2=Factor() { e1=new  TimesExp(e1,e2); }
      | "/" e2=Factor() { e1=new DivideExp(e1,e2); }
      )*
    { return e1; }
  }
Exp Factor() :
  { Token t; Exp e; }
  { ( t=<IDENTIFIER>      { return new Identifier(t.image); } |
      t=<INTEGER_LITERAL> { return new IntegerLiteral(t.image); } |
      "(" e=Exp() ")"           { return e; } )
  }
```

POSITIONS

In a one-pass compiler, lexical analysis, parsing, and semantic analysis (typechecking) are all done simultaneously. If there is a type error that must be reported to the user, the *current* position of the lexical analyzer is a reasonable approximation of the source position of the error. In such a compiler, the lexical analyzer keeps a "current position" global variable, and the errormessage routine just prints the value of that variable with each message.

A compiler that uses abstract-syntax-tree data structures need not do all the parsing and semantic analysis in one pass. This makes life easier in many ways, but slightly complicates the production of semantic error messages. The lexer reaches the end of file before semantic analysis even begins; so if a semantic error is detected in traversing the abstract syntax tree, the *current* position of the lexer (at end of file) will not be useful in generating a line number for the error message. Thus, the source-file position of each node of the abstract syntax tree must be remembered, in case that node turns out to contain a semantic error.

To remember positions accurately, the abstract-syntax data structures must be sprinkled with `pos` fields. These indicate the position, within the original source file, of the characters from which these abstract-syntax structures were derived. Then the type-checker can produce useful error messages. (The syntax constructors we will show in Figure 4.9 do not have `pos` fields; any compiler that uses these exactly as given will have a hard time producing accurately located error messages.)

```
package syntaxtree;

Program(MainClass m, ClassDeclList cl)
MainClass(Identifier i1, Identifier i2, Statement s)
```

```
abstract class ClassDecl
ClassDeclSimple(Identifier i, VarDeclList vl, MethodDeclList ml)
ClassDeclExtends(Identifier i, Identifier j,
                                  VarDeclList vl, MethodDeclList ml) see
Ch.14

VarDecl(Type t, Identifier i)
MethodDecl(Type t, Identifier i, FormalList fl, VarDeclList vl,
                           StatementList sl, Exp e)
Formal(Type t, Identifier i)

abstract class Type
IntArrayType() BooleanType() IntegerType() IdentifierType(String s)

abstract class Statement
Block(StatementList sl)
If(Exp e, Statement s1, Statement s2)
While(Exp e, Statement s)
Print(Exp e)
Assign(Identifier i, Exp e)
ArrayAssign(Identifier i, Exp e1, Exp e2)

abstract class Exp
And(Exp e1, Exp e2)
LessThan(Exp e1, Exp e2)
Plus(Exp e1, Exp e2) Minus(Exp e1, Exp e2) Times(Exp e1, Exp e2)
ArrayLookup(Exp e1, Exp e2)
ArrayLength(Exp e)
Call(Exp e, Identifier i, ExpList el)
IntegerLiteral(int i)
True()
False()
IdentifierExp(String s)
This()
NewArray(Exp e)
NewObject(Identifier i)
Not(Exp e)

Identifier(String s)
list classes ClassDeclList() ExpList() FormalList() MethodDeclList()
StatementList() VarDeclList()
```

Figure 4.9: Abstract syntax for the MiniJava language.

The lexer must pass the source-file positions of the beginning and end of each token to the parser. We can augment the types `Exp`, etc. with a `position` field; then each constructor must take a `pos` argument to initialize this field. The positions of leaf nodes of the syntax tree can be obtained from the tokens returned by the lexical analyzer; internal-node positions can be derived from the positions of their subtrees. This is tedious but straightforward.

4.3 VISITORS

Each abstract syntax class of Program 4.5 has a constructor for building syntax trees, and an `eval` method for returning the value of the represented expression. This is an *object-oriented* style of programming. Let us consider an alternative.

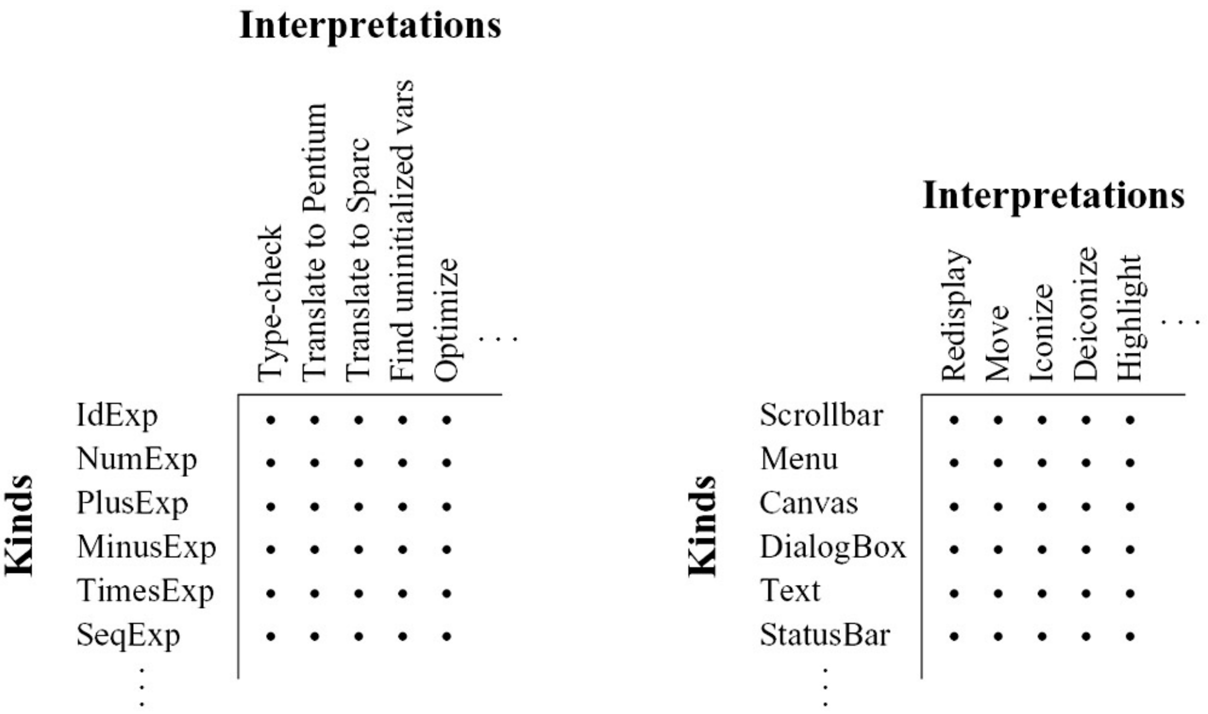
PROGRAM 4.5: Exp class for Program 4.4.

```
public abstract class Exp {
   public abstract int eval();
}
public class PlusExp extends Exp {
   private Exp e1,e2;
   public PlusExp(Exp a1, Exp a2) { e1=a1; e2=a2; }
   public int eval() {
       return e1.eval()+e2.eval();
   }
}
public class MinusExp extends Exp {
   private Exp e1,e2;
   public MinusExp(Exp a1, Exp a2) { e1=a1; e2=a2; }
   public int eval() {
       return e1.eval()-e2.eval();
   }
}
public class TimesExp extends Exp {
   private Exp e1,e2;
   public TimesExp(Exp a1, Exp a2) { e1=a1; e2=a2; }
   public int eval() {
       return e1.eval()*e2.eval();
   }
}
public class DivideExp extends Exp {
   private Exp e1,e2;
   public DivideExp(Exp a1, Exp a2) { e1=a1; e2=a2; }
   public int eval() {
       return e1.eval()/e2.eval();
   }
}
public class Identifier extends Exp {
   private String f0;
   public Identifier(String n0) { f0 = n0; }
   public int eval() {
       return lookup(f0);
   }
}
public class IntegerLiteral extends Exp {
   private String f0;
   public IntegerLiteral(String n0) { f0 = n0; }
   public int eval() {
       return Integer.parseInt(f0);
   }
}
```

Suppose the code for evaluating expressions is written *separately* from the abstract syntax classes. We might do that by examining the syntax-tree data structure by using `instanceof` and by fetching public class variables that represent subtrees. This is a *syntax separate from interpretations* style of programming.

The choice of style affects the modularity of the compiler. In a situation such as this, we have several *kinds* of objects: compound statements, assignment statements, print statements, and so on. And we also may have several different *interpretations* of these objects: type-check, translate to Pentium code, translate to Sparc code, optimize, interpret, and so on.

Each *interpretation* must be applied to each *kind*; if we add a new kind, we must implement each interpretation for it; and if we add a new interpretation, we must implement it for each kind. Figure 4.6 illustrates the orthogonality of kinds and interpretations - for compilers, and for graphic user interfaces, where the *kinds* are different widgets and gadgets, and the *interpretations* are move, hide, and redisplay commands.



(a) Compiler

(b) Graphic user interface

Figure 4.6: Orthogonal directions of modularity.

If the *syntax separate from interpretations* style is used, then it is easy and modular to add a new *interpretation:* One new function is written, with clauses for the different kinds all grouped logically together. On the other hand, it will not be modular to add a new *kind*, since a new clause must be added to every interpretation function.

With the *object-oriented* style, each interpretation is just a *method* in all the classes. It is easy and modular to add a new *kind:* All the interpretations of that kind are grouped together as methods of the new class. But it is not modular to add a new *interpretation:* A new method must be added to every class.

For graphic user interfaces, each application will want to make its own kinds of widgets; it is impossible to predetermine one set of widgets for everyone to use. On the other hand, the set of common operations (interpretations) is fixed: The window manager demands that each widget support only a certain interface. Thus, the *object-oriented* style works well, and the *syntax separate from interpretations* style would not be as modular.

For programming languages, on the other hand, it works very well to fix a syntax and then provide many interpretations of that syntax. If we have a compiler where one interpretation is *translate to Pentium* andwewishtoport that compiler to the Sparc, then not only must we add operations for generating Sparc code but we might also want to remove (in this configuration) the Pentium code-generation functions. This would be very inconvenient in the object-oriented style, requiring each class to be edited. In the *syntax separate from interpretations* style, such a change is modular: We remove a Pentiumrelated module and add a Sparc module.

We prefer a syntax-separate-from-interpretations style. Fortunately, we can use this style without employing `instanceof` expressions for accessing syntax trees. Instead, we can use a technique known as the Visitor pattern. A visitor implements an interpretation; it is an object which contains a `visit` method for each syntax-tree class. Each syntax-tree class should contain an `accept` method. An `accept` method serves as a hook for all interpretations. It is called by a visitor and it has just one task: It passes control back to an appropriate method of the visitor. Thus, control goes back and forth between a visitor and the syntax-tree classes.

Intuitively, the visitor calls the `accept` method of a node and asks "what is your class?" The `accept` method answers by calling the corresponding `visit` method of the visitor. Code for the running example, using visitors, is given in Programs 4.7 and 4.8. Every visitor implements the interface `Visitor`. Notice that each `accept` method takes a visitor as an argument, and that each `visit` method takes a syntax-tree-node object as an argument.

PROGRAM 4.7: Syntax classes with accept methods.

```
public abstract class Exp {
   public abstract int accept(Visitor v);
}
public class PlusExp extends Exp {
   public Exp e1,e2;
   public PlusExp(Exp a1, Exp a2) { e1=a1; e2=a2; }
   public int accept(Visitor v) {
       return v.visit(this);
   }
}
public class MinusExp extends Exp {
   public Exp e1,e2;
   public MinusExp(Exp a1, Exp a2) { e1=a1; e2=a2; }
   public int accept(Visitor v) {
       return v.visit(this);
   }
}
public class TimesExp extends Exp {
   public Exp e1,e2;
   public TimesExp(Exp a1, Exp a2) { e1=a1; e2=a2; }
   public int accept(Visitor v) {
       return v.visit(this);
   }
}
public class DivideExp extends Exp {
   public Exp e1,e2;
   public DivideExp(Exp a1, Exp a2) { e1=a1; e2=a2; }
   public int accept(Visitor v) {
       return v.visit(this);
   }
}
public class Identifier extends Exp {
   public String f0;
   public Identifier(String n0) { f0 = n0; }
   public int accept(Visitor v) {
       return v.visit(this);
   }
}
public class IntegerLiteral extends Exp {
   public String f0;
   public IntegerLiteral(String n0) { f0 = n0; }
   public int accept() {
       return v.visit(this);
   }
}
```

PROGRAM 4.8: An interpreter visitor.

```
public interface Visitor {
   public int visit(PlusExp n);
   public int visit(MinusExp n);
   public int visit(TimesExp n);
   public int visit(DivideExp n);
   public int visit(Identifier n);
   public int visit(IntegerLiteral n);
}
public class Interpreter implements Visitor {
   public int visit(PlusExp n) {
       return n.e1.accept(this)+n.e2.accept(this);
   }
   public int visit(MinusExp n) {
       return n.e1.accept(this)-n.e2.accept(this);
   }
   public int visit(TimesExp n) {
       return n.e1.accept(this)*n.e2.accept(this);
   }
   public int visit(DivideExp n) {
       return n.e1.accept(this)/n.e2.accept(this);
   }
   public int visit(Identifier n) {
       return lookup(n.f0);
   }
   public int visit(IntegerLiteral n) {
       return Integer.parseInt(n.f0);
   }
}
```

In Programs 4.7 and 4.8, the `visit` and `accept` methods all return `int`. Suppose we want instead to return `String`. In that case, we can add an appropriate `accept` method to each syntax tree class, and we can write a new visitor class in which all `visit` methods return `String`.

The main difference between the object-oriented style and the syntaxseparate-from-interpretations style is that, for example, the interpreter code in Program 4.5 is in the `eval` methods while in Program 4.8 it is in the `Interpreter` visitor.

In summary, with the Visitor pattern we can add a new interpretation without editing and recompiling existing classes, provided that each of the appropriate classes has an `accept` method. The following table summarizes some advantages of the Visitor pattern:

	Frequent type casts?	**Frequent recompilation?**
Instanceof and type casts	Yes	No
Dedicated methods	No	Yes
The Visitor pattern	No	No

ABSTRACT SYNTAX FOR MiniJava

Figure 4.9 shows classes for the abstract syntax of MiniJava. The meaning of each constructor in the abstract syntax should be clear after a careful study of Appendix A, but there are a few points that merit explanation.

Only the constructors are shown in Figure 4.9; the object field variables correspond exactly to the names of the constructor arguments. Each of the six list classes is implemented in the same way, for example:

```
public class ExpList {
   private Vector list;
   public ExpList() {
      list = new Vector();
   }
   public void addElement(Exp n) {
      list.addElement(n);
   }
   public Exp elementAt(int i) {
      return (Exp)list.elementAt(i);
   }
   public int size() {
      return list.size();
   }
}
```

Each of the nonlist classes has an accept method for use with the visitor pattern. The interface `Visitor` is shown in Program 4.10.

PROGRAM 4.10: MiniJava visitor

```
public interface Visitor {
  public void visit(Program n);
  public void visit(MainClass n);
  public void visit(ClassDeclSimple n);
  public void visit(ClassDeclExtends n);
  public void visit(VarDecl n);
  public void visit(MethodDecl n);
  public void visit(Formal n);
  public void visit(IntArrayType n);
  public void visit(BooleanType n);
  public void visit(IntegerType n);
  public void visit(IdentifierType n);
  public void visit(Block n);
  public void visit(If n);
  public void visit(While n);
  public void visit(Print n);
  public void visit(Assign n);
  public void visit(ArrayAssign n);
  public void visit(And n);
  public void visit(LessThan n);
  public void visit(Plus n);
  public void visit(Minus n);
  public void visit(Times n);
  public void visit(ArrayLookup n);
  public void visit(ArrayLength n);
  public void visit(Call n);
  public void visit(IntegerLiteral n);
  public void visit(True n);
  public void visit(False n);
  public void visit(IdentifierExp n);
  public void visit(This n);
  public void visit(NewArray n);
  public void visit(NewObject n);
  public void visit(Not n);
  public void visit(Identifier n);
}
```

We can construct a syntax tree by using nested `new` expressions. For example, we can build a syntax tree for the MiniJava statement:

```
x = y.m(1,4+5);
```

using the following Java code:

```
ExpList el = new ExpList();
el.addElement(new IntegerLiteral(1));
el.addElement(new Plus(new IntegerLiteral(4),
                       new IntegerLiteral(5)));
Statement s = new Assign(new Identifier("x"),
                         new Call(new IdentifierExp("y"),
                                  new Identifier("m"),
                                  el));
```

SableCC enables automatic generation of code for syntax tree classes, code for building syntax trees, and code for template visitors. For JavaCC, a companion tool called the Java Tree Builder (JTB) enables the generation of similar code. The advantage of using such tools is that once the grammar is written, one can go straight on to writing visitors that operate on syntax trees. The disadvantage is that the syntax trees supported by the generated code may be less abstract than one could desire.

PROGRAM ABSTRACT SYNTAX

Add semantic actions to your parser to produce abstract syntax for the MiniJava language. Syntax-tree classes are available in `$MINIJAVA/chap4`, together with a `PrettyPrintVisitor`. If you use JavaCC, you can use JTB to generate the needed code automatically. Similarly, with SableCC, the needed code can be generated automatically.

FURTHER READING

Many compilers mix recursive-descent parsing code with semantic-action code, as shown in Program 4.1; Gries [1971] and Fraser and Hanson [1995] are ancient and modern examples. Machine-generated parsers with semantic actions (in special-purpose "semantic-action mini-languages") attached to the grammar productions were tried out in 1960s [Feldman and Gries 1968]; Yacc [Johnson 1975] was one of the first to permit semantic action fragments to be written in a conventional, general-purpose programming language.

The notion of *abstract syntax* is due to McCarthy [1963], who designed the abstract syntax for Lisp [McCarthy et al. 1962]. The abstract syntax was intended to be used for writing programs until designers could get around to creating a concrete syntax with human-readable punctuation (instead of **L**ots of **I**rritating **S**illy **P**arentheses), but programmers soon got used to programming directly in abstract syntax.

The search for a theory of programming-language semantics, and a notation for expressing semantics in a compiler-compiler, led to ideas such as *denotational semantics* [Stoy 1977]. The semantic interpreter shown in Programs 4.4 and 4.5 is inspired by ideas from denotational semantics, as is the idea of separating concrete syntax from semantics using the abstract syntax as a clean interface.

EXERCISES

4.1 Write a package of Java classes to express the abstract syntax of regular expressions.

4.2 Extend Grammar 3.15 such that a program is a sequence of either assignment statements or print statements. Each assignment statement assigns an expression to an implicitly-declared variable; each print statement prints the value of an expression. Extend the interpreter in Program 4.1 to handle the new language.

4.3 Write a JavaCC version of the grammar from Exercise 4.2. Insert Java code for interpreting programs, in the style of Program 4.2.

4.4 Modify the JavaCC grammar from Exercise 4.3 to contain Java code for building syntax trees, in the style of Program 4.4. Write two interpreters for the language: one in object-oriented style and one that uses visitors.

4.5 In `$MINIJAVA/chap4/handcrafted/visitor`, there is a file with a visitor `PrettyPrintVisitor.java` for pretty printing syntax trees. Improve the pretty printing of nested `if` and `while` statements.

4.6 The visitor pattern in Program 4.7 has `accept` methods that return `int`. Ifone wanted to write some visitors that return integers, others that return class *A*, and yet others that return class *B*, one could modify all the classes in Program 4.7 to add two more `accept` methods, but this would not be very modular. Another way is to make the visitor return `Object` and cast each result, but this loses the benefit of compile-time type-checking. But there is a third way.

Modify Program 4.7 so that all the `accept` methods return `void`, and write two extensions of the `Visitor` class: one that computes an `int` for each `Exp`, and the other that computes a `float` for each `Exp`. Since the `accept` method will return `void`, the visitor object must have an instance variable into which each `accept` method can place its result. Explain why, if one then wanted to write a visitor that computed an object of class *C* for each `Exp`, no more modification of the `Exp` subclasses would be necessary.

Chapter 5: Semantic Analysis

OVERVIEW

se-man-tic: of or relating to meaning in language

Webster's Dictionary

The *semantic analysis* phase of a compiler connects variable definitions to their uses, checks that each expression has a correct type, and translates the abstract syntax into a simpler representation suitable for generating machine code.

5.1 SYMBOL TABLES

This phase is characterized by the maintenance of *symbol tables* (also called *environments*) mapping identifiers to their types and locations. As the declarations of types, variables, and functions are processed, these identifiers are bound to "meanings" in the symbol tables. When *uses* (nondefining occurrences) of identifiers are found, they are looked up in the symbol tables.

Each local variable in a program has a *scope* in which it is visible. For example, in a MiniJava method `m`, all formal parameters and local variables declared in `m` are visible only until the end of `m`. As the semantic analysis reaches the end of each scope, the identifier bindings local to that scope are discarded.

An environment is a set of *bindings* denoted by the ↦ arrow. For example, we could say that the environment σ_0 contains the bindings {`g` ↦ `string`, `a` ↦ `int`}, meaning that the identifier `a` is an integer variable and `g` is a string variable.

Consider a simple example in the Java language:

```
1 class C {
2   int a; int b; int c;
3   public void m(){
4     System.out.println(a+c);
5     int j = a+b;
6     String a = "hello";
7     System.out.println(a);
8     System.out.println(j);
9     System.out.println(b);
10    }
11 }
```

Suppose we compile this class in the environment σ_0. The field declarations on line 2 give us the table σ_1 equal to σ_0 + {`a` ↦ `int`, `b` ↦ `int`, `c` ↦ `int`}, that is, σ_0 extended with new bindings for `a`, `b`, and `c`. The identifiers in line 4 can be looked up in σ_1. At line 5, the table σ_2 = σ_1 + {`j` ↦ `int`} is created; and at line 6, σ_3 = σ_2 + {`a` ↦ `String`} is created.

How does the + operator for tables work when the two environments being "added" contain different bindings for the same symbol? When σ_2 and {`a` ↦ `String`} map `a` to `int` and

`String`, respectively? To make the scoping rules work the way we expect them to in real programming languages, we want `{a` ↦ `String}` to take precedence. So we say that $X + Y$ for tables is not the same as $Y + X$; bindings in the right-hand table override those in the left.

The identifiers in lines 7, 8, and 9 can be looked up in σ_3. Finally, at line 10, we discard σ_3 and go back to σ_1. And at line 11 we discard σ_1 and go back to σ_0.

How should this be implemented? There are really two choices. In a *functional* style, we make sure to keep σ_1 in pristine condition while we create σ_2 and σ_3. Then when we need σ_1 again, it's rested and ready.

In an *imperative* style, we modify σ_1 until it becomes σ_2. This *destructive update* "destroys" σ_1; while σ_2 exists, we cannot look things up in σ_1. But when we are done with σ_2, we can *undo* the modification to get σ_1 back again. Thus, there is a single global environment σ which becomes σ_0, σ_1, σ_2, σ_3, σ_1, σ_0 at different times and an "undo stack" with enough information to remove the destructive updates. When a symbol is added to the environment, it is also added to the undo stack; at the end of scope (e.g., at line 10), symbols popped from the undo stack have their latest binding removed from σ (and their previous binding restored).

Either the functional or imperative style of environment management can be used regardless of whether the language being compiled or the implementation language of the compiler is a "functional" or "imperative" or "objectoriented" language.

MULTIPLE SYMBOL TABLES

In some languages there can be several active environments at once: Each module, or class, or record in the program has a symbol table σ of its own.

In analyzing Figure 5.1, let σ_0 be the base environment containing predefined functions, and let

$$\sigma_1 = \{a \mapsto int\}$$

$$\sigma_2 = \{E \mapsto \sigma_1\}$$

$$\sigma_3 = \{b \mapsto int, a \mapsto int\}$$

$$\sigma_4 = \{N \mapsto \sigma_3\}$$

$$\sigma_5 = \{d \mapsto int\}$$

$$\sigma_6 = \{D \mapsto \sigma_5\}$$

$$\sigma_7 = \sigma_2 + \sigma_4 + \sigma_6$$

```
structure M = struct     package M;
   structure E = struct  class E {
      val a = 5;             static int a = 5;
   end                   }
   structure N = struct  class N {
      val b = 10             static int b = 10;
      val a = E.a + b        static int a = E.a + b;
   end                   }
   structure D = struct  class D {
      val d = E.a + N.a      static int d = E.a +
   end                 N.a;
end                      }
```

(a) An example in ML   (b) An example in Java

Figure 5.1: Several active environments at once.

In ML, the *N* is compiled using environment $\sigma_0 + \sigma_2$ to look up identifiers; *D* is compiled using $\sigma_0 + \sigma_2 + \sigma_4$, and the result of the analysis is $\{M \mapsto \sigma_7\}$.

In Java, forward reference is allowed (so inside *N* the expression *D.d* would be legal), so *E*, *N*, and *D* are all compiled in the environment σ_7; for this program the result is still $\{M \mapsto \sigma_7\}$.

EFFICIENT IMPERATIVE SYMBOL TABLES

Because a large program may contain thousands of distinct identifiers, symbol tables must permit efficient lookup.

Imperative-style environments are usually implemented using hash tables, which are very efficient. The operation $\sigma' = \sigma + \{a \mapsto \tau\}$ is implemented by inserting τ in the hash table with key *a*. A simple *hash table with external chaining* works well and supports deletion easily (we will need to delete $\{a \mapsto \tau\}$ to recover σ at the end of the scope of *a*).

Program 5.2 implements a simple hash table. The *i*th bucket is a linked list of all the elements whose keys hash to *i* mod `SIZE`.

PROGRAM 5.2: Hash table with external chaining.

```
class Bucket {String key; Object binding; Bucket next;
        Bucket(String k, Object b, Bucket n) {key=k; binding=b; next=n;}
}

class HashT {
   final int SIZE = 256;
   Bucket table[] = new Bucket[SIZE];

   private int hash(String s) {
        int h=0;
        for(int i=0; i<s.length(); i++)
           h=h*65599+s.charAt(i);
        return h;
   }

   void insert(String s, Binding b) {
        int index=hash(s)%SIZE
        table[index]=new Bucket(s,b,table[index]);
   }

   Object lookup(String s) {
        int index=hash(s)%SIZE
        for (Binding b = table[index]; b!=null; b=b.next)
          if (s.equals(b.key)) return b.binding;
        return null;
   }
   void pop(String s) {
        int index=hash(s)%SIZE
        table[index]=table[index].next;
   }
}
```

Consider $\sigma + \{a \mapsto \tau_2\}$ when σ contains $a \mapsto \tau_1$ already. The `insert` function leaves $a \mapsto \tau_1$ in the bucket and puts $a \mapsto \tau_2$ earlier in the list. Then, when `pop`(a) is done at the end of a's scope, σ is restored. Of course, `pop` works only if bindings are inserted and popped in a stacklike fashion.

An industrial-strength implementation would improve on this in several ways; see Exercise 5.1.

EFFICIENT FUNCTIONAL SYMBOL TABLES

In the functional style, we wish to compute $\sigma' = \sigma + \{a \mapsto \tau\}$ in such a way that we still have σ available to look up identifiers. Thus, instead of "altering" a table by adding a binding to it we create a new table by computing the "sum" of an existing table and a new binding. Similarly, when we add 7 + 8 we don't alter the 7 by adding 8 to it; we create a new value, 15 − and the 7 is still available for other computations.

However, nondestructive update is not efficient for hash tables. Figure 5.3a shows a hash table implementing mapping m_1. It is fast and efficient to add *mouse* to the fifth slot; just make the *mouse* record point at the (old) head of the fifth linked list, and make the fifth slot point to the *mouse* record. But then we no longer have the mapping m_1:Wehavedestroyedittomake m_2. The other alternative is to copy the array, but still share all the old buckets, as shown in Figure 5.3b. But this is not efficient: The array in a hash table should be quite large, proportional in size to the number of elements, and we cannot afford to copy it for each new entry in the table.

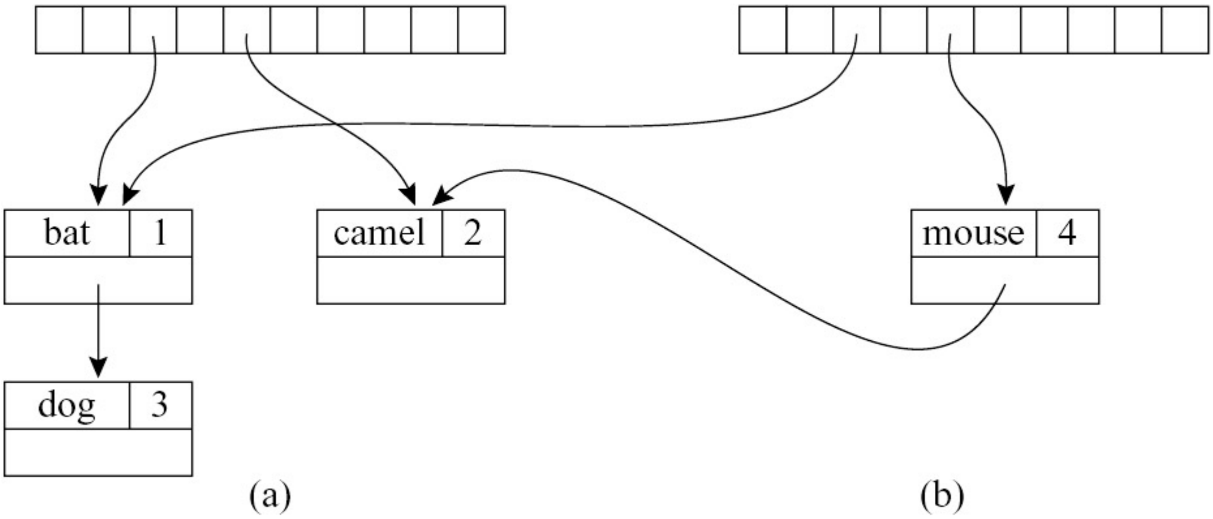


Figure 5.3: Hash tables.

By using binary search trees we can perform such "functional" additions to search trees efficiently. Consider, for example, the search tree in Figure 5.4, which represents the mapping $m_1 = \{bat \mapsto 1,\ camel \mapsto 2,\ dog \mapsto 3\}$.

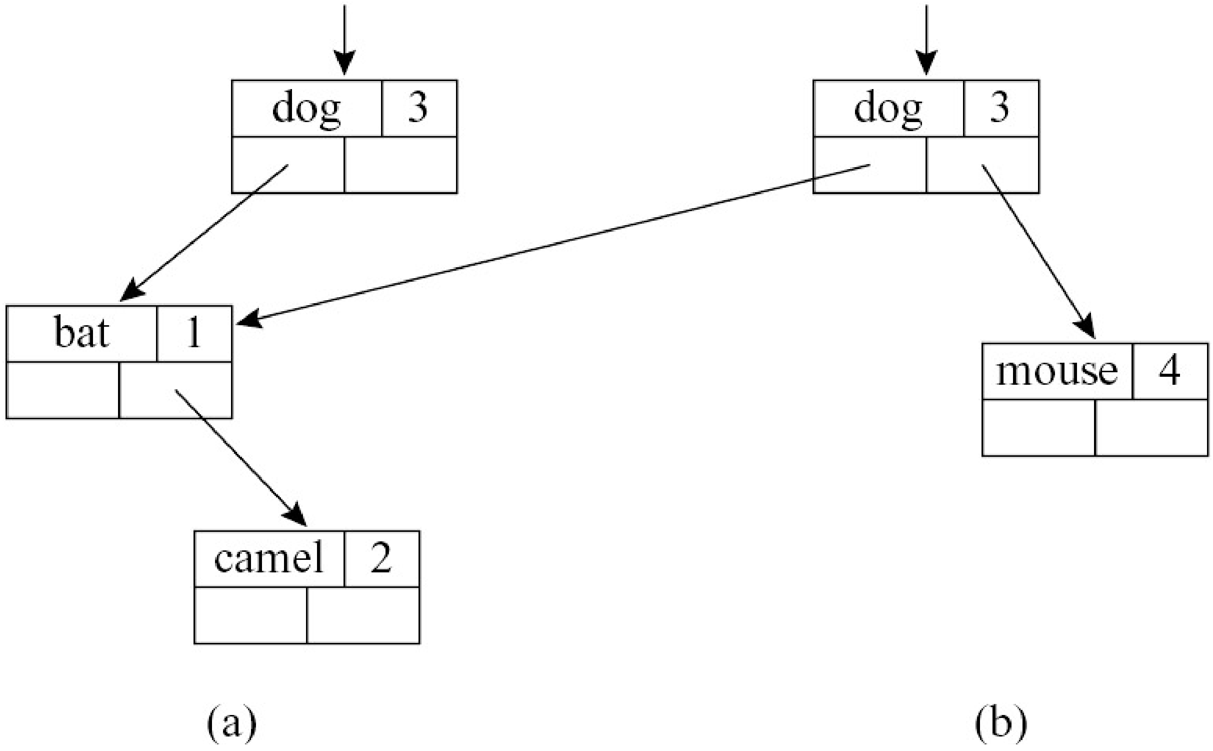


Figure 5.4: Binary search trees.

We can add the binding *mouse* ↦ 4, creating the mapping m_2 without destroying the mapping m_1, as shown in Figure 5.4b. If we add a new node at depth *d* of the tree, we must create *d* new nodes - but we don't need to copy the whole tree. So creating a new tree (that shares some structure with the old one) can be done as efficiently as looking up an element: in log(*n*) time for a balanced tree of *n* nodes. This is an example of a *persistent data structure;* a persistent *red-black* tree can be kept balanced to guarantee log(*n*) access time (see Exercise 1.1c, and also page 276).

SYMBOLS

The hash table of Program 5.2 must examine every character of the string *s* for the `hash` operation, and then again each time it compares *s* against a string in the *i*th bucket. To avoid unnecessary string comparisons, we can convert each string to a `symbol`, so that all the different occurrences of any given string convert to the same symbol object.

The `Symbol` module implements symbols and has these important properties:

- Comparing symbols for equality is fast (just pointer or integer comparison).
- Extracting an integer hash key is fast (in case we want to make a hash table mapping symbols to something else).
- Comparing two symbols for "greater-than" (in some arbitrary ordering) is fast (in case we want to make binary search trees).

Even if we intend to make functional-style environments mapping symbols to bindings, we can use a destructive-update hash table to map strings to symbols: We need this to make sure the second occurrence of "abc" maps to the same symbol as the first occurrence. Program 5.5 shows the interface of the `Symbol` module.

PROGRAM 5.5: The interface of package `Symbol`.

```
package Symbol;

public class Symbol {
  public String toString();
  public static Symbol symbol(String s);
}
public class Table {
  public Table();
  public void put(Symbol key, Object value);
  public Object get(Symbol key);
  public void beginScope();
  public void endScope();
  public java.util.Enumeration keys();
}
```

Environments are implemented in the `Symbol.Table` class as `Table`s mapping `Symbol`s to bindings. We want different notions of `binding` for different purposes in the compiler - type bindings for types, value bindings for variables and functions - so we let the bindings be `Object`, though in any given table every binding should be a type binding, or every binding should be a value binding, and so on.

To implement the `Symbol` class (Program 5.6), we rely on the `intern()` method of the `java.lang.String` class to give us a unique object for any given character sequence; we can map from `Symbol` to `String` by having each symbol contain a string variable, but the reverse mapping must be done using a hash table (we use `java.util.Hashtable`).

PROGRAM 5.6: Symbol table implementation.

```
package Symbol;
public class Symbol {
  private String name;
  private Symbol(String n) {name=n; }
  private static java.util.Dictionary dict = new java.util.Hashtable();

  public String toString() {return name;}

  public static Symbol symbol(String n) {
        String u = n.intern();
        Symbol s = (Symbol)dict.get(u);
        if (s==null) {s = new Symbol(u); dict.put(u,s); }
        return s;
  }
}
```

To handle the "undo" requirements of destructive update, the interface function `beginScope` remembers the current state of the table, and `endScope` restores the table to where it was at the most recent `beginScope` that has not already been ended.

An imperative table is implemented using a hash table. When the binding $x \mapsto b$ is entered (`table.put(x,b)`), x is hashed into an index i, and a `Binder` object $x \mapsto b$ is placed at the head of the linked list for the ith bucket. If the table had already contained a binding $x \mapsto b'$, that would still be in the bucket, hidden by $x \mapsto b$. This is important because it will support the implementation of *undo* (`beginScope` and `endScope`).

The key x is not a character string, but is the `Symbol` object itself.

There must also be an auxiliary stack, showing in what order the symbols were "pushed" into the symbol table. When $x \mapsto b$ is entered, then x is pushed onto this stack. A `beginScope` operation pushes a special marker onto the stack. Then, to implement `endScope`, symbols are popped off the stack down to and including the topmost marker. As each symbol is popped, the head binding in its bucket is removed.

The auxiliary stack can be integrated into the `Binder` by having a global variable `top` showing the most recent `Symbol` bound in the table. Then "pushing" is accomplished by copying `top` into the `prevtop` field of the `Binder`. Thus, the "stack" is threaded through the binders.

If we wanted to use functional-style symbol tables, the `Table` interface might look like this:

```
public class Table {
  public Table();
  public Table put(Symbol key, Object value);
  public Object get(Symbol key);
  public java.util.Enumeration keys();
}
```

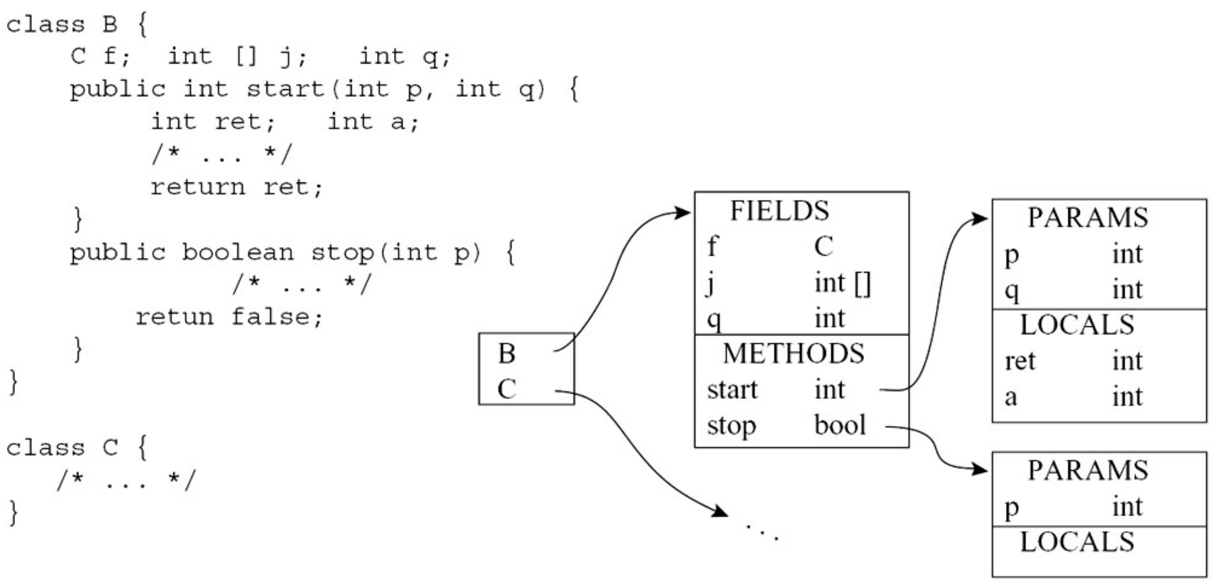
The `put` function would return a new table without modifying the old one. We wouldn't need `beginScope` and `endScope`, because we could keep an old version of the table even as we use the new version.

5.2 TYPE-CHECKING MiniJava

With what should a symbol table be filled - that is, what is a `binding`? To enable type-checking of MiniJava programs, the symbol table should contain all declared type information:

- each variable name and formal-parameter name should be bound to its type;
- each method name should be bound to its parameters, result type, and local variables; and
- each class name should be bound to its variable and method declarations.

For example, consider Figure 5.7, which shows a program and its symbol table. The two class names `B` and `C` are each mapped to two tables for fields and methods. In turn, each method is mapped to both its result type and tables with its formal parameters and local variables.



```
class B {
    C f;  int [] j;   int q;
    public int start(int p, int q) {
         int ret;   int a;
         /* ... */
         return ret;
    }
    public boolean stop(int p) {
              /* ... */
        retun false;
    }
}

class C {
   /* ... */
}
```

Figure 5.7: A MiniJava Program and its symbol table

The primitive types in MiniJava are `int` and `boolean`; all other types are either integer array, written `int []`, or class names. For simplicity, we choose to represent each type as a string, rather than as a symbol; this allows us to test type equality by doing string comparison.

Type-checking of a MiniJava program proceeds in two phases. First, we build the symbol table, and then we type-check the statements and expressions. During the second phase, the symbol table is consulted for each identifier that is found. It is convenient to use two phases because, in Java and MiniJava, the classes are mutually recursive. If we tried to do type-checking in a single phase, then we might need to type-check a call to a method that is not yet entered into the symbol table. To avoid such situations, we use an approach with two phases.

The first phase of the type-checker can be implemented by a visitor that visits nodes in a MiniJava syntaxtree and builds a symbol table. For instance, the visit method in Program 5.8 handles variable declarations. It will add the variable name and type to a data structure for the current class which later will be added to the symbol table. Notice that the visit method checks whether a variable is declared more than once and, if so, then it prints an appropriate error message.

PROGRAM 5.8: A `visit` method for variable declarations

```
class ErrorMsg {
  boolean anyErrors;
  void complain(String msg) {
     anyErrors = true;
     System.out.println(msg);
  }
}

// Type t;
// Identifier i;
public void visit(VarDecl n) {

  Type t = n.t.accept(this);
  String id = n.i.toString();

  if (currMethod == null) {
      if (!currClass.addVar(id,t))
         error.complain(id + "is already defined in " + currClass.getId());
  } else if (!currMethod.addVar(id,t))
         error.complain(id + "is already defined in "
              + currClass.getId() + "." + currMethod.getId());
}
```

The second phase of the type-checker can be implemented by a visitor that type-checks all statements and expressions. The result type of each visit method is `String`, for representing MiniJava types. The idea is that when the visitor visits an expression, then it returns the type of that expression. If the expression does not type-check, then the type-check is terminated with an error message.

Let's take a simple case: an addition expression $e_1 + e_2$. In MiniJava, both operands must be integers (the type-checker must check this) and the result will be an integer (the type-checker will return this type). The visit method for addition is easy to implement; see Program 5.9.

PROGRAM 5.9: A `visit` method for plus expressions

```
// Exp e1,e2;
public Type visit(Plus n) {
  if (! (n.e1.accept(this) instanceof IntegerType) )
     error.complain("Left side of LessThan must be of type integer");
  if (! (n.e2.accept(this) instanceof IntegerType) )
     error.complain("Right side of LessThan must be of type integer");
  return new IntegerType();
}
```

In most languages, addition is *overloaded*: The + operator stands for either integer addition or real addition. If the operands are both integers, the result is integer; if the operands are both real, the result is real. And in many languages if one operand is an integer and the other is real, the integer is implicitly converted into a real, and the result is real. Of course, the compiler will have to make this conversion explicit in the machine code it generates.

For an assignment statement, it must be checked that the left-hand side and the right-hand side have the same type. When we allow extension of classes, the requirement is less strict: It is sufficient to check that the right-hand side is a subtype of the left-hand side.

For method calls, it is necessary to look up the method identifier in the symbol table to get the parameter list and the result type. For a call `e.m(…)`, where `e` has type `C`, we look up the definition of `m` in class `C`. The parameter types must then be matched against the arguments in the function-call expression. The result type of the method becomes the type of the method call as a whole.

Every kind of statement and expression has its own type-checking rules, but in all the cases we have not already described, the rules can be derived by reference to the Java Language Specification.

ERROR HANDLING

When the type-checker detects a type error or an undeclared identifier, it should print an error message and continue - because the programmer would like to be told of all the errors in the program. To recover after an error, it's often necessary to build data structures as if a valid expression had been encountered. For example, type-checking

```
{int i = new C();
 int j = i + i;
 ...
}
```

even though the expression `new C()` doesn't match the type required to initialize an integer, it is still useful to enter `i` in the symbol table as an integer so that the rest of the program can be type-checked.

If the type-checking phase detects errors, then the compiler should not produce a compiled program as output. This means that the later phases of the compiler - translation, register allocation, etc. - will not be executed. It will be easier to implement the later phases of the compiler if they are not required to handle invalid inputs. Thus, if at all possible, all errors in the input program should be detected in the front end of the compiler (parsing and type-checking).

PROGRAM TYPE-CHECKING

Design a set of visitors which type-checks a MiniJava program and produces any appropriate error messages about mismatching types or undeclared identifiers.

EXERCISES

- **5.1** Improve the hash table implementation of Program 5.2: Double the size of the array when the average bucket length grows larger than 2 (so `table` is now a pointer to a dynamically allocated array). To double an array, allocate a bigger one and rehash the contents of the old array; then discard the old array.
- *****5.2** In many applications, we want a + operator for environments that does more than add one new binding; instead of $\sigma' = \sigma + \{a \mapsto \tau\}$, we want $\sigma' = \sigma_1 + \sigma_2$, where σ_1 and σ_2 are arbitrary environments (perhaps overlapping, in which case bindings in σ_2 take precedence).

  We want an efficient algorithm and data structure for environment "adding." Balanced trees can implement $\sigma + \{a \mapsto \tau\}$ efficiently (in log(N) time, where N is the size of σ), but take $O(N)$ to compute $\sigma_1 + \sigma_2$ if σ_1 and σ_2 are both about size N.

  To abstract the problem, solve the general nondisjoint integer-set union problem. The input is a set of commands of the form,

$$\begin{array}{ll}
s_1 = \{4\} & \textit{(define singleton set)} \\
s_2 = \{7\} & \\
s_3 = s_1 \cup s_2 & \textit{(nondestructive union)} \\
6 \overset{?}{\in} s_3 & \textit{(membership test)} \\
s_4 = s_1 \cup s_3 & \\
s_5 = \{9\} & \\
s_6 = s_4 \cup s_5 & \\
7 \overset{?}{\in} s_2 &
\end{array}$$

An efficient algorithm is one that can process an input of N commands, answering all membership queries, in less than $o(N^2)$ time.

- *a. Implement an algorithm that is efficient when a typical set union $a \leftarrow b \cup c$ has b much smaller than c [Brown and Tarjan 1979].
- ***b. Design an algorithm that is efficient even in the worst case, or prove that this can't be done (see Lipton et al. [1997] for a lower bound in a restricted model).

Chapter 6: Activation Records

stack: an orderly pile or heap

Webster's Dictionary

OVERVIEW

In almost any modern programming language, a function may have *local* variables that are created upon entry to the function. Several invocations of the function may exist at the same time, and each invocation has its own *instantiations* of local variables.

In the Java method

```
int f(int x) {
  int y = x+x;
  if (y<10)
     return f(y);
  else
     return y-1;
}
```

a new instantiation of `x` is created (and initialized by `f`'s caller) each time that `f` is called. Because there are recursive calls, many of these `x`'s exist simultaneously. Similarly, a new instantiation of `y` is created each time the body of `f` is entered.

In many languages (including C, Pascal, and Java), local variables are destroyed when a function returns. Since a function returns only after all the functions it has called have returned, we say that function calls behave in last-in-first-out (LIFO) fashion. If local variables are created on function entry and destroyed on function exit, then we can use a LIFO data structure - a stack - to hold them.

HIGHER-ORDER FUNCTIONS

But in languages supporting both nested functions *and* function-valued variables, it may be necessary to keep local variables after a function has returned! Consider Program 6.1: This is legal in ML, but of course in C one cannot really nest the function *g* inside the function *f*.

PROGRAM 6.1: An example of higher-order functions.

