CALCOLARE LA DERIVATA.

$$f(x)=\sqrt{x+2}=(x+2)^{\frac{1}{2}}=\frac{1}{2}(x+2)^{-\frac{1}{2}}=\frac{1}{2}\frac{1}{\sqrt{x+2}}=\frac{1}{2\sqrt{x+2}}$$

STUDIARE LA FUNZIONE $f(x)=x^3-4x$

- DOMINIO $\mathbb{R}$
- SIMMETRIE $f(-x)=?$

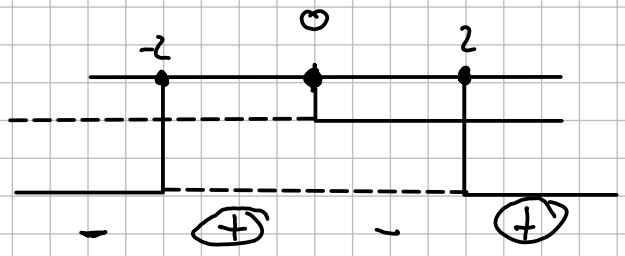
$$f(-x)=(-x)^3-4(-x)=-x^3+4x \quad (\text{DISPARI})$$

- SEGNO E INTERSEZ. ASSI

$x^3-4x\geq 0 \qquad x(x^2-4)\geq 0$

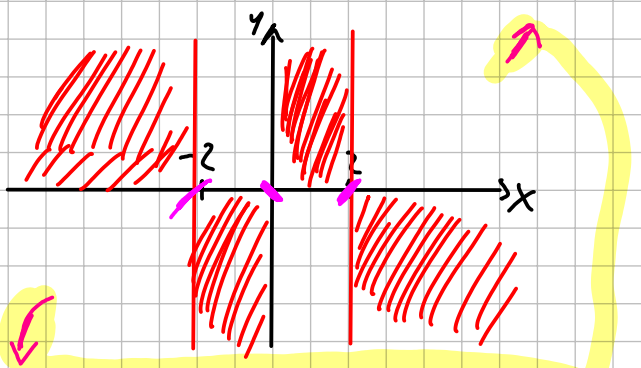
$$\begin{cases} x\geq 0 \\ x^2-4\geq 0 \rightarrow x\leq -2 \vee x\geq 2 \end{cases} \qquad x^2-4=0,\ x^2=4 \rightarrow x=\pm 2$$

$$\begin{cases} -2\leq x\leq 0 \\ x\geq 2 \end{cases}$$



INTERSEZ. ASSI. $x=-2$, $x=0$, $x=2$, $(0,0)$

BOZZA



4) LIMITI $\pm\infty$

$$\lim_{x\to+\infty} x^3-4x = x^3\left(1-\frac{4x}{x^3}\right)=+\infty$$

($x^3\to+\infty$, $\frac{4x}{x^3}\to 0$)

$$\lim_{x\to-\infty} x^3-4x = x^3\left(1-\frac{4x}{x^3}\right)=-\infty$$

($x^3\to-\infty$, $\frac{4x}{x^3}\to 0$)

5) $f'(x) > 0$

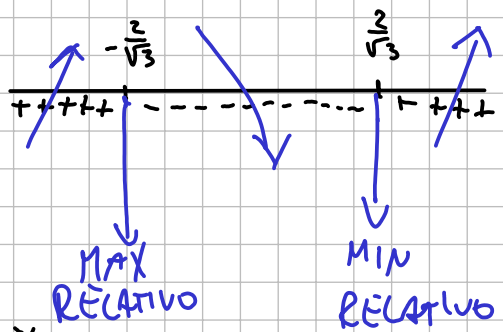
$f(x) = x^3 - 4x \longrightarrow f'(x) = 3x^2 - 4$

$3x^2 - 4 > 0$ — $3x^2 - 4 = 0$

$3x^2 = 4$

$x = \pm\sqrt{\frac{4}{3}} \quad \pm \frac{2}{\sqrt{3}}$

$x > \frac{2}{\sqrt{3}} \wedge x < -\frac{2}{\sqrt{3}}$



BOZZA

TROVO PUNTO MAX/MIN REL.

