MAT 230 EXAM TWO

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Your Name Here

Directions: Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.

PROBLEM 1

This question has 2 parts.

Part 1: Suppose that F and X are events from a common sample space with $P(F) \neq 0$ and $P(X) \neq 0$.

(a) Prove that $P(X) = P(X|F)P(F) + P(X|\bar{F})P(\bar{F})$. Hint: Explain why $P(X|F)P(F) = P(X \cap F)$ is another way of writing the definition of conditional probability, and then use that with the logic from the proof of Theorem 4.1.1.

(b) Explain why $P(F|X) = P(X|F)P(F)/P(X)$ is another way of stating Theorem 4.2.1 Bayes Theorem.

Part 2: A website reports that 70% of its users are from outside a certain country. Out of their users from outside the country, 60% of them log on every day. Out of their users from inside the country, 80% of them log on every day.

(a) What percent of all users log on every day? Hint: Use the equation from Part 1 (a).

(b) Using Bayes Theorem, out of users who log on every day, what is the probability that they are from inside the country?

PROBLEM 2

This question has 2 parts.

Part 1: The drawing below shows a Hasse diagram for a partial order on the set:
$\{A, B, C, D, E, F, G, H, I, J\}$
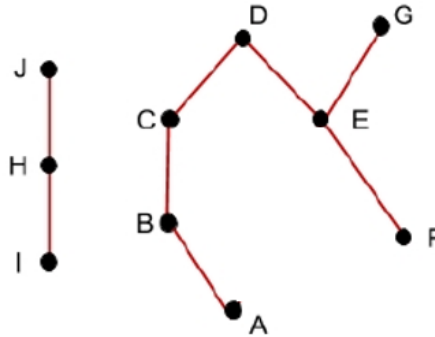
Figure 1: *A Hasse diagram shows 10 vertices and 8 edges. The vertices, represented by dots, are as follows: vertex J is upward of vertex H; vertex H is upward of vertex I; vertex B is inclined upward to the left of vertex A; vertex C is upward of vertex B; vertex D is inclined upward to the right of vertex C; vertex E is inclined upward to the left of vertex F; vertex G is inclined upward to the right of vertex E. The edges, represented by line segments between the vertices are as follows: 3 vertical edges connect the following vertices: B and C, H and I, and H and J; 5 inclined edges connect the following vertices: A and B, C and D, D and E, E and F, and E and G.*

Determine the properties of the Hasse diagram based on the following questions:
 (a) What are the minimal elements of the partial order?

 (b) What are the maximal elements of the partial order?

 (c) Which of the following pairs are comparable?
$$(A, D), \ (J, F), \ (B, E), \ (G, F), \ (D, B), \ (C, F), \ (H, I), (C, E)$$

Part 2: Consider the partial order with domain $\{3, 5, 6, 7, 10, 14, 20, 30, 60, 70\}$ and with $x \le y$ if x evenly divides y. Select the correct Hasse diagram for the partial order.
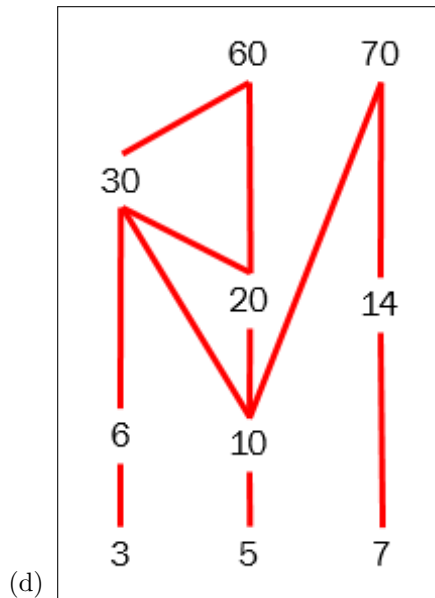

(a)
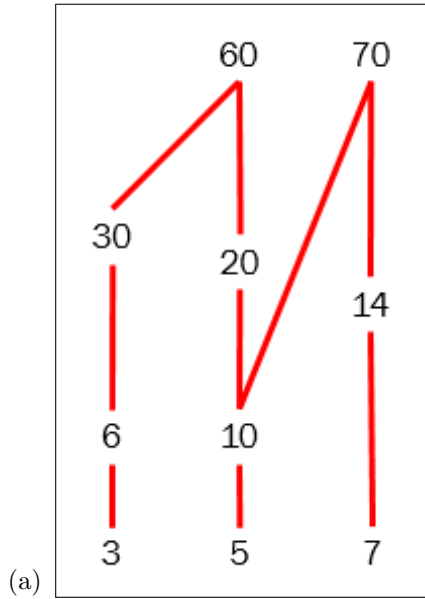
Figure 2: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 20, 20 and 60, 10 and 70, 7 and 14, 14 and 70.*