```
fun f(x) =             int (*)() f(int x) {
  let fun g(y) = x+y     int g(int y) {return
   in g              x+y;}
  end                    return g;
                       }

val h = f(3)
val j = f(4)           int (*h)() = f(3);
                       int (*j)() = f(4);

val z = h(5)
val w = j(7)           int z = h(5);
                       int w = j(7);
```

(a) Written in ML (b) Written in pseudo-C

When *f*(3) is executed, a new local variable *x* is created for the activation of function *f*. Then *g* is returned as the result of *f(x)*; but *g* has not yet been called, so *y* is not yet created.

At this point *f* has returned, but it is too early to destroy *x*, because when *h*(5) is eventually executed it will need the value *x* = 3. Meanwhile, *f*(4) is entered, creating a *different* instance of *x*, and it returns a *different* instance of *g* in which *x* = 4.

It is the combination of *nested functions* (where inner functions may use variables defined in the outer functions) and *functions returned as results* (or stored into variables) that causes local variables to need lifetimes longer than their enclosing function invocations.

Pascal has nested functions, but it does not have functions as returnable values. C has functions as returnable values, but not nested functions. So these languages can use stacks to hold local variables.

ML, Scheme, and several other languages have both nested functions and functions as returnable values (this combination is called *higher-order functions*). So they cannot use stacks to hold all local variables. This complicates the implementation of ML and Scheme - but the added expressive power of higher-order functions justifies the extra implementation effort.

For the remainder of this chapter we will consider languages with stackable local variables and postpone discussion of higher-order functions to Chapter 15. Notice that while Java allows nesting of functions (via inner classes), MiniJava does not.

6.1 STACK FRAMES

The simplest notion of a *stack* is a data structure that supports two operations, *push* and *pop*. However, it turns out that local variables are pushed in large batches (on entry to functions) and popped in large batches (on exit). Furthermore, when local variables are created they are not always initialized right away. Finally, after many variables have been pushed, we want to continue accessing variables deep within the stack. So the abstract *push* and *pop* model is just not suitable.

Instead, we treat the stack as a big array, with a special register - the *stack pointer* - that points at some location. All locations beyond the stack pointer are considered to be garbage, and all locations before the stack pointer are considered to be allocated. The stack usually grows only at the entry to a function, by an increment large enough to hold all the local variables for that function, and, just before the exit from the function, shrinks by the same amount. The area on the stack devoted to the local variables, parameters, return address, and other temporaries for a function is called the function's *activation record* or *stack frame*. For historical reasons, run-time stacks usually start at a high memory address and grow toward smaller addresses. This can be rather confusing: Stacks grow downward and shrink upward, like icicles.

The design of a frame layout takes into account the particular features of an instruction set architecture and the programming language being compiled. However, the manufacturer of a computer often prescribes a "standard" frame layout to be used on that architecture, where possible, by all compilers for all programming languages. Sometimes this layout is not the most convenient one for a particular programming language or compiler. But by using the "standard" layout, we gain the considerable benefit that functions written in one language can call functions written in another language.

Figure 6.2 shows a typical stack frame layout. The frame has a set of *incoming arguments* (technically these are part of the previous frame but they are at a known offset from the frame pointer) passed by the caller. The *return address* is created by the CALL instruction and tells where (within the calling function) control should return upon completion of the current function. Some *local variables* are in this frame; other local variables are kept in machine registers. Sometimes a local variable kept in a register needs to be *saved* into the frame to make room for other uses of the register; there is an area in the frame for this purpose. Finally, when the current function calls other functions, it can use the *outgoing argument* space to pass parameters.

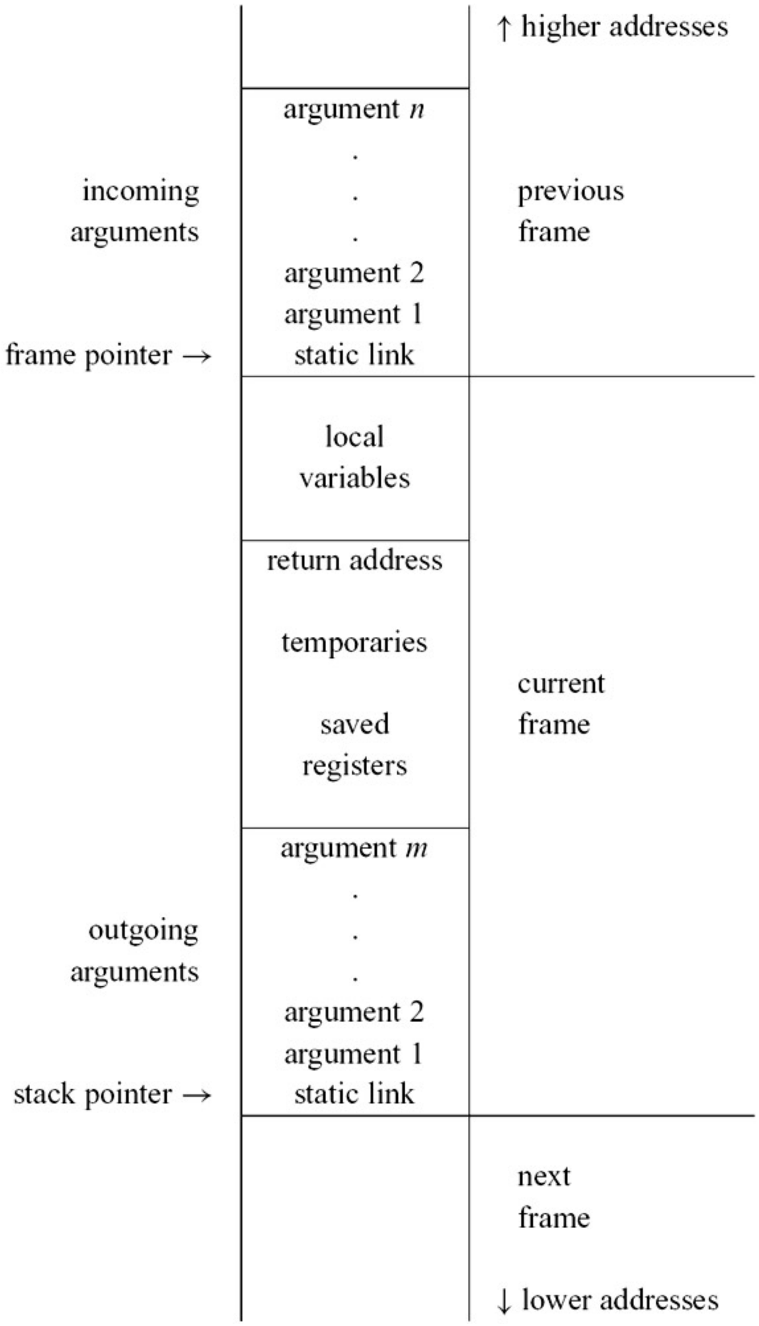


Figure 6.2: A stack frame.

THE FRAME POINTER

Suppose a function $g(\ldots)$ calls the function $f(a_1,\ldots, a_n)$. We say g is the *caller* and f is the *callee*. On entry to f, the stack pointer points to the first argument that g passes to f . On entry, f allocates a frame by simply subtracting the frame size from the stack pointer SP.

The old SP becomes the current *frame pointer* FP. In some frame layouts, FP is a separate register; the old value of FP is saved in memory (in the frame) and the new FP becomes the old SP. When f exits, it just copies FP back to SP and fetches back the saved FP. This arrangement is useful if f 's frame size can vary, or if frames are not always contiguous on the stack. But if the frame size is fixed, then for each function f the FP will always differ from SP by a known constant, and it is not necessary to use a register for FP at all − FP is a "fictional" register whose value is always SP + *framesize*.

Why talk about a frame pointer at all? Why not just refer to all variables, parameters, etc., by their offset from SP, if the frame size is constant? The frame size is not known until quite late in the compilation process, when the number of memory-resident temporaries and saved registers is determined. But it is useful to know the offsets of formal parameters and local variables much earlier. So, for convenience, we still talk about a frame pointer. And we put the formals and locals right near the frame pointer at offsets that are known *early*; the temporaries and saved registers go farther away, at offsets that are known *later*.

REGISTERS

A modern machine has a large set of *registers* (typically 32 of them). To make compiled programs run fast, it's useful to keep local variables, intermediate results of expressions, and other values in registers instead of in the stack frame. Registers can be directly accessed by arithmetic instructions; on most machines, accessing memory requires separate *load* and *store* instructions. Even on machines whose arithmetic instructions can access memory, it is faster to access registers.

A machine (usually) has only one set of registers, but many different procedures and functions need to use registers. Suppose a function f is using register r to hold a local variable and calls procedure g, which also uses r for its own calculations. Then r must be saved (stored into a stack frame) before g uses it and restored (fetched back from the frame) after g is finished using it. But is it f 's responsibility to save and restore the register, or g's? We say that r is a *caller-save* register if the caller (in this case, f) must save and restore the register, and r is *callee-save* if it is the responsibility of the callee (in this case, g).

On most machine architectures, the notion of caller-save or callee-save register is not something built into the hardware, but is a convention described in the machine's reference manual. On the MIPS computer, for example, registers 16-23 are preserved across procedure calls (callee-save), and all other registers are not preserved across procedure calls (caller-save).

Sometimes the saves and restores are unnecessary. If f knows that the value of some variable x will not be needed after the call, it may put x in a caller-save register *and not save it* when calling g. Conversely, if f hasalocal variable i that is needed before and after several function calls, it may put i in some callee-save register r_i, andsave r_i just once (upon entry to f)and fetch it back just once (before returning from f). Thus, the wise selection of a caller-save or callee-save register for each local variable and temporary can reduce the number of stores and fetches a program executes. We will rely on our register allocator to choose the appropriate kind of register for each local variable and temporary value.

PARAMETER PASSING

On most machines whose calling conventions were designed in the 1970s, function arguments were passed on the stack.[1] But this causes needless memory traffic. Studies of actual programs have shown that very few functions have more than four arguments, and almost none have more than six. Therefore, parameter-passing conventions for modern machines specify that the first k arguments (for $k = 4$ or $k = 6$, typically) of a function are passed in registers $r_p, \ldots, r_{p+k-1}$, and the rest of the arguments are passed in memory.

Now, suppose $f(a_1, \ldots, a_n)$ (which received its parameters in $r_1, \ldots, r_n$, for example) calls $h(z)$. It must pass the argument z in r_1; so f saves the old contents of r_1 (the value a_1) in its stack frame before calling h. But there is the memory traffic that was supposedly avoided by passing arguments in registers! How has the use of registers saved any time?

There are four answers, any or all of which can be used at the same time:

1. Some procedures don't call other procedures - these are called *leaf* procedures. What proportion of procedures are leaves? Well, if we make the (optimistic) assumption that the average procedure calls either no other procedures or calls at least two others, then we can describe a "tree" of procedure calls in which there are more leaves than internal nodes. This means that *most* procedures called are leaf procedures.

   Leaf procedures need not write their incoming arguments to memory. In fact, often they don't need to allocate a stack frame at all. This is an important savings.

2. Some optimizing compilers use *interprocedural register allocation*, analyzing all the functions in an entire program at once. Then they assign different procedures different registers in which to receive parameters and hold local variables. Thus $f(x)$ might receive x in r_1, but call $h(z)$ with z in r_7.
3. Even if f is not a leaf procedure, it might be finished with all its use of the argument x by the time it calls h (technically, x is a dead variable at the point where h is called). Then f can overwrite r_1 without saving it.
4. Some architectures have *register windows*, so that each function invocation can allocate a fresh set of registers without memory traffic.

If f needs to write an incoming parameter into the frame, where in the frame should it go? Ideally, f's frame layout should matter only in the implementation of f. A straightforward approach would be for the caller to pass arguments $a_1, \ldots; a_k$ in registers and $a_{k+1}, \ldots; a_n$ at the end of its own frame - the place marked *outgoing arguments* in Figure 6.2 - which become the *incoming arguments* of the callee. If the callee needed to write any of these arguments to memory, it would write them to the area marked *local variables*.

The C programming language actually allows you to take the address of a formal parameter and guarantees that all the formal parameters of a function are at consecutive addresses! This is the `varargs` feature that `printf` uses. Allowing programmers to take the address of a parameter can lead to a *dangling reference* if the address outlives the frame - as the address of `x` will in `int *f(int x){return &x;}` - and even when it does not lead to bugs, the consecutive-address rule for parameters constrains the compiler and makes stack-frame layout more complicated. To resolve the contradiction that parameters are passed in registers, but have addresses too, the first *k* parameters are passed in registers; but any parameter whose address is taken must be written to a memory location on entry to the function. To satisfy `printf`, the memory locations into which register arguments are written must all be consecutive with the memory locations in which arguments *k* + 1, *k* + 2, etc., are written. Therefore, C programs can't have some of the arguments saved in one place and some saved in another - they must all be saved contiguously.

So in the standard calling convention of many modern machines the *calling* function reserves space for the register arguments in its own frame, next to the place where it writes argument *k* + 1. But the calling function does not actually write anything there; that space is written into *by the called function*, and only if the called function needs to write arguments into memory for any reason.

A more dignified way to take the address of a local variable is to use *call-by-reference*. With call-by-reference, the programmer does not explicitly manipulate the address of a variable *x*. Instead, if *x* is passed as the argument to *f*(*y*), where *y* is a "by-reference" parameter, the compiler generates code to pass the address of *x* instead of the contents of *x*. At any use of *y* within the function, the compiler generates an extra pointer dereference. With call-by-reference, there can be no "dangling reference", since *y* must disappear when *f* returns, and *f* returns before *x*'s scope ends.

RETURN ADDRESSES

When function *g* calls function *f*, eventually *f* must return. It needs to know where to go back to. If the *call* instruction within *g* is at address *a*, then (usually) the right place to return to is *a* + 1, the next instruction in *g*. This is called the *return address*.

On 1970s machines, the return address was pushed on the stack by the *call* instruction. Modern science has shown that it is faster and more flexible to pass the return address in a register, avoiding memory traffic and also avoiding the need to build any particular stack discipline into the hardware.

On modern machines, the *call* instruction merely puts the return address (the address of the instruction after the call) in a designated register. A nonleaf procedure will then have to write it to the stack (unless interprocedural register allocation is used), but a leaf procedure will not.

FRAME-RESIDENT VARIABLES

So a modern procedure-call convention will pass function parameters in registers, pass the return address in a register, and return the function result in a register. Many of the local variables will be allocated to registers, as will the intermediate results of expression evaluation. Values are written to memory (in the stack frame) only when necessary for one of these reasons:

- the variable will be passed by reference, so it must have a memory address (or, in the C language the `&` operator is anywhere applied to the variable);
- the variable is accessed by a procedure nested inside the current one;[2]
- the value is too big to fit into a single register;[3]
- the variable is an array, for which address arithmetic is necessary to extract components;

- the register holding the variable is needed for a specific purpose, such as parameter passing (as described above), though a compiler may move such values to other registers instead of storing them in memory;
- or there are so many local variables and temporary values that they won't all fit in registers, in which case some of them are "spilled" into the frame.

We will say that a variable *escapes* if it is passed by reference, its address is taken (using C's `&` operator), or it is accessed from a nested function.

When a formal parameter or local variable is declared, it's convenient to assign it a location - either in registers or in the stack frame - right at that point in processing the program. Then, when occurrences of that variable are found in expressions, they can be translated into machine code that refers to the right location. Unfortunately, the conditions in our list don't manifest themselves early enough. When the compiler first encounters the declaration of a variable, it doesn't yet know whether the variable will ever be passed by reference, accessed in a nested procedure, or have its address taken; and it doesn't know how many registers the calculation of expressions will require (it might be desirable to put some local variables in the frame instead of in registers). An industrial-strength compiler must assign provisional locations to all formals and locals, and decide later which of them should really go in registers.

STATIC LINKS

In languages that allow nested function declarations (such as Pascal, ML, and Java), the inner functions may use variables declared in outer functions. This language feature is called *block structure*. For example, consider Program 6.3, which is written with a Pascal-like syntax. The function `write` refers to the outer variable `output`, and `indent` refers to outer variables `n` and `output`. To make this work, the function `indent` must have access not only to its own frame (for variables `i` and `s`) but also to the frames of `show` (for variable `n`) and `prettyprint` (for variable `output`).

PROGRAM 6.3: Nested functions.

```
type tree = {key: string, left: tree, right: tree}

function prettyprint(tree: tree) : string =
 let
      var output := ""

      function write(s: string) =
          output := concat(output,s)

      function show(n:int, t: tree) =
         let function indent(s: string) =
                 (for i := 1 to n
                   do write(" ");
                  output := concat(output,s); write("\n"))
          in if t=nil then indent(".")
             else (indent(t.key);
                   show(n+1,t.left);
                   show(n+1,t.right))
      end

 in show(0,tree); output
end
```

There are several methods to accomplish this:

- Whenever a function *f* is called, it can be passed a pointer to the frame of the function statically enclosing *f* ; this pointer is the *static link*.
- A global array can be maintained, containing - in position *i* - a pointer to the frame of the most recently entered procedure whose *static nesting depth* is *i*. This array is called a *display*.
- When *g* calls *f*, each variable of *g* that is actually accessed by *f* (or by any function nested inside *f*) is passed to *f* as an extra argument. This is called *lambda lifting*.

We will describe in detail only the method of static links. Which method should be used in practice? See Exercise 6.6.

Whenever a function *f* is called, it is passed a pointer to the stack frame of the "current" (most recently entered) activation of the function *g* that *immediately encloses* *f* in the text of the program.

For example, in Program 6.3:

Line#

21 `prettyprint` calls `show`, passing `prettyprint's` own frame pointer as `show`'s static link.

10 `show` stores its static link (the address of `prettyprint's` frame) into its own frame.

15 `show` calls `indent`, passing its own frame pointer as `indent`'s static link.

17 `show` calls `show`, passing its own static link (not its own frame pointer) as the static link.

12 `indent` uses the value *n* from `show`'s frame. To do so, it fetches at an appropriate offset from `indent`'s static link (which points at the frame of `show`).

13 `indent` calls `write`. It must pass the frame pointer of `prettyprint` as the static link. To obtain this, it first fetches at an offset from its own static link (from `show`'s frame), the static link that had been passed to `show`.

14 `indent` uses the variable `output` from `prettyprint`'sframe.Todosoit starts with its own static link, then fetches `show`'s, then fetches `output`.[4]

So on each procedure call or variable access, a chain of zero or more fetches is required; the length of the chain is just the *difference* in static nesting depth between the two functions involved.

6.2 FRAMES IN THE MiniJava COMPILER

What sort of stack frames should the MiniJava compiler use? Here we face the fact that every target-machine architecture will have a different standard stack frame layout. If we want MiniJava functions to be able to call C functions, we should use the standard layout. But we don't want the specifics of any particular machine intruding on the implementation of the translation module of the MiniJava compiler.

Thus we must use *abstraction*. Just as the Symbol module provides a clean interface, and hides the internal representation of Symbol.Table from its clients, we must use an abstract representation for frames.

The frame interface will look something like this:

```
package Frame;
import Temp.Temp; import Temp.Label;

public abstract class Access { ... }
public abstract class AccessList {...head;...tail;... }
public abstract class Frame {
  abstract public Frame newFrame(Label name,
                               Util.BoolList formals);
           public Label name;
           public AccessList formals;
  abstract public Access allocLocal(boolean escape);
  /* ... other stuff, eventually ... *
}
```

The abstract class Frame is implemented by a module specific to the target machine. For example, if compiling to the MIPS architecture, there would be

```
package Mips;
class Frame extends Frame.Frame { ... }
```

In general, we may assume that the machine-independent parts of the compiler have access to this implementation of Frame; for example,

```
// in class Main.Main:
Frame.Frame frame = new Mips.Frame(...);
```

In this way the rest of the compiler may access frame without knowing the identity of the target machine (except an occurrence of the word Mips here and there).

The class Frame holds information about formal parameters and local variables allocated in this frame. To make a new frame for a function *f* with *k* formal parameters, call newFrame(*f*, *l*), where *l* is a list of *k* booleans: true for each parameter that escapes and false for each parameter that does not. (In MiniJava, no parameters ever escape.) The result will be a Frame object. For example, consider a three-argument function named *g* whose first argument escapes (needs to be kept in memory). Then

```
frame.newFrame(g,new BoolList(true,
                      new BoolList(false,
                       new BoolList(false, null))))
```

returns a new frame object.

The Access class describes formals and locals that may be in the frame or in registers. This is an *abstract data type*, so its implementation as a pair of subclasses is visible only inside the Frame module:

```
package Mips;
class InFrame extends Frame.Access {int offset; ... }
class InReg   extends Frame.Access {Temp temp; ... }
```

`InFrame`(X) indicates a memory location at offset X from the frame pointer; `InReg`(t_{84}) indicates that it will be held in "register" t_{84}. `Frame.Access` is an abstract data type, so outside of the module the `InFrame` and `InReg` constructors are not visible. Other modules manipulate accesses using interface functions to be described in the next chapter.

The `formals` field is a list of k "accesses" denoting the locations where the formal parameters will be kept at run time, as seen from inside the callee. Parameters may be seen differently by the caller and the callee. For example, if parameters are passed on the stack, the caller may put a parameter at offset 4 from the stack pointer, but the callee sees it at offset 4 from the frame pointer. Or the caller may put a parameter into register 6, but the callee may want to move it out of the way and always access it from register 13. On the Sparc architecture, with register windows, the caller puts a parameter into register `o1`, butthe `save` instruction shifts register windows so the callee sees this parameter in register `i1`.

Because this "shift of view" depends on the calling conventions of the target machine, it must be handled by the `Frame` module, starting with `newFrame`. For each formal parameter, `newFrame` must calculate two things:

- How the parameter will be seen from inside the function (in a register, or in a frame location).
- What instructions must be produced to implement the "view shift."

For example, a frame-resident parameter will be seen as "memory at offset X from the frame pointer", and the view shift will be implemented by copying the stack pointer to the frame pointer on entry to the procedure.

REPRESENTATION OF FRAME DESCRIPTIONS

The implementation module `Frame` is supposed to keep the representation of `Frame` objects secret from any clients of the `Frame` module. But really it's an object holding:

- the locations of all the formals,
- instructions required to implement the "view shift",
- the number of locals allocated so far,
- and the `label` at which the function's machine code is to begin (see page 131).

Table 6.4 shows the formals of the three-argument function g (see page 127) as `newFrame` would allocate them on three different architectures: the Pentium, MIPS, and Sparc. The first parameter escapes, so it needs to be `InFrame` on all three machines. The remaining parameters are `InFrame` on the Pentium, but `InReg` on the other machines.

Table 6.4: Formal parameters for $g(x_1, x_2, x_3)$ where x_1 escapes.

	Pentium	**MIPS**	**Sparc**
1	`InFrame`(8)	`InFrame`(0)	`InFrame`(68)
Formals 2	`InFrame`(12)	`InReg`(t_{157})	`InReg`(t_{157})
3	`InFrame`(16)	`InReg`(t_{158})	`InReg`(t_{158})
	$M[\mathrm{sp} + 0] \leftarrow \mathrm{fp}$	$\mathrm{sp} \leftarrow \mathrm{sp} - K$	`save %sp,-K,%sp`

Table 6.4: Formal parameters for $g(x_1, x_2, x_3)$ where x_1 escapes.

	Pentium	MIPS	Sparc
View Shift	fp ← sp	M[sp + K + 0] ← `r4`	M[fp + 68] ← `i0`
	sp ← sp − K	t_{157} ← `r5`	t_{157} ← `i1`
		t_{158} ← `r6`	t_{158} ← `i2`

The freshly created temporaries t_{157} and t_{158}, and the *move* instructions that copy `r4` and `r5` into them (or on the Sparc, `i1` and `i2`), may seem superfluous. Why shouldn't the body of *g* just access these formals directly from the registers in which they arrive? To see why not, consider

```
void m(int x, int y) { h(y,y); h(x,x); }
```

If *x* stays in "parameter register 1" throughout *m*, and *y* is passed to *h* in parameter register 1, then there is a problem.

The register allocator will eventually choose which machine register should hold t_{157}. If there is no interference of the type shown in function `m`, then (on the MIPS) the allocator will take care to choose register `r4` to hold t_{157} and `r5` to hold t_{158}. Then the *move* instructions will be unnecessary and will be deleted at that time.

See also pages 157 and 251 for more discussion of the view shift.

LOCAL VARIABLES

Some local variables are kept in the frame; others are kept in registers. To allocate a new local variable in a frame *f*, the translation phase calls

```
f.allocLocal(false)
```

This returns an `InFrame` access with an offset from the frame pointer. For example, to allocate two local variables on the Sparc, `allocLocal` would be called twice, returning successively `InFrame(-4)` and `InFrame(-8)`, which are standard Sparc frame-pointer offsets for local variables.

The boolean argument to `allocLocal` specifies whether the new variable escapes and needs to go in the frame; if it is false, then the variable can be allocated in a register. Thus, `f.allocLocal(false)` might create `InReg`(t_{481}).

For MiniJava, that no variables escape. This is because in MiniJava:

- there is no nesting of classes and methods;
- it is not possible to take the address of a variable;
- integers and booleans are passed by value; and
- objects, including integer arrays, can be represented as pointers that are also passedbyvalue.

The calls to `allocLocal` need not come immediately after the frame is created. In many languages, there may be variable-declaration blocks nested inside the body of a function. For example,

```
void f()
{int v=6;
 print(v);
 {int v=7;
  print(v);
 }
 print(v);
 {int v=8;
  print(v);
 }
 print(v);
}
```

In this case there are three different variables *v*. The program will print the sequence 6 7 6 8 6. As each variable declaration is encountered in processing the program, we will allocate a temporary or new space in the frame, associated with the name *v*. As each `end` (or closing brace) is encountered, the association with *v* will be forgotten - but the space is still reserved in the frame. Thus, there will be a distinct temporary or frame slot for every variable declared within the entire function.

The register allocator will use as few registers as possible to represent the temporaries. In this example, the second and third *v* variables (initialized to 7 and 8) could be held in the same register. A clever compiler might also optimize the size of the frame by noticing when two frame-resident variables could be allocated to the same slot.

TEMPORARIES AND LABELS

The compiler's translation phase will want to choose registers for parameters and local variables, and choose machine-code addresses for procedure bodies. But it is too early to determine exactly which registers are available, or exactly where a procedure body will be located. We use the word *temporary* to mean a value that is temporarily held in a register, and the word *label* to mean some machine-language location whose exact address is yet to be determined - just like a label in assembly language.

`Temps` are abstract names for local variables; `labels` are abstract names for static memory addresses. The `Temp` module manages these two distinct sets of names.

```
package Temp;
public class Temp {
  public String toString();
  public Temp();
}
public class Label {
  public String toString();
  public Label();
  public Label(String s);
  public Label(Symbol s);
}
public class TempList  {...}
public class LabelList {...}
```

new Temp.Temp() returns a new temporary from an infinite set of temps. new Temp.Label() returns a new label from an infinite set of labels. And new Temp.Label(*string*)) returns a new label whose assembly-language name is *string*.

When processing the declaration m(...), a label for the address of m's machine code can be produced by new Temp.Label(). It's tempting to call new Temp.Label("m") instead - the assembly-language program will be easier to debug if it uses the label m instead of L213 - but unfortunately there could be two different methods named m in different classes. A better idea is to call new Temp.Label("C"+"$"+"m"), where C is the name of the class in which the method m occurs.

MANAGING STATIC LINKS

The Frame module should be independent of the specific source language being compiled. Many source languages, including MiniJava, do not have nested function declarations; thus, Frame should not know anything about static links. Instead, this is the responsibility of the translation phase. The translation phase would know that each frame contains a static link. The static link would be passed to a function in a register and stored into the frame. Since the static link behaves so much like a formal parameter, we can treat it as one (as much as possible).

[1] Before about 1960, they were passed not on the stack but in statically allocated blocks of memory, which precluded the use of recursive functions.

[2] However, with register allocation across function boundaries, local variables accessed by inner functions can sometimes go in registers, as long as the inner function knows where to look.

[3] However, some compilers spread out a large value into several registers for efficiency.

[4] This program would be cleaner if show called write here instead of manipulating output directly, but it would not be as instructive.

PROGRAM FRAMES

If you are compiling for the Sparc, implement the Sparc package containing Sparc/SparcFrame.java. If compiling for the MIPS, implement the Mips package, and so on.

If you are working on a RISC machine (such as MIPS or Sparc) that passes the first *k* parameters in registers and the rest in memory, keep things simple by handling *only* the case where there are *k* or fewer parameters.

Supporting files available in $MINIJAVA/chap6 include:

Temp/* The module supporting temporaries and labels.

Util/BoolList.java The class for lists of booleans.

Optional: Handle functions with more than *k* formal parameters.

FURTHER READING

The use of a single contiguous stack to hold variables and return addresses dates from Lisp [McCarthy 1960] and Algol [Naur et al. 1963]. Block structure (the nesting of functions) and the use of static links are also from Algol.

Computers and compilers of the 1960s and '70s kept most program variables in memory, so that there was less need to worry about which variables escaped (needed addresses). The VAX, built in 1978, had a procedure-call instruction that assumed all arguments were pushed on the stack, and itself pushed program counter, frame pointer, argument pointer, argument count, and callee-save register mask on the stack [Leonard 1987].

With the RISC revolution [Patterson 1985] came the idea that procedure calling can be done with much less memory traffic. Local variables should be kept in registers by default; storing and fetching should be done *as needed*, driven by "spilling" in the register allocator [Chaitin 1982].

Most procedures don't have more than five arguments and five local variables [Tanenbaum 1978]. To take advantage of this, Chow et al. [1986] and Hopkins [1986] designed calling conventions optimized for the common case: The first four arguments are passed in registers, with the (rare) extra arguments passed in memory; compilers use both caller- and callee-save registers for local variables; leaf procedures don't even need stack frames of their own if they can operate within the caller-save registers; and even the return address need not always be pushed on the stack.

EXERCISES

- **6.1** Using the C compiler of your choice (or a compiler for another language), compile some small test functions into assembly language. On Unix this is usually done by `cc -S`. Turn on all possible compiler optimizations. Then evaluate the compiled programs by these criteria:

    a. a. Are local variables kept in registers?

    b. b. If local variable *b* is live across more than one procedure call, is it kept in a callee-save register? Explain how doing this would speed up the following program:

    c. `int f(int a) {int b; b=a+1; g(); h(b); return b+2;}`

    d. c. If local variable *x* is never live across a procedure call, is it properly kept in a caller-save register? Explain how doing this would speed up the following program:

    e. `void h(int y) {int x; x=y+1; f(y); f(2);}`

- **6.2** If you have a C compiler that passes parameters in registers, make it generate assembly language for this function:
- `extern void h(int, int);`
- `void m(int x, int y) {h(y,y); h(x,x);}`

  Clearly, if arguments to $m(x, y)$ arrive in registers r_{arg1} and r_{arg2}, and arguments to h must be passed in r_{arg1} and r_{arg2}, then x cannot stay in r_{arg1} during the marshalling of arguments to $h(y, y)$. Explain when and how your C compiler moves x out of the r_{arg1} register so as to call $h(y, y)$.

- **6.3** For each of the variables *a, b, c, d, e* in this C program, say whether the variable should be kept in memory or a register, and why.
- `int f(int a, int b)`

```
  { int c[3], d, e;
    d=a+1;
    e=g(c, &b);
    return e+c[1]+b;
  }
```

- ***6.4** How much memory should this program use?

```
  int f(int i) {int j,k; j=i*i; k=i?f(i-1):0; return k+j;}
  void main() {f(100000);}
```

   a. Imagine a compiler that passes parameters in registers, wastes no space providing "backup storage" for parameters passed in registers, does not use static links, and in general makes stack frames as small as possible. How big should each stack frame for *f* be, in words?
   b. What is the maximum memory use of this program, with such a compiler?
   c. Using your favorite C compiler, compile this program to assembly language and report the size of *f* 's stack frame.
   d. Calculate the total memory use of this program with the real C compiler.
   e. Quantitatively and comprehensively explain the discrepancy between (a) and (c).
   f. Comment on the likelihood that the designers of this C compiler considered deeply recursive functions important in real programs.

- ***6.5** Instead of (or in addition to) using static links, there are other ways of getting access to nonlocal variables. One way is just to leave the variable in a register!

```
  function f() : int =
    let var a := 5
        function g() : int =
           (a+1)
     in g()+g()
    end
```

If *a* is left in register r_7 (for example) while *g* is called, then *g* can just access it from there.

What properties must a local variable, the function in which it is defined, and the functions in which it is used, have for this trick to work?

- ***6.6** A display is a data structure that may be used as an alternative to static links for maintaining access to nonlocal variables. It is an array of frame pointers, indexed by static nesting depth. Element *Di* of the display always points to the most recently called function whose static nesting depth is *i*.

The bookkeeping performed by a function *f*, whose static nesting depth is *i*, looks like:

```
Copy Di to save locationin stack frame
Copy frame pointer to Di
   ... body of f ...
Copy save locationback to Di
```

In Program 6.3, function `prettyprint` is at depth 1, `write` and `show` are at depth 2, and so on.

   a. Show the sequence of machine instructions required to fetch the variable `output` into a register at line 14 of Program 6.3, using static links.
   b. Show the machine instructions required if a display were used instead.

c. When variable *x* is declared at depth d_1 and accessed at depth d_2, how many instructions does the static-link method require to fetch *x*?
d. How many does the display method require?
e. How many instructions does static-link maintenance require for a procedure entry and exit (combined)?
f. How many instructions does display maintenance require for procedure entry and exit?

Should we use displays instead of static links? Perhaps; but the issue is more complicated. For languages such as Pascal with block structure but no function variables, displays work well.

But the full expressive power of block structure is obtained when functions can be returned as results of other functions, as in Scheme and ML. For such languages, there are more issues to consider than just variable-access time and procedure entry-exit cost: there is closure-building cost, and the problem of avoiding useless data kept live in closures. Chapter 15 explains some of the issues.

Chapter 7: Translation to Intermediate Code

trans-late: to turn into one's own or another language

Webster's Dictionary

OVERVIEW

The semantic analysis phase of a compiler must translate abstract syntax into abstract machine code. It can do this after type-checking, or at the same time.

Though it is possible to translate directly to real machine code, this hinders portability and modularity. Suppose we want compilers for *N* different source languages, targeted to *M* different machines. In principle this is *N* · *M* compilers (Figure 7.1a), a large implementation task.

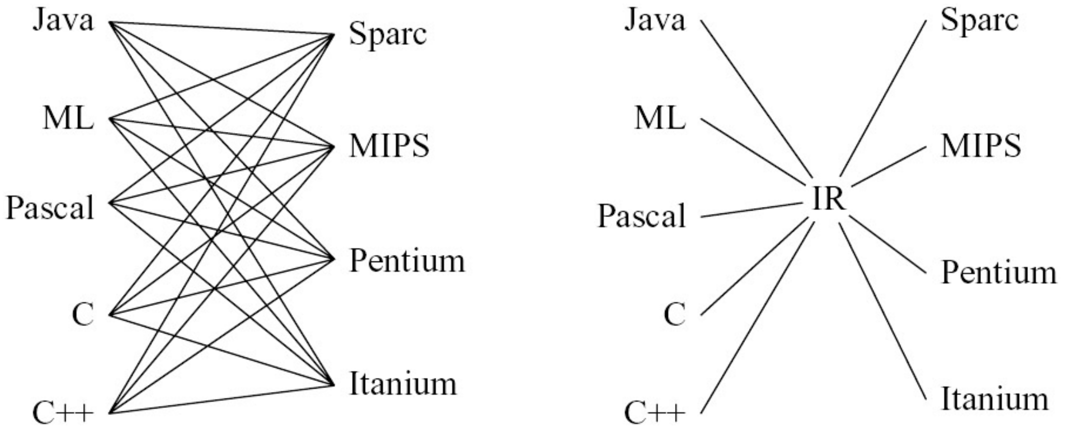


Figure 7.1: Compilers for five languages and four target machines: (a) without an IR, (b) with an IR.

An *intermediate representation* (IR) is a kind of abstract machine language that can express the target-machine operations without committing to too much machine-specific detail. But it is also independent of the details of the source language. The *front end* of the compiler does lexical analysis, parsing, semantic analysis, and translation to intermediate representation. The *back end* does optimization of the intermediate representation and translation to machine language.

A portable compiler translates the source language into IR and then translates the IR into machine language, as illustrated in Figure 7.1b. Now only *N* front ends and *M* back ends are required. Such an implementation task is more reasonable.

Even when only one front end and one back end are being built, a good IR can modularize the task, so that the front end is not complicated with machine-specific details, and the back end is not bothered with information specific to one source language. Many different kinds of IR are used in compilers; for this compiler we have chosen simple expression trees.

7.1 INTERMEDIATE REPRESENTATION TREES

The intermediate representation tree language is defined by the package `Tree`, containing abstract classes `Stm` and `Exp` and their subclasses, as shown in Figure 7.2.

```
package Tree;

abstract class Exp
CONST(int value)
NAME(Label label)
TEMP(Temp.Temp temp)
BINOP(int binop, Exp left, Exp right)
MEM(Exp exp)
CALL(Exp func, ExpList args)
ESEQ(Stm stm, Exp exp)

abstract class Stm
MOVE(Exp dst, Exp src)
EXP(Exp exp)
JUMP(Exp exp, Temp.LabelList targets)
CJUMP(int relop, Exp left, Exp right, Label iftrue, Label iffalse)
SEQ(Stm left, Stm right)
LABEL(Label label)

other classes:
ExpList(Exp head, ExpList tail)
StmList(Stm head, StmList tail)

other constants:
final static int BINOP.PLUS, BINOP.MINUS, BINOP.MUL, BINOP.DIV, BINOP.AND,
    BINOP.OR, BINOP.LSHIFT, BINOP.RSHIFT, BINOP.ARSHIFT, BINOP.XOR;

final static int CJUMP.EQ, CJUMP.NE, CJUMP.LT, CJUMP.GT, CJUMP.LE,
        CJUMP.GE, CJUMP.ULT, CJUMP.ULE, CJUMP.UGT, CJUMP.UGE;
```

Figure 7.2: Intermediate representation trees.

A good intermediate representation has several qualities:

- It must be convenient for the semantic analysis phase to produce. ∊ It must be convenient to translate into real machine language, for all the desired target machines.
- Each construct must have a clear and simple meaning, so that optimizing transformations that rewrite the intermediate representation can easily be specified and implemented.

Individual pieces of abstract syntax can be complicated things, such as array subscripts, procedure calls, and so on. And individual "real machine" instructions can also have a complicated effect (though this is less true of modern RISC machines than of earlier architectures). Unfortunately, it is not always the case that complex pieces of the abstract syntax correspond exactly to the complex instructions that a machine can execute.

Therefore, the intermediate representation should have individual components that describe only extremely simple things: a single fetch, store, add, move, or jump. Then any "chunky" piece of abstract syntax can be translated into just the right set of abstract machine instructions; and groups of abstract machine instructions can be clumped together (perhaps in quite different clumps) to form "real" machine instructions.

Here is a description of the meaning of each tree operator. First, the expressions (`Exp`), which stand for the computation of some value (possibly with side effects):

- CONST(*i*) The integer constant *i*.
- NAME(*n*) The symbolic constant *n* (corresponding to an assembly language label).
- TEMP(*t*) Temporary *t*. A temporary in the abstract machine is similar to a register in a real machine. However, the abstract machine has an infinite number of temporaries.
- BINOP(o, e_1, e_2) The application of binary operator *o* to operands e_1, e_2. Subexpression e_1 is evaluated before e_2. The integer arithmetic operators are PLUS, MINUS, MUL, DIV; the integer bitwise logical operators are AND, OR, XOR; the integer logical shift operators are LSHIFT, RSHIFT; the integer arithmetic right-shift is ARSHIFT. The MiniJava language has only one logical operator, but the intermediate language is meant to be independent of any source language; also, the logical operators might be used in implementing other features of MiniJava.
- MEM(*e*) The contents of *wordSize* bytes of memory starting at address *e* (where *wordSize* is defined in the `Frame` module). Note that when MEM is used as the left child of a MOVE, it means "store", but anywhere else it means "fetch."
- CALL(*f, l*) A procedure call: the application of function *f* to argument list *l*. The subexpression *f* is evaluated before the arguments which are evaluated left to right.
- ESEQ(*s, e*) The statement *s* is evaluated for side effects, then *e* is evaluated for a result.

The statements (`stm`) of the tree language perform side effects and control flow:

- MOVE(TEMP *t*, *e*) Evaluate *e* and move it into temporary *t*.
- MOVE(MEM(e_1) e_2) Evaluate e_1, yielding address *a*. The nevaluate e_2, and store the result into *wordSize* bytes of memory starting at *a*.
- EXP(*e*) Evaluate *e* and discard the result.
- JUMP(*e, labs*) Transfer control (jump) to address *e*. The destination *e* may be a literal label, as in NAME(*lab*), or it may be an address calculated by any other kind of expression. For example, a C-language `switch(i)` statement may be implemented by doing arithmetic on *i*. The list of labels `labs` specifies all the possible locations that the expression *e* can evaluate to; this is necessary for dataflow analysis later. The common case of jumping to a known label *l* is written as JUMP(NAME *l*, `new LabelList(`*l*, `null`)), but the JUMP class has an extra constructor so that this can be abbreviated as JUMP(*l*).
- CJUMP(o, e_1, e_2, t, f) Evaluate e_1, e_2 in that order, yielding values *a, b*. Then compare *a, b* using the relational operator *o*. If the result is `true`, jump to *t*; otherwise jump to *f* . The relational operators are EQ and NE for integer equality and nonequality (signed or unsigned); signed integer inequalities LT, GT, LE, GE; and unsigned integer inequalities ULT, ULE, UGT, UGE.
- SEQ(s_1, s_2) The statement s_1 followed by s_2.
- LABEL(*n*) Define the constant value of name *n* to be the current machine code address. This is like a label definition in assembly language. The value NAME(*n*) may be the target of jumps, calls, etc.

It is almost possible to give a formal semantics to the `Tree` language. However, there is no provision in this language for procedure and function definitions - we can specify only the body of each function. The procedure entry and exit sequences will be added later as special "glue" that is different for each target machine.

7.2 TRANSLATION INTO TREES

Translation of abstract syntax expressions into intermediate trees is reasonably straightforward; but there are many cases to handle. We will cover the translation of various language constructs, including many from MiniJava.

KINDS OF EXPRESSIONS

The MiniJava grammar has clearly distinguished statements and expressions. However, in languages such as C, the distinction is blurred; for example, an assignment in C can be used as an expression. When translating such languages, we will have to ask the following question. What should the representation of an abstract-syntax expression be in the `Tree` language? At first it seems obvious that it should be `Tree.Exp`. However, this is true only for certain kinds of expressions, the ones that compute a value. Expressions that return no value are more naturally represented by `Tree.Stm`. And expressions with boolean values, such as $a > b$, might best be represented as a conditional jump - a combination of `Tree.Stm` and a pair of destinations represented by `Temp.Labels`.

It is better instead to ask, "how might the expression be used?" Then we can make the right kind of *methods* for an object-oriented interface to expressions. Both for MiniJava and other languages, we end up with `Translate.Exp`, not the same class as `Tree.Exp`, having three methods:

```
package Translate;
public abstract class Exp {
   abstract Tree.Exp unEx();
   abstract Tree.Stm unNx();
   abstract Tree.Stm unCx(Temp.Label t, Temp.Label f);
}
```

- `Ex` stands for an "expression", represented as a `Tree.Exp`.
- `Nx` stands for "no result", represented as a `Tree` statement.
- `Cx` stands for "conditional", represented as a function from label-pair to statement. If you pass it a true destination and a false destination, it will make a statement that evaluates some conditionals and then jumps to one of the destinations (the statement will never "fall through").

For example, the MiniJava statement

```
if (a<b && c<d) {
        // true block
}
else {
      // false block
}
```

might translate to a `Translate.Exp` whose `unCx` method is roughly like

```
Tree.Stm unCx(Label t, Label f) {
  Label z = new Label();
  return new SEQ(new CJUMP(CJUMP.LT,a,b,z,f),
                 new SEQ(new LABEL(z),
                 new CJUMP(CJUMP.LT,c,d,t,f)));
}
```

The abstract class `Translate.Exp` can be instantiated by several subclasses: `Ex` for an ordinary expression that yields a single value, `Nx` for an expression that yields no value, and `Cx` for a "conditional" expression that jumps to either *t* or *f* :

```
class Ex extends Exp {
   Tree.Exp exp;
   Ex(Tree.Exp e) {exp=e;}
   Tree.Exp unEx() {return exp;}
   Tree.Stm unNx() { ... ?... }
   Tree.Stm unCx(Label t, Label f) { ... ?... }
}
class Nx extends Exp {
   Tree.Stm stm;
   Nx(Tree.Stm s) {stm=s;}
   Tree.Exp unEx() { ... ?... }
   Tree.Stm unNx() {return stm;}
   Tree.Stm unCx(Label t, Label f) { ... ?... }
}
```

But what does the `unNx` method of an `Ex` do? We have a simple `Tree.Exp` that yields a value, and we are asked to produce a `Tree.Stm` that produces no value. There is a conversion operator `Tree.EXP`, and `unNx` must apply it:

```
class Ex extends Exp {
  Tree.Exp exp;
  ⋮
  Tree.Stm unNx() {return new Tree.EXP(exp); }
  ⋮
}
```

Each kind of `Translate.Exp` class must have similar conversion methods. For example, the MiniJava statement

```
flag = (a<b && c<d);
```

requires the `unEx` method of a `Cx` object so that a 1 (for true) or 0 (for false) can be stored into `flag`.

Program 7.3 shows the class `Cx`. The `unEx` method is of particular interest. To convert a "conditional" into a "value expression", we invent a new temporary *r* and new labels *t* and *f*. Then we make a `Tree.Stm` that moves the value 1 into *r*, and a conditional jump `unCx`(*t*, *f*) that implements the conditional. If the condition is false, then 0 is moved into *r*; if it is true, then execution proceeds at *t* and the second move is skipped. The result of the whole thing is just the temporary *r* containing zero or one.

PROGRAM 7.3: The `Cx` class.

