CALCOLARE LA DERIVATA.

$$f(x) = \sqrt{x+2} = (x+2)^{\frac{1}{2}} = \frac{1}{2}(x+2)^{-\frac{1}{2}} = \frac{1}{2}\,\frac{1}{\sqrt{x+2}} = \frac{1}{2\sqrt{x+2}}$$

STUDIARE LA FUNZIONE $f(x) = x^3 - 4x$

- DOMINIO $\mathbb{R}$
- SIMMETRIE $f(-x) = ?$

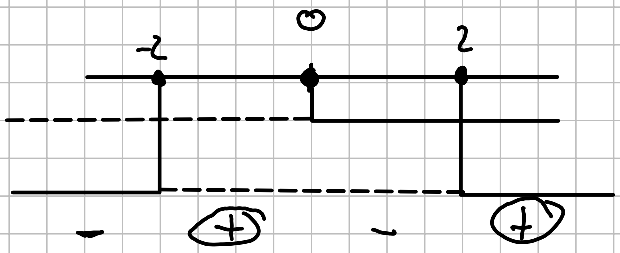
$$f(-x) = (-x)^3 - 4(-x) = -x^3 + 4x \quad (\text{DISPARI})$$

- SEGNO E INTERSEZ. ASSI

$$x^3 - 4x \geq 0 \qquad x(x^2-4) \geq 0$$

$$\begin{cases} x \geq 0 \\ x^2 - 4 \geq 0 \end{cases} \qquad x^2 - 4 = 0 \quad x^2 = 4 \rightarrow x = \pm 2$$

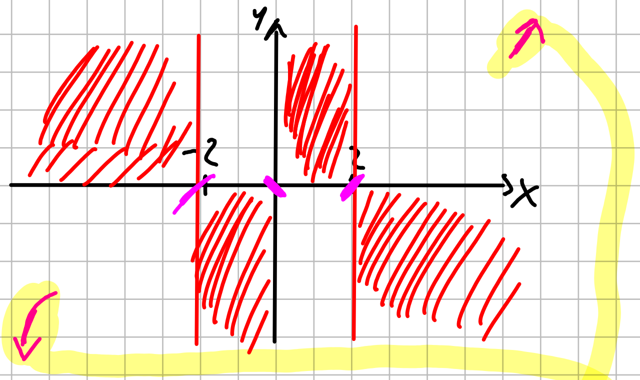
$$\rightarrow x \leq 2 \ \wedge \ x \geq 2$$



$$\begin{cases} -2 \leq x \leq 0 \\ x \geq 2 \end{cases}$$

INTERSEZ. ASSI, $x = -2$
$x = 0$
$x = 2$
$(0,0)$

BOZZA



4) LIMITI $\pm\infty$

$$\lim_{x\to+\infty} x^3 - 4x = \underbrace{x^3}_{\to +\infty}\left(1 - \underbrace{\frac{4x}{x^3}}_{\to 0}\right) = +\infty$$

$$\lim_{x\to-\infty} x^3 - 4x = \underbrace{x^3}_{\to -\infty}\left(1 - \underbrace{\frac{4x}{x^3}}_{\to 0}\right) = -\infty$$

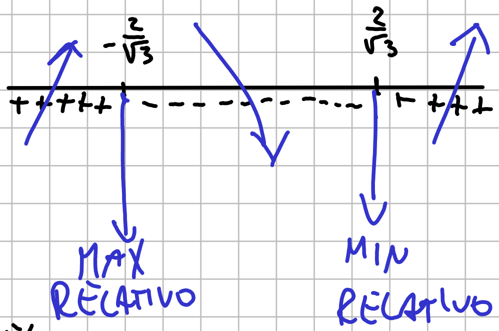
5) $f'(x) > 0$

$$f(x) = x^3 - 4x \longrightarrow f'(x) = 3x^2 - 4$$

$3x^2 - 4 > 0$ — $3x^2 - 4 = 0$

$\cancel{3}x^2 = \pm\sqrt{\frac{4}{3}}$ $\quad \pm\frac{2}{\sqrt{3}}$

$x > \frac{2}{\sqrt{3}} \wedge x < -\frac{2}{\sqrt{3}}$



BOZZA

TROVO PUNTO MAX/MIN REL.

$f(x) = x^3 - 4x \to \pm\frac{2}{\sqrt{3}}$

$$f\left(\frac{2}{\sqrt{3}}\right) = \left(\frac{2}{\sqrt{3}}\right)^3 - 4\left(\frac{2}{\sqrt{3}}\right) =$$

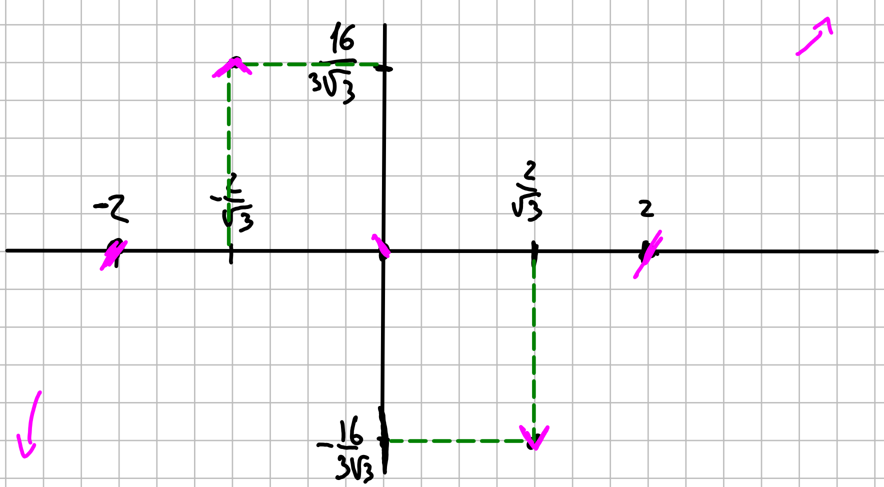
$$= \frac{8}{3\sqrt{3}} - \frac{8}{\sqrt{3}} = \frac{8-24}{3\sqrt{3}} = -\frac{16}{3\sqrt{3}} \quad (y)$$

$$\left(\frac{2}{\sqrt{3}}\right)^3 = \frac{8}{\sqrt{27}} = \frac{8}{\sqrt{9\cdot 3}} \to \frac{8}{3\sqrt{3}}$$

MAX REL. $\left(\frac{2}{\sqrt{3}}, -\frac{16}{3\sqrt{3}}\right)$

MIN REL. $\left(-\frac{2}{\sqrt{3}}, \frac{16}{3\sqrt{3}}\right)$

→ non calcolato perché la $f(x)$ è DISPARI.



CONCAVITA'

$f(x) = x^3 - 4x \qquad f'(x) = 3x^2 - 4 \qquad f''(x) = 6x$

$6x > 0 \qquad x > 0$

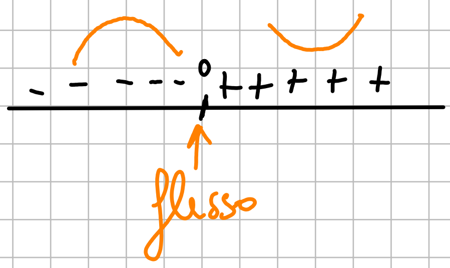
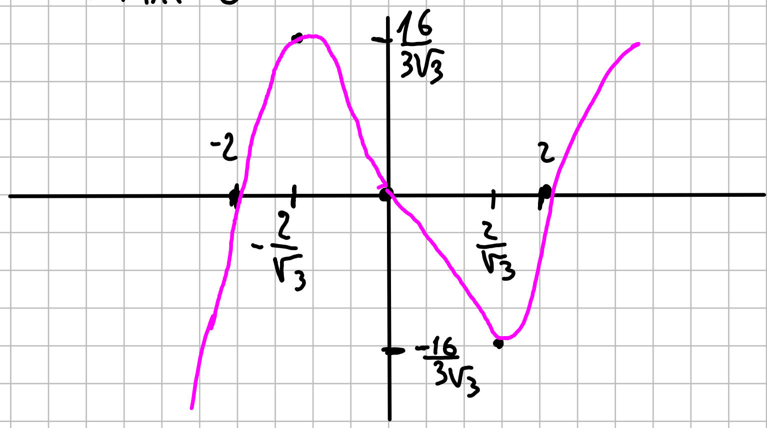


GRAFICO QUALITATIVO



STUDIARE LA FUNZIONE $f(x)=\dfrac{2x^3}{x^2-1}$

1) DOMINIO $x^2-1\neq 0$

$$\boxed{x\neq 1 \quad \wedge \quad x\neq -1}$$

2) Simmetrie

$$f(-x)=\frac{2(-x)^3}{x^2-1}=-\frac{2x^3}{x^2-1}= \text{DISPARI}$$

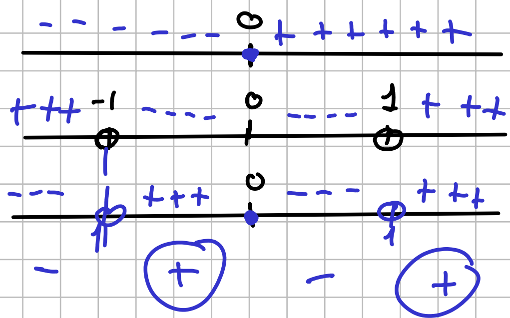
3) Segno e intercett. assi.

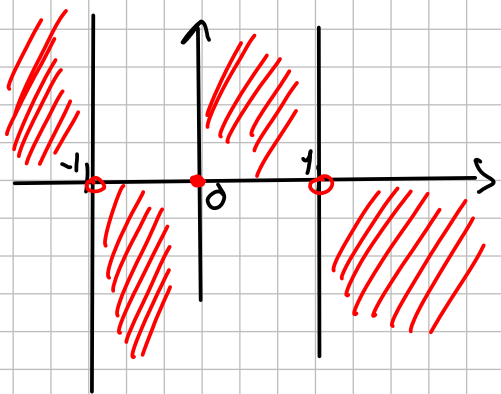
$$\frac{2x^3}{x^2-1}\geq 0$$

$2x^3\geq 0$

$x^2-1>0$

$\dfrac{2x^3}{x^2-1}$





4) LIMITI

$$\lim_{x\to+\infty}\frac{2x^3}{x^2-1}=\frac{\overbrace{2x^{\cancel{3}}}^{\to+\infty}}{\cancel{x^2}_1\left(1-\underbrace{\frac{1}{x^2}}_{\to 0}\right)}=+\infty$$

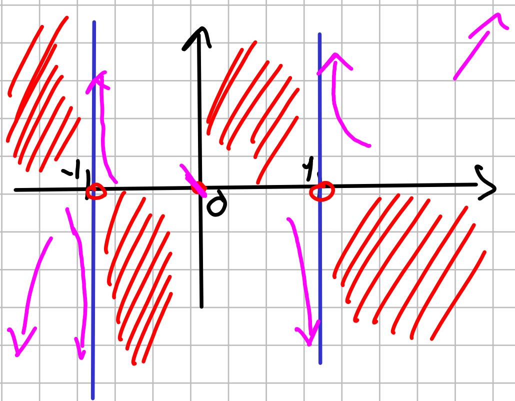
$$\lim_{x\to-\infty}\frac{2x^3}{x^2-1}=\frac{\overbrace{2x^{\cancel{3}}}^{\to-\infty}}{\cancel{x^2}_1\left(1-\underbrace{\frac{1}{x^2}}_{\to 0}\right)}=-\infty$$

$$\lim_{x\to1^+}\frac{2x^3}{x^2-1}=\frac{+2}{0^+}=+\infty$$

$$\lim_{x\to1^-}\frac{2x^3}{x^2-1}=\frac{2}{0^-}=-\infty$$

$$\lim_{x\to-1^+}\frac{2x^3}{x^2-1}=\frac{-2}{0^-}=+\infty$$

$$\lim_{x\to-1^-}\frac{2x^3}{x^2-1}\ \frac{-2}{0^+}=-\infty$$



5) $f'(x)$ di $\frac{2x^3}{x^2-1}$

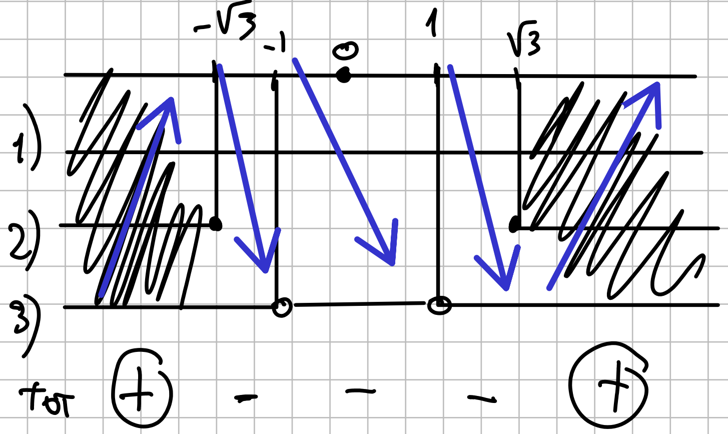
$$\frac{6x^2(x^2-1)-2x^3\cdot 2x}{(x^2-1)^2}=\frac{6x^4-6x^2-4x^4}{(x^2-1)^2}=\frac{2x^4-6x^2}{(x^2-1)^2}$$

$$f'(x)=\frac{2x^2(x^2-3)}{(x^2-1)^2}>0 \qquad \begin{cases} 1) & 2x^2>0 \\ 2) & x^2-3>0 \\ 3) & (x^2-1)^2>0 \end{cases}$$