(b)
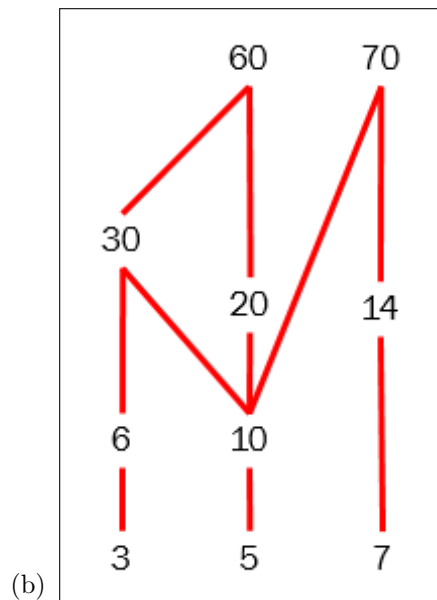
Figure 3: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 30, 10 and 20, 20 and 60, 10 and 70, 7 and 14, 14 and 70.*

(c)
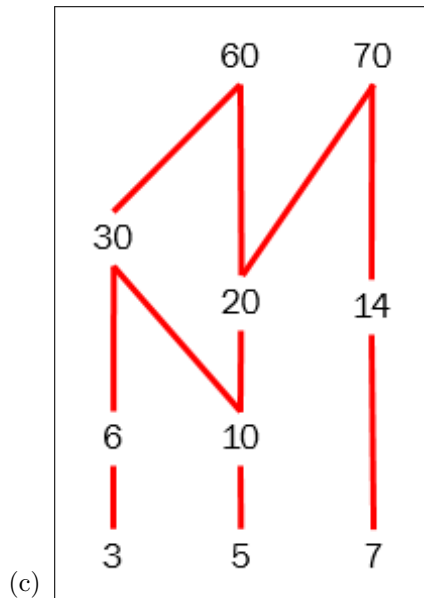
Figure 4: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 30, 10 and 20, 20 and 60, 20 and 70, 7 and 14, 14 and 70.*

(d)

Figure 5: *A Hasse diagram shows a set of elements 3; 5; 6; 7; 10; 14; 20; 30; 60, 70. There are lines connecting 3 and 6, 6 and 30, 30 and 60, 5 and 10, 10 and 30, 10 and 20, 20 and 30, 20 and 60, 10 and 70, 7 and 14, 14 and 70.*

Problem 3

A car dealership sells cars that were made in 2015 through 2020. Let the cars for sale be the domain of a relation R where two cars are related if they were made in the same year.

(a) Prove that this relation is an equivalence relation.

(b) Describe the partition defined by the equivalence classes.

Problem 4

Analyze each graph below to determine whether it has an Euler circuit and/or an Euler trail.

- If it has an Euler circuit, specify the nodes for one.
- If it does not have an Euler circuit, justify why it does not.
- If it has an Euler trail, specify the nodes for one.
- If it does not have an Euler trail, justify why it does not.

(a)
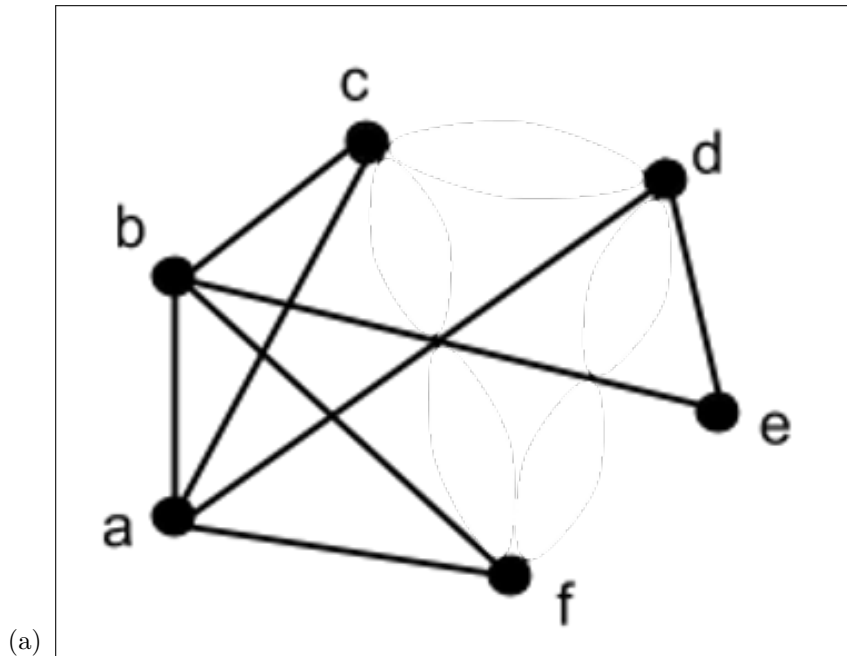
Figure 6: *An undirected graph has 6 vertices, a through f. There are 8-line segments that are between the following vertices: a and b, a and c, a and d, a and f, b and c, b and e, b and f, d and e.*

