MODULE ONE PROBLEM SET

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Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

Problem 1

In the following question, the domain of **discourse** is a set of male patients in a clinical study. Define the following predicates:

- $P(x):$ x was given the placebo
- $D(x):$ x was given the medication
- $M(x):$ x had migraines

Translate each of the following statements into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

Sample question: Some patient was given the placebo and the medication.

- $\exists x\ (P(x)\ \wedge\ D(x))$
- *Negation:* $\neg\exists x\ (P(x)\ \wedge\ D(x))$
- *Applying De Morgan's law:* $\forall x\ (\neg P(x)\ \vee\ \neg D(x))$
- *English: Every patient was either not given the placebo or not given the medication (or both).*

(a) Every patient was given the medication or the placebo or both.

Logical expression: $\forall x(D(x) \vee P(x))$
Negation: $\neg\forall x(D(x) \vee P(x))$
Applying De Morgan's law: $\exists x(\neg D(x) \wedge \neg P(x)$
English: Some patient was not given the medication and not given the placebo.

(b) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \to q \equiv \neg p \vee q$.)

Logical Expression: $\forall x(P(x) \longrightarrow M(x))$
Negation: $\neg\forall x(P(x) \longrightarrow M(x))$
Conditional Identity: $\neg\forall x(\neg P(x) \vee M(x))$
Applying De Morgan's law: $\exists x(\neg\neg P(x) \wedge \neg M(x))$
Double negation law: $\exists x(P(x) \wedge \neg M(x))$
English: Some patient was given the placebo but does not have a migraines.

(c) There is a patient who had migraines and was given the placebo.

Logical Expression: $\exists x(M(x) \wedge P(x))$
Negation: $\neg\exists x(M(x) \wedge P(x))$
Applying De Morgan's law: $\forall x(\neg M(x) \vee \neg P(x))$
English: Every patient who does not have migraines was not given the placebo.

Problem 2

Use De Morgan's law for quantified statements and the laws of propositional logic to show the following equivalences:

(a) $\neg\forall x\ (P(x) \wedge \neg Q(x)) \ \equiv\ \exists x\ (\neg P(x) \vee Q(x))$

$\exists x(\neg P(x) \vee \neg\neg Q(x))$ **De Morgan's Law**
$\exists x(\neg P(x) \vee Q(x))$ **Double negation law**

(b) $\neg\forall x\ (\neg P(x) \to Q(x)) \ \equiv\ \exists x\ (\neg P(x) \wedge \neg Q(x))$

$\exists x(\neg\neg P(x) \longrightarrow \neg Q(x))$ **De Morgan's Law**
$\exists x(P(x) \longrightarrow \neg Q(x))$ **Double negation law**
$\exists x(\neg P(x) \vee \neg Q(x))$ **Conditional identity**

(c) $\neg\exists x\ \big(\neg P(x) \vee (Q(x) \wedge \neg R(x))\big) \ \equiv\ \forall x\ \big(P(x) \wedge (\neg Q(x) \vee R(x))\big)$

$\forall x(\neg\neg P(x) \wedge \neg(Q(x) \wedge \neg R(x)))$**De Morgan's law**
$\forall x(P(x) \wedge \neg(Q(x) \wedge \neg R(x)))$**Double negation laws**
$\forall x(P(x) \wedge (\neg Q(x) \vee \neg\neg R(x)))$**De Morgan's law**
$\forall x(P(x) \wedge (\neg Q(x) \vee R(x)))$**Double negation laws**

Problem 3

The domain of **discourse** for this problem is a group of three people who are working on a project. To make notation easier, the people are numbered 1, 2, 3. The predicate $M(x,\ y)$ indicates whether x has sent an email to y, so $M(2,\ 3)$ is read "Person 2 has sent an email to person 3." The table below shows the value of the predicate $M(x,\ y)$ for each $(x,\ y)$ pair. The truth value in row x and column y gives the truth value for $M(x,\ y)$.

M	1	2	3
1	T	T	T
2	T	F	T
3	T	T	F

Determine if the quantified statement is true or false. Justify your answer.

(a) $\forall x\, \forall y\, (x \neq y) \ \rightarrow\ M(x,\ y))$

(true) No matter the person, when not including themselves, they had send the email to everyone else.

(b) $\forall x\, \exists y\ \ \neg M(x,\ y)$

(false) The table is negated, leaving person 1 not sending an email to anyone.

(c) $\exists x\, \forall y\ \ M(x,\ y)$

(true) Person 1 has sent the email to everyone.

Problem 4

Translate each of the following English statements into logical expressions. The domain of **discourse** is the set of all real numbers.

(a) The reciprocal of every positive number less than one is greater than one.

$\forall x(1 > x > 0 \to \frac{1}{x} > 1)$

(b) There is no smallest number.

$\neg\exists x\forall y(x \leq y)$

(c) Every number other than 0 has a multiplicative inverse.

$\forall x\exists y(x \neq 0 \to (xy = 1))$

PROBLEM 5

The sets A, B, and C are defined as follows:

$$A = tall, grande, venti$$
$$B = foam, no - foam$$
$$C = non - fat, whole$$

Use the definitions for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

(a) Write an element from the set $A \times B \times C$.

(grande,foam,whole)

(b) Write an element from the set $B \times A \times C$.

(no-foam,tall,non-fat)

(c) Write the set $B \times C$ using roster notation.

$B \times C = \{(foam, non - fat), (foam, whole), (no - foam, non - fat), (no - foam, whole)\}$