

Solution of Question 1 & 2

The Key to get the letters is :

Only adjacent letters can be connected by a line SEGMENT directly.

e.g. look at T & Z in FASTEN and FROZEN there is only 1 place for E to make ZEN and TEN.

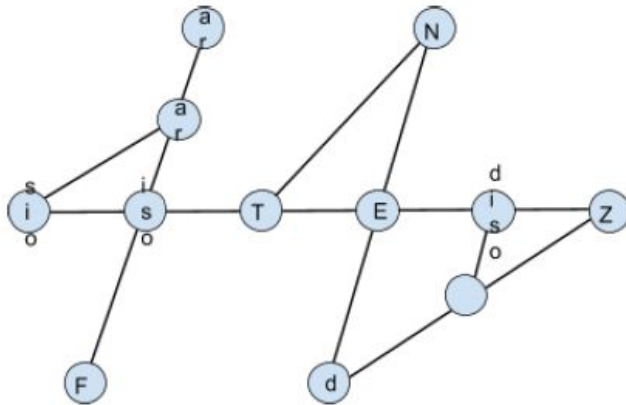
Look at the neighbours of T in DENT-IST & LIS-TED => I & S have to be in line with T.

similarly,

F-ASTEN & F-ROZEN => E & R should be in line with F.

LISTE-D => D has to be in a line with E.

FRO-ZEN => O has to be in line with Z.



Now, see that a,r must be only in the upper two circles. ...(sudoku logic same 2 numbers in 2 boxes)

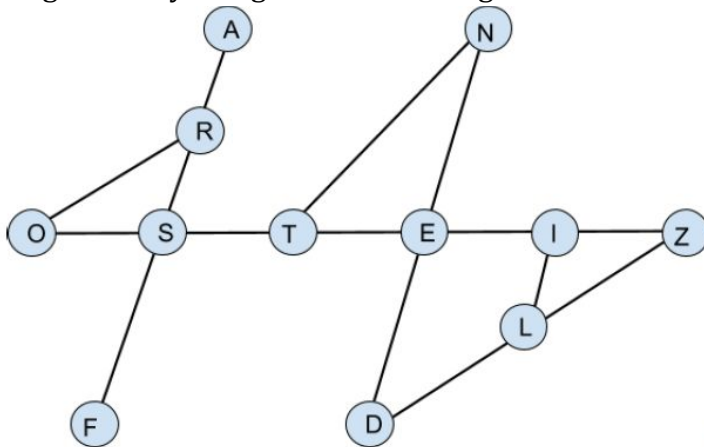
similarly, i,s,o will be in middle line.=> d has to be in the lower circle &

o will be in line of r (FROZEN),

which leaves it in 2 circles on middle left.

Now, **FA-STEN** => s shall be in line with a, so it can only in middle left. hence I is fixed on the middle right.

Solving similarly will give the following answer



ANSWERS- Q1 : AROSE, Q2 : 4 letters O S T E

Solution of Question 3 - 4

Q3) As quacky didn't come to know after knowing the product, then there will be atleast 1 more pair of numbers having the same product. Possibilities are :

$$1 \times 4 = 2 \times 2$$

$$1 \times 6 = 2 \times 3$$

$$1 \times 8 = 2 \times 4$$

$$2 \times 6 = 3 \times 4$$

$$2 \times 8 = 4 \times 4$$

$$3 \times 8 = 4 \times 6$$

After this Mario tells he doesn't know, that means there are atleast two pair remaining in the possibilities with the same sum:

$$1 + 4 = 2 + 3$$

$$2 + 8 = 4 + 6$$

$$2 + 6 = 4 + 4$$

$$1 + 6 = 3 + 4$$

$$1 + 8$$

$$3 + 8$$

$$2 + 2$$

$$2 + 4$$

Now in these 8 there are still 2 pairs with same product....

$$1 \times 6 = 2 \times 3$$

$$2 \times 6 = 3 \times 4$$

$$2 \times 8 = 4 \times 4$$

$$1 \times 4$$

$$4 \times 6$$

Now in the 6 there are atleast 2 pairs with same sum...

$$1 + 6 = 3 + 4$$

$$2 + 6 = 4 + 4$$

$$2 + 8$$

$$2 + 3$$

Now there are atleast 2 pairs with same sum...

$$2, 6 \text{ } 3, 4$$

$$1, 6$$

$$4, 4$$

The product of the possibilities is 12 which will be the answer

Q4) If you have the first move to make, what will be your strategy? One thing is for sure that you can not always win. For example in a configuration (1,2) it is impossible for you to

win no matter what move you make. In a sense (1,2) is a losing state. Your job is to find out whether given configuration is a winning state or not and if it is a winning state, what will be the sequence of moves that will lead you to victory. There cannot be any losing state which has a difference of 1 since they can be brought to (1,2) which is a losing state and there is no other losing state with one of the 2 numbers i.e 1,2 since they can be brought to losing state.