```
abstract class Cx extends Exp {
  Tree.Exp unEx() {
    Temp r = new Temp();
    Label t = new Label();
    Label f = new Label();
    return new Tree.ESEQ(
           new Tree.SEQ(new Tree.MOVE(new Tree.TEMP(r),
                                      new Tree.CONST(1)),
                     new Tree.SEQ(unCx(t,f),
                      new Tree.SEQ(new Tree.LABEL(f),
                       new Tree.SEQ(new Tree.MOVE(new Tree.TEMP(r),
                                                  new Tree.CONST(0)),
                        new Tree.LABEL(t))))),
                    new Tree.TEMP(r));
}

abstract Tree.Stm unCx(Label t, Label f);

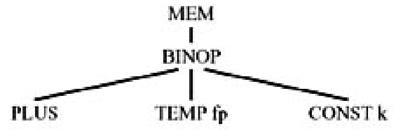
Tree.Stm unNx() { ... }
}
```

The unCx method is still abstract: We will discuss this later, with the translation of comparison operators. But the unEx and unNx methods can still be implemented in terms of the unCx method. We have shown unEx; we will leave unNx (which is simpler) as an exercise.

The unCx method of class Ex is left as an exercise. It's helpful to have unCx treat the cases of CONST 0and CONST 1 specially, since they have particularly simple and efficient translations. Class Nx's unEx and unCx methods need not be implemented, since these cases should never occur in compiling a well-typed MiniJava program.

SIMPLE VARIABLES

For a simple variable *v* declared in the current procedure's stack frame, we translate it as



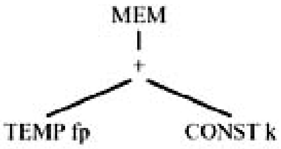
MEM(BINOP(PLUS, TEMP fp, CONST *k*))

where *k* is the offset of *v* within the frame and TEMP fp is the frame-pointer register. For the MiniJava compiler we make the simplifying assumption that all variables are the same size - the natural word size of the machine.

The Frame class holds all machine-dependent definitions; here we add to it a frame-pointer register FP and a constant whose value is the machine's natural word size:

```
package Frame;
public class Frame {
   ⋮
  abstract public Temp FP();
  abstract public int wordSize();
}
public abstract class Access {
    public abstract Tree.Exp exp(Tree.Exp framePtr);
}
```

In this and later chapters, we will abbreviate BINOP(PLUS, e_1, e_2) as + (e_1, e_2), so the tree above would be shown as



+(TEMP fp, CONST *k*)

The `exp` method of `Frame.Access` is used by `Translate` to turn a `Frame.Access` into the `Tree` expression. The `Tree.Exp` argument is the address of the stack frame that the `Access` lives in. Thus, for an access *a* such as `InFrame`(*k*), we have

```
a.exp(new TEMP(frame.FP())) =
             MEM(BINOP(PLUS,TEMP(frame.FP()),CONST(k)))
```

If *a* is a register access such as `InReg`(t_{832}), then the frame-address argument to *a*`.exp()` will be discarded, and the result will be simply TEMP t_{832}.

An *l*-value such as *v* or *a*[*i*] or *p:next* can appear either on the left side or the right side of an assignment statement - *l* stands for *left*, to distinguish from *r*-values, which can appear only on the right side of an assignment. Fortunately, both MEM and TEMP nodes can appear on the left of a MOVE node.

ARRAY VARIABLES

For the rest of this chapter we will not specify all the interface functions of `Translate`, as we have done for `simpleVar`. But the rule of thumb just given applies to all of them: There should be a `Translate` function to handle array subscripts, one for record fields, one for each kind of expression, and so on.

Different programming languages treat array-valued variables differently.

In Pascal, an array variable stands for the contents of the array - in this case all 12 integers. The Pascal program

```
var a,b : array[1..12] of integer
begin

      a:=b
end;
```

copies the contents of array *a* into array *b*.

In C, there is no such thing as an array variable. There are pointer variables; arrays are like "pointer constants." Thus, this is illegal:

```
{int a[12], b[12];
 a=b;
}
```

but this is quite legal:

```
{int a[12], *b;
 b=a;
}
```

The statement `b=a` does not copy the elements of *a*; instead, it means that *b* now points to the beginning of the array *a*.

In MiniJava (as in Java and ML), array variables behave like pointers. MiniJava has no named array constants as in C, however. Instead, new array values are created (and initialized) by the construct `new int[n];` where *n* is the number of elements, and 0 is the initial value of each element. In the program

```
int [] a;
a = new int[12];
b = new int[12];
a = b;
```

the array variable *a* ends up pointing to the same 12 zeros as the variable *b*; the original 12 zeros allocated for *a* are discarded.

MiniJava objects are also pointers. Object assignment, like array assignment, is pointer assignment and does not copy all the fields (see below). This is also true of other object-oriented and functional programming languages, which try to blur the syntactic distinction between pointers and objects. In C or Pascal, however, a record value is "big" and record assignment means copying all the fields.

STRUCTURED *L*-VALUES

An *l*-value is the result of an expression that can occur on the *left* of an assignment statement, such as `x` or `p.y` or `a[i+2]`. An *r*-value is one that can only appear on the *right* of an assignment, such as `a+3` or `f(x)`. That is, an *l*-value denotes a *location* that can be assigned to, and an *r*-value does not.

Of course, an *l*-value can occur on the right of an assignment statement; in this case the *contents* of the location are implicitly taken.

We say that an integer or pointer value is a "scalar", since it has only one component. Such a value occupies just one word of memory and can fit in a register. All the variables and *l*-values in MiniJava are scalar. Even a MiniJava array or object variable is really a pointer (a kind of scalar); the *Java Language Reference Manual* may not say so explicitly, because it is talking about Java semantics instead of Java implementation.

In C or Pascal there are structured *l*-values - structs in C, arrays and records in Pascal - that are not scalar. To implement a language with "large" variables such as the arrays and records in C or Pascal requires a bit of extra work. In a C compiler, the `access` type would require information about the size of the variable. Then, the MEM operator of the `TREE` intermediate language would need to be extended with a notion of size:

```
package Tree;
abstract class Exp
MEM(Exp exp, int size)
```

The translation of a local variable into an IR tree would look like

MEM(+(TEMP `fp`, CONST k_n), *S*)

where the *S* indicates the size of the object to be fetched or stored (depending on whether this tree appears on the left or right of a MOVE).

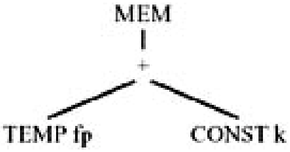
Leaving out the size on MEM nodes makes the MiniJava compiler easier to implement, but limits the generality of its intermediate representation.

SUBSCRIPTING AND FIELD SELECTION

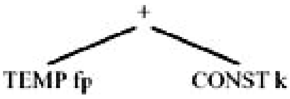
To subscript an array in Pascal or C (to compute *a*[*i*]), just calculate the address of the *i*th element of *a*: $(i - l) \times s + a$, where *l* is the lower bound of the index range, *s* is the size (in bytes) of each array element, and *a* is the base address of the array elements. If *a* is global, with a compile-time constant address, then the subtraction $a - s \times l$ can be done at compile time.

Similarly, to select field *f* of a record *l*-value *a* (to calculate *a*. *f*), simply add the constant field offset of *f* to the address *a*.

An array variable *a* is an *l*-value; so is an array subscript expression *a*[*i*], even if *i* is not an *l*-value. To calculate the *l*-value *a*[*i*] from *a*, we do arithmetic on the address of *a*. Thus, in a Pascal compiler, the translation of an *l*-value (particularly a structured *l*-value) should *not* be something like



but should instead be the `Tree` expression representing the base address of the array:

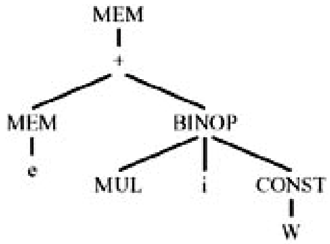


What could happen to this *l*-value?

- A particular element might be subscripted, yielding a (smaller) *l*-value. A "+" node would add the index times the element size to the *l*-value for the base of the array.
- The *l*-value (representing the entire array) might be used in a context where an *r*-value is required (e.g., passed as a by-value parameter, or assigned to another array variable). Then the *l*-value is *coerced* into an *r*-value by applying the MEM operator to it.

In the MiniJava language, there are no structured, or "large", *l*-values. This is because all object and array values are really pointers to object and array structures. The "base address" of the array is really the contents of a pointer variable, so MEM is required to fetch this base address.

Thus, if *a* is a memory-resident array variable represented as MEM(*e*), then the contents of address *e* will be a one-word pointer value *p*. The contents of addresses *p*, $p + W$, $p + 2W$, … (where *W* is the word size) will be the elements of the array (all elements are one word long). Thus, *a*[*i*] is just



MEM(+(MEM(*e*), BINOP(MUL, *i*, CONST *W*)))

l-values and MEM nodes. Technically, an *l*-value (or *assignable variable*) should be represented as an *address* (without the top MEM node in the diagram above). Converting an *l*-value to an *r*-value (when it is used in an expression) means *fetching* from that address; assigning to an *l*-value means *storing* to that address. We are attaching the MEM node to the *l*-value before knowing whether it is to be fetched or stored; this works only because in the `Tree` intermediate representation, MEM means both *store* (when used as the left child of a MOVE)and *fetch* (when used elsewhere).

A SERMON ON SAFETY

Life is too short to spend time chasing down irreproducible bugs, and money is too valuable to waste on the purchase of flaky software. When a program has a bug, it should detect that fact as soon as possible and announce that fact (or take corrective action) before the bug causes any harm.

Some bugs are very subtle. But it should not take a genius to detect an outof-bounds array subscript: If the array bounds are *L* ..*H*, and the subscript is *i*, then *i* < *L* or *i* > *H* is an array bounds error. Furthermore, computers are well-equipped with hardware able to compute the condition *i* > *H*. For several decades now, we have known that compilers can automatically emit the code to test this condition. There is no excuse for a compiler that is unable to emit code for checking array bounds. Optimizing compilers can often *safely* remove the checks by compile-time analysis; see Section 18.4.

One might say, by way of excuse, "but the language in which I program has the kind of address arithmetic that makes it impossible to know the bounds of an array." Yes, and the man who shot his mother and father threw himself upon the mercy of the court because he was an orphan.

In some rare circumstances, a portion of a program demands blinding speed, and the timing budget does not allow for bounds checking. In such a case, it would be best if the optimizing compiler could analyze the subscript expressions and prove that the index will always be within bounds, so that an explicit bounds check is not necessary. If that is not possible, perhaps it is reasonable in these rare cases to allow the programmer to explicitly specify an unchecked subscript operation. But this does not excuse the compiler from checking all the other subscript expressions in the program.

Needless to say, the compiler should check pointers for `nil` before dereferencing them, too.[1]

ARITHMETIC

Integer arithmetic is easy to translate: Each arithmetic operator corresponds to a `Tree` operator.

The `Tree` language has no unary arithmetic operators. Unary negation of integers can be implemented as subtraction from zero; unary complement can be implemented as XOR with all ones.

Unary floating-point negation cannot be implemented as subtraction from zero, because many floating-point representations allow a *negative zero*. The negation of negative zero is positive zero, and vice versa. Thus, the `Tree` language does not support unary negation very well.

Fortunately, the MiniJava language doesn't support floating-point numbers; but in a real compiler, a new operator would have to be added for floating negation.

CONDITIONALS

The result of a comparison operator will be a `Cx` expression: a statement *s* that will jump to any true-destination and false-destination you specify.

Making "simple" `Cx` expressions from comparison operators is easy with the CJUMP operator. However, the whole point of the `Cx` representation is that conditional expressions can be combined easily with the MiniJava operator

`&&`. Therefore, an expression such as `x<5` will be translated as `Cx`(s_1), where

$$s_1(t, f) = \mathrm{CJUMP}(\mathrm{LT}, x, \mathrm{CONST}(5), t, f)$$

for any labels *t* and *f*.

To do this, we extend the `Cx` class to make a subclass `RelCx` that has private fields to hold the left and right expressions (in this case *x* and 5) and the comparison operator (in this case `Tree.CJUMP.LT`). Then we override the `unCx` method to generate the CJUMP from these data. It is not necessary to make `unEx` and `unNx` methods, since these will be inherited from the parent `Cx` class.

The most straightforward thing to do with an `if`-expression

if e_1 **then** e_2 **else** e_3

is to treat e_1 as a `Cx` expression, and e_2 and e_3 as `Ex` expressions. That is, use the `unCx` method of e_1 and the `unEx` of e_2 and e_3. Make two labels *t* and *f* to which the conditional will branch. Allocate a temporary *r*, and after label *t*, move e_2 to *r*; after label *f*, move e_3 to *r*. Both branches should finish by jumping to a newly created "join" label.

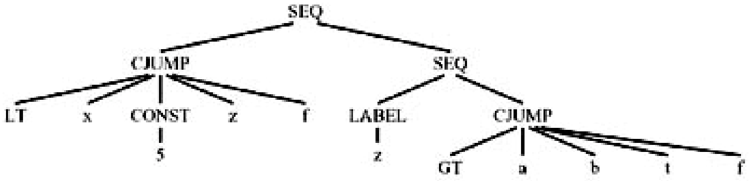
This will produce perfectly correct results. However, the translated code may not be very efficient at all. If e_2 and e_3 are both "statements" (expressions that return no value), then their representation is likely to be `Nx`, not `Ex`. Applying `unEx` to them will work - a coercion will automatically be applied - but it might be better to recognize this case specially.

Even worse, if e_2 or e_3 is a `Cx` expression, then applying the `unEx` coercion to it will yield a horrible tangle of jumps and labels. It is much better to recognize this case specially.

For example, consider

if $x < 5$ **then** $a > b$ **else** 0

As shown above, $x < 5$ translates into `Cx`(s_1); similarly, $a > b$ will be translated as `Cx`(s_2) for some s_2. The whole `if`-statement should come out approximately as



SEQ(s_1(z, f), SEQ(LABEL z, s_2(t, f)))

for some new label z.

Therefore, the translation of an **if** requires a new subclass of `Exp`:

```
class IfThenElseExp extends Exp {
   Exp cond, a, b;
   Label t = new Label();
   Label f = new Label();
   Label join = new Label();
   IfThenElseExp(Exp cc, Exp aa, Exp bb) {
            cond=cc; a=aa; b=bb;
   }
   Tree.Stm unCx(Label tt, Label ff) { ... }
   Tree.Exp unEx() { ... }
   Tree.Stm unNx() { ... }
}
```

The labels `t` and `f` indicate the beginning of the then-clause and elseclause, respectively. The labels `tt` and `ff` are quite different: These are the places to which conditions inside the then-clause (or else-clause) must jump, depending on the truth of those subexpressions.

STRINGS

A string literal is typically implemented as the constant address of a segment of memory initialized to the proper characters. In assembly language, a label is used to refer to this address from the middle of some sequence of instructions. At some other place in the assembly-language program, the *definition* of that label appears, followed by the assembly-language pseudo-instruction to reserve and initialize a block of memory to the appropriate characters.

For each string literal `lit`, a translator must make a new label `lab`, and return the tree `Tree.NAME(lab)`. It should also put the assembly-language fragment `frame.string(lab,lit)` onto a global list of such fragments to be handed to the code emitter. "Fragments" are discussed further on page 157.

All string operations are performed in functions provided by the runtime system; these functions heap-allocate space for their results, and return pointers. Thus, the compiler (almost) doesn't need to know what the representation is, as long as it knows that each string pointer is exactly one word long. We say "almost" because string literals must be represented.

In Pascal, strings are fixed-length arrays of characters; literals are padded with blanks to make them fit. This is not very useful. In C, strings are pointers to variable-length, zero-terminated sequences. This is much more useful, though a string containing a zero byte cannot be represented.

RECORD AND ARRAY CREATION

Imagine a language construct $\{e_1, e_2, \ldots, e_n\}$ which creates an n-element record initialized to the values of expressions e_i. This is like an object constructor that initializes all the instance variables of the object. Such a record may outlive the procedure activation that creates it, so it cannot be allocated on the stack. Instead, it must be allocated on the *heap*. If there is no provision for freeing records (or strings), industrial-strength systems should have a *garbage collector* to reclaim unreachable records (see Chapter 13).

The simplest way to create a record is to call an external memory-allocation function that returns a pointer to an n-word area into a new temporary r. Then a series of MOVE trees can initialize offsets 0, $1W; 2W, \ldots, (n-1)W$ from r with the translations of expressions e_i. Finally, the result of the whole expression is TEMP(r), as shown in Figure 7.4.

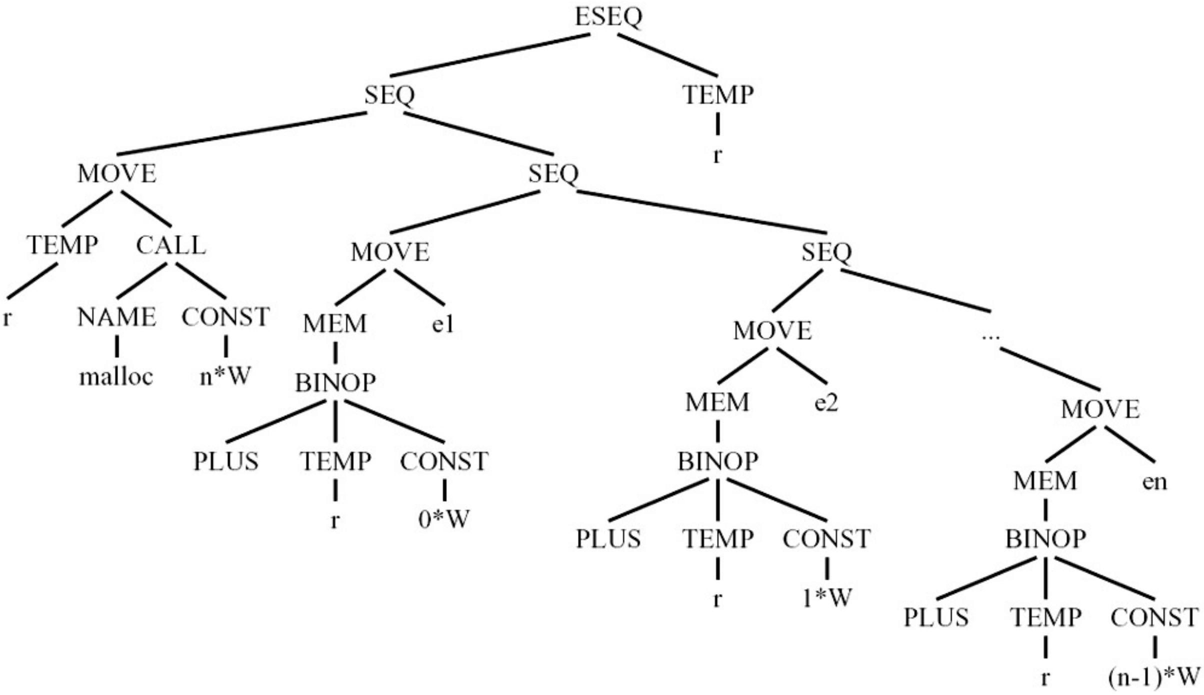


Figure 7.4: Object initialization.

In an industrial compiler, calling `malloc` (or its equivalent) on every record creation might be too slow; see Section 13.7.

Array creation is very much like record creation, except that all the fields are initialized to the same value. The external `initArray` function can take the array length and the initializing value as arguments, see later.

In MiniJava we can create an array of integers by the construct

```
new int [exp]
```

where `exp` is an expression that evaluates to an integer. This will create an integer array of a length determined by the value of `exp` and with each value initialized to zero.

To translate array creation, the compiler needs to perform the following steps:

1. Determine how much space is needed for the array. This can be calculated by:

$$((\text{length of the array}) + 1) \times (\text{size of an integer, e.g., } 4).$$

The reason we add one to the length of the array is that we want to store the length of the array along with the array. This is needed for bounds checking and to determine the length at run time.

2. Call an external function to get space on the heap. The call will return a pointer to the beginning of the array.
3. Generate code for saving the length of the array at offset 0.
4. Generate code for initializing each of the values in the array to zero starting at offset 4.

Calling runtime-system functions. To call an external function named `init-Array` with arguments *a*, *b*, simply generate a CALL such as

```
static Label initArray = new Label("initArray");
new CALL(new NAME(initArray),
         new Tree.ExpList(a, new Tree.ExpList(b, null)));
```

This refers to an external function `initArray` which is written in a language such as C or assembly language - it cannot be written in MiniJava because MiniJava has no mechanism for manipulating raw memory.

But on some operating systems, the C compiler puts an underscore at the beginning of each label; and the calling conventions for C functions may differ from those of MiniJava functions; and C functions don't expect to receive a static link, and so on. All these target-machine-specific details should be encapsulated into a function provided by the `Frame` structure:

```
public abstract class Frame {
   ⋮
   abstract public Tree.Exp externalCall(String func,
                                         Tree.ExpList args);
}
```

where `externalCall` takes the name of the external procedure and the arguments to be passed.

The implementation of `externalCall` depends on the relationship between MiniJava's procedure-call convention and that of the external function. The simplest possible implementation looks like

```
Tree.Exp externalCall(String s, Tree.ExpList args) {
     return new Tree.CALL(new Tree.NAME(new Temp.Label(s)),
                          args);
}
```

but may have to be adjusted for static links, or underscores in labels, and so on. Also, calling `new Label(s)` repeatedly with the same *s* makes several label objects that all mean the same thing; this may confuse other parts of the compiler, so it might be useful to maintain a string-to-label table to avoid duplication.

WHILE LOOPS

The general layout of a **while** loop is

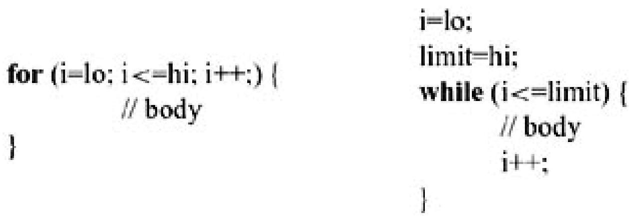
```
test:
            if not(condition) goto done body
            goto test
done:
```

If a **break** statement occurs within the *body* (and not nested within any interior **while** statements), the translation is simply a JUMP to *done*.

Translation of **break** statements needs to have a new formal parameter `break` that is the *done* label of the nearest enclosing loop. In translating a **while** loop, the translator will be called recursively upon *body* with the *done* label passed as the `break` parameter. When the translator is recursively calling itself in nonloop contexts, it can simply pass down the same `break` parameter that was passed to it.

FOR LOOPS

A **for** statement can be expressed using other kinds of statements:



```
                                    i=lo;
                                    limit=hi;
for (i=lo; i<=hi; i++;) {           while (i<=limit) {
        // body                             // body
}                                           i++;
                                    }
```

A straightforward approach to the translation of **for** statements is to rewrite the *abstract syntax* into the abstract syntax of the **while** statement shown, and then translate the result.

This is almost right, but consider the case where *limit*=*maxint*. Then $i + 1$ will overflow; either a hardware exception will be raised, or $i \leq limit$ will always be true! The solution is to put the test at the *bottom* of the loop, where $i < limit$ can be tested *before* the increment. Then an extra test will be needed before entering the loop to check $lo \leq hi$.

FUNCTION CALL

Translating a function call $f(a_1, \ldots a_n)$ is simple:

$$\text{CALL(NAME } l_f, [e_1, e_2, \ldots, e_n])$$

where l_f is the label for *f*. In an object-oriented language, the implicit variable `this` must be made an explicit argument of the call. That is, $p.m(a_1, \ldots a_n)$ is translated as

$$\text{CALL(NAME } l_{c\$m}, [p, e_1, e_2, \ldots, e_n])$$

where *p* belongs to class *c*, and *c*$*m* is the *m* method of class *c*. For a static method, the computation of address $l_{c\$m}$ can be done at compile time - it's a simple label, as it is in MiniJava. For dynamic methods, the computation is more complicated, as explained in Chapter 14.

STATIC LINKS

Some programming languages (such as Pascal, Scheme, and ML) support nesting of functions so that the inner functions can refer to variables declared in the outer functions. When building a compiler for such a language, frame representations and variable access are a bit more complicated.

When a variable *x* is declared at an outer level of static scope, static links must be used. The general form is

$$\begin{aligned}&\text{MEM}(+(\text{CONST } k_n,\ \text{MEM}(+(\text{CONST } k_{n-1}, \ldots\\ &\qquad\qquad\qquad \text{MEM}(+(\text{CONST } k_1,\ \text{TEMP FP}))\ldots))))\end{aligned}$$

where the $k_1, \ldots, k_{n-1}$ are the various static-link offsets in nested functions, and k_n is the offset of *x* in its own frame.

In creating `TEMP` variables, those temporaries that escape (i.e., are called from within an inner function) must be allocated in the stack frame, not in a register. When accessing such a temporary from either the same function or an inner function, we must pass the appropriate static link. The `exp` method of `Frame.Access` would need to calculate the appropriate chain of dereferences.

Translating a function call $f(a_1, \ldots a_n)$ using static links requires that the static link must be added as an implicit extra argument:

$$\text{CALL}(\text{NAME } l_f,\ [sl, e_1, e_2, \ldots, e_n])$$

Here l_f is the label for *f*, and *sl* is the static link, computed as described in Chapter 6. To do this computation, both the `level` of *f* and the `level` of the function calling *f* are required. A chain of (zero or more) offsets found in successive `level` descriptors is fetched, starting with the frame pointer TEMP(FP) defined by the `Frame` module.

[1] A different way of checking for `nil` is to unmap page 0 in the virtual-memory page tables, so that attempting to fetch/store fields of a `nil` record results in a page fault.

7.3 DECLARATIONS

For each variable declaration within a function body, additional space will be reserved in the `frame`. Also, for each function declaration, a new "fragment" of `Tree` code will be kept for the function's body.

VARIABLE DEFINITION

The translation of a variable declaration should return an augmented type environment that is used in processing the remainder of the function body.

However, the initialization of a variable translates into a `Tree` expression that must be put just before the body of the function. Therefore, the translator must return a `Translate.Exp` containing assignment expressions that accomplish these initializations.

If the translator is applied to function and type declarations, the result will be a "no-op" expression such as `Ex(CONST(0))`.

FUNCTION DEFINITION

A function is translated into a segment of assembly language with a *prologue*, a *body*, andan *epilogue*. The body of a function is an expression, and the *body* of the translation is simply the translation of that expression.

The *prologue,* which precedes the body in the assembly-language version of the function, contains

1. pseudo-instructions, as needed in the particular assembly language, to announce the beginning of a function;
2. a label definition for the function name;
3. an instruction to adjust the stack pointer (to allocate a new frame);
4. instructions to save "escaping" arguments into the frame, and to move nonescaping arguments into fresh temporary registers;
5. store instructions to save any callee-save registers - including the return address register - used within the function.
   Then comes
6. the function *body*.
   The *epilogue* comes after the body and contains
7. an instruction to move the return value (result of the function) to the register reserved for that purpose;
8. load instructions to restore the callee-save registers;
9. an instruction to reset the stack pointer (to deallocate the frame);
10. a *return* instruction (JUMP to the return address);
11. pseudo-instructions, as needed, to announce the end of a function.

Some of these items (1, 3, 9, and 11) depend on exact knowledge of the frame size, which will not be known until after the register allocator determines how many local variables need to be kept in the frame because they don't fit in registers. So these instructions should be generated very late, in a `FRAME` function called `procEntryExit3` (see also page 252). Item 2 (and 10), nestled between 1 and 3 (and 9 and 11, respectively) are also handled at that time.

To implement 7, the `Translate` phase should generate a move instruction

```
MOVE(RV, body)
```

that puts the result of evaluating the body in the return value (`RV`) location specified by the machine-specific `frame` structure:

```
package Frame;
public abstract class Frame {
   ⋮
   abstract public Temp RV();
}
```

Item 4 (moving incoming formal parameters), and 5 and 8 (the saving and restoring of callee-save registers), are part of the *view shift* described on page 128. They should be done by a function in the `Frame` module:

```
package Frame;
public abstract class Frame {
   ⋮
   abstract public Tree.Stm procEntryExit1(Tree.Stm body);
}
```

The implementation of this function will be discussed on page 251. `Translate` should apply it to each procedure body (items 5-7) as it is translated.

FRAGMENTS

Given a function definition comprising an already-translated `body` expression, the `Translate` phase should produce a descriptor for the function containing this necessary information:

- **frame:** The frame descriptor containing machine-specific information about local variables and parameters;
- **body:** The result returned from `procEntryExit1`.

Call this pair a *fragment* to be translated to assembly language. It is the second kind of fragment we have seen; the other was the assembly-language pseudo-instruction sequence for a string literal. Thus, it is useful to define (in the `Translate` interface) a `frag` datatype:

```
package Translate;
public abstract class Frag { public Frag next; }
public ProcFrag(Tree.Stm body, Frame.Frame frame);
public DataFrag(String data);
```

PROGRAM 7.5: A MiniJava program.

```
class Vehicle {
    int position;
    int gas;
    int move (int x) {
        position = position + x;
        return position;
    }
    int fill (int y) {
        gas = gas + y;
        return gas;
    }
}
```

```
public class Translate {
   ⋮
   private Frag frags; // linked list of accumulated fragments
   public void procEntryExit(Exp body);
   public Frag getResult();
}
```

The semantic analysis phase calls upon `new Translate.Level(...)` in processing a function header. Later it calls other methods of `Translate` to translate the body of the function. Finally the semantic analyzer calls `procEntryExit`, which has the *side effect* of remembering a `ProcFrag`.

All the remembered fragments go into a private fragment list within `Translate`;then `getResult` can be used to extract the fragment list.

CLASSES AND OBJECTS

Figure 7.5 shows a MiniJava class `Vehicle` with two instance variables `position` and `gas`, and two methods `move` and `fill`. We can create multiple `Vehicle` objects. Each `Vehicle` object will have its own `position` and `gas` variables. Two `Vehicle` objects can have different values in their variables, and in MiniJava, only the methods of an object can access the variables of that object. The translation of `new Vehicle()` is much like the translation of record creation and can be done in two steps:

1. Generate code for allocating heap space for all the instance variables; in this case we need to allocate 8 bytes (2 integers, each of size, say, 4).
2. Iterate through the memory locations for those variables and initialize them- in this case, they should both be initialized to 0.

Methods and the *this* pointer. Method calls in MiniJava are similar to function calls; but first, we must determine the class in which the method is declared and look up the method in that class. Second, we need to address the following question. Suppose we have multiple `Vehicle` objects and we want to call a method on one of them; how do we ensure that the implementation knows for which object we are calling the method? The solution is to pass that object as an extra argument to the method; that argument is the *this* pointer. For a method call

```
Vehicle v;
...
v.move();
```

the `Vehicle` object in variable `v` will be the *this* pointer when calling the `move` method.

The translation of method declarations is much like the translation of functions, but we need to avoid name clashes among methods with the same name that are declared in different classes. We can do that by choosing a naming scheme such that the name of the translated method is the concatenation of the class name and the method name. For example, the translation of `move` can be given the name `Vehicle move`.

Accessing variables In MiniJava, variables can be accessed from methods in the same class. Variables are accessed via the *this* pointer; thus, the translation of a variable reference is like field selection for records. The position of the variable in the object can be looked up in the symbol table for the class.

PROGRAM TRANSLATION TO TREES

Design a set of visitors which translate a MiniJava program into intermediate representation trees.

Supporting files in `$MINIJAVA/chap7` include:

```
Tree/* Data types for the Tree language.
Tree/Print.java Functions to display trees for debugging.
```

A simpler translator To simplify the implementation of the translator, you may do without the `Ex`, `Nx`, `Cx` constructors. The entire translation can be done with ordinary value expressions. This means that there is only one `Exp` class (without subclasses); this class contains one field of type `Tree.Exp` and only an `unEx()` method. Instead of `Nx`(*s*), use `Ex`(ESEQ(*s*, CONST 0)). For conditionals, instead of a `Cx`, use an expression that just evaluates to 1 or 0.

The intermediate representation trees produced from this kind of naive translation will be bulkier and slower than a "fancy" translation. But they *will* work correctly, and in principle a fancy back-end optimizer might be able to clean up the clumsiness. In any case, a clumsy but correct translator is better than a fancy one that doesn't work.

EXERCISES

- **7.1** Suppose a certain compiler translates all statements and expressions into Tree.Exp trees, and does not use the `Nx` and `Cx` constructors to represent expressions in different ways. Draw a picture of the IR tree that results from each of the following MiniJava statements and expressions.
  a. `a+5`
  b. `b[i+1]`
  c. `a<b`, which should be implemented by making an ESEQ whose left-hand side moves a 1 or 0 into some newly defined temporary, and whose right-hand side is the temporary.
  d. `a = x+y;` which should be translated with an EXP node at the top.
  e. `if (a<b) c=a; else c=b;` translated using the `a<b` tree from part (c) above; the whole statement will therefore be rather clumsy and inefficient.
  f. `if (a<b) c=a; else c=b;` translated in a less clumsy way.
- **7.2** Translate each of these MiniJava statements and expressions into IR trees, but using the `Ex`, `Nx`, and `Cx` constructors as appropriate. In each case, just draw pictures of the trees; an `Ex` tree will be a Tree `Exp`, an `Nx` tree will be a Tree `Stm`, anda `Cx` tree will be a `Stm` with holes labeled trueand falseinto which labels can later be placed.
  a. `a+5;`
  b. `b[i+1]=0;`
  c. `while (a<0) a=a+1;`
  d. `a<b` moves a 1 or 0 into some newly defined temporary, and whose right-hand side is the temporary.
  e. `a = x+y;`
  f. `if (a<b) c=a; else c=b;`
- **7.3** Using the C compiler of your choice (or a compiler for another language), translate some functions to assembly language. On Unix this is done with the `-S` option to the C compiler.

  Then identify all the components of the calling sequence (items 1-11), and explain what each line of assembly language does (especially the pseudoinstructions that comprise items 1 and 11). Try one small function that returns without much computation (a leaffunction), and one that calls another function before eventually returning.

- **7.4** The `Tree` intermediate language has no operators for floating-point variables. Show how the language would look with new binops for floating-point arithmetic, and new relops for floating-point comparisons. You may find it useful to introduce a variant of MEM nodes to describe fetching and storing floating-point values.

- ***7.5** The `Tree` intermediate language has no provision for data values that are not exactly one word long. The C programming language has signed and unsigned integers of several sizes, with conversion operators among the different sizes. Augment the intermediate language to accommodate several sizes of integers, with conversions among them.

  Hint: Do not distinguish signed values from unsigned values in the intermediate trees, but do distinguish between signed operators and unsigned operators. See also Fraser and Hanson [1995], Sections 5.5 and 9.1.

Chapter 8: Basic Blocks and Traces

ca-non-i-cal: reduced to the simplest or clearest schema possible

Webster's Dictionary

OVERVIEW

The trees generated by the semantic analysis phase must be translated into assembly or machine language. The operators of the `Tree` language are chosen carefully to match the capabilities of most machines. However, there are certain aspects of the tree language that do not correspond exactly with machine languages, and some aspects of the `Tree` language interfere with compiletime optimization analyses.

For example, it's useful to be able to evaluate the subexpressions of an expression in any order. But the subexpressions of `Tree.exp` can contain side effects - ESEQ and CALL nodes that contain assignment statements and perform input/output. If tree expressions did not contain ESEQ and CALL nodes, then the order of evaluation would not matter.

Some of the mismatches between `Trees` and machine-language programs are

- The CJUMP instruction can jump to either of two labels, but real machines' conditional jump instructions fall through to the next instruction if the condition is false.
- ESEQ nodes within expressions are inconvenient, because they make different orders of evaluating subtrees yield different results.
- CALL nodes within expressions cause the same problem.
- CALL nodes within the argument-expressions of other CALL nodes will cause problems when trying to put arguments into a fixed set of formal-parameter registers.

Why does the `Tree` language allow ESEQ and two-way CJUMP, if they are so troublesome? Because they make it much more convenient for the `Translate` (translation to intermediate code) phase of the compiler.

We can take any tree and rewrite it into an equivalent tree without any of the cases listed above. Without these cases, the only possible parent of a SEQ node is another SEQ; all the SEQ nodes will be clustered at the top of the tree. This makes the SEQs entirely uninteresting; we might as well get rid of them and make a linear list of `Tree.Stms`.

The transformation is done in three stages: First, a tree is rewritten into a list of *canonical trees* without SEQ or ESEQ nodes; then this list is grouped into a set of *basic blocks*, which contain no internal jumps or labels; then the basic blocks are ordered into a set of *traces* in which every CJUMP is immediately followed by its `false` label.

Thus the module `Canon` has these tree-rearrangement functions:

```
package Canon;
public class Canon {
 static public Tree.StmList linearize(Tree.Stm s);
}
public class BasicBlocks {
  public StmListList blocks;
  public Temp.Label done;
  public BasicBlocks(Tree.StmList stms);
}
StmListList(Tree.StmList head, StmListList tail);
public class TraceSchedule {
  public TraceSchedule(BasicBlocks b);
  public Tree.StmList stms;
}
```

`Linearize` removes the ESEQs and moves the CALLs to top level. Then `BasicBlocks` groups statements into sequences of straight-line code. Finally, `TraceSchedule` orders the blocks so that every CJUMP is followed by its `false` label.

8.1 CANONICAL TREES

Let us define *canonical trees* as having these properties:

1. No SEQ or ESEQ.
2. The parent of each CALL is either EXP(…) or MOVE(TEMP *t*,…).

TRANSFORMATIONS ON ESEQ

How can the ESEQ nodes be eliminated? The idea is to lift them higher and higher in the tree, until they can become SEQ nodes.

Figure 8.1 gives some useful identities on trees.

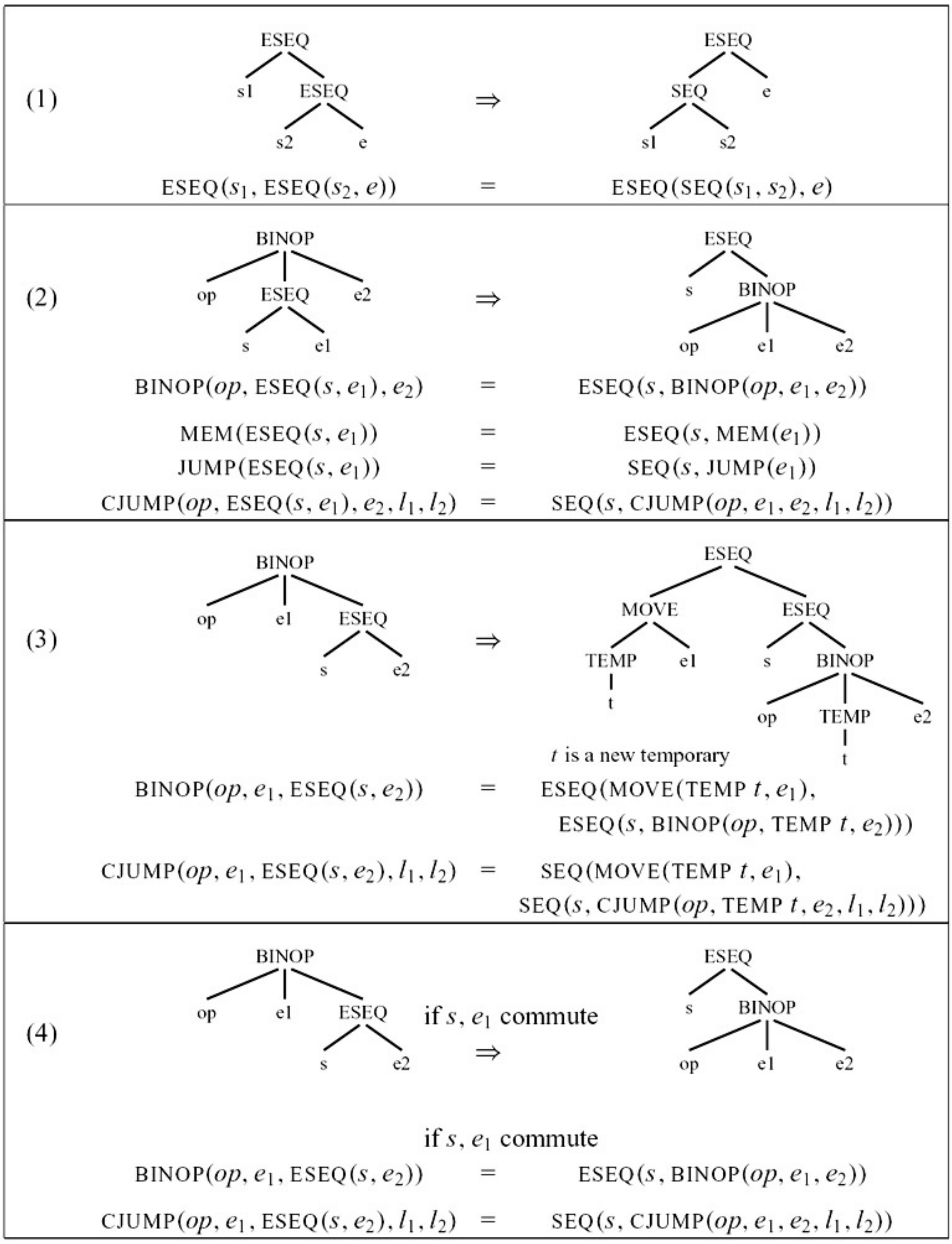

(1)
ESEQ(s1, ESEQ(s2, e)) = ESEQ(SEQ(s1, s2), e)
(2)
BINOP(op, ESEQ(s, e1), e2) = ESEQ(s, BINOP(op, e1, e2))
MEM(ESEQ(s, e1)) = ESEQ(s, MEM(e1))
JUMP(ESEQ(s, e1)) = SEQ(s, JUMP(e1))
CJUMP(op, ESEQ(s, e1), e2, l1, l2) = SEQ(s, CJUMP(op, e1, e2, l1, l2))
(3)
t is a new temporary
BINOP(op, e1, ESEQ(s, e2)) = ESEQ(MOVE(TEMP t, e1), ESEQ(s, BINOP(op, TEMP t, e2)))
CJUMP(op, e1, ESEQ(s, e2), l1, l2) = SEQ(MOVE(TEMP t, e1), SEQ(s, CJUMP(op, TEMP t, e2, l1, l2)))
(4)
if s, e1 commute
BINOP(op, e1, ESEQ(s, e2)) = ESEQ(s, BINOP(op, e1, e2))
CJUMP(op, e1, ESEQ(s, e2), l1, l2) = SEQ(s, CJUMP(op, e1, e2, l1, l2))
ESEQ
SEQ
BINOP
MOVE
TEMP
op
s
s1
s2
e
e1
e2
t

Figure 8.1: Identities on trees (see also Exercise 8.1).

Identity (1) is obvious. So is identity (2): Statement *s* is to be evaluated; then e_1; then e_2; then the sum of the expressions is returned. If *s* has side effects that affect e_1 or e_2, then either the left-hand side or the right-hand side of the first equation will execute those side effects before the expressions are evaluated.

Identity (3) is more complicated, because of the need not to interchange the evaluations of *s* and e_1. For example, if *s* is MOVE(MEM(*x*), *y*) and e_1 is BINOP(PLUS, MEM(*x*), *z*), then the program will compute a different result if *s* is evaluated before e_1 instead of after. Our goal is simply to pull *s* out of the BINOP expression; but now (to preserve the order of evaluation) we must pull e_1 out of the BINOP with it. To do so, we assign e_1 into a new temporary *t*, and put *t* inside the BINOP.

It may happen that *s* causes no side effects that can alter the result produced by e_1. This will happen if the temporaries and memory locations assigned by *s* are not referenced by e_1 (and *s* and e_1 don't both perform external I/O). In this case, identity (4) can be used.

We cannot always tell if two expressions commute. For example, whether MOVE(MEM(*x*), *y*) commutes with MEM(*z*) depends on whether *x* = *z*, which we cannot always determine at compile time. So we *conservatively approximate* whether statements commute, saying either "they definitely do commute" or "perhaps they don't commute." For example, we know that any statement "definitely commutes" with the expression CONST(*n*), so we can use identity (4) to justify special cases like

$$\mathrm{BINOP}(op, \mathrm{CONST}(n), \mathrm{ESEQ}(s, e)) = \mathrm{ESEQ}(s, \mathrm{BINOP}(op, \mathrm{CONST}(n), e)).$$

The `commute` function estimates (very naively) whether a statement commutes with an expression:

```
static boolean commute(Tree.Stm a, Tree.Exp b) {
    return isNop(a)
        || b instanceof Tree.NAME
        || b instanceof Tree.CONST;
}
static boolean isNop(Tree.Stm a) {
   return a instanceof Tree.EXP
         && ((Tree.EXP)a).exp instanceof Tree.CONST;
}
```

A constant commutes with any statement, and the empty statement commutes with any expression. Anything else is assumed not to commute.

GENERAL REWRITING RULES

In general, for each kind of `Tree` statement or expression we can identify the subexpressions. Then we can make rewriting rules, similar to the ones in Figure 8.1, to pull the ESEQs out of the statement or expression.

For example, in [e_1, e_2, ESEQ(*s*, e_3)], the statement *s* must be pulled leftward past e_2 and e_1. If they commute, we have (*s*; [e_1, e_2, e_3]). But suppose e_2 does not commute with *s*; then we must have

$$(\mathrm{SEQ}(\mathrm{MOVE}(t_1, e_1), \mathrm{SEQ}(\mathrm{MOVE}(t_2, e_2), s)); \quad [\mathrm{TEMP}(t_1), \mathrm{TEMP}(t_2), e_3])$$

Or if e_2 commutes with *s* but e_1 does not, we have

$$(\mathrm{SEQ}(\mathrm{MOVE}(t_1, e_1), s); \quad [\mathrm{TEMP}(t_1),\ e_2,\ e_3])$$

The `reorder` function takes a list of expressions and returns a pair of (statement, expression-list). The statement contains all the things that must be executed before the expression-list. As shown in these examples, this includes all the statement-parts of the ESEQs, as well as any expressions to their left with which they did not commute. When there are no ESEQs at all we will use EXP(CONST 0), which does nothing, as the statement.

Algorithm Step one is to make a "subexpression-extraction" method for each kind. Step two is to make a "subexpression-insertion" method: Given an ESEQ-clean version of each subexpression, this builds a new version of the expression or statement.

These will be methods of the `Tree.Exp` and `Tree.Stm` classes:

```
package Tree;
abstract public class Exp {
        abstract public ExpList kids();
        abstract public Exp build(ExpList kids);
}
abstract public class Stm {
        abstract public ExpList kids();
        abstract public Stm build(ExpList kids);
}
```

Each subclass `Exp` or `Stm` must implement the methods; for example,

```
package Tree;
public class BINOP extends Exp {
  public int binop;
  public Exp left, right;
  public BINOP(int b, Exp l, Exp r) {binop=b; ...}
  public final static int PLUS=0, MINUS=1, MUL=2, DIV=3,
           AND=4,OR=5,LSHIFT=6,RSHIFT=7,ARSHIFT=8,XOR=9;
  public ExpList kids() {return new ExpList(left,
                                 new ExpList(right,null));}
  public Exp build(ExpList kids) {
    return new BINOP(binop,kids.head,kids.tail.head);
  }
}
```

Other subclasses have similar (or even simpler) `kids` and `build` methods. Using these `build` methods, we can write functions

```
static Tree.Stm do_stm(Tree.Stm s)
static Tree.ESEQ do_exp (Tree.Exp e)
```

that pull all the ESEQs out of a statement or expression, respectively. That is, `do_stm` uses `s.kids()` to get the immediate subexpressions of `s`, which will be an expression-list *l*. It then pulls all the ESEQs out of *l* recursively, yielding a clump of side-effecting statements s_1 and a cleaned-up list l'. Then SEQ(s_1, `s.build(`l'`)`) constructs a new statement, like the original `s` but with no ESEQs. These functions rely on auxiliary functions `reorder_stm` and `reorder_exp` for help; see also Exercise 8.3.

The left-hand operand of the MOVE statement is not considered a subexpression, because it is the *destination* of the statement - its value is not used by the statement. However, if the destination is a memory location, then the *address* acts like a source. Thus we have,

```
public class MOVE extends Stm {
  public Exp dst, src;
  public MOVE(Exp d, Exp s) {dst=d; src=s;}
  public ExpList kids() {
    if (dst instanceof MEM)
       return new ExpList(((MEM)dst).exp,
                          new ExpList(src,null));
    else return new ExpList(src,null);
  }
  public Stm build(ExpList kids) {
    if (dst instanceof MEM)
     return new MOVE(new MEM(kids.head), kids.tail.head);
    else return new MOVE(dst, kids.head);
```

```
   }
}
```

Now, given a list of "kids", we pull the ESEQs out, from right to left.

MOVING CALLS TO TOP LEVEL

The `Tree` language permits CALL nodes to be used as subexpressions. However, the actual implementation of CALL will be that each function returns its result in the same dedicated return-value register TEMP(`RV`). Thus, if we have

$$\mathrm{BINOP}(\mathrm{PLUS}, \mathrm{CALL}(\ldots), \mathrm{CALL}(\ldots))$$

the second call will overwrite the `RV` register before the PLUS can be executed.

We can solve this problem with a rewriting rule. The idea is to assign each return value immediately into a fresh temporary register, that is

$$\mathrm{CALL}(\mathit{fun}, \mathit{args}) \quad \rightarrow \quad \mathrm{ESEQ}(\mathrm{MOVE}(\mathrm{TEMP}\ t, \mathrm{CALL}(\mathit{fun}, \mathit{args})), \mathrm{TEMP}\ t)$$

Now the ESEQ-eliminator will percolate the MOVE up outside of its containing BINOP (etc.) expressions.

This technique will generate a few extra MOVE instructions, which the register allocator (Chapter 11) can clean up.

The rewriting rule is implemented as follows: `reorder` replaces any occurrence of CALL(*f*, *args*) by

$$\mathrm{ESEQ}(\mathrm{MOVE}(\mathrm{TEMP}\ t_{\mathrm{new}}, \mathrm{CALL}(f, \mathit{args})),\ \mathrm{TEMP}\ t_{\mathrm{new}})$$

and calls itself again on the ESEQ. But `do_stm` recognizes the pattern

$$\mathrm{MOVE}(\mathrm{TEMP}\ t_{\mathrm{new}}, \mathrm{CALL}(f, \mathit{args}))$$

and does not call `reorder` on the CALL node in that case, but treats the *f* and *args* as the children of the MOVE node. Thus, `reorder` never "sees" any CALL that is already the immediate child of a MOVE. Occurrences of the pattern EXP(CALL(*f*, *args*)) are treated similarly.

A LINEAR LIST OF STATEMENTS

Once an entire function body s_0 is processed with `do_stm`, the result is a tree s_0' where all the SEQ nodes are near the top (never underneath any other kind of node). The `linearize` function repeatedly applies the rule

$$\mathrm{SEQ}(\mathrm{SEQ}(a, b), c) \;=\; \mathrm{SEQ}(a, \mathrm{SEQ}(b, c))$$

The result is that s'_0 is linearized into an expression of the form

$$\mathrm{SEQ}(s_1, \mathrm{SEQ}(s_2, \ldots, \mathrm{SEQ}(s_{n-1}, s_n) \ldots))$$

Here the SEQ nodes provide no structuring information at all, and we can just consider this to be a simple list of statements,

$$s_1, s_2, \ldots, s_{n-1}, s_n$$

where none of the s_i contain SEQ or ESEQ nodes.

These rewrite rules are implemented by `linearize`, with an auxiliary function `linear`:

```
static Tree.StmList linear(Tree.SEQ s, Tree.StmList l) {
  return linear(s.left,linear(s.right,l));
}
static Tree.StmList linear(Tree.Stm s, Tree.StmList l) {
  if (s instanceof Tree.SEQ) return linear((Tree.SEQ)s,l);
  else return new Tree.StmList(s,l);
}
static public Tree.StmList linearize(Tree.Stm s) {
  return linear(do_stm(s), null);
}
```

8.2 TAMING CONDITIONAL BRANCHES

Another aspect of the `Tree` language that has no direct equivalent in most machine instruction sets is the two-way branch of the CJUMP instruction. The `Tree` language CJUMP is designed with two target labels for convenience in translating into trees and analyzing trees. On a real machine, the conditional jump either transfers control (on a `true` condition) or "falls through" to the next instruction.

To make the trees easy to translate into machine instructions, we will rearrange them so that every CJUMP(*cond*, l_t, l_f) is immediately followed by LABEL(l_f), its "false branch." Each such CJUMP can be directly implemented on a real machine as a conditional branch to label l_t.

We will make this transformation in two stages: First, we take the list of canonical trees and form them into *basic blocks*; then we order the basic blocks into a *trace*. The next sections will define these terms.

BASIC BLOCKS

In determining where the jumps go in a program, we are analyzing the program's *control flow*. Control flow is the sequencing of instructions in a program, ignoring the data values in registers and memory, and ignoring the arithmetic calculations. Of course, not knowing the data values means we cannot know whether the conditional jumps will go to their true or false labels; so we simply say that such jumps can go either way.

In analyzing the control flow of a program, any instruction that is not a jump has an entirely uninteresting behavior. We can lump together any sequence of nonbranch instructions into a basic block and analyze the control flow between basic blocks.

A *basic block* is a sequence of statements that is always entered at the beginning and exited at the end, that is:

- The first statement is a LABEL.
- The last statement is a JUMP or CJUMP.

- There are no other LABELs, JUMPs, or CJUMPs.

The algorithm for dividing a long sequence of statements into basic blocks is quite simple. The sequence is scanned from beginning to end; whenever a LABEL is found, a new block is started (and the previous block is ended); whenever a JUMP or CJUMP is found, a block is ended (and the next block is started). If this leaves any block not ending with a JUMP or CJUMP, then a JUMP to the next block's label is appended to the block. If any block has been left without a LABEL at the beginning, a new label is invented and stuck there.

We will apply this algorithm to each function-body in turn. The procedure "epilogue" (which pops the stack and returns to the caller) will not be part of this body, but is intended to follow the last statement. When the flow of program execution reaches the end of the last block, the epilogue should follow. But it is inconvenient to have a "special" block that must come last and that has no JUMP at the end. Thus, we will invent a new label `done` - intended to mean the beginning of the epilogue - and put a JUMP(NAME `done`) at the end of the last block.

In the MiniJava compiler, the class `Canon.BasicBlocks` implements this simple algorithm.

TRACES

Now the basic blocks can be arranged in any order, and the result of executing the program will be the same - every block ends with a jump to the appropriate place. We can take advantage of this to choose an ordering of the blocks satisfying the condition that each CJUMP is followed by its false label.

At the same time, we can also arrange that many of the unconditional JUMPs are immediately followed by their target label. This will allow the deletion of these jumps, which will make the compiled program run a bit faster.

A *trace* is a sequence of statements that could be consecutively executed during the execution of the program. It can include conditional branches. A program has many different, overlapping traces. For our purposes in arranging CJUMPs and false-labels, we want to make a set of traces that exactly covers the program: Each block must be in exactly one trace. To minimize the number of JUMPs from one trace to another, we would like to have as few traces as possible in our covering set.

A very simple algorithm will suffice to find a covering set of traces. The idea is to start with some block - the beginning of a trace - and follow a possible execution path - the rest of the trace. Suppose block b_1 ends with a JUMP to b_4, and b_4 has a JUMP to b_6. Then we can make the trace b_1, b_4, b_6.

But suppose b_6 ends with a conditional jump CJUMP(*cond*, b_7, b_3). We cannot know at compile time whether b_7 or b_3 will be next. But we can assume that some execution will follow b_3, so let us imagine it is that execution that we are simulating. Thus, we append b_3 to our trace and continue with the rest of the trace after b_3. The block b_7 will be in some other trace.

Algorithm 8.2 (which is similar to `Canon.TraceSchedule`) ordersthe blocks into traces as follows: It starts with some block and follows a chain of jumps, marking each block and appending it to the current trace. Eventually it comes to a block whose successors are all marked, so it ends the trace and picks an unmarked block to start the next trace.

ALGORITHM 8.2: Generation of traces.

