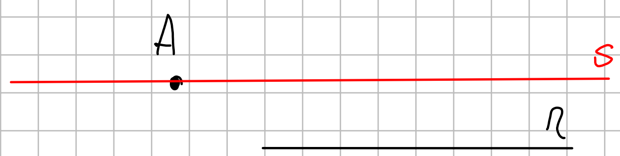
31-01-22

È assegnato nello spazio un sistema di riferimento cartesiano ortogonale $O, \vec{x}, \vec{y}, \vec{z}, u$.

1) Dato un punto $A = (-2, 0, -1)$ e una retta

$$r: \begin{cases} x + z = 0 \\ y + z - 1 = 0, \end{cases}$$

determinare la retta s passante per A e parallela alla retta r e il piano π passante per A ed ortogonale alla retta s.



$\bar{V}_s = \bar{V}_r$

$$\begin{cases} x+z=0 \\ y+z=0 \\ t=0 \end{cases} \quad \begin{cases} x=-z \\ y=-z \\ t=0 \end{cases} \quad \forall z \quad \begin{aligned} P_\infty &= (-z, -z, z, 0) \\ " &= (-1, -1, 1, 0) \end{aligned}$$

$$\bar{V}_r = (-1, -1, 0) \qquad A = (-2, 0, -1)$$

$$\frac{x+2}{-1} = \frac{y-0}{-1} = \frac{z+1}{0}$$

$$\begin{cases} \frac{x+2}{-1} = \frac{y-0}{-1} \\ \frac{y-0}{-1} = \frac{z+1}{0} \end{cases} \quad \begin{cases} -x-2=-y \\ z+1=0 \end{cases} \quad \begin{cases} x-y+2=0 \\ z+1=0 \end{cases} \quad s$$

$\pi \perp s$ PER A

$$v_\pi = \bar{v}_s \qquad v_\pi = (\underset{a}{-1}, \underset{b}{-1}, \underset{c}{0}) \qquad A = (\underset{x}{-2}, \underset{y}{0}, \underset{z}{-1})$$

$$ax+by+cz+d=0$$

$$(-1)(-2)+(-1)(0)+(0)(-1)+d=0$$

$$2-0-0+d=0 \qquad -x-by-2=0$$

$$d=-2 \qquad x+y+2=0 \; (\pi)$$

2) Studiare, al variare del parametro reale $h \in \mathbb{R}$, le coniche di equazione:

$$x^2 + (h-1)y^2 - 2x + (h-2)y = 0.$$

calcolando in particolare i punti base e le coniche spezzate.

$$B = \begin{pmatrix} 1 & 0 & -1 \\ 0 & h-1 & \frac{h-2}{2} \\ -1 & \frac{h-2}{2} & 0 \end{pmatrix}$$

$$|B| = -\left[\left(\frac{h-2}{2}\right)^2 + (h-1)\right] = -\left[\frac{(h-2)^2}{4} + h - 1\right]$$
$$= -\left[\frac{h^2+4-4h}{4} + h - 1\right] =$$
$$= \left[\frac{h^2 \cancel{+4} \cancel{-4h} \cancel{+4h} \cancel{-4}}{4}\right] =$$
$$= -\frac{h^2}{4}$$

$|B| = 0$

$-\frac{h^2}{4} = 0 \qquad h^2 = 0 \qquad \boxed{\text{Per } h=0}$ Coniche spezzate

Sost. h nel fascio $\qquad x^2 + (h-1)y^2 - 2x + (h-2)y = 0.$

$$B = \begin{pmatrix} 1 & 0 & -1 \\ 0 & h-1 & \frac{h-2}{2} \\ -1 & \frac{h-2}{2} & 0 \end{pmatrix} \qquad \begin{pmatrix} \boxed{\begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}} & \begin{matrix} -1 \\ -1 \end{matrix} \\ \begin{matrix} -1 & -1 \end{matrix} & 0 \end{pmatrix}$$

$\det_{2\times 2} = -1 \qquad \rho = 2$ QUALI RETTE distinte?

$$x^2 - y^2 - 2x - 2y = 0$$
$$(x+y)(x-y) - 2x - 2y = 0$$
$$(x+y)(x-y) - 2(x+y)$$
$$\boxed{(x+y)(x-y-2)} = 0 \qquad \cap \begin{cases} x+y = 0 \\ x-y-2 = 0 \end{cases}$$

