MAT 230 EXAM TWO

Tiffany McDonnell

Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

This question has 2 parts.

Part 1: Suppose that F and X are events from a common sample space with $P(F) \neq 0$ and $P(X) \neq 0$.

(a) Prove that $P(X) = P(X|F)P(F) + P(X|\bar{F})P(\bar{F})$. Hint: Explain why $P(X|F)P(F) = P(X \cap F)$ is another way of writing the definition of conditional probability, and then use that with the logic from the proof of Theorem 4.1.1.

$P(X|F) = \frac{P(X \cap F)}{P(F)}$
therefore $P(X \cap F) = P(X|F) * P(F)$
$X = (X \cap \overline{F}) \cup (x \cap F)$
$P(X) = P(X \cap \overline{F}) + P(X \cap F)$
$P(X) = P(x|\overline{F}) * P(\overline{F}) + P(X|F) * P(F)$
$P(X) = P(X|F) * P(F) + P(X|\overline{F}) * P(\overline{F})$

(b) Explain why $P(F|X) = P(X|F)P(F)/P(X)$ is another way of stating Theorem 4.2.1 Bayes Theorem.

$P(F|X) = \frac{F \cap X}{P(X)}$
Therefore $P(F|X) = \frac{P(X|F) * P(F)}{P(X|F) * P(F) + P(X|\overline{F}) * P(\overline{F})}$

Part 2: A website reports that 70% of its users are from outside a certain country. Out of their users from outside the country, 60% of them log on every day. Out of their users from inside the country, 80% of them log on every day.

(a) What percent of all users log on every day? Hint: Use the equation from Part 1 (a).

Let C be the users in the country
Let $\overline{C}$ be the users not in the country
Let E be the the users that log in each day.

$P(C) = 70\%$ and $P(\overline{C}) = 30\%$
$P(E|C) = 60\%$ and $P(E|\overline{C}) = 80\%$

$P(E) = P(E|C) * P(C) + P(E|\overline{C}) * P(\overline{C})$
$P(E) = \frac{60}{100} * \frac{70}{100} + \frac{80}{100} * \frac{30}{100}$
$P(E) = .42 + .24 = .66$ or 66%
so **P(E)=66%** of users log in each day.

(b) Using Bayes Theorem, out of users who log on every day, what is the probability that they are from inside the country?

Bayes Theorem consist of $P(\overline{C}|E) = \frac{P(\overline{C} \cap E)}{P(E)} = \frac{P(E|\overline{C}) * P(\overline{C})}{P(E|C) * P(C) + P(E|\overline{C}) * P(\overline{C})}$

Therefore $P(\overline{C}|E) = 36.36\%$ of the users who log in each day are from inside the country.

Problem 2

This question has 2 parts.

Part 1: The drawing below shows a Hasse diagram for a partial order on the set:
$\{A, B, C, D, E, F, G, H, I, J\}$

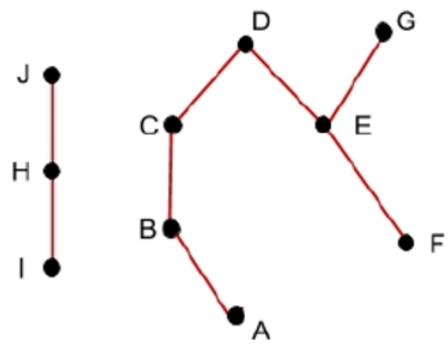


Figure 1: *A Hasse diagram shows 10 vertices and 8 edges. The vertices, represented by dots, are as follows: vertex J is upward of vertex H; vertex H is upward of vertex I; vertex B is inclined upward to the left of vertex A; vertex C is upward of vertex B; vertex D is inclined upward to the right of vertex C; vertex E is inclined upward to the left of vertex F; vertex G is inclined upward to the right of vertex E. The edges, represented by line segments between the vertices are as follows: 3 vertical edges connect the following vertices: B and C, H and I, and H and J; 5 inclined edges connect the following vertices: A and B, C and D, D and E, E and F, and E and G.*

Determine the properties of the Hasse diagram based on the following questions:

(a) What are the minimal elements of the partial order?

I,A,F

(b) What are the maximal elements of the partial order?

