2) E' assegnato l'endomorfismo $g\colon \mathbb{R}^3 \to \mathbb{R}^3$ definito dalle assegnazioni:

$$\begin{cases} g(1,1,0) = (-2,0,-1) \\ g(0,1,1) = (-1,h,0) \\ g(0,1,0) = (0,-1,0) \end{cases} \quad \text{con } h \text{ parametro reale}$$

Studiare la semplicità di g al variare di $h \in \mathbb{R}$, determinando, nei casi in cui è possibile, una base di autovettori per g.

TROVIAMO LA MATRICE. metodo standard

$$\begin{cases} g(e_1)+g(e_2) = (-2,0,-1) \to g(e_1) = (-2,0,-1)-(0,-1,0) = (-2,1,-1) \\ g(e_2)+g(e_3) = (-1,h,0) \to g(e_3) = (-1,h,0)-(0,-1,0) = (-1,h+1,0) \\ g(e_2) = (0,-1,0) \end{cases}$$

$$M(g) \begin{pmatrix} -2 & 0 & -1 \\ 1 & -1 & h+1 \\ -1 & 0 & 0 \end{pmatrix}$$

Polinomio caratteristico

$$\begin{pmatrix} -2-T & 0 & -1 \\ 1 & -1-T & h+1 \\ -1 & 0 & 0-T \end{pmatrix} \Rightarrow \begin{pmatrix} -2-T & 0 & -1 \\ 1 & -1-T & h+1 \\ -1 & 0 & -T \end{pmatrix} \quad \det = 0$$

$$\det = (-2-T)(-1-T)(-T) - (-1-T) = 0$$

$$(-1-T)\left[(-2-T)(-T)-1\right] = 0$$

$$(-1-T)\left[2T+T^2-1\right] = 0$$

$-1-T=0 \to T=-1 \quad m_{-1}=1$

$T^2+2T-1=0$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{8}}{2}$$

$-1+\sqrt{2} \quad m_{-1+\sqrt{2}}=1$

$-1-\sqrt{2} \quad m_{-1-\sqrt{2}}=1$

$m_{-1}=1 \to g_{-1}=1$

$m_{-1+\sqrt{2}}=1 \to g_{-1+\sqrt{2}}=1$

$m_{-1-\sqrt{2}}=1 \to g_{-1-\sqrt{2}}=1$

g È SEMPLICE $\forall h \in \mathbb{R}$

Cerco gli autovettori vedendo gli autospazi:

V_{-1}, $V_{-1-\sqrt{2}}$, $V_{-1+\sqrt{2}}$.

$$V_{-1} = \dim \ker(g_{-1}) \rightarrow \begin{pmatrix} -2-(-1) & 0 & -1 \\ 1 & -1-(-1) & h+1 \\ -1 & 0 & 0-(-1) \end{pmatrix} \Rightarrow \begin{pmatrix} -1 & 0 & -1 \\ 1 & 0 & h+1 \\ -1 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -x - z = 0 \\ x + (h+1)z = 0 \\ -x + z = 0 \end{cases} \quad \begin{cases} z + (h+1)z = 0 \rightarrow (\cancel{h+2})z = 0 \\ x = z \rightarrow x = 0 \end{cases} \quad \forall y$$

$\vec{u}_1 = (0, 1, 0)$ 1° AUTOVETTORE

$$V_{-1-\sqrt{2}} = \dim \ker g_{-1-\sqrt{2}} \begin{pmatrix} -2-(-1-\sqrt{2}) & 0 & -1 \\ 1 & -1-(-1-\sqrt{2}) & h+1 \\ -1 & 0 & 0-(-1-\sqrt{2}) \end{pmatrix}$$

$$\begin{pmatrix} -1+\sqrt{2} & 0 & -1 \\ 1 & \sqrt{2} & h+1 \\ -1 & 0 & 1+\sqrt{2} \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

SOMMA × DIFFER.