Next (3,5) is a losing state and then (4,7) (5,9) (6,11) (8,14) (10,17) (11,19) (12,21) (13,23) (15,26)

From (12,15), the only losing state you can bring to is (4,7) which requires Mario to kill 16 dogs(8 from each)

Solution of Question 5

Q5) If we see from the centre then the the maximum distance horizontally and vertically can be 4 and minimum can be 0.

Possibilities with $A^2 + B^2$ for finding the order,

0,4 16
 1,4 17
 2,4 20
 3,4 25
 4,4 32
 0,3 09
 1,3 10
 2,3 13
 3,3 18
 0,2 04
 1,2 05
 2,2 08
 0,1 01
 1,1 02

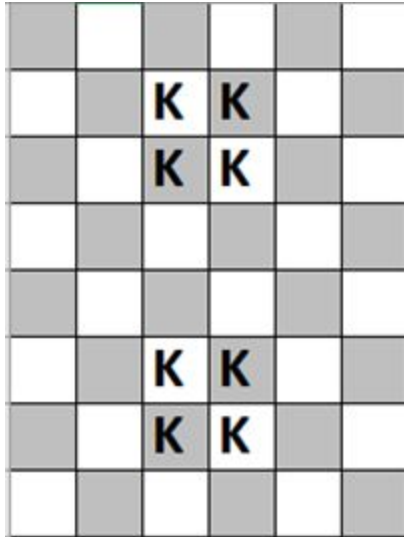
which are 14 possibilities and that means all will be used in this puzzle as no two distance can be equal.

So there will be 4,4 difference in 14,15... 3,4 in 13 ,14 and so on...

Now we start from the sides and write variable for which positions are possible, then we see different positions possible and continue till we reach one solution. WE go back from it to fill all the remaining squares and we will get $a=6$ and $b=10$ Answer will be 16

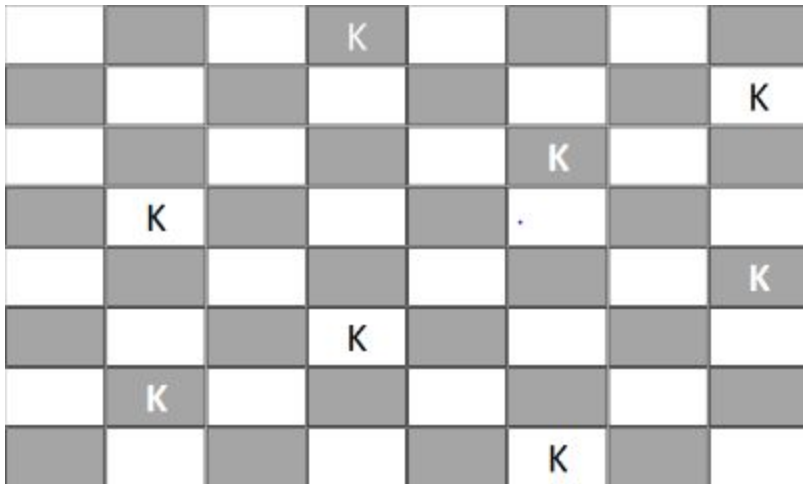
Solution of Question 6 - 8

Q6) The end position will look like the figure given below. This is optimum because if the chess board is divided into 2 parts, the group of knights on the right will dominate the right half while the other group will dominate the left half.



Q7) Arrange a queen in 4 of the 5 rows available preferably adjacent ones. So this leaves us with the last row unoccupied by queen. We have now successfully dominated 4 out of the 5 rows. Now, the positions of the queens determine the domination of the last row. Also, the row left unoccupied by queens can be changed to any of the remaining 4 rows which increase the number of solutions. On successfully trying out the different cases out, the number of cases of arrangement of queens to dominate the board turn out to be more than 6.

