2015 HAUTS SOLUTIONS

CHESS WITH A TWIST:

1. D

To complete the following task the knight must alternate white and black.
It must also alternate between outer rows (row 1&4) and inner rows (row 2&3), else the number of squares covered on inner rows will be more than that covered on outer rows.
Suppose it starts at a black square on inner row, then squares visited on even-numbered turns are white squares on outer rows, and those visited on odd-numbered turns are black squares on inner rows. It never hits the other squares. Similar argument is valid if you start at some other square.

2. C

You can easily convince yourself that N=1 or 2 won't work.
N=4 won't work by an argument similar to that in Question: 1. When we go from the eighth square to the ninth square we must change colour; we must go from the second to the third row vertically (or vice versa); and we must go from the second column to the third column (or vice versa).

For N=3 the following example suffices:

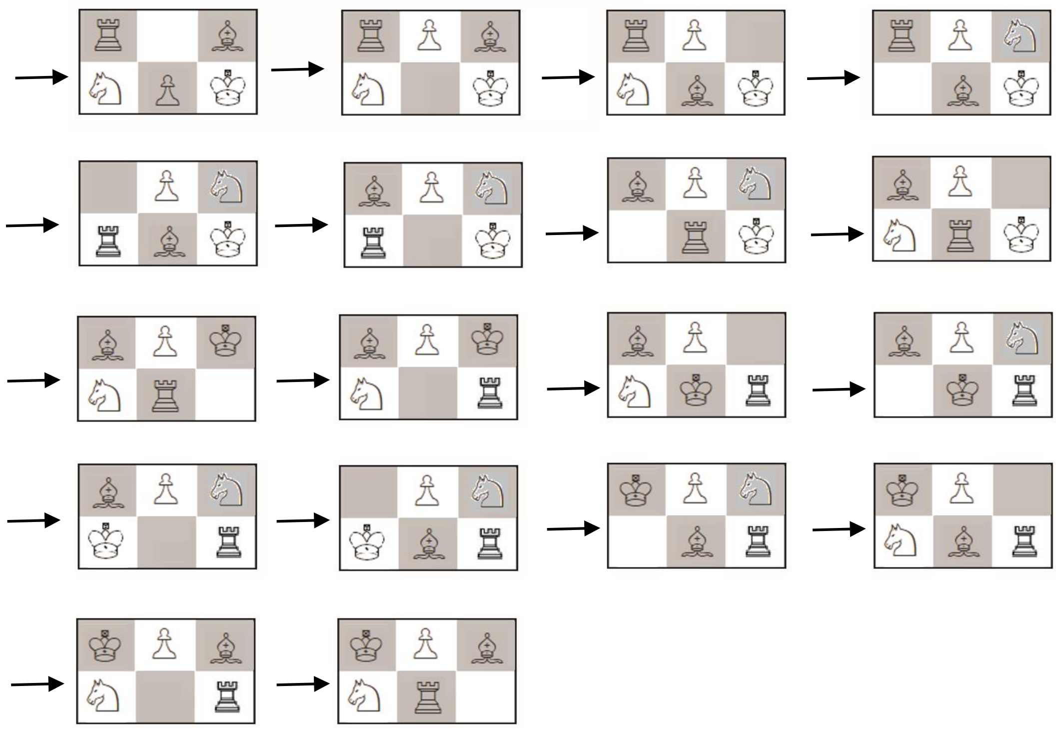
1	8	3
4	11	6
7	2	9
10	5	12

For N=5:

14	5	18	1	12
19	10	13	6	17
4	15	8	11	2
9	20	3	16	7

Similarly, for every value of N>4 we can derive a knight's tour.

3. B



Hence, Total number of steps =18.

TROUBLE WITH FLOWERS:

4. D
5. B

Exchange: $R_n - Y_n = (R_{n-1} - 3) - (Y_{n-1} + 2) = (R_{n-1} - Y_{n-1} - 5)$
Swap: $R_n - Y_n = (Y_{n-1} - R_{n-1})$

From the above two statements it can be declared that (R, Y) can never become (5,5) because the initial value of R-Y is 3. Hence the value of ((R-Y) mod 5) remains either 3 or 2 but never goes to 0.

CRACK THE CODE:

6. A
According to the given hints
1. Chessboard (8 x 8 board)
2. The poetry

It means the letters have to be filled in a table as shown.

a	b	c	d	e	f	g	h
i	j	k	l	m	n	o	p
q	r	s	t	u	v	w	x
y	z	a	b	c	d	e	f
g	h	i	j	k	l	m	n
o	p	q	r	s	t	u	v
w	x	y	z	a	b	c	d
e	f	g	h	i	j	k	l

So, according to the hint given by the clock, the letters are selected in anti-clockwise direction. So “Powder” is coated as “hfvkma” and thus “Magic” will be coded as “erfpk”.