J,D,G

(c) Which of the following pairs are comparable?

$$(A, D),\ (J, F),\ (B, E),\ (G, F),\ (D, B),\ (C, F),\ (H, I), (C, E)$$

The pairs that are comparable are:
(A,D), (G,F), (D,B), and (H,I)
because they are connected on the graph

Part 2: Consider the partial order with domain $\{3, 5, 6, 7, 10, 14, 20, 30, 60, 70\}$ and with $x \leq y$ if x evenly divides y. Select the correct Hasse diagram for the partial order.

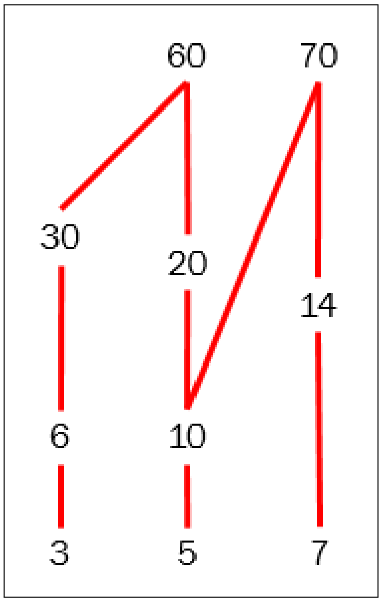
(a) 

Figure 2: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 20, 20 and 60, 10 and 70, 7 and 14, 14 and 70.*

10 divides 30 and in the graph they are not connected therefore this graph does not represent the given set.

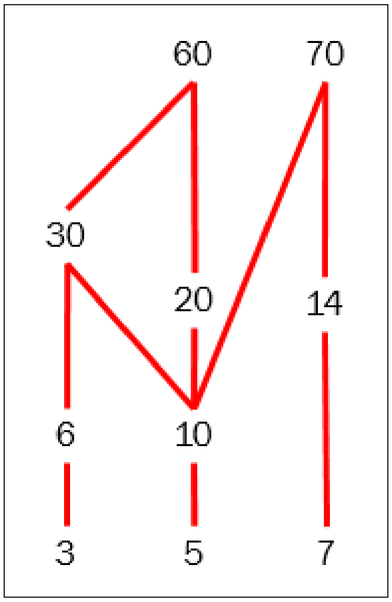
(b) 

Figure 3: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 30, 10 and 20, 20 and 60, 10 and 70, 7 and 14, 14 and 70.*

The Diagram that represents the set of numbers given is
graph B
x has to be $\neq$ y and x has to evenly divide y
3 can divide 6 which can divide 30 which can divide 60
5 can divide 10 which can divide 30,20 and 70
20 can divide 60
7 can divide 14 which can divide by 70

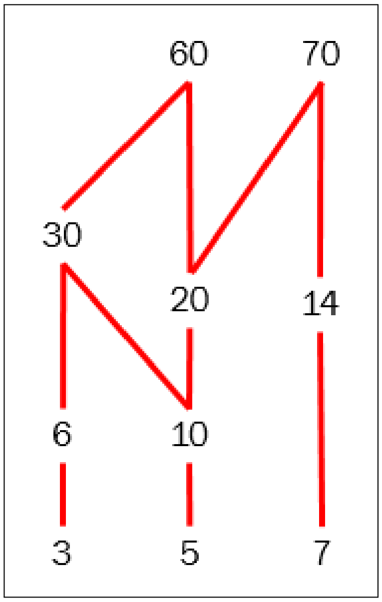
(c) 

Figure 4: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 30, 10 and 20, 20 and 60, 20 and 70, 7 and 14, 14 and 70.*

In this graph they have 20 and 70 connected. 20 cannot divide 70 evenly, therefore this graph does not represent the given set.

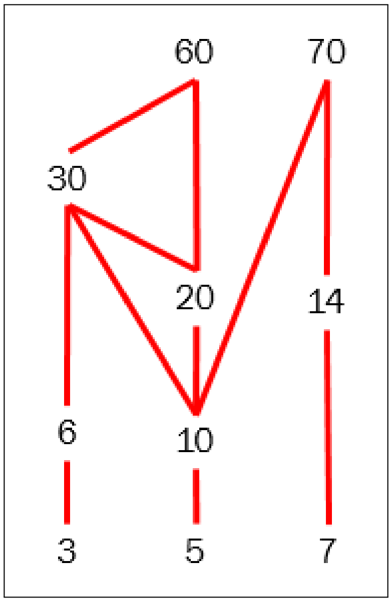
(d) 

Figure 5: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 30, 10 and 20, 20 and 30, 20 and 60, 10 and 70, 7 and 14, 14 and 70.*

In the graph 30 and 20 are connected but 20 cannot divide 30 evenly, therefore this graph does not represent the given set.