$x^2-3>0$

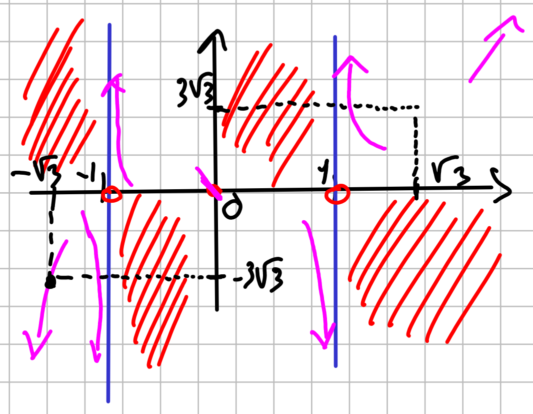
$x^{\cancel{2}}=\pm\sqrt{3}$

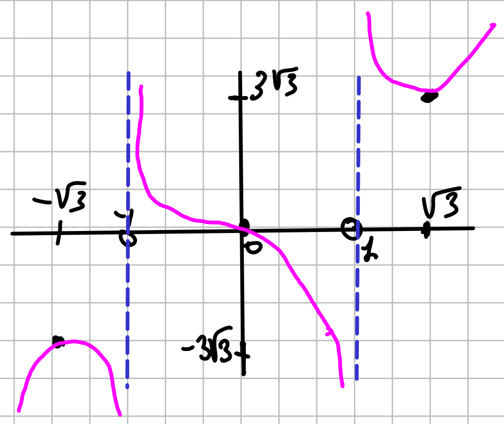
$x^2-1)^2$



$x\leq-\sqrt{3}$, $x\geq\sqrt{3}$, $-1<x<1$

MAX. REL. $-\sqrt{3}$ / MIN REL. $\sqrt{3}$





MAX. REL. $f(-\sqrt{3})$

$\hookrightarrow(-\sqrt{3},-3\sqrt{3})$

$$\frac{2x^3}{x^2-1}\rightarrow\frac{2(-\sqrt{3})^3}{\sqrt{3}^{\cancel{x}}-1}=\cancel{2}\left(\frac{-\sqrt{27}}{\cancel{2}}\right)-3\sqrt{3}$$

MIN REL $(\sqrt{3},3\sqrt{3})$

6) CONCAVITÀ

$$f''(x)\Rightarrow f'(x)=\frac{2x^2(x^2-3)}{(x^2-1)^2}=\frac{2x^4-6x^2}{(x^2-1)^2}$$

$(x^2-1)^2=$

x^4+1-2x^2

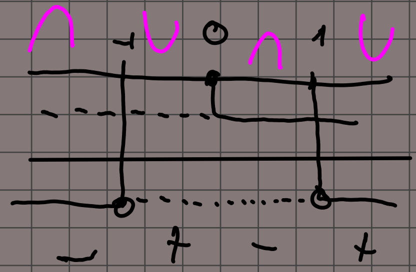
$4x^3-4x$

$$f''(x) = \frac{(8x^3-12x)\cdot(x^2-1)^2-(2x^4-6x^2)\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4} =$$

$$= \frac{\cancel{(x^2-1)}\cdot\left[(8x^3-12x)(x^2-1) - \underbrace{2(2x^4-6x^2)(2x)}_{(-4x)}\right]}{(x^2-1)^{\cancel{4}\,3}} =$$

$$= \frac{\cancel{8x^5} - 8x^3 - 12x^3 + 12x - \cancel{8x^5} + 24x^3}{(x^2-1)^3} = \frac{4x^3+12x}{(x^2-1)^3} = \boxed{\frac{4x(x^2+3)}{(x^2-1)^3}}$$

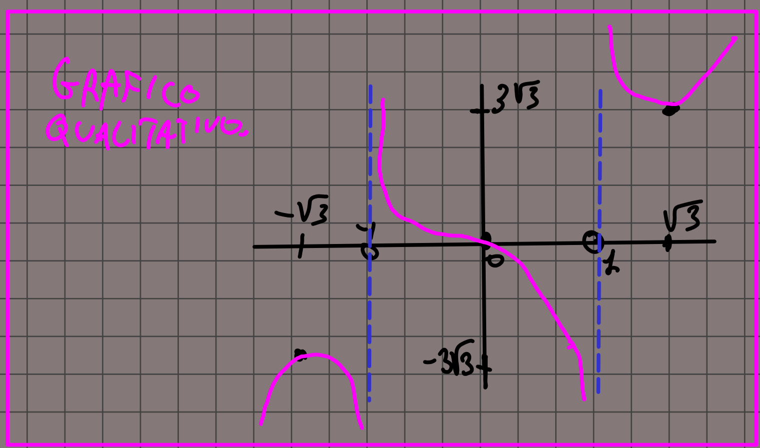
$$f''(x) > 0 \qquad \begin{cases} 1)\ 4x > 0 \\ 2)\ x^2+3 > 0 \\ 3)\ (x^2-1)^3 > 0 \end{cases}$$



$(x^2-1)^3 = 0$

$x^2 - 1 = 0$

$x = \pm 1$



$f(x) = x^3 - 3x + 2$

STUDIARE LA FUNZIONE

1) DOMINIO $= \mathbb{R}$

2) $f(-x) = (-x)^3 - 3(-x) + 2$

$\hookrightarrow -x^3 + 3x + 2$

3) SEGNO $f(x) \geq 0 \rightarrow x^3 - 3x + 2 \geq 0$

	1	0	-3	2
1		1	1	-2
	1	1	-2	0

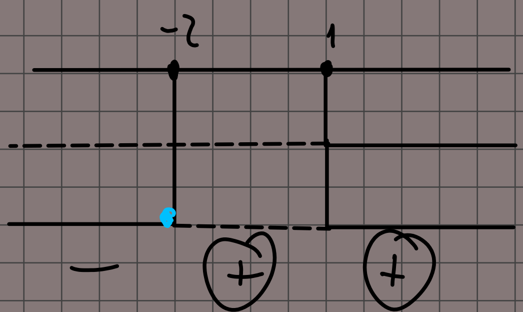
$(x-1)(\underline{x^2 + x - 2}) \geq 0$

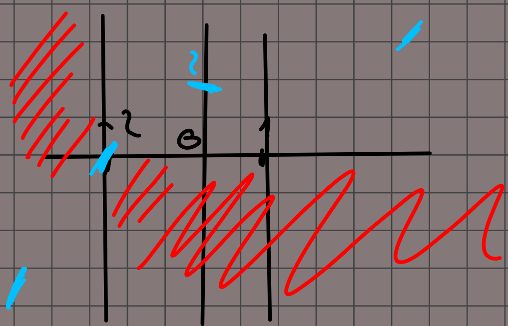
$x \geq 1$

$\frac{-1 \pm 3}{2}$: 1, -2

$x \leq -2 \wedge x \geq 1$

$1 - 4(-2) = 9$





4) LIMITI $f(x) = x^3 - 3x + 2$

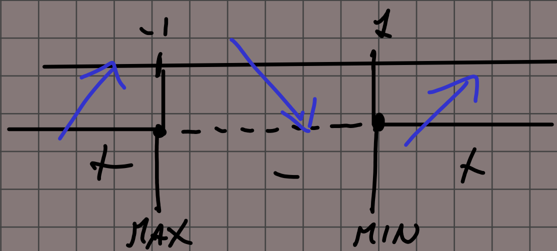
$$\lim_{x \to +\infty} x^3 - 3x + 2 = \underbrace{x^3}_{+\infty}\left(1 - \underbrace{\frac{3x}{x^2}}_{0} + \underbrace{\frac{2}{x^3}}_{0}\right) = +\infty$$

$$\lim_{x \to -\infty} x^3 - 3x + 2 = \underbrace{x^3}_{-\infty}\left(1 - \underbrace{\frac{3x}{x^2}}_{0} + \underbrace{\frac{2}{x^3}}_{0}\right) = -\infty$$

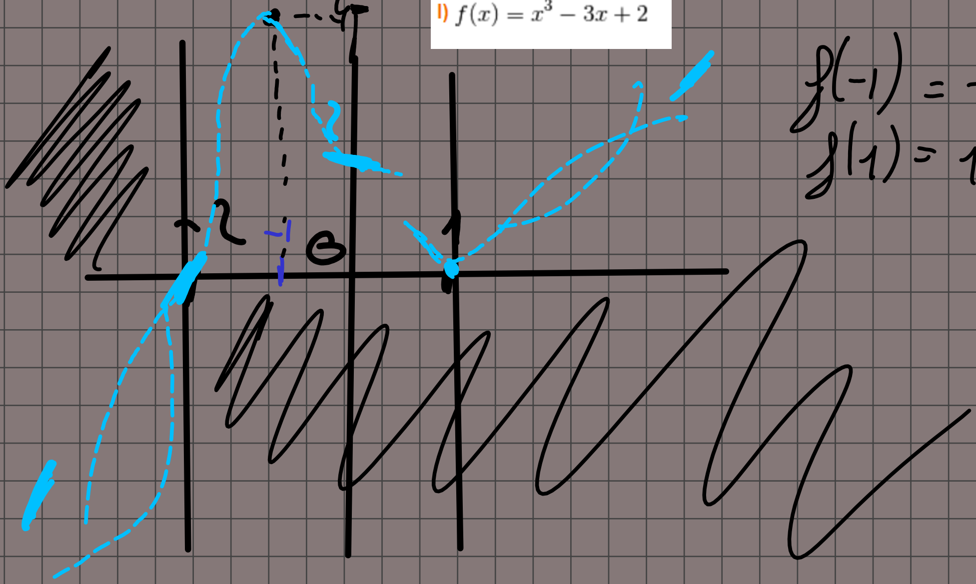
5) $f'(x) > 0$ $\qquad f'(x) = 3x^2 - 3$

$3x^2 - 3 > 0$ $\qquad \boxed{x < -1 \wedge x > 1}$

$3x^2 - 3 = 0$ $\qquad \cancel{3}x^2 = \pm\cancel{3}1$



1) $f(x) = x^3 - 3x + 2$



$f(-1) = -1 + 3 + 2 = 4$

$f(1) = 1 - 3 + 2 = 0$

6) $f''(x) > 0$ $\quad f''(x) = 6x$

$3x^2 - 3$

$6x > 0$

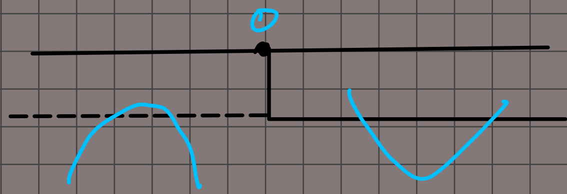
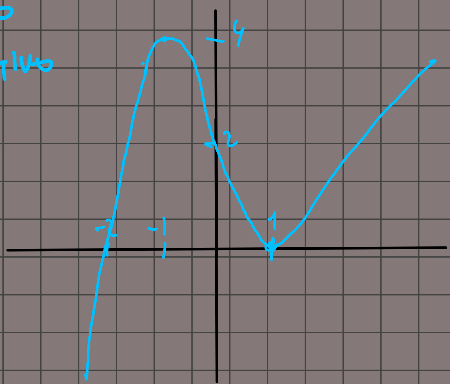


GRAFICO QUALITATIVO



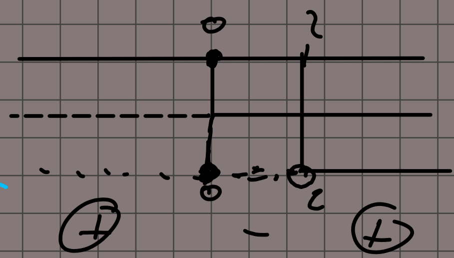
II) $f(x) = \dfrac{6x}{x-2}$

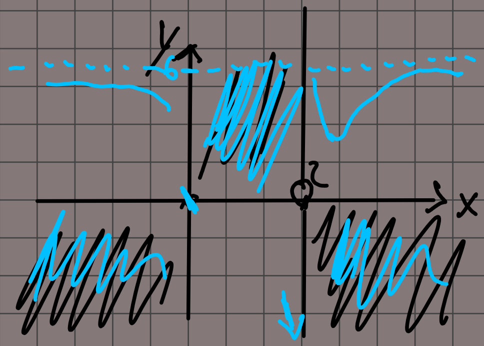
1) DOMINIO $\boxed{x \neq 2}$

2) $f(-x) \rightarrow \dfrac{6(-x)}{-x-2} = \dfrac{-6x}{-x-2} \quad - \dfrac{6x}{x+2}$

$x-2>0$
$x>2$

3) SEGNO $\dfrac{6x}{x-2} \geq 0$





$x=2$ ASINTOTO VERT.
$y=6$ " " ORIZZ.