$f(x) = x^3 - 4x$

$\pm \frac{2}{\sqrt{3}}$

$f\left(\frac{2}{\sqrt{3}}\right) = \left(\frac{2}{\sqrt{3}}\right)^3 - 4\left(\frac{2}{\sqrt{3}}\right) =$

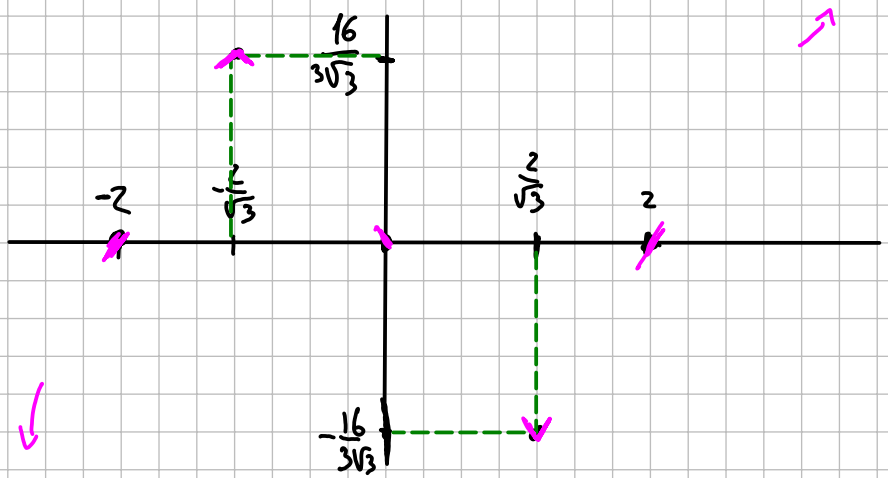
$\left(\frac{2}{\sqrt{3}}\right)^3 = \frac{8}{\sqrt{27}} = \frac{8}{\sqrt{9 \cdot 3}} = \frac{8}{3\sqrt{3}}$

$= \frac{8}{3\sqrt{3}} - \frac{8}{\sqrt{3}} = \frac{8 - 24}{3\sqrt{3}} = -\frac{16}{3\sqrt{3}}$ (4)

MAX REL. $\left(\frac{2}{\sqrt{3}}, -\frac{16}{3\sqrt{3}}\right)$

MIN REL. $\left(-\frac{2}{\sqrt{3}}, \frac{16}{3\sqrt{3}}\right)$

→ non calcolato perché la $f(x)$ è DISPARI.



CONCAVITÀ

$f(x) = x^3 - 4x \qquad f'(x) = 3x^2 - 4 \qquad f''(x) = 6x$

$6x > 0 \qquad x > 0$

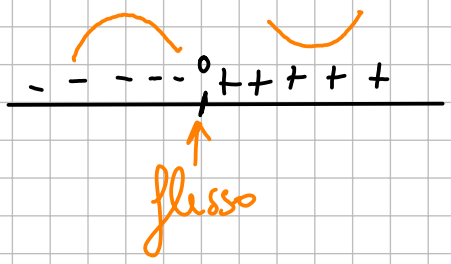
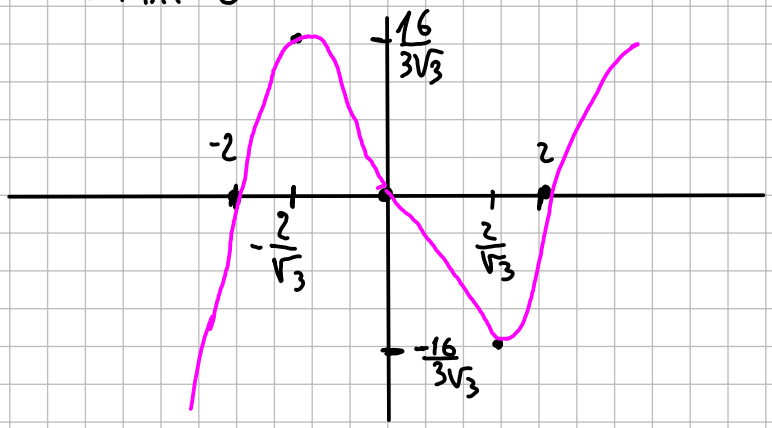