Q8) The torus can be unfolded to obtain an 8x8 chess board in which a knight can move from to left end and appear at the right end and disappear at the top and reappear from the bottom of the chess board.



Solution of Question 9

Q9) The key is that there are 26 columns which can be considered as the 26 alphabets,

where, each word fits in each row and each column corresponds to an alphabet in the order.

eodC dorW +*-, when descrambled gives us, Code Word !*+.

So, when one follows the pattern of !*+ in each row, it literally translates to:

Wise is the code word.

Hence, the Answer is option: C)I

Solution of Questions 10 - 13

Q10) First note that a jar shall not become zero with other jars being nonzero.(Think !).

Yes mario can always empty the jars. Basic idea is to reduce one from every jar until a jar contains only one, then double that jar and continue. This will ensure emptying, though not in minimum steps.

For minimum steps- we shall reduce the unnecessary steps.

The method is to first double each jar until it reaches a number less than or equal to the highest number, then deduct all jars by one until a jar becomes one OR if a jar can again be doubled under above condition (note that this doubling will not add any extra work & infact reduce some.). Then check above rule after every deduction & double the necessary jars.

For eg, consider only 2 jars- with **1 4** as initial counts. Then he shall follow these steps-

1. Double the jar 1 (gives **2 4**)
2. Double the jar 1 (gives **4 4**)
3. Reduce both jars one by one (**3 3 -> 2 2 -> 1 1 -> 0 0**)(**4 times**)

Which gives **6** as the minimum steps to empty the jars.

eg 2: in the case **2 6 8**, the number shall drop as follows (11 steps)-

2 6 8 >> 4 6 8 >> 8 6 8 (double each jar close to highest number)

8 6 8 -> 7 5 7 -> 6 4 6 -> 5 3 5 -> 4 2 4 (**reduce all ONLY until there is a double possible again!**)

4 2 4 >> 4 4 4 -> 3 3 3 >> 2 2 2 >> 1 1 1 >> 0 0 0 (then follow same procedure)

Now that you have an idea, the original question will be easy to answer.

4	2	6	7	3	4	5	8	3
---	---	---	---	---	---	---	---	---

(reach following in **6 steps** by doubling respective jars)

8	8	6	7	6	8	5	8	6
---	---	---	---	---	---	---	---	---

(reach following in **2 steps** by reducing the jars)

6	6	4	5	4	6	3	6	4
---	---	---	---	---	---	---	---	---

(**1 step** - double the 3 jar)

6	6	4	5	4	6	6	6	4
---	---	---	---	---	---	---	---	---

(2 reductions)

4	4	2	3	2	4	4	4	2
---	---	---	---	---	---	---	---	---

(3 doubles)

4	4	4	3	4	4	4	4	4
---	---	---	---	---	---	---	---	---

(2 reductions)

2	2	2	1	2	2	2	2	2
---	---	---	---	---	---	---	---	---

(1 double & 2 reductions)

2	2	2	2	2	2	2	2	2
0	0	0	0	0	0	0	0	0

Which gives that minimum **19 steps** are required for emptying the jars.

Q11) **Two extra people** are needed to return correct mushrooms **using the following method-**

Let the extra people have names E1 and E2.

Step 1: Arrange the 64 people into lines such that - In each line, every person is having the mushroom of the person directly behind. Except the last person whose mushroom is with the first person in that line.

Note : If two people have each others' mushrooms, they will make a separate line of only two people.

Step 2:: For a single line, first E1 swaps mushrooms with the last person. Then E2 swaps mushrooms with each person in middle one at a time from front to back. Finally E1 will swap mushrooms with the first person. Then E1 & E2 swap each others' mushrooms. This way everyone in this line along with E1 & E2 will have their own mushrooms.

Step 3: Repeat Step 2 for each line.

Q12) "Buy 1, Get 2 Identical Stones Free" means we can have 3 units of every kind of stone we buy.

Answer is 3. We can measure weight of any randomly given mushroom using 3 stones of weights 1 kg, 7 kg & 49 kg ($7^0, 7^1, 7^2$ resp.)

Suppose we could place stones only in the right pan while a mushroom is in left pan of the balance.

Say we have stones of weight $S_1 = 1\text{kg}$, $S_2 = 2\text{kg}$ & $S_3 = 4\text{kg}$ to balance a mushroom M of weight 6, then only S_2 & S_3 are required to balance M .