PYRAMID WALLS:

7. C
The minimum number of moves required are 8. Here is one of the possible solutions –

4		6		8		7		5		1		2		3		1
2		1		5		6		4		3		3		2		2
3		3		4		1		2		2		1		1		3
1	➔	2	➔	2	➔	3	➔	3	➔	4	➔	4	➔	4	➔	4
6	Move 1	4	Move 2	3	Move 3	2	Move 4	1	Move 5	5	Move 6	5	Move 7	5	Move 8	5
5		5		1		4		6		6		6		6		6
8		8		6		5		7		7		7		7		7
7		7		7		8		8		8		8		8		8

8. C
We will work backwards from the second last layer. Suppose that a block of second last layer was the starting position, then the maximum path sum from that block would be the sum of that block’s value and the greater value among the two blocks below it. Similarly, maximum sum from any third last layer’s block would be sum of itself and greater of the answer of the blocks below it. As we move up, no of blocks in a layer decreases and we get the answer for the topmost block. Here is the value of answers if each block were the starting block worked out as above said.

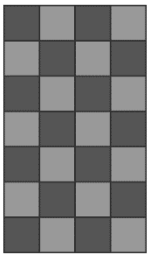
249						
224	223					
157	145	160				
149	122	126	116			
96	96	94	74	83		
90	21	38	39	27	38	
7	13	11	3	10	21	5

9. C

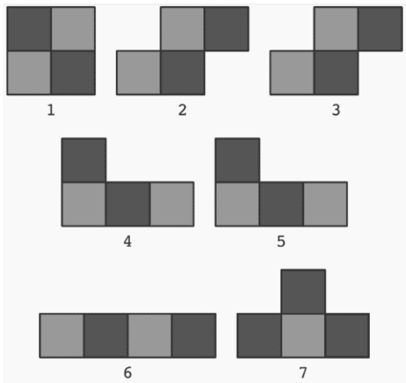
We will break this into 2 parts. First we count the number of paths from topmost block to '55'and then from '55' to bottom layer. It is easy to see that there are 6 paths from topmost block to '55' (one passing from '8'; 4 from '19'; 1 from '34'). Now, observe that from any block of 2nd most bottom layer, there are 2 paths to the bottom layer. Similarly, from any block of 3rd most bottom layer, we can go to 2 different blocks of 2nd most bottom layer hence from each block of 3rd most bottom layer, there are 4 paths to the bottom. Hence, there are 6 paths from top-most block to '55' and 4 paths from '55' to bottom layer. Now we multiply these two numbers to get 24.

FRAGMENTS:

10. D



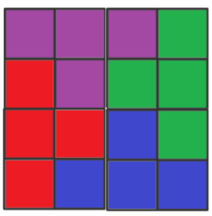
The 7 individual pieces, add up to a total of 28 squares. Therefore, assuming we can indeed form it into a rectangle, it would have to be 7x4 or 14x2 squares in size. I'm using the former case here simply because it's a more natural shape, however this proof applies equally as well to the latter. Now imagine that we label each of these squares with a colour - either black or white - such that they form a checkerboard pattern as shown beside. Notice that the number of black squares must be equal to the number of white, a property we'll exploit.
So that's 14 black squares, and 14 white. Looking at each of the pieces individually, the issue with our assumption quickly appears.



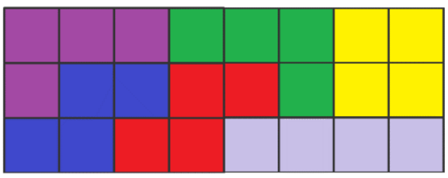
As shown beside, for pieces 1-6, the number of black squares within the piece is equal to the number of white. Clearly which squares are black and which are white depends on the actual placement of the piece within the rectangle, but the shapes themselves dictate the count of each colour (since adjacent squares must be of different colours).
However, piece 7 disrupts the trend. Irrelevant of how it's located, it must be comprised of 3 squares of one colour, and 1 of the other, a property that is purely down to its shape.
So, taking that into account along with the other 6 pieces, in total they're comprised of 13 squares of one colour, and 15 of the other, with no assumptions about how they're located within the rectangle. We needed 14 of each, and since we've just shown that we can't get that, our original assumption is overturned and our proof is complete.