$h \neq 0 \rightarrow \det B \neq 0$ quindi $|A|$

$h \neq 0$

$$B = \begin{pmatrix} 1 & 0 & -1 \\ 0 & h-1 & \frac{h-2}{2} \\ -1 & \frac{h-2}{2} & 0 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 0 \\ 0 & h-1 \end{pmatrix}$$

$|A| = h-1$

$|A| > 0$ ellisse, $h-1 > 0$ $h > 1$

$|A| < 0$ iperboli, $h < 1$

$|A| = 0$ $h = 1$ PARABOLA

$h=1$ parabola $\quad \beta Y^2 = 2\gamma X$

$$\begin{array}{c} h=1 \\ |A| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \end{array}$$

$\mathrm{Tr}A = \beta = 1 + 0 = 1$

$$\gamma = +\sqrt{-\frac{|B|}{\mathrm{Tr}A}} \Rightarrow \gamma = +\sqrt{\frac{-\frac{-h^2}{4}}{1}} = +\sqrt{\frac{h^2}{4}} = \frac{h}{2} = \frac{1}{2}$$

$Y^2 = X$

parabola $Y^2 - X = 0$

$$r \begin{cases} x + y = 0 \\ x - y - 2 = 0 \end{cases}$$

Punti base?

$x^2 + (h-1)y^2 - 2x + (h-2)y = 0.$

$x^2 + hy^2 - y^2 - 2x + hy - 2y = 0$

$x^2 - y^2 - 2x - 2y + hy^2 + hy = 0$

$x^2 - y^2 - 2x - 2y + h(y^2 + y) = 0$

$y^2 + y = 0 \quad \nexists \mathbb{R}$ La seconda generatrice del fascio non soddisfa i requisiti

Vado a prendere la parabola che si ha per h=1

$$x^2 + (h-1)y^2 - 2x + (h-2)y = 0.$$

$x^2 - 2x - y = 0$ PARABOLA

$r: (x+y)(x-y-2) = 0$

$$\begin{cases}(x+y)(x-y-2) = 0 \\ x^2 - 2x - y = 0\end{cases} \quad \begin{cases}(x+y) = 0 \\ y = x^2 - 2x\end{cases} \quad \begin{cases}x + x^2 - 2x = 0 \\ y = .\end{cases} \quad \begin{cases}x^2 - x = 0 \\ y = x^2 - 2x\end{cases} \quad \begin{cases}x(x-1) = 0 \\ y = x^2 - 2x\end{cases}$$

$$\begin{cases}x - y - 2 = 0 \\ x^2 - 2x - y = 0\end{cases} \qquad \begin{cases}x = 0 \\ y = 0\end{cases} \quad A = (0,0) \qquad \begin{cases}x = 1 \\ y = -1\end{cases} \quad B = (1,-1)$$

$$\begin{cases}y = x - 2 \\ x^2 - 2x - x + 2 = 0\end{cases} \qquad \begin{cases}y = 2 - 2 = 0 \\ x^2 - 3x + 2 = 0\end{cases} \longrightarrow 9 - 8 = 1 \qquad y = -1$$

$$\frac{3 \pm 1}{2} \begin{cases}\frac{4}{2} = 2 \\ \frac{2}{2} = 1\end{cases}$$

$C(2,0)$
$D(1,-1)$

$A = (0,0)$
$B = (1,-1)$
$C = (2,0)$
$D = (1,-1)$

$A = (0,0)$
$B \equiv D = (1,-1)$
$C = (2,0)$

$\begin{cases}\text{1 punto doppio} \\ \text{2 DISTINTI}\end{cases}$

Coniche tangenti.

È assegnato nello spazio un sistema di riferimento cartesiano ortogonale $O, \vec{x}, \vec{y}, \vec{z}, u$.

1) Sono dati il punto $A = (-1,1,1)$, la retta

$$r: \begin{cases} x+3z=2 \\ y+2z-1=0, \end{cases}$$

e il piano $\pi : \; x+y-z-1=0$. Determinare la retta s passante per A, parallela al piano π e perpendicolare alla retta r. Verificare che r e s sono sghembe.

