Esercizi Geometria

1) Data una retta nello spazio, determinare i parametri direttori e scriverla come intersezione tra due piani:

a) $\frac{x-2}{2} = \frac{y+1}{0} = z-2$

$$\begin{cases} \frac{x-2}{2} = \frac{y+1}{0} \\ \frac{x-2}{2} = z-2 \end{cases} \quad \begin{cases} y+1=0 \\ x-2=2z-4 \end{cases} \quad \begin{cases} y+1=0 \\ x+2-2z=0 \end{cases} \quad \begin{cases} y+1=0 \\ x-2z+2=0 \end{cases}$$

$$\begin{cases} y+1=0 \\ x-2z+2=0 \\ t=0 \end{cases} \quad \begin{cases} y=0 \\ x-2z=0 \\ t=0 \end{cases} \quad \begin{cases} y=0 \\ x=2z \\ t=0 \end{cases} \quad \forall z$$

$$P_{\infty} = (2z, 0, z, 0)$$

$$\Downarrow$$

$$P_{\infty} = (2, 0, 1, 0)$$

$$\vec{v}_r = (2, 0, 1)$$

2) Data una retta e un piano, stabilire se la retta r è contenuta nel piano π:

a) $r: \begin{cases} 2y - z & = -1 \\ x - y & = -3 \end{cases}, \pi: x+y-z-2=0$

Tutti i punti della retta giacciono nel piano

$$\begin{cases} 2y-z=-1 \\ x-y=-3 \end{cases} \quad \begin{cases} z=2y+1 \\ x=y-3 \end{cases} \quad \forall y \qquad P_0 = (y-3, y, 2y+1)$$

$$x+y-z-2=0$$

$$(y-3)+y-2y-1-2=0$$

$$\cancel{y}-3+\cancel{y}-\cancel{2y}-1-2=0$$

$$-6=0 \quad \text{NO} \quad r \notin \pi$$

3) Dato un punto fissato $P_0 = (1, 1, -1)$, determinare la retta passante per P_0e avente parametri direttori (5,0,5).

$$\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n} \qquad \bar{v}_r = (5,0,5)$$

$$\frac{x-1}{5} = \frac{y-1}{0} = \frac{z+1}{5} \qquad \begin{cases} \frac{x-1}{5} = \frac{y-1}{0} \\ \frac{x-1}{5} = \frac{z+1}{5} \end{cases} \qquad \begin{cases} y-1=0 \\ x-1=z+1 \end{cases} \qquad \begin{cases} y-1=0 \\ x-z-2=0 \end{cases}$$

4) Date le seguente rette e i seguenti piani, trovare i parametri direttori di ciascuno:

a) $r_1 : \begin{cases} x+y & = 2 \\ x+y+z & = 2 \end{cases}; r_2 : \begin{cases} 2x-3z+1 & = 0 \\ x-y+2z-1 & = 0 \end{cases}; r_3 \begin{cases} x & = 0 \\ y & = z-3 \end{cases}$

$\bar{v}_{r_1} =$

$$\begin{cases} x+y=2 \\ x+y+z=2 \\ t=0 \end{cases} \quad \begin{cases} x+y=0 \\ x+y+z=0 \\ t=0 \end{cases} \quad \begin{cases} x=-y \\ \cancel{-y+y}+z=0 \\ t=0 \end{cases} \quad \begin{cases} x=-y \\ z=0 \\ t=0 \end{cases} \quad \forall y$$

$$P_{\infty} = (-\overset{1}{\cancel{y}}, \overset{1}{\cancel{y}}, 0, 0) \rightarrow \bar{v}_r = (-1, 1, 0)$$

b) $\pi_1 : 2x + y - 3z + 1 = 0; \pi_2 : x - z = 0; \pi_3 : x + y + 4 = 0$

$$v_{\pi_1} = (2, 1, -3) \qquad \bar{v}_{\pi_2} = (1, 0, -1) \qquad \bar{v}_{\pi_3} = (1, 1, 0)$$