We can represent above statement using 0 & 1 as the coefficients of S_1 , S_2 & S_3 .

M (on left pan) $= 1 \times S_3 + 1 \times S_2 + 0 \times S_1$ (on right pan)

Note that 110 is also the binary conversion of 7 i.e. we represent 7 in terms of powers of 2 : $1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 7$.

This way we can measure ANY weight from 1 to 7kg. For range of 171kg, we would require 8 types of weights (i.e. 1, $2, 2^2 \dots$ upto 2^7). But there is a shorter way-

We can instead represent the numbers in powers of 3. Here the coefficients will be 0, 1 or 2. This can do as we have 3 identical copies of each stone.

for eg, $16 = 1 \times 9 + 2 \times 3 + 1 \times 1$. Giving 121 as the representation.

For 171kg, this would require 5 types (1, 3, 9, 27 & 81).

If we increase to powers of 4, we require 1, 4, 16 & 64. (4 kinds)

To reach lesser kinds, we'd use powers of 5, coefficients will be 0, 1, 2, 3, 4. But we don't have 4 copies of the stones. **But still we can overcome this problem-**

Now, in the actual question, we can put stones on **both sides of the balance**.

To represent this, we use '-1' to denote that a stone is in the left pan, '-2' to denote 2 stones in left pan and so on.

for eg, $16 = 1 \times 9 + 2 \times 3 + 1 \times 1$

$$= 1 \times 9 + (3-1) \times 3 + 1 \times 1$$

$$= 2 \times 9 + (-1) \times 3 + 1 \times 1$$

In this way, we are putting 3kg on left pan making the weight 19kg on left. Also on right side, weight is $2 \times 9 + 1 \times 1 = 19\text{kg}$.

The advantage of using both sides is that- we can do with only 3 identical copies of each stone, rather than 6. Which means we can use powers of 7 to write the weights.

For eg, writing 48 in powers of 7 is-

$48 = 6 \times 7^1 + 6 \times 7^0$ (Whenever coefficient crosses 3, convert it to -ve number.)

$$= (7-1) \times 7^1 + (7-1) \times 7^0$$

$$= 1 \times 7^2 + (-1) \times 7^1 + 1 \times 7^1 + (-1) \times 7^0$$

$$= 1 \times 7^2 + -1 \times 7^0$$

So we weigh 48kg by putting 1kg on left & 49 kg on right,

Similarly, for 153kg -

$$153 = 3 \times 49 + 0 \times 7 + 6 \times 1 = 3 \times 49 + 1 \times 7 - 1 \times 1$$

We can weigh 153 by putting 3 stones of 49kg & one 7kg stone on right and 1kg stone on left with the mushroom.

The range of this will be $6 \times 7^2 + 6 \times 7^1 + 6 \times 7^0 = 171$.

So, we can measure all weights till 171kg using only **3 kinds of stones** : 1kg, 7kg & 49kg.

Q13) **Answer is 7.**

Note : The face-offs are independent of each other (their hungers will have their initial values before each faceoff).

at least 7 matches are needed.

Denote the players by A, B, C, D and E.

1. First, A plays against B, while C plays against D. Without loss of generality, suppose A and C are the winners.

2. Then, let A play C in the winners' match. Without loss of generality, suppose A wins. Up to this point, we have determined that $A > B$ and $A > C > D$.

3. Now, we determine the position of E within the $A > C > D$ chain. This can be achieved in two matches. First let E play against C. If E wins, then let him play against A. Otherwise let him play against D. After this, we have a complete ordering of A, C, D and E.

4. Finally we have to find the position of B using only two more matches.

So far we only have $A > B$. There are two cases. If the previous step produced $E > A > C > D$. Then we can simply play B against C and D to complete the ordering. If $A > E$ occurs instead, then we may, without loss of generality, assume that $A > E > C > D$ since none of E, C or D have played against B. Now we can simply repeat the method used in the previous step to find the position of B amongst $E > C > D$, by first matching B against C, then E or D depending on the outcome.

In all cases, we have determined the complete ordering in 7 matches.

Solution of Question 14-15

Q14) answer is (a).

Case (i) : If they play in a straight line, there will always be atleast two squids on one side of the shark. (try partitioning three in all ways). Now since mario can move only one squid at a time, they together can move only at half the rate than the shark (or one squid will be stuck back while other moves),so if shark keeps moving in that direction, in any case it will catch it in some move.