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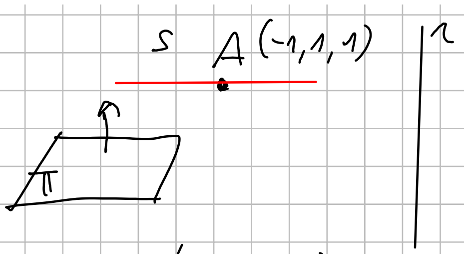


$\bar{u}_\pi = (1,1,-1) \qquad s: \dfrac{x-x_0}{l} = \dfrac{y-y_0}{m} = \dfrac{z-z_0}{n}$

$\begin{cases} \bar{V}_s \perp \bar{u}_\pi \\ \bar{V}_s \perp \bar{V}_r \end{cases} \qquad \vec{V}_r \; ? \qquad \begin{cases} x+3z=0 \\ y+2z=0 \\ t=0 \end{cases} \qquad \begin{cases} x=-3z \\ y=-2z \\ t=0 \end{cases} \; \forall z$

$P_\infty(-3\cancel{z}, -2\cancel{z}, \cancel{z}, 0) \rightsquigarrow \bar{V}_r = (-3,-2,1)$

PROD. scalare $=0$

$\begin{cases} \bar{V}_s \perp \bar{u}_\pi \\ \bar{V}_s \perp \bar{V}_r \end{cases} \to \begin{cases} \bar{V}_s \cdot (1,1,-1) = 0 \\ \bar{V}_s \cdot (-3,-2,1) = 0 \end{cases} \qquad \begin{cases} (l,m,n)\cdot(1,1,-1)=0 \\ (l,m,n)\cdot(-3,-2,-1)=0 \end{cases}$

$\begin{cases} l+m-n=0 \\ -3l-2m-n=0 \end{cases} \qquad \begin{cases} n=l+m \\ -3l-2m-l-m=0 \end{cases} \qquad \begin{cases} n= \_\_\_ \\ -4l-3m=0 \end{cases} \qquad \begin{cases} n=-\frac{3}{4}m+m \\ 4l=-3m \;\; \frac{}{4} \end{cases}$

$\begin{cases} n=\dfrac{-3+4}{4}m \\ l=-\frac{3}{4}m \end{cases} \qquad \begin{cases} n=-\frac{7}{4}m \\ l=-\frac{3}{4}m \end{cases} \qquad \bar{V}_s = \left(-\frac{3}{4}\cancel{m}, \cancel{m}, -\frac{7}{4}\cancel{m}\right)$

$\bar{V}_s = (\underset{l}{-3}, \underset{m}{4}, \underset{n}{-7})$

$(-1,1,1) = A \qquad \dfrac{x+1}{-3} = \dfrac{y-1}{4} = \dfrac{z-1}{-7}$

$\begin{cases} \dfrac{x+1}{-3} = \dfrac{y-1}{4} \\ \dfrac{y-1}{4} = \dfrac{z-1}{-7} \end{cases} \qquad \begin{cases} 4x+4=-3y+3 \\ 7y-7=4z-4 \end{cases} \qquad \boxed{\begin{cases} 4x+3y+1=0 \\ 7y-4z-3=0 \end{cases}} \;\; s$

r e s SGHEMBE?

$\boxed{r} = \begin{cases} x+3z=2 \\ y+2z-1=0 \end{cases} \qquad \boxed{\begin{cases} x+3z-2=0 \\ y+2z-1=0 \end{cases}}$

$$\begin{array}{cccc} x & y & z & \text{noto} \end{array}$$

$$\begin{vmatrix} 1 & 0 & 3 & -2 \\ 0 & 1 & 2 & -1 \\ 4 & 3 & 0 & 1 \\ 0 & 7 & -4 & -3 \end{vmatrix}$$

Se det $\neq 0$ allora sghembe.

1° colonna

$\underset{=1}{a_{11}} \underset{=0}{A_{11}} + \underset{4}{a_{31}} \underset{=24}{A_{31}} = 4 \cdot 24 = 96$ (Rette sghembe)

$$\begin{array}{r} 1 \\ 24 \\ 4 \\ \hline 96 \end{array}$$

$A_{11} = (-1)^2 \cdot \det \begin{vmatrix} 0 & 3 & -2 \\ 1 & 2 & -1 \\ 3 & 0 & 1 \end{vmatrix} = +[-9-[3-12]] =$

$= -9-3+12 = 0$

$$\begin{array}{r} 37- \\ 21 \\ \hline 16 \end{array}$$

$A_{31} = (-1)^4 \cdot \det \begin{vmatrix} 0 & 3 & -2 \\ 1 & 2 & -1 \\ 7 & -4 & -3 \end{vmatrix} = -21+8-[-9-28]$

$-21+8+9+28 = 24$

2) Studiare, al variare del parametro reale $h \in \mathbb{R}$, il fascio di coniche del piano $z = 0$ di equazione:

$$hx^2 + hy^2 + 2xy + 2x + 2hy = 0.$$

calcolando, in particolare, i suoi punti base e le coniche spezzate.

