



$$\begin{aligned}
 & \left\langle \alpha r \alpha' r' \omega \middle| \chi_j^{(2)} \right\rangle_{\hat{R}} \\
 &= \int d\hat{R} \left\langle \alpha r \hat{\mathbf{r}} \middle| \hat{R} \middle| \chi_j \right\rangle \left\langle \alpha' r' \hat{\mathbf{r}}'(\omega) \middle| \hat{R} \middle| \chi_j \right\rangle
 \end{aligned}$$