MODULE TWO PROBLEM SET

Your Name Here

Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

Part 1. Indicate whether the argument is valid or invalid. For valid arguments, prove that the argument is valid using a truth table. For invalid arguments, give truth values for the variables showing that the argument is not valid.

(1)

$$\begin{array}{c} (p \wedge q) \to r \\ \\ \therefore (p \vee q) \to r \end{array}$$

Part 2. Converse and inverse errors are typical forms of invalid arguments. Prove that each argument is invalid by giving truth values for the variables showing that the argument is invalid. You may find it easier to find the truth values by constructing a truth table.

(a) Converse error

$$\begin{array}{c} p \to q \\ q \\ \\ \therefore p \end{array}$$

(b) Inverse error

$$\begin{array}{c} p \to q \\ \neg p \\ \\ \therefore \neg q \end{array}$$

Part 3. Which of the following arguments are invalid and which are valid? Prove your answer by replacing each proposition with a variable to obtain the form of the argument. Then prove that the form is valid or invalid.

(a)

The patient has high blood pressure or diabetes or both.
The patient has diabetes or high cholesterol or both.
∴ The patient has high blood pressure or high cholesterol.

PROBLEM 2

Part 1. Which of the following arguments are valid? Explain your reasoning.

(a) I have a student in my class who is getting an A. Therefore, John, a student in my class, is getting an A.

(b) Every Girl Scout who sells at least 30 boxes of cookies will get a prize. Suzy, a Girl Scout, got a prize. Therefore, Suzy sold at least 30 boxes of cookies.

Part 2. Determine whether each argument is valid. If the argument is valid, give a proof using the laws of logic. If the argument is invalid, give values for the predicates P and Q over the domain a, b that demonstrate the argument is invalid.

(a)

$$\exists x\,(P(x) \;\wedge\; Q(x))$$

$$\therefore \exists x\, Q(x) \;\wedge\; \exists x\, P(x)$$

(b)

$$\forall x\,(P(x) \;\vee\; Q(x))$$

$$\therefore \forall x\, Q(x) \;\vee\; \forall x\, P(x)$$

Problem 3

Prove the following using a direct proof. Your proof should be expressed in complete English sentences.

If a, b, and c are integers such that b is a multiple of a^3 and c is a multiple of b^2, then c is a multiple of a^6.

Problem 4

Prove the following using a direct proof:

The sum of the squares of 4 consecutive integers is an even integer.

Problem 5

Prove the following using a proof by contrapositive:

Let x be a rational number. Prove that if xy is irrational, then y is irrational.

Problem 6

Prove the following using a proof by contradiction:

The average of four real numbers is greater than or equal to at least one of the numbers.

Problem 7

Let $q = \dfrac{a}{b}$ and $r = \dfrac{c}{d}$ be two rational numbers written in lowest terms. Let $s = q + r$ and $s = \dfrac{e}{f}$ be written in lowest terms. Assume that s is not 0.

Prove or disprove the following two statements.

a. If b and d are odd, then f is odd.

b. If b and d are even, then f is even.

Problem 8

Define $P(n)$ to be the assertion that:

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

(a) Verify that $P(3)$ is true.

(b) Express $P(k)$.

(c) Express $P(k+1)$.

(d) In an inductive proof that for every positive integer n,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

what must be proven in the base case?

(e) In an inductive proof that for every positive integer n,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

what must be proven in the inductive step?

(f) What would be the inductive hypothesis in the inductive step from your previous answer?

(g) Prove by induction that for any positive integer n,

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$