$B = \begin{pmatrix} h & 1 & 1 \\ 1 & h & h \\ 1 & h & 0 \end{pmatrix}$ $\quad A = \begin{pmatrix} h & 1 \\ 1 & h \end{pmatrix}$

$\det B = h + h - [h^3 + h]$

$2h - h^3 - h = h - h^3$

$|B| = 0$

$h^3 - h = 0$

$h(h^2-1) = 0 \quad \begin{cases} h = 0 \\ h = 1 \end{cases}$

$\boxed{h=0}$ $\quad hx^2 + hy^2 + 2xy + 2x + 2hy = 0.$

$2xy + 2x = 0$

$2x(y+1) = 0 \quad \begin{cases} \cancel{2}x = 0 \\ y+1 = 0 \end{cases}$ $\quad \Gamma_1$

$h = 1$ $\quad \underbrace{x^2+y^2+2xy}+2x+2y = 0$

$(x+y)^2 + 2x + 2y$

$(x+y)^2 + 2(x+y) \quad (x+y)(x+y+2) = 0$

$\begin{cases} x+y = 0 \\ x+y+2 = 0 \end{cases}$ $\quad \Gamma_2$

$$\begin{cases} 2x(y+1)=0 \\ (x+y)(x+y+2)=0 \end{cases} \quad ① \begin{cases} 2x=0 \\ x+y=0 \end{cases} \quad ② \begin{cases} 2x=0 \\ x+y+2=0 \end{cases}$$

$$③ \begin{cases} y+1=0 \\ x+y=0 \end{cases} \quad ④ \begin{cases} y+1=0 \\ x+y+2=0 \end{cases}$$

$$① \begin{cases} 2x=0 \\ x+y=0 \end{cases} = \begin{cases} x=0 \\ y=0 \end{cases} \qquad A=(0,0)$$

$$② \begin{cases} 2x=0 \\ x+y+2=0 \end{cases} \quad \begin{cases} x=0 \\ y=-2 \end{cases} \qquad B=(0,-2)$$

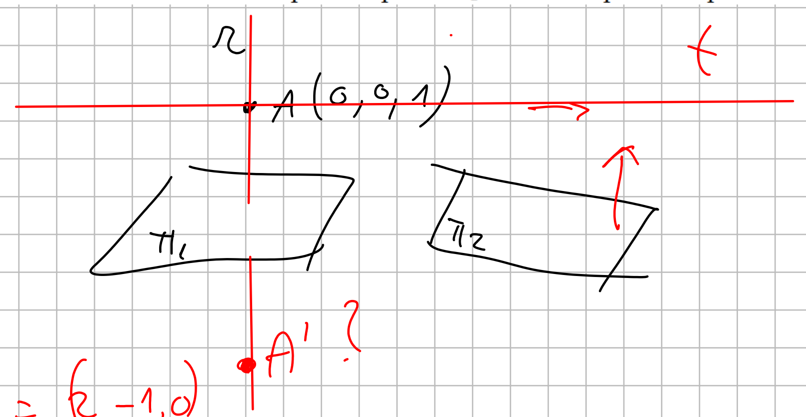
$$③ \begin{cases} y+1=0 \\ x+y=0 \end{cases} \quad \begin{cases} y=-1 \\ x=1 \end{cases} \qquad C=(1,-1)$$

$$4 \begin{cases} y+1=0 \\ x+y+2=0 \end{cases} \quad \begin{cases} y=-1 \\ x-1+2=0 \end{cases} \quad \begin{cases} y=-1 \\ x=-1 \end{cases} \qquad D=(-1,-1)$$

4 punti DISTINTI CASO 1

1) Sono dati il punto $A = (0,0,1)$ e i piani $\pi_1 : 2x - y + 3 = 0$ e $\pi_2 :\ x + y - z - 3 = 0$. Determinare il punto A' simmetrico di A rispetto al piano π_1 e la retta t passante per A e ortogonale π_2.