```
Put all the blocks of the program into a list Q.
while Q is not empty
      Start a new (empty) trace, call it T.
      Remove the head element b from Q.
      while b is not marked
            Mark b; append b to the end of the current trace T.
            Examine the successors of b (the blocks to which b branches);
            if there is any unmarked successor c
               $b \leftarrow c$
      End the current trace T.
```

FINISHING UP

An efficient compiler will keep the statements grouped into basic blocks, because many kinds of analysis and optimization algorithms run faster on (relatively few) basic blocks than on (relatively many) individual statements. For the MiniJava compiler, however, we seek simplicity in the implementation of later phases. So we will flatten the ordered list of traces back into one long list of statements.

At this point, most (but not all) CJUMPs will be followed by their true or false label. We perform some minor adjustments:

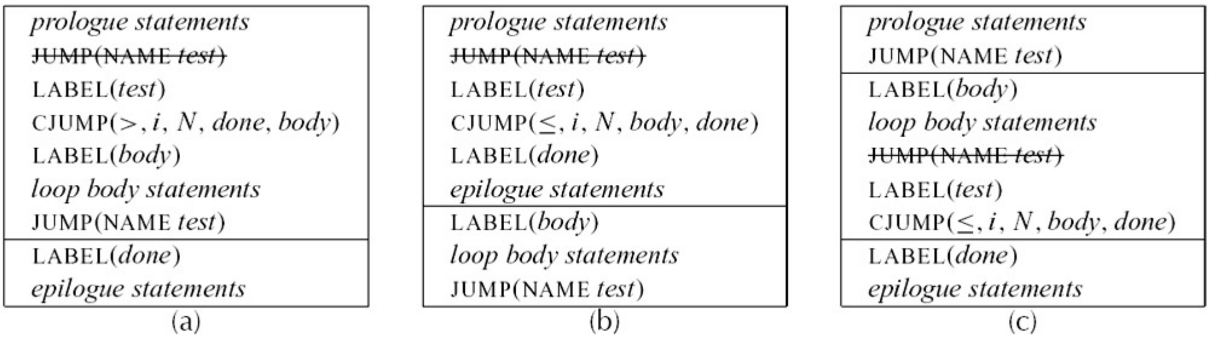
- Any CJUMP immediately followed by its false label we let alone (there will be many of these).
- For any CJUMP followed by its true label, we switch the true and false labels and negate the condition.
- For any CJUMP(*cond*, *a*, *b*, l_t, l_f) followed by neither label, we invent a new false label l_f' and rewrite the single CJUMP statement as three statements, just to achieve the condition that the CJUMP is followed by its false label:
- CJUMP($cond$, a, b, l_t, l_f')
- LABEL l_f'
- JUMP(NAME l_f)

The trace-generating algorithm will tend to order the blocks so that many of the unconditional JUMPs are immediately followed by their target labels. We can remove such jumps.

OPTIMAL TRACES

For some applications of traces, it is important that any frequently executed sequence of instructions (such as the body of a loop) should occupy its own trace. This helps not only to minimize the number of unconditional jumps, but also may help with other kinds of optimizations, such as register allocation and instruction scheduling.

Figure 8.3 shows the same program organized into traces in different ways. Figure 8.3a has a CJUMP and a JUMP in every iteration of the **while**-loop; Figure 8.3b uses a different trace covering, also with CJUMP and a JUMP in every iteration. But Figure 8.3c shows a better trace covering, with no JUMP in each iteration.



(a)

prologue statements
~~JUMP(NAME *test*)~~
LABEL(*test*)
CJUMP(>, *i*, *N*, *done*, *body*)
LABEL(*body*)
loop body statements
JUMP(NAME *test*)
LABEL(*done*)
epilogue statements

(b)

prologue statements
~~JUMP(NAME *test*)~~
LABEL(*test*)
CJUMP(≤, *i*, *N*, *body*, *done*)
LABEL(*done*)
epilogue statements
LABEL(*body*)
loop body statements
JUMP(NAME *test*)

(c)

prologue statements
JUMP(NAME *test*)
LABEL(*body*)
loop body statements
~~JUMP(NAME *test*)~~
LABEL(*test*)
CJUMP(≤, *i*, *N*, *body*, *done*)
LABEL(*done*)
epilogue statements

Figure 8.3: Different trace coverings for the same program.

The MiniJava compiler's `Canon` module doesn't attempt to optimize traces around loops, but it is sufficient for the purpose of cleaning up the `Tree`-statement lists for generating assembly code.

FURTHER READING

The rewrite rules of Figure 8.1 are an example of a *term rewriting system*; such systems have been much studied [Dershowitz and Jouannaud 1990].

Fisher [1981] shows how to cover a program with traces so that frequently executing paths tend to stay within the same trace. Such traces are useful for program optimization and scheduling.

EXERCISES

- ***8.1** The rewriting rules in Figure 8.1 are a subset of the rules necessary to eliminate all ESEQs from expressions. Show the right-hand side for each of the following incomplete rules:

  a. MOVE(TEMP t, ESEQ(s, e)) ⇒

  b. MOVE(MEM(ESEQ(s, e_1)), e_2) ⇒

  c. MOVE(MEM(e_1), ESEQ(s, e_2)) ⇒

  d. EXP(ESEQ(s, e)) ⇒

  e. EXP(CALL(ESEQ(s, e), *args*)) ⇒

  f. MOVE(TEMP t, CALL(ESEQ(s, e), *args*)) ⇒

  g. EXP(CALL(e_1, [e_2, ESEQ(s, e_3), e_4])) ⇒

  In some cases, you may need two different right-hand sides depending on whether something commutes (just as parts (3) and (4) of Figure 8.1 have different right-hand sides for the same left-hand sides).

- **8.2** Draw each of the following expressions as a tree diagram, and then apply the rewriting rules of Figure 8.1 and Exercise 8.1, as well as the CALL rule on page 168.
  a. MOVE(MEM(ESEQ(SEQ(CJUMP(LT, TEMP$_i$, CONST$_0$, L_{out}, L_{ok}), LABEL$_{ok}$) TEMP$_i$)), CONST$_1$)
  b. MOVE(MEM(MEM(NAME$_a$)), MEM(CALL(TEMP$_f$, [])))
  c. BINOP(PLUS, CALL(NAME$_f$, [TEMP$_x$]), CALL(NAME$_g$, [ESEQ(MOVE(TEMPx, CONST$_0$), TEMP$_x$)]))
- ***8.3** The directory `$MINIJAVA/chap8` contains an implementation of every algorithm described in this chapter. Read and understand it.

- **8.4** A primitive form of the `commute` test is shown on page 164. This function is conservative: If interchanging the order of evaluation of the expressions will change the result of executing the program, this function will definitely return false; but if an interchange is harmless, `commute` might return true or false.

  Write a more powerful version of `commute` that returns true in more cases, but is still conservative. Document your program by drawing pictures of (pairs of) expression trees on which it will return true.

- ***8.5** The left-hand side of a MOVE node really represents a destination, not an expression. Consequently, the following rewrite rule is nota good idea:
- MOVE(e_1, ESEQ(s, e_2)) → SEQ(s, MOVE(e_1, e_2))   if s, e_1 commute

  Write a statement matching the left side of this rewrite rule that produces a different result when rewritten.

  Hint: It is very reasonable to say that the statement MOVE(TEMP$_a$, TEMP$_b$) commutes with expression TEMP$_b$ (if a and b are not the same), since TEMP$_b$ yields the same value whether executed before or after the MOVE.

  Conclusion: The only subexpression of MOVE(TEMP$_a$, e) is e, and the subexpressions of MOVE(MEM(e_1), e_2) are [e_1, e_2]; we should not say that a is a subexpression of MOVE(a, b).

- **8.6** Break this program into basic blocks.

```
1.   $m \leftarrow 0$
2.   $v \leftarrow 0$
3.   if $v \geq n$ goto 15
4.   $r \leftarrow v$
5.   $s \leftarrow 0$
6.   if $r < n$ goto 9
7.   $v \leftarrow v + 1$
8.   goto 3
9.   $x \leftarrow M[r]$
10.  $s \leftarrow s + x$
11.  if $s \leq m$ goto 13
12.  $m \leftarrow s$
13.  $r \leftarrow r + 1$
14.  goto 6
15.  return $m$
```

- **8.7** Express the basic blocks of Exercise 8.6 as statements in the `Tree` intermediate form, and use Algorithm 8.2 to generate a set of traces.

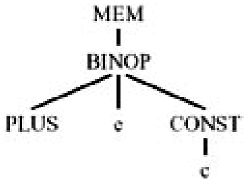
Chapter 9: Instruction Selection

in-struc-tion: a code that tells a computer to perform a particular operation

Webster's Dictionary

OVERVIEW

The intermediate representation (`Tree`) language expresses only one operation in each tree node: memory fetch or store, addition or subtraction, conditional jump, and so on. A real machine instruction can often perform several of these primitive operations. For example, almost any machine can perform an add and a fetch in the same instruction, corresponding to the tree



Finding the appropriate machine instructions to implement a given intermediate representation tree is the job of the *instruction selection* phase of a compiler.

TREE PATTERNS

We can express a machine instruction as a fragment of an IR tree, called a *tree pattern*. Then instruction selection becomes the task of tiling the tree with a minimal set of tree patterns.

For purposes of illustration, we invent an instruction set: the *Jouette* architecture. The arithmetic and memory instructions of *Jouette* are shown in Figure 9.1. On this machine, register r_0 always contains zero.

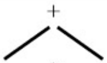
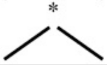
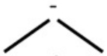
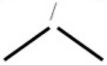
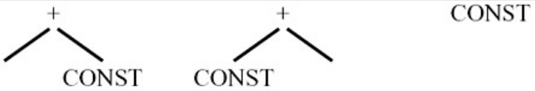
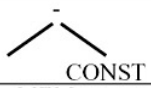
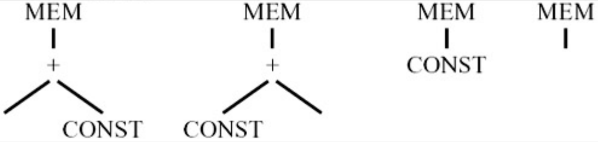
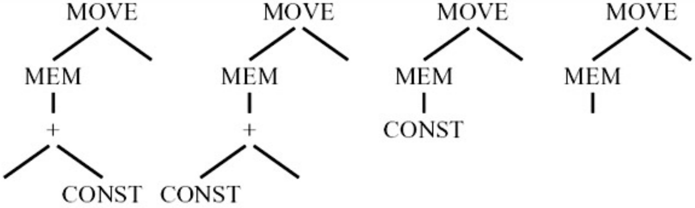
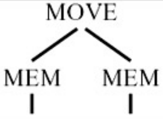
Name	*Effect*	*Trees*
—	r_i	TEMP
ADD	$r_i \leftarrow r_j + r_k$	
MUL	$r_i \leftarrow r_j \times r_k$	
SUB	$r_i \leftarrow r_j - r_k$	
DIV	$r_i \leftarrow r_j / r_k$	
ADDI	$r_i \leftarrow r_j + c$	
SUBI	$r_i \leftarrow r_j - c$	
LOAD	$r_i \leftarrow M[r_j + c]$	 MEM + CONST MEM + CONST MEM CONST MEM
STORE	$M[r_j + c] \leftarrow r_i$	 MOVE MEM + CONST MOVE MEM + CONST MOVE MEM CONST MOVE MEM
MOVEM	$M[r_j] \leftarrow M[r_i]$	

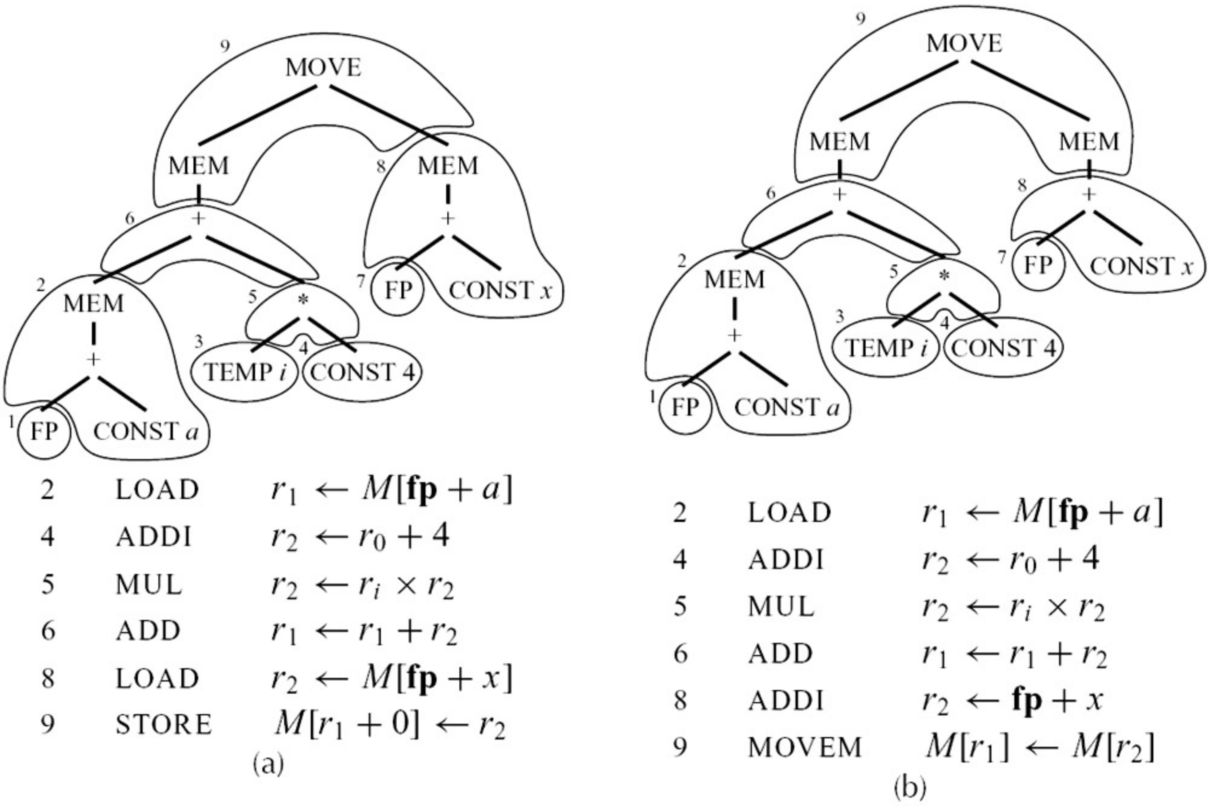
Figure 9.1: Arithmetic and memory instructions. The notation $M[x]$ denotes the memory word at address x.

Each instruction above the double line in Figure 9.1 produces a result in a register. The very first entry is not really an instruction, but expresses the idea that a TEMP node is implemented as a register, so it can "produce a result in a register" without executing any instructions at all. The instructions below the double line do not produce results in registers, but are executed only for side effects on memory.

For each instruction, the tree patterns it implements are shown. Some instructions correspond to more than one tree pattern; the alternate patterns are obtained for commutative operators (+ and *), and in some cases where a register or constant can be zero (LOAD and STORE). In this chapter we abbreviate the tree diagrams slightly: BINOP(PLUS, *x*, *y*) nodes will be written as +(*x*, *y*), and the actual values of CONST and TEMP nodes will not always be shown.

The fundamental idea of instruction selection using a tree-based intermediate representation is *tiling* the IR tree. The *tiles* are the set of tree patterns corresponding to legal machine instructions, and the goal is to cover the tree with nonoverlapping tiles.

For example, the MiniJava-language expression such as *a*[*i*] := *x*, where *i* is a register variable and *a* and *x* are frame-resident, results in a tree that can be tiled in many different ways. Two tilings, and the corresponding instruction sequences, are shown in Figure 9.2 (remember that *a* is really the frame offset of the pointer to an array). In each case, tiles 1, 3, and 7 do not correspond to any machine instructions, because they are just registers (TEMPs) already containing the right values.



2	LOAD	$r_1 \leftarrow M[\mathbf{fp} + a]$
4	ADDI	$r_2 \leftarrow r_0 + 4$
5	MUL	$r_2 \leftarrow r_i \times r_2$
6	ADD	$r_1 \leftarrow r_1 + r_2$
8	LOAD	$r_2 \leftarrow M[\mathbf{fp} + x]$
9	STORE	$M[r_1 + 0] \leftarrow r_2$

2	LOAD	$r_1 \leftarrow M[\mathbf{fp} + a]$
4	ADDI	$r_2 \leftarrow r_0 + 4$
5	MUL	$r_2 \leftarrow r_i \times r_2$
6	ADD	$r_1 \leftarrow r_1 + r_2$
8	ADDI	$r_2 \leftarrow \mathbf{fp} + x$
9	MOVEM	$M[r_1] \leftarrow M[r_2]$

Figure 9.2: A tree tiled in two ways.

Finally - assuming a "reasonable" set of tile patterns - it is always possible to tile the tree with tiny tiles, each covering only one node. In our example, such a tiling looks like this:

ADDI	$r_1 \leftarrow r_0 + a$
ADD	$r_1 \leftarrow \mathbf{fp} + r_1$
LOAD	$r_1 \leftarrow M[r_1 + 0]$
ADDI	$r_2 \leftarrow r_0 + 4$
MUL	$r_2 \leftarrow r_i \times r_2$
ADD	$r_1 \leftarrow r_1 = r_2$
ADDI	$r_2 \leftarrow r_0 + x$
ADD	$r_2 \leftarrow \mathbf{fp} + r_2$
LOAD	$r_2 \leftarrow M[r_2 + 0]$
STORE	$M[r_1 + 0] \leftarrow r_2$

For a reasonable set of patterns, it is sufficient that each individual `Tree` node correspond to some tile. It is usually possible to arrange for this; for example, the LOAD instruction can be made to cover just a single MEM node by using a constant of 0, and so on.

OPTIMAL AND OPTIMUM TILINGS

The best tiling of a tree corresponds to an instruction sequence of least cost: the shortest sequence of instructions. Or if the instructions take different amounts of time to execute, the least-cost sequence has the lowest total time.

Suppose we could give each kind of instruction a cost. Then we could define an *optimum* tiling as the one whose tiles sum to the lowest possible value. An *optimal* tiling is one where no two adjacent tiles can be combined into a single tile of lower cost. If there is some tree pattern that can be split into several tiles of lower combined cost, then we should remove that pattern from our catalog of tiles before we begin.

Every *optimum* tiling is also *optimal*, but not vice versa. For example, suppose every instruction costs one unit, except for MOVEM, which costs m units. Then either Figure 9.2a is optimum (if $m > 1$) or Figure 9.2b is optimum (if $m < 1$) or both (if $m = 1$); but both trees are optimal.

Optimum tiling is based on an idealized cost model. In reality, instructions are not self-contained with individually attributable costs; nearby instructions interact in many ways, as discussed in Chapter 20.

9.1 ALGORITHMS FOR INSTRUCTION SELECTION

There are good algorithms for finding optimum and optimal tilings, but the algorithms for optimal tilings are simpler, as you might expect.

Complex instruction set computers (CISC) have instructions that accomplish several operations each. The tiles for these instructions are quite large, and the difference between optimum and optimal tilings - while never very large - is at least sometimes noticeable.

Most architectures of modern design are *reduced instruction set computers (RISC)*. Each RISC instruction accomplishes just a small number of operations (all the *Jouette* instructions except MOVEM are typical RISC instructions). Since the tiles are small and of uniform cost, there is usually no difference at all between optimum and optimal tilings. Thus, the simpler tiling algorithms suffice.

MAXIMAL MUNCH

The algorithm for optimal tiling is called *maximal munch*. It is quite simple. Starting at the root of the tree, find the largest tile that fits. Cover the root node - and perhaps several other nodes near the root - with this tile, leaving several subtrees. Now repeat the same algorithm for each subtree.

As each tile is placed, the instruction corresponding to that tile is generated. The maximal munch algorithm generates the instructions *in reverse order* - after all, the instruction at the root is the first to be generated, but it can only execute after the other instructions have produced operand values in registers.

The "largest tile" is the one with the most nodes. For example, the tile for ADD has one node, the tile for SUBI has two nodes, and the tiles for STORE and MOVEM have three nodes each.

If two tiles of equal size match at the root, then the choice between them is arbitrary. Thus, in the tree of Figure 9.2, STORE and MOVEM both match, and either can be chosen.

Maximal munch is quite straightforward to implement in Java. Simply write two recursive functions, `munchStm` for statements and `munchExp` for expressions. Each clause of `munchExp` will match one tile. The clauses are ordered in order of tile preference (biggest tiles first).

Program 9.3 is a partial example of a *Jouette* code generator based on the maximal munch algorithm. Executing this program on the tree of Figure 9.2 will match the first clause of `munchStm`; this will call `munchExp` to emit all the instructions for the operands of the STORE, followed by the STORE itself. Program 9.3 does not show how the registers are chosen and operand syntax is specified for the instructions; we are concerned here only with the pattern-matching of tiles.

PROGRAM 9.3: Maximal Munch in Java.

```
void munchMove(MEM dst, Exp src) {
  // MOVE(MEM(BINOP(PLUS, e1, CONST(i))), e2)
  if (dst.exp instanceof BINOP && ((BINOP)dst.exp).oper==BINOP.PLUS
           && ((BINOP)dst.exp).right instanceof CONST)
     {munchExp(((BINOP)dst.exp).left); munchExp(src); emit("STORE");}
  // MOVE(MEM(BINOP(PLUS, CONST(i), e1)), e2)
  else if (dst.exp instanceof BINOP && ((BINOP)dst.exp).oper==BINOP.PLUS
           && ((BINOP)dst.exp).left instanceof CONST)
     {munchExp(((BINOP)dst.exp).right); munchExp(src); emit("STORE");}
  // MOVE(MEM(e1), MEM(e2))
  else if (src instanceof MEM)
     {munchExp(dst.exp); munchExp(((MEM)src).exp); emit("MOVEM");}
  // MOVE(MEM(e1, e2)
  else
     {munchExp(dst.exp); munchExp(src); emit("STORE");}
}
void munchMove(TEMP dst, Exp src) {
  // MOVE(TEMP(t1), e)
  munchExp(src); emit("ADD");
}
void munchMove(Exp dst, Exp src) {
  // MOVE(d, e)
  if (dst instanceof MEM) munchMove((MEM)dst,src);
  else if (dst instanceof TEMP) munchMove((TEMP)dst,src);
}
void munchStm(Stm s) {
  if (s instanceof MOVE) munchMove(((MOVE)s).dst, ((MOVE)s).src);
  ⋮ // CALL, JUMP, CJUMP unimplemented here
}
void munchExp(Exp)
MEM(BINOP(PLUS, e1, CONST(i))) ⇒ munchExp(e1); emit("LOAD");
MEM(BINOP(PLUS, CONST(i), e1)) ⇒ munchExp(e1); emit("LOAD");
MEM(CONST(i)) ⇒ emit("LOAD");
MEM(e1)  ⇒ munchExp(e1); emit("LOAD");
BINOP(PLUS, e1, CONST(i)) ⇒ munchExp(e1); emit("ADDI");
BINOP(PLUS, CONST(i, e1) ⇒ munchExp(e1); emit("ADDI");
CONST(i) ⇒ munchExp(e1); emit("ADDI");
BINOP(PLUS, e1, CONST(i)) ⇒ munchExp(e1); emit("ADD");
TEMP(t) ⇒ {}
```

If, for each node-type in the `Tree` language, there exists a single-node tile pattern, then maximal munch cannot get "stuck" with no tile to match some subtree.

DYNAMIC PROGRAMMING

Maximal munch always finds an optimal tiling, but not necessarily an optimum. A dynamic-programming algorithm can find the optimum. In general, dynamic programming is a technique for finding optimum solutions for a whole problem based on the optimum solution of each subproblem; here the subproblems are the tilings of the subtrees.

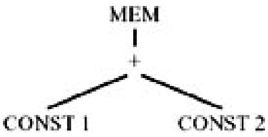
The dynamic-programming algorithm assigns a *cost* to every node in the tree. The cost is the sum of the instruction costs of the best instruction sequence that can tile the subtree rooted at that node.

This algorithm works bottom-up, in contrast to maximal munch, which works top-down. First, the costs of all the children (and grandchildren, etc.) of node *n* are found recursively. Then, each tree pattern (tile kind) is matched against node *n*.

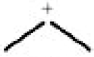
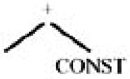
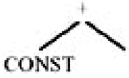
Each tile has zero or more *leaves*. In Figure 9.1 the leaves are represented as edges whose bottom ends exit the tile. The leaves of a tile are places where subtrees can be attached.

For each tile *t* of cost *c* that matches at node *n*, there will be zero or more subtrees s_i corresponding to the leaves of the tile. The cost c_i of each subtree has already been computed (because the algorithm works bottom-up). So the cost of matching tile *t* is just $c + \Sigma c_i$.

Of all the tiles t_j that match at node *n*, the one with the minimum-cost match is chosen, and the (minimum) cost of node *n* is thus computed. For example, consider this tree:

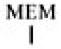
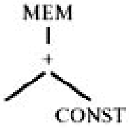
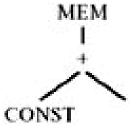


The only tile that matches CONST 1 is an ADDI instruction with cost 1. Similarly, CONST 2 has cost 1. Several tiles match the + node:

Tile	Instruction	Tile Cost	Leaves Cost	Total Cost
	ADD	1	1+1	3
	ADDI	1	1	2
	ADDI	1	1	2

The ADD tile has two leaves, but the ADDI tile has only one leaf. In matching the first ADDI pattern, we are saying "though we computed the cost of tiling CONST 2, we are not going to use that information." If we choose to use the first ADDI pattern, then CONST 2 will not be the root of any tile, and its cost will be ignored. In this case, either of the two ADDI tiles leads to the minimum cost for the + node, and the choice is arbitrary. The + node gets a cost of 2.

Now, several tiles match the MEM node:

Tile	Instruction	Tile Cost	Leaves Cost	Total Cost
	LOAD	1	2	3
	LOAD	1	1	2
	LOAD	1	1	2

Either of the last two matches will be optimum.

Once the cost of the root node (and thus the entire tree) is found, the *instruction emission* phase begins. The algorithm is as follows:

Emission(node *n*): for each leaf l_i of the tile selected at node *n*, perform Emission(l_i). Then emit the instruction matched at node *n*.

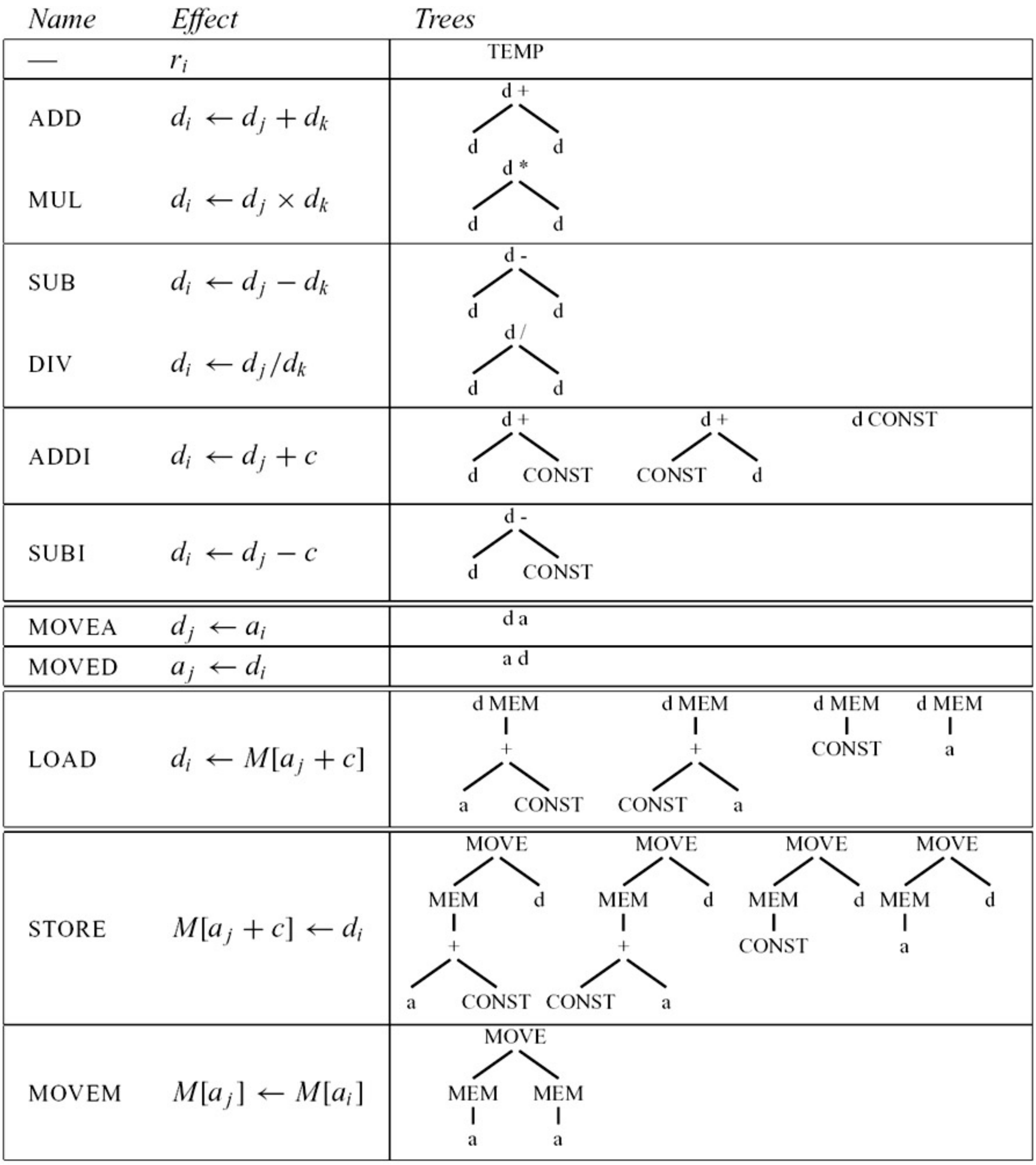
Emission(*n*) does *not* recur on the children of node *n*, but on the *leaves of the tile* that matched at *n*. For example, after the dynamic-programming algorithm finds the optimum cost of the simple tree above, the emission phase emits

ADDI $r_1 \leftarrow r_0 + 1$
LOAD $r_1 \leftarrow M[r_1 + 2]$

but no instruction is emitted for any tile rooted at the + node, because this was not a leaf of the tile matched at the root.

TREE GRAMMARS

For machines with complex instruction sets and several classes of registers and addressing modes, there is a useful generalization of the dynamic-programming algorithm. Suppose we make a brain-damaged version of *Jouette* with two classes of registers: *a* registers for addressing, and *d* registers for "data." The instruction set of the *Schizo-Jouette* machine (loosely based on the Motorola 68000) is shown in Figure 9.4.



Name	Effect	Trees
—	r_i	TEMP
ADD	$d_i \leftarrow d_j + d_k$	
MUL	$d_i \leftarrow d_j \times d_k$	
SUB	$d_i \leftarrow d_j - d_k$	
DIV	$d_i \leftarrow d_j / d_k$	
ADDI	$d_i \leftarrow d_j + c$	
SUBI	$d_i \leftarrow d_j - c$	
MOVEA	$d_j \leftarrow a_i$	d a
MOVED	$a_j \leftarrow d_i$	a d
LOAD	$d_i \leftarrow M[a_j + c]$	
STORE	$M[a_j + c] \leftarrow d_i$	
MOVEM	$M[a_j] \leftarrow M[a_i]$	

d +
d d
d *
d d
d -
d d
d /
d d
d +
d CONST
d +
CONST d
d CONST
d -
d CONST
d MEM
+
a CONST
d MEM
+
CONST a
d MEM
CONST
d MEM
a
MOVE
MEM d
+
a CONST
MOVE
MEM d
+
CONST a
MOVE
MEM d
CONST
MOVE
MEM d
a
MOVE
MEM MEM
a a

Figure 9.4: The *Schizo-Jouette* architecture.

The root and leaves of each tile must be marked with *a* or *d* to indicate which kind of register is implied. Now, the dynamic-programming algorithm must keep track, for each node, of the min-cost match as an *a* register, *and also* the min-cost match as a *d* register.

At this point it is useful to use a context-free grammar to describe the tiles; the grammar will have nonterminals *s* (for statements), *a* (for expressions calculated into an *a* register), and *d* (for expressions calculated into a *d* register). Section 3.1 describes the use of context-free grammars for source-language syntax; here we use them for quite a different purpose.

The grammar rules for the LOAD, MOVEA, and MOVED instructions might look like this:

```
d → MEM(+(a, CONST))
d → MEM(+(CONST, a))
d → MEM(CONST)
```

```
d → MEM(a)
d → a
a → d
```

Such a grammar is highly ambiguous: There are many different parses of the same tree (since there are many different instruction sequences implementing the same expression). For this reason, the parsing techniques described in Chapter 3 are not very useful in this application. However, a generalization of the dynamic-programming algorithm works quite well: The minimum-cost match at each node *for each nonterminal of the grammar* is computed.

Though the dynamic-programming algorithm is conceptually simple, it becomes messy to write down directly in a general-purpose programming language such as Java. Thus, several tools have been developed. These *codegenerator generators* process grammars that specify machine instruction sets; for each rule of the grammar, a cost and an action are specified. The costs are used to find the optimum tiling, and then the actions of the matched rules are used in the *emission* phase.

Like Yacc and Lex, the output of a code-generator generator is usually a program in C or Java that operates a table-driven matching engine with the action fragments (written in C or Java) inserted at the appropriate points.

Such tools are quite convenient. Grammars can specify addressing modes of treelike CISC instructions quite well. A typical grammar for the VAX has 112 rules and 20 nonterminal symbols; and one for the Motorola 68020 has 141 rules and 35 nonterminal symbols. However, instructions that produce more than one result - such as autoincrement instructions on the VAX -are difficult to express using tree patterns.

Code-generator generators are probably overkill for RISC machines. The tiles are quite small, there aren't very many of them, and there is little need for a grammar with many nonterminal symbols.

FAST MATCHING

Maximal munch and the dynamic-programming algorithm must examine, for each node, all the tiles that match at that node. A tile matches if each nonleaf node of the tile is labeled with the same operator (MEM, CONST, etc.) as the corresponding node of the tree.

The naive algorithm for matching would be to examine each tile in turn, checking each node of the tile against the corresponding part of the tree. However, there are better approaches. To match a tile at node *n* of the tree, the label at *n* can be used in a `case` statement:

```
match(n) {
 switch (label(n)) {
  case MEM: ...
  case BINOP: ...
  case CONST: ...
}
```

Once the clause for one label (such as MEM) is selected, only those patterns rooted in that label remain in consideration. Another `case` statement can use the label of the child of *n* to begin distinguishing among those patterns.

The organization and optimization of decision trees for pattern matching is beyond the scope of this book. However, for better performance the naive sequence of clauses in function `munchExp` should be rewritten as a sequence of comparisons that never looks twice at the same tree node.

EFFICIENCY OF TILING ALGORITHMS

How expensive are maximal munch and dynamic programming?

Let us suppose that there are T different tiles, and that the average matching tile contains K nonleaf (labeled) nodes. Let K' be the largest number of nodes that ever need to be examined to see which tiles match at a given subtree; this is approximately the same as the size of the largest tile. And suppose that, on the average, T' different patterns (tiles) match at each tree node. For a typical RISC machine we might expect $T = 50$, $K = 2$, $K' = 4$, $T' = 5$.

Suppose there are N nodes in the input tree. Then maximal munch will have to consider matches at only $N=K$ nodes because, once a "munch" is made at the root, no pattern-matching needs to take place at the nonleaf nodes of the tile.

To find all the tiles that match at one node, at most K' tree nodes must be examined; but (with a sophisticated decision tree) each of these nodes will be examined only once. Then each of the successful matches must be compared to see if its cost is minimal. Thus, the matching at each node costs $K' + T'$, for a total cost proportional to $(K' + T')N/K$.

The dynamic-programming algorithm must find all the matches at *every* node, so its cost is proportional to $(K' + T')N$. However, the constant of proportionality is higher than that of maximal munch, since dynamic programming requires two tree-walks instead of one.

Since K, K', and T' are constant, the running time of all of these algorithms is linear. In practice, measurements show that these instruction selection algorithms run very quickly compared to the other work performed by a real compiler - even lexical analysis is likely to take more time than instruction selection.

9.2 CISC MACHINES

A typical modern RISC machine has

1. 32 registers,
2. only one class of integer/pointer registers,
3. arithmetic operations only between registers,
4. "three-address" instructions of the form $r_1 \leftarrow r_2 \oplus r_3$,
5. load and store instructions with only the M[reg+const] addressing mode,
6. every instruction exactly 32 bits long,
7. one result or effect per instruction.

Many machines designed between 1970 and 1985 are *complex instruction set computers (CISC).* Such computers have more complicated addressing modes that encode instructions in fewer bits, which was important when computer memories were smaller and more expensive. Typical features found on CISC machines include

1. few registers (16, or 8, or 6);
2. registers divided into different classes, with some operations available only on certain registers;
3. arithmetic operations can access registers or memory through "addressing modes";
4. "two-address" instructions of the form $r_1 \leftarrow r_1 \oplus r_2$;
5. several different addressing modes;
6. variable-length instructions, formed from variable-length opcode plus variablelength addressing modes;
7. instructions with side effects such as "autoincrement" addressing modes.

Most computer architectures designed since 1990 are RISC machines, but most general-purpose computers installed since 1990 are CISC machines: the Intel 80386 and its descendants (486, Pentium).

The Pentium, in 32-bit mode, has six general-purpose registers, a stack pointer, and a frame pointer. Most instructions can operate on all six registers, but the multiply and divide instructions operate only on the `eax` register. In contrast to the "three-address" instructions found on RISC machines, Pentium arithmetic instructions are generally "two-address", meaning that the destination register must be the same as the first source register. Most instructions can have either two register operands ($r_1 \leftarrow r_1 \oplus r_2$), or one register and one memory operand, for example $M[r_1 + c] \leftarrow M[r_1 + c] \oplus r_2$ or $r_1 \leftarrow r_1 \oplus M[r_2 + c]$, but not $M[r_1 + c_1] \leftarrow M[r_1 + c_1] \oplus M[r_2 + c_2]$

We will cut through these Gordian knots as follows:

1. **Few registers:** We continue to generate TEMP nodes freely, and assume that the register allocator will do a good job.
2. **Classes of registers:** The multiply instruction on the Pentium requires that its left operand (and therefore destination) must be the `eax` register. The highorder bits of the result (useless to a MiniJava program) are put into register `edx`. The solution is to move the operands and result explicitly; to implement $t_1 \leftarrow t_2 \times t_3$:

```
mov eax, $t_2$      eax $t_2$
mul $t_3$           eax ← eax × $t_3$;               edx ← garbage
mov $t_1$, eax      $t_1$ ← eax
```

   This looks very clumsy; but one job that the register allocator performs is to eliminate as many move instructions as possible. If the allocator can assign t_1 or t_3 (or both) to register `eax`, then it can delete one or both of the move instructions.

3. **Two-address instructions:** We solve this problem in the same way as we solve the previous one: by adding extra move instructions. To implement $t_1 \leftarrow t_2 + t_3$ we produce

```
mov $t_1$,$t_2$        $t_1$ ← $t_2$
add $t_1$, $t_3$       $t_1$ ← $t_1$ + $t_3$
```

   Then we hope that the register allocator will be able to allocate t_1 and t_2 to the same register, so that the move instruction will be deleted.

4. **Arithmetic operations can address memory:** The instruction selection phase turns every TEMP node into a "register" reference. Many of these "registers" will actually turn out to be memory locations. The *spill* phase of the register allocator must be made to handle this case efficiently; see Chapter 11.

   The alternative to using memory-mode operands is simply to fetch all the operands into registers before operating and store them back to memory afterwards. For example, these two sequences compute the same thing:

   ```
   mov eax, [ebp - 8]
   add eax, ecx                 add [ebp - 8], ecx
   mov [ebp - 8], eax
   ```

   The sequence on the right is more concise (and takes less machine-code space), but *the two sequences are equally fast*. The load, register-register add, and store take 1 cycle each, and the memory-register add takes 3 cycles. On a highly pipelined machine such as the Pentium Pro, simple cycle counts are not the whole story, but the result will be the same: The processor has to perform the load, add, and store, no matter how the instructions specify them.

   The sequence on the left has one significant disadvantage: It trashes the value in register eax. Therefore, we should try to use the sequence on the right when possible. But the issue here turns into one of register allocation, not of instruction speed; so we defer its solution to the register allocator.

5. **Several addressing modes:** An addressing mode that accomplishes six things typically takes six steps to execute. Thus, these instructions are often no faster than the multi-instruction sequences they replace. They have only two advantages: They "trash" fewer registers (such as the register eax in the previous example), and they have a shorter instruction encoding. With some work, treematching instruction selection can be made to select CISC addressing modes, but programs can be just as fast using the simple RISC-like instructions.
6. **Variable-length instructions:** This is not really a problem for the compiler; once the instructions are selected, it is a trivial (though tedious) matter for the assembler to emit the encodings.
7. **Instructions with side effects:** Some machines have an "autoincrement" memory fetch instruction whose effect is

$r_2 \leftarrow M[r_1];\ r_1 \leftarrow r_1 + 4$

This instruction is difficult to model using tree patterns, since it produces two results. There are three solutions to this problem:

a. Ignore the autoincrement instructions, and hope they go away. This is an increasingly successful solution, as few modern machines have multiple-side-effect instructions.
b. Try to match special idioms in an ad hoc way, within the context of a tree pattern-matching code generator.
c. Use a different instruction algorithm entirely, one based on DAG patterns instead of tree patterns.

Several of these solutions depend critically on the register allocator to eliminate move instructions and to be smart about spilling; see Chapter 11.

9.3 INSTRUCTION SELECTION FOR THE MiniJava COMPILER

Pattern-matching of "tiles" is simple (if tedious) in Java, as shown in Program 9.3. But this figure does not show what to do with each pattern match. It is all very well to print the name of the instruction, but which registers should these instructions use?

In a tree tiled by instruction patterns, the root of each tile will correspond to some intermediate result held in a register. Register allocation is the act of assigning register numbers to each such node.

The instruction selection phase can simultaneously do register allocation. However, many aspects of register allocation are independent of the particular target-machine instruction set, and it is a shame to duplicate the registerallocation algorithm for each target machine. Thus, register allocation should come either before or after instruction selection.

Before instruction selection, it is not even known which tree nodes will need registers to hold their results, since only the roots of tiles (and not other labeled nodes within tiles) require explicit registers. Thus, register allocation before instruction selection cannot be very accurate. But some compilers do it anyway, to avoid the need to describe machine instructions without the real registers filled in.

We will do register allocation after instruction selection. The instruction selection phase will generate instructions without quite knowing which registers the instructions use.

ABSTRACT ASSEMBLY LANGUAGE INSTRUCTIONS

We will invent a data type for "assembly language instruction without register assignments", called `Assem.Instr`:

```
package Assem;
import Temp.TempList;

public abstract class Instr {
  public String assem;
  public abstract TempList use();
  public abstract TempList def();
  public abstract Targets jumps();
  public String format(Temp.TempMap m);
}
public Targets(Temp.LabelList labels);

public OPER(String assem, TempList dst, TempList src,
            Temp.LabelList jump);
public OPER(String assem, TempList dst, TempList src);
public MOVE(String assem, Temp dst, Temp src);
public LABEL(String assem, Temp.Label label);
```

An OPER holds an assembly language instruction `assem`, a list of operand registers `src`, and a list of result registers `dst`. Any of these lists may be empty. Operations that always fall through to the next instruction are constructed with `OPER(assem,dst,src)` and the `jumps()` method will return `null`; other operations have a list of "target" labels to which they may jump (this list must explicitly include the next instruction if it is possible to fall through to it). The `use()` method returns the `src` list, and the `def()` method returns the `dst` list, either of which may be `null`.

A LABEL is a point in a program to which jumps may go. It has an `assem` component showing how the label will look in the assembly language program and a `label` component identifying which label symbol was used.

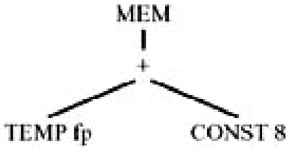
A MOVE is like an OPER, but must perform only data transfer. Then, if the `dst` and `src` temporaries are assigned to the same register, the MOVE can later be deleted. The `use()` method returns a singleton list `src`, and the `def()` method returns a singleton list `dst`.

Calling *i*.`format`(*m*) formats an assembly instruction as a string; *m* is an object implementing the `TempMap` interface, which contains a method to give the register assignment (or perhaps just the name) of every temp.

```
package Temp;
public interface TempMap {
    public String tempMap(Temp.Temp t);
}
```

Machine independence. The `Assem.Instr` class is *independent* of the chosen target-machine assembly language (though it is tuned for machines with only one class of register). If the target machine is a Sparc, then the `assem` strings will be Sparc assembly language. We will use *Jouette* assembly language for illustration.

For example, the tree



could be translated into *Jouette* assembly language as

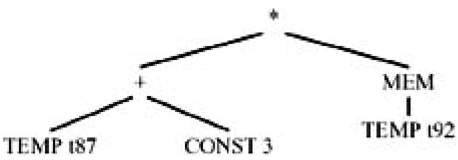
```
new OPER("LOAD 'd0 <- M['s0+8]",
         new TempList(new Temp(), null),
         new TempList(frame.FP(), null));
```

This instruction needs some explanation. The actual assembly language of *Jouette*, after register allocation, might be

```
LOAD r1 <- M[r27+8]
```

assuming that register r_{27} is the frame pointer `fp` and that the register allocator decided to assign the new temp to register r_1.Butthe `Assem` instruction does not know about register assignments; instead, it just talks of the sources and destination of each instruction. This LOAD instruction has one source register, which is referred to as `‘s0`, and one destination register, referred to as `‘d0`.

Another example will be useful. The tree



could be translated as

assem	*dst*	*src*
`ADDI ‘d0 <- ‘s0+3`	t908	t87
`LOAD ‘d0 <- M[‘s0+0]`	t909	t92
`MUL ‘d0 <- ‘s0*‘s1`	t910	t908,t909

where t908, t909, and t910 are temporaries newly chosen by the instruction selector.

After register allocation the assembly language might look like:

```
ADDI    r1 <- r12+3
LOAD    r2 <- M[r13+0]
MUL     r1 <- r1 * r2
```

The `string` of an `instr` may refer to *source registers* `‘s0`, `‘s1`, … `‘s`(k − 1), and *destination* registers `‘d0`, `‘d1`, etc. Jumps are OPER instructions that refer to labels `‘j0`, `‘j1`, etc. Conditional jumps, which may branch away or fall through, typically have two labels in the `jump` list but refer to only one of them in the `assem` string.

Two-address instructions Some machines have arithmetic instructions with two operands, where one of the operands is both a source and a destination. The instruction `add t1,t2`, which has the effect of $t_1 \leftarrow t_1 + t_2$, can be described as

```
assem          dst   src
 add ‘d0,‘s1   t1    t1, t2
```

where `‘s0` is implicitly, but not explicitly, mentioned in the `assem` string.

PRODUCING ASSEMBLY INSTRUCTIONS

Now it is a simple matter to write the right-hand sides of the pattern-matching clauses that "munch" `Tree` expressions into `Assem` instructions. We will show some examples from the *Jouette* code generator, but the same ideas apply to code generators for real machines.

The functions `munchStm` and `munchExp` will produce `Assem` instructions, bottom-up, as side effects. `MunchExp` returns the temporary in which the result is held.