(b)
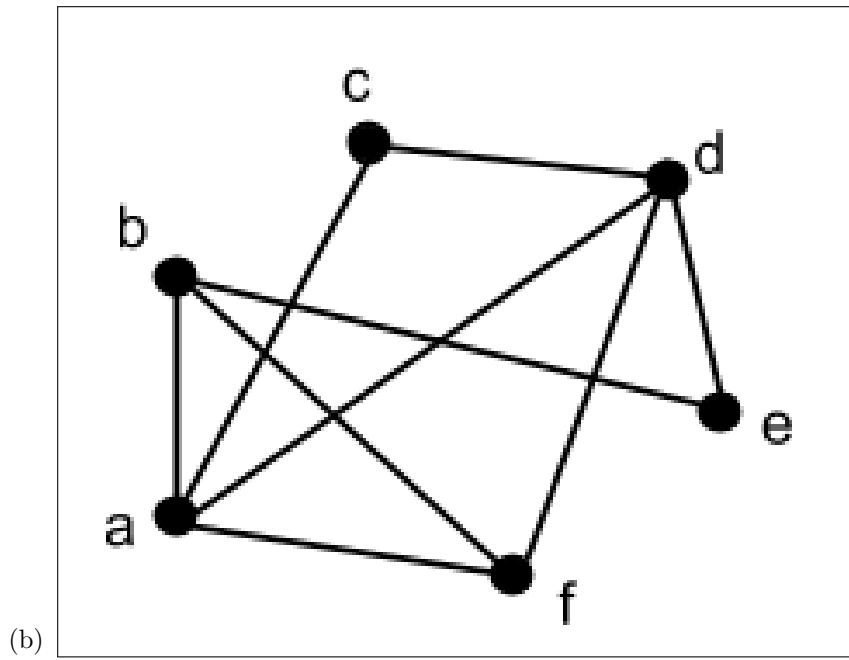
Figure 7: *An undirected graph has 6 vertices, a through f. There are 9-line segments that are between the following vertices: a and b, a and c, a and d, a and f, b and e, b and f, c and d, d and e, d and f.*

(c)
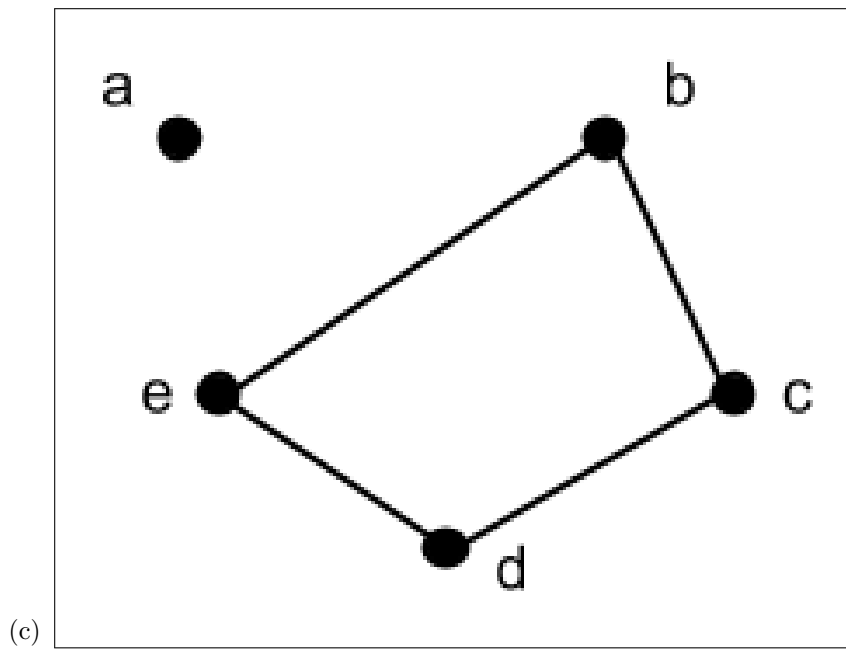
Figure 8: *An undirected graph has 5 vertices, a through e. There are 4-line segments that are between the following vertices: b and c, b and e, c and d, d and e.*