Case (ii) : No, octopops cannot always win the two-dimensional version. Place the 20 squids on the 20 horizontal lines described by $y = 0, y = 3, y = 6, \dots, y = 57$. Place the shark so that it is initially more than 1 unit away from all of the squids. For each line $y = 3i$ (i is $0, 1, 2, \dots, 19$), define its trigger zone to be the region that is within 1 unit of the line, or $\{3i - 1 \leq y \leq 3i + 1\}$. It is not possible to reach the line without being in its trigger zone in the previous turn. Also it is clear that the 20 trigger zones are pairwise disjoint.

So we can deal with one trigger zone at a time.

The strategy is as follows:

Whenever the shark enters a trigger zone, move the corresponding squid along its horizontal line by 1 unit away (left or right only) from the shark. Since the shark cannot catch the squid the moment it enters the trigger zone, and it certainly cannot outrun the squid in a one-on-one chase, the shark will never be able to catch any of the squids.

Q15) answer is (b). There is no enough space to move a 4x1 tile when 2x2 tile is broken and vice versa.

Tip : you may look at your 3x3 numpad on the keyboard(imagine each number key is equally divided in 4 parts) for reference of 6x6 hole.

Solution of Question 16-17

Q16) **answer is (d).**

All squares have to be of equal weight (Think !).

Suppose they have unequal weights, then there has to be a maximum weight present in those 64 squares. Then that square must be surrounded by smaller weights, but the smaller weights cannot average to a higher number (since average of any numbers is less than their maximum). So only possibility is for the weights to be equal.

Q17) **answer is (c) 16days.**

Tip: Look at the horizontal number-line on keyboard for reference (with '0' as room no 10). Let the rooms be numbered from 1 to 10.

Mario can open any door, one in a day. While ghost has to keep sliding to an adjacent room (i.e. **Each day ghost's room number follows odd-even pattern**).

This means if mario starts from an even numbered room, while ghost is initially in odd numbered room, on next day mario will be in odd & ghost will be in even i.e. it is impossible to catch the ghost without switching the even-odd pattern.

The ghost always has two doors to shift into, except when at end rooms 1 & 10. We can block only one room at a time for it. So in the worst case, it may escape us by shifting to the other(which we didn't block) room every time. But we can surely catch it when it is at end rooms 1 or 10 & we are at two rooms distance from it (3 or 8), then ghost will be caught in room 2 or room 9.

The way to ensure this is : **sequentially open rooms from 2 to 9(both inclusive) and then go backwards from rooms 9 to 2**. Note that we open room 9 twice consecutively so that we switch our odd-even pattern.

If ghost is in even room number in the start .. You will catch in the forward move

If ghost is in odd room number in the start .. You will catch in the backward move

- Why did we start from 2 & not other number, say 3 ? -> Starting from 3 onwards will give it a chance to oscillate in between 1 & 2 (starting from even no= 2), so we have to start from 2.
- Why did we come backwards from 9 to 2? -> If ghost was in odd room at start, we won't catch it in forward move, then in such cases the backward move is required.

Solution of Question 18

Q18) As given in the question, arrange the clocks in increasing order of time and then check the letter pointed by Hour's hand

LUCIFER is the answer.

Solution of Question 19-20

Q19) Toggle all the doors except 5th, 10th, 15th, 20th, 25th, 30th, 35th, 40th, 45th, 49th

This way, Only Door 1 will be opened and all others closed.

That means minimum steps required are 39 So any odd number equal to or greater than 39 is the possible steps

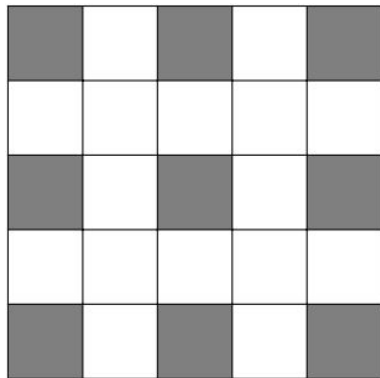
So 49 steps is the answer

Q20) Number of doors and 15 should not have any common factor.