GRAFICO QUALITATIVO



STUDIARE LA FUNZIONE $f(x) = \frac{2x^3}{x^2-1}$

1) DOMINIO $x^2 - 1 \neq 0$

$$x \neq 1 \quad \wedge \quad x \neq -1$$

2) Simmetrie

$$f(-x) = \frac{2(-x)^3}{x^2-1} = -\frac{2x^3}{x^2-1} = \text{DISPARI}$$

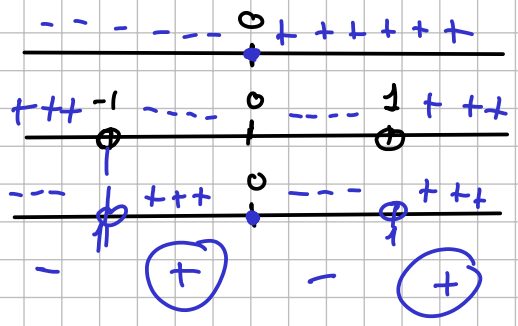
3) Segno e intercett. assi.

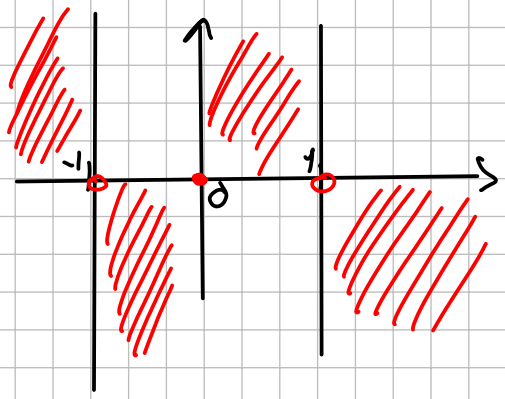
$$\frac{2x^3}{x^2-1} \geq 0$$

$2x^3 \geq 0$

$x^2 - 1 > 0$

$\frac{2x^3}{x^2-1}$





4) LIMITI

$$\lim_{x \to +\infty} \frac{2x^3}{x^2-1} = \frac{2x^3}{x^2\left(1-\frac{1}{x^2}\right)} = +\infty$$

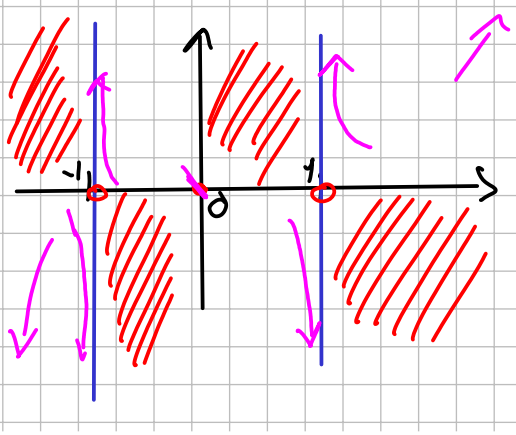
$$\lim_{x \to -\infty} \frac{2x^3}{x^2-1} = \frac{2x^3}{x^2\left(1-\frac{1}{x^2}\right)} = -\infty$$

$$\lim_{x \to 1^+} \frac{2x^3}{x^2-1} = \frac{+2}{0^+} = +\infty$$

$$\lim_{x \to 1^-} \frac{2x^3}{x^2-1} = \frac{2}{0^-} = -\infty$$

$$\lim_{x \to -1^+} \frac{2x^3}{x^2-1} = \frac{-2}{0^-} = +\infty$$

$$\lim_{x \to -1^-} \frac{2x^3}{x^2-1} \quad \frac{-2}{0^+} = -\infty$$



5) $f'(x)$ di $\frac{2x^3}{x^2-1}$

$$\frac{6x^2(x^2-1)-2x^3\cdot 2x}{(x^2-1)^2}=\frac{6x^4-6x^2-4x^4}{(x^2-1)^2}=\frac{2x^4-6x^2}{(x^2-1)^2}$$

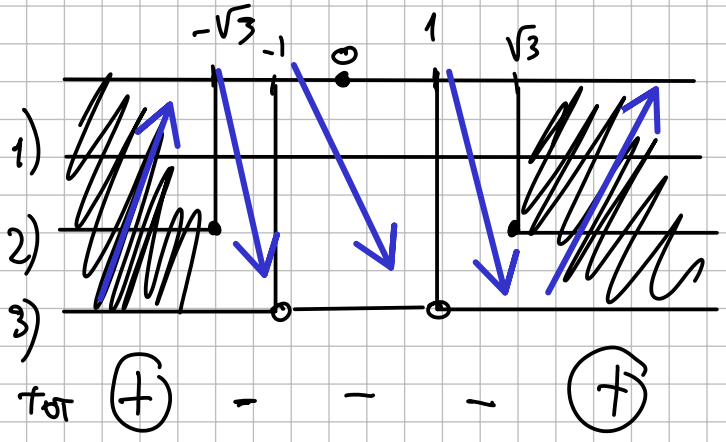
$$f'(x)=\frac{2x^2(x^2-3)}{(x^2-1)^2}>0$$