4) LIMITI

$$\lim_{x\to+\infty} \frac{6x}{x-2} = \left[\frac{\infty}{\infty}\right] \quad \frac{6\cancel{x}}{\cancel{x}\left(1-\frac{2}{x}\right)} = 6 \qquad \frac{2}{x}\to 0$$

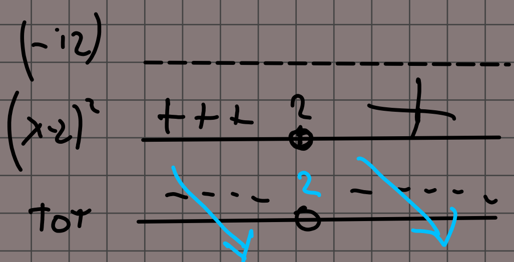
$$\lim_{x\to-\infty} \frac{6x}{x-2} = \frac{6\cancel{x}}{\cancel{x}\left(1-\frac{2}{x}\right)} = 6 \qquad \frac{2}{x}\to 0$$

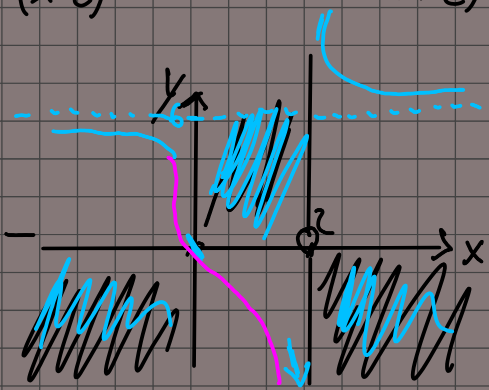
$$\lim_{x\to 2^+} \frac{6x}{x-2} = \frac{6\cancel{x}}{\cancel{x}\left(1-\frac{2}{x}\right)} = \frac{6}{0^+} = +\infty$$

$$\lim_{x\to 2^-} \frac{6x}{x-2} = -\infty$$

5) DERIVATE $f'(x) > 0$

$$f(x) = \frac{6x}{x-2} \rightarrow f'(x) = \frac{6\cdot(x-2)-(6x)}{(x-2)^2} = \frac{\cancel{6x}-12-\cancel{6x}}{(x-2)^2} \quad \frac{-12}{(x-2)^2}$$





6) $f''(x) \rightarrow f'(x) = \frac{-12}{(x-2)^2}$

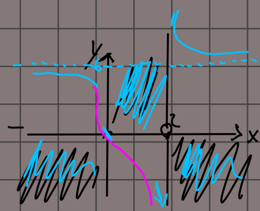
$(x-2)^2 \xrightarrow{\frac{d}{dx}} 2(x-2)\cdot 1$

$$f''(x) = \frac{-(-12)\cdot 2(x-2)}{(x-2)^4}$$

$$f''(x) = \frac{12\cdot(2x-4)}{(x-2)^4} > 0$$

$2x-4>0$

$2x = 4/2$



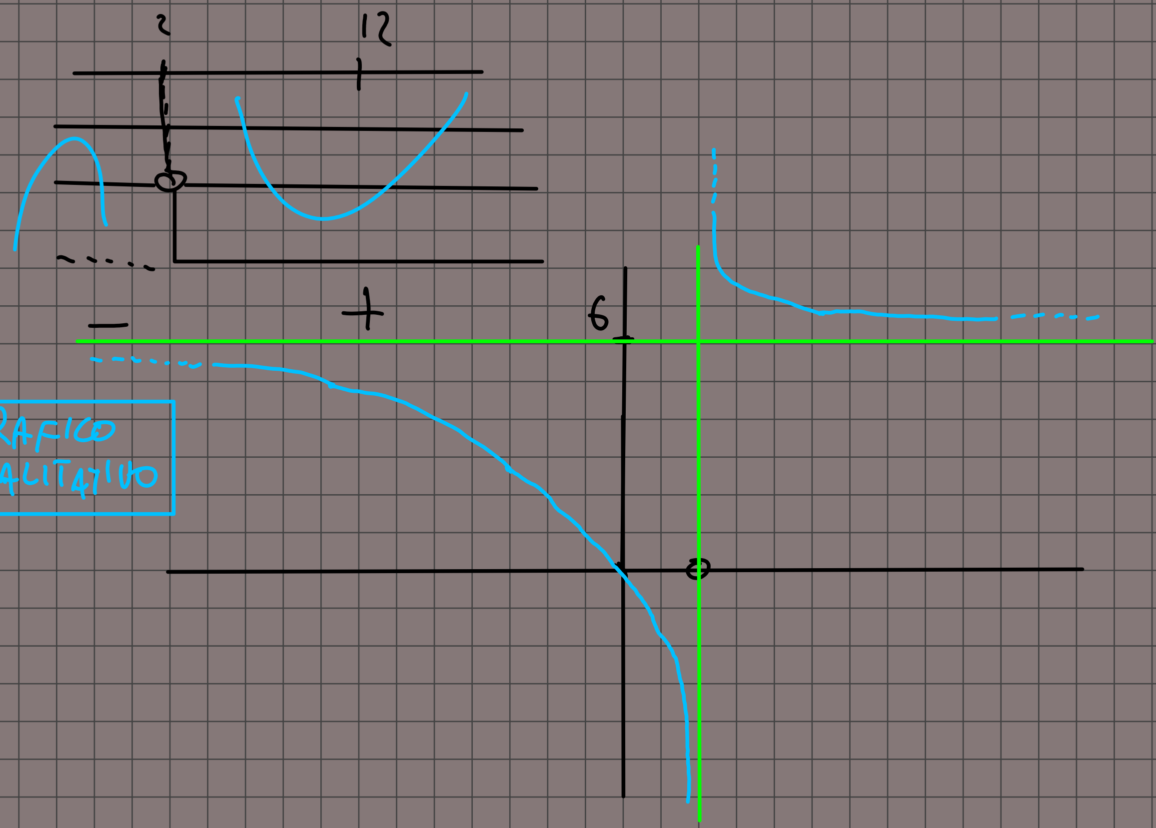


GRAFICO QUALITATIVO

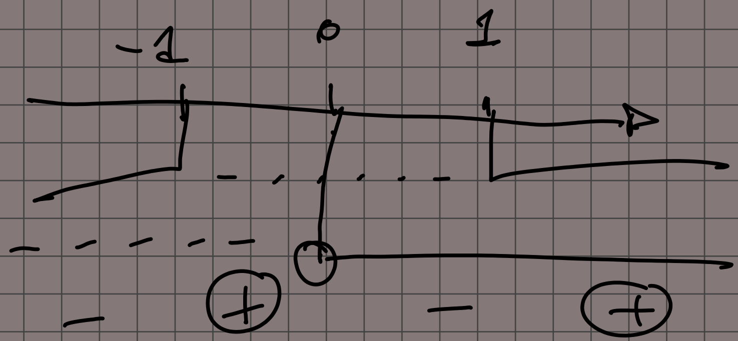
$$f(x) = \frac{x^2-1}{x}$$

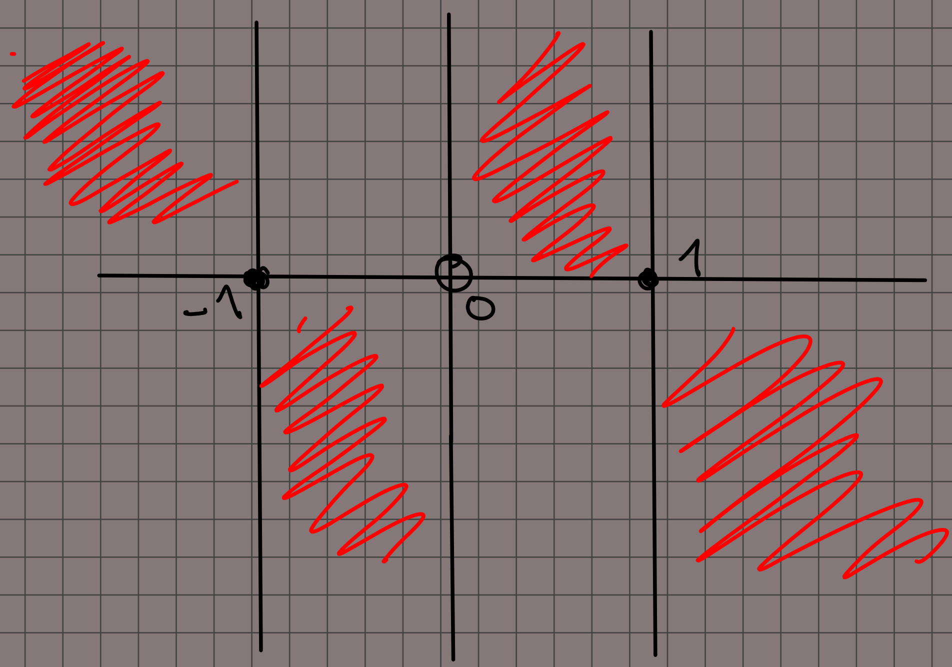
1) DOMINIO → $x \in \mathbb{R} \mid x \neq 0$

2) SIMMETRIA → $f(-x) = -\frac{x^2-1}{x}$

3) SEGNO

$$\frac{x^2-1}{x} \geq 0 \qquad \begin{cases} x^2-1 \geq 0 \\ x > 0 \end{cases} \qquad \begin{matrix} x = \pm 1 \text{ ext} \\ x > 0 \end{matrix}$$





4) limiti

$0\ (+/-)$

$+\infty$

$-\infty$

$$\lim_{x\to+\infty} \frac{x^2-1}{x} = \frac{\cancel{x^2}\left(1-\frac{1}{x^2}\right)}{\cancel{x}} = x = +\infty$$

$-\infty$

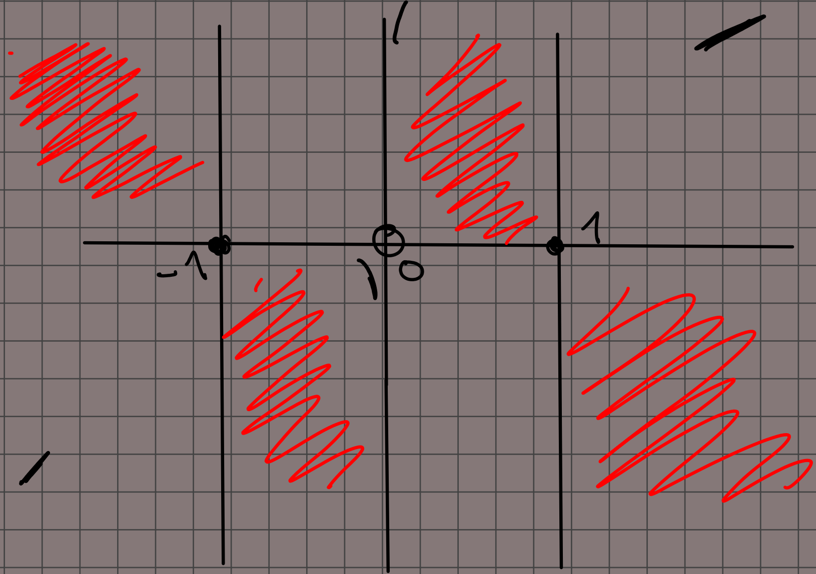
$\lim_{x\to-\infty}$

$$\lim_{x\to 0^+} \frac{x^2-1}{x} = \frac{x^2\left(1-\frac{1}{x^2}\right)}{x} \to \infty = -\infty$$

$$\lim_{x\to 0^-} [\ //\ ] = +\infty$$

$\lim \frac{1}{\ }$

- $+\infty \to +\infty$
- $-\infty \to -\infty$
- $0^+ \to -\infty$
- $0^- \to +\infty$