11. No Answer (137x4)

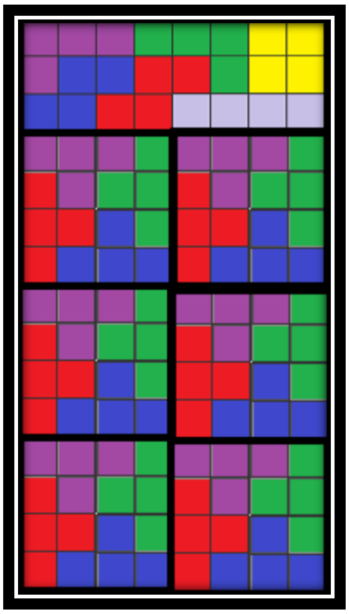
The smallest rectangle/square that can be made using the most valued piece is a 4X4 square as shown below.



So of the 30 pieces we can use 4*n pieces to get the maximum score possible. n comes out to be 6 not 7,since we need to use the rest of the pieces atleast once. As for the remaining 6 pieces , the maximum scores possible is using the following arrangement-



Thus the resulting arrangement is-



Thus the maximum points possible is: 6*(4*4*5) + 4*4*2 + 4*3*2 + 4*2 + 4*1 = 137*4 = 548.

ENIGMATHIC:

12. 52

Here is how you get to the answer. You may call it a greedy strategy.

As when you place a 2dinar coin at position *x*, position *x*+6 cannot be a 2dinar coin, but position *x*+12 can be a 2dinar coin, and position *x*+18 cannot…

So, place 2dinar coins on position 1 to 6. This means positions 7 to 12 are 1dinar coins, 13 to 18 are 2dinar coins, and so on.

So for every 12 consecutive coins, 6 of them are 2dinar coins. The last 4 (position 97 to 100) are also 2dinar coins. Hence the number of 2dinar coins is 52.

Now we can also prove that *x>52* is infeasible:

We will assume a configuration *C* has at most 47 1dinar coins, and will show *C* is infeasible, as follows. First, note that 100 = 8⅓ dozen. If at most 47 1dinar coins are placed somewhere in the first eight consecutive-dozens of a hundred, then at least one of those consecutive-dozens contains fewer than six 1dinar coins (because 47/8 < 6).

Next, note that if a consecutive-dozen has fewer than six 1dinar coins, then at least two 2dinar coins are separated by exactly five coins. This is because there are six ways to have five-separation, involving places 1+7; 2+8; 3+9; 4+10; 5+11; and 6+12. Five 1dinar coins can block up at most five of those six ways; it takes at least six 1dinar coins to block them all.

This shows that any configuration *C* with fewer than 48 1dinar coins is infeasible.

13. 17

Out of 6 marks the number of 2 mark pairs you can make is 15(5+4+3+2+1). Therefore, the minimum length of scale required is 15. With a very few logical derivations you can come to the conclusion that the answer is 17 and here is one of the possible solution:

0,3,5,9,16,17

14. 40

'd' =5 is the most optimized solution and the following is the only possible combination for d=5

1+4+6+14+15.

COALESCE:

15. A

The maximum time for which you can survive is till the below situation occurs:

2^{16}	2^{9}	2^{8}	2
2^{15}	2^{10}	2^{7}	2^{2}
2^{14}	2^{11}	2^{6}	2^{3}
2^{13}	2^{12}	2^{5}	2^{4}

The time required for the above situation to occur is = $(2 + 2^2 + 2^3 + \ldots\ldots + 2^{16})/2 = 2^{16} - 1$

16. A

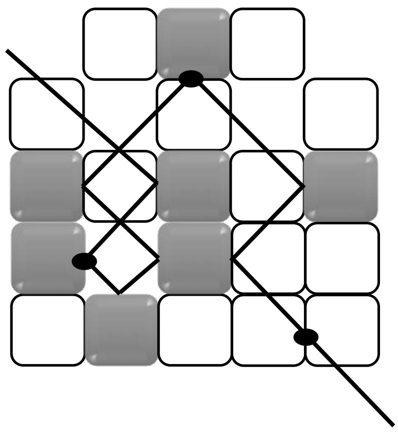
The largest stone that can be made if both 2 and 4kg stones fall would be when the below situation occurs:

2^{17}	2^{10}	2^{9}	2^{2}
2^{16}	2^{11}	2^{8}	2^{3}
2^{15}	2^{12}	2^{7}	2^{4}
2^{14}	2^{13}	2^{6}	2^{5}

Therefore, the largest stone that can be made is of weight = 2^{17} kg.

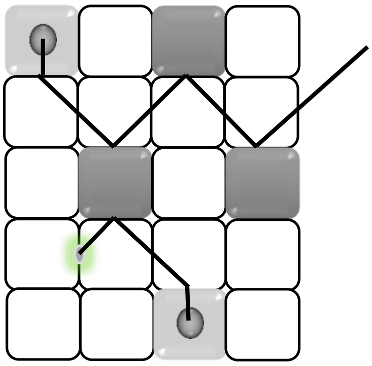
KABULIWALA'S TREASURE:

17. 71



From the above figure it is clear that the final positions of the rocks are 2, 7, 9, 11, 12, 13, 17.
Therefore, the final answer = 2 + 7 + 9 + 11 + 12 + 13 + 17 = 71.

