2015 JUNIOR SOLUTIONS

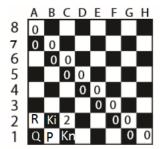
CHESS QUIZZER:

1. C

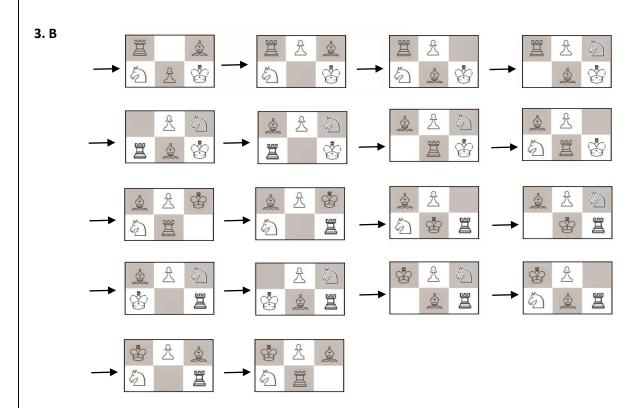


The above figure represents the first '8' moves which leads to 90° rotation of initial arrangement. Therefore, on performing a total of '16' moves required arrangement is achieved.

2. C



The above diagram shows one of the possible solution but clearly the Queens's position must be A1 in every possible solution.



Hence, Total number of steps =18.

TROUBLE WITH FLOWERS:

4. D

5. B

Exchange:
$$R_n - Y_n = (R_{n-1} - 3) - (Y_{n-1} + 2) = (R_{n-1} - Y_{n-1} - 5)$$

Swap: $R_n - Y_n = (Y_{n-1} - R_{n-1})$

From the above two statements it can be declared that (R, Y) can never become (5,5) because the initial value of R-Y is 3. Hence the value of ((R-Y) mod 5) remains either 3 or 2 but never goes to 0.

CRACK THE CODE:

6. A

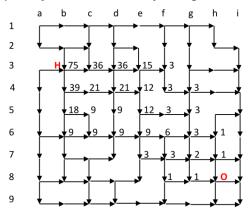
With the number of circles in keychain =26, and given hint that begins with a, the possible coding pattern is as shown.

The letters are substituted by their upper neighbours (Alohomora = "eszrzxzbe") So, our answer is "syimylsz".

SAFE PATHS:

7. C

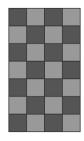
Let us work backward from 'O' to 'H'. Since we can move only in right or down direction, there can be a route to the 'O' from only those points which are to the above and left of 'O'. From the point h7, there is only 1 route i.e. move down. Similarly, 1 route from g8. From g7, we can go to g8 or h7 and then to 'O' hence two routes. Take any point 'P' and assume that 'm' routes are possible from the point just below 'P' and 'n' routes are possible from the point just to the right of 'P'. Then we can go 1 down from 'P' and take any of the 'm' routes. Hence there are m+n routes possible from 'P'. So, the number of routes from any point is the sum of number of routes from the point just below it and just right of it. Doing this from 'O' to 'H' we get 75 routes as the answer.



FRAGMENTS:

8. D

The 7 individual pieces, add up to a total of 28 squares. Therefore, assuming we can indeed form it into a rectangle, it would have to be 7x4 or 14x2 squares in size. I'm using the former case here simply because it's a more natural shape, however this proof applies equally as well to the latter. Now imagine that we label each of these squares with a colour - either black or white - such that they form a checkerboard pattern as shown beside. Notice that the number of black squares must be equal to the number of white, a property we'll exploit.

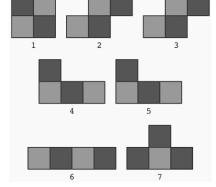


So that's 14 black squares, and 14 white. Looking at each of the pieces individually, the issue with our assumption quickly appears.

As shown beside, for pieces 1-6, the number of black squares within the piece is equal to the number of white. Clearly which squares are black and which are white depends on the actual placement of the piece within the rectangle, but the shapes themselves dictate the count of each colour (since adjacent squares must be of different colours).

However, piece 7 disrupts the trend. Irrelevant of how it's located, it must be comprised of 3 squares of one colour, and 1 of the other, a property that is purely down to its shape.

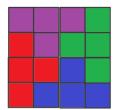
So, taking that into account along with the other 6 pieces, in total they're comprised of 13 squares of one colour, and 15 of the other, with no assumptions



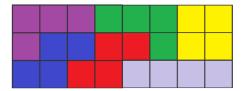
about how they're located within the rectangle. We needed 14 of each, and since we've just shown that we can't get that, our original assumption is overturned and our proof is complete.

9. No Answer (137x4)

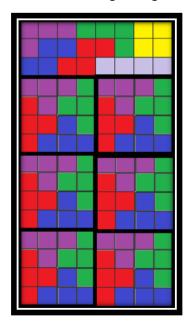
The smallest rectangle/square that can be made using the most valued piece is a 4X4 square as shown below.



So of the 30 pieces we can use 4*n pieces to get the maximum score possible. n comes out to be 6 not 7, since we need to use the rest of the pieces atleast once. As for the remaining 6 pieces, the maximum scores possible is using the following arrangement-



Thus the resulting arrangement is-



Thus the maximum points possible is: 6*(4*4*5) + 4*4*2 + 4*3*2 + 4*2 + 4*1 = 137*4 = 548.

ENIGMATHIC:

10.32

1st ranked person will disagree on dividing in every case

2nd ranked will only say agree on dividing when 2 pirates are remaining

3rd will only say to divide when 4 pirates are remaining because if 4th get killed, 3rd will also get killed as 1st and 2nd will not be willing to divide

4th will only say to divide when 4 pirates are remaining.