14 Aprile 2022



$\bar{u}_{\pi_1} = (2,-1,0)$

$$\frac{x-0}{2} = \frac{y-0}{-1} = \frac{z-1}{0} \qquad \begin{cases} \frac{x}{2} = -y \\ \cancel{-y} = \frac{z-1}{\cancel{0}} \end{cases} \quad \begin{cases} x + 2y = 0 \\ z - 1 = 0 \end{cases}$$

$$\begin{matrix} r \\ \\ \pi_1 \end{matrix}\begin{cases} x+2y=0 \\ z-1=0 \\ 2x-y+3=0 \end{cases} \quad \begin{cases} x=2y \\ z=1 \\ 2(2y)-y+3=0 \end{cases} \quad \begin{cases} x=2y \\ z=1 \\ 4y-\cancel{y}+3=0 \end{cases} \quad \begin{cases} x=2y \\ z=1 \\ 3y+3=0 \end{cases} \quad \begin{cases} \_\_\_ \\ \_\_\_ \\ \cancel{3}y = \frac{-3}{3} \end{cases}$$

$$\begin{cases} x=-2 \\ z=1 \\ y=-1 \end{cases} \qquad H = (\underset{x_H}{\overset{\|}{-2}}, \underset{y_H}{\overset{\|}{-1}}, \underset{z_H}{\overset{\|}{1}})$$

$$-2 = \frac{0+x}{2} \Rightarrow x = -4$$

$$-1 = \frac{0+y}{2} \rightarrow y = -2$$

$$1 = \frac{1+z}{2} \rightarrow z = 1$$

$$A = (0,0,1)$$

$$A' = (-4,-2,1)$$

t che passa per A e $\perp$ π_2

$$\bar{u}_{\pi_2} = (1,1,-1) \qquad A = (0,0,1)$$

$$\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-1}{-1} \qquad t: \begin{cases} x = y \\ y = -z+1 \end{cases} \quad \begin{cases} x-y=0 \\ y+z+1=0 \end{cases} \quad (t)$$

2) E' assegnato nel piano un sistema di riferimento cartesiano ortogonale $O, \vec{x}, \vec{y}, u$. Studiare il fascio di coniche di equazione:

$$x^2 + hy^2 + 2hxy + hx - hy = 0.$$

Studiare l'iperbole equilatera del fascio determinando una forma ridotta, centro, assi, vertici e asintoti.

$$B = \begin{pmatrix} 1 & h & \frac{1}{2}h \\ h & h & -\frac{1}{2}h \\ \frac{1}{2}h & -\frac{1}{2}h & 0 \end{pmatrix}$$

$$\det B = (h)\left(-\frac{1}{2}h\right)\left(\frac{1}{2}h\right) + \left(\frac{1}{2}h\right)(h)\left(-\frac{1}{2}h\right) - \left[\left(-\frac{1}{2}h\right)^2 + \left(\frac{1}{2}h\right)^2(h)\right]$$

$$\det B = h\left(-\frac{1}{2}h\right)\left(\frac{1}{2}h\right) + \left(\frac{1}{2}h\right)(h)\left(-\frac{1}{2}h\right) - \left(-\frac{1}{2}h\right)^2 - \left(\frac{1}{2}h\right)^2(h) =$$

$$h\left(-\frac{1}{4}h^2\right) + \left(-\frac{1}{4}h^2\right)(h) - \frac{1}{4}h^2 - \left(\frac{1}{4}h^2\right)(h)$$

$$\boxed{h\left(-\frac{1}{4}h^2 - \frac{1}{4}h^2 - \frac{1}{4}h - \frac{1}{4}h^2\right)} \neq 0 \qquad h \neq 0$$

$\det B$

$$-\frac{3}{\cancel{4}}h^2 - \frac{1}{\cancel{4}}h \neq 0 \qquad 3h^2 + h \neq 0$$

$$h(3h+1) \neq 0 \qquad h \neq 0 \qquad h \neq -\frac{1}{3}$$

Iperbole per $h \neq 0$ e $h \neq -\frac{1}{3}$

Equilatera per $\text{Tr} A = 0 \qquad 1 + h = 0 \qquad h = -1 \to$ EQUILATERA

$$\alpha X^2 + \beta Y^2 = \gamma$$

$$A = \begin{pmatrix} 1 & h \\ h & h \end{pmatrix} \qquad \det A = h - h^2$$

$$\gamma = -\frac{|B|}{|A|}$$

$$\gamma = \frac{h\left(-\frac{1}{4}h^2 - \frac{1}{4}h^2 - \frac{1}{4}h - \frac{1}{4}h^2\right)}{h - h^2}$$

