MAT 230 EXAM ONE

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Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

(a) The domain for all variables in the expressions below is the set of real numbers. **Determine whether each statement is true or false.**

(i) $\forall\, x\; \exists\, y\; (x\,+\,y\;\geq\;0)$

∀ x ∈ R ∃ y = -x + 1 ∈ R
so
x+y = (x(-x+1))
= x-x+1
= 1 ¿ 0
Therefore it is
True

(ii) $\exists\, x\; \forall\, y\; (x\,\cdot\,y\;>\;0)$

Let y = -x such that
xy = x(-x)
= -x^2 < 0 any negative number squared is a positive number Therefore The statement is
False

(b) **Translate each of the following English statements into logical expressions.**

(i) There are two numbers whose ratio is less than 1.

$\exists x, y(\frac{x}{y} < 1, \frac{y}{x} < 1)$

(ii) The reciprocal of every positive number is also positive.

$\forall x(x > 0, \frac{1}{x} > 0)$

Problem 2

Prove the following using the specified technique:

(a) Let x and y be two real numbers such that $x + y$ is rational. Prove by contrapositive that if x is irrational, then $x - y$ is irrational.

Let x - y be a rational number
x - y = $\frac{m}{n}$, n $\neq$ 0,for some integers m,n
x + y is rational so,
x + y = $\frac{j}{k}$, k $\neq$ 0, for some integers j,k
x-y+x+y = $\frac{m}{n} + \frac{j}{k}$
2x = $\frac{mk+jn}{nk}$
x = $\frac{1}{2}(\frac{mk+jn}{kn})$
since Mk+jn and 2kn are both integers and kn is not 0, x is then a rational number
Thus proving if x is irrational then x-y is irrational

(b) Prove by contradiction that for any positive two real numbers, x and y, if $x \cdot y \leq 50$, then either $x < 8$ or $y < 8$.

Assuming xy $\leq$ 50 is false
we can substitute x with 7 when x $<$ 8
and y can be any number (x+1)
the the case x = 7 then 7 * 8 $\leq$ 50 this statement is false proving the given statement is **true** by contradiction

Problem 3

Let $n \geq 1$, x be a real number, and $x \geq -1$. **Prove the following statement using mathematical induction.**

$$(1 + x)^n \geq 1 + nx$$

base case is n=1 is true
Assume n=a is true
(1+x)
a $\geq$ 1 + ax $\rightarrow$ $\forall$ x $\geq$ -1
Now
(1+x)
a + 1 $\geq$ (1+x)(1+ax) $\rightarrow$ $\forall$ $\geq$ -1
= 1 + ax + x + ax2 $\geq$ 1 + (a+1)x $\rightarrow$ $\forall$ $\geq$ -1
therefore n=a+1 is true
proving n $\geq$ 1 is true by induction

Problem 4

Solve the following problems:

(a) How many ways can a store manager arrange a group of 1 team leader and 3 team workers from his 25 employees?

There are 4 members in group 1 out of 25 employees
25 x 24 x 23 x 22 =
303,600 ways to arrange group 1

(b) A states license plate has 7 characters. Each character can be a capital letter $(A - Z)$, or a non-zero digit $(1 - 9)$. How many license plates start with 3 capital letters and end with 4 digits with no letter or digit repeated?

letters = 26
digits = 9
so
$\mathbf{(}\binom{7}{3} * 26^3 * 9^4\mathbf{)}$

(c) How many binary strings of length 5 have at least 2 adjacent bits that are the same ("00" or "11") somewhere in the string?

There are 2 combinations with no same adjacent bit.
2^5 two different bits at length 5
so
2^5 - 2 =
30 possible strings

PROBLEM 5

A class with n kids lines up for recess. The order in which the kids line up is random with each ordering being equally likely. There are two kids in the class named Betty and Mary. The use of the word "*or*" in the description of the events, should be interpreted as the inclusive or. That is "*A or B*" means that A is true, B is true, or both A and B are true.

What is the probability that Betty is first in line or Mary is last in line as a function of n? Simplify your final expression as much as possible and include an explanation of how you calculated this probability.

Total ways kids line up = n!
Event Betty is in first place = A
Event Mary is in last place = B
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$BothBettyisfirstandMaryislast = A \cap B$
$P(A) = \frac{(n-1)!}{n!}$
$P(B) = \frac{(n-1)!}{n!}$
$P(A \cup B) = \frac{(n-2)!}{n!}$
so
$\frac{(n-1)!}{n!} + \frac{(n-1)!}{n!} - \frac{(n-2)!}{n!}(n-1)(n-2)$
$= \frac{1}{n} * \frac{2n-3}{n-1} or \frac{2n-3}{n(n-1)}$
$= 2n - \frac{3}{n^2-n}$

PROBLEM 6

The general manager, marketing director, and 3 other employees of Company A are hosting a visit by the vice president and 2 other employees of Company B. The eight people line up in a random order to take a photo. Every way of lining up the people is equally likely.

(a) What is the probability that the general manager is next to the vice president?

letting both general manager and vice president as 1(solve for 7 people)
$7! = 5{,}040$ now solving for general and vice president
$2! = 2$ $P(A) = \frac{(5040*2)}{40{,}320}$
$= \frac{1}{4}$

(b) What is the probability that the marketing director is in the leftmost position?

If the marketing director is in the left most spot I solve for the remaining 7 people
$\frac{7!}{8!} = \frac{5040}{40320}$
$= \frac{1}{8}$

(c) Determine whether the two events are independent. Prove your answer by showing that one of the conditions for independence is either true or false.

If we have the marketing directer in the left most position
then neither the vice president nor the general manager can have that postion.
In which case then lowers the probability and making the events **dependent** on each other