Problem 3

A car dealership sells cars that were made in 2015 through 2020. Let the cars for sale be the domain of a relation R where two cars are related if they were made in the same year.

(a) Prove that this relation is an equivalence relation.

To prove if the relation is an equivalence relation we check if it is reflective, symmetric, and transitive.

Let R=the relation
for any car for sale within the domain it will be related to itself because it was made the same year as itself.
$\rightarrow$ R is reflective

Let a and b be any two cars for sale within the domain, such that aRb.
a and b are made the same year so b and a have to be made the same year.
$\rightarrow$bRa so R is symmetric.

Let a,b, and c be any three cars being sold within the same domain such that aRb and bRc.
if a and b are made the same year and b and c are made the same year then a and c are made the same year.
$\rightarrow$ aRc so R is transitive

Thus proving the relation is an equivalence relation.

(b) Describe the partition defined by the equivalence classes.

The partition defined the domain for the cars fro sale being made in the years(2015,2016,2017,2018,2019,2020) were as (x) represents the equivalence class for year (x).

Problem 4

Analyze each graph below to determine whether it has an Euler circuit and/or an Euler trail.

- If it has an Euler circuit, specify the nodes for one.
- If it does not have an Euler circuit, justify why it does not.
- If it has an Euler trail, specify the nodes for one.
- If it does not have an Euler trail, justify why it does not.

(a)

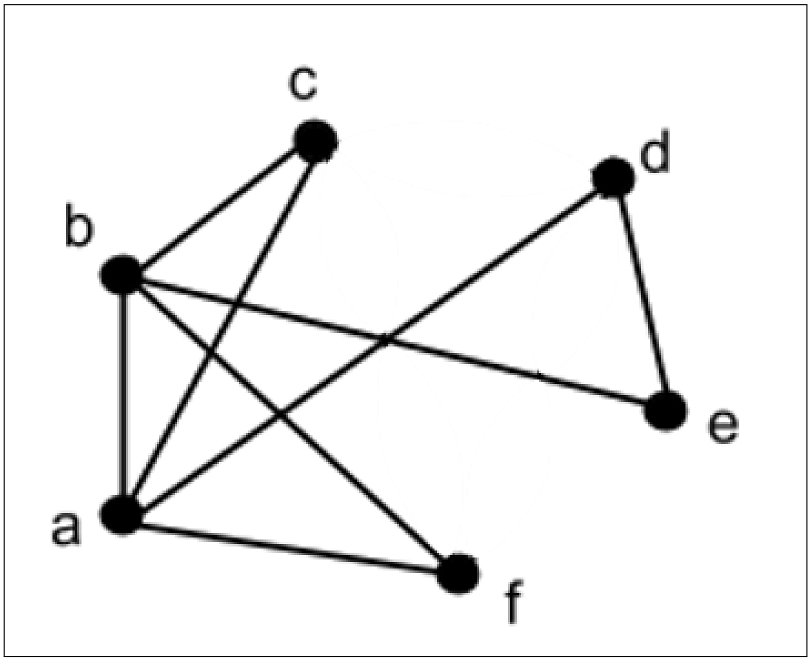


Figure 6: *An undirected graph has 6 vertices, a through f. There are 8-line segments that are between the following vertices: a and b, a and c, a and d, a and f, b and c, b and e, b and f, d and e.*

The degree of every vertex is even thus must have a **Euler circuit**
f→ b→e→d→a→c→b→a→f