$h = -1$

$$= \frac{-1\left(-\frac{1}{4} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \frac{1}{4}\right)}{-2} = \frac{-\left(-\frac{2}{4}\right)}{-2} = -\frac{\frac{2}{4}}{-2} = \frac{\cancel{2}}{4} \cdot \left(-\frac{1}{\cancel{2}}\right)$$

$$= -\left(-\frac{1}{4}\right) = \frac{1}{4}$$

$$\gamma = \frac{1}{4}$$

α, β sol. del. P.C(A)

$$\begin{pmatrix} 1-T & -1 \\ -1 & -1-T \end{pmatrix}$$

$\det = (1-T)(-1-T) - 1 = 0$ $\qquad 1-T=1 \quad T=0$

$(-T+1)(-T-1) - 1$ $\qquad -1-T=1 \quad T=-1$

$T^2 - 1 - 1 = 0$

$T^2 - 2 = 0 \qquad T = \pm\sqrt{2}$

$T = \sqrt{2} \quad \alpha$

$T = -\sqrt{2} \quad \beta$

$\sqrt{2}X^2 - \sqrt{2}Y^2 = \frac{1}{4}$

MATRICE IPERBOLE EQUILATERO

$$\begin{pmatrix} 1 & -1 & -\frac{1}{2} \\ -1 & -1 & +\frac{1}{2} \\ -\frac{1}{2}h & \frac{1}{2} & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & -1 & -\frac{1}{2} \\ -1 & -1 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} x_c \\ y_c \\ 1 \end{pmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$$\begin{cases} x_c - y_c - \frac{1}{2} = 0 \\ -x_c - y_c + \frac{1}{2} = 0 \end{cases} \begin{cases} y_c = x_c - \frac{1}{2} \\ -x_c - x_c + \frac{1}{2} + \frac{1}{2} = 0 \end{cases}$$

$$\begin{cases} y_c = x_c - \frac{1}{2} \\ -2x_c + 1 = 0 \end{cases} \begin{cases} y_c = \dots \\ 2x_c - 1 = 0 \end{cases} \begin{cases} y_c = \frac{1}{2} - \frac{1}{2} \\ \cancel{2}x_c = \frac{1}{2} \end{cases} \begin{cases} y_c = 0 \\ x_c = \frac{1}{2} \end{cases}$$

Centro $\left(\frac{1}{2}, 0\right)$

ASSI $\qquad m_1 = -\frac{a_{11} - \alpha}{a_{12}}$ $\qquad y - y_c = m_1(x - x_c)$

$y = 1-\sqrt{2}\left(x - \frac{1}{2}\right)$

$m_1 = -\frac{1-\sqrt{2}}{-1}$ $\qquad y = 1 - \sqrt{2}x - \frac{1-\sqrt{2}}{2}$

$m_1 = -\left[-1 + \sqrt{2}\right] = 1 - \sqrt{2}$ $\qquad 2y = 2 - 2\sqrt{2}x$

$m_2 = -\frac{1}{m_1} = -\frac{1}{1-\sqrt{2}}$ $\qquad 2\sqrt{2}x + 2y - 2 = 0$

Asse 2

$y - 0 = -\frac{1}{1-\sqrt{2}}\left(x - \frac{1}{2}\right)$

$y = -\frac{1}{1-\sqrt{2}}x + \frac{1}{2-2\sqrt{2}}$

$\frac{1}{1-\sqrt{2}}x - y - \frac{1}{2-2\sqrt{2}} = 0$ ASSE 2

$$x^2 + hy^2 + 2hxy + hx - hy = 0.$$

Vertici. iperbole $h = -1 \Rightarrow x^2 - y^2 - 2xy - x + y = 0$

$\begin{cases} \text{iperbole} \\ \text{ASSE } 1 \end{cases}$ $\begin{cases} \text{iperbole} \\ \text{Asse } 2 \end{cases}$

$$\begin{cases} x^2 - y^2 - 2xy - x + y = 0 \\ 2\sqrt{2}x + 2y - 2 = 0 \end{cases}$$

ASINTOTI

$y = \pm \frac{b}{a} x$