5) Dato un punto P_0e una retta r, determinare la retta s passante per P_0 e parallela alla retta r e una retta t passante per P_0e ortogonale alla retta s:

a) dato $P_0 = (2, -2, 0), r : \begin{cases} x + y & = 2 \\ x - z & = 3 \end{cases}$

$$\bar{V}_r = ? \quad \begin{cases} x+y = 2 \\ x-z = 3 \\ t = 0 \end{cases} \quad \begin{cases} x+y = 0 \\ x-z = 0 \\ t = 0 \end{cases} \quad \begin{cases} x = -y \\ -y-z = 0 \\ t = 0 \end{cases} \quad \begin{cases} x = -y \\ z = -y \\ t = 0 \end{cases} \quad \forall y$$

$$P_{\infty} = (-\cancel{y}, \cancel{y}, -\cancel{y}, 0) \rightarrow (-1, 1, -1, 0)$$

$\bar{V}_r = (-1, 1, -1)$ s passa per $(2, -2, 0)$ $s \parallel \bar{V}_r$

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n} \qquad \frac{x-2}{-1} = \frac{y+2}{1} = \frac{z-0}{-1}$$

$$s: \begin{cases} \frac{x-2}{-1} = y+2 \\ y+2 = \frac{z}{-1} \end{cases} \quad \begin{cases} -x + \cancel{2} = y + \cancel{2} \\ y+2+z = 0 \end{cases} \quad \begin{cases} x+y = 0 \\ y+z+2 = 0 \end{cases} \qquad V_s = (-1, 1, -1)$$

t passa per $(2, -2, 0)$ e $t \perp s$

$$\bar{V}_t \cdot \bar{V}_s = 0 \qquad (l, m, n) \cdot (-1, 1, -1) = 0$$

$$\vec{V}_t \quad -l + m - n = 0 \qquad l = m - n \quad \forall m, n$$

$$\frac{x-2}{m-n} = \frac{y+2}{m} = \frac{z-0}{n}$$ infinite rette.

6) Dato un punto P_0e un piano π, determinare il piano $\pi_\parallel$ passante per P_0 e parallela al piano π

a) dato $P_0 = (0,2,0)$, $\pi : x+y=0$

$n_\pi = (1,1,0)$ $\quad ax+by+cz+d=0$

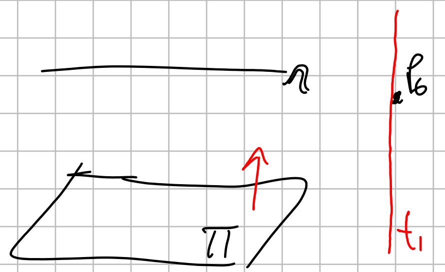
$\cancel{1(0)}+1(2)+\cancel{0\cdot(0)}+d=0$

$2+d=0 \quad d=-2$

$x+y-2=0$

7) Data una retta r un punto P_0e un piano π, determinare la retta t_1 passante per P_0, ortogonale al piano π e il piano $\pi_\perp$ passante per P_0, ortogonale alla retta r

7a) Data una retta $r : \begin{cases} x-y+z & =0 \\ x+y+z-2 & =0 \end{cases}$, $P_0 = (0,0,1)$, $\pi : x+3y-1=0$



$v_t = u_\pi = \vec{u_\pi} = (1,3,0)$

$\vec{v_t} = (1,3,0)$

$\pi_\perp$ per P_0 $\perp r$

$v_r = ?$

$$\frac{x-\overset{0}{x_0}}{1} = \frac{y-\overset{0}{y_0}}{3} = \frac{z-\overset{1}{z_0}}{0}$$

$$\begin{cases} x = \frac{y}{3} \\ \frac{y}{3} = \frac{z-1}{0} \end{cases} \quad \begin{cases} y=3x \\ z-1=0 \end{cases} \quad \boxed{\begin{cases} 3x-y=0 \\ z-1=0 \end{cases} (t)}$$

$$\begin{cases} x-y+z=0 \\ x+y+z\cancel{-2}=0 \\ t=0 \end{cases} \quad \begin{cases} y=x+z \\ x+x+z+z=0 \\ t=0 \end{cases}$$

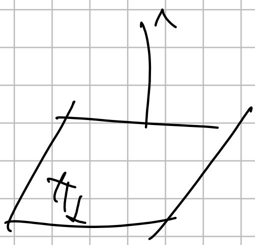
$$\begin{cases} y=x+z \\ 2x+2z=0 \\ t=0 \end{cases} \quad \begin{cases} y=0 \\ x=-z \\ t=0 \end{cases}$$