1) $2x^2>0$
2) $x^2-3>0$
3) $(x^2-1)^2>0$

$x^2-3>0$

$x^2=\pm\sqrt{3}$

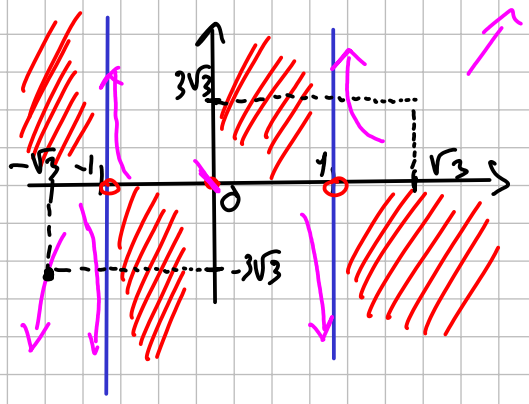
$(x^2-1)^2$

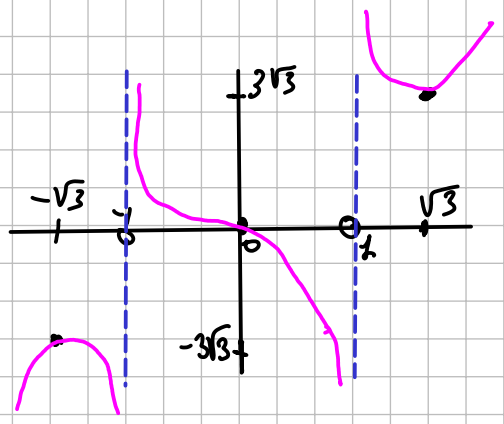


tot + − − − +

$x\le -\sqrt{3}$, $x\ge\sqrt{3}$, $-1<x<1$

MAX. REL. $-\sqrt{3}$ / MIN REL. $\sqrt{3}$





MAX. REL. $f(-\sqrt{3})$

↳ $(-\sqrt{3}, -3\sqrt{3})$

$\frac{2x^3}{x^2-1} \to \frac{2(-\sqrt{3})^3}{\sqrt{3}^2-1}=\frac{(-\sqrt{27})}{2}-3\sqrt{3}$

MIN REL $(\sqrt{3}, 3\sqrt{3})$

6) CONCAVITÀ.

$f''(x) \Rightarrow f'(x)=\frac{2x^2(x^2-3)}{(x^2-1)^2}=\frac{2x^4-6x^2}{(x^2-1)^2}$

$(x^2-1)^2=$

x^4+1-2x^2

$4x^3-4x$

$$f''(x) = \frac{(8x^3-12x)\cdot(x^2-1)^2 - (2x^4-6x^2)\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4} =$$

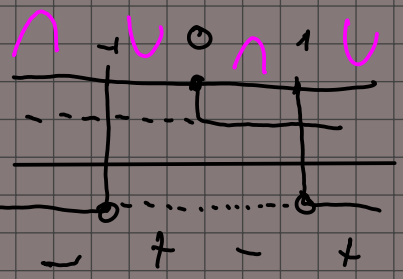
$$= \frac{(x^2-1)\cdot\left[(8x^3-12x)(x^2-1) - 2(2x^4-6x^2)(2x)\right]}{(x^2-1)^{4\cdot 3}}$$

$(-4x)$

$$= \frac{8x^5 - 8x^3 - 12x^3 + 12x - 8x^5 + 24x^3}{(x^2-1)^3} = \frac{4x^3+12x}{(x^2-1)^3} = \frac{4x(x^2+3)}{(x^2-1)^3}$$