$$\begin{cases} (-1+\sqrt{2})x - z = 0 \\ x + \sqrt{2}y + (h+1)z = 0 \\ -x + (1+\sqrt{2})z = 0 \end{cases} \quad \begin{cases} (-1+\sqrt{2})(1+\sqrt{2})z - z = 0 \rightarrow (\sqrt{2}^2 - (1)^2)z - z = 0 = 0 = 0 \\ x = (1+\sqrt{2})z \end{cases}$$

$$((\sqrt{2})^2 - (1)^2)z - z = 0 \rightarrow$$
$$(2-1)z - z = 0$$
$$z - z = 0$$

continuo sistema qui

$$\begin{cases} 0=0 \\ x + \sqrt{2}y + (h+1)z = 0 \\ x = (1+\sqrt{2})z \end{cases} \quad \begin{cases} (1+\sqrt{2})z + \sqrt{2}y + (h+1)z = 0 \\ \end{cases}$$

$$\begin{cases} -- \\ (2+h+\sqrt{2})z + \sqrt{2}y = 0 \\ -- \end{cases} \quad \begin{cases} \cancel{\sqrt{2}}y = \dfrac{(-2-h-\sqrt{2})z}{\sqrt{2}} \\ x = (1+\sqrt{2})z \quad \forall z \end{cases}$$

$$V_{-1-\sqrt{2}} = \left(1+\sqrt{2}z, \frac{-2-h-\sqrt{2}}{\sqrt{2}}z, z\right)$$

$$u_2 = \left(1+\sqrt{2}, \frac{-2-h-\sqrt{2}}{\sqrt{2}}, 1\right) \quad 2° \text{ AUTOVETTORE}$$

$$V_{-1+\sqrt{2}} = \dim \operatorname{Ker} g_{-1+\sqrt{2}} \begin{pmatrix} -2-(-1+\sqrt{2}) & 0 & -1 \\ 1 & -1-(-1+\sqrt{2}) & h+1 \\ -1 & 0 & 0-(-1+\sqrt{2}) \end{pmatrix}$$

$$\begin{pmatrix} -1-\sqrt{2} & 0 & -1 \\ 1 & -\sqrt{2} & h+1 \\ -1 & 0 & 1-\sqrt{2} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} (-1-\sqrt{2})x - z = 0 \\ x - (-\sqrt{2})y + (h+1)z = 0 \\ -x + (1-\sqrt{2})z = 0 \end{cases} \quad \begin{cases} (-1-\sqrt{2})(1-\sqrt{2})z - z = 0 \\ \sim\ --\ \sim \\ x = (1-\sqrt{2})z \end{cases}$$

$$\begin{cases} (\sqrt{2}^2 - 1^2)z - z = 0 \\ \dots \\ x = \dots \end{cases} \quad \begin{cases} (2-1)z - z = 0 \\ -- \\ x = \dots \end{cases} \quad \begin{cases} 0=0 \quad \forall z \\ (1-\sqrt{2})z - (-\sqrt{2})y + (h+1)z = 0 \\ x = (1-\sqrt{2})z \end{cases}$$

$$\begin{cases} (1-\sqrt{2}+h+1)z+\sqrt{2}y=0 \\ x=\dots \quad \forall z \end{cases} \quad \begin{cases} (2-\sqrt{2}+h)z+\sqrt{2}y=0 \\ x=(1-\sqrt{2})z \end{cases}$$

$$\cancel{\sqrt{2}}\, y=\left(\frac{\sqrt{2}-2-h}{\sqrt{2}}\right)z \qquad V_{1+\sqrt{2}}=\left((1-\sqrt{2})z,\ \frac{\sqrt{2}-2-h}{\sqrt{2}}z,\ z\right)$$

$$x=(1-\sqrt{2})z \qquad \forall z$$

$$u_3=\left((1-\sqrt{2})z,\ \frac{\sqrt{2}-2-h}{\sqrt{2}}z,\ 1\right)$$ 3° AUTOVETTORE

BASE DI AUTOVETTORI $=\{\underline{u}_1, \underline{u}_2, \underline{u}_3\}$ SOLUZIONE