$P_\infty = (-\overset{1}{z},0,\overset{1}{z},0)\ \forall z.$

$P_\infty = (-1,0,1,0)$

$\vec{v_r} = (-1,0,1)$

π_1 passa per P_0 e $\perp r \rightarrow \bar{v}_2 = (-1,0,1)$
$(0,0,1)$ $\qquad a \; b \; c$



$ax + by + cz + d = 0$

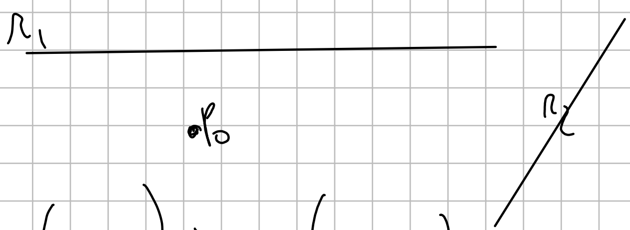
$-1(\cancel{0}) + 0(\cancel{0}) + 1(1) + d = 0$

$1 + d = 0 \qquad d = -1$

$-x + z - 1 = 0$

8) Date due rette r_1, r_2 e un punto P_0, determinare la retta t che passa per P_0 ed è ortogonale ad entrambe le rette

8a) $r_1 : \begin{cases} x + y - z & = 0 \\ 2x - y & = 1 \end{cases}, r_2 : \begin{cases} y + z & = 0 \\ x & = 0 \end{cases}, P_0 = (1, 0, -3)$



$t \perp r_1 \wedge t \perp r_2$

$\bar{v}_{r_1} = (1,2,3) \quad \bar{v}_{r_2} = (0,-1,1)$

$\begin{cases} x + y - z = 0 \\ 2x - y = 1 \\ t = 0 \end{cases} \quad \begin{cases} z = x + y \\ y = 2x \\ t = 0 \end{cases} \quad \begin{cases} z = 3x \\ y = 2x \\ t = 0 \end{cases} \quad \forall x \qquad P_{\infty r} = (\cancel{x}^{1}, 2\cancel{x}^{1}, 3\cancel{x}^{1}, 0)$

$\begin{cases} y + z = 0 \\ x = 0 \\ t = 0 \end{cases} \quad \begin{cases} y = -z \\ x = 0 \\ t = 0 \end{cases} \quad \forall z \qquad P_{\infty} = (0, -\cancel{z}^{1}, \cancel{z}^{1}, 0)$

$P_0 = (1, 0, -3)$

$\bar{v}_{r_1} = (1,2,3) \quad \bar{v}_{r_2} = (0,-1,1)$

$t : \frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$

$\begin{cases} \bar{v}_t \cdot \bar{v}_{r_1} = 0 \\ \bar{v}_t \cdot v_{r_2} = 0 \end{cases} \quad \begin{cases} (l, m, n) \cdot (1,2,3) = 0 \\ (l, m, n) \cdot (0,-1,1) = 0 \end{cases}$

$$\begin{cases} l+2m+3n=0 \\ -m+n=0 \end{cases} \quad \begin{cases} l+2m+3n=0 \\ m=n \end{cases} \quad \begin{cases} l+5n=0 \\ m=n \end{cases} \quad \begin{cases} l=-5n \\ m=n \end{cases} \ \forall n$$

$$P_0=(1,0,-3)$$

$$V_t=(-5\cancel{n},\cancel{n},\cancel{n})$$
$$V_t=(-5,1,1)$$

$$t: \frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n} \longrightarrow \frac{x-1}{-5}=\frac{y-0}{1}=\frac{z+3}{1}$$

$$\begin{cases} \frac{x-1}{-5}=\frac{y}{1} \\ \frac{y}{1}=\frac{z+3}{1} \end{cases} \quad \begin{cases} x-1=-5y \\ y-z-3=0 \end{cases} \quad \begin{cases} x+5y-1=0 \\ y-z-3=0 \end{cases} \ t$$

9) Date due rette r_1, r_2 e un punto P_0, determinare il piano $\pi_{\parallel}$ che passa per P_0 ed è parallelo ad entrambe le rette