```
Temp.Temp munchExp(Tree.Exp e);
void      munchStm(Tree.Stm s);
```

The "actions" of the `munchExp` clauses of Program 9.3 can be written as shown in Programs 9.5 and 9.6.

PROGRAM 9.5: `Assem`-instructions for `munchStm`.

```
TempList L(Temp h, TempList t) {return new TempList(h,t);}

munchStm(SEQ(a,b))
    {munchStm(a); munchStm(b);}
munchStm(MOVE(MEM(BINOP(PLUS,e1,CONST(i))),e2))
    emit(new OPER("STORE M['s0+" + i + "] <- 's1\n",
                null, L(munchExp(e1), L(munchExp(e2), null))));
```

```
munchStm(MOVE(MEM(BINOP(PLUS,CONST(i),e1)),e2))
    emit(new OPER("STORE M['s0+" + i + "] <- 's1\n",
                null, L(munchExp(e1), L(munchExp(e2), null))));
munchStm(MOVE(MEM(e1),MEM(e2)))
    emit(new OPER("MOVE M['s0] <- M['s1]\n",
                null, L(munchExp(e1), L(munchExp(e2), null))));
munchStm(MOVE(MEM(CONST(i)),e2))
    emit(new OPER("STORE M[r0+" + i + "] <- 's0\n",
                null, L(munchExp(e2), null)));
munchStm(MOVE(MEM(e1),e2))
    emit(new OPER("STORE M['s0] <- 's1\n",
                null, L(munchExp(e1), L(munchExp(e2), null))));
munchStm(MOVE(TEMP(i), e2))
    emit(new OPER("ADD 'd0 <- 's0 + r0\n",
                L(i,null), L(munchExp(e2), null)));
munchStm(LABEL(lab))
    emit(new Assem.LABEL(lab.toString() + ":\n", lab));
```

PROGRAM 9.6: Assem-instructions for munchExp.

```
munchExp(MEM(BINOP(PLUS,e1,CONST(i))))
   Temp r = new Temp();
   emit(new OPER("LOAD 'd0 <- M['s0+" + i + "]\n",
                  L(r,null), L(munchExp(e1),null)));
   return r;
munchExp(MEM(BINOP(PLUS,CONST(i),e1)))
   Temp r = new Temp();
   emit(new OPER("LOAD 'd0 <- M['s0+" + i + "]\n",
                  L(r,null), L(munchExp(e1),null)));
   return r;
munchExp(MEM(CONST(i)))
   Temp r = new Temp();
   emit(new OPER("LOAD 'd0 <- M[r0+" + i + "]\n",
          L(r,null), null));
   return r;
munchExp(MEM(e1))
   Temp r = new Temp();
   emit(new OPER("LOAD 'd0 <- M['s0+0]\n",
                  L(r,null), L(munchExp(e1),null)));
   return r;
munchExp(BINOP(PLUS,e1,CONST(i)))
   Temp r = new Temp();
   emit(new OPER("ADDI 'd0 <- 's0+" + i + "\n",
                  L(r,null), L(munchExp(e1),null)));
   return r;
munchExp(BINOP(PLUS,CONST(i),e1))
   Temp r = new Temp();
   emit(new OPER("ADDI 'd0 <- 's0+" + i + "\n",
                  L(r,null), L(munchExp(e1),null)));
   return r;
munchExp(CONST(i))
   Temp r = new Temp();
   emit(new OPER("ADDI 'd0 <- r0+" + i + "\n",
                  null, L(munchExp(e1),null)));
   return r;
munchExp(BINOP(PLUS,e1,e2))
   Temp r = new Temp();
   emit(new OPER("ADD 'd0 <- 's0+'s1\n",
                  L(r,null), L(munchExp(e1),L(munchExp(e2),null))));
   return r;
munchExp(TEMP(t))
   return t;
```

The `emit` function just accumulates a list of instructions to be returned later, as shown in Program 9.7.

PROGRAM 9.7: The `Codegen` class.

```
package Jouette;
public class Codegen {
   Frame frame;
   public Codegen(Frame f) {frame=f;}

   private Assem.InstrList ilist=null, last=null;

   private void emit(Assem.Instr inst) {
      if (last!=null)
         last = last.tail = new Assem.InstrList(inst,null);
      else last = ilist = new Assem.InstrList(inst,null);
   }
   void      munchStm(Tree.Stm s) { ... }
   Temp.Temp munchExp(Tree.Exp s)  { ... }

   Assem.InstrList codegen(Tree.Stm s) {
         Assem.InstrList l;
         munchStm(s);
         l=ilist;
         ilist=last=null;
         return l;
   }
}
package Frame;
public class Frame {
   ...
   public Assem.InstrList codegen(Tree.Stm stm); {
        return (new Codegen(this)).codegen(stm);
   }
}
```

PROCEDURE CALLS

Procedure calls are represented by EXP(CALL(*f*, *args*)), and function calls by MOVE(TEMP *t*, CALL(*f*, *args*)). These trees can be matched by tiles such as

```
munchStm(EXP(CALL(e,args)))
   {Temp r = munchExp(e); TempList l = munchArgs(0,args);
     emit(new OPER("CALL 's0\n",calldefs,L(r,l)));}
```

In this example, `munchArgs` generates code to move all the arguments to their correct positions, in outgoing parameter registers and/or in memory. The integer parameter to `munchArgs` is *i* for the *i*th argument; `munchArgs` will recur with *i* + 1 for the next argument, and so on.

What `munchArgs` returns is a list of all the temporaries that are to be passed to the machine's `CALL` instruction. Even though these temps are never written explicitly in assembly language, they should be listed as "sources" of the instruction, so that liveness analysis (Chapter 10) can see that their values need to be kept up to the point of call.

A CALL is expected to "trash" certain registers - the caller-save registers, the return-address register, and the return-value register. This list of calldefs should be listed as "destinations" of the CALL, so that the later phases of the compiler know that something happens to them here.

In general, any instruction that has the side effect of writing to another register requires this treatment. For example, the Pentium's multiply instruction writes to register edx with useless high-order result bits, so edx and eax are both listed as destinations of this instruction. (The high-order bits can be very useful for programs written in assembly language to do multiprecision arithmetic, but most programming languages do not support any way to access them.)

IF THERE'S NO FRAME POINTER

In a stack frame layout such as the one shown in Figure 6.2, the frame pointer points at one end of the frame and the stack pointer points at the other. At each procedure call, the stack pointer register is copied to the frame pointer register, and then the stack pointer is incremented by the size of the new frame.

Many machines' calling conventions do not use a frame pointer. Instead, the "virtual frame pointer" is always equal to stack pointer plus frame size. This saves time (no copy instruction) and space (one more register usable for other purposes). But our Translate phase has generated trees that refer to this fictitious frame pointer. The codegen function must replace any reference to FP+*k* with SP + *k* + *fs*, where *fs* is the frame size. It can recognize these patterns as it munches the trees.

However, to replace them it must know the value of *fs*, which cannot yet be known because register allocation is not known. Assuming the function *f* is to be emitted at label L14 (for example), codegen can just put sp+L14_framesize in its assembly instructions and hope that the prologue for *f* (generated by Frame.procEntryExit3) will include a definition of the assembly language constant L14_framesize. Codegen is passed the frame argument (Program 9.7) so that it can learn the name L14.

Implementations that have a "real" frame pointer won't need this hack and can ignore the frame argument to codegen. But why would an implementation use a real frame pointer when it wastes time and space to do so? The answer is that this permits the frame size to grow and shrink even after it is first created; some languages have permitted dynamic allocation of arrays within the stack frame (e.g., using alloca in C). Calling-convention designers now tend to avoid dynamically adjustable frame sizes, however.

PROGRAM INSTRUCTION SELECTION

Implement the translation to Assem-instructions for your favorite instruction set (let μ stand for *Sparc*, *Mips*, *Alpha*, *Pentium*, etc.) using maximal munch. If you would like to generate code for a RISC machine, but you have no RISC computer on which to test it, you may wish to use SPIM (a MIPS simulator implemented by James Larus), described on the Web page for this book.

First write the class μ.Codegen implementing the "maximal munch" translation algorithm from IR trees to the Assem data structure.

Use the Canon module (described in Chapter 8) to simplify the trees before applying your Codegen module to them. Use the format function to translate the resulting Assem trees to μ assembly language. Since you won't have done register assignment, just pass new Temp.DefaultMap() to format as the translation function from temporaries to strings.

```
package Temp;
public class DefaultMap implements TempMap {
        public String tempMap(Temp.Temp t) {
           return t.toString();
        }
}
```

This will produce "assembly" language that does not use register names at all: The instructions will use names such as t3, t283, and so on. But some of these temps are the "built-in" ones created by the Frame module to stand for particular machine registers (see page 143), such as Frame.FP. The assembly language will be easier to read if these registers appear with their natural names (e.g., fp instead of t1).

The Frame module must provide a mapping from the special temps to their names, and nonspecial temps to null:

```
package Frame;
public class Frame implements Temp.TempMap {
   ⋮
   abstract public String tempMap(Temp temp);
}
```

Then, for the purposes of displaying your assembly language prior to register allocation, make a new TempMap function that first tries frame.tempMap, and if that returns null, resorts to Temp.toString().

REGISTER LISTS

Make the following lists of registers; for each register, you will need a string for its assembly language representation and a Temp.Temp for referring to it in Tree and Assem data structures.

- specialregs a list of μ registers used to implement "special" registers such as RV and FP and also the stack pointer SP, the return-address register RA, and (on some machines) the zero register ZERO. Some machines may have other special registers;
- argregs a list of μ registers in which to pass outgoing arguments (including the static link);
- calleesaves a list of μ registers that the called procedure (callee) must preserve unchanged (or save and restore);
- callersaves a list of μ registers that the callee may trash.

The four lists of registers must not overlap, and must include any register that might show up in Assem instructions. These lists are not public, but they are useful internally for both Frame and Codegen - for example, to implement munchArgs and to construct the calldefs list.

Implement the procEntryExit2 function of the μ.Frame class.

```
package Frame;
```

```
class Frame implements Temp.TempMap {
   ⋮
   abstract public Assem.InstrList procEntryExit2(
                                Assem.InstrList body);
}
```

This function appends a "sink" instruction to the function body to tell the register allocator that certain registers are live at procedure exit. In the case of the *Jouette* machine, this is simply:

```
package Jouette;
class Frame extends Frame.Frame {
    ⋮
  static TempList returnSink =
                        L(ZERO, L(RA, L(SP, calleeSaves)));

  static Assem.InstrList append(Assem.InstrList a,
                                Assem.InstrList b) {
        if (a==null) return b;
        else {Assem.InstrList p;
              for(p=a; p.tail!=null; p=p.tail) {}
              p.tail=b;
              return a;
        }
  }
public Assem.InstrList procEntryExit2(
                             Assem.InstrList body) {
  return append(body,
    new Assem.InstrList(
          new Assem.OPER("", null, returnSink),null));
 }
}
```

meaning that the temporaries *zero*, *return-address*, *stack pointer*, and all the callee-saves registers are still live at the end of the function. Having *zero* live at the end means that it is live throughout, which will prevent the register allocator from trying to use it for some other purpose. The same trick works for any other special registers the machine might have.

Files available in `$MINIJAVA/chap9` include:

`Canon/*` Canonicalization and trace generation.

`Assem/*` The `Assem` module.

`Main/Main.java` A `Main` module that you may wish to adapt.

Your code generator will handle only the body of each procedure or function, but not the procedure entry/exit sequences. Use a "scaffold" version of `Frame.procEntryExit3` function:

```
package μ;
class Frame extends Frame.Frame {
   ⋮
   public Frame.Proc procEntryExit3(Assem.InstrList body) {
        return new Frame.Proc(
            "PROCEDURE " + name.toString() + "\n",
            body,
            "END " + name.toString() + "\n");
```

```
    }
}
```

FURTHER READING

Cattell [1980] expressed machine instructions as tree patterns, invented the maximal munch algorithm for instruction selection, and built a *code-generator generator* to produce an instruction selection function from a tree-pattern description of an instruction set. Glanville and Graham [1978] expressed the tree patterns as productions in LR(1) grammars, which allows the maximal munch algorithm to use multiple nonterminal symbols to represent different classes of registers and addressing modes. But grammars describing instruction sets are inherently ambiguous, leading to problems with the LR(1) approach; Aho et al. [1989] use dynamic programming to parse the tree grammars, which solves the ambiguity problem, and describe the *Twig* automatic code-generator generator. The dynamic programming can be done at compiler-construction time instead of code-generation time [Pelegri-Llopart and Graham 1988]; using this technique, the *BURG* tool [Fraser et al. 1992] has an interface similar to Twig's but generates code much faster.

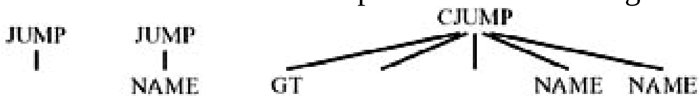
EXERCISES

- **9.1** For each of the following expressions, draw the tree and generate *Jouette*-machine instructions using maximal munch. Circle the tiles (as in Figure 9.2), but number them in the order that they are munched, and show the sequence of *Jouette* instructions that results.
    a. MOVE(MEM(+(+(CONST$_{1000}$, MEM(TEMP$_x$)), TEMP*fp*)), CONST$_0$)
    b. BINOP(MUL, CONST$_5$, MEM(CONST$_{100}$))
- ***9.2** Consider a machine with the following instruction:
- `mult const1(src1), const2(src2), dst3`
- $r_3 \leftarrow M[r_1 + \mathrm{const}_1] * M[r_2 + \mathrm{const}_2]$

  On this machine, r_0 is always 0, and *M*[1] always contains 1.

    a. Draw all the tree patterns corresponding to this instruction (and its special cases).
    b. Pick **one** of the bigger patterns and show how to write a Java if-statement to match it, with the `Tree` representation used for the MiniJava compiler.
- **9.3** The *Jouette* machine has control-flow instructions as follows:

BRANCHGE	if $ri \geq 0$ goto *L*
BRANCHLT	if $ri < 0$ goto *L*
BRANCHEQ	if $ri = 0$ goto *L*
BRANCHNE	if $ri \neq 0$ goto *L*
JUMP	goto r_i

- where the JUMP instruction goes to an address contained in a register.
- Use these instructions to implement the following tree patterns:
- 

- Assume that a CJUMP is always followed by its false label. Show the best way to implement each pattern; in some cases you may need to use more than one instruction or make up a new temporary. How do you implement CJUMP(GT, …) without a BRANCHGT instruction?

Chapter 10: Liveness Analysis

live: of continuing or current interest

Webster's Dictionary

OVERVIEW

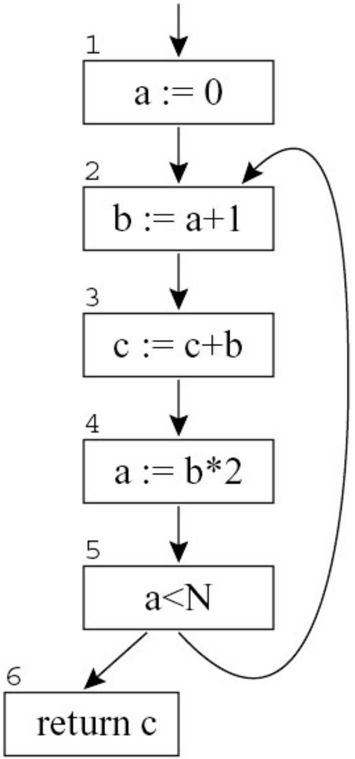
The front end of the compiler translates programs into an intermediate language with an unbounded number of temporaries. This program must run on a machine with a bounded number of registers. Two temporaries *a* and *b* can fit into the same register, if *a* and *b* are never "in use" at the same time. Thus, many temporaries can fit in few registers; if they don't all fit, the excess temporaries can be kept in memory.

Therefore, the compiler needs to analyze the intermediate-representation program to determine which temporaries are in use at the same time. We say a variable is *live* if it holds a value that may be needed in the future, so this analysis is called *liveness* analysis.

To perform analyses on a program, it is often useful to make a *control-flow graph*. Each statement in the program is a node in the flow graph; if statement *x* can be followed by statement *y*, there is an edge from *x* to *y*. Graph 10.1 shows the flow graph for a simple loop.

GRAPH 10.1: Control-flow graph of a program.

```
      $a \leftarrow 0$
$L_1: b \leftarrow a + 1$
      $c \leftarrow c + b$
      $a \leftarrow b * 2$
      if $a < N$ goto $L_1$
      return $c$
```



Let us consider the liveness of each variable (Figure 10.2). A variable is live if its current value will be used in the future, so we analyze liveness by working from the future to the past. Variable *b* is used in statement 4, so *b* is live on the 3 → 4 edge. Since statement 3 does not assign into *b*, then *b* is also live on the 2 → 3 edge. Statement 2 assigns into *b*. That means that the contents of *b* on the 1 → 2 edge are not needed by anyone; *b* is dead on this edge. So the *live range* of *b* is {2 → 3, 3 → 4}.

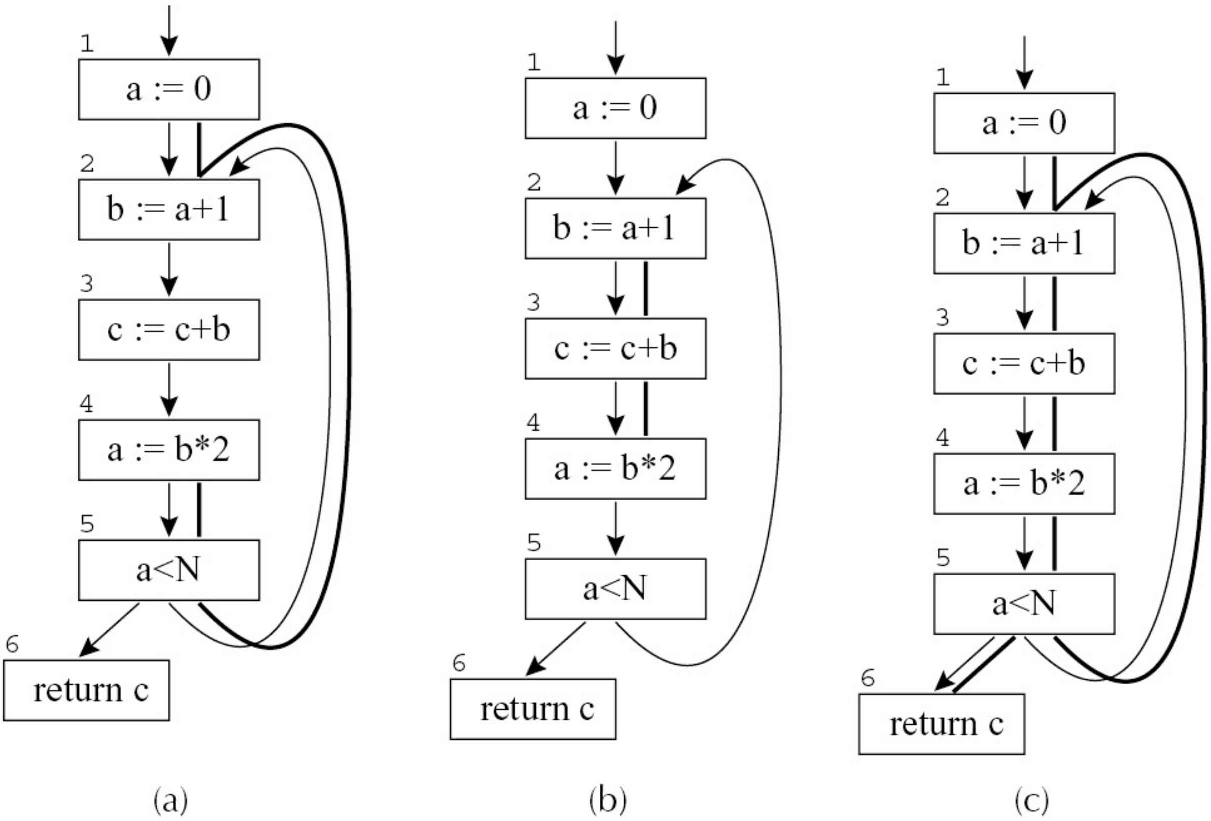


Figure 10.2: Liveness of variables *a*, *b*, *c*.

The variable *a* is an interesting case. It's live from 1 → 2, and again from 4 → 5 → 2, but not from 2 → 3 → 4. Although *a* has a perfectly well-defined value at node 3, that value will not be needed again before *a* is assigned a new value.

The variable *c* is live on entry to this program. Perhaps it is a formal parameter. If it is a local variable, then liveness analysis has detected an uninitialized variable; the compiler could print a warning message for the programmer.

Once all the live ranges are computed, we can see that only two registers are needed to hold *a*, *b*, and *c*, since *a* and *b* are never live at the same time. Register 1 can hold both *a* and *b*, and register 2 can hold *c*.

10.1 SOLUTION OF DATAFLOW EQUATIONS

Liveness of variables "flows" around the edges of the control-flow graph; determining the live range of each variable is an example of a *dataflow* problem. Chapter 17 will discuss several other kinds of dataflow problems.

Flow-graph terminology A flow-graph node has *out-edges* that lead to *successor* nodes, and *in-edges* that come from *predecessor* nodes. The set *pred*[*n*] is all the predecessors of node *n*, and *succ*[*n*] is the set of successors.

In Graph 10.1 the out-edges of node 5 are 5 → 6 and 5 → 2, and *succ*[5] = {2, 6}. The in-edges of 2 are 5 → 2 and 1 → 2, and *pred*[2] = {1, 5}.

Uses and defs An assignment to a variable or temporary *defines* that variable. An occurrence of a variable on the right-hand side of an assignment (or in other expressions) *uses* the variable. We can speak of the *def* of a variable as the set of graph nodes that define it; or the *def* of a graph node as the set of variables that it defines; and similarly for the *use* of a variable or graph node. In Graph 10.1, *def*(3) = {*c*}, *use*(3) = {*b, c*}.

Liveness A variable is *live* on an edge if there is a directed path from that edge to a *use* of the variable that does not go through any *def*. A variable is *live-in* at a node if it is live on any of the in-edges of that node; it is *live-out* at a node if it is live on any of the out-edges of the node.

CALCULATION OF LIVENESS

Liveness information (*live-in* and *live-out*) can be calculated from *use* and *def* as follows:

1. If a variable is in *use*[*n*], then it is *live-in* at node *n*. That is, if a statement uses a variable, the variable is live on entry to that statement.
2. If a variable is *live-in* at a node *n*, then it is *live-out* at all nodes *m* in *pred*[*n*].
3. If a variable is *live-out* at node *n*, and not in *def* [*n*], then the variable is also *live-in* at *n*. That is, if someone needs the value of *a* at the end of statement *n*, and *n* does not provide that value, then *a*'s value is needed even on entry to *n*.

These three statements can be written as Equations 10.3 on sets of variables. The live-in sets are an array *in*[*n*] indexed by node, and the live-out sets are an array *out*[*n*]. That is, *in*[*n*] is all the variables in *use*[*n*], plus all the variables in *out*[*n*] and not in *def* [*n*]. And *out*[*n*] is the union of the live-in sets of all successors of *n*.

EQUATIONS 10.3: Dataflow equations for liveness analysis.

$$in[n] = use[n] \cup (out[n] - def[n])$$

$$out[n] = \bigcup_{s \in succ[n]} in[s]$$

Algorithm 10.4 finds a solution to these equations by iteration. As usual, we initialize *in*[*n*] and *out*[*n*] to the the empty set {}, forall *n*, then repeatedly treat the equations as assignment statements until a fixed point is reached.

ALGORITHM 10.4: Computation of liveness by iteration.

```
for each n
   in[n] {}; out[n] {}
repeat
   for each n
      in′[n] → in[n]; out′[n] ← out[n]
      in[n] ← use[n] ∪ (out[n] – def[n])
      out[n] ← $\bigcup_{s\ in\ succ[n]}$in[s]
until in′[n] = in[n] and out′[n] = out[n] for all n
```

Table 10.5 shows the results of running the algorithm on Graph 10.1. The columns 1st, 2nd, etc., are the values of in and out on successive iterations of the **repeat** loop. Since the 7th column is the same as the 6th, the algorithm terminates.

Table 10.5: Liveness calculation following forward control-flow edges.

			1st		**2nd**		**3rd**		**4th**		**5th**		**6th**		**7th**	
	use	***def***	***in***	***out***	***in***	***out***	***in***	***out***	***in***	***out***	***in***	***out***	***in***	***out***	***in***	***out***
1		a				a		a		ac	c	ac	c	ac	c	ac
2	a	b	a		a	bc	ac	bc	ac	bc	ac	bc	ac	bc	ac	bc
3	bc	c	bc		bc	b	bc	b	bc	b	bc	b	bc	bc	bc	bc
4	b	a	b		b	a	b	a	b	ac	bc	ac	bc	ac	bc	ac
5	a		a	a	a	ac	ac	ac	ac	ac	ac	ac	ac	ac	ac	ac
6	c		c		c		c		c		c		c		c	

We can speed the convergence of this algorithm significantly by ordering the nodes properly. Suppose there is an edge 3 → 4 in the graph. Since *in*[4] is computed from *out*[4], and *out*[3] is computed from *in*[4], and so on, we should compute the in and out sets in the order *out*[4] → *in*[4] → *out*[3] → *in*[3]. But in Table 10.5, just the opposite order is used in each iteration! We have waited as long as possible (in each iteration) to make use of information gained from the previous iteration.

Table 10.6 shows the computation, in which each **for** loop iterates from 6 to 1 (approximately following the *reversed* direction of the flow-graph arrows), and in each iteration the out sets are computed before the in sets. By the end of the second iteration, the fixed point has been found; the third iteration just confirms this.

Table 10.6: Liveness calculation following reverse control-flow edges.

			1st		**2nd**		**3rd**	
	use	***def***	***out***	***in***	***out***	***in***	***out***	***in***
6	c			c		c		c
5	a		c	ac	ac	ac	ac	ac
4	b	a	ac	bc	ac	bc	ac	bc
3	bc	c	bc	bc	bc	bc	bc	bc
2	a	b	bc	ac	bc	ac	bc	ac
1		a	ac	c	ac	c	ac	c

When solving dataflow equations by iteration, the order of computation should follow the "flow." Since liveness flows *backward* along control-flow arrows, and from "out" to "in", so should the computation.

Ordering the nodes can be done easily by depth-first search, as shown in Section 17.4.

Basic blocks Flow-graph nodes that have only one predecessor and one successor are not very interesting. Such nodes can be merged with their predecessors and successors; what results is a graph with many fewer nodes, where each node represents a basic block. The algorithms that operate on flow graphs, such as liveness analysis, go much faster on the smaller graphs. Chapter 17 explains how to adjust the dataflow equations to use basic blocks. In this chapter we keep things simple.

One variable at a time Instead of doing dataflow "in parallel" using set equations, it can be just as practical to compute dataflow for one variable at a time as information about that variable is needed. For liveness, this would mean repeating the dataflow traversal once for each temporary. Starting from each *use* site of a temporary *t*, and tracing backward (following *predecessor* edges of the flow graph) using depth-first search, we note the liveness of *t* at each flow-graph node. The search stops at any definition of the temporary. Although this might seem expensive, many temporaries have very short live ranges, so the searches terminate quickly and do not traverse the entire flow graph for most variables.

REPRESENTATION OF SETS

There are at least two good ways to represent sets for dataflow equations: as arrays of bits or as sorted lists of variables.

If there are *N* variables in the program, the bit-array representation uses *N* bits for each set. Calculating the union of two sets is done by *or*-ing the corresponding bits at each position. Since computers can represent *K* bits per word (with $K = 32$ typical), one set-union operation takes N/K operations.

A set can also be represented as a linked list of its members, sorted by any totally ordered key (such as variable name). Calculating the union is done by merging the lists (discarding duplicates). This takes time proportional to the size of the sets being unioned.

Clearly, when the sets are sparse (fewer than N/K elements, on the average), the sorted-list representation is asymptotically faster; when the sets are dense, the bit-array representation is better.

TIME COMPLEXITY

How fast is iterative dataflow analysis?

A program of size *N* has at most *N* nodes in the flow graph, and at most *N* variables. Thus, each live-in set (or live-out set) has at most *N* elements; each set-union operation to compute live-in (or live-out) takes $O(N)$ time.

The **for** loop computes a constant number of set operations per flow-graph node; there are $O(N)$ nodes; thus, the **for** loop takes $O(N^2)$ time.

Each iteration of the **repeat** loop can only make each in or out set larger, never smaller. This is because the in and out sets are *monotonic* with respect to each other. That is, in the equation $in[n] = use[n] \cup (out[n] - def[n])$, a larger $out[n]$ can only make $in[n]$ larger. Similarly, in $out[n] = \cup_{s \text{ in } succ[n]} in[s]$, a larger $in[s]$ can only make $out[n]$ larger.

Each iteration must add something to the sets; but the sets cannot keep growing infinitely; at most every set can contain every variable. Thus, the sum of the sizes of all in and out sets is $2N^2$, which is the most that the repeat loop can iterate.

Thus, the worst-case run time of this algorithm is $O(N^4)$. Ordering the nodes using depth-first search (Algorithm 17.5, page 363) usually brings the number of **repeat**-loop iterations to two or three, and the live sets are often sparse, so the algorithm runs between $O(N)$ and $O(N^2)$ in practice.

Section 17.4 discusses more sophisticated ways of solving dataflow equations quickly.

LEAST FIXED POINTS

Table 10.7 illustrates two solutions (and a nonsolution!) to the Equations 10.3; assume there is another program variable *d* not used in this fragment of the program.

Table 10.7: *X* and *Y* are solutions to the liveness equations; *Z* is not a solution.

			X		Y		Z	
	use	***def***	***in***	***out***	***in***	***out***	***in***	***out***
1		a	c	ac	cd	acd	c	ac
2	a	b	ac	bc	acd	bcd	ac	b
3	bc	c	bc	bc	bcd	bcd	b	b
4	b	a	bc	ac	bcd	acd	b	ac
5	a		ac	ac	acd	acd	ac	ac
6	c		c		c		c	

In solution *Y*, the variable *d* is carried uselessly around the loop. But in fact, *Y* satisfies Equations 10.3 just as *X* does. What does this mean? Is *d* live or not?

The answer is that any solution to the dataflow equations is a *conservative approximation*. If the value of variable *a* will truly be needed in some execution of the program when execution reaches node *n* of the flow graph, then we can be assured that *a* is live-out at node *n* in any solution of the equations. But the converse is not true; we might calculate that *d* is live-out, but that doesn't mean that its value will really be used.

Is this acceptable? We can answer that question by asking what use will be made of the dataflow information. In the case of liveness analysis, if a variable is *thought to be live*, then we will make sure to have its value in a register. A conservative approximation of liveness is one that may erroneously believe a variable is live, but will never erroneously believe it is dead. The consequence of a conservative approximation is that the compiled code might use more registers than it really needs; but it will compute the right answer.

Consider instead the live-in sets *Z*, which fail to satisfy the dataflow equations. Using this *Z* we think that *b* and *c* are never live at the same time, and we would assign them to the same register. The resulting program would use an optimal number of registers but *compute the wrong answer*.

A dataflow equation used for compiler optimization should be set up so that any solution to it provides conservative information to the optimizer; imprecise information may lead to suboptimal but never incorrect programs.

Theorem Equations 10.3 have more than one solution.

Proof *X* and *Y* are both solutions.

Theorem All solutions to Equations 10.3 contain solution *X*. That is, if $in_X[n]$ and $in_Y[n]$ are the live-in sets for some node *n* in solutions *X* and *Y*, then $in_X[n] \subseteq in_Y[n]$.

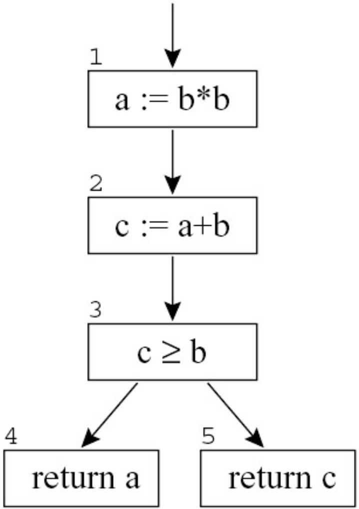
Proof See Exercise 10.2.

We say that *X* is the *least solution* to Equations 10.3. Clearly, since a bigger solution will lead to using more registers (producing suboptimal code), we want to use the least solution. Fortunately, Algorithm 10.4 always computes the least fixed point.

STATIC VS. DYNAMIC LIVENESS

A variable is live "if its value will be used in the future." In Graph 10.8, we know that $b \times b$ must be nonnegative, so that the test $c \geq b$ will be true. Thus, node 4 will never be reached, and *a*'s value will not be used after node 2; *a* is not live-out of node 2.

GRAPH 10.8: Standard static dataflow analysis will not take advantage of the fact that node 4 can never be reached.



But Equations 10.3 say that *a* is live-in to node 4, and therefore live-out of nodes 3 and 2. The equations are ignorant of which way the conditional branch will go. "Smarter" equations would permit *a* and *c* to be assigned the same register.

Although we can prove here that $b*b \geq 0$, and we could have the compiler look for arithmetic identities, no compiler can ever fully understand how all the control flow in every program will work. This is a fundamental mathematical theorem, derivable from the halting problem.

Theorem There is no program *H* that takes as input any program *P* and input *X* and (without infinite-looping) returns true if *P*(*X*) halts and false if *P*(*X*) infinite-loops.

Proof Suppose that there were such a program *H*; then we could arrive at a contradiction as follows. From the program *H*, construct the function *F*,

$$F(Y) = \textbf{if } H(Y, Y) \textbf{ then } (\textbf{while true do } ()) \textbf{ else true}$$

By the definition of *H*, if *F*(*F*) halts, then *H*(*F, F*) is true; so the **then** clause is taken; so the **while** loop executes forever; so *F*(*F*) does not halt. But if *F*(*F*) loops forever, then *H*(*F, F*) is false; so the **else** clause is taken; so *F*(*F*) halts. The program *F*(*F*) halts if it doesn't halt, and doesn't halt if it halts: a contradiction. Thus there can be no program *H* that tests whether another program halts (and always halts itself).

Corollary No program $H'(X, L)$ can tell, for any program *X* and label *L* within *X*, whether the label *L* is ever reached on an execution of *X*.

Proof From H' we could construct *H*. In some program that we want to test for halting, just let *L* be the end of the program, and replace all instances of the **halt** command with **goto** *L*.

Conservative approximation This theorem does not mean that we can *never* tell if a given label is reached or not, just that there is not a general algorithm that can *always* tell. We could improve our liveness analysis with some special-case algorithms that, in some cases, calculate more information about run-time control flow. But any such algorithm will come up against many cases where it simply cannot tell exactly what will happen at run time.

Because of this inherent limitation of program analysis, no compiler can really tell if a variable's value is truly needed - whether the variable is truly live. Instead, we have to make do with a conservative approximation. We assume that any conditional branch goes both ways. Thus, we have a dynamic condition and its static approximation:

- **Dynamic liveness** Avariable *a* is dynamically live at node *n* if some execution of the program goes from *n* to a use of *a* without going through any definition of *a*.
- **Static liveness** Avariable *a* is statically live at node *n* if there is some path of control-flow edges from *n* to some use of *a* that does not go through a definition of *a*.

Clearly, if *a* is dynamically live, it is also statically live. An optimizing compiler must allocate registers, and do other optimizations, on the basis of static liveness, because (in general) dynamic liveness cannot be computed.

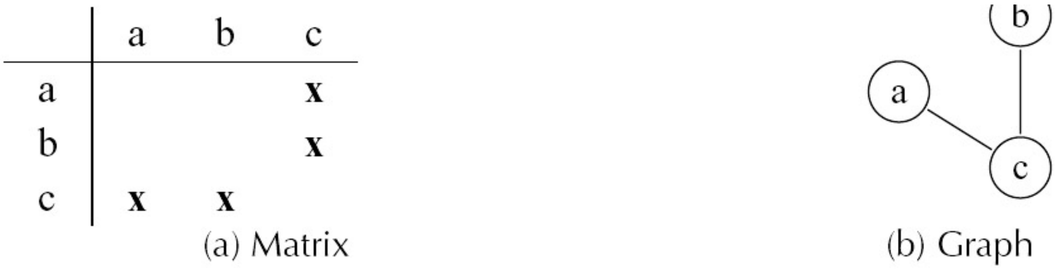
INTERFERENCE GRAPHS

Liveness information is used for several kinds of optimizations in a compiler. For some optimizations, we need to know exactly which variables are live at each node in the flow graph.

One of the most important applications of liveness analysis is for register allocation: We have a set of temporaries *a, b, c,…* that must be allocated to registers $r_1,\ldots, r_k$. A condition that prevents *a* and *b* from being allocated to the same register is called an *interference*.

The most common kind of interference is caused by overlapping live ranges: When *a* and *b* are both live at the same program point, then they cannot be put in the same register. But there are some other causes of interference: for example, when *a* must be generated by an instruction that cannot address register r_1, then *a* and r_1 interfere.

Interference information can be expressed as a matrix; Figure 10.9a has an **x** marking interferences of the variables in Graph 10.1. The interference matrix can also be expressed as an undirected graph (Figure 10.9b), with a node for each variable, and edges connecting variables that interfere.



	a	b	c
a			x
b			x
c	x	x	

(a) Matrix

(b) Graph

Figure 10.9: Representations of interference.

Special treatment of MOVE instructions In static liveness analysis, we can give MOVE instructions special consideration. It is important not to create artifical interferences between the source and destination of a move. Consider the program:

```
t ← s                     (copy)
   ⋮
x ← ... s ...             (use of s)
   ⋮
y ← ... t ...             (use of t)
```

After the copy instruction both *s* and *t* are live, and normally we would make an interference edge (*s*, *t*) since *t* is being defined at a point where *s* is live. But we do not need separate registers for *s* and *t*, since they contain the same value. The solution is just not to add an interference edge (*t*, *s*) in this case. Of course, if there is a later (nonmove) definition of *t* while *s* is still live, that will create the interference edge (*t, s*).

Therefore, the way to add interference edges for each new definition is

1. At any nonmove instruction that *defines* avariable *a*, where the *live-out* variables are $b_1,\dots, b_j$, add interference edges $(a, b_1),\dots,(a, b_j)$.
2. At a move instruction $a \leftarrow c$, where variables $b_1,\dots, b_j$ are *live-out*, add interference edges $(a, b_1),\dots,(a, b_j)$ for any b_i that is *not* the same as *c*.

10.2 LIVENESS IN THE MiniJava COMPILER

The flow analysis for the MiniJava compiler is done in two stages: First, the control flow of the `Assem` program is analyzed, producing a control-flow graph; then, the liveness of variables in the control-flow graph is analyzed, producing an interference graph.

GRAPHS

To represent both kinds of graphs, let's make a `Graph` abstract data type (Program 10.10).

PROGRAM 10.10: The `Graph` abstract data type.

```
package Graph;
```

```
public class Graph {
  public Graph();
  public NodeList nodes();
  public Node newNode();
  public void addEdge(Node from, Node to);
  public void rmEdge(Node from, Node to);
  public void show(java.io.PrintStream out);
}

public class Node {
    public Node(Graph g);
    public NodeList succ();
    public NodeList pred();
    public NodeList adj();
    public int outDegree();
    public int inDegree();
    public int degree();
    public boolean goesTo(Node n);
    public boolean comesFrom(Node n);
    public boolean adj(Node n);
    public String toString();
}
```

The constructor `Graph()` creates an empty directed graph; `g.newNode()` makes a new node within a graph *g*. A directed edge from *n* to *m* is created by `g.addEdge(n,m)`; after that, *m* will be found in the list `n.succ()` and *n* will be in `m.pred()`. When working with undirected graphs, the function `adj` is useful: *m*.`adj`() = *m*.`succ`() ∪ *m*.`pred`().

To delete an edge, use `rmEdge`. To test whether *m* and *n* are the same node, use `m==n`.

When using a graph in an algorithm, we want each node to represent something (an instruction in a program, for example). To make mappings from nodes to the things they are supposed to represent, we use a `Hashtable`. The following idiom associates information *x* with node *n* in a mapping `mytable`.

```
java.util.Dictionary mytable = new java.util.Hashtable();
  ... mytable.put(n,x);
```

CONTROL-FLOW GRAPHS

The `FlowGraph` package manages control-flow graphs. Each instruction (or basic block) is represented by a node in the flow graph. If instruction *m* can be followed by instruction *n* (either by a jump or by falling through), then there will be an edge (*m*, *n*) in the graph.

```
public abstract class FlowGraph extends Graph.Graph {
   public abstract TempList def(Node node);
   public abstract TempList use(Node node);
   public abstract boolean isMove(Node node);
   public void show(java.io.PrintStream out);
}
```

Each `Node` of the flow graph represents an instruction (or, perhaps, a basic block). The `def()` method tells what temporaries are defined at this node (destination registers of the instruction). `use()` tells what temporaries are used at this node (source registers of the instruction). isMove tells whether this instruction is a MOVE instruction, one that could be deleted if the def and use were identical.

The AssemFlowGraph class provides an implementation of FlowGraph for Assem instructions.

```
package FlowGraph;
public class AssemFlowGraph extends FlowGraph {
     public Instr instr(Node n);
     public AssemFlowGraph(Assem.InstrList instrs);
}
```

The constructor AssemFlowGraph takes a list of instructions and returns a flow graph. In making the flow graph, the jump fields of the instrs are used in creating control-flow edges, and the use and def information (obtained from the src and dst fields of the instrs) is attached to the nodes by means of the use and def methods of the flowgraph.

Information associated with the nodes For a flow graph, we want to associate some *use* and *def* information with each node in the graph. Then the liveness-analysis algorithm will also want to remember *live-in* and *live-out* information at each node. We could make room in the Node class to store all of this information. This would work well and would be quite efficient. However, it may not be very modular. Eventually we may want to do other analyses on flow graphs, which remember other kinds of information about each node. We may not want to modify the data structure (which is a widely used interface) for each new analysis.

Instead of storing the information *in* the nodes, a more modular approach is to say that a graph is a graph, and that a flow graph is a graph along with separately packaged auxiliary information (tables, or functions mapping nodes to whatever). Similarly, a dataflow algorithm on a graph does not need to modify dataflow information *in* the nodes, but modifies its own privately held mappings.

There may be a trade-off here between efficiency and modularity, since it may be faster to keep the information *in* the nodes, accessible by a simple pointer-traversal instead of a hash-table or search-tree lookup.

LIVENESS ANALYSIS

The RegAlloc package has an abstract class InterferenceGraph to indicate which pairs of temporaries cannot share a register:

```
package RegAlloc;
abstract public class InterferenceGraph extends Graph.Graph{
   abstract public Graph.Node tnode(Temp.Temp temp);
   abstract public Temp.Temp gtemp(Node node);
   abstract public MoveList moves();
   public int spillCost(Node node);
}
```

The method tnode relates a Temp to a Node, and gtemp is the inverse map. The method moves tells what MOVE instructions are associated with this graph (this is a hint about what pairs of temporaries to try to allocate to the same register). The spillCost(n) is an estimate of how many extra instructions would be executed if *n* were kept in memory instead of in registers; for a naive spiller, it suffices to return 1 for every *n*.

The class Liveness produces an interference graph from a flow graph:

```
package RegAlloc;
public class Liveness extends InterferenceGraph {
   public Liveness(FlowGraph flow);
}
```

In the implementation of the `Liveness` module, it is useful to maintain a data structure that remembers what is live at the exit of each flow-graph node:

```
private java.util.Dictionary liveMap =
                         new java.util.Hashtable();
```

where the keys are nodes and objects are `TempLists`. Given a flow-graph node *n*, the set of live temporaries at that node can be looked up in a global `liveMap`.

Having calculated a complete `liveMap`, we can now construct an interference graph. At each flow node *n* where there is a newly defined temporary $d \in def(n)$, and where temporaries $\{t_1, t_2; \ldots\}$ are in the `liveMap`, we just add interference edges (d, t_1), (d, t_2),…. For MOVEs, these edges will be safe but suboptimal; pages 213-214 describe a better treatment.

What if a newly defined temporary is not live just after its definition? This would be the case if a variable is defined but never used. It would seem that there's no need to put it in a register at all; thus it would not interfere with any other temporaries. But if the defining instruction is going to execute (perhaps it is necessary for some other side effect of the instruction), then it *will* write to some register, and that register had better not contain any other live variable. Thus, zero-length live ranges *do* interfere with any live ranges that overlap them.

PROGRAM CONSTRUCTING FLOW GRAPHS

Implement the `AssemFlowGraph` class that turns a list of `Assem` instructions into a flow graph. Use the abstract classes `Graph.Graph` and `Flow- Graph.FlowGraph` provided in `$MINIJAVA/chap10`.

PROGRAM LIVENESS

Implement the `Liveness` module. Use either the set-equation algorithm with the array-of-boolean or sorted-list-of-temporaries representation of sets, or the one-variable-at-a-time method.

EXERCISES

- **10.1** Perform flow analysis on the program of Exercise 8.6:
    a. Draw the control-flow graph.
    b. Calculate live-in and live-out at each statement.
    c. Construct the register interference graph.
- ****10.2** Prove that Equations 10.3 have a least fixed point and that Algorithm 10.4 always computes it.

  Hint: We know the algorithm refuses to terminate until it has a fixed point. The questions are whether (a) it must eventually terminate, and (b) the fixed point it computes is smaller than all other fixed points. For (a) show that the sets can only get bigger. For (b) show by induction that at any time the *in* and *out* sets are subsets of those in any possible fixed point. This is clearly true initially, when *in* and *out* are both empty; show that each step of the algorithm preserves the invariant.

- ***10.3** Analyze the asymptotic complexity of the one-variable-at-a-time method of computing dataflow information.
- ***10.4** Analyze the worst-case asymptotic complexity of making an interference graph, for a program of size *N* (with at most *N* variables and at most *N* control-flow nodes). Assume the dataflow analysis is already done and that *use*, *def*, and *live-out* information for each node can be queried in constant time. What representation of graph adjacency matrices should be used for efficiency?
- **10.5** The DEC Alpha architecture places the following restrictions on floating-point instructions, for programs that wish to recover from arithmetic exceptions:

1. Within a basic block (actually, in any sequence of instructions not separated by a trap-barrier instruction), no two instructions should write to the same destination register.
2. A source register of an instruction cannot be the same as the destination register of that instruction or any later instruction in the basic block.

$r_1 + r_5 \rightarrow r_4$	$r_1 + r_5 \rightarrow r_4$	$r_1 + r_5 \rightarrow r_3$	$r_1 + r_5 \rightarrow r_4$
$r_3 \times r_2 \rightarrow r_4$	$r_4 \times r_2 \rightarrow r_1$	$r_4 \times r_2 \rightarrow r_4$	$r_4 \times r_2 \rightarrow r_6$
violates rule 1.	*violates rule 2.*	*violates rule 2.*	*OK*

3. Show how to express these restrictions in the register interference graph.

Chapter 11: Register Allocation

reg-is-ter: a device for storing small amounts of data
al-lo-cate: to apportion for a specific purpose

Webster's Dictionary

OVERVIEW

The `Translate`, `Canon`, and `Codegen` phases of the compiler assume that there are an infinite number of registers to hold temporary values and that MOVE instructions cost nothing. The job of the register allocator is to assign the many temporaries to a small number of machine registers, and, where possible, to assign the source and destination of a MOVE to the same register so that the MOVE can be deleted.

From an examination of the control and dataflow graph, we derive an *interference graph*. Each node in the interference graph represents a temporary value; each edge (t_1, t_2) indicates a pair of temporaries that cannot be assigned to the same register. The most common reason for an interference edge is that t_1 and t_2 are live at the same time. Interference edges can also express other constraints; for example, if a certain instruction $a \leftarrow b \oplus c$ cannot produce results in register r_{12} on our machine, we can make a interfere with r_{12}.

Next we *color* the interference graph. We want to use as few colors as possible, but no pair of nodes connected by an edge may be assigned the same color. Graph coloring problems derive from the old mapmakers' rule that adjacent countries on a map should be colored with different colors. Our "colors" correspond to registers: If our target machine has K registers, and we can K -color the graph (color the graph with K colors), then the coloring is a valid register assignment for the interference graph. If there is no K -coloring, we will have to keep some of our variables and temporaries in memory instead of registers; this is called *spilling*.

11.1 COLORING BY SIMPLIFICATION

Register allocation is an *NP*-complete problem (except in special cases, such as expression trees); graph coloring is also *NP*-complete. Fortunately there is a linear-time approximation algorithm that gives good results; its principal phases are **Build**, **Simplify**, **Spill**, and **Select**.

Build: Construct the interference graph. We use dataflow analysis to compute the set of temporaries that are simultaneously live at each program point, and we add an edge to the graph for each pair of temporaries in the set. We repeat this for all program points.

Simplify: We color the graph using a simple heuristic. Suppose the graph G contains a node m with fewer than K neighbors, where K is the number of registers on the machine. Let G' be the graph $G - \{m\}$ obtained by removing m. If G' can be colored, then so can G, for when m is added to the colored graph G', the neighbors of m have at most $K - 1$ colors among them, so a free color can always be found for m. This leads naturally to a stack-based (or recursive) algorithm for coloring: We repeatedly remove (and push on a stack) nodes of degree less than K. Each such simplification will decrease the degrees of other nodes, leading to more opportunity for simplification.

Spill: Suppose at some point during simplification the graph *G* has nodes only of *significant degree*, that is, nodes of degree $\geq K$. Then the *simplify* heuristic fails, and we mark some node for spilling. That is, we choose some node in the graph (standing for a temporary variable in the program) and decide to represent it in memory, not registers, during program execution. An optimistic approximation to the effect of spilling is that the spilled node does not interfere with any of the other nodes remaining in the graph. It can therefore be removed and pushed on the stack, and the simplify process continued.

Select: We assign colors to nodes in the graph. Starting with the empty graph, we rebuild the original graph by repeatedly adding a node from the top of the stack. When we add a node to the graph, there must be a color for it, as the premise for removing it in the simplify phase was that it could always be assigned a color provided the remaining nodes in the graph could be successfully colored.

When *potential spill* node *n* that was pushed using the *Spill* heuristic is popped, there is no guarantee that it will be colorable: Its neighbors in the graph may be colored with *K* different colors already. In this case, we have an *actual spill*. We do not assign any color, but we continue the *Select* phase to identify other actual spills.

But perhaps some of the neighbors are the same color, so that among them there are fewer than *K* colors. Then we can color *n*, and it does not become an actual spill. This technique is known as *optimistic coloring*.

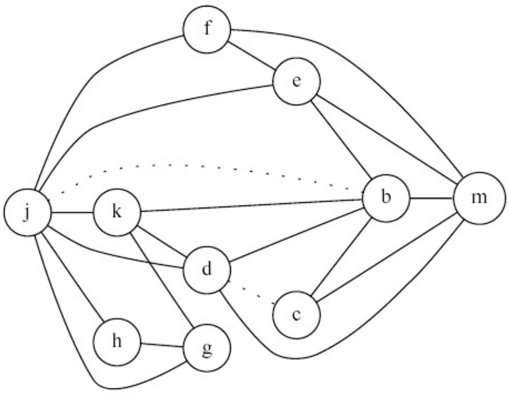
Start over: If the **Select** phase is unable to find a color for some node(s), then the program must be rewritten to fetch them from memory just before each use, and store them back after each definition. Thus, a spilled temporary will turn into several new temporaries with tiny live ranges. These will interfere with other temporaries in the graph. So the algorithm is repeated on this rewritten program. This process iterates until *simplify* succeeds with no spills; in practice, one or two iterations almost always suffice.

EXAMPLE

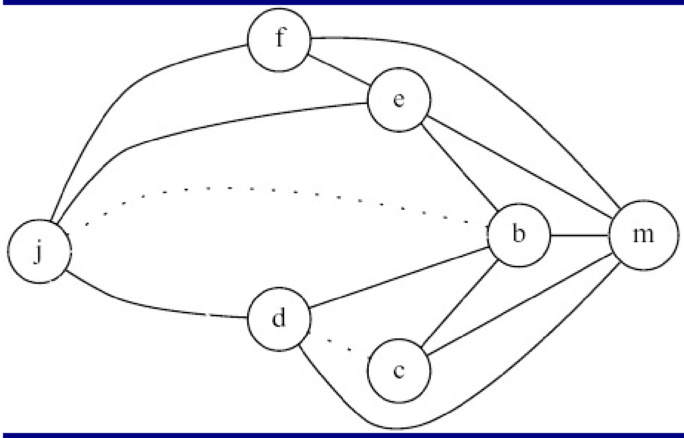
Graph 11.1 shows the interferences for a simple program. The nodes are labeled with the temporaries they represent, and there is an edge between two nodes if they are simultaneously live. For example, nodes `d`, `k`, and `j` are all connected since they are live simultaneously at the end of the block. Assuming that there are four registers available on the machine, then the simplify phase can start with the nodes `g`, `h`, `c`, and `f` in its working set, since they have less than four neighbors each. A color can always be found for them if the remaining graph can be successfully colored. If the algorithm starts by removing `h` and `g` and all their edges, then node `k` becomes a candidate for removal and can be added to the work list. Graph 11.2 remains after nodes `g`, `h`, and `k` have been removed. Continuing in this fashion a possible order in which nodes are removed is represented by the stack shown in Figure 11.3a, where the stack grows upward.

GRAPH 11.1: Interference graph for a program. Dotted lines are not interference edges but indicate move instructions.

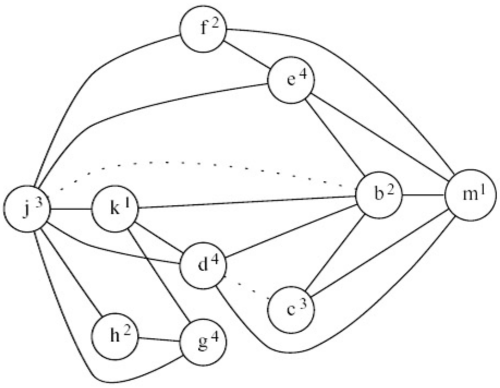
```
live-in:  k  j
       g  :=  mem[j+12]
       h  :=  k  -  1
       f  :=  g  *  h
       e  :=  mem[j+8]
       m  :=  mem[j+16]
       b  :=  mem[f]
       c  :=  e  +  8
       d  :=  c
       k  :=  m  +  4
       j  :=  b
live-out:  d  k  j
```



GRAPH 11.2: After removal of h, g, k.



m	1
c	3
b	2
f	2
e	4
j	3
d	4
k	1
h	2
g	4



(a) stack (b) assignment

Figure 11.3: Simplification stack, and a possible coloring.

The nodes are now popped off the stack and the original graph reconstructed and colored simultaneously. Starting with m, a color is chosen arbitrarily since the graph at this point consists of a singleton node. The next node to be put into the graph is c. The only constraint is that it be given a color different from m, since there is an edge from m to c. A possible assignment of colors for the reconstructed original graph is shown in Figure 11.3b.

11.2 COALESCING

It is easy to eliminate redundant move instructions with an interference graph. If there is no edge in the interference graph between the source and destination of a move instruction, then the move can be eliminated. The source and destination nodes are *coalesced* into a new node whose edges are the union of those of the nodes being replaced.

In principle, any pair of nodes not connected by an interference edge could be coalesced. This aggressive form of copy propagation is very successful at eliminating move instructions. Unfortunately, the node being introduced is more constrained than those being removed, as it contains a union of edges. Thus, it is quite possible that a graph, colorable with *K* colors before coalescing, may no longer be *K* -colorable after reckless coalescing. We wish to coalesce only where it is *safe* to do so, that is, where the coalescing will not render the graph uncolorable. Both of the following strategies are safe:

Briggs: Nodes *a* and *b* can be coalesced if the resulting node *ab* will have fewer than *K* neighbors of significant degree (i.e., having ≥ *K* edges). The coalescing is guaranteed not to turn a *K* -colorable graph into a non-*K* -colorable graph, because after the simplify phase has removed all the insignificantdegree nodes from the graph, the coalesced node will be adjacent only to those neighbors that were of significant degree. Since there are fewer than *K* of these, *simplify* can then remove the coalesced node from the graph. Thus if the original graph was colorable, the conservative coalescing strategy does not alter the colorability of the graph.

George: Nodes *a* and *b* can be coalesced if, for every neighbor *t* of *a*, either *t* already interferes with *b* or *t* is of insignificant degree. This coalescing is safe, by the following reasoning. Let *S* be the set of insignificant-degree neighbors of *a* in the original graph. If the coalescing were not done, *simplify* could remove all the nodes in *S*, leaving a reduced graph G_1. If the coalescing is done, then *simplify* can remove all the nodes in *S*, leaving a graph G_2. But G_2 is a subgraph of G_1 (the node *ab* in G_2 corresponds to the node *b* in G_1), and thus must be at least as easy to color.

These strategies are *conservative*, because there are still safe situations in which they will fail to coalesce. This means that the program may perform some unnecessary MOVE instructions - but this is better than spilling!

Interleaving simplification steps with conservative coalescing eliminates most move instructions, while still guaranteeing not to introduce spills. The coalesce, simplify, and spill procedures should be alternated until the graph is empty, as shown in Figure 11.4.

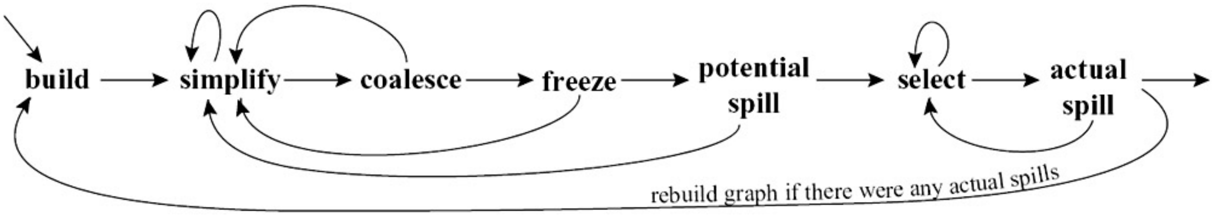


Figure 11.4: Graph coloring with coalescing.

These are the phases of a register allocator with coalescing:

Build: Construct the interference graph, and categorize each node as either *move-related* or *non-move-related*. A move-related node is one that is either the source or destination of a move instruction.

Simplify: One at a time, remove non-move-related nodes of low (< *K*) degree from the graph.

Coalesce: Perform conservative coalescing on the reduced graph obtained in the simplification phase. Since the degrees of many nodes have been reduced by *simplify*, the conservative strategy is likely to find many more moves to coalesce than it would have in the initial interference graph. After two nodes have been coalesced (and the move instruction deleted), if the resulting node is no longer move-related, it will be available for the next round of simplification. *Simplify* and *coalesce* are repeated until only significant-degree or move-related nodes remain.

Freeze: If neither *simplify* nor *coalesce* applies, we look for a move-related node of low degree. We *freeze* the moves in which this node is involved: That is, we give up hope of coalescing those moves. This causes the node (and perhaps other nodes related to the frozen moves) to be considered non-move-related, which should enable more simplification. Now, *simplify* and *coalesce* are resumed.

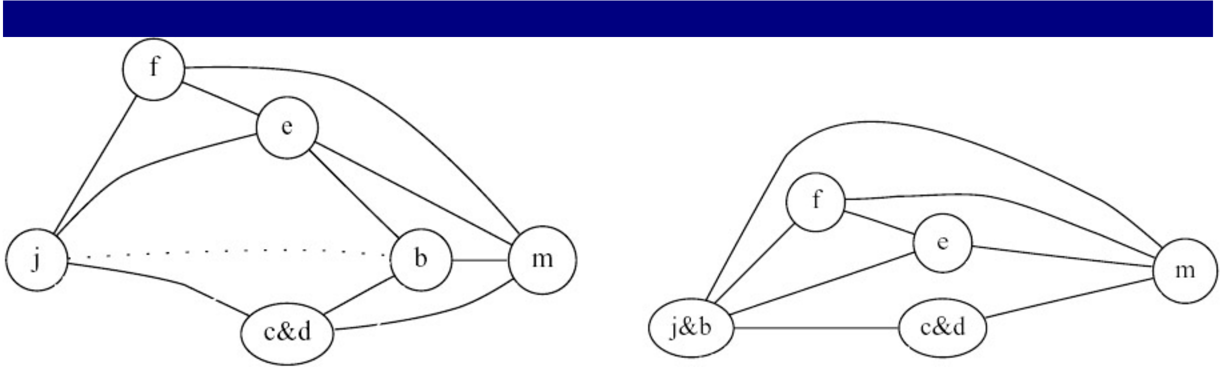
Spill: If there are no low-degree nodes, we select a significant-degree node for potential spilling and push it on the stack.

Select: Pop the entire stack, assigning colors.

Consider Graph 11.1; nodes b, c, d, and j are the only move-related nodes. The initial work list used in the simplify phase must contain only non-moverelated nodes and consists of nodes g, h, and f. Once again, after removal of g, h, and k we obtain Graph 11.2.

We could continue the simplification phase further; however, if we invoke a round of coalescing at this point, we discover that c and d are indeed coalesceable as the coalesced node has only two neighbors of significant degree: m and b. The resulting graph is shown in Graph 11.5a, with the coalesced node labeled as c&d.

GRAPH 11.5: (a) after coalescing c and d; (b) after coalescing b and j.



From Graph 11.5a we see that it is possible to coalesce b and j as well. Nodes b and j are adjacent to two neighbors of significant degree, namely m and e. The result of coalescing b and j is shown in Graph 11.5b.

After coalescing these two moves, there are no more move-related nodes, and therefore no more coalescing is possible. The simplify phase can be invoked one more time to remove all the remaining nodes. A possible assignment of colors is shown in Figure 11.6.

stack	coloring
e	1
m	2
f	3
j&b	4
c&d	1
k	2
h	2
g	1

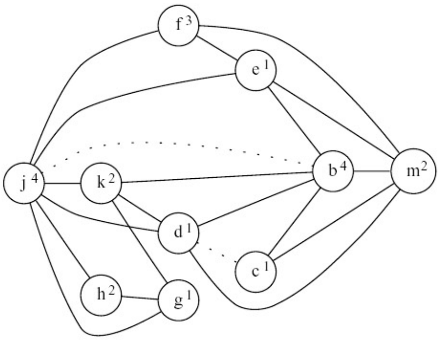


Figure 11.6: A coloring, with coalescing, for Graph 11.1.

Some moves are neither coalesced nor frozen. Instead, they are *constrained*. Consider the graph *x*, *y*, *z*, where (*x*, *z*) is the only interference edge and there are two moves $x \leftarrow y$ and $y \leftarrow z$. Either move is a candidate for coalescing. But after *x* and *y* are coalesced, the remaining move $xy \leftarrow z$ cannot be coalesced because of the interference edge (*xy*, *z*). We say this move is *constrained*, and we remove it from further consideration: It no longer causes nodes to be treated as move-related.

SPILLING

If spilling is necessary, *build* and *simplify* must be repeated on the whole program. The simplest version of the algorithm discards any coalescences found if *build* must be repeated. Then it is easy to see that coalescing does not increase the number of spills in any future round of *build*. A more efficient algorithm preserves any coalescences done *before the first potential spill was discovered*, but discards (uncoalesces) any coalescences done after that point.

Coalescing of spills On a machine with many registers (> 20), there will usually be few spilled nodes. But on a six-register machine (such as the Intel Pentium), there will be many spills. The front end may have generated many temporaries, and transformations such as SSA (described in Chapter 19) may split them into many more temporaries. If each spilled temporary lives in its own stack-frame location, then the frame may be quite large.

Even worse, there may be many move instructions involving pairs of spilled nodes. But to implement $a \leftarrow b$ when *a* and *b* are both spilled temporaries requires a fetch-store sequence, $t \leftarrow M[b_{loc}]$; $M[a_{loc}] \leftarrow t$. This is expensive, and also defines a temporary *t* that itself may cause other nodes to spill.

But many of the spill pairs are never live simultaneously. Thus, they may be graph-colored, with coalescing! In fact, because there is no fixed limit to the number of stack-frame locations, we can coalesce aggressively, without worrying about how many high-degree neighbors the spill nodes have. The algorithm is thus:

1. Use liveness information to construct the interference graph for spilled nodes.
2. While there is any pair of noninterfering spilled nodes connected by a move instruction, coalesce them.
3. Use *simplify* and *select* to color the graph. There is no (further) spilling in this coloring; instead, *simplify* just picks the lowest-degree node, and *select* picks the first available color, without any predetermined limit on the number of colors.
4. The colors correspond to activation-record locations for the spilled variables.

This should be done *before* generating the spill instructions and regenerating the register-temporary interference graph, so as to avoid creating fetch-store sequences for coalesced moves of spilled nodes.

11.3 PRECOLORED NODES

Some temporaries are *precolored* - they represent machine registers. The front end generates these when interfacing to standard calling conventions across module boundaries, for example. For each actual register that is used for some specific purpose, such as the frame pointer, standard-argument-1-register, standard-argument-2-register, and so on, the `Codegen` or `Frame` module should use the particular temporary that is permanently bound to that register (see also page 251). For any given color (that is, for any given machine register) there should be only one precolored node of that color.

The *select* and *coalesce* operations can give an ordinary temporary the same color as a precolored register, as long as they don't interfere, and in fact this is quite common. Thus, a standard calling-convention register can be reused inside a procedure as a temporary variable. Precolored nodes may be coalesced with other (non-precolored) nodes using conservative coalescing.

For a K-register machine, there will be K precolored nodes that all interfere with each other. Those of the precolored nodes that are not used explicitly (in a parameter-passing convention, for example) will not interfere with any ordinary (non-precolored) nodes; but a machine register used explicitly will have a live range that interferes with any other variables that happen to be live at the same time.

We cannot *simplify* a precolored node - this would mean pulling it from the graph in the hope that we can assign it a color later, but in fact we have no freedom about what color to assign it. And we should not spill precolored nodes to memory, because the machine registers are by definition *registers*. Thus, we should treat them as having "infinite" degree.

TEMPORARY COPIES OF MACHINE REGISTERS

The coloring algorithm works by calling *simplify*, *coalesce*, and *spill* until only the precolored nodes remain, and then the *select* phase can start adding the other nodes (and coloring them).

Because precolored nodes do not spill, the front end must be careful to keep their live ranges short. It can do this by generating MOVE instructions to move values to and from precolored nodes. For example, suppose r_7 is a callee-save register; it is "defined" at procedure entry and "used" at procedure exit. Instead of being kept in a precolored register throughout the procedure (Figure 11.7a), it can be moved into a fresh temporary and then moved back (Figure 11.7b). If there is *register pressure* (a high demand for registers) in this function, t_{231} will spill; otherwise t_{231} will be coalesced with r_7 and the MOVE instructions will be eliminated.

$$
\begin{array}{lll}
 & \text{enter:} & \text{def}(r_7) \\
 & & \\
(a) & & \vdots \\
 & & \\
 & \text{exit:} & \text{use}(r_7)
\end{array}
\qquad\qquad
\begin{array}{lll}
 & \text{enter:} & \text{def}(r_7) \\
 & & t_{231} \leftarrow r_7 \\
(b) & & \vdots \\
 & & r_7 \leftarrow t_{231} \\
 & \text{exit:} & \text{use}(r_7)
\end{array}
$$

Figure 11.7: Moving a callee-save register to a fresh temporary.

CALLER-SAVE AND CALLEE-SAVE REGISTERS

A local variable or compiler temporary that is not live across any procedure call should usually be allocated to a caller-save register, because in this case no saving and restoring of the register will be necessary at all. On the other hand, any variable that is live across several procedure calls should be kept in a callee-save register, since then only one save/restore will be necessary (on entry/exit from the calling procedure).

How can the register allocator allocate variables to registers using this criterion? Fortunately, a graph-coloring allocator can do this very naturally, as a byproduct of ordinary coalescing and spilling. All the callee-save registers are considered live on entry to the procedure, and are *used* by the return instruction. The CALL instructions in the `Assem` language have been annotated to *define* (interfere with) all the caller-save registers. If a variable is not live across a procedure call, it will tend to be allocated to a caller-save register.

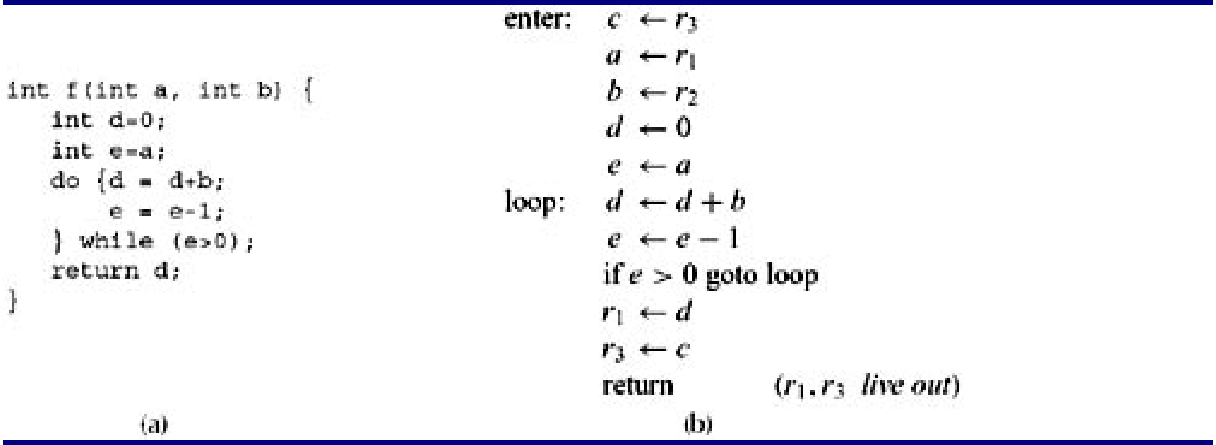
If a variable x is live across a procedure call, then it interferes with all the caller-save (precolored) registers, *and* it interferes with all the new temporaries (such as t_{231} in Figure 11.7) created for callee-save registers. Thus, a spill will occur. Using the common spill-cost heuristic that spills a node with high degree but few uses, the node chosen for spilling will not be x but t_{231}. Since t_{231} is spilled, r_7 will be available for coloring x (or some other variable). Essentially, the callee saves the callee-save register by spilling t_{231}.

EXAMPLE WITH PRECOLORED NODES

A worked example will illustrate the issues of register allocation with precolored nodes, callee-save registers, and spilling.

A C compiler is compiling Program 11.8a for a target machine with three registers; r_1 and r_2 are caller-save, and r_3 is callee-save. The code generator has therefore made arrangements to preserve the value of r_3 explicitly, by copying it into the temporary c and back again.

PROGRAM 11.8: A C function and its translation into instructions



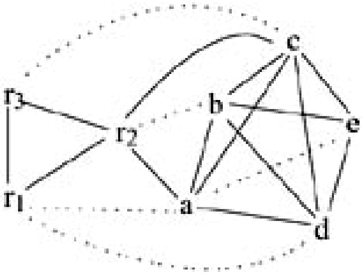
```
int f(int a, int b) {
   int d=0;
   int e=a;
   do {d = d+b;
       e = e-1;
   } while (e>0);
   return d;
}
```

(a)

```
enter:  c ← r3
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop:   d ← d + b
        e ← e − 1
        if e > 0 goto loop
        r1 ← d
        r3 ← c
        return          (r1, r3 live out)
```

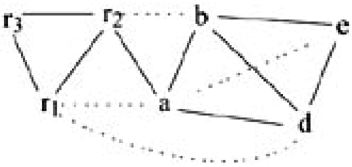
(b)

The instruction-selection phase has produced the instruction list of Program 11.8b. The interference graph for this function is shown at right.



The register allocation proceeds as follows (with $K = 3$):

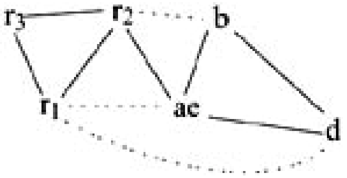
1. In this graph, there is no opportunity for *simplify* or *freeze* (because all the non-precolored nodes have degree $\geq K$). Any attempt to *coalesce* would produce a coalesced node adjacent to K or more significant-degree nodes. Therefore we must *spill* some node. We calculate spill priorities as follows:



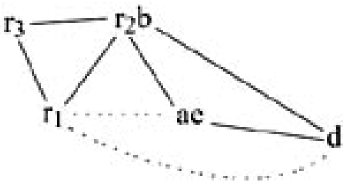
Node		Uses+Defs outside loop		Uses+Defs within loop		Degree		Spill priority
a	(	2	+ 10 ×	0	) /	4	=	0.50
b	(	1	+ 10 ×	1	) /	4	=	2.75
c	(	2	+ 10 ×	0	) /	6	=	0.33
d	(	2	+ 10 ×	2	) /	4	=	5.50
e	(	1	+ 10 ×	3	) /	3	=	10.33

2. Node *c* has the lowest priority - it interferes with many other temporaries but is rarely used - so it should be spilled first. Spilling *c*, we obtain the graph at right.

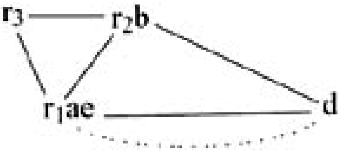
2. We can now coalesce *a* and *e*, since the resulting node will be adjacent to fewer than K significant-degree nodes (after coalescing, node *d* will be low-degree, though it is significant-degree right now). No other *simplify* or *coalesce* is possible now.



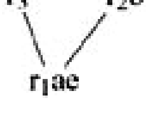
3. Now we could coalesce ae&r_1 or coalesce b&r_2. Let us do the latter.



4. We can now coalesce either ae&r_1 or coalesce d&r_1. Let us do the former.

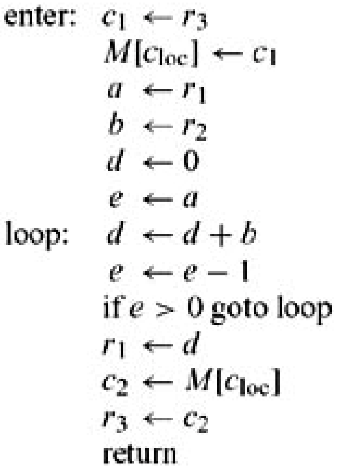


5. We cannot now coalesce r_1ae&d because the move is *constrained:* The nodes r_1ae and *d* interfere. We must *simplify d*.



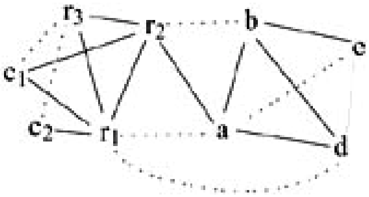
6. Now we have reached a graph with only precolored nodes, so we pop nodes from the stack and assign colors to them. First we pick *d*, which can be assigned color r_3. Nodes *a*, *b*, and *e* have already been assigned colors by coalescing. But node *c*, which was a *potential spill*, turns into an *actual spill* when it is popped from the stack, since no color can be found for it.

7. Since there was spilling in this round, we must rewrite the program to include spill instructions. For each use (or definition) of *c*, we make up a new temporary, and fetch (or store) it immediately beforehand (or afterward).

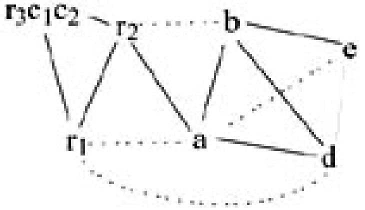


```
enter:  c1 ← r3
        M[c_loc] ← c1
        a ← r1
        b ← r2
        d ← 0
        e ← a
loop:   d ← d + b
        e ← e − 1
        if e > 0 goto loop
        r1 ← d
        c2 ← M[c_loc]
        r3 ← c2
        return
```

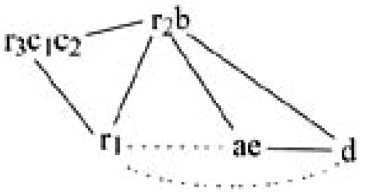
8. Now we build a new interference graph:



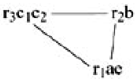
9. Graph-coloring proceeds as follows. We can immediately coalesce c_1&r_3 and then c_2&r_3.



10. Then, as before, we can coalesce *a*&*e* and then *b*&r_2.



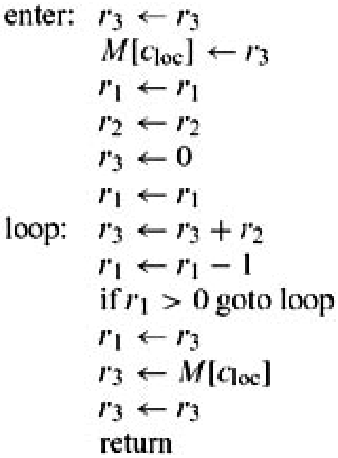
11. As before, we can coalesce *ae*&r_1 and then simplify *d*.



12. Now we start popping from the stack: We select color r_3 for *d*, and this was the only node on the stack - all other nodes were coalesced or precolored. The coloring is shown at right.

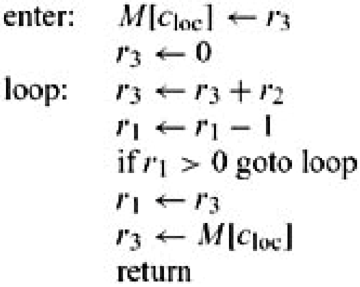
Node	**Color**
a	r_1
b	r_2
c	r_3
d	r_3
e	r_1

13. Now we can rewrite the program using the register assignment.



```
enter:  $r_3 \leftarrow r_3$
        $M[c_{loc}] \leftarrow r_3$
        $r_1 \leftarrow r_1$
        $r_2 \leftarrow r_2$
        $r_3 \leftarrow 0$
        $r_1 \leftarrow r_1$
loop:   $r_3 \leftarrow r_3 + r_2$
        $r_1 \leftarrow r_1 - 1$
        if $r_1 > 0$ goto loop
        $r_1 \leftarrow r_3$
        $r_3 \leftarrow M[c_{loc}]$
        $r_3 \leftarrow r_3$
        return
```

14. Finally, we can delete any move instruction whose source and destination are the same; these are the result of coalescing.



```
enter:  $M[c_{loc}] \leftarrow r_3$
        $r_3 \leftarrow 0$
loop:   $r_3 \leftarrow r_3 + r_2$
        $r_1 \leftarrow r_1 - 1$
        if $r_1 > 0$ goto loop
        $r_1 \leftarrow r_3$
        $r_3 \leftarrow M[c_{loc}]$
        return
```

The final program has only one uncoalesced move instruction.

11.4 GRAPH-COLORING IMPLEMENTATION

The graph-coloring algorithm needs to query the interference-graph data structure frequently. There are two kinds of queries:

1. Get all the nodes adjacent to node *X*; and
2. Tell if *X* and *Y* are adjacent.

An adjacency list (per node) can answer query 1 quickly, but not query 2 if the lists are long. A two-dimensional bit matrix indexed by node numbers can answer query 2 quickly, but not query 1. Therefore, we need both data structures to (redundantly) represent the interference graph. If the graph is very sparse, a hash table of integer pairs may be better than a bit matrix.

The adjacency lists of machine registers (precolored nodes) can be very large; because they're used in standard calling conventions, they interfere with any temporaries that happen to be live near *any* of the procedure-calls in the program. But we don't need to represent the adjacency list for a precolored node, because adjacency lists are used only in the *select* phase (which does not apply to precolored nodes) and in the Briggs coalescing test. To save space and time, we do not explicitly represent the adjacency lists of the machine registers. We coalesce an ordinary node *a* with a machine register *r* using the George coalescing test, which needs the adjacency list of *a* but not of *r*.

To test whether two ordinary (non-precolored) nodes can be coalesced, the algorithm shown here uses the Briggs coalescing test.

Associated with each move-related node is a count of the moves it is involved in. This count is easy to maintain and is used to test if a node is no longer move-related. Associated with all nodes is a count of the number of neighbors currently in the graph. This is used to determine whether a node is of significant degree during coalescing, and whether a node can be removed from the graph during simplification.

It is important to be able to quickly perform each *simplify* step (removing a low-degree non-move-related node), each *coalesce* step, and each *freeze* step. To do this, we maintain four work lists:

- Low-degree non-move-related nodes *(simplifyWorklist)*;
- Move instructions that might be coalesceable *(worklistMoves)*;
- Low-degree move-related nodes *(freezeWorklist)*;
- High-degree nodes *(spillWorklist)*.

Using these work lists, we avoid quadratic time blowup in finding coalesceable nodes.

DATA STRUCTURES

The algorithm maintains these data structures to keep track of graph nodes and move edges:

Node work lists, sets, and stacks The following lists and sets are always *mutually disjoint* and every node is always in exactly one of the sets or lists.

- **precolored:** machine registers, preassigned a color.
- **initial:** temporary registers, not precolored and not yet processed.
- **simplifyWorklist:** list of low-degree non-move-related nodes.
- **freezeWorklist:** low-degree move-related nodes.
- **spillWorklist:** high-degree nodes.
- **spilledNodes:** nodes marked for spilling during this round; initially empty.
- **coalescedNodes:** registers that have been coalesced; when $u \leftarrow v$ is coalesced, v is added to this set and u put back on some work list (or vice versa).
- **coloredNodes:** nodes successfully colored.
- **selectStack:** stack containing temporaries removed from the graph.

Since membership in these sets is often tested, the representation of each node should contain an enumeration value telling which set it is in. Since nodes must frequently be added to and removed from these sets, each set can be represented by a doubly linked list of nodes. Initially (on entry to Main), and on exiting RewriteProgram, only the sets *precolored* and *initial* are nonempty.

Move sets There are five sets of move instructions, and every move is in exactly one of these sets (after Build through the end of Main).

- **coalescedMoves:** moves that have been coalesced.
- **constrainedMoves:** moves whose source and target interfere.
- **frozenMoves:** moves that will no longer be considered for coalescing.
- **worklistMoves:** moves enabled for possible coalescing.
- **activeMoves:** moves not yet ready for coalescing.

Like the node work lists, the move sets should be implemented as doubly linked lists, with each move containing an enumeration value identifying which set it belongs to.

When a node *x* changes from significant to low-degree, the moves associated with its neighbors must be added to the move work list. Moves that were blocked with too many significant neighbors might now be enabled for coalescing.

Other data structures.

- **adjSet:** the set of interference edges (*u*, *v*) in the graph; if (*u*, *v*) 2 adjSet, then (*v*, *u*) ∈ adjSet.
- **adjList:** adjacency list representation of the graph; for each non-precolored temporary *u*, adjList[*u*] is the set of nodes that interfere with *u*.
- **degree:** an array containing the current degree of each node.
- **moveList:** a mapping from a node to the list of moves it is associated with.
- **alias:** when a move (*u*, *v*) has been coalesced, and *v* put in coalescedNodes, then alias (*v*) = *u*.
- **color:** the color chosen by the algorithm for a node; for precolored nodes this is initialized to the given color.

INVARIANTS

After Build, the following invariants always hold:

Degree invariant

$$\begin{aligned}&(u \in \mathrm{simplifyWorklist} \cup \mathrm{freezeWorklist} \cup \mathrm{spillWorklist}) \quad \Rightarrow \\ &\quad \mathrm{degree}(u) = |\mathrm{adjList}(u) \cap (\mathrm{precolored} \cup \mathrm{simplifyWorklist} \\ &\qquad\qquad\qquad\qquad\qquad \cup\, \mathrm{freezeWorklist} \cup \mathrm{spillWorklist})|\end{aligned}$$

Simplify worklist invariant Either *u* has been selected for spilling, or

$$\begin{aligned}&(u \in \mathrm{simplifyWorklist}) \quad \Rightarrow \\ &\quad \mathrm{degree}(u) < K \;\wedge\; \mathrm{moveList}[u] \cap (\mathrm{activeMoves} \cup \mathrm{worklistMoves}) = \{\}\end{aligned}$$

Freeze worklist invariant

$$\begin{aligned}&(u \in \mathrm{freezeWorklist}) \quad \Rightarrow \\ &\quad \mathrm{degree}(u) < K \;\wedge\; \mathrm{moveList}[u] \cap (\mathrm{activeMoves} \cup \mathrm{worklistMoves}) \neq \{\}\end{aligned}$$

Spill worklist invariant.

$$(u \in \mathrm{spillWorklist}) \quad \Rightarrow \quad \mathrm{degree}(u) \geq K$$

PROGRAM CODE

The algorithm is invoked using the procedure *Main*, which loops (via tail recursion) until no spills are generated.

```
procedure Main()
    LivenessAnalysis()
    Build() MakeWorklist()
    repeat
        if simplifyWorklist ≠ {} then Simplify()
        else if worklistMoves ≠ {} then Coalesce()
        else if freezeWorklist ≠ {} then Freeze()
        else if spillWorklist ≠ {} then SelectSpill()
    until simplifyWorklist = {} ∧ worklistMoves = {}
        ∧ freezeWorklist = {} ∧ spillWorklist = {}
    AssignColors()
    if spilledNodes ≠ {} then
        RewriteProgram(spilledNodes)
        Main()
```

If *AssignColors* spills, then *RewriteProgram* allocates memory locations for the spilled temporaries and inserts store and fetch instructions to access them. These stores and fetches are to newly created temporaries (with tiny live ranges), so the main loop must be performed on the altered graph.

```
procedure Build ()
    forall b ∈ blocks in program
        let live = liveOut(b)
        forall I ∈ instructions(b) in reverse order
            if isMoveInstruction(I) then
                live ← live/use(I
                forall n ∈ def(I) ∪ use(I)
                    moveList[n] ← moveList[n] ∪ {I}
                worklistMoves ← worklistMoves ∪ {I}
            live ← live ∪ def(I)
            forall d ∈ def(I)
                forall l ∈ live
                    AddEdge(l, d)
            live ← use(I) ∪ (live/def(I))
```

Procedure *Build* constructs the interference graph (and bit matrix) using the results of static liveness analysis, and also initializes the *worklistMoves* to contain all the moves in the program.

```
procedure AddEdge(u, v)
    if ((u, v) ∉ adjSet) ∧ (u ≠ v) then
        adjSet ← adjSet ∪[(u, v), (v, u)]
        if u ∉ precolored then
             adjList[u] ← adjList[u] ∪ {v}
             degree[u] ← degree[u] + 1
        if v ∉ precolored then
             adjList[v] ← adjList[v] ∪ {u}
             degree[v] ← degree[v] + 1

procedure MakeWorklist()
    forall n ∈ initial
        initial ← initial / {n}
        if degree[n] ≥ K then
            spillWorklist ← spillWorklist ∪ {n}
        else if MoveRelated(n) then
            freezeWorklist ← freezeWorklist ∪ {n}
        else
```

```
            simplifyWorklist ← simplifyWorklist ∪ {n}

function Adjacent(n)
    adjList[n] / (selectStack ∪ coalescedNodes)

function NodeMoves (n)
    moveList[n] ∩ (activeMoves ∪ worklistMoves)

function MoveRelated(n)
    NodeMoves(n) ≠ {}

procedure Simplify()
    let n ∈ simplifyWorklist
    simplifyWorklist ← simplifyWorklist / {n}
    push(n, selectStack)
    forall m ∈ Adjacent(n)
        DecrementDegree(m)
```

Removing a node from the graph involves decrementing the degree of its *current* neighbors. If the *degree* of a neighbor is already less than $K - 1$, then the neighbor must be move-related, and is not added to the `simplifyWorklist`. When the degree of a neighbor transitions from K to $K - 1$, moves associated with *its* neighbors may be enabled.

```
procedure DecrementDegree(m)
    let d = degree[m]
    degree[m] ← d-1
    if d = K then
        EnableMoves({m} ∪ Adjacent(m))
        spillWorklist ← spillWorklist / {m}
        if MoveRelated(m) then
             freezeWorklist ← freezeWorklist ∪ {m}
        else
             simplifyWorklist ← simplifyWorklist ∪ {m}

procedure EnableMoves(nodes)
    forall n ∈ nodes
        forall m ∈ NodeMoves(n)
            if m ∈ activeMoves then
                activeMoves ← activeMoves / {m}
                worklistMoves worklistMoves ∪ {m}
```

Only moves in the *worklistMoves* are considered in the coalesce phase. When a move is coalesced, it may no longer be move-related and can be added to the simplify work list by the procedure *AddWorkList*. *OK* implements the heuristic used for coalescing a precolored register. *Conservative* implements the conservative coalescing heuristic.

```
procedure AddWorkList(u)
    if (u ≠ precolored ∧ not(MoveRelated(u)) ∧ degree[u] < K) then
     freezeWorklist ← freezeWorklist / {u}
     simplifyWorklist ← simplifyWorklist ∪ {u}

function OK(t, r)
    degree[t] < K ∩ t ∈ precolored ∩ (t, r) ∈ adjSet
```

```
function Conservative(nodes)
    let k = 0
    forall n ∈ nodes
         if degree[n] ≥ K then k ← k + 1
    return (k < K)
procedure Coalesce()
    let m=copy(x, y)) ∈ worklistMoves
    x ← GetAlias(x)
    y ← GetAlias(y)
    if y ∈ precolored then
         let (u, v) = (y, x)
    else
         let (u, v) = (x, y)
    worklistMoves ← worklistMoves / {m}
    if (u = v) then
         coalescedMoves ← coalescedMoves ∪ {m}
         AddWorkList(u)
    else if v ∈ precolored ∩ (u, v) ∈ adjSet then
         constrainedMoves ← constrainedMoves ∪ {m}
         AddWorkList(u)
         AddWorkList(v)
    else if u ∈ precolored ∧ (∀t ∈ Adjacent(v, OK(t, u/)
              ∩  u ∉ precolored ∧
                 Conservative(Adjacent(u) ∪ Adjacent(v) then
         coalescedMoves ← coalescedMoves ∪ {m}
         Combine(u, v)
         AddWorkList(u)
    else
         activeMoves ← activeMoves ∪ {m}
procedure Combine(u, v)
    if v ∈ freezeWorklist then
         freezeWorklist freezeWorklist / {v}
    else
         spillWorklist ← spillWorklist / {v}
    coalescedNodes ← coalescedNodes ∪ {v}
    alias[v] ← u
    moveList[u] ← moveList[u] ∪ moveList[v]
    EnableMoves(v)
    forall t ∈ Adjacent(v)
         AddEdge(t,u)
         DecrementDegree(t)
    if degree[u] ≥ K ∧ u ∈ freezeWorkList
         freezeWorkList ← freezeWorkList / {u}
         spillWorkList ← spillWorkList ∪ {u}
function GetAlias (n)
    if n ∈ coalescedNodes then
         GetAlias(alias[n])
    else n

procedure Freeze()
    let u ∈ freezeWorklist
    freezeWorklist ← freezeWorklist / {u}
    simplifyWorklist ← simplifyWorklist ∪ {u}
    FreezeMoves(u)

procedure FreezeMoves(u)
    forall $m_{(=copy(x,\ y))}$ ∈ NodeMoves(u)
         if GetAlias(y)=GetAlias(u) then
             v ← GetAlias(x)
         else
             v ← GetAlias(y)
         activeMoves ← activeMoves / {m}
         frozenMoves ← frozenMoves ∪ {m}
         if v ∈ freezeWorklist ∧ NodeMoves(v) = {} then
             freezeWorklist ← freezeWorklist / {v}
             simplifyWorklist simplifyWorklist ∪ {v}

procedure SelectSpill()
     let m ∈ spillWorklist selected using favorite heuristic
         Note: avoid choosing nodes that are the tiny live ranges
         resulting from the fetches of previously spilled registers
    spillWorklist ← spillWorklist / {m}
    simplifyWorklist ← simplifyWorklist ∪ {m}
    FreezeMoves(m)
procedure AssignColors()
    while SelectStack not empty
         let n = pop(SelectStack)
         okColors ← {0,...,K-1}
         forall w ∈ adjList[n]
             if GetAlias(w) ∈ (coloredNodes ∪ precolored) then
                  okColors ← okColors / {color[GetAlias(w)]}
         if okColors Dfg then
             spilledNodes ← spilledNodes ∪ {n}
         else
             coloredNodes ← coloredNodes ∪ {n}
             let c ∈ okColors
             color[n] c
    forall n ∈ coalescedNodes
         color[n] ← color[GetAlias(n)]

procedure RewriteProgram()
    Allocate memory locations for each v ∈ spilledNodes,
    Create a new temporary $v_i$ for each definition and each use,
    In the program (instructions), insert a store after each
    definition of a $v_i$, a fetch before each use of a $v_i$.
    Put all the $v_i$ into a set newTemps.
    spilledNodes ← {}
    initial ← coloredNodes ∪ coalescedNodes ∪ newTemps
    coloredNodes ← {}
    coalescedNodes ← {}
```

We show a variant of the algorithm in which all coalesces are discarded if the program must be rewritten to incorporate spill fetches and stores. For a faster algorithm, keep all the coalesces found before the first call to `SelectSpill` and rewrite the program to eliminate the coalesced move instructions and temporaries.

In principle, a heuristic could be used to select the freeze node; the *Freeze* shown above picks an arbitrary node from the freeze work list. But freezes are not common, and a selection heuristic is unlikely to make a significant difference.

11.5 REGISTER ALLOCATION FOR TREES

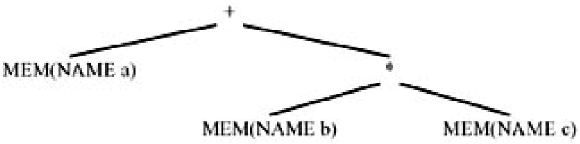
Register allocation for expression trees is much simpler than for arbitrary flow graphs. We do not need global dataflow analysis or interference graphs. Suppose we have a tiled tree such as in Figure 9.2a. This tree has two *trivial* tiles, the TEMP nodes *fp* and *i*, which we assume are already in registers r_{fp} and r_i . We wish to label the roots of the nontrivial tiles (the ones corresponding to instructions, i.e., 2, 4, 5, 6, 8) with registers from the list $r_1, r_2, \ldots, r_k$.

Algorithm 11.9 traverses the tree in postorder, assigning a register to the root of each tile. With *n* initialized to zero, this algorithm applied to the root (tile 9) produces the allocation {tile2 ↦ r_1, tile4 ↦ r_2, tile5 ↦ r_2, tile6 ↦ r_1, tile8 ↦ r_2, tile9 ↦ r_1}. The algorithm can be combined with Maximal Munch, since both algorithms are doing the same bottom-up traversal.

ALGORITHM 11.9: Simple register allocation on trees.

```
function SimpleAlloc(t)
   for each nontrivial tile u that is a child of t
       SimpleAlloc(u)
   for each nontrivial tile u that is a child of t
       n ← n - 1
   n ← n + 1
   assign $r_n$ to hold the value at the root of t
```

But this algorithm will not always lead to an optimal allocation. Consider the following tree, where each tile is shown as a single node:



The SimpleAlloc function will use three registers for this expression (as shown at left on the next page), but by reordering the instructions we can do the computation using only two registers (as shown at right):

$r_1 \leftarrow M[a]$ $r_1 \leftarrow M[b]$

$r_2 \leftarrow M[b]$ $r_2 \leftarrow M[c]$

$r_3 \leftarrow M[c]$ $r_1 \leftarrow r_1 \times r_2$

$r_2 \leftarrow r_2 \times r_3$ $r_2 \leftarrow M[a]$

$r_1 \leftarrow r_1 + r_2$ $r_1 \leftarrow r_2 + r_1$

Using dynamic programming, we can find the optimal ordering for the instructions. The idea is to label each tile with the number of registers it needs during its evaluation. Suppose a tile *t* has two nontrivial children u_{left} and u_{right} that require n and m registers, respectively, for their evaluation. If we evaluate u_{left} first, and hold its result in one register while we evaluate u_{right}, then we have needed max(n, 1 + m) registers for the whole expression rooted at t. Conversely, if we evaluate u_{right} first, then we need max(1 + n, m) registers. Clearly, if $n > m$, we should evaluate u_{left} first, and if $n < m$, we should evaluate u_{right} first. If $n = m$, we will need $n + 1$ registers no matter which subexpression is evaluated first.

Algorithm 11.10 labels each tile *t* with *need*[*t*], the number of registers needed to evaluate the subtree rooted at *t*. It can be generalized to handle tiles with more than two children. Maximal Munch should identify - but not emit - the tiles, simultaneously with the labeling of Algorithm 11.10. The next pass emits *Assem* instructions for the tiles; wherever a tile has more than one child, the subtrees must be emitted in decreasing order of register *need*.

ALGORITHM 11.10: Sethi-Ullman labeling algorithm.

```
function Label($t$)
   for each tile $u$ that is a child of $t$
       Label($u$)
   if $t$ is trivial
       then $need[t] \leftarrow 0$
   else if $t$ has two children, $u_{left}$ and $u_{right}$
       then if $need[u^{left}] = need[u_{right}]$
               then $need[t] \leftarrow 1 + need[u_{left}]$
               else $need[t] \leftarrow \max(1, need[u_{left}], need[u_{right}])$
   else if $t$ has one child, $u$
       then $need[t] \leftarrow \max(1, need[u]$
   else if $t$ has no children
       then $need[t] \leftarrow 1$
```

Algorithm 11.10 can profitably be used in a compiler that uses graph-coloring register allocation. Emitting the subtrees in decreasing order of *need* will minimize the number of simultaneously live temporaries and reduce the number of spills.

In a compiler without graph-coloring register allocation, Algorithm 11.10 is used as a pre-pass to Algorithm 11.11, which assigns registers as the trees are emitted and also handles spilling cleanly. This takes care of register allocation for the internal nodes of expression trees; allocating registers for explicit TEMPsofthe *Tree* language would have to be done in some other way. In general, such a compiler would keep almost all program variables in the stack frame, so there would not be many of these explicit TEMPs to allocate.

ALGORITHM 11.11: Sethi-Ullman register allocation for trees.