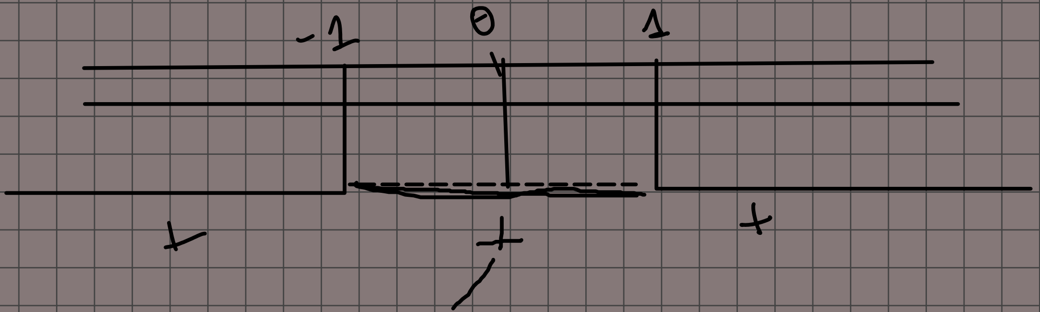


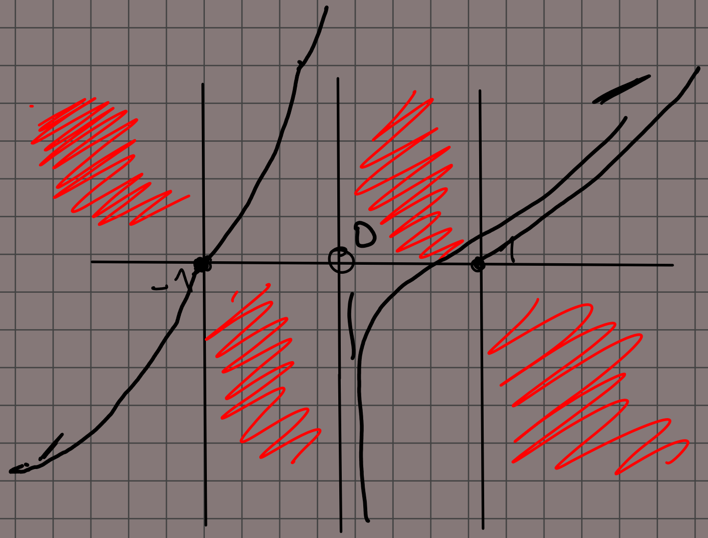
5) DERIVATA PRIMA

D. 1° → $2x$
D. 2° → 1

$$f(x)\ \frac{x^2-1}{x} \rightsquigarrow f'(x) = \frac{2x^2 - x^2 + 1}{x^2} \rightsquigarrow f'(x) = \frac{x^2+1}{x^2}$$

- $x^2 > 0 \quad \forall x$
- $x^2 + 1 > 0 \to x^2 + 1 = 0 \rightsquigarrow x = \pm 1 \rightsquigarrow$ ext.





6) DERIVATA SECONDA

$x^2+1 \to 2x$

$x^2 \to 2x$

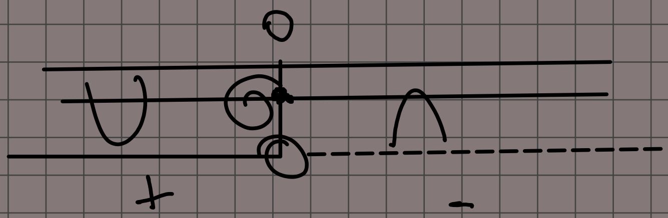
$$f'(x) = \frac{x^2+1}{x^2} = \frac{2x(x^2) - 2x(x^2+1)}{x^4}$$

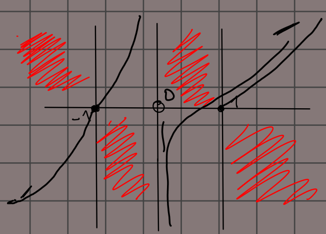
$$2\cancel{x^3} - 2\cancel{x^3} \quad \frac{-2\cancel{x}}{x^{\cancel{4}3}} = f''(x)$$

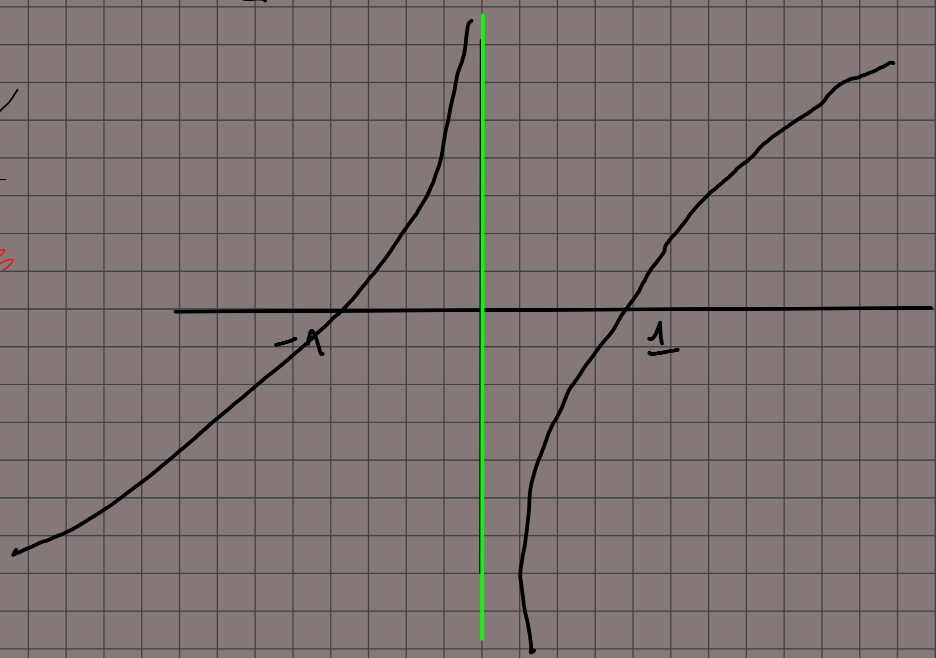
$\underline{x^4 > 0 \quad \forall x \mid x \neq 0}$

$-2x > 0$

$x < 0$







VI) $f(x)=\dfrac{1}{\sqrt{x+3}}$

1) DOMINIO: $x+3>0 \rightarrow x>-3$

$x \neq -3$

2) SIMMETRIA:

$$f(-x)=\frac{1}{\sqrt{-x+3}}$$

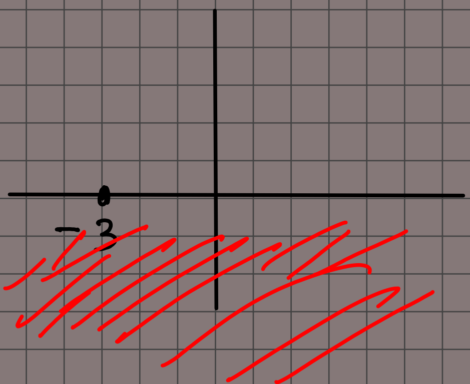
NB

3) $\dfrac{1}{\sqrt{x+3}} \geq 0$

$1>0$ SEMPRE

$\sqrt[\text{pari}]{} > 0$ SEMPRE

$\rightarrow$ MAI SOTTO X.



4) limiti

$+\infty$

$-\infty$

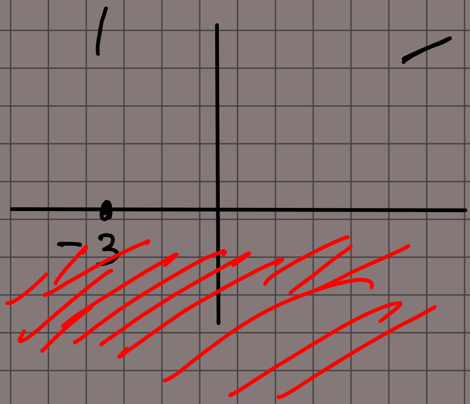
-3^+

$\cancel{3}$

$$\lim_{x\to\infty}\frac{1}{\sqrt{x+3}}=\frac{1}{\sqrt{x\left(1+\frac{3}{x}\right)}}=\frac{1}{\sqrt{\infty}}=\frac{1}{\infty}=0.$$

$$\lim_{x\to-\infty} \nexists$$

$$\lim_{x\to-3^+}\frac{1}{\sqrt{x+3}}=\frac{1}{\sqrt{0}}=\frac{1}{0}=+\infty$$



5) DERIVATA PRIMA

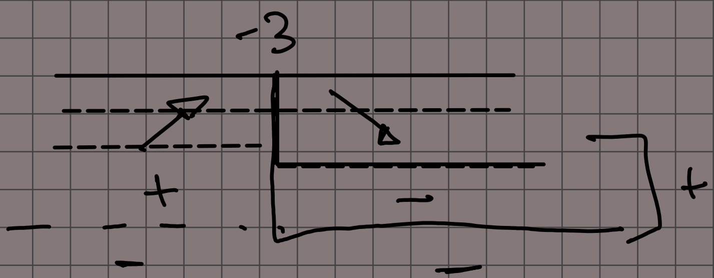
$$\frac{1}{2}(x+3)\cdot 1$$

$$f(x) = \frac{1}{\sqrt{x+3}} \quad = \frac{0\cancel{(x)} - \frac{1}{2}(x+3)}{x+3}$$

$$-\frac{1}{2}(x+3) > 0$$

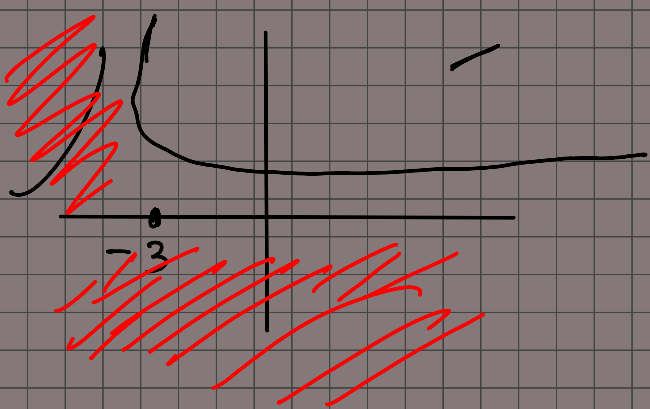
$$-\frac{1}{2} > 0 \quad \nexists$$

$$= \begin{cases} x+3 > 0 \to x > -3 \\ x+3 \geq 0 \to x \geq -3 \end{cases}$$



$$\frac{\sqrt{(x+3)}^{\frac{1}{2}-1}}{}$$

$$\frac{1}{2}(x+3)^{-\frac{1}{2}} \cdot 1$$



$$\frac{\frac{1}{2}\cdot\frac{1}{x+3^{\frac{1}{2}}}}{x+3} \qquad \frac{1}{2(x+3)^{\frac{1}{2}}}\cdot\frac{1}{x+3}$$

$$\frac{1}{2(x+3)^{\frac{3}{2}}} = -\frac{1}{2\sqrt{(x+3)}^3}$$

6) DERIVATA SECONDA

$$\frac{-\frac{1}{2}(x+3)}{x+3}$$

$$f(x) = -\frac{1}{2}(x+3) \to f'(x) = \cancel{0(x+3)} + \left(-\frac{1}{2}\right)\cdot 1 = -\frac{1}{2}$$

$$\frac{3}{2} - 1 \quad \frac{3-2}{2} = -\frac{1}{2}$$

$\frac{1}{2(x+3)^{\frac{3}{2}}} = f''(x) = \frac{\cancel{0\cdot(\ldots)} - 1\cdot 3(x+3)^{-\frac{1}{2}}}{2(x+3)^3} =$ $\frac{1}{2}+3=\frac{1+6}{2}$

$2(x+3)^{\frac{3}{2}} \xrightarrow{D} 3(x+3)^{-\frac{1}{2}}$ $= -1\cdot\frac{3}{(x+3)^{\frac{1}{2}}}\cdot\frac{1}{2(x+3)^3} = -\frac{3}{2(x+3)^{\frac{7}{2}}}$

$\cup \to \cap$

$= -\frac{3}{2(x+3)^{7/2}} \to (-)\frac{3}{2\sqrt{x+3}^{7}}$

$(x+3)^{4}(x+3)^{3} = (x+3)^{7}$

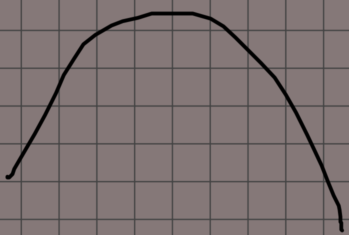
$\hookrightarrow 2(x+3)^2\sqrt{(x+3)}^{3}$