9a) $r_1: \begin{cases} x+y-z &= 0 \\ 2x-y &= 1 \end{cases}, r_2: \begin{cases} y+z &= 0 \\ x &= 0 \end{cases}, P_0=(1,0,-3)$

$$\bar{V}_{r_1}=(1,2,3) \quad \bar{V}_{r_2}=(0,-1,1) \quad \pi \parallel r_1 \parallel r_2$$

$$\begin{cases} x+y-z=0 \\ 2x-y=1 \\ t=0 \end{cases} \quad \begin{cases} z=x+y \\ y=2x \\ t=0 \end{cases} \quad \begin{cases} z=3x \\ y=2x \\ t=0 \end{cases} \ \forall x \quad P_\infty=(\cancel{x},2\cancel{x},3\cancel{x},0)$$

$$\begin{cases} y+z=0 \\ x=0 \\ t=0 \end{cases} \quad \begin{cases} y=-z \\ x=0 \\ t=0 \end{cases} \quad P_\infty=(0,-\cancel{z},\cancel{z},0) \ \forall z$$

$$\bar{V}_{r_1}=(1,2,3) \quad \bar{V}_{r_2}=(0,-1,1) \quad \pi \parallel r_1 \parallel r_2 \quad P_0(1,0,-3)$$

$$\pi: ax+by+cz+d=0 \qquad \begin{cases} \bar{v}_\pi \cdot \bar{V}_{r_1}=0 \\ \bar{v}_\pi \cdot \bar{V}_{r_2}=0 \end{cases}$$

$$\begin{cases} (a,b,c)\cdot(1,2,3)=0 \\ (a,b,c)\cdot(0,-1,1)=0 \end{cases} \begin{cases} a+2b+3c=0 \\ -b+c=0 \end{cases} \begin{cases} a+2b+3b=0 \\ c=b \end{cases} \begin{cases} a+5b=0 \\ c=b \end{cases} \begin{cases} a=-5b \\ c=b \\ \forall b \end{cases}$$

$$v_{\pi_\parallel}=(-5\cancel{b},\cancel{b},\cancel{b})=(-5,1,1)$$

$$-5(x_0)+1(y_0)+1(z_0)+d=0 \qquad P=(1,0,-3)$$

$$-5(1)+\cancel{1(0)}+1(-3)+d=0$$

$$-5-3+d=0 \qquad d=8$$

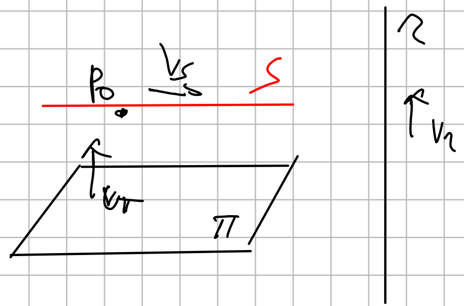
$$-5x+y+z+8=0 \qquad 5x-y-z-8=0$$

10) Data una retta, un punto e un piano, determinare la retta s parallela al piano, che passa per il punto ed è ortogonale alla retta e il piano π' parallelo alla retta, ortogonale al piano e che passa per il punto

10a) Data una retta $r : \begin{cases} 4x-y & =0 \\ y+z-2 & =0 \end{cases}$, $P_0=(0,0,0)$, $\pi : x+y-1=0$

$s // \pi \wedge s \perp r$

$\hookrightarrow P_0$



$$v_\pi=(1,1,0) \qquad v_r=(1,4,-4)$$

$$r: \begin{cases} 4x-y=0 \\ y+z=0 \\ t=0 \end{cases} \quad \begin{cases} y=4x \\ 4x+z=0 \\ t=0 \end{cases} \quad \begin{cases} y=4x \\ z=-4x \\ t=0 \end{cases} \quad P_\infty=(\cancel{x},4\cancel{x},-4\cancel{x},0)$$

$$v_r=(1,4,-4)$$

$$\begin{cases} v_s // v_\pi \\ v_s \cdot v_r=0 \end{cases} \quad \begin{cases} (l,m,n)\cdot(1,1,0)=0 \\ (l,m,n)\cdot(1,4,-4)=0 \end{cases}$$

$$\begin{cases} l+m=0 \\ l+4m-4n=0 \end{cases} \quad \begin{cases} l=-m \\ -m+4m-4n=0 \end{cases} \quad \begin{cases} l=-m \\ 3m-4n=0 \end{cases} \quad \begin{cases} l=-m \quad \forall m \\ n=\frac{3}{4}m \end{cases}$$

$$v_s=\left(-\cancel{m},\ \cancel{m},\ \frac{3\cancel{m}}{4}\right) \qquad v_s=(-4,4,3)$$

$$\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n} \qquad P_0=(0,0,0)$$

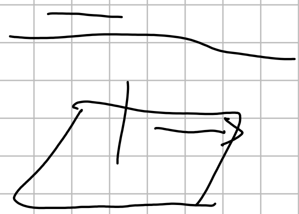
$$\frac{x}{-4}=\frac{y}{4}=\frac{z}{3} \quad \begin{cases} \frac{x}{-4}=\frac{y}{4} \\ \frac{y}{4}=\frac{z}{3} \end{cases} \quad \begin{cases} 4x+4y=0 \\ 3y-4z=0 \end{cases} \quad s$$

