MODULE THREE PROBLEM SET

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Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

A 125-page document is being printed by five printers. Each page will be printed exactly once.

(a) Suppose that there are no restrictions on how many pages a printer can print. How many ways are there for the 125 pages to be assigned to the five printers?

One possible combination: printer A prints out pages 2-50, printer B prints out pages 1 and 51-60, printer C prints out 61-80 and 86-90, printer D prints out pages 81-85 and 91-100, and printer E prints out pages 101-125.

125 pages, and 5 printers
5^{125} ways to be printed

(b) Suppose the first and the last page of the document must be printed in color, and only two printers are able to print in color. The two color printers can also print black and white. How many ways are there for the 125 pages to be assigned to the five printers?

colored pages is 2 and only 2 printers print color
$2^2 = 4$
remainder pages = 123 and 5 printers print black and white
5^{123}
$(5^{123})4$ ways to print

(c) Suppose that all the pages are black and white, but each group of 25 consecutive pages (1-25, 26-50, 51-75, 76-100, 101-125) must be assigned to the same printer. Each printer can be assigned 0, 25, 50, 75, 100, or 125 pages to print.

How many ways are there for the 125 pages to be assigned to the five printers?

5 groups of pages to print and 5 printers
5^5 ways or 3125 ways to print

Problem 2

Ten kids line up for recess. The names of the kids are:

$$\{\text{Alex, Bobby, Cathy, Dave, Emy, Frank, George, Homa, Ian, Jim}\}.$$

Let S be the set of all possible ways to line up the kids. For example, one order might be:

(Frank, George, Homa, Jim, Alex, Dave, Cathy, Emy, Ian, Bobby)

The names are listed in order from left to right, so Frank is at the front of the line and Bobby is at the end of the line.

Let T be the set of all possible ways to line up the kids in which George is ahead of Dave in the line. Note that George does not have to be immediately ahead of Dave. For example, the ordering shown above is an element in T.

Now define a function f whose domain is S and whose target is T. Let x be an element of S, so x is one possible way to order the kids. If George is ahead of Dave in the ordering x, then $f(x) = x$. If Dave is ahead of George in x, then $f(x)$ is the ordering that is the same as x, except that Dave and George have swapped places.

(a) What is the output of f on the following input?
(Frank, George, Homa, Jim, Alex, Dave, Cathy, Emy, Ian, Bobby)

x ∈ S, x = {Frank, George, Homa, Jim, Alex, Dave, Cathy, Emy, Ian, Bobby}
George is in front of Dave so..
$f(x) = x$ output is
f = {Frank, George, Homa, Jim, Alex, Dave, Cathy, Emy, Ian, Bobby}

(b) What is the output of f on the following input?
(Emy, Ian, Dave, Homa, Jim, Alex, Bobby, Frank, George, Cathy)

y ∈ S, y = {Emy, Ian, Dave, Homa, Jim, Alex, Bobby, Frank, George, Cathy}
Dave is in front of George so..
output is y = {Emy, Ian, Dave, Homa, Jim, Alex, Bobby, Frank, George, Cathy}

(c) Is the function f a k-to-1 correspondence for some positive integer k? If so, for what value of k? Justify your answer.

It is able to be a (k-1)because both line ups have the same output.
x =(Ian, Alex, Dave, Homa, Jim, Emy, Frank, Bobby, George, Cathy)
y = (Ian, Alex, George, Homa, Jim, Emy, Frank, Bobby, Dave, Cathy)
$x \neq y for, x, y \in S$

$f(x) = f(y) = x for x \neq y$

(d) There are 3628800 ways to line up the 10 kids with no restrictions on who comes before whom. That is, $|S| = 3628800$. Use this fact and the answer to the previous question to determine $|T|$.

$|S| = 3628800$
$f : S \longrightarrow T$ is 2-1 So,
$|T| = \frac{|S|}{2}$
$\frac{3628800}{2} = 1814400$

PROBLEM 3

Consider the following definitions for sets of characters:

- Digits $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- Letters $= \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$
- Special characters $= \{*, \&, \$, \#\}$

Compute the number of passwords that satisfy the given constraints.

(i) Strings of length 7. Characters can be special characters, digits, or letters, with no repeated characters.

Digits = 10, Letters = 26, Special characters = 4
Total characters is 40
Since each character can only be used once, once a character is used 1 is subtracted by the total characters.
The string length is 7 so
40*39*38*37*36*35*34 = 93,963,542,400 different passwords

(ii) Strings of length 6. Characters can be special characters, digits, or letters, with no repeated characters. The first character can not be a special character.

first spot cannot be a special character so first letter has 36 possible characters.
second spot then has 39 possible with 1 being subtracted with each addition spot.So...
36*39*38*37*36*35 = 2,487,270,240 possible passwords

Problem 4

A group of four friends goes to a restaurant for dinner. The restaurant offers 12 different main dishes.

(i) Suppose that the group collectively orders four different dishes to share. The waiter just needs to place all four dishes in the center of the table. How many different possible orders are there for the group?