$-\frac{3}{4(x+3)^{5/2}}$

$-\frac{3}{4}$

$\sqrt{(x+3)}^{5}$

$-\frac{3}{4\cdot(x+3)^2\sqrt{x+3}}$



$\sqrt{(x+3)^5} = \sqrt{(x+3)^4\cdot(x+3)} =$

$\sqrt[2]{(x+3)^{4}}\cdot\sqrt{(x+3)} = (x+3)^2\sqrt{(x+3)}$

$-\frac{3}{4\sqrt{(x+3)^5}}$

3

4

$\sqrt{m}$

$D(N) : -\frac{1}{2}$

$D(D) : 1$

$$\frac{\cancel{0 \cdot (x+3)} - \left(-\frac{1}{2}(x+3)\cancel{x}\right)}{(x+3)^2} = -\left(-\frac{1}{2}x - \frac{3}{2}\right)$$

$$\frac{\frac{1}{2}x + \frac{3}{2}}{(x+3)^2} = f''(x)$$

$$\frac{-\frac{1}{2}\cancel{(x+3)} - 1\left(-\frac{1}{2}(x+3)\right)}{(x+3)^{\cancel{2}}}$$

$$\frac{-\frac{1}{2} + \frac{1}{2}\cancel{(x+3)}}{x\cancel{+3}} = 0$$

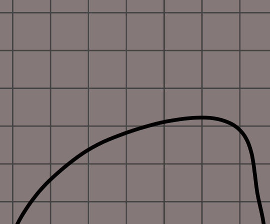
$$f(x) = \frac{1}{\sqrt{x+3}}$$

$$-\frac{1}{2}(x+3) \xrightarrow{D} -\frac{1}{2}$$

$$f'(x) = \frac{-\frac{1}{2}(x+3)}{x+3}$$

$$f''(x) = \frac{-\frac{1}{2}(x+3) - \left(-\frac{1}{2}(x+3)\right)}{(x+3)^2} = \frac{-\frac{1}{2}(x+3) + \frac{1}{2}(x+3)}{(x+3)^2}$$

$$= \frac{\cancel{(x+3)}\left(\overset{0}{\cancel{-\frac{1}{2} + \frac{1}{2}}}\right)}{(x+3)^2} = 0$$



$$f(x) = |x^3 - x|$$

1) $D = \mathbb{R}$

$$\begin{cases} x^3 - x \geq 0 \\ x^3 - x \end{cases}$$

$$\begin{cases} x(x^2-1) \geq 0 \\ x^3 - x \end{cases}$$

$$\begin{cases} -1 \leq x \leq 0 \quad \wedge \quad x \geq 1 \\ x^3 - x \end{cases}$$

$$\begin{cases} x^3 - x < 0 \\ -x^3 + x \end{cases}$$

$$\begin{cases} x(x^2-1) < 0 \\ -x^3 + x \end{cases}$$

$$\begin{cases} x < -1 \quad 0 < x < 1 \\ -x^3 + x \end{cases}$$

$x \geq 0 \quad \vee \; x^2 - 1 \geq 0$

$x \geq 0 \quad \vee (x \leq -1 \wedge x \geq 1)$

$-1 \leq x \leq 0 \quad \wedge \quad x \geq 1$

$x < 0 \quad \vee \; x^2 - 1 < 0$

$x < 0 \quad \vee (-1 < x < 1)$

$x < -1 \qquad 0 < x < 1$

$$\begin{cases} x^3 - x & \text{SE} \quad -1 \leq x \leq 0 \quad \wedge \quad x \geq 1 \\ -x^3 + x & \text{SE} \quad x < -1 \quad 0 < x < 1 \end{cases}$$

2) DISPARI

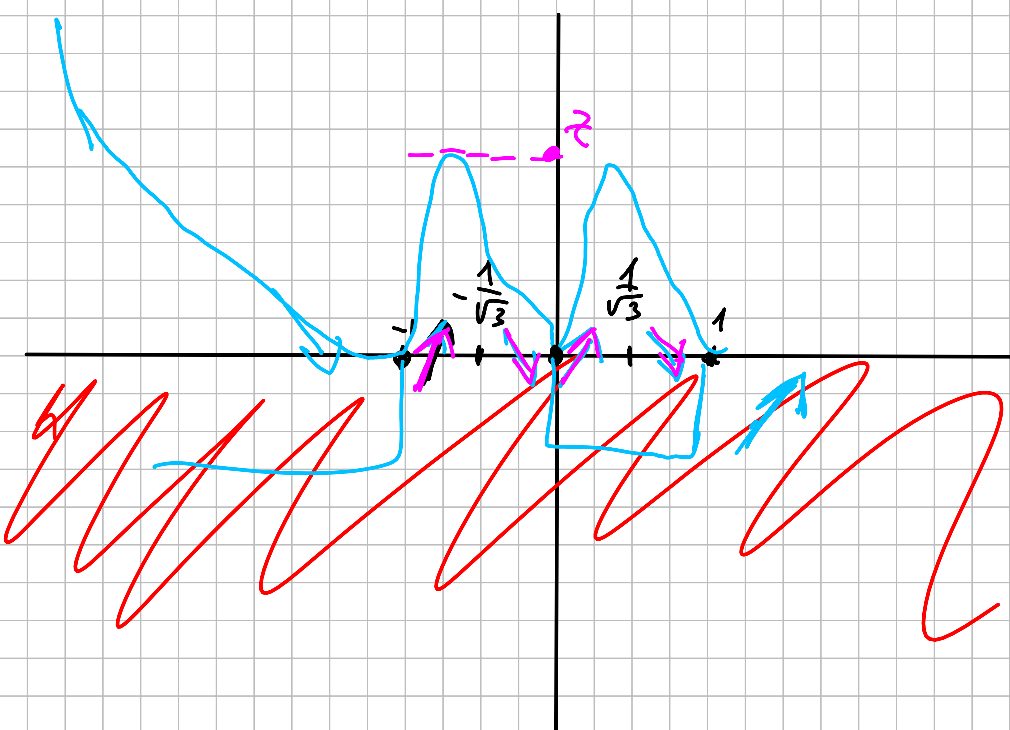
3) $f(x) \geq 0$

4) LIMITI

$$\lim_{x \to +\infty} x^3 - x = +\infty$$

$$\lim_{x \to -\infty} -x^3 + x = +\infty$$

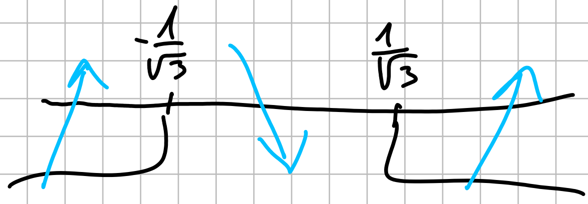
5) DERIVATA.



$$\begin{cases} x^3 - x & \text{SE } -1\le x\le 0 \ \wedge\ x\ge 1 \\ -x^3+x & \text{SE } x<-1 \quad 0<x<1 \end{cases}$$

$$\begin{cases} 3x^2-1 & \text{se } -1\le x\le 0 \ \wedge\ x\ge 1 \\ -3x^2+1 & \text{se } x<-1 \quad 0<x<1 \end{cases}$$

$$\begin{cases} 3x^2-1>0 \qquad 3x^2-1=0 \qquad \cancel{3}x^2 = \pm\frac{1}{\sqrt{3}} \\ -3x^2+1>0 \qquad -3x^2+1=0 \qquad -3x^2=-1 \qquad 3x^2=1 \end{cases}$$



6) PER'

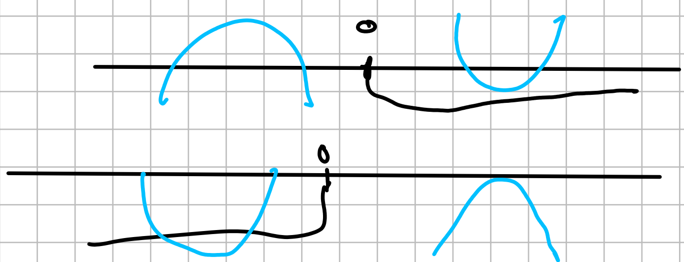
$$\begin{cases} 3x^2-1 & \text{se } -1\le x\le 0 \ \wedge\ x\ge 1 \\ -3x^2+1 & \text{se } x<-1 \quad 0<x<1 \end{cases}$$

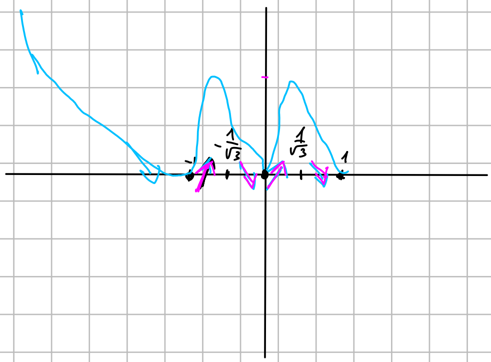
$\cancel{6}x>0$

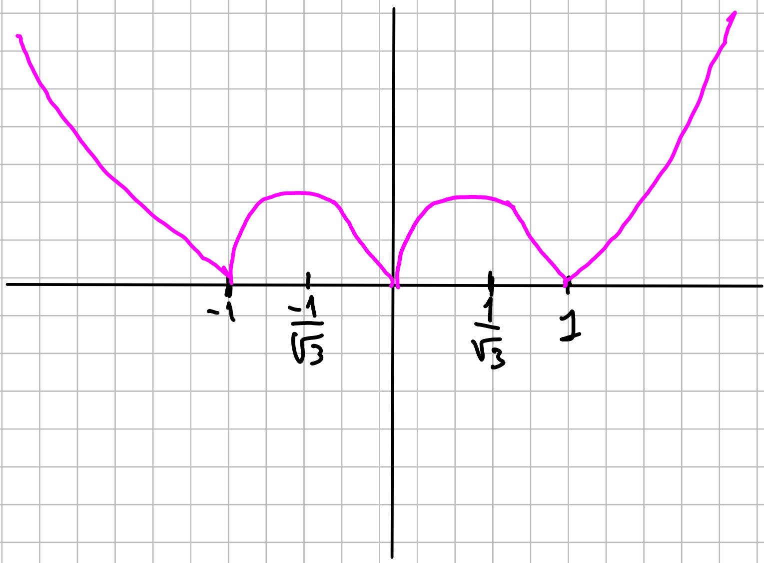
$-6x>0 \quad 6x<0$

x

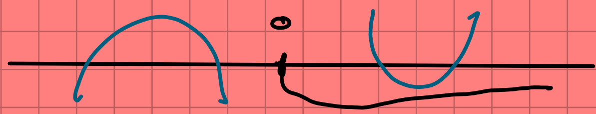
$$\begin{cases} 6x & \text{se } -1\le x\le 0 \ \wedge\ x\ge 1 \\ -6x & \text{se } x<-1 \quad 0<x<1 \end{cases}$$



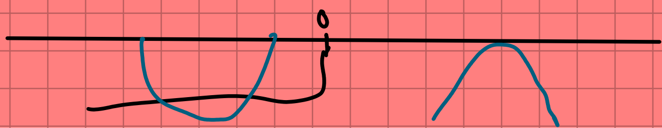




$-1 \leq x \leq 0 \;\wedge\; x \geq 1$



$x < -1 \quad 0 < x < 1$



Studiare e tracciare il grafico della funzione $f(x) = \dfrac{x}{\sqrt{x-2}}$

1) DOMINIO

$x > 2$

$$\begin{cases} x-2 \geq 0 \\ \sqrt{x-2} \neq 0 \end{cases} \quad \begin{cases} x-2 > 0 \\ x \neq 2 \end{cases} \quad \begin{cases} x > 2 \\ x \neq 2 \end{cases}$$

2) simmetrie

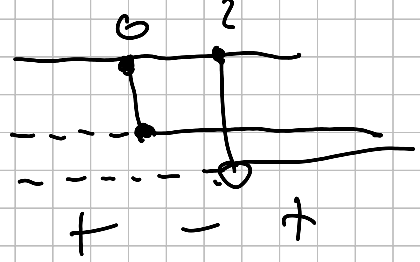
$$f(-x) = \frac{-x}{\sqrt{-x-2}} \neq -f(x) \vee f(x) \quad \left(\begin{matrix}\text{NÉ PARI} \\ \text{NÉ DISPARI}\end{matrix}\right)$$

3) SEGNO.