$\pi' \parallel r$, $\pi' \perp \pi$ } per $(0,0,0)$

a $r: \begin{cases} 4x - y & = 0 \\ y + z - 2 & = 0 \end{cases}$, $P_0 = (0,0,0)$, $\pi : x + y - 1 = 0$

$v_\pi = (1,1,0) \qquad v_r = (1,4,-4)$

$\pi' = ax + by + cz + d = 0$



$\left.\begin{cases} v_{\pi} \cdot (1,4,-4) = 0 \\ v_{\pi} \cdot (1,1,0) = 0 \end{cases}\right\} v_{\pi'}$

$\begin{cases} (a,b,c) \cdot (1,4,-4) = 0 \\ (a,b,c) \cdot (1,1,0) = 0 \end{cases} \quad \begin{cases} a + 4b - 4c = 0 \\ a + b = 0 \end{cases} \quad \begin{cases} -b + 4b - 4c = 0 \\ a = -b \end{cases} \quad \begin{cases} 3b - 4c = 0 \\ a = -b \end{cases}$

$\begin{cases} \cancel{4}c = \frac{3}{4}b \\ a = -b \end{cases} \quad \forall b$

$v = (-b, b, \frac{3}{4}b)$

$v = (-1, 1, \frac{3}{4}) \rightarrow \boxed{(-4, 4, 3) \ v_{\pi'}}$

$\boxed{P_0 = (0,0,0)}$

$\pi' = ax + by + cz + d = 0$

$-4(0) + 4(0) + 3(0) + d = 0 \quad \boxed{d = 0}$

$-4x + 4y + 3z = 0$

$4x - 4y - 3z = 0$

11) Dati tre punti, determinare l'unico piano passante per i tre punti:

11a) Dati $A = (1,1,2), B = (0,0,1), C = (2,-2,0)$

$\pi' = ax + y + cz + d = 0$

$\begin{cases} a(1) + b(1) + c(2) + d = 0 \\ \cancel{a(0)} + \cancel{b(0)} + c(1) + d = 0 \\ a(2) + b(-2) + \cancel{c(0)} + d = 0 \end{cases} \quad \begin{cases} a + b + 2c + d = 0 \\ c + d = 0 \\ 2a - 2b + d = 0 \end{cases} \quad \begin{cases} a + b + 2c - c = 0 \\ d = -c \\ 2a - 2b - c = 0 \end{cases}$

$\begin{cases} a + b + c = 0 \\ d = -c \\ \end{cases} \rightarrow \begin{cases} a = -b - c \\ d = -c \\ 2(-b-c) - 2b - c = 0 \end{cases} \quad \begin{cases} a = -b - c \\ d = -c \\ -2b - 2c - 2b - c = 0 \end{cases} \quad \begin{cases} a = -b - c \\ d = -c \\ -4b - 3c = 0 \end{cases}$

$$\begin{cases} \text{—} \\ d = -c \\ \cancel{4}b = -\frac{3c}{4} \end{cases} \qquad \begin{cases} a = \frac{3}{4}c - c \\ d = -c \\ b = -\frac{3}{4}c \end{cases} \rightarrow \quad a = \frac{3-4}{4}c \qquad \begin{cases} a = -\frac{1}{4}c \\ d = -c \\ b = -\frac{3}{4}c \end{cases}$$

$$\left(-\frac{1}{\cancel{4}}\cancel{d},\ -\frac{3}{4}d,\ 4\cancel{d},\ -4\cancel{d}\right)$$

$$(-1, -3, 4, -4) \qquad -1x - 3y + 4z - 4 = 0$$

$A = (1,1,2), B = (0,0,1), C = (2,-2,0)$

$$\begin{array}{cccc} x & +3y & -4z & +4 = 0 \\ 1 & 1 & 2 & \end{array} \qquad \begin{array}{cccc} x & +3y & -4z & +4 = 0 \\ 0 & 0 & 1 & \text{ok} \end{array} \qquad \begin{array}{cccc} x & +3y & -4z & +4 = 0 \\ 2 & -2 & 0 & \end{array}$$