12 dishes and 4 friends
$\frac{12!}{4!(12-4)!} = \frac{12!}{4!8!} = \frac{12*11*10*9}{4*3*2*1} = 495$ possible orders

(ii) Suppose that each individual orders a main course. The waiter must remember who ordered which dish as part of the order. It's possible for more than one person to order the same dish. How many different possible orders are there for the group?

4 friends and 12 dishes
12^4 or 20,736 possible orders

How many different passwords are there that contain only digits and lower-case letters and satisfy the given restrictions?

(iii) Length is 7 and the password must contain at least one digit.

digits $= 10$, lowercase letters $= 26$
total characters is 36
10! for the one that has to be a digit $= 10$
remaining 6 spots for the password 36^6
$10 * 36^6 =$ 783,641,640,960 possible passwords

(iv) Length is 7 and the password must contain at least one digit and at least one letter.

at least one digit 10! $= 10$
at least one letter 26! $= 26$
remaining 5 spots in password 36^5
$26 * 10 * 36^5 =$ 15,721,205,760 possible passwords

PROBLEM 5

A university offers a Calculus class, a Sociology class, and a Spanish class. You are given data below about two groups of students.

(i) Group 1 contains 170 students, all of whom have taken at least one of the three courses listed above. Of these, 61 students have taken Calculus, 78 have taken Sociology, and 72 have taken Spanish. 15 have taken both Calculus and Sociology, 20 have taken both Calculus and Spanish, and 13 have taken both Sociology and Spanish. How many students have taken all three classes?

A = Calculus = 61
B = Sociology = 78
C = Spanish = 72
$A \cap B = 15$
$A \cap C = 20$
$B \cap C = 13$
$A \cup B \cup C = 170$
$A \cup B \cup C = A + B + C - (A \cap B) - (A \cap C) - (B \cap C) + (A \cap B \cap C)$
$170 = 61+78+72-15-20-13 + (A \cap B \cap C)$
$170 = 163 + (A \cap B \cap C)$
$7 = (A \cap B \cap C)$
7 students have taken all 3

(ii) You are given the following data about Group 2. 32 students have taken Calculus, 22 have taken Sociology, and 16 have taken Spanish. 10 have taken both Calculus and Sociology, 8 have taken both Calculus and Spanish, and 11 have taken both Sociology and Spanish. 5 students have taken all three courses while 15 students have taken none of the courses. How many students are in Group 2?

A = Calculus = 32
B = Sociology = 22
C = Spanish = 16
$A \cap B = 10$
$A \cap C = 8$
$B \cap C = 11$
$A \cap B \cap C = 5$ $A \cup B \cup C = A + B + C - (A \cap B) - (A \cap C) - (B \cap C) + (A \cap B \cap C)$
$A \cup B \cup C = 32 + 22 + 16 - 10 - 8 - 11 + 5$ $A \cup B \cup C = 46$
15 students had none
$46 + 15 =$
61 students in group 2

Problem 6

A coin is flipped five times. For each of the events described below, express the event as a set in roster notation. Each outcome is written as a string of length 5 from $\{H, T\}$, such as $HHHTH$. Assuming the coin is a fair coin, give the probability of each event.

{TTTTT, TTTTH, TTHTH, TTHTT, TTTHH, TTTHT, THTTH, THTTT, TTHHH, TTHHT, THHTH, THHTT, THTHH, THTHT, HTTHT, HTTTH, HTTTT, THHHH, THHHT, HTHHT, HTHTH, HTHTT, HTTHH, HHTHT, HHTTH, HHTTT, HTHHH, HHTHH, HHHTT, HHHTH, HHHHT, HHHHH}

(a) The first and last flips come up heads.

{HHHHH, HTHHH, HHTHH, HHHTH, HTTHH, HTHTH, HHTTH, HTTTH}

$\frac{8}{24}$
probability is $\frac{1}{4}$

(b) There are at least two consecutive flips that come up heads.

There are 19 cases in which there is at least two consecutive flips that are heads
probability is $\frac{19}{32}$

(c) The first flip comes up tails and there are at least two consecutive flips that come up heads.

there are 8 cases that meet the requirements
$\frac{8}{32}$
probability is $\frac{1}{4}$

Problem 7

An editor has a stack of k documents to review. The order in which the documents are reviewed is random with each ordering being equally likely. Of the k documents to review, two are named “Relaxation Through Mathematics” and “The Joy of Calculus.” Give an expression for each of the probabilities below as a function of k. Simplify your final expression as much as possible so that your answer does not include any expressions in the form
$\left(\begin{array}{c} a \\ b \end{array} \right)$.

(a) What is the probability that “Relaxation Through Mathematics” is first to review?

k!
$\frac{1(k-1)!}{k(k-1)!} = \frac{1}{k}$

(b) What is the probability that “Relaxation Through Mathematics” and “The Joy of Calculus” are next to each other in the stack?

There are two possible cases for this so
$\frac{2-(k-1)!}{k!} = \frac{2}{k}$