So, 53 is the answer

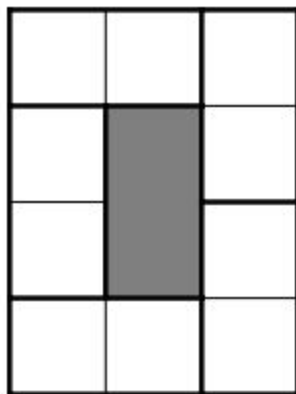
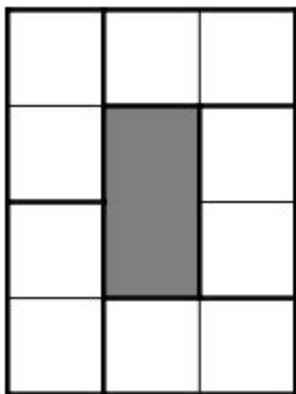
Solution of Question 21-22

Q21) Colour nine of the squares as follows.



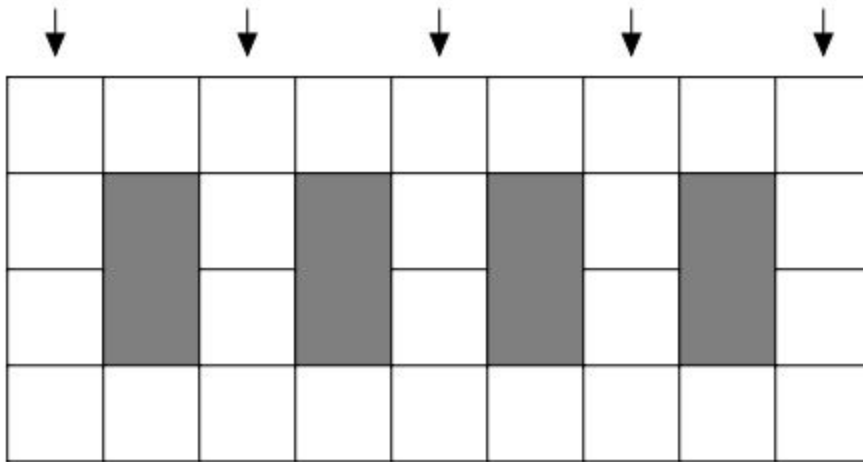
Each L-tromino can cover at most one of the coloured squares. Since there are only eight L-trominoes, the 1×1 square tile must also cover a coloured square. It turns out that all nine positions are possible. The constructions are quite straightforward and will be left to the reader.

Q22) For $n = 1$, a 2×1 rectangle can be tiled by a domino in exactly one way. For $n = 2$, the following shape can be tiled in exactly two ways. Note that there is a 2×1 rectangular hole in the middle which is not part of the shape.

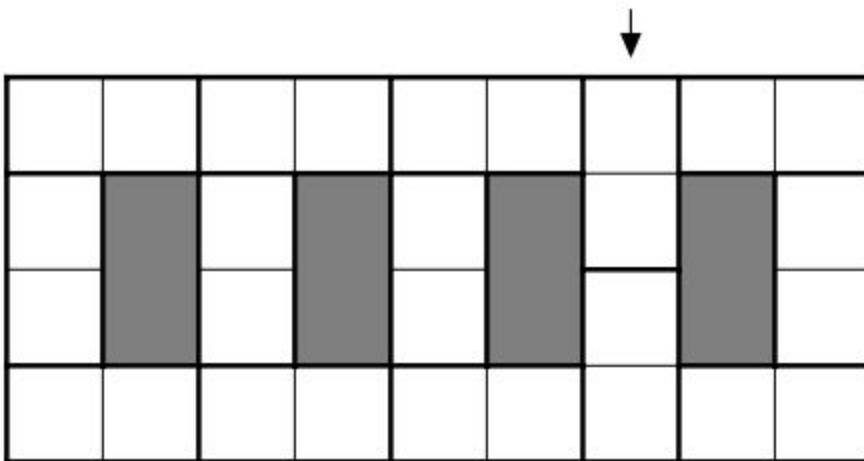


This construction can actually be generalised to any n . For $n = 5$ we have

the following shape (with four holes).



Call a column strong if it has two vertical dominoes. Out of the five columns indicated, at least one must be strong. But as soon as we choose a strong column, the tiling is forced and no other columns can also be strong. For example, if the fourth column is strong, the following tiling is forced.



Since there were five possible choices of strong columns, there are exactly five ways to tile the shape. By similar arguments, for all natural numbers n , there exists a shape which can be tiled by dominoes in exactly n ways.