18. 45

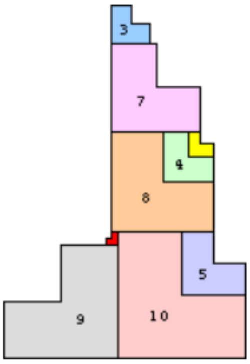


Therefore, the sum of positions where the rocks are present = 1 + 3 + 10 + 12 + 19 = 45.

TILING:

19. 56
20. 21

The figure shown beside is the stack with the maximum height possible.
Hence, 6+14+16+20 = 56 is the maximum height.



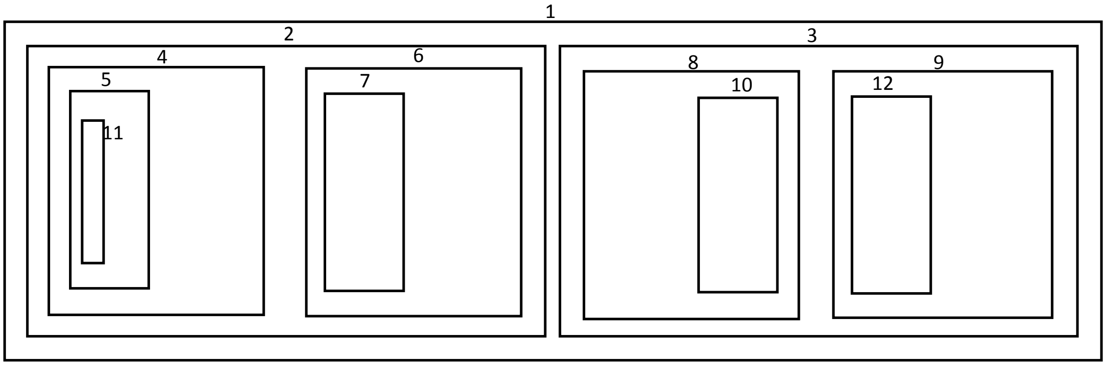
FINAL HUNT:

21. B

Observe the two orders. It is clear that the number present at the beginning in the second order i.e., ‘1’ corresponds to the initial largest box. Now, locate the position of ‘1’ in the first order. All the numbers on the left of ‘1’ which are (5,11,4,2,7,6) correspond to the boxes in the left cavity of ‘1’ and all the numbers present on right which are (8,10,3,12,9) correspond to the boxes in the right cavity of ‘1’.

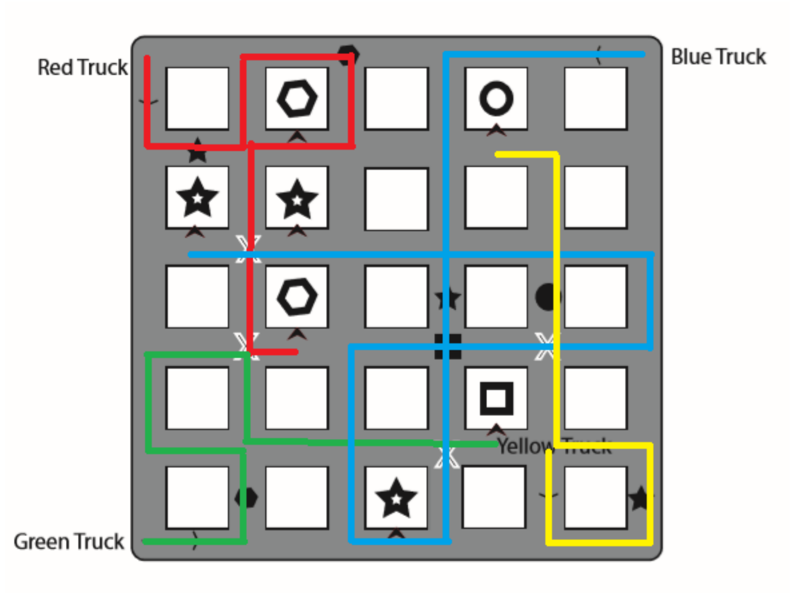
Now, among (5,11,4,2,7,6) locate which number comes first in the second order and similarly for the right side numbers (8,10,3,12,9) and then continue the same procedure till you obtain the entire arrangement.

Below is the final arrangement:



22. No Answer (1)
From the above figure it is clear that the number of boxes of smallest size is 1.

23. C
24. B



From the above figure,
No. of turns taken by blue truck = 6.
No. of articles carried by red truck = 2 (1red + 1blue).