Similarly 5th, 6th, 7th, 8th will say to divide only when 8 are remaining

9th to 16th ranked pirates will say to divide only when 16 are remaining and it continues

So, finally 32 pirates will divide the treasure.

11.40

'd' =5 is the most optimized solution and the following is the only possible combination for d=5 1+4+6+14+15.

12.35

We will have a knockout tournament such that 16 pairs of keys are taken and heavier in each case is retained for next round. Next round will have 8 pairs hence 8 weighings. Next round 4, then 2 and finally 1 more. Thus in 31 weighings, we will get the heaviest key. Now the second heaviest key is the one which has lost only to the heaviest key. But since it could have lost in any of the 5 rounds, the second heaviest key is the heaviest among the 5 keys which were defeated by the heaviest key. This will further need 4 weighings and hence 35 weighings in total are needed.

(Many are expected to get '61' as answer as the common thinking might be that find the heaviest key in 31 steps, remove it and find the heaviest out of remaining in 30 more steps)

13.9

302 dinars can be divided into 9 pouches in the following way 1, 2, 4, 8, 16, 32, 64, 128, 47

You can also find other combination but the minimum number of pouches required will be 9.

COALESCE:

14. A

The maximum time for which you can survive is till the below situation occurs:

2 ¹⁶	2 ⁹	28	2
2 ¹⁵	2 ¹⁰	27	2 ²
2 ¹⁴	211	2 ⁶	2 ³
2 ¹³	2 ¹²	2 ⁵	2 ⁴

The time required for the above situation to occur is = $(2 + 2^2 + 2^3 + \dots + 2^{16})/2 = 2^{16}-1$

15. A

The largest stone that can be made if both 2 and 4kg stones fall would be when the below situation occurs:

2 ¹⁷	2 ¹⁰	2 ⁹	2 ²
2 ¹⁶	2 ¹¹	2 ⁸	2 ³
2 ¹⁵	2 ¹²	2 ⁷	24
2 ¹⁴	2 ¹³	2 ⁶	2 ⁵

Therefore, the largest stone that can be made is of weight = 2^{17} kg.

ON THE BANKS OF HARI:

16. B

The procedure has been clearly described in the question. So, find out the 3 digit alphabet corresponding to all the English alphabets. Therefore, the final answer should be SPIWD.

17. C

It is clear that if we know who made the sixth move, we also know who can make the seventh move and wins.

If Shahzia (circles) has made the sixth move, there are three possibilities for the situation after five moves:

Possibility 1:

Possibility 2:



Based on the rules of the game, only possibility 1 could have resulted in the situation after six moves. In that case, there are three possibilities for the situation after four moves:

Possibility 1:

Possibility 2:

Possibility 3:







Based on the rules of the game, Khaled (crosses) would have made the winning move, which however did not happen. From this, we can conclude that Khaled did not make the fifth move and Shahzia did not make the sixth move. Therefore, Khaled must have made the sixth move and Shahzia can make the seventh, winning move!

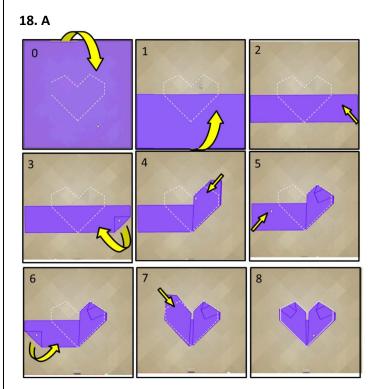
To check that Khaled could indeed have made the sixth move, we look at the following three possibilities after five moves:



Based on the rules of the game, only possibility 3 can result in the situation after six moves. Therefore, Khaled could indeed have made the sixth move.

Hence, Khaled has made the sixth move, and Shahzia will make the seventh move and win.

TREASURE OF KABUL:

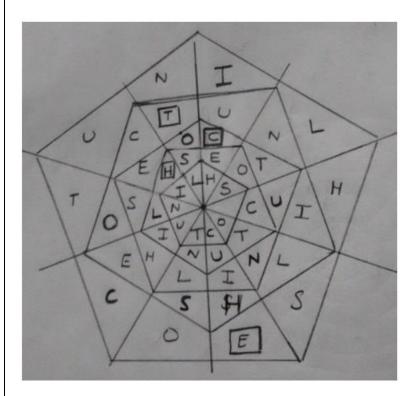


Therefore, total number of steps required are '8'.

FINAL HUNT:

19. C

You can take the following question as a pentagon Sudoku. Therefore, after completing the task the pentagon look something like this:



The letters in the boxes starting from outer ring are E, T, C, H. Hence the answer is ETCH.

20. D

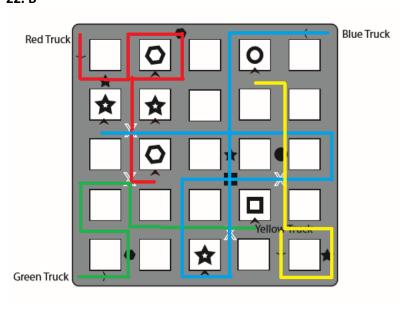
After decoding the poem, you realize that you must go around 'H' in the matrix in the direction given by arrow and note down the word formed. On finding out all such words and combining the you will get the following question

square/ of/larg est/one /digit/ number/

So, the answer is = $9^2 = 81$.

21. C

22. B



From the above figure, No. of turns taken by blue truck = 6. No. of articles carried by red truck = 2 (1red + 1blue).	