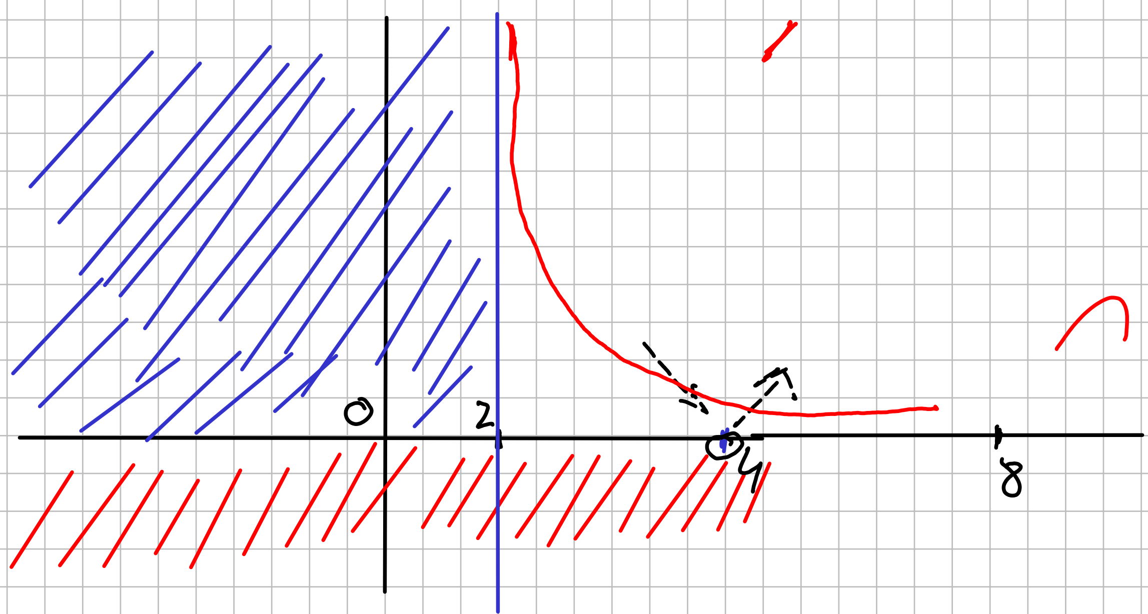
$$\frac{x}{\sqrt{x-2}} \geq 0$$

$$\frac{x}{\sqrt{x-2}} = 0$$

$$\begin{cases} x \geq 0 \\ x > 2 \end{cases}$$



$$\begin{cases} \cancel{x < 0} \\ x > 2 \end{cases} \Rightarrow D = (x > 2)$$



4) LIMITI

$$\lim_{x \to 2^+} \frac{x}{\sqrt{x-2}} = \frac{2}{0^+} = +\infty$$

$$\lim_{x \to +\infty} \frac{x}{\sqrt{x-2}} = \left[\frac{\infty}{\infty}\right] = +\infty$$

$$\frac{x}{\sqrt{x-2}} \cdot \frac{\sqrt{x-2}}{\sqrt{x-2}} = \frac{x\sqrt{x-2}}{x-2} =$$

$$= \frac{\cancel{x}\sqrt{x-2}}{\cancel{x}\left(1-\frac{2}{x}\right)} = +\infty$$

$\frac{2}{x} \to 0$

5) DERIVATA 1

$$\frac{x}{\sqrt{x-2}}$$

$$(x-2)^{\frac{1}{2}} \qquad \frac{1}{2}(x-2)^{-\frac{1}{2}}\cdot 1$$

$$\frac{1}{2}\cdot\frac{1}{\sqrt{x-2}} = \frac{1}{2\sqrt{x-2}}$$

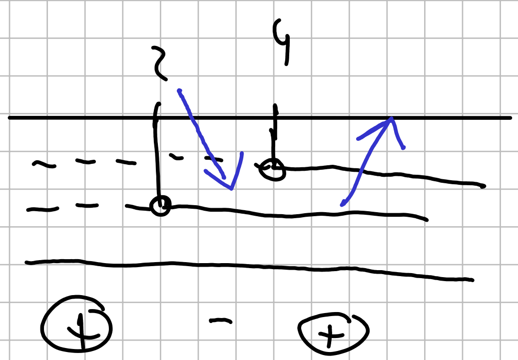
$$f'(x) = \frac{1\cdot\left(\sqrt{x-2}\right) - x\cdot\left(\frac{1}{2\sqrt{x-2}}\right)}{x-2} = \frac{\left(\sqrt{x-2} - x\cdot\frac{1}{2\sqrt{x-2}}\right)}{x-2}$$

$$= \frac{\left(\sqrt{x-2} - \frac{x}{2\sqrt{x-2}}\right)}{x-2} = \frac{\frac{2(x-2)-x}{2\sqrt{x-2}}}{x-2} =$$

$$\frac{2(x-2)-x}{2\sqrt{x-2}}\cdot\frac{1}{x-2} = \frac{2x-4-x}{2\sqrt{x-2}}\cdot\frac{1}{x-2} = \frac{x-4}{2\sqrt{x-2}}\cdot\frac{1}{x-2} =$$

$$\frac{x-4}{2(x-2)^{\frac{1}{2}}\cdot(x-2)} = \boxed{\frac{x-4}{2(x-2)^{\frac{3}{2}}}} \rightarrow f'(x)$$

$$\frac{x-4}{2(x-2)^{\frac{3}{2}}} > 0 \qquad \begin{cases} x-4>0 \\ 2>0 \\ (x-2)^{\frac{3}{2}}>0 \end{cases} \quad \begin{cases} x>4 \\ \text{—} \\ \left(\sqrt{x-2}\right)^3>0 \end{cases} \quad \begin{cases} x>4 \\ \text{—} \\ \sqrt{x-2}>0 \end{cases} \quad \begin{cases} \text{—} \\ \text{—} \\ x>2 \end{cases}$$



$$\begin{cases} \cancel{x>2} \\ x>4 \end{cases} \rightarrow D = (x>2)$$

6) DER. 2

$$\boxed{\frac{x-4}{2(x-2)^{\frac{3}{2}}}} \longrightarrow f'(x)$$

$$(x-4) = 1$$

$$\frac{3}{2} - 1 = \frac{3-2}{2} = +\frac{1}{2}$$

$$f''(x) = \frac{1 \cdot 2(x-2)^{\frac{3}{2}} - (x-4) \cdot 3(x-2)^{\frac{1}{2}}}{\left(2(x-2)^{\frac{3}{2}}\right)^2} =$$

$$2(x-2)^{\frac{3}{2}} \xrightarrow{D} 0 \cdot \cancel{(x-2)^{\frac{3}{2}}} + 2 \cdot \left(\frac{3}{2}(x-2)^{\frac{1}{2}} \cdot 1\right)$$

$$= 2\left(\frac{3}{2}(x-2)^{\frac{1}{2}}\right) = 3\sqrt{x-2}$$

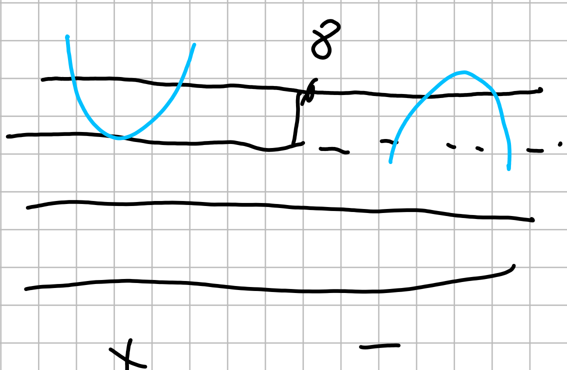
$$\frac{2(x-2)^{3/2} - (x-4) \cdot 3(x-2)^{\frac{1}{2}}}{4(x-2)^3} = \frac{\cancel{(x-2)^{\frac{1}{2}}}\left[2(x-2) - 3x + 12\right]}{4(x-2)^{3-\frac{5}{2}}} =$$

$$\frac{1}{2} - 3 = \frac{1-6}{2} = -\frac{5}{2}$$

$$= \frac{2x-4-3x+12}{4(x-2)^{-5/2}} = \frac{8-x}{4(x-2)^{-5/2}}$$

$$f''(x) > 0 \quad \begin{cases} 8 - x > 0 \\ 4(x-2)^{-5/2} > 0 \end{cases} \quad \begin{cases} x < 8 \\ 4 > 0 \checkmark \\ (x-2)^{-\frac{5}{2}} > 0 \end{cases}$$

$$\left(\frac{1}{\cancel{\sqrt{x-2}}}\right)^{5} > 0 \quad = 1 > 0 \checkmark$$



Esercizio 2. Studiare la funzione definita dalla legge

5 settembre 2022

$$-2x + \sqrt{|x-1|}$$

e tracciarne un grafico qualitativo.

1) $D = \mathbb{R}$

① $\begin{cases} x-1 \geq 0 \\ -2x+\sqrt{x-1} \end{cases}$ $\begin{cases} x \geq 1 \\ -2x+\sqrt{x-1} \end{cases}$

② $\begin{cases} x-1 < 0 \\ -2x+\sqrt{-x+1} \end{cases}$ $\begin{cases} x < 1 \\ -2x+\sqrt{-x+1} \end{cases}$

FORMA A TRATTI

$$\begin{cases} -2x+\sqrt{x-1} & \text{se} \quad x \geq 1 \\ -2x+\sqrt{-x+1} & \text{se} \quad x < 1 \end{cases}$$

2) SIMMETRIA $-2x + \sqrt{|x-1|}$

$f(-x) = -2(-x)+\sqrt{|-x-1|} = 2x+\sqrt{|-x-1|}$ NO PARI/DISPARI

3) SEGNO $f(x) \geq 0$

$\boxed{-2x+\sqrt{|x-1|} \geq 0}$ $\rightarrow$ $\left(-2x+\sqrt{|x-1|}\right)^2 \geq 0$

$= 4x^2 + |x-1| - 2(-2x)\cdot\left(\sqrt{|x-1|}\right) = 0$

$\downarrow$

$-2x+\sqrt{|x-1|} \geq 0$

$\sqrt{|x-1|} \geq 2x$

$|x-1| \geq 4x^2$

$$\begin{cases} x \geq 1 \\ x - 1 \geq 4x^2 \end{cases} \qquad \begin{cases} x < 1 \\ -x + 1 \geq 4x^2 \end{cases}$$

$$\begin{cases} x \geq 1 \\ 4x^2 - x + 1 \leq 0 \end{cases} \qquad \begin{cases} x < 1 \\ 4x^2 + x - 1 \leq 0 \end{cases}$$

$\downarrow$

$b^2 - 4ac = 1 - 4(4) = -15$

DISCORDI $\emptyset$

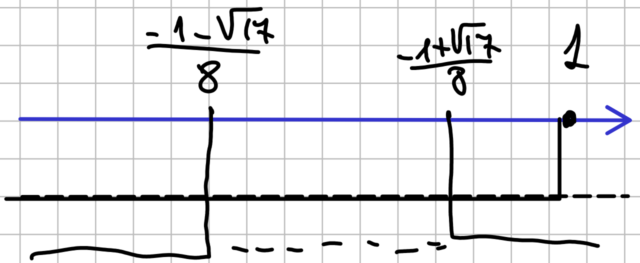
$b^2 - 4ac =$

$1 - (4)(4)(-1) =$

$= 1 - (-16) = 17$

$$\begin{cases} x < 1 \\ \dfrac{-1-\sqrt{17}}{8} < x < \dfrac{-1+\sqrt{17}}{8} \end{cases}$$

$-0,64$ $\quad$ $0,39$



Studiare la seguente funzione:

$$f(x)=\sqrt{1+|x|}$$

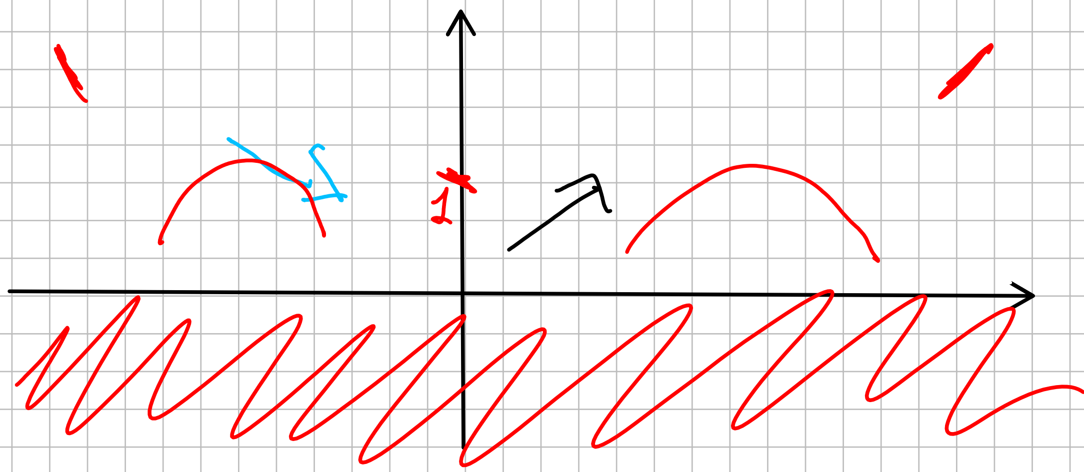
$f(0)=\sqrt{1+|0|}=\pm 2$

1) DOMINIO $\mathbb{R}$

2) $f(-x)=\sqrt{1+|-x|}=f(x)$ PARI

3) SEGNO $\rightsquigarrow$ intersez. ASSE $\vec{y}$

$\sqrt{1+|x|}>0$ SEMPRE $f(0)=\sqrt{1+0}=1$ (y)



4) LIMITI e forma a tratti.