```
function SethiUllman($t$, $n$)
   if $t$ has two children, $u_{left}$ and $u_{right}$
      if $need[u_{left}] \geq K$ and $need[u_{right}] \geq K$
          SethiUllman($u_{right}$, 0)
          $n \leftarrow n - 1$
          spill: emit instruction to store $reg[u_{right}]$
          SethiUllman($u_{left}$, 0)
          unspill: $reg[u_{right}] \leftarrow$ "$r_1$"; emit instruction to fetch $reg[u_{right}]$
      else if $need[u_{left}] \geq need[u_{right}]$
          SethiUllman($u_{left}$, $n$)
          SethiUllman($u_{right}$, $n + 1$)
      else $need[u_{left}] < need[u_{right}]$
          SethiUllman($u_{right}$, $n$)
          SethiUllman($u_{left}$, $n$)
      $reg[t] \leftarrow$ "$r_n$"
      emit OPER($instruction[t]$, $reg[t]$, [ $reg[u_{left}]$, $reg[u_{right}]$])
   else if $t$ has one child, $u$
      SethiUllman($u$, $n$)
      $reg[t] \leftarrow$ "$r_n$"
      emit OPER($instruction[t]$, $reg[t]$, [$reg[u]$])
   else if $t$ is nontrivial but has no children
      $reg[t] \leftarrow$ "$r_n$"
      emit OPER($instruction[t]$, $reg[t]$, [])
   else if $t$ is a trivial node TEMP($r_i$)
      $reg[t] \leftarrow$ "$r_i$"
```

PROGRAM GRAPH COLORING

Implement graph-coloring register allocation as two modules: `Color`, which does just the graph coloring itself, and `RegAlloc`, which manages spilling and calls upon `Color` as a subroutine. To keep things simple, do not implement spilling or coalescing; this simplifies the algorithm considerably.

```
package RegAlloc;

public class RegAlloc implements Temp.TempMap {
  public Assem.InstrList instrs;
  public String tempMap(Temp temp);
  public RegAlloc(Frame.Frame f, Assem.InstrList il);
}

class Color implements TempMap {
  public TempList spills();
  public String tempMap(Temp t);
  public Color(InterferenceGraph ig,
               TempMap initial,
               TempList registers);
}
```

Given an interference graph, an `initial` allocation (precoloring) of some temporaries imposed by calling conventions, and a list of colors (`registers`), `color` produces an extension of the `initial` allocation. The resulting allocation assigns all temps used in the flow graph, making use of registers from the `registers` list.

The `initial` allocation is the `frame` (which implements a `TempMap` describing precolored temporaries); the `registers` argument is just the list of all machine registers, `Frame.registers` (see page 251). The registers in the `initial` allocation can also appear in the `registers` argument to `Color`, since it's OK to use them to color other nodes as well.

The result of `Color` is a `TempMap` (that is, `Color implements TempMap`) describing the register allocation, along with a list of spills. The result of `RegAlloc` - if there were no spills - is an identical `TempMap`, which can be used in final assembly-code emission as an argument to `Assem.format`.

A better `Color` interface would have a `spillCost` argument that specifies the spilling cost of each temporary. This can be just the number of uses and defs, or better yet, uses and defs weighted by occurrence in loops and nested loops. A naive `spillCost` that just returns 1 for every temporary will also work.

A simple implementation of the coloring algorithm without coalescing requires only one work list: the `simplifyWorklist`, which contains all non-precolored, nonsimplified nodes of degree less than K . Obviously, no `freezeWorklist` is necessary. No `spillWorklist` is necessary either, if we are willing to look through all the nodes in the original graph for a spill candidate every time the `simplifyWorklist` becomes empty.

With only a `simplifyWorklist`, the doubly linked representation is not necessary: This work list can be implemented as a singly linked list or a stack, since it is never accessed "in the middle."

ADVANCED PROJECT: SPILLING

Implement spilling, so that no matter how many parameters and locals a MiniJava program has, you can still compile it.

ADVANCED PROJECT: COALESCING

Implement coalescing, to eliminate practically all the MOVE instructions from the program.

FURTHER READING

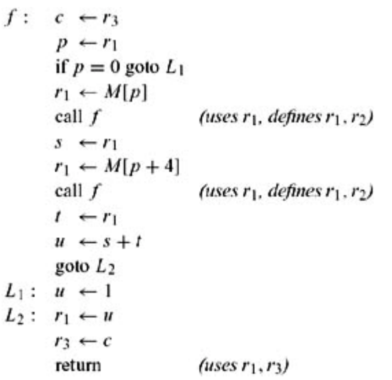
Kempe [1879] invented the simplification algorithm that colors graphs by removing vertices of degree < K. Chaitin [1982] formulated register allocation as a graph-coloring problem - using Kempe's algorithm to color the graph - and performed copy propagation by (nonconservatively) coalescing nonin- terfering move-related nodes before coloring the graph. Briggs et al. [1994] improved the algorithm with the idea of optimistic spilling, and also avoided introducing spills by using the conservative coalescing heuristic before coloring the graph. George and Appel [1996] found that there are more opportunities for coalescing if conservative coalescing is done during simplification instead of beforehand, and developed the work-list algorithm presented in this chapter.

Ershov [1958] developed the algorithm for optimal register allocation on expression trees; Sethi and Ullman [1970] generalized this algorithm and showed how it should handle spills.

EXERCISES

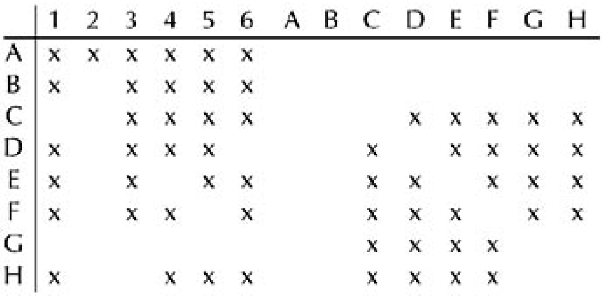
- **11.1** The following program has been compiled for a machine with three registers r_1, r_2, r_3; r_1 and r_2 are (caller-save) argument registers and r_3 is a callee-save register. Construct the interference graph and show the steps of the register allocation process in detail, as on pages 229−232. When you coalesce two nodes, say whether you are using the Briggs or George criterion.

  Hint: When two nodes are connected by an interference edge andamove edge, you may delete the move edge; this is called constrain and is accomplished by the first **else if** clause of procedure Coalesce.



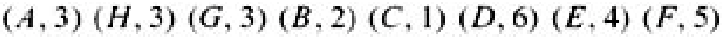
```
f :    $c \leftarrow r_3$
       $p \leftarrow r_1$
       if $p = 0$ goto $L_1$
       $r_1 \leftarrow M[p]$
       call $f$                (uses $r_1$, defines $r_1$, $r_2$)
       $s \leftarrow r_1$
       $r_1 \leftarrow M[p+4]$
       call $f$                (uses $r_1$, defines $r_1$, $r_2$)
       $t \leftarrow r_1$
       $u \leftarrow s + t$
       goto $L_2$
$L_1$ :  $u \leftarrow 1$
$L_2$ :  $r_1 \leftarrow u$
       $r_3 \leftarrow c$
       return                 (uses $r_1$, $r_3$)
```

- **11.2** The table below represents a register-interference graph. Nodes 1−6 are precolored (with colors 1−6), and nodes A−H are ordinary (non-precolored). Every pair of precolored nodes interferes, and each ordinary node interferes with nodes where there is an × in the table.

	1	2	3	4	5	6	A	B	C	D	E	F	G	H
A	x	x	x	x	x	x								
B	x		x	x	x	x								
C			x	x	x	x				x	x	x	x	x
D	x		x	x	x				x		x	x	x	x
E	x		x		x	x			x	x		x	x	x
F	x		x	x		x			x	x	x		x	x
G									x	x	x	x		
H	x			x	x	x			x	x	x	x		

  The following pairs of nodes are related by MOVE instructions:

  $(A,3)$ $(H,3)$ $(G,3)$ $(B,2)$ $(C,1)$ $(D,6)$ $(E,4)$ $(F,5)$

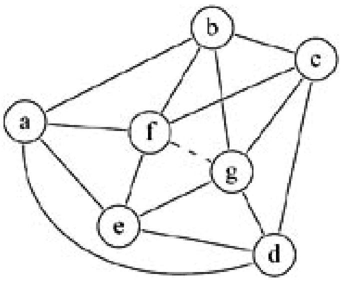
  Assume that register allocation must be done for an 8-register machine.

  - a. Ignoring the MOVE instructions, and without using the *coalesce* heuristic, color this graph using *simplify* and *spill*. Record the sequence (stack) of *simplify* and *potential-spill* decisions, show which potential spills become actual spills, and show the coloring that results.
  - b. Color this graph using coalescing. Record the sequence of *simplify, coalesce, freeze,* and *spill* decisions. Identify each *coalesce* as Briggs- or George-style. Show how many MOVE instructions remain.
  - *c. Another coalescing heuristic is biased *coloring*. Instead of using a *conservative coalescing* heuristic during simplification, run the *simplify-spill* part of the algorithm as in part (a), but in the selectpart of the algorithm,
    - i. When selecting a color for node *X* that is move-related to node *Y*, when a color for *Y* has already been selected, use the same color if possible (to eliminate the MOVE).
    - ii. When selecting a color for node *X* that is move-related to node *Y*, when a color for *Y* has not yet been selected, use a color that is not the same as the color of any of *Y* 's neighbors (to increase the chance of heuristic (i) working when *Y* is colored).

    Conservative coalescing (in the *simplify* phase) has been found to be more effective than biased coloring, in general; but it might not be on this particular graph. Since the two coalescing algorithms are used in different phases, they can both be used in the same register allocator.

- *d. Use both conservative coalescing and biased coloring in allocating registers. Show where biased coloring helps make the right decisions.

- **11.3** *Conservative coalescing* is so called because it will not introduce any (potential) spills. But can it avoid spills? Consider this graph, where the solid edges represent interferences and the dashed edge represents a MOVE:



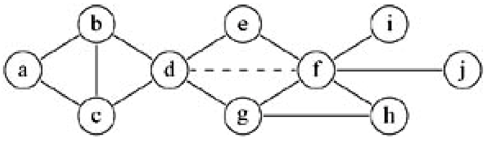
a. 4-color the graph without coalescing. Show the *select-stack*, indicating the order in which you removed nodes. Is there a potential spill? Is there an actual spill?

b. 4-color the graph with conservative coalescing. Did you use the Briggs or George criterion? Is there a potential spill? Is there an actual spill?

- **11.4** It has been proposed that the conservative coalescing heuristic could be simplified. In testing whether MOVE(*a*, *b*) can be coalesced, instead of asking whether the combined node *ab* is adjacent to < *K* nodes of significant degree, we could simply test whether *ab* is adjacent to < *K* nodes of any degree. The theory is that if *ab* is adjacent to many low-degree nodes, they will be removed by simplification anyway.

. Show that this kind of coalescing cannot create any new potential spills.

a. Demonstrate the algorithm on this graph (with *K* = 3):



b. *Show that this test is less effective than standard conservative coalescing.

Hint: Use the graph of Exercise 11.3, with *K* = 4.

Chapter 12: Putting It All Together

de-bug: to eliminate errors in or malfunctions of

Webster's Dictionary

OVERVIEW

Chapters 2-11 have described the fundamental components of a good compiler: a *front end*, which does lexical analysis, parsing, construction of abstract syntax, type-checking, and translation to intermediate code; and a *back end*, which does instruction selection, dataflow analysis, and register allocation.

What lessons have we learned? We hope that the reader has learned about the algorithms used in different components of a compiler and the interfaces used to connect the components. But the authors have also learned quite a bit from the exercise.

Our goal was to describe a good compiler that is, to use Einstein's phrase, "as simple as possible - but no simpler." we will now discuss the thorny issues that arose in designing the MiniJava compiler.

Structured l-values Java (and MiniJava) have no record or array variables, as C, C++, and Pascal do. Instead, all object and array values are really just pointers to heap-allocated data. Implementing structured l-values requires some care but not too many new insights.

Tree intermediate representation The `Tree` language has a fundamental flaw: It does not describe procedure entry and exit. These are handled by opaque procedures inside the `Frame` module that generate `Tree` code. This means that a program translated to `Trees` using, for example, the `Pentium-Frame` version of `Frame` will be different from the same program translated using `SparcFrame` - the `Tree` representation is not completely machine-independent.

Also, there is not enough information in the trees themselves to simulate the execution of an entire program, since the *view shift* (page 128) is partly done implicitly by procedure prologues and epilogues that are not represented as `Trees`. Consequently, there is not enough information to do whole-program optimization (across function boundaries).

The `Tree` representation is a *low-level* intermediate representation, useful for instruction selection and intraprocedural optimization. A *high-level* intermediate representation would preserve more of the source-program semantics, including the notions of nested functions (if applicable), nonlocal variables, object creation (as distinguished from an opaque external function call), and so on. Such a representation would be more tied to a particular family of source languages than the general-purpose `Tree` language is.

Register allocation Graph-coloring register allocation is widely used in real compilers, but does it belong in a compiler that is supposed to be "as simple as possible"? After all, it requires the use of global dataflow (liveness) analysis, construction of interference graphs, and so on. This makes the back end of the compiler significantly bigger.

It is instructive to consider what the MiniJava compiler would be like without it. We could keep all local variables in the stack frame, fetching them into temporaries only when they are used as operands of instructions. The redundant loads within a single basic block can be eliminated by a simple intrablock liveness analysis. Internal nodes of `Tree` expressions could be assigned registers using Algorithms 11.10 and 11.9. But other parts of the compiler would become much uglier: The TEMPs introduced in canonicalizing the trees (eliminating ESEQs) would have to be dealt with in an ad hoc way, by augmenting the `Tree` language with an operator that provides explicit scope for temporary variables; the `Frame` interface, which mentions registers in many places, would now have to deal with them in more complicated ways. To be able to create arbitrarily many `temps` and `moves`, and rely on the register allocator to clean them up, greatly simplifies procedure-calling sequences and code generation.

PROGRAM PROCEDURE ENTRY/EXIT

Implement the rest of the `Frame` module, which contains all the machine-dependent parts of the compiler: register sets, calling sequences, and activation record (frame) layout.

Program 12.1 shows the `Frame` class. Most of this interface has been described elsewhere. What remains is

PROGRAM 12.1: Package `Frame`.

```
package Frame;
import Temp.Temp;

public abstract class Frame implements Temp.TempMap {
abstract public Temp RV();    (see p. 157)
abstract public Temp FP();    (p. 143)
abstract public Temp.TempList registers();
abstract public String tempMap(Temp temp);
abstract public int wordSize();  (p. 143)
abstract public Tree.Exp externalCall(String func,Tree.ExpList args);  (p.
153)
abstract public Frame newFrame(Temp.Label name,
                               Util.BoolList formals);   (p. 127)
public AccessList formals;   (p. 128)
public Temp.Label name;      (p. 127)
abstract public Access allocLocal(boolean escape);    (p. 129)
abstract public Tree.Stm procEntryExit1(Tree.Stm body);   (p. 251)
abstract public Assem.InstrList procEntryExit2(Assem.InstrList body);  (p.
199)
abstract public Proc procEntryExit3(Assem.InstrList body);
abstract public Assem.InstrList codegen(Tree.Stm stm);   (p. 196)
}
```

- **registers** A list of all the register names on the machine, which can be used as "colors" for register allocation.
- **tempMap** For each machine register, the `Frame` module maintains a particular `Temp` that serves as the "precolored temporary" that stands for the register. These temps appear in the `Assem` instructions generated from CALL nodes, in procedure entry sequences generated by `procEntryExit1`, and so on. The `tempMap` tells the "color" of each of these precolored temps.
- **procEntryExit1** For each incoming register parameter, move it to the place from which it is seen from within the function. This could be a fresh temporary. One good way to handle this is for `newFrame` to create a sequence of `Tree.MOVE` statements as it

creates all the formal parameter "accesses." `newFrame` can put this into the `frame` data structure, and `procEntryExit1` can just concatenate it onto the procedure `body`.

Also concat enated to the body are statements for saving and restoring of callee-save registers (including the return-address register). If your register allocator does not implement spilling, all the callee-save (and return-address) registers should be written to the frame at the beginning of the procedure body and fetched back afterward. Therefore, `procEntryExit1` should call `allocLocal` for each register to be saved, and generate `Tree.MOVE` instructions to save and restore the registers. With luck, saving and restoring the callee-save registers will give the register allocator enough headroom to work with, so that some nontrivial programs can be compiled. Of course, some programs just cannot be compiled without spilling.

If your register allocator implements spilling, then the callee-save registers should not always be written to the frame. Instead, if the register allocator needs the space, it may choose to spill only some of the callee-save registers. But "precolored" temporaries are never spilled; so `procEntryExit1` should make up new temporaries for each callee-save (and return-address) register. On entry, it should move all these registers to their new temporary locations, and on exit, it should move them back. Of course, these moves (for nonspilled registers) will be eliminated by register coalescing, so they cost nothing.

- **procEntryExit3** Creates the procedure prologue and epilogue assembly language. First (for some machines) it calculates the size of the *outgoing parameter space* in the frame. This is equal to the maximum number of outgoing parameters of any CALL instruction in the procedure body. Unfortunately, after conversion to `Assem` trees the procedure calls have been separated from their arguments, so the outgoing parameters are not obvious. Either `procEntryExit2` should scan the body and record this information in some new component of the `frame` type, or `procEntryExit3` should use the maximum legal value.

  Once this is known, the assembly language for procedure entry, stackpointer adjustment, and procedure exit can be put together; these are the `prologue` and `epilogue`.

PROGRAM MAKING IT WORK

Make your compiler generate working code that runs.

The file `$MINIJAVA/chap12/runtime.c` is a C-language file containing several external functions useful to your MiniJava program. These are generally reached by `externalCall` from code generated by your compiler. You may modify this as necessary.

Write a module `Main` that calls on all the other modules to produce an assembly language file `prog.s` for each input program `prog.java`. This assembly language program should be assembled (producing `prog.o`) and linked with `runtime.o` to produce an executable file.

Programming projects

After your MiniJava compiler is done, here are some ideas for further work:

- **12.1** Write a garbage collector (in C) for your MiniJava compiler. You will need to make some modifications to the compiler itself to add descriptors to records and stack frames (see Chapter 13).
- **12.2** Implement inner classes is MiniJava.
- **12.3** Implement dataflow analyses such as *reaching definitions* and *available expressions* and use them to implement some of the optimizations discussed in Chapter 17.
- **12.4** Figure out other approaches to improving the assembly language generated by your compiler. Discuss; perhaps implement.
- **12.5** Implement instruction scheduling to fill branch-delay and load-delay slots in the assembly language (for a machine such as the Sparc). Or discuss how such a module could be integrated into the existing compiler; what interfaces would have to change, and in what ways?
- **12.6** Implement "software pipelining" (instruction scheduling around loop iterations) in your compiler (see Chapter 20).
- **12.7** Analyze how adequate the MiniJava language itself would be for writing a compiler. What are the smallest possible additions/changes that would make it a much more useful language?
- **12.8** In the MiniJava language, some object types are recursive and *must* be implemented as pointers; that is, a value of that type might contain a pointer to another value of the same type (directly or indirectly). But some object types are not recursive, so they could be implemented without pointers. Modify your compiler to take advantage of this by keeping nonrecursive records in the stack frame instead of on the heap.
- **12.9** Similarly, some arrays have bounds that are known at compile time, are not recursive, and are not assigned to other array variables. Modify your compiler so that these arrays are implemented right in the stack frame.
- **12.10** Implement inline expansion of functions (see Section 15.4).
- **12.11** Suppose an ordinary MiniJava program were to run on a parallel machine (a multiprocessor)? How could the compiler automatically make a parallel program out of the original sequential one? Research the approaches.

Part Two: Advanced Topics

Chapter List

Chapter 13: Garbage Collection

gar-bage: unwanted or useless material

Webster's Dictionary

OVERVIEW

Heap-allocated records that are not reachable by any chain of pointers from program variables are *garbage*. The memory occupied by garbage should be reclaimed for use in allocating new records. This process is called *garbage collection*, and is performed not by the compiler but by the runtime system (the support programs linked with the compiled code).

Ideally, we would say that any record that is not dynamically live (will not be used in the future of the computation) is garbage. But, as Section 10.1 explains, it is not always possible to know whether a variable is live. So we will use a conservative approximation: We will require the compiler to guarantee that any *live* record is *reachable*; we will ask the compiler to minimize the number of reachable records that are *not* live; and we will preserve all reachable records, even if some of them might not be live.

Figure 13.1 shows a Java program ready to undergo garbage collection (at the point marked *garbage-collect here*). There are only three program variables in scope: `p`, `q`, and `r`.

```
class list {list link;
            int key; }
class tree {int key;
             tree left;
             tree right; }
class main {
 static tree maketree() { ··· }
 static void showtree(tree t) { ··· }
 static void main() {
   {list x = new list(nil,7);
    list y = new list(x,9);
    x.link = y;
   }
   {tree p = maketree();
    tree r = p.right;
    int q = r.key;
     garbage-collect here
    showtree(r);
   }
}
```

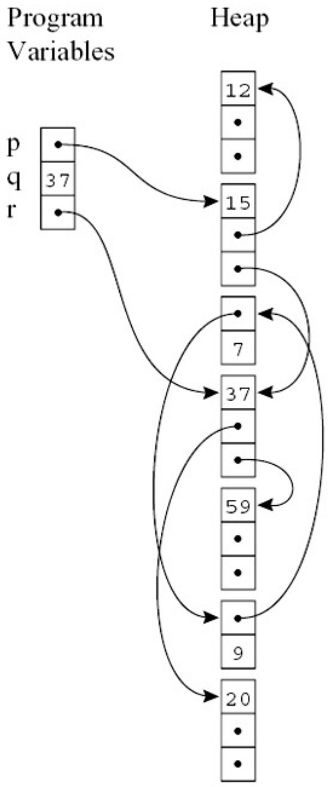


Figure 13.1: A heap to be garbage collected. Class descriptors are not shown in the diagram.

13.1 MARK-AND-SWEEP COLLECTION

Program variables and heap-allocated records form a directed graph. The variables are *roots* of this graph. A node *n* is reachable if there is a path of directed edges *r* → … → *n* starting at some root *r*. A graph-search algorithm such as *depth-first search* (Algorithm 13.2) can *mark* all the reachable nodes.

ALGORITHM 13.2: Depth-first search.

```
function DFS(x)
  if x is a pointer into the heap
     if record x is not marked
        mark x
       for each field $f_i$ of record x
           DFS(x. $f_i$)
```

Any node not marked must be garbage, and should be reclaimed. This can be done by a *sweep* of the entire heap, from its first address to its last, looking for nodes that are not marked (Algorithm 13.3). These are garbage and can be linked together in a linked list (the *freelist*). The sweep phase should also unmark all the marked nodes, in preparation for the next garbage collection.

ALGORITHM 13.3: Mark-and-sweep garbage collection.

```
Mark phase:                     Sweep phase:
 for each root v              p ← first address in heap
     DFS(v)                       while p < last address in heap
                                        if record p is marked
                                                  unmark p
                                        else let $f_1$ be the first field in p
                                                  p. $f_1$ ← freelist
                                                  freelist ← p
                                        p ← p+(size of record p)
```

After the garbage collection, the compiled program resumes execution. Whenever it wants to heap-allocate a new record, it gets a record from the freelist. When the freelist becomes empty, that is a good time to do another garbage collection to replenish the freelist.

Cost of garbage collection Depth-first search takes time proportional to the number of nodes it marks, that is, time proportional to the amount of reachable data. The sweep phase takes time proportional to the size of the heap. Suppose there are R words of reachable data in a heap of size H. Then the cost of one garbage collection is $c_1R + c_2H$ for some constants c_1 and c_2; for example, c_1 might be 10 instructions and c_2 might be 3 instructions.

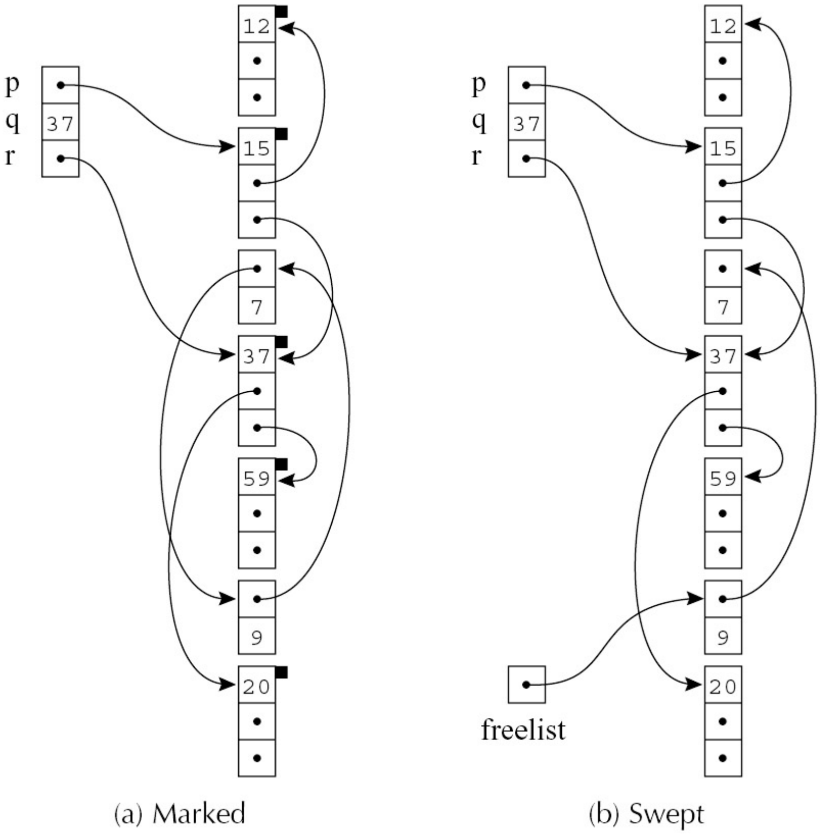
The "good" that collection does is to replenish the freelist with $H - R$ words of usable memory. Therefore, we can compute the *amortized cost* of collection by dividing the *time spent collecting* by the *amount of garbage reclaimed*. That is, for every word that the compiled program allocates, there is an eventual garbage-collection cost of

$$\frac{c_1 R + c_2 H}{H - R}$$

If R is close to H, this cost becomes very large: Each garbage collection reclaims only a few words of garbage. If H is much larger than R, then the cost per allocated word is approximately c_2, or about 3 instructions of garbage-collection cost per word allocated.

The garbage collector can measure H (the heap size) and $H - R$ (the freelist size) directly. After a collection, if R/H is larger than 0.5 (or some other criterion), the collector should increase H by asking the operating system for more memory. Then the cost per allocated word will be approximately $c_1 + 2c_2$, or perhaps 16 instructions per word.

Using an explicit stack The DFS algorithm is recursive, and the maximum depth of its recursion is as long as the longest path in the graph of reachable data. There could be a path of length H in the worst case, meaning that the stack of activation records would be larger than the entire heap!



(a) Marked

(b) Swept

Figure 13.4: Mark-and-sweep collection.

To attack this problem, we use an explicit stack (instead of recursion), as in Algorithm 13.5. Now the stack could still grow to size H, but at least this is H words and not H activation records. Still, it is unacceptable to require auxiliary stack memory as large as the heap being collected.

ALGORITHM 13.5: Depth-first search using an explicit stack.

```
function DFS(x)
 if x is a pointer and record x is not marked
  mark x
  t 1
  stack[t] ← x
  while t > 0
        x ← stack[t]; t ← t - 1
        for each field $f_i$ of record x
```

```
if $x.\ f_i$ is a pointer and record $x.\ f_i$ is not marked
  mark $x.\ f_i$
  $t$ ← $t$ + 1; stack[$t$] ← $x.\ f_i$
```

Pointer reversal After the contents of field *x. f$_i$* have been pushed on the stack, Algorithm 13.5 will never again look the original location *x. f$_i$*. This means we can use *x. f$_i$* to store one element of the stack itself! This all-too-clever idea is called *pointer reversal*, because *x. f$_i$* will be made to point back to the record from which *x* was reached. Then, as the stack is popped, the field *x. f$_i$* will be restored to its original value.

Algorithm 13.6 requires a field in each record called *done*, which indicates how many fields in that record have been processed. This takes only a few bits per record (and it can also serve as the mark field).

ALGORITHM 13.6: Depth-first search using pointer reversal.

```
function DFS($x$)
 if $x$ is a pointer and record $x$ is not marked
  $t$ ← nil
  mark $x$; done[$x$] 0
  while true
        $i$ done[$x$]
        if $i$ < # of fields in record $x$
          $y$ ← $x.\ f_i$
          if $y$ is a pointer and record $y$ is not marked
            $x.\ f_i$ ← $t$; $t$ ← $x$; $x$ ← $y$
            mark $x$; done[$x$] 0
          else
            done[$x$] ← $i$ + 1
        else
          $y$ ← $x$; $x$ ← $t$
          if $x$ = nil then return
          $i$ ← done[$x$]
          $t$ ← $x.\ f_i$; $x.\ f^i$ ← $y$
          done[$x$] ← $i$ + 1
```

The variable *t* serves as the top of the stack; every record *x* on the stack is already marked, and if *i* = done[*x*], then *x. f$_i$* is the "stack link" to the next node down. When popping the stack, *x. f$_i$* is restored to its original value.

An array of freelists The sweep phase is the same no matter which marking algorithm is used: It just puts the unmarked records on the freelist, and unmarks the marked records. But if records are of many different sizes, a simple linked list will not be very efficient for the allocator. When allocating a record of size *n*, it may have to search a long way down the list for a free block of that size.

A good solution is to have an array of several freelists, so that freelist[*i*] is a linked list of all records of size *i*. The program can allocate a node of size *i* just by taking the head of freelist[*i*]; the sweep phase of the collector can put each node of size *j* at the head of freelist[*j*].

If the program attempts to allocate from an empty freelist[*i*], it can try to grab a larger record from freelist[*j*] (for $j > i$) and split it (putting the unused portion back on freelist[$j - i$]). If this fails, it is time to call the garbage collector to replenish the freelists.

Fragmentation It can happen that the program wants to allocate a record of size *n*, and there are many free records smaller than *n* but none of the right size. This is called *external fragmentation*. On the other hand, *internal fragmentation* occurs when the program uses a too-large record without splitting it, so that the unused memory is inside the record instead of outside.

13.2 REFERENCE COUNTS

One day a student came to Moon and said: "I understand how to make a better garbage collector. We must keep a reference count of the pointers to each cons."

Moon patiently told the student the following story:

"One day a student came to Moon and said: ‘I understand how to make a better garbage collector …'"

(MIT-AI koan by Danny Hillis)

Mark-sweep collection identifies the garbage by first finding out what is reachable. Instead, it can be done directly by keeping track of how many pointers point to each record: This is the *reference count* of the record, and it is stored with each record.

The compiler emits extra instructions so that whenever *p* is stored into $x.\ f_i$, the reference count of *p* is incremented, and the reference count of what $x.\ f_i$ previously pointed to is decremented. If the decremented reference count of some record *r* reaches zero, then *r* is put on the freelist and all the other records that *r* points to have their reference counts decremented.

Instead of decrementing the counts of $r.\ f_i$ when *r* is put on the freelist, it is better to do this "recursive" decrementing when *r* is removed from the freelist, for two reasons:

1. It breaks up the "recursive decrementing" work into shorter pieces, so that the program can run more smoothly (this is important only for interactive or real-time programs).
2. The compiler must emit code (at each decrement) to check whether the count has reached zero and put the record on the freelist, but the recursive decrementing will be done only in one place, in the allocator.

Reference counting seems simple and attractive. But there are two major problems:

1. Cycles of garbage cannot be reclaimed. In Figure 13.1, for example, there is a loop of list cells (whose keys are 7 and 9) that are not reachable from program variables; but each has a reference count of 1.
2. Incrementing the reference counts is very expensive indeed. In place of the single machine instruction $x.\ f_i \leftarrow p$, the program must execute

```
$z$         $\leftarrow x.f_i$
$c$         $\leftarrow z$.count
$c$         $\leftarrow c - 1$
$z$.count   $\leftarrow c$
if $c = 0$ call *putOnFreelist*
$x.f_i$     $\leftarrow p$
$c$         $\leftarrow p$.count
$c$         $\leftarrow c + 1$
$p$.count   $\leftarrow c$
```

A naive reference counter will increment and decrement the counts on every assignment to a program variable. Because this would be extremely expensive, many of the increments and decrements are eliminated using dataflow analysis: As a pointer value is fetched and then propagated through local variables, the compiler can aggregate the many changes in the count to a single increment, or (if the net change is zero) no extra instructions at all. However, even with this technique there are many ref-count increments and decrements that remain, and their cost is very high.

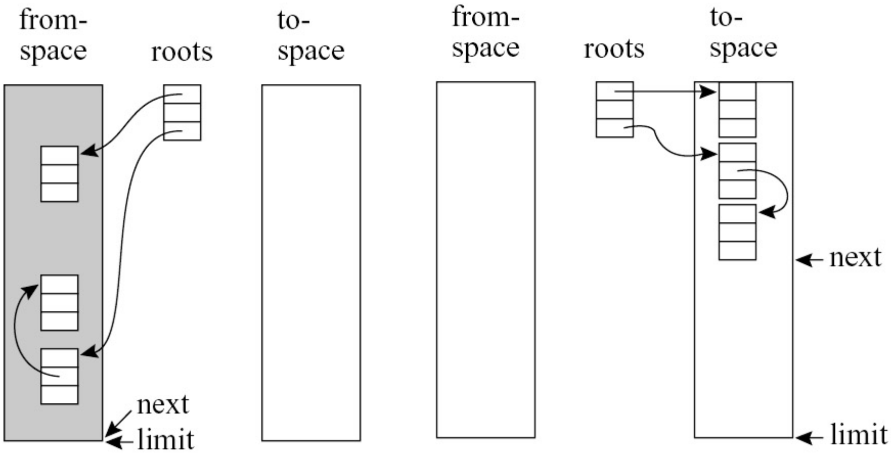
There are two possible solutions to the "cycles" problem. The first is simply to require the programmer to explicitly break all cycles when she is done with a data structure. This is less annoying than putting explicit *free* calls (as would be necessary without any garbage collection at all), but it is hardly elegant. The other solution is to combine reference counting (for eager and nondisruptive reclamation of garbage) with an occasional mark-sweep collection (to reclaim the cycles).

On the whole, the problems with reference counting outweigh its advantages, and it is rarely used for automatic storage management in programming language environments.

13.3 COPYING COLLECTION

The reachable part of the heap is a directed graph, with records as nodes, and pointers as edges, and program variables as roots. Copying garbage collection traverses this graph (in a part of the heap called *from-space*), building an isomorphic copy in a fresh area of the heap (called *to-space*). The to-space copy is *compact*, occupying contiguous memory without fragmentation (that is, without free records interspersed with the reachable data). The roots are made to point at the to-space copy; then the entire from-space (garbage, plus the previously reachable graph) is unreachable.

Figure 13.7 illustrates the situation before and after a copying collection. Before the collection, the from-space is full of reachable nodes and garbage; there is no place left to allocate, since `next` has reached `limit`. After the collection, the area of to-space between `next` and `limit` is available for the compiled program to allocate new records. Because the new-allocation area is contiguous, allocating a new record of size *n* into pointer `p` is very easy: Just copy `next` to `p`, and increment `next` by *n*. Copying collection does not have a fragmentation problem.



(a) Before collection

(b) After collection

Figure 13.7: Copying collection.

Eventually, the program will allocate enough that `next` reaches `limit`; then another garbage collection is needed. The roles of from-space and to-space are swapped, and the reachable data are again copied.

Initiating a collection To start a new collection, the pointer `next` is initialized to point at the beginning of to-space; as each reachable record in from-space is found, it is copied to to-space at position `next`, and `next` incremented by the size of the record.

Forwarding The basic operation of copying collection is *forwarding* a pointer; that is, given a pointer `p` that points to from-space, make `p` point to to-space (Algorithm 13.8).

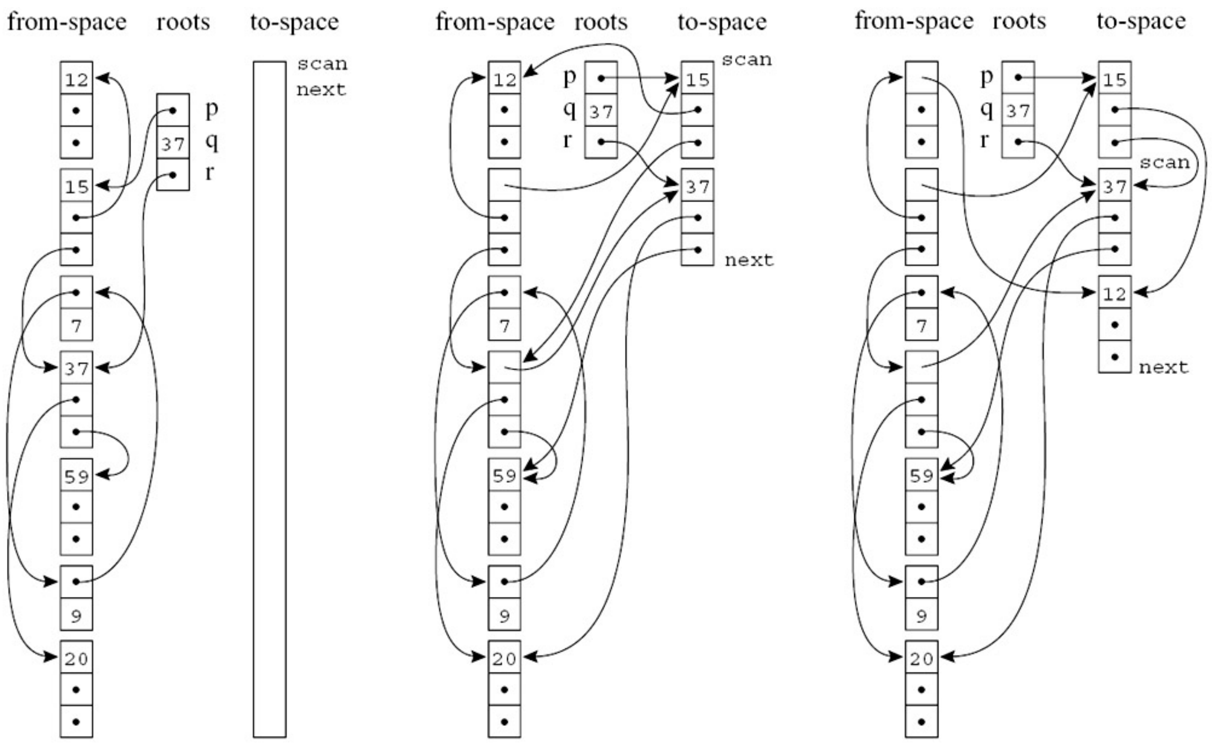
ALGORITHM 13.8: Forwarding a pointer.

```
function Forward($p$)
  if $p$ points to from-space
     then if $p.f_1$ points to to-space
          then return $p.f_1$
          else for each field $f_i$ of $p$
                   next.$f_i$ ← $p.f_i$
               $p.f_1$ ← next
               next ← next+ size of record $p$
               return $p.f_1$
    else return $p$
```

There are three cases:

1. If *p* points to a from-space record that has already been copied, then $p.f_1$ is a special *forwarding pointer* that indicates where the copy is. The forwarding pointer can be identified just by the fact that it points within the to-space, as no ordinary from-space field could point there.
2. If *p* points to a from-space record that has not yet been copied, then it is copied to location `next`; and the forwarding pointer is installed into $p.f_1$. It's all right to overwrite the f_1 field of the old record, because all the data have already been copied to the to-space at `next`.
3. If *p* is not a pointer at all, or if it points outside from-space (to a record outside the garbage-collected arena, or to to-space), then forwarding *p* does nothing.

Cheney's algorithm The simplest algorithm for copying collection uses breadth-first search to traverse the reachable data (Algorithm 13.9, illustrated in Figure 13.10). First, the roots are forwarded. This copies a few records (those reachable *directly* from root pointers) to to-space, thereby incrementing `next`.



(a) Before collection (b) Roots forwarded (c) One record scanned

Figure 13.10: Breadth-first copying collection.

ALGORITHM 13.9: Breadth-first copying garbage collection.

```
scan ← next ← beginning of to-space
for each root r

    r ← Forward(r)
while scan < next
    for each field $f_i$ of record at scan
        scan. $f_i$ Forward(scan. $f_i$)

    scan ← scan+ size of record at scan
```

The area between `scan` and `next` contains records that have been copied to to-space, but whose fields have not yet been forwarded: In general, these fields point to from-space. The area between the beginning of to-space and `scan` contains records that have been copied *and* forwarded, so that all the pointers in this area point to to-space. The **while** loop (of Algorithm 13.9) moves `scan` toward `next`, but copying records will cause `next` to move also. Eventually, `scan` catches up with `next` after all the reachable data are copied to to-space.

Cheney's algorithm requires no external stack, and no pointer reversal: It uses the to-space area between `scan` and `next` as the queue of its breadth-first search. This makes it considerably simpler to implement than depth-first search with pointer reversal.

Locality of reference However, pointer data structures copied by breadth-first have poor locality of reference: If a record at address *a* points to another record at address *b*, it is likely that *a* and *b* will be far apart. Conversely, the record at *a* + 8 is likely to be unrelated to the one at *a*. Records that are copied near each other are those whose distance from the roots are equal.

In a computer system with virtual memory, or with a memory cache, good locality of reference is important. After the program fetches address *a*, then the memory subsystem expects addresses near *a* to be fetched soon. So it ensures that the entire page or cache line containing *a* and nearby addresses can be quickly accessed.

Suppose the program is fetching down a chain of *n* pointers in a linked list. If the records in the list are scattered around memory, each on a page (or cache line) containing completely unrelated data, then we expect *n* difference pages or cache lines to be active. But if successive records in the chain are at adjacent addresses, then only *n*/*k* pages (cache lines) need to be active, where *k* records fit on each page (cache line).

Depth-first copying gives better locality, since each object *a* will tend to be adjacent to its first child *b*, unless *b* is adjacent to another "parent" a'. Other children of *a* may not be adjacent to *a*, but if the subtree *b* is small, then they should be nearby.

But depth-first copy requires pointer-reversal, which is inconvenient and slow. A hybrid, partly depth-first and partly breadth-first algorithm can provide acceptable locality. The basic idea is to use breadth-first copying, but whenever an object is copied, see if some child can be copied near it (Algorithm 13.11).

ALGORITHM 13.11: Semi-depth-first forwarding.