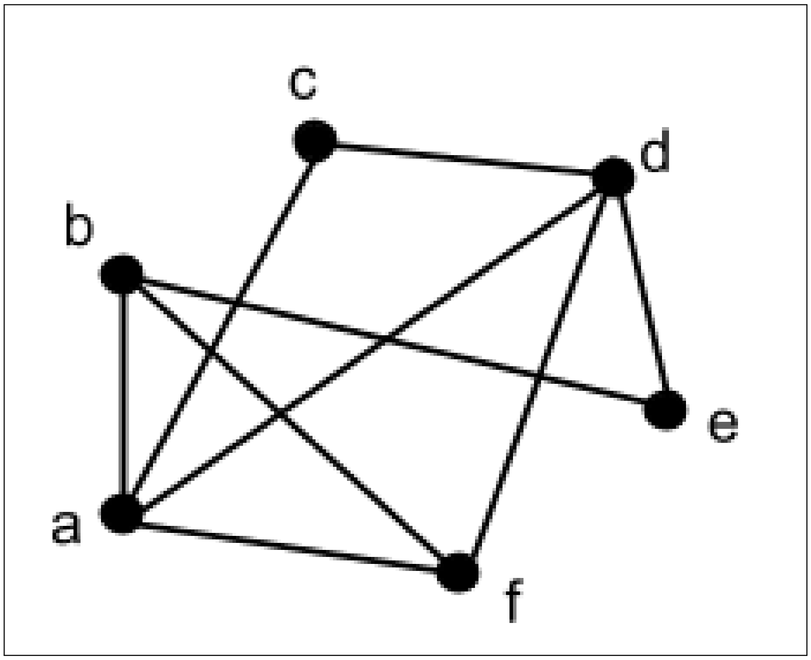
(b) 

Figure 7: *An undirected graph has 6 vertices, a through f. There are 9-line segments that are between the following vertices: a and b, a and c, a and d, a and f, b and e, b and f, c and d, d and e, d and f.*

This has a **Euler trail** because it has exactly 2 vertices with an odd number of degree.
b→f→a→d→c→a→b→e

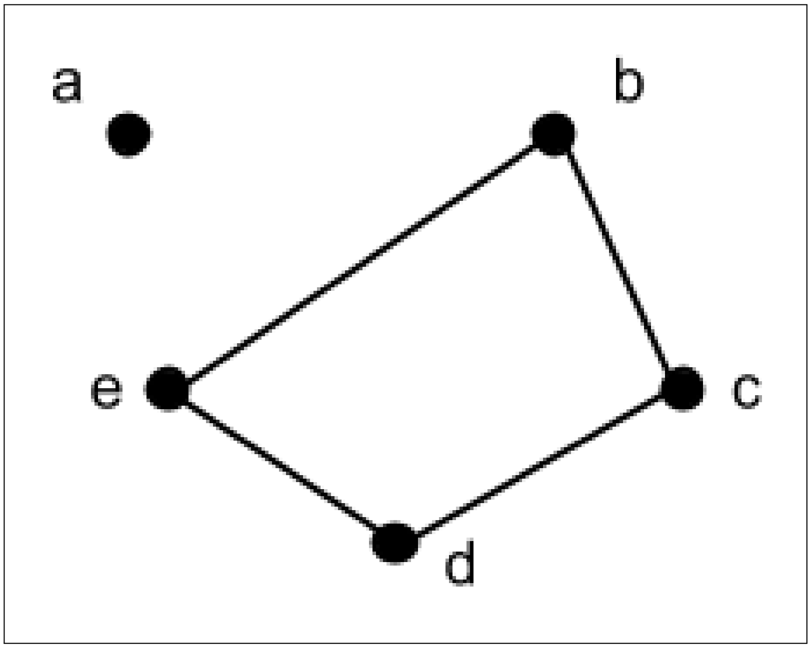
(c) 

Figure 8: *An undirected graph has 5 vertices, a through e. There are 4-line segments that are between the following vertices: b and c, b and e, c and d, d and e.*

vertex a is disconnected from the graph making it a disconnected graph.
Therefore **it cannot have a Euler Circuit or Euler Trail**