$$\begin{cases} x\geq 0 \\ \sqrt{1+x} \end{cases} \qquad \begin{cases} x<0 \\ \sqrt{1-x} \end{cases}$$

$$\lim_{x\to+\infty}\sqrt{1+x}=+\infty$$

$$\lim_{x\to-\infty}\sqrt{1-x}=+\infty$$

5) DERIVATA

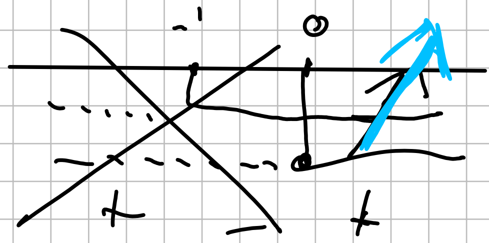
$$\begin{cases} x\geq 0 \\ \dfrac{1}{2\sqrt{x+1}} \end{cases} \qquad f'(x) \qquad \begin{cases} x<0 \\ -\dfrac{1}{2\sqrt{1-x}} \end{cases}$$

$\boxed{x \geq 0}$

↗

$\frac{1}{2\sqrt{x+1}} > 0 \qquad \frac{2\sqrt{x+1}}{4(x+1)} = \frac{2\sqrt{x+1}}{4x+4} > 0$

$\begin{cases} 2\sqrt{x+1} > 0 \\ 4x+4>0 \end{cases} \qquad \begin{cases} x+1>0 \\ \cancel{4}x > -\cancel{4}1 \end{cases} \qquad \begin{cases} x>-1 \\ x>-1 \end{cases}$



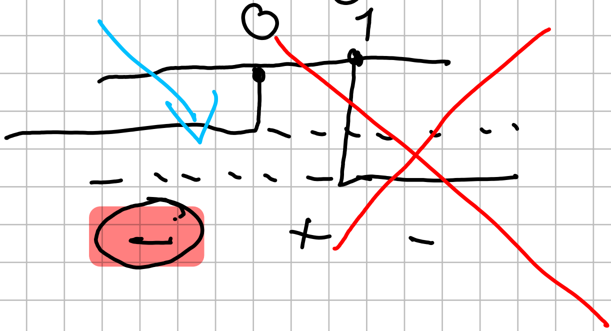
$\frac{1}{2\sqrt{1-x}} > 0 \qquad < 0$

$\cancel{2}\sqrt{1-x} < 0$

$1-x<0$

$-x<-1$

$\begin{cases} x>1 \\ x<0 \end{cases}$



6) DERIVATA 2

$2\sqrt{x+1} = 2(x+1)^{\frac{1}{2}} =$

$(x+1)^{-\frac{1}{2}} = \frac{1}{\sqrt{x+1}}$

$\begin{cases} x \geq 0 \\ -\frac{1}{\sqrt{x+1}} \cdot \frac{1}{4(x+1)} \end{cases}$

$4(x+1)^{\frac{1}{2}} \cdot (x+1)$

$\begin{cases} x \geq 0 \\ \frac{1}{2\sqrt{x+1}} \end{cases} \qquad f'(x) \qquad \begin{cases} x<0 \\ -\frac{1}{2\sqrt{1-x}} \end{cases}$

$f''(x) = \frac{0 \cdot \cancel{(2\sqrt{x+1})} - 1 \cdot \frac{1}{\sqrt{x+1}}}{4(x+1)} =$

$$\begin{cases} x \geq 0 \\ -\dfrac{1}{4(x+1)^{3/2}} \end{cases}$$

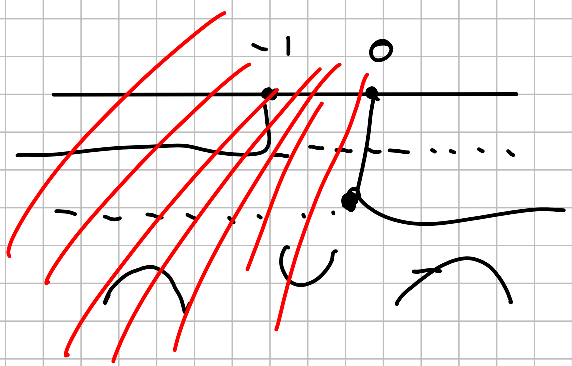
$(\sqrt{x+1})^3$

$\hookrightarrow -\dfrac{1}{4(x+1)^{\frac{3}{2}}} > 0 \rightarrow 4(x+1)^{3/2} < 0$

$4 < 0$ MAI

$(x+1)^{3/2} < 0$

$\hookrightarrow \boxed{x < -1}$



$2\sqrt{1-x} = 2(1-x)^{\frac{1}{2}}$

$(1-x)$

$$\begin{cases} x < 0 \\ -\dfrac{1}{2\sqrt{1-x}} \end{cases} \qquad \begin{cases} x < 0 \\ -\left[\dfrac{\cancel{0 \cdot (2\sqrt{1-x})} - 1 \cdot (1-x)^{-\frac{1}{2}} \cdot (-1)}{4(1-x)}\right] \end{cases}$$

$$\begin{cases} x < 0 \\ -\left[-\dfrac{\frac{1}{\sqrt{1-x}} \cdot (-1)}{4(1-x)}\right] \end{cases} \qquad \begin{cases} x < 0 \\ -\left[-\dfrac{1}{\sqrt{1-x}} \cdot \dfrac{1}{4(1-x)} \cdot (-1)\right] \end{cases} \qquad \begin{cases} x < 0 \\ \boxed{-\dfrac{1}{4(1-x)^{\frac{3}{2}}}} \end{cases}$$

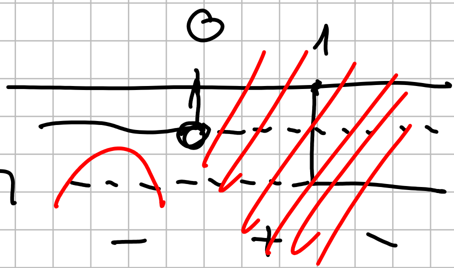
$-\dfrac{1}{4(1-x)^{3/2}} > 0$

$(1-x)^{3/2} < 0$

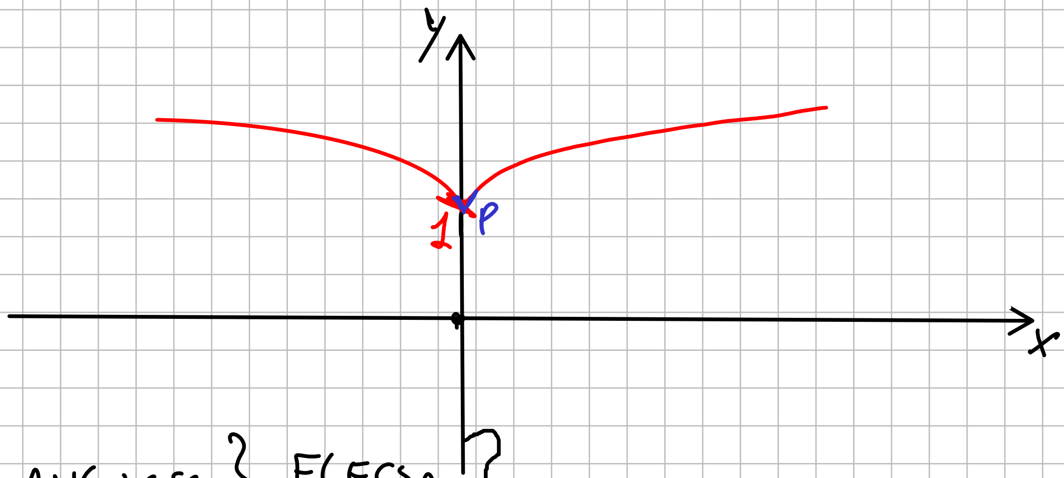
$1 - x < 0$

$-x < -1$

$x > 1$



7) GRAFICO QUALITATIVO



P(0,1) E' ANGOLOSO? FLESSO?

$$\lim_{x \to 0^-} \frac{1}{2\sqrt{x+1}} = -\frac{1}{2}$$

$$\lim_{x \to 0^+} -\frac{1}{2\sqrt{1-x}} = \frac{1}{2}$$

DIVERSI

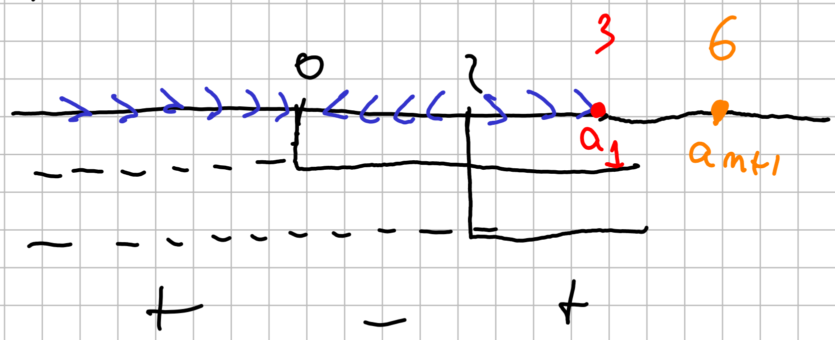
Il punto (0,1) e' un punto angoloso

Esercizio 3. Calcolare il limite della successione

$$\begin{cases} a_1 = 3 \\ a_{n+1} = a_n^2 - a_n \end{cases}$$

$f(t) = t^2 - t$ $\qquad$ $f(t) = t^2 - 2t > 0$

$t(t-2) > 0$ $\qquad$ $t > 0$ $\qquad$ $t > 2$



$]-\infty, 0[$ $\qquad$ $]0, 2[$ $\qquad$ $]2, +\infty[$

$a_n \to 0$ $\qquad$ $a_n \to 0$ $\qquad$ $a_n \to +\infty$

$a_1 \in]2, +\infty]$, $a_n \to +\infty$

LOCALIZZAZIONE

$3^2 - 3 = 6$, $a_{n+1} \in]2, +\infty[$, $a_{n+1} \to +\infty$

Esercizio 3. Determinare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \dfrac{1+a_n}{1+a_n^2} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

$$f(t) = \frac{1+t}{1+t^2}$$

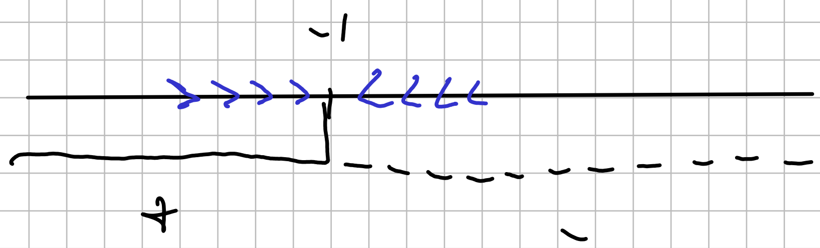
$$\varphi(t) = \frac{1+t}{1+t^2} - t$$

$$\hookrightarrow \varphi(t) = \frac{1 + \cancel{t} - \cancel{t} - t^3}{1+t^2}$$

$$\hookrightarrow \varphi(t) = \frac{-t^3+1}{1+t^2} > 0$$

N: $-t^3+1 > 0 \to -t^3 > -1$, $t^3 < -1$, $t < -1$

D: $1+t^2 > 0 \to t^2 > -1$, $\forall t$



$]-\infty, 1[$ $\quad$ $]1, +\infty[$

$a_n \to 1$ $\quad$ $a_n \to 1$

$a_{n+1} \to 1 \quad \forall n$ Visto che l'intera successione tende a 1.

Quindi il limite è uguale $\forall \lambda$.