```
function Forward(p)
  if p points to from-space
     then if p. $f_1$ points to to-space
          then return p. $f_1$
          else Chase(p); return p. $f_1$
     else return p

function Chase(p)
  repeat
    q next
    next ← next+ size of record p
    r ← nil
    for each field $f_i$ of record p
        q. $f_i$ ← p. $f_i$
        if q. $f_i$ points to from-space and q. $f_i$ . $f_1$ does not point to to-space
        then r ← q. $f_i$
    p. $f_1$ ← q
    p ← r
  until p = nil
```

Cost of garbage collection Breadth-first search (or the semi-depth-first variant) takes time proportional to the number of nodes it marks, that is, c_3 *R* for some constant c_3 (perhaps equal to 10 instructions). There is no sweep phase, so c_3 *R* is the total cost of collection. The heap is divided into two semi-spaces, so each collection reclaims $H = 2 - R$ words that can be allocated before the next collection. The amortized cost of collection is thus

$$\frac{c_3 R}{\frac{H}{2} - R}$$

instructions per word allocated.

As H grows much larger than R, this cost approaches zero. That is, *there is no inherent lower bound to the cost of garbage collection*. In a more realistic setting, where $H = 4R$, the cost would be about 10 instructions per word allocated. This is rather costly in space and time: It requires four times as much memory as reachable data, and requires 40 instructions of overhead for every 4-word object allocated. To reduce both space and time costs significantly, we use *generational* collection.

13.4 GENERATIONAL COLLECTION

In many programs, newly created objects are likely to die soon; but an object that is still reachable after many collections will probably survive for many collections more. Therefore the collector should concentrate its effort on the "young" data, where there is a higher proportion of garbage.

We divide the heap into *generations*, with the youngest objects in generation G_0; every object in generation G_1 is older than any object in G_0; everything in G_2 is older than anything in G_1, and so on.

To collect (by mark-and-sweep or by copying) just G_0, just start from the roots and do either depth-first marking or breadth-first copying (or semidepth-first copying). But now the roots are not just program variables: They include any pointer within $G_1, G_2, \ldots$ that points into G_0. If there are too many of these, then processing the roots will take longer than the traversal of reachable objects within G_0!

Fortunately, it is rare for an older object to point to a much younger object. In many common programming styles, when an object a is created its fields are immediately initialized; for example, they might be made to point to b and c. But b and c already exist; they are older than a. So we have a newer object pointing to an older object. The only way that an older object b could point to a newer object a is if some field of b is updated long after b is created; this turns out to be rare.

To avoid searching all of $G_1, G_2, \ldots$ for roots of G_0, we make the compiled program *remember* where there are pointers from old objects to new ones. There are several ways of remembering:

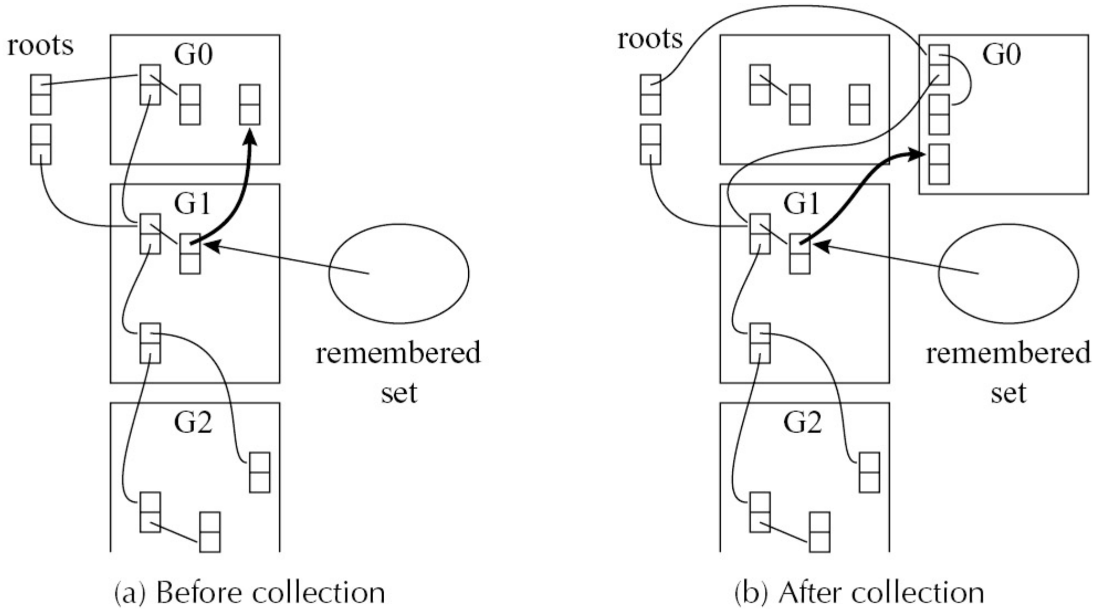


Figure 13.12: Generational collection. The bold arrow is one of the rare pointers from an older generation to a newer one.

- **Remembered list:** The compiler generates code, after each *update* store of the form $b.f_i \leftarrow a$, to put b into a vector of *updated objects*. Then, at each garbage collection, the collector scans the remembered list looking for old objects b that point into G_0.
- **Remembered set:** Like the remembered list, but uses a bit within object b to record that b is already in the vector. Then the code generated by the compiler can check this bit to avoid duplicate references to b in the vector.
- **Card marking:** Divide memory into logical "cards" of size 2^k bytes. An object can occupy part of a card or can start in the middle of one card and continue onto the next. Whenever address b is updated, the card containing that address is *marked*. There is an array of bytes that serve as marks; the byte index can be found by shifting address b right by k bits.
- **Page marking:** This is like card marking, but if 2^k is the page size, then the computer's virtual memory system can be used instead of extra instructions generated by the compiler. Updating an old generation sets a *dirty bit* for that page. If the operating system does not make dirty bits available to user programs, then the user program can implement this by write-protecting the page and asking the operating system to refer protection violations to a usermode fault handler that records the dirtiness and unprotects the page.

When a garbage collection begins, the remembered set tells which objects (or cards, or pages) of the old generation can possibly contain pointers into G_0; these are scanned for roots.

Algorithm 13.3 or 13.9 can be used to collect G_0: "heap" or "from-space" means G_0, "to-space" means a new area big enough to hold the reachable objects in G_0, and "roots" include program variables *and* the remembered set. Pointers to older generations are left unchanged: The marking algorithm does not mark old-generation records, and the copying algorithm copies them verbatim without forwarding them.

After several collections of G_0, generation G_1 may have accumulated a significant amount of garbage that should be collected. Since G_0 may contain many pointers into G_1, it is best to collect G_0 and G_1 together. As before, the remembered set must be scanned for roots contained in $G_2, G_3;\ldots$. Even more rarely, G_2 will be collected, and so on.

Each older generation should be exponentially bigger than the previous one. If G_0 is half a megabyte, then G_1 should be two megabytes, G_2 should be eight megabytes, and so on. An object should be promoted from G_i to G_{i+1} when it survives two or three collections of G_i.

Cost of generational collection Without detailed empirical information about the distribution of object lifetimes, we cannot analyze the behavior of generational collection. In practice, however, it is common for the youngest generation to be less than 10% live data. With a copying collector, this means that H / R is 10 *in this generation*, so that the amortized cost per word reclaimed is $c_3 R / (10R - R)$, or about 1 instruction. If the amount of reachable data in G_0 is about 50 to 100 kilobytes, then the amount of space "wasted" by having $H = 10R$ in the youngest generation is about a megabyte. In a 50-megabyte multigeneration system, this is a small space cost.

Collecting the older generations can be more expensive. To avoid using too much space, a smaller H / R ratio can be used for older generations. This increases the time cost of an older-generation collection, but these are sufficiently rare that the overall amortized time cost is still good.

Maintaining the remembered set also takes time, approximately 10 instructions per pointer update to enter an object into the remembered set and then process that entry in the remembered set. If the program does many more updates than fresh allocations, then generational collection may be more expensive than nongenerational collection.

13.5 INCREMENTAL COLLECTION

Even if the overall garbage collection time is only a few percent of the computation time, the collector will occasionally interrupt the program for long periods. For interactive or real-time programs this is undesirable. Incremental or concurrent algorithms interleave garbage collection work with program execution to avoid long interruptions.

Terminology The *collector* tries to collect the garbage; meanwhile, the compiled program keeps changing (mutating) the graph of reachable data, so it is called the *mutator*. An *incremental* algorithm is one in which the collector operates only when the mutator requests it; in a *concurrent* algorithm the collector can operate between or during any instructions executed by the mutator.

Tricolor marking In a mark-sweep or copying garbage collection, there are three classes of records:

- **White** objects are not yet visited by the depth-first or breadth-first search.
- **Grey** objects have been visited (marked or copied), but their children have not yet been examined. In mark-sweep collection, these objects are on the stack; in Cheney's copying collection, they are between `scan` and `next`.
- **Black** objects have been marked, and their children also marked. In mark-sweep collection, they have already been popped off the stack; in Cheney's algorithm, they have already been scanned.

The collection starts with all objects white; the collector executes Algorithm 13.13, blackening grey objects and greying their white children. Implicit in changing an object from grey to black is *removing it from the stack or queue*; implicit in greying an object is *putting it into the stack or queue*. When there are no grey objects, then all white objects must be garbage.

ALGORITHM 13.13: Basic tricolor marking

```
while there are any grey objects
 select a grey record p
 for each field $f_i$ of p
     if record p. $f_i$ is white
        color record p. $f_i$ grey
 color record p black
```

Algorithm 13.13 generalizes all of the mark-sweep and copying algorithms shown so far: Algorithms 13.2, 13.3, 13.5, 13.6, and 13.9. All these algorithms preserve two natural invariants:

1. No black object points to a white object.
2. Every grey object is on the collector's (stack or queue) data structure (which we will call the *grey-set*).

While the collector operates, the mutator creates new objects (of what color?) and updates pointer fields of existing objects. If the mutator breaks one of the invariants, then the collection algorithm will not work.

Most incremental and concurrent collection algorithms are based on techniques to allow the mutator to get work done while preserving the invariants. For example:

- **Dijkstra, Lamport, et al.** Whenever the mutator stores a white pointer *a* into a black object *b*, it colors *a* grey. (The compiler generates extra instructions at each store to check for this.)
- **Steele.** Whenever the mutator stores a white pointer *a* into a black object *b*, it colors *b* grey (using extra instructions generated by the compiler).
- **Boehm, Demers, Shenker.** All-black pages are marked read-only in the virtual memory system. Whenever the mutator stores *any* value into an all-black page, a page fault marks all objects on that page grey (and makes the page writable).
- **Baker.** Whenever the mutator fetches a pointer *b* to a white object, it colors *b* grey. The mutator never possesses a pointer to a white object, so it cannot violate invariant 1. The instructions to check the color of *b* are generated by the compiler after every fetch.
- **Appel, Ellis, Li.** Whenever the mutator fetches a pointer *b* from any virtual-memory page containing any nonblack object, a page-fault handler colors every object on the page black (making children of these objects grey). Thus the mutator never possesses a pointer to a white object.

The first three of these are *write-barrier* algorithms, meaning that each *write* (store) by the mutator must be checked to make sure an invariant is preserved. The last two are *read-barrier* algorithms, meaning that *read* (fetch) instructions are the ones that must be checked. We have seen write barriers before, for generational collection: Remembered lists, remembered sets, card marking, and page marking are all different implementations of the write barrier. Similarly, the read barrier can be implemented in software (as in Baker's algorithm) or using the virtual-memory hardware.

Any implementation of a write or read barrier must synchronize with the collector. For example, a Dijkstra-style collector might try to change a white node to grey (and put it into the grey-set) at the same time the mutator is also greying the node (and putting it into the grey-set). Thus, software implementations of the read or write barrier will need to use explicit synchronization instructions, which can be expensive.

But implementations using virtual-memory hardware can take advantage of the synchronization implicit in a page fault: If the mutator faults on a page, the operating system will ensure that no other process has access to that page before processing the fault.

13.6 BAKER'S ALGORITHM

Baker's algorithm illustrates the details of incremental collection. It is based on Cheney's copying collection algorithm, so it forwards reachable objects from from-space to to-space. Baker's algorithm is compatible with generational collection, so that the from-space and to-space might be for generation G_0, or might be $G_0 + \ldots + G_k$.

To initiate a garbage collection (which happens when an *allocate* request fails for lack of unused memory), the roles of the (previous) from-space and to-space are swapped, and all the roots are forwarded; this is called the *flip*. Then the mutator is resumed; but each time the mutator calls the allocator to get a new record, a few pointers at `scan` are scanned, so that `scan` advances toward `next`. Then a new record is allocated *at the end of the to-space* by decrementing `limit` by the appropriate amount.

The invariant is that the mutator has pointers only to to-space (never to from-space). Thus, when the mutator allocates and initializes a new record, that record need not be scanned; when the mutator stores a pointer into an old record, it is only storing a to-space pointer.

If the mutator fetches a field of a record, it might break the invariant. So each fetch is followed by two or three instructions that check whether the fetched pointer points to from-space. If so, that pointer must be *forwarded* immediately, using the standard *forward* algorithm.

For every word allocated, the allocator must advance `scan` by at least one word. When `scan=next`, the collection terminates until the next time the allocator runs out of space. If the heap is divided into two semi-spaces of size $H / 2$, and $R < H / 4$, then `scan` will catch up with `next` before `next` reaches halfway through the to-space; also by this time, no more than half the to-space will be occupied by newly allocated records.

Baker's algorithm copies no more data than is live at the flip. Records allocated during collection are not scanned, so they do not add to the cost of collection. The collection cost is thus $c_3 R$. But there is also a cost to check (at every allocation) whether incremental scanning is necessary; this is proportional to $H / 2 - R$.

But the largest cost of Baker's algorithm is the extra instructions after every fetch, required to maintain the invariant. If one in every 10 instructions fetches from a heap record, and each of these fetches requires two extra instructions to test whether it is a from-space pointer, then there is at least a 20% overhead cost just to maintain the invariant. All of the incremental or concurrent algorithms that use a software write or read barrier will have a significant cost in overhead of ordinary mutator operations.

13.7 INTERFACE TO THE COMPILER

The compiler for a garbage-collected language interacts with the garbage collector by generating code that allocates records, by describing locations of roots for each garbage-collection cycle, and by describing the layout of data records on the heap. For some versions of incremental collection, the compiler must also generate instructions to implement a read or write barrier.

FAST ALLOCATION

Some programming languages, and some programs, allocate heap data (and generate garbage) very rapidly. This is especially true of programs in functional languages, where updating old data is discouraged.

The most allocation (and garbage) one could imagine a reasonable program generating is one word of allocation per store instruction; this is because each word of a heap-allocated record is usually initialized. Empirical measurements show that about one in every seven instructions executed is a store, almost regardless of programming language or program. Thus, we have (at most) 1/7 word of allocation per instruction executed.

Supposing that the cost of garbage collection can be made small by proper tuning of a generational collector, there may still be a considerable cost to create the heap records. To minimize this cost, *copying collection* should be used so that the allocation space is a contiguous free region; the next free location is `next` and the end of the region is `limit`. To allocate one record of size N, the steps are

1. Call the allocate function.
2. Test `next` + N < `limit` ? (If the test fails, call the garbage collector.)
3. Move `next` into `result`
4. Clear M[`next`], M[`next + 1`],..., M[`next + N − 1`]
5. `next` ← `next` + N
6. Return from the allocate function.

A. Move `result` into some computationally useful place.
B. Store useful values into the record.

Steps 1 and 6 should be eliminated by *inline expanding* the allocate function at each place where a record is allocated. Step 3 can often be eliminated by combining it with step A, and step 4 can be eliminated in favor of step B (steps A and B are not numbered because they are part of the useful computation; they are not allocation overhead).

Steps 2 and 5 cannot be eliminated, but if there is more than one allocation in the same basic block (or in the same *trace*; see Section 8.2), the comparison and increment can be shared among multiple allocations. By keeping `next` and `limit` in registers, steps 2 and 5 can be done in a total of three instructions.

By this combination of techniques, the cost of allocating a record - and then eventually garbage collecting it - can be brought down to about four instructions. This means that programming techniques such as the *persistent binary search tree* (page 108) can be efficient enough for everyday use.

DESCRIBING DATA LAYOUTS

The collector must be able to operate on records of all types: `list`, `tree`, or whatever the program has declared. It must be able to determine the number of fields in each record, and whether each field is a pointer.

For statically typed languages such as Pascal, or for object-oriented languages such as Java or Modula-3, the simplest way to identify heap objects is to have the first word of every object point to a special type- or class-descriptor record. This record tells the total size of the object and the location of each pointer field.

For statically typed languages this is an overhead of one word per record to serve the garbage collector. But object-oriented languages need this descriptor pointer in every object just to implement dynamic method lookup, so that there is no additional per-object overhead attributable to garbage collection.

The type- or class-descriptor must be generated by the compiler from the static type information calculated by the semantic analysis phase of the compiler. The descriptor pointer will be the argument to the runtime system's `alloc` function.

In addition to describing every heap record, the compiler must identify to the collector every pointer-containing temporary and local variable, whether it is in a register or in an activation record. Because the set of live temporaries can change at every instruction, the *pointer map* is different at every point in the program. Therefore, it is simpler to describe the pointer map only at points where a new garbage collection can begin. These are at calls to the `alloc` function; and also, since any function call might be calling a function which in turn calls `alloc`, the pointer map must be described at each function call.

The pointer map is best keyed by return addresses: A function call at location *a* is best described by its return address immediately after *a*, because the return address is what the collector will see in the very next activation record. The data structure maps return addresses to live-pointer sets; for each pointer that is live immediately after the call, the pointer map tells its register or frame location.

To find all the roots, the collector starts at the top of the stack and scans downward, frame by frame. Each return address keys the pointer-map entry that describes the next frame. In each frame, the collector marks (or forwards, if copying collection) from the pointers in that frame.

Callee-save registers need special handling. Suppose function *f* calls *g*, which calls *h*. Function *h* knows that it saved some of the callee-save registers in its frame and mentions this fact in its pointer map; but *h does not know which of these registers are pointers*. Therefore the pointer map for *g* must describe which of its callee-save registers contain pointers at the call to *h* and which are "inherited" from *f*.

DERIVED POINTERS

Sometimes a compiled program has a pointer that points into the middle of a heap record, or that points before or after the record. For example, the expression `a[i-2000]` can be calculated internally as `M[a-2000+i]`:

$$
\begin{aligned}
t_1 &\leftarrow a - 2000\\
t_2 &\leftarrow t_1 + i\\
t_3 &\leftarrow M[t_2]
\end{aligned}
$$

If the expression `a[i-2000]` occurs inside a loop, the compiler might choose to hoist $t_1 \leftarrow a - 2000$ outside the loop to avoid recalculating it in each iteration. If the loop also contains an `alloc`, and a garbage collection occurs while t_1 is live, will the collector be confused by a pointer t_1 that does not point to the beginning of an object, or (worse yet) that points to an unrelated object?

We say that the t_1 is *derived* from the *base* pointer a. The pointer map must identify each *derived pointer* and tell the base pointer from which it is derived. Then, when the collector relocates a to address a', it must adjust t_1 to point to address $t_1 + a' - a$.

Of course, this means that a must remain live as long as t_1 is live. Consider the loop at left, implemented as shown at right:

```
let                                                  r1 ← 100
  var a := intarray[100] of 0                        r2 ← 0
                                                     call alloc
                                                     a ← r1
 in                                                  t1 ← a - 2000
  for i := 1930 to 1990                              i ← 1930
    do f(a[i-2000])                                  L1 : r1 ← M[t1 + i]
                                                     call f
end                                                  L2 : if i ≤ 1990 goto
L1
```

If there are no other uses of `a`, then the temporary a appears dead after the assignment to t_1. But then the pointer map associated with the return address L_2 would not be able to "explain" t_1 adequately. Therefore, for purposes of the compiler's liveness analysis, *a derived pointer implicitly keeps its base pointer live*.

PROGRAM DESCRIPTORS

Implement record descriptors and pointer maps for the MiniJava compiler.

For each record-type declaration, make a string literal to serve as the record descriptor. The length of the string should be equal to the number of fields in the record. The ith byte of the string should be `p` if the ith field of the record is a pointer (string, record, or array), or `n` if the ith field is a nonpointer.

The `allocRecord` function should now take the record descriptor string (pointer) instead of a length; the allocator can obtain the length from the string literal. Then `allocRecord` should store this descriptor pointer at field zero of the record. Modify the runtime system appropriately.

The user-visible fields of the record will now be at offsets 1, 2, 3,… instead of 0, 1, 2,…; adjust the compiler appropriately.

Design a descriptor format for arrays, and implement it in the compiler and runtime system.

Implement a temp-map with a boolean for each temporary: Is it a pointer or not? Also make a similar map for the offsets in each stack frame, for frame-resident pointer variables. You will not need to handle derived pointers, as your MiniJava compiler probably does not keep derived pointers live across function calls.

For each procedure call, put a new return-address label L_{ret} immediately after the `call` instruction. For each one, make a data fragment of the form

$L_{\mathrm{ptrmap327}}$:	.word	$L_{\mathrm{ptrmap326}}$	*link to previous ptr-map entry*
	.word	L_{ret327}	*key for this entry*
	.word	…	*pointer map for this return address*
	⋮		

and then the runtime system can traverse this linked list of pointer-map entries, and perhaps build it into a data structure of its own choosing for fast lookup of return addresses. The data-layout pseudo-instructions (`.word`, etc.) are, of course, machine-dependent.

PROGRAM GARBAGE COLLECTION

Implement a mark-sweep or copying garbage collector in the C language, and link it into the runtime system. Invoke the collector from `allocRecord` or `initArray` when the free space is exhausted.

FURTHER READING

Reference counting [Collins 1960] and mark-sweep collection [McCarthy 1960] are almost as old as languages with pointers. The pointer-reversal idea is attributed by Knuth [1967] to Peter Deutsch and to Herbert Schorr and W. M. Waite.

Fenichel and Yochelson [1969] designed the first two-space copying collector, using depth-first search; Cheney [1970] designed the algorithm that uses the unscanned nodes in to-space as the queue of a breadth-first search, and also the semi-depth-first copying that improves the locality of a linked list.

Steele [1975] designed the first concurrent mark-and-sweep algorithm. Dijkstra et al. [1978] formalized the notion of tricolor marking, and designed a concurrent algorithm that they could prove correct, trying to keep the synchronization requirements as weak as possible. Baker [1978] invented the incremental copying algorithm in which the mutator sees only to-space pointers.

Generational garbage collection, taking advantage of the fact that newer objects die quickly and that there are few old-to-new pointers, was invented by Lieberman and Hewitt [1983]; Ungar [1986] developed a simpler and more efficient *remembered set* mechanism.

The Symbolics Lisp Machine [Moon 1984] had special hardware to assist with incremental and generational garbage collection. The microcoded memory-fetch instructions enforced the invariant of Baker's algorithm; the microcoded memory-store instructions maintained the remembered set for generational collection. This collector was the first to explicitly improve locality of reference by keeping related objects on the same virtual-memory page.

As modern computers rarely use microcode, and a modern general-purpose processor embedded in a general-purpose memory hierarchy tends to be an order of magnitude faster and cheaper than a computer with special-purpose instructions and memory tags, attention turned in the late 1980s to algorithms that could be implemented with standard RISC instructions and standard virtual-memory hardware. Appel et al. [1988] use virtual memory to implement a read barrier in a truly concurrent variant of Baker's algorithm. Shaw [1988] uses virtual-memory *dirty bits* to implement a write barrier for generational collection, and Boehm et al. [1991] make the same simple write barrier serve for concurrent generational mark-and-sweep. Write barriers are cheaper to implement than read barriers, because stores to old pages are rarer than fetches from to-space, and a write barrier merely needs to set a dirty bit and continue with minimal interruption of the mutator. Sobalvarro [1988] invented the card marking technique, which uses ordinary RISC instructions without requiring interaction with the virtual-memory system.

Appel and Shao [1996] describe techniques for fast allocation of heap records and discuss several other efficiency issues related to garbage-collected systems.

Branquart and Lewi [1971] describe pointer maps communicated from a compiler to its garbage collector; Diwan et al. [1992] tie pointer maps to return addresses, show how to handle derived pointers, and compress the maps to save space.

Appel [1992, Chapter 12] shows that compilers for functional languages must be careful about closure representations; using simple static links (for example) can keep enormous amounts of data reachable, preventing the collector from reclaiming it.

Boehm and Weiser [1988] describe *conservative collection*, where the compiler does not inform the collector which variables and record fields contain pointers, so the collector must "guess." Any bit pattern pointing into the allocated heap is assumed to be a possible pointer and keeps the pointed-to record live. However, since the bit pattern might really be meant as an integer, the object cannot be moved (which would change the possible integer), and some garbage objects may not be reclaimed. Wentworth [1990] points out that such an integer may (coincidentally) point to the root of a huge garbage data structure, which therefore will not be reclaimed; so conservative collection will occasionally suffer from a disastrous space leak. Boehm [1993] describes several techniques for making these disasters unlikely: For example, if the collector ever finds an integer pointing to address *X* that is not a currently allocated object, it should *blacklist* that address so that the allocator will never allocate an object there. Boehm [1996] points out that even a conservative collector needs some amount of compiler assistance: If a derived pointer can point outside the bounds of an object, then its base pointer must be kept live as long as the derived pointer exists.

Page 481 discusses some of the literature on improving the cache performance of garbage-collected systems.

Cohen [1981] comprehensively surveys the first two decades of garbagecollection research; Wilson [1997] describes and discusses more recent work. Jones and Lins [1996] offer a comprehensive textbook on garbage collection.

EXERCISES

- *__13.1__ Analyze the cost of mark-sweep versus copying collection. Assume that every record is exactly two words long, and every field is a pointer. Some pointers may point outside the collectible heap, and these are to be left unchanged.

a. Analyze Algorithm 13.6 to estimate c_1, the cost (in instructions per reachable word) of depth-first marking.
b. Analyze Algorithm 13.3 to estimate c_2, the cost (in instructions per word in the heap) of sweeping.
c. Analyze Algorithm 13.9 to estimate c_3, the cost per reachable word of copying collection.
d. There is some ratio γ so that with $H = \gamma R$ the cost of copying collection equals the cost of mark-sweep collection. Find γ.
e. For $H > \gamma R$, which is cheaper, mark-sweep or copying collection?

- **13.2** Run Algorithm 13.6 (pointer reversal) on the heap of Figure 13.1. Show the state of the heap; the `done` flags; and variables `t`, `x`, and `y` at the time the node containing 59 is first marked.
- ***13.3** Assume `main` calls `f` with callee-save registers all containing 0. Then `f` saves the callee-save registers it is going to use; puts pointers into some callee-save registers, integers into others, and leaves the rest untouched; and then it calls `g`. Now `g` saves some of the callee-save registers, puts some pointers and integers into them, and calls `alloc`, which starts a garbage collection.
  a. Write functions `f` and `g` matching this description.
  b. Illustrate the pointer maps of functions `f` and `g`.
  c. Show the steps that the collector takes to recover the exact locations of all the pointers.
- ****13.4** Every object in the Java language supports a `hashCode()` method that returns a "hash code" for that object. Hash codes need not be unique ± different objects can return the same hash code – but each object must return the same hash code every time it is called, and two objects selected at random should have only a small chance of having the same hash code.

  The Java language specification says that "This is typically implemented by converting the address of the object to an integer, but this implementation technique is not required by the Java language."

  Explain the problem in implementing `hashCode()` this way in a Java system with copying garbage collection, and propose a solution.

Chapter 14: Object-Oriented Languages

OVERVIEW

ob-ject: to feel distaste for something

Webster's Dictionary

An important characteristic of object-oriented languages is the notion of *extension* or *inheritance*. If some program context (such as the formal parameter of a function or method) expects an object that supports methods m_1, m_2, m_3, then it will also accept an object that supports m_1, m_2, m_3, m_4.

14.1 CLASS EXTENSION

Program 14.1 illustrates the use of class extension in Java. Every `Vehicle` is an `Object`;every `Car` is a `Vehicle`; thus every `Car` is also an `Object`. Every `Vehicle` (and thus every `Car` and `Truck`) has an integer `position` field and a `move` method.

PROGRAM 14.1: An object-oriented program.

```
class Vehicle {
    int position;
    void move (int x) { position = position + x; }
}
class Car extends Vehicle{
    int passengers;
    void await(Vehicle v) {
        if (v.position < position)
            v.move(position - v.position);
        else
            this.move(10);
    }
}
class Truck extends Vehicle{
    void move(int x) {
        if (x <= 55) { position = position + x; }
    }
}
class Main{
    public static void main(String args[]) {
        Truck t = new Truck();
        Car c = new Car();
        Vehicle v = c;
        c.passengers = 2;
        c.move(60);
        v.move(70);
        c.await(t);
    }
}
```

In addition, a `Car` has an integer `passengers` field and an `await` method. The variables in scope on entry to `await` are

```
passengers because it is a field of Car,
position because it is (implicitly) a field of Car,
v because it is a formal parameter of await,
this because it is (implicitly) a formal parameter of await.
```

At the call to `c.await(t)`, the truck `t` is bound to the formal parameter `v` of the `await` method. Then when `v.move` is called, this activates the `Truck_move` method body, not `Vehicle_move`.

We use the notation `A_m` to indicate a *method instance* `m` declared within aclass `A`. This is not part of the Java syntax, it is just for use in discussing the semantics of Java programs. Each different declaration of a method is a different method instance. Two different method instances could have the same method name if, for example, one overrides the other.

14.2 SINGLE INHERITANCE OF DATA FIELDS

To evaluate the expression `v.position`, where `v` belongs to class `Vehicle`, the compiler must generate code to fetch the field `position` from the object (record) that `v` points to.

This seems simple enough: The environment entry for variable `v` contains (among other things) a pointer to the type (class) description of `Vehicle`; this has a list of fields and their offsets. But at run time the variable `v` could also contain a pointer to a `Car` or `Truck`; where will the `position` field be in a `Car` or `Truck` object?

Single inheritance For *single-inheritance* languages, in which each class extends just one parent class, the simple technique of *prefixing* works well. Where `B` extends `A`, those fields of `B` that are inherited from `A` are laid out in a `B` record *at the beginning, in the same order they appear in* `A` *records*. Fields of `B` not inherited from `A` are placed afterward, as shown in Figure 14.2.

```
class A {          int a = 0;}
class B extends A {int b = 0;
                   int c = 0;}
class C extends A {int d = 0;}
class D extends B {int e = 0;}
```

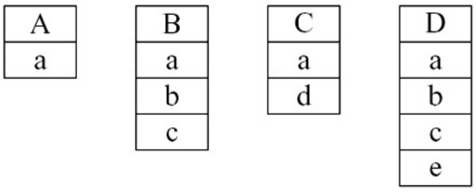


Figure 14.2: Single inheritance of data fields.

METHODS

A method instance is compiled much like a function: It turns into machine code that resides at a particular address in the instruction space. Let us say, for example, that the method instance `Truck_move` has an entry point at machine-code label `Truck_move`. In the semantic analysis phase of the compiler, each variable's environment entry contains a pointer to its class descriptor; each class descriptor contains a pointer to its parent class, and also a list of method instances; and each method instance has a machine-code label.

Static methods Some object-oriented languages allow some methods to be declared *static*. The machine code that executes when `c.f()` is called depends on the type of the *variable* `c`, not the type of the *object* that `c` holds. To compile a method-call of the form `c.f()`, the compiler finds the class of `c`; let us suppose it is class `C`. Then it searches in class `C` for a method `f`; suppose none is found. Then it searches the parent class of `C`, class `B`, for a method `f`; then the parent class of `B`; and so on. Suppose in some ancestor class `A` it finds a static method `f`; then it can compile a function call to label `A_f`.

Dynamic methods This technique will not work for dynamic methods. If method `f` in `A` is a dynamic method, then it might be overridden in some class `D` which is a subclass of `C` (see Figure 14.3). But there is no way to tell at compile time if the variable `c` is pointing to an object of class `D` (in which case `D_f` should be called) or class `C` (in which case `A_f` should be called).

```
class A           {int x = 0;
                   int f() {...} }
class B extends A {int g() {...} }
class C extends B {int g() {...} }
class D extends C {int y = 0;
                   int f() {...} }
```

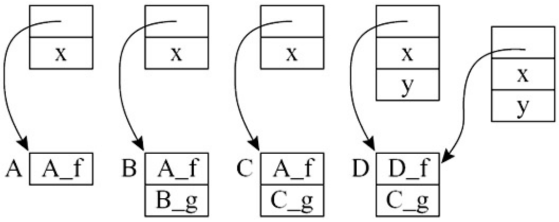


Figure 14.3: Class descriptors for dynamic method lookup.

To solve this problem, the class descriptor must contain a vector with a method instance for each (nonstatic) method name. When class `B` inherits from `A`, the method table *starts with* entries for all method names known to `A`, and then continues with `new` methods declared by `B`. This is very much like the arrangement of fields in objects with inheritance.

Figure 14.3 shows what happens when class `D` overrides method `f`. Although the entry for `f` is at the beginning of `D`'s method table, as it is also at the beginning of the ancestor class `A`'s method table, it points to a different method-instance label because `f` has been overridden.

To execute `c.f()`, where `f` is a dynamic method, the compiled code must execute these instructions:

1. Fetch the class descriptor *d* at offset 0 from object `c`.
2. Fetch the method-instance pointer *p* from the (constant) `f` offset of *d*.
3. Jump to address *p*, saving return address (that is, call *p*).

14.3 MULTIPLE INHERITANCE

In languages that permit a class `D` to extend several parent classes `A, B, C` (that is, where `A` is not a subclass of `B`, or vice versa), finding field offsets and method instances is more difficult. It is impossible to put all the `A` fields at the beginning of `D` *and* to put all the `B` fields at the beginning of `D`.

Global graph coloring One solution to this problem is to statically analyze all classes at once, finding some offset for each field name that can be used in every record containing that field. We can model this as a graph-coloring problem: There is a node for each distinct field name, and an edge for any two fields which coexist (perhaps by inheritance) in the same class.[1] The offsets 0, 1, 2;… are the colors. Figure 14.4 shows an example.

```
class A { int a = 0; }
class B { int b = 0;
          int c = 0; }
class C extends A { int d = 0; }
class D extends A,B,C { int e = 0; }
```

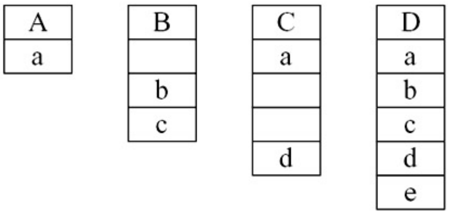


Figure 14.4: Multiple inheritance of data fields.

The problem with this approach is that it leaves empty slots in the middle of objects, since it cannot always color the *N* fields of each class with colors with the first *N* colors. To eliminate the empty slots in objects, we pack the fields of each object and have the class descriptor tell where each field is. Figure 14.5 shows an example. We have done graph coloring on all the field names, as before, but now the "colors" are not the offsets of those fields within the *objects* but within the *descriptors*. To fetch a field *a* of object *x*, we fetch the *a*-word from *x*'s descriptor; this word contains a small integer telling the position of the actual *a* data within *x*.

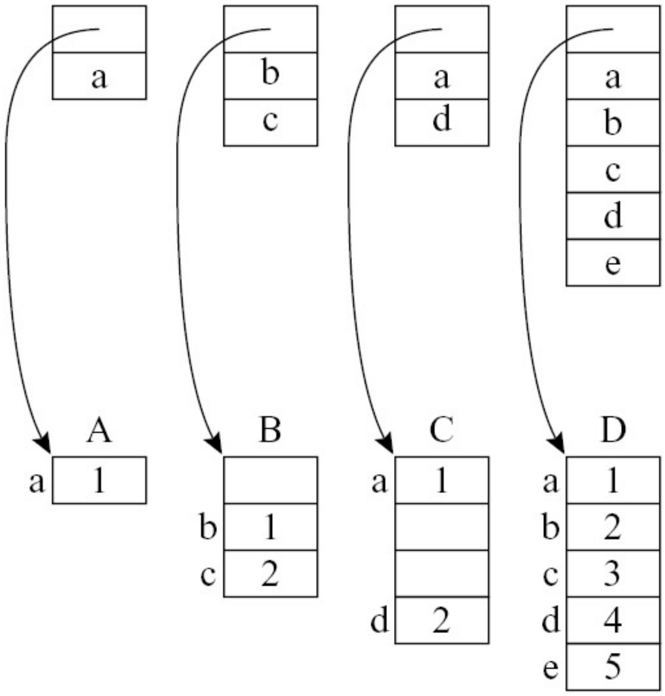


Figure 14.5: Field offsets in descriptors for multiple inheritance.

In this scheme, class descriptors have empty slots, but the objects do not; this is acceptable because a system with millions of objects is likely to have only dozens of class descriptors. But each data fetch (or store) requires three instructions instead of one:

1. Fetch the descriptor pointer from the object.
2. Fetch the field-offset value from the descriptor.
3. Fetch (or store) the data at the appropriate offset in the object.

In practice, it is likely that other operations on the object will have fetched the descriptor pointer already, and multiple operations on the same field (e.g., fetch then store) won't need to refetch the offset from the descriptor; commonsubexpression elimination can remove much of this redundant overhead.

Method lookup Finding method instances in a language with multiple inheritance is just as complicated as finding field offsets. The global graph-coloring approach works well: The method names can be mixed with the field names to form nodes of a large interference graph. Descriptor entries for fields give locations within the objects; descriptor entries for methods give machine-code addresses of method instances.

Problems with dynamic linking Any global approach suffers from the problem that the coloring (and layout of class descriptors) can be done only at link time; the job is certainly within the capability of a special-purpose linker.

However, many object-oriented systems have the capability to load new classes into a running system; these classes may be extensions (subclasses) of classes already in use. Link-time graph coloring poses many problems for a system that allows dynamic incremental linking.

Hashing Instead of global graph coloring, we can put a hash table in each class descriptor, mapping field names to offsets and method names to method instances. This works well with separate compilation and dynamic linking.

The characters of the field names are not hashed at run time. Instead, each field name *a* is hashed at compile time to an integer hash_a in the range $[0, N - 1]$. Also, for each field name a unique run-time record (pointer) ptr_a is made.

Each class descriptor has a field-offset table `Ftab` of size *N* containing field-offsets and method instances, and (for purposes of collision detection) a parallel *key* table `Ktab` containing field-name pointers. If the class has a field *x*, then field-offset-table slot number hash_x contains the offset for *x*, and key-table slot number hash_x contains the pointer ptr_x.

To fetch a field *x* of object *c*, the compiler generates code to

1. Fetch the class descriptor *d* at offset 0 from object `c`.
2. Fetch the field name *f* from the address offset *d* + `Ktab` + hash_x.
3. Test whether $f = \text{ptr}_x$; if so
4. Fetch the field offset *k* from *d* + `Ftab` + hash_x.
5. Fetch the contents of the field from *c* + *k*.

This algorithm has four instructions of overhead, which may still be tolerable. A similar algorithm works for dynamic method-instance lookup.

The algorithm as described does not say what to do if the test at line 3 fails. Any hash-table collision-resolution technique can be used.

[1] *Distinct field name* does not mean simple equivalence of strings. Each fresh declaration of field or method *x* (where it is not overriding the *x* of a parent class) is really a distinct name.

14.4 TESTING CLASS MEMBERSHIP

Some object-oriented languages allow the program to test membership of an object in a class at run time, as summarized in Table 14.6.

Table 14.6. Facilities for type testing and safe casting.

	Modula-3	**Java**
Test whether object `x` belongs class `C`, or to any subclass of `C`.	ISTYPE(`x,C`)	`x instanceof C`
Given a variable `x` of class `C`, where `x` actually points to an object of class `D` that extends `C`, yield an expression whose compile-time type is class `D`.	NARROW(`x,D`)	`(D)x`

Since each object points to its class descriptor, the address of the class descriptor can serve as a "type-tag." However, if `x` is an instance of `D`, and `D` extends `C`, then `x` is also an instance of `C`. Assuming there is no multiple inheritance, a simple way to implement `x instanceof C` is to generate code that performs the following loop at run time:

```
      $t_1 \leftarrow x$.descriptor
$L_1$ :  if $t_1 = C$ goto $true$
      $t_1 \leftarrow t_1$.super
      if $t_1$ = nil goto $false$

      goto $L_1$
```

where t_1.super is the superclass (parent class) of class t_1.

However, there is a faster approach using a *display* of parent classes. Assume that the class nesting depth is limited to some constant, such as 20. Reserve a 20-word block in each class descriptor. In the descriptor for a class `D` whose nesting depth is *j*, put a pointer to descriptor `D` in the *j*th slot, a pointer to `D.super` in the ($j - 1$)th slot, a pointer to `D.super.super` in slot $j - 2$, andsoonupto `Object` in slot 0. In all slots numbered greater than *j*, put nil.

Now, if `x` is an instance of `D`, or of any subclass of `D`, then the *j*th slot of `x`'s class descriptor will point to the class descriptor `D`. Otherwise it will not. So `x instanceof D` requires

1. Fetch the class descriptor *d* at offset 0 from object `c`.
2. Fetch the *j*th class-pointer slot from *d*.
3. Compare with the class descriptor *D*.

This works because the class-nesting depth of *D* is known at compile time.

Type coercions Given a variable *c* of type *C*, it is always legal to treat *c* as any supertype of *C* - if *C* extends *B*, and variable *b* has type *B*, then the assignment *b* ← *c* is legal and safe.

But the reverse is not true. The assignment *c* ← *b* is safe only if *b* is really (at run time) an instance of *C*, which is not always the case. If we have *b* ← `new` *B*, *c* ← *b*, followed by fetching some field of *c* that is part of class *C* but not class *B*, then this fetch will lead to unpredictable behavior.

Thus, safe object-oriented languages (such as Modula-3 and Java) accompany any coercion from a superclass to a subclass with a run-time type-check that raises an exception unless the run-time value is really an instance of the subclass (e.g., unless `b instanceof C`).

It is a common idiom to write

```
Modula-3:                                   Java:
IF ISTYPE(b,C)                                if (b instanceof C)
  THEN f(NARROW(b,C))                             f((C)b)
  ELSE ...                                    else ...
```

Now there are two consecutive, identical type tests: one explicit (ISTYPE or `instanceof`) and one implicit (in NARROW or the cast). A good compiler will do enough flow analysis to notice that the **then**-clause is reached only if `b` is in fact an instance of `C`, so that the type-check in the narrowing operation can be eliminated.

C++ is an unsafe object-oriented language. It has a *static cast* mechanism without run-time checking; careless use of this mechanism can make the program "go wrong" in unpredictable ways. C++ also has `dynamic_cast` with run-time checking, which is like the mechanisms in Modula-3 and Java.

Typecase Explicit `instanceof` testing, followed by a narrowing cast to a subclass, is not a wholesome "object-oriented" style. Instead of using this idiom, programmers are expected to use dynamic methods that accomplish the right thing in each subclass. Nevertheless, the test-then-narrow idiom is fairly common.

Modula-3 has a **typecase** facility that makes the idiom more beautiful and efficient (but not any more "object-oriented"):

```
TYPECASE expr
OF C1 (v1) => S1
 | C2 (v2) => S2
    ⋮
 | Cn (vn) => Sn
ELSE S0
END
```

If the *expr* evaluates to an instance of class C_i, then a new variable v_i of type C_i points to the result of the *expr*, and statement S_i is executed. The declaration of v_i is implicit in the TYPECASE, and its scope covers only S_i.

If more than one of the C_i match (which can happen if, for example, one is a superclass of another), then only the first matching clause is taken. If none of the C_i match, then the ELSE clause is taken (statement S_0 is executed).

Typecase can be converted straightforwardly to a chain of **else-if**s, with each **if** doing an instance test, a narrowing, and a local variable declaration. However, if there are very many clauses, then it can take a long time to go through all the **else-if**s. Therefore it is attractive to treat it like a case (switch) statement on integers, using an indexed jump (computed goto).

That is, an ordinary case statement on integers:

```
ML:                                 C, Java:
case i                             switch (i) {
 of 0 => s0                              case 0: s0; break;
  | 1=> s1                               case 1: s1; break;
  | 2=> s2                               case 2: s2; break;
  | 3=> s3                               case 3: s3; break;
  | 4=> s4                               case 4: s4; break;
  |_=> sd                                default: sd;

}
```

is compiled as follows: First a range-check comparison is made to ensure that *i* is within the range of case labels (0-4, in this case); then the address of the *i*th statement is fetched from the *i*th slot of a table, and control jumps to s_i.

This approach will not work for **typecase**, because of subclassing. That is, even if we could make class descriptors be small integers instead of pointers, we cannot do an indexed jump based on the class of the object, because we will miss clauses that match superclasses of that class. Thus, Modula-3 **typecase** is implemented as a chain of **else-if**s.

Assigning integers to classes is not trivial, because separately compiled modules can each define their own classes, and we do not want the integers to clash. But a sophisticated linker might be able to assign the integers at link time.

If all the classes in the **typecase** were `final` classes (in the sense used by Java, that they cannot be extended), then this problem would not apply. Modula-3 does not have final classes; and Java does not have **typecase**. But a clever Java system might be able to recognize a chain of **else-if**s that do **instanceof** tests for a set of final classes, and generate a indexed jump.

14.5 PRIVATE FIELDS AND METHODS

True object-oriented languages can protect fields of objects from direct manipulation by other objects' methods. A *private* field is one that cannot be fetched or updated from any function or method declared outside the object; a private method is one that cannot be called from outside the object.

Privacy is enforced by the type-checking phase of the compiler. In the symbol table of `C`, along with each field offset and method offset, is a boolean flag indicating whether the field is private. When compiling the expression `c.f()` or `c.x`, it is a simple matter to check that field and reject accesses to private fields from any method outside the object declaration.

There are many varieties of privacy and protection. Different languages allow

- Fields and methods which are accessible only to the class that declares them.
- Fields and methods accessible to the declaring class, and to any subclasses of that class.
- Fields and methods accessible only within the same module (package, namespace) as the declaring class.
- Fields that are read-only from outside the declaring class, but writable by methods of the class.

In general, these varieties of protection can be statically enforced by compiletime type-checking, for class-based languages.

14.6 CLASSLESS LANGUAGES

Some object-oriented languages do not use the notion of **class** at all. In such a language, each object implements whatever methods and has whatever data fields it wants. Type-checking for such languages is usually *dynamic* (done at run time) instead of *static* (done at compile time).

Many objects are created by *cloning*: copying an existing object (or *template* object) and then modifying some of the fields. Thus, even in a classless language there will be groups ("pseudo-classes") of similar objects that can share descriptors. When *b* is created by cloning *a*, it can share a descriptor with *a*. Only if a new field is added or a method field is updated (overridden) does *b* require a new descriptor.

The techniques used in compiling classless languages are similar to those for class-based languages with multiple inheritance and dynamic linking: Pseudo-class descriptors contain hash tables that yield field offsets and method instances.

The same kinds of global program analysis and optimization that are used for class-based languages - finding which method instance will be called from a (dynamic) method call site - are just as useful for classless languages.

14.7 OPTIMIZING OBJECT-ORIENTED PROGRAMS

An optimization of particular importance to object-oriented languages (which also benefit from most optimizations that apply to programming languages in general) is the conversion of dynamic method calls to static method-instance calls.

Compared with an ordinary function call, at each method call site there is a dynamic method lookup to determine the method instance. For single-inheritance languages, method lookup takes only two instructions. This seems like a small cost, but:

- Modern machines can jump to constant addresses more efficiently than to addresses fetched from tables. When the address is manifest in the instruction stream, the processor is able to pre-fetch the instruction cache at the destination and direct the instruction-issue mechanism to fetch at the target of the jump. Unpredictable jumps stall the instruction-issue and -execution pipeline for several cycles.
- An optimizing compiler that does inline expansion or interprocedural analysis will have trouble analyzing the consequences of a call if it doesn't even know which method instance is called at a given site.

For multiple-inheritance and classless languages, the dynamic method-lookup cost is even higher.

Thus, optimizing compilers for object-oriented languages do global program analysis to determine those places where a method call is always calling the same method instance; then the dynamic method call can be replaced by a static function call.

For a method call `c.f()`, where `c` is of class `C`, *type hierarchy analysis* is used to determine which subclasses of `C` contain methods `f` that may override `C_f`. If there is no such method, then the method instance must be `C_f`.

This idea is combined with *type propagation*, a form of static dataflow analysis similar to *reaching definitions* (see Section 17.2). After an assignment $c \leftarrow$ `new` C, the exact class of c is known. This information can be propagated through the assignment $d \leftarrow c$, and so on. When `d.f()` is encountered, the type-propagation information limits the range of the type hierarchy that might contribute method instances to `d`.

Suppose a method `f` defined in class `C` calls method `g` on `this`. But `g` is a dynamic method and may be overridden, so this call requires a dynamic method lookup. An optimizing compiler may make a different copy of a method instance `C_f` for each subclass (e.g., `D,E`) that extends `C`. Then when the (new copy) `D_f` calls `g`, the compiler knows to call the instance `D_g` without a dynamic method lookup.

PROGRAM MiniJava WITH CLASS EXTENSION

Implement class extension in your MiniJava compiler.

FURTHER READING

Dahl and Nygaard's Simula-67 language [Birtwistle et al. 1973] introduced the notion of classes, objects, single inheritance, static methods, instance testing, typecase, and the *prefix* technique to implement static single inheritance. In addition it had coroutines and garbage collection.

Cohen [1991] suggested the *display* for constant-time testing of class membership.

Dynamic methods and multiple inheritance appeared in Smalltalk [Goldberg et al. 1983], but the first implementations used slow searches of parent classes to find method instances. Rose [1988] and Connor et al. [1989] discuss fast hash-based field- and method-access algorithms for multiple inheritance. The use of graph coloring in implementing multiple inheritance is due to Dixon et al. [1989]. Lippman [1996] shows how C++-style multiple inheritance is implemented.

Chambers et al. [1991] describe several techniques to make classless, dynamically typed languages perform efficiently: pseudo-class descriptors, multiple versions of method instances, and other optimizations. Diwan et al. [1996] describe optimizations for statically typed languages that can replace dynamic method calls by static function calls.

Conventional object-oriented languages choose a method instance for a call `a.f(x,y)` based only on the class of the method *receiver* (`a`) and not other arguments (`x,y`). Languages with *multimethods* [Bobrow et al. 1989] allow dynamic method lookup based on the types of all arguments. This would solve the problem of *orthogonal directions of modularity* discussed on page 93. Chambers and Leavens [1995] show how to do static type-checking for multimethods; Amiel et al. [1994] and Chen and Turau [1994] show how to do efficient dynamic multimethod lookup.

Nelson [1991] describes Modula-3, Stroustrup [1997] describes C++, and Arnold and Gosling [1996] describe Java.

EXERCISES

- ***14.1** A problem with the *display* technique (as explained on page 290) for testing class membership is that the maximum class-nesting depth N must be fixed in advance, and every class descriptor needs N words of space even if most classes are not deeply nested. Design a variant of the *display* technique that does not suffer from these problems; it will be a couple of instructions more costly than the one described on page 290.
- **14.2** The hash-table technique for finding field offsets and method instances in the presence of multiple inheritance is shown incompletely on page 289 − the case of $f \neq \text{ptr}_x$ is not resolved. Choose a collision-resolution technique, explain how it works, and analyze the extra cost (in instructions) in the case that $f = \text{ptr}_x$(no collision) and $f \neq \text{ptr}_x$ (collision).
- ***14.3** Consider the following class hierarchy, which contains five method-call sites. The task is to show which of the method-call sites call known method instances, and

(in each case) show which method instance. For example, you might say that "method-instance `X_g` always calls `Y_f`; method `Z_g` may call more than one instance of `f`."

```
class A           { int f() { return 1; } }
class B extends A { int g() { this.f(); return 2; } }
class C extends B { int f() { this.g(); return 3; } }
class D extends C { int g() { this.f(); return 4; } }
class E extends A { int g() { this.f(); return 5; } }
class F extends E { int g() { this.f(); return 6; } }
```

Do this analysis for each of the following assumptions:

a. This is the entire program, and there are no other subclasses of these modules.
b. This is part of a large program, and any of these classes may be extended elsewhere.
c. Classes `C` and `E` are local to this module, and cannot be extended elsewhere; the other classes may be extended.

- ***14.4** Use method *replication* to improve your analysis of the program in Exercise 14.3. That is, make *every* class override `f` and `g`. For example, in class `B` (which does not already override `f`), put a copy of method `A_f`, and in `D` put acopyof `C_F`:

```

class B extends A { ... int f() { return 1; } }
class D extends C { ... int f() { this.g(); return 3; } }
```

Similarly, add new instances `E_f`, `F_f`, and `C_g`. Now, for each set of assumptions (a), (b), and (c), show which method calls go to known static instances.

- ****14.5** Devise an efficient implementation mechanism for any **typecase** that only mentions `final` classes. A `final` class is one that cannot be extended. (In Java, there is a `final` keyword; but even in other object-oriented languages, a class that is not exported from a module is effectively `final`, and a link-time whole-program analysis can discover which classes are never extended, whether declared `final` or not.)

You may make any of the following assumptions, but state which assumptions you need to use:

a. The linker has control over the placement of class-descriptor records.
b. Class descriptors are integers managed by the linker that index into a table of descriptor records.
c. The compiler explicitly marks `final` classes (in their descriptors).
d. Code for **typecase** can be generated at link time.
e. After the program is running, no other classes and subclasses are dynamically linked into the program.

Appendix A: MiniJava Language Reference Manual

MiniJava is a subset of Java. The meaning of a MiniJava program is given by its meaning as a Java program. Overloading is not allowed in MiniJava. The MiniJava statement `System.out.println( …);` can only print integers. The MiniJava expression `e.length` only applies to expressions of type `int[]`.

A.1 LEXICAL ISSUES

Identifiers: An *identifier* is a sequence of letters, digits, and underscores, starting with a letter. Uppercase letters are distinguished from lowercase. In this appendix the symbol *id* stands for an identifier.

Integer literals: A sequence of decimal digits is an *integer constant* that denotes the corresponding integer value. In this appendix the symbol *INTEGER_LITERAL* stands for an integer constant.

Binary operators: A *binary operator* is one of

```
&& < + - *
```

In this appendix the symbol *op* stands for a binary operator.

Comments: A comment may appear between any two tokens. There are two forms of comments: One starts with `/*`, ends with `*/`, and may be nested; another begins with `//` and goes to the end of the line.

A.2 GRAMMAR

In the MiniJava grammar, we use the notation *N**, where *N* is a nonterminal, to mean 0, 1, or more repetitions of *N*.

GRAMMAR A.2

```
   Program → MainClass ClassDecl*
 MainClass → class id { public static void main ( String [] id )
               { Statement }}
 ClassDecl → class id { VarDecl* MethodDecl* }
           → class id extends id { VarDecl* MethodDecl* }
   VarDecl → Type id ;
MethodDecl → public Type id ( FormalList )
               { VarDecl* Statement* return Exp ;}
FormalList → Type id FormalRest*
           →
FormalRest →, Type id
      Type → int []
           → boolean
           → int
           → id
 Statement → { Statement* }
           → if ( Exp ) Statement else Statement
           → while ( Exp ) Statement
           → System.out.println ( Exp ) ;
           → id = Exp ;
           → id [ Exp ]= Exp ;
       Exp → Exp op Exp
           → Exp [ Exp ]
           → Exp . length
           → Exp . id ( ExpList )
           → INTEGER LITERAL
           → true
           → false
           → id
           → this
           → new int [ Exp ]
           → new id ()
           → ! Exp
           → ( Exp )
   ExpList → Exp ExpRest*
           →
  ExpRest  →  ,Exp
```

A.3 SAMPLE PROGRAM

```
class Factorial {
    public static void main(String[] a) {
        System.out.println(new Fac().ComputeFac(10));
    }
}
class Fac {
    public int ComputeFac(int num) {
        int num_aux;
        if (num < 1)
            num_aux = 1;
        else
            num_aux = num * (this.ComputeFac(num-1));
        return num_aux;
    }
```