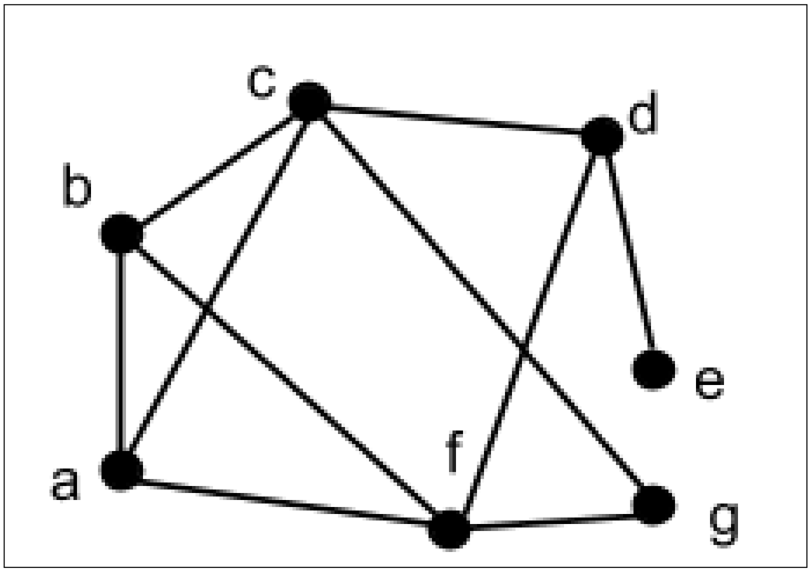
(d) 

Figure 9: *An undirected graph has 7 vertices, a through g. There are 10-line segments that are between the following vertices: a and b, a and c, a and f, b and c, b and f, c and d, c and g, d and e, d and f, f and g.*

In order to have a Euler curcuit every degree on the graph has to have an even number. Vertex e has a degree of 1 which is an odd number **therefore the graph does not have a Euler Circuit**.
In order to have a Euler trail the graph has to have exactly 2 vertices that have an odd degree. Vertices a,b,d, and e have odd degrees. That is a total of 4 odd degree vertices. $4 \neq 2$ **therefor it can not have a Euler Trail.**

Problem 5

Use Prim's algorithm to compute the minimum spanning tree for the weighted graph. Start the algorithm at vertex A. Explain and justify each step as you add an edge to the tree.

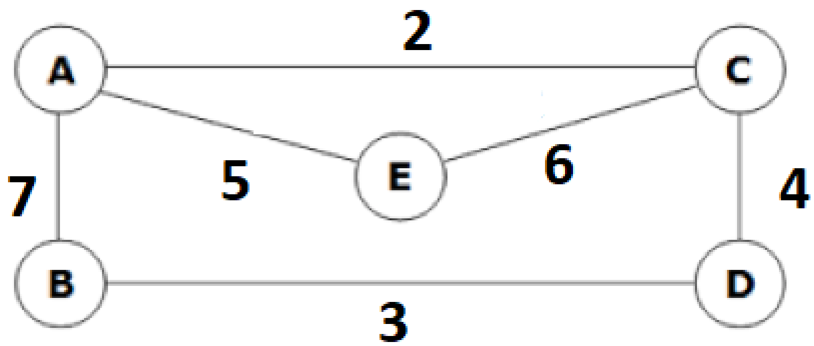


Figure 10: *A weighted graph shows 5 vertices, represented by circles, and 6 edges, represented by line segments. Vertices A, B, C, and D are placed at the corners of a rectangle, whereas vertex E is at the center of the rectangle. The edges, A B, B D, A C, C D, A E, and E C, have the weights, 7, 3, 2, 4, 5, and 6, respectively.*

In the graph start at vertex A. Follow the edges with the lowest value without repeating a vertex.If you run into a dead end start to back track to the last vertex with an available edge that leads to a vertex that has not been touched. Repeat until every vertex has been touched and add the value of the edges used to get to each vertex.
(A→C)(C→D)(D→B)(C→E)
2+4+3+5=14
→the weight of the minimum spanning tree is 14

Problem 6

A lake initially contains 1000 fish. Suppose that in the absence of predators or other causes of removal, the fish population increases by 10% each month. However, factoring in all causes, 80 fish are lost each month.

Give a recurrence relation for the population of fish after n months. How many fish are there after 5 months? If your fish model predicts a non-integer number of fish, round down to the next lower integer.

You start with 1000 fish and it increases by 10 percent so we have to multiply 1000 by 1.1 in order to increase it by 10 percent. 80 fish are lost each month therefore has to be subtracted.
(1000)(1.1)-80 = 1020
Since only the first month is 1000 fish the equation above would represent the second month and can be rewritten as a_1= (1000)(1.1)-80. In order to represt all months going forward it would be written as a_n=(a_{n-1})(1.1)-80

a_1=1000
a_2=(1000)(1.1)-80=1020
a_3=(1020)(1.1)-80=1042
a_4=(1042)(1.1)-80=1066
a_5=(1066)(1.1)-80=1092

Thus there will be 1092 fish after 5 months