Esercizio 3. Calcolare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = a_n|a_n| \end{cases}$$

$f(t) = t|t|$

$\varphi(t) = t|t| - t > 0$

$\Downarrow$

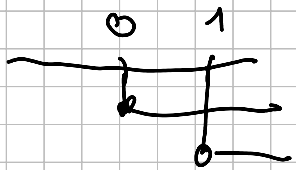
$\varphi(t) = t(|t| - 1) > 0$

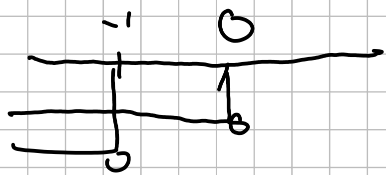
$t > 0$

$|t| - 1 > 0$

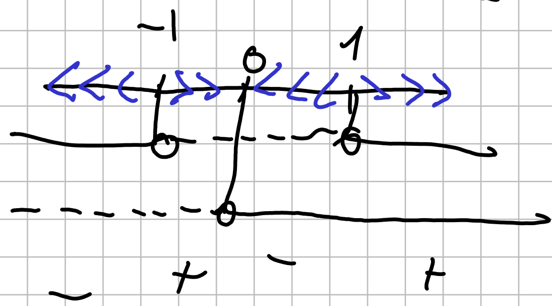
$$\begin{cases} t \geq 0 \\ t - 1 > 0 \end{cases} \qquad \begin{cases} t < 0 \\ -t - 1 > 0 \end{cases}$$

$$\begin{cases} t \geq 0 \\ t > 1 \end{cases} \qquad \begin{cases} t < 0 \\ t < -1 \end{cases}$$





$t > 1$ $\qquad$ $t < -1$



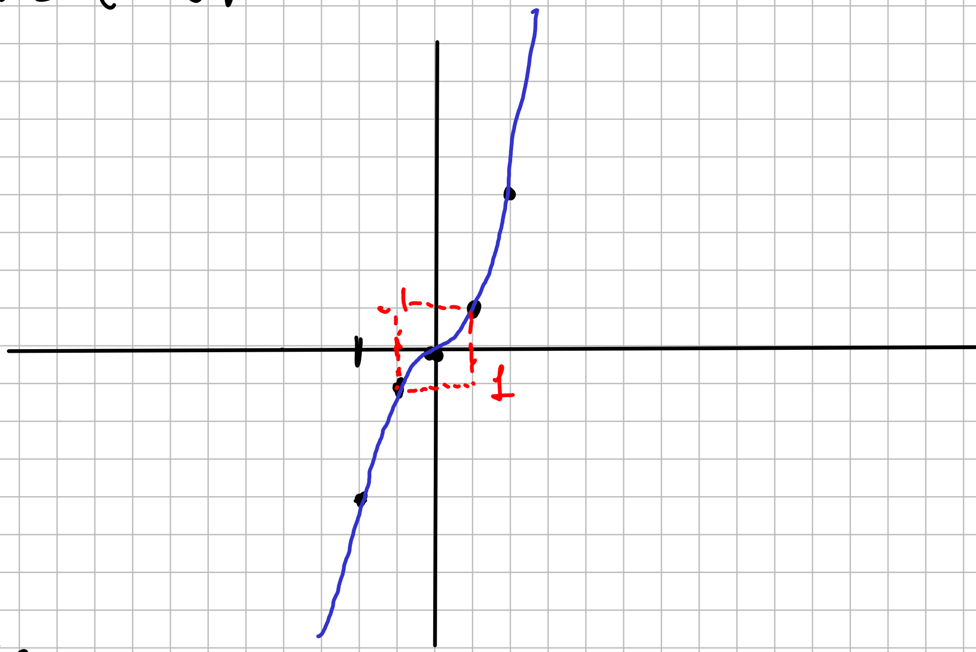
$]-\infty, -1[$ $\quad$ $]-1, 0[$ $\quad$ $]0, 1[$ $\quad$ $]1, +\infty[$

$a_n \to -\infty$ $\quad$ $a_n \to 0$ $\quad$ $a_n \to 0$ $\quad$ $a_n \to +\infty$

λ? $\quad$ λ? $\quad$ λ? $\quad$ λ?

GRAFICO di $f(t)=t|t|$

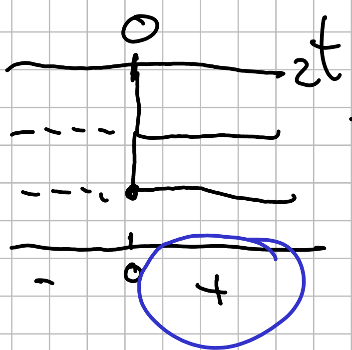
t	$f(t)$
-2	-4
-1	-1
0	0
1	1
2	4



$\begin{cases} t \geq 0 \\ t^2 \end{cases}$

$\downarrow f'(t)$

$\begin{cases} t \geq 0 \\ 2t > 0 \end{cases}$



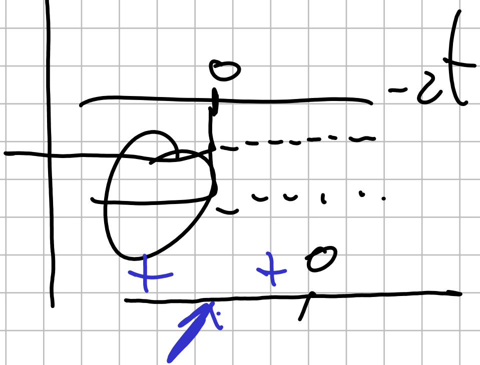
$\begin{cases} t < 0 \\ -t^2 \end{cases}$

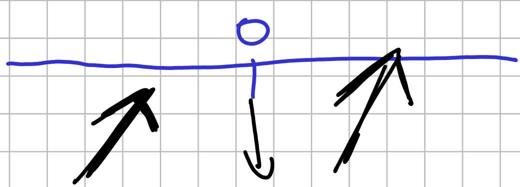
$f'(t)$

$\begin{cases} t < 0 \\ -2t > 0 \end{cases}$

$-2t > 0$

$2t < 0$





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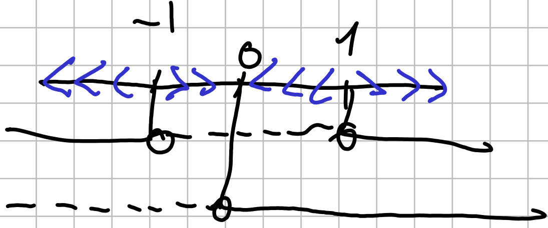
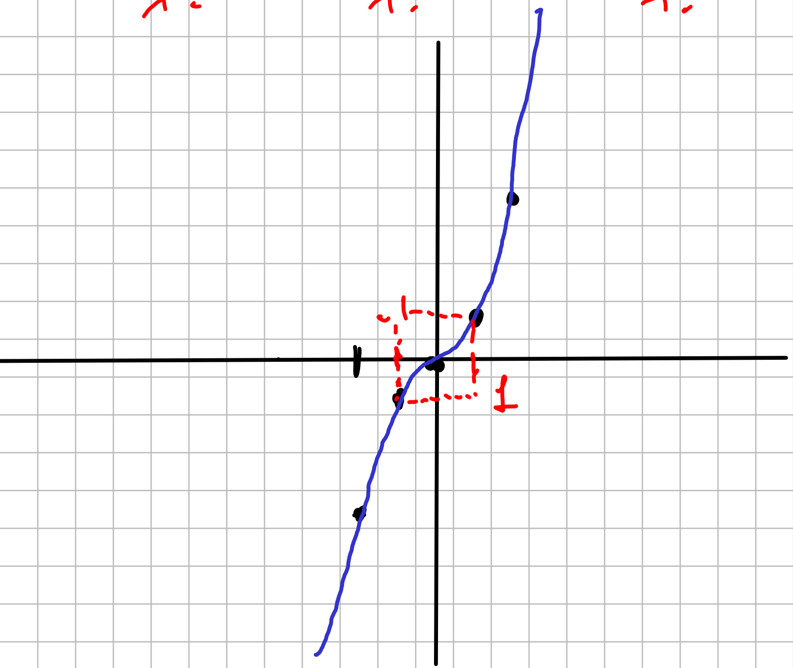


$]-\infty,-1[$ $\quad]-1,0[$ $\quad]0,1[$ $\quad]1,+\infty[$

$a_n \to -\infty$ $\quad a_n \to 0$ $\quad a_n \to 0$ $\quad a_n \to +\infty$

λ? $\quad \lambda$? $\quad \lambda$? $\quad \lambda$?



$\boxed{\text{se } t < -1 \Rightarrow f(t) < -1}$

$\boxed{\text{se } -1 < t < 0 \Rightarrow -1 < f(t) < 0}$

$\boxed{\text{se } 0 < t < 1 \Rightarrow 0 < f(t) < 1}$

$\boxed{\text{se } t > 1 \Rightarrow f(t) > 1}$

1) $\lambda < -1 \Rightarrow a_2 < -1$, $a_n < 1$, $a_n \to -\infty$

2) $-1 < \lambda < 0 \Rightarrow -1 < a_2 < 0$, $-1 < a_n < 0$, $a_n \to 0$

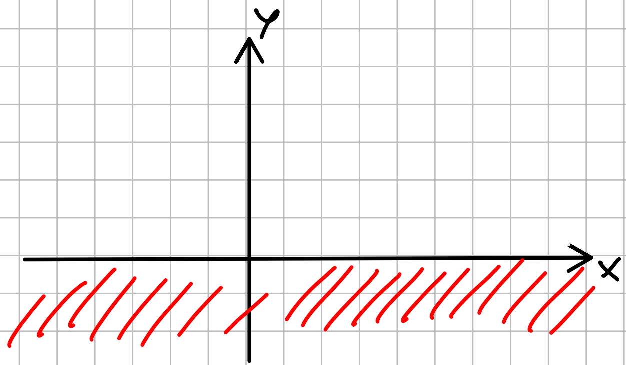
3) $0 < \lambda < 1 \Rightarrow 0 < a_2 < 1$, $0 < a_n < 1$, $a_n \to 0$

4) $\lambda > 1$, $a_2 > 1$, $a_n > 1$, $a_n \to +\infty$

$$f(x) = \sqrt{x^2+1} + \sqrt{(x-3)^2+1}$$

1) DOMINIO $]-\infty, +\infty[$ $\forall x$

2) SEGNO $\sqrt{x^2+1} + \sqrt{(x-3)^2+1} \geq 0$ VERA $\forall x$



3) SIMMETRIE $f(-x) = \sqrt{x^2+1} + \sqrt{(-x-3)^2+1} \neq f(x) \neq -f(x)$

NE' PARI NE' DISPARI

4) LIMITI $+\infty$, $-\infty$

$$\lim_{x\to+\infty} \sqrt{x^2+1} + \sqrt{(x-3)^2+1} =$$

$$\lim_{x\to+\infty} \sqrt{x^2\left(1+\frac{1}{x^2}\right)} + \sqrt{x^2+9-6x+1} =$$

$$\lim_{x\to+\infty} \sqrt{x^2\left(1+\frac{1}{x^2}\right)} + \sqrt{x^2-6x+1} =$$

$$\lim_{x\to+\infty} \sqrt{\cancel{x^2}\left(1+\underbrace{\frac{1}{x^2}}_{0}\right)} + \sqrt{\cancel{x^2}\left(1-\underbrace{\frac{6}{x}}_{0}+\underbrace{\frac{1}{x^2}}_{0}\right)}$$

$$\lim_{x\to+\infty} 2x = +\infty$$

$$\lim_{x\to-\infty} \sqrt{x^2+1} + \sqrt{(x-3)^2+1}$$

$$\lim_{x\to-\infty} \sqrt{x^2\left(1+\frac{1}{x^2}\right)} + \sqrt{x^2+9-6x+1} =$$

$$\lim_{x\to-\infty} \sqrt{x^2\left(1+\frac{1}{x^2}\right)} + \sqrt{x^2-6x+1} =$$

$$\lim_{x\to-\infty} \sqrt{\underbrace{x^2}_{+\infty}\left(1+\underbrace{\frac{1}{x^2}}_{0}\right)} + \sqrt{\underbrace{x^2}_{+\infty}\left(1-\underbrace{\frac{6}{x}}_{0}+\underbrace{\frac{1}{x^2}}_{0}\right)}$$

$$\lim_{x\to+\infty} \sim = +\infty$$

5) DERIVATA

$$f(x) = \sqrt{x^2+1} + \sqrt{(x-3)^2+1} \qquad (x-3)^2 \quad 2(x-3).$$

$$f(x) = (x^2+1)^{\frac{1}{2}} + ((x-3)^2+1)^{\frac{1}{2}}$$

$$f'(x) = \left[\frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x\right] + \left[\frac{1}{2}\left(((x-3)^2+1)^{-\frac{1}{2}}(2x-6)\right)\right]$$

$$f'(x) = \left[\frac{1}{2} \cdot \frac{1}{\sqrt{x^2+1}} \cdot 2x\right] + \left[\frac{1}{2} \cdot \frac{1}{\sqrt{(x-3)^2+1}} \cdot (2x-6)\right]$$

$$f'(x) = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+1}} + \frac{\cancel{2}x-\cancel{6}^{\,3}}{\cancel{2}\sqrt{x-3^2+1}} = \frac{x}{\sqrt{x^2+1}} + \frac{x-3}{\sqrt{(x-3)^2+1}} =$$

$$= \frac{x\overbrace{\sqrt{(x-3)^2+1}}^{+} + (x-3)\overbrace{\sqrt{x^2+1}}^{+}}{\underbrace{(\sqrt{x^2+1})}_{+}\underbrace{(\sqrt{(x-3)^2+1})}_{+}} > 0$$

$$\begin{cases} x\sqrt{(x-3)^2+1} + (x-3)\sqrt{x^2+1} > 0 \\ \sqrt{x^2+1} > 0 \end{cases} \qquad \begin{cases} x\sqrt{(x-3)^2+1} + (x-3)\sqrt{x^2+1} > 0 \\ \sqrt{(x-3)^2+1} > 0 \end{cases}$$

$$\begin{cases} x \quad (x-3)^2+1 + (x-3)(x^2+1) > 0 \\ \forall x \end{cases}$$

$$x(x^2+9-6x)+1 + x^3+x-3x^2-3 > 0$$

$$x^3+9x-6x^2+1+x^3+x-3x^2-3 > 0$$

$$2x^3-9x^2+10x-2 > 0$$

BASTA