$1 + 3 - 8 + 4 = 0$ ✓ $\qquad 2 - 6 + 4$ ok

12) Verificare se due rette sono sghembe (cioè determinante diverso da zero)

12a) $r_1 : \begin{cases} 4x - y - z &= 0 \\ x - y - 1 &= 0 \end{cases}, r_2 : \begin{cases} z &= 0 \\ x &= 0 \end{cases},$

12b) $r_1 : \begin{cases} x - z &= 0 \\ x - y - 1 &= 0 \end{cases}, r_2 : \begin{cases} x + z &= 0 \\ z &= 0 \end{cases}$

a)
$$\begin{array}{cccc} x & y & z & \text{noto} \end{array}$$
$$\begin{pmatrix} 4 & -1 & -1 & 0 \\ 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \qquad \det. = a_{41} \cdot A_{41} \qquad \det \neq 0$$

$$A_{41} = (-1)^5 \cdot \det \begin{pmatrix} -1 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} -1$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \det \quad a_{43} \cdot A_{43}$$

$$A_{43} = -\det \begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$-\left(-(0)\right) \det = 0$$

13) Date due rette calcolare l'angolo individuato dalle due rette:

13a) $r_1 : \begin{cases} 4x - y - z = 0 \\ x - y = 1 \end{cases}, r_2 : \begin{cases} z = 0 \\ x = 0 \end{cases},$

$\bar{V}_{r_1} \cdot \bar{V}_{r_2} = |\bar{v}_{r_1}| \cdot |v_{r_2}| \cdot \cos\alpha$

$\begin{cases} z = 0 \\ x = 0 \\ t = 0 \end{cases} \quad \forall y \qquad P_\infty = (0, y, 0, 0)$

$\bar{v}_{r_2} = (0, 1, 0)$

$\begin{cases} 4x - y - z = 0 \\ x - y = 0 \\ t = 0 \end{cases} \quad \begin{cases} 4x - x - z = 0 \\ y = x \\ t = 0 \end{cases}$

$\begin{cases} 3x - z = 0 \\ y = x \\ t = 0 \end{cases} \quad \begin{cases} z = 3x \\ y = x \\ t = 0 \end{cases} \quad \forall x$

$P_\infty = (\cancel{x}, \cancel{x}, 3\cancel{x}, 0)$

$(1 \;\; 1 \;\; 3 \;\; 0)$

$\bar{V}_{r_1} = (1, 1, 3)$

$\dfrac{\bar{V}_{r_1} \cdot \bar{V}_{r_2}}{|\bar{v}_{r_1}| \cdot |v_{r_2}|} = \cos\alpha$

$\cos\alpha = \dfrac{(0, 1, 0) \cdot (1, 1, 3)}{\sqrt{0^2 + 1^2 + 0^2}\sqrt{1^2 + 1^2 + 9}} = \dfrac{1}{\sqrt{1}\sqrt{11}} = = \dfrac{1}{\sqrt{11}} \quad \dfrac{\sqrt{11}}{11}$

$\arccos\left(\dfrac{\sqrt{11}}{11}\right) = \alpha \ldots$

14) Calcolare la distanza punto-piano:

14a) $P_0 = (2, 5, -1), \pi : x - z = 0$

$$d = \frac{|ax+by+cz+d|}{\sqrt{a^2+b^2+c^2}} =$$

$$a(2) + b(5) + c(-1) + d = 0$$

$a = 1$, $b = 0$, $c = -1$, $d = 0$

$\bar{v}_\pi = (1, 0, -1)$

$$\frac{2+0+1+0}{\sqrt{1^2+0^2+(-1)^2}} = \frac{3}{\sqrt{2}} \quad \frac{3\sqrt{2}}{2}$$

15) Dati due punti calcolare il punto medio M di A e B:

15a) $A = (2, 5, -1), B = (2.0, 0)$

$$M = \left(2, \frac{5}{2}, -\frac{1}{2}\right)$$

16) Dati due punti P_0 e il punto P,trovare il simmetrico P_0'di P rispetto ad P:

16a) $P_0 = (2, 5, -1), P = (2.0, 0)$

$$P = \left(\underbrace{\frac{2+x_{P_0'}}{2}}_{x_P}, \underbrace{\frac{5+y_{P_0}}{2}}_{y_P}, \underbrace{\frac{-1+z_{P_0}}{2}}_{z_P}\right)$$

$$2 = \frac{2+x}{2} \qquad 0 = \frac{5+y}{2} \qquad 0 = \frac{-1+z}{2}$$

$$4 = 2+x \qquad 5+y=0 \qquad z-1=0$$

$$x = 2 \qquad y = -5 \qquad z = 1$$

$$P_0' = (2, -5, 1)$$

18) Dati due vettori v_1, v_2,calcolare il prodotto scalare $v_1 \cdot v_2$, i moduli

di v_1e di v_2,e l'angolo individuato dai due vettori. Inoltre individuare un vettore parallelo a v_1e uno ortogonale a v_2

18a) $v_1 = (2, 5, -1), v_2 = i + j - k$

$V_1 \cdot V_2 = ? \quad |V_1|, |V_2|$

$(2, 5, -1) \cdot (1, 1, -1) = \quad 2 + 5 + 1 = 8$

$\sqrt{2^2 + 5^2 + (1^2)} = \sqrt{4 + 25 + 1} = \sqrt{30}$

$\sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$

$V_x = \lambda V_1 \qquad \bar{V}_x \cdot \bar{V}_2 = 0$

$(x, y, z) = \lambda (2, 5, -1) \quad \forall \lambda$

$(x, y, z) \cdot (1, 1, -1) = 0 \qquad x + y - z = 0$

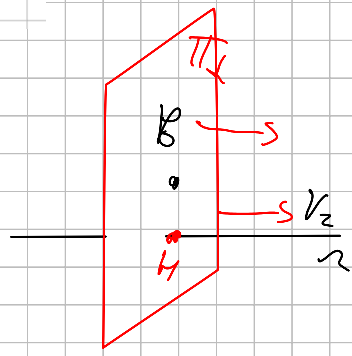
$z = x + y$

$V_x \perp V_2 = (x, y, x+y) \quad \forall x, y$

20) SOSTITUITO DA QUESTO:

$P_0=(3,6,1)$ $\quad r \begin{cases} x=2 & (\pi_1) \\ y+z-1=0 & (\pi_2) \end{cases}$ $\quad r=\pi_1\cap\pi_2$ (nello spazio)

TROVARE LA DISTANZA PUNTO-RETTA nello spazio



$$\begin{cases} x=0 \\ y+z=0 \\ t=0 \end{cases} \quad \begin{cases} x=0 \\ y=-z \\ t=0 \end{cases} \quad \forall z \qquad P_{00}=(0,-\overset{1}{\cancel{z}},\overset{1}{\cancel{z}},0)$$

$$\vec{V_2}=(\underset{a}{0},\underset{b}{-1},\underset{c}{1})$$

$$\pi_1 = ax+by+cz+d=0 \qquad P_0(\underset{x}{3},\underset{y}{6},\underset{z}{1})$$

$$\cancel{0(3)}+(-1)(6)+1+d=0$$

$$-6+1+d=0 \qquad d=5$$

$$0(x)+(-1)y+z+5=0$$

$$-y+z+5=0 \qquad H=r\cap\pi$$

$$y-z-5=0$$

$$\begin{cases} y-z-5=0 \\ x=2 \\ y+z-1=0 \end{cases} \quad \begin{cases} 1-z-z-5=0 \\ x=2 \\ y=1-z \end{cases} \quad \begin{cases} -2z-4=0 \\ x=2 \\ y=1-z \end{cases} \quad \begin{cases} \cancel{-2}z=\frac{4}{-2} \quad = z=-2 \\ x=2 \\ y=1+2=3 \end{cases}$$

$$H: (2,3,-2) \qquad d=\overline{P_0H}=\sqrt{(\underset{2}{x_H}-\underset{3}{x_{P_0}})^2+(\underset{3}{y_H}-\underset{6}{y_{P_0}})^2+(\underset{-2}{z_H}-\underset{1}{z_{P_0}})^2}$$

$$\sqrt{-1^2+(-3)^2+(-3)^2}=$$

$$\sqrt{1+9+9}=\sqrt{19}$$