(d)
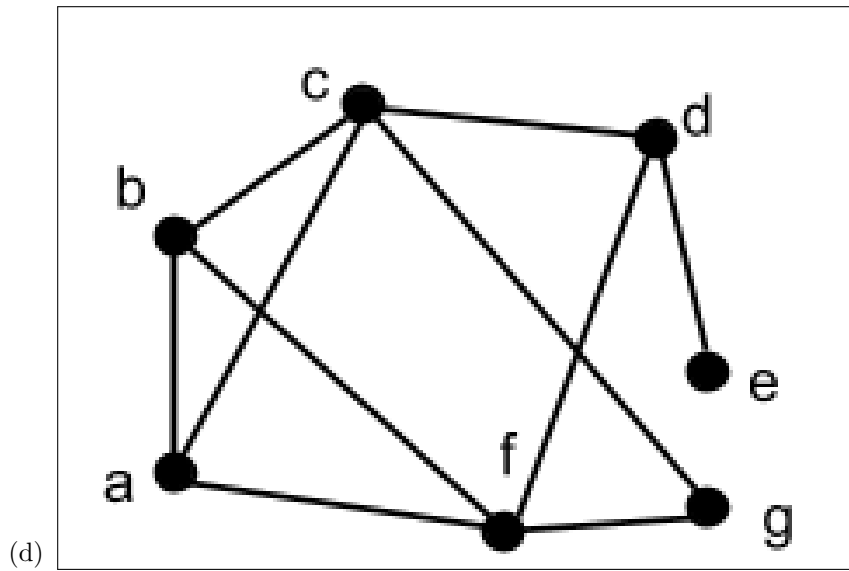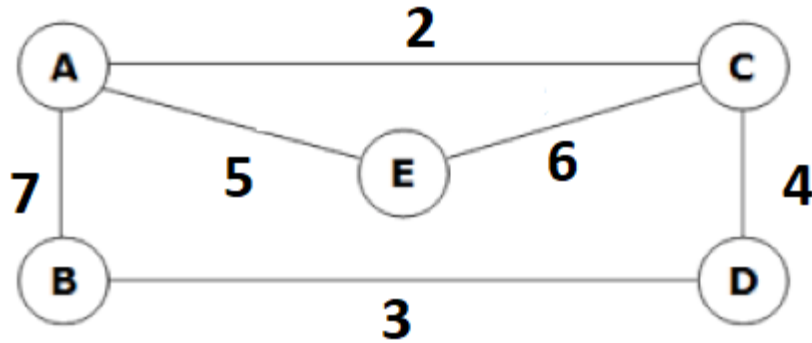
Figure 9: *An undirected graph has 7 vertices, a through g. There are 10-line segments that are between the following vertices: a and b, a and c, a and f, b and c, b and f, c and d, c and g, d and e, d and f, f and g.*

PROBLEM 5

Use Prim's algorithm to compute the minimum spanning tree for the weighted graph. Start the algorithm at vertex A. Explain and justify each step as you add an edge to the tree.

Figure 10: *A weighted graph shows 5 vertices, represented by circles, and 6 edges, represented by line segments. Vertices A, B, C, and D are placed at the corners of a rectangle, whereas vertex E is at the center of the rectangle. The edges, A B, B D, A C, C D, A E, and E C, have the weights, 7, 3, 2, 4, 5, and 6, respectively.*

Problem 6

A lake initially contains 1000 fish. Suppose that in the absence of predators or other causes of removal, the fish population increases by 10% each month. However, factoring in all causes, 80 fish are lost each month.

Give a recurrence relation for the population of fish after n months. How many fish are there after 5 months? If your fish model predicts a non-integer number of fish, round down to the next lower integer.