$f''(x) > 0$

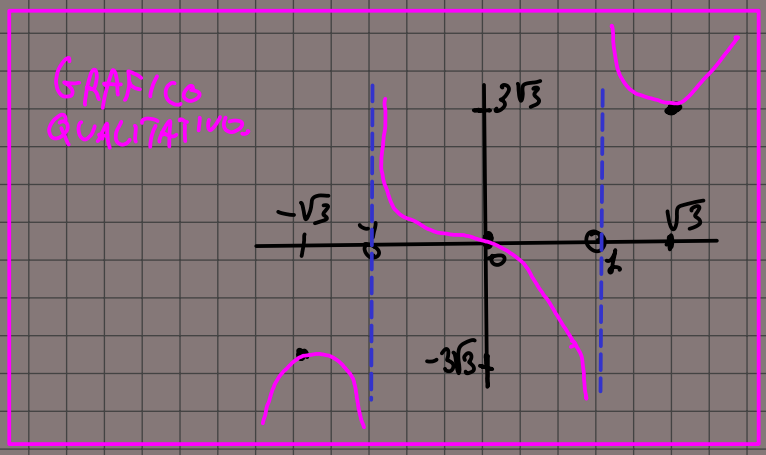
$$\begin{cases} 1)\ 4x > 0 \\ 2)\ x^2+3 > 0 \\ 3)\ (x^2-1)^3 > 0 \end{cases}$$



$(x^2-1)^3 = 0$

$x^2 - 1 = 0$

$x = \pm 1$



I) $f(x) = x^3 - 3x + 2$ STUDIARE LA FUNZIONE

1) DOMINIO $= \mathbb{R}$

2) $f(-x) = (-x)^3 - 3(-x) + 2$

$\hookrightarrow -x^3 + 3x + 2$

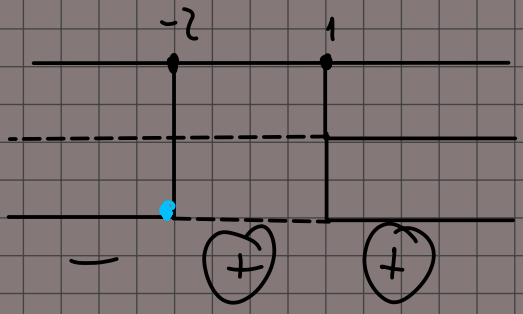
3) SEGNO $f(x) \geq 0 \rightarrow x^3 - 3x + 2 \geq 0$

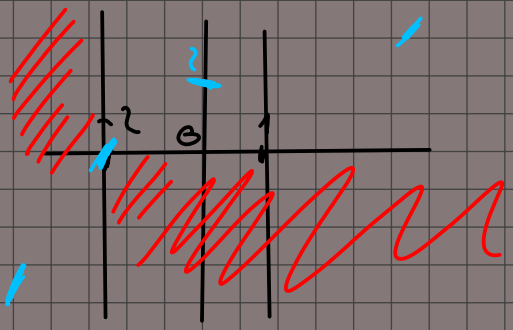
	1	0	-3	2
1		1	1	-2
	1	1	-2	0

$(x-1)(x^2+x-2) \geq 0$ — $x \geq 1$

$\frac{-1 \pm 3}{2}$ $\begin{cases} 1 \\ -2 \end{cases}$ $x \leq -2 \wedge x \geq 1$

$1 - 4(-2) = 9$





4) LIMITI

I) $f(x) = x^3 - 3x + 2$

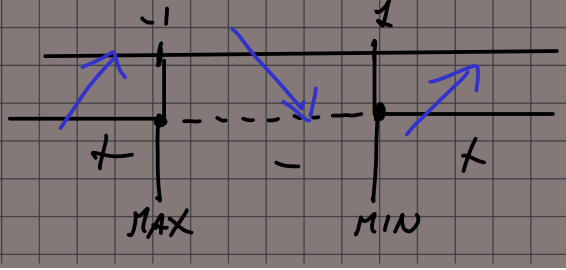
$\lim_{x \to +\infty} x^3 - 3x + 2 = x^3\left(1 - \frac{3x}{x^2} + \frac{2}{x^3}\right) = +\infty$ ($x^3 \to +\infty$, $\frac{3x}{x^2} \to 0$, $\frac{2}{x^3} \to 0$)

$\lim_{x \to -\infty} x^3 - 3x + 2 = x^3\left(1 - \frac{3x}{x^2} + \frac{2}{x^3}\right) = -\infty$ ($x^3 \to -\infty$, $\frac{3x}{x^2} \to 0$, $\frac{2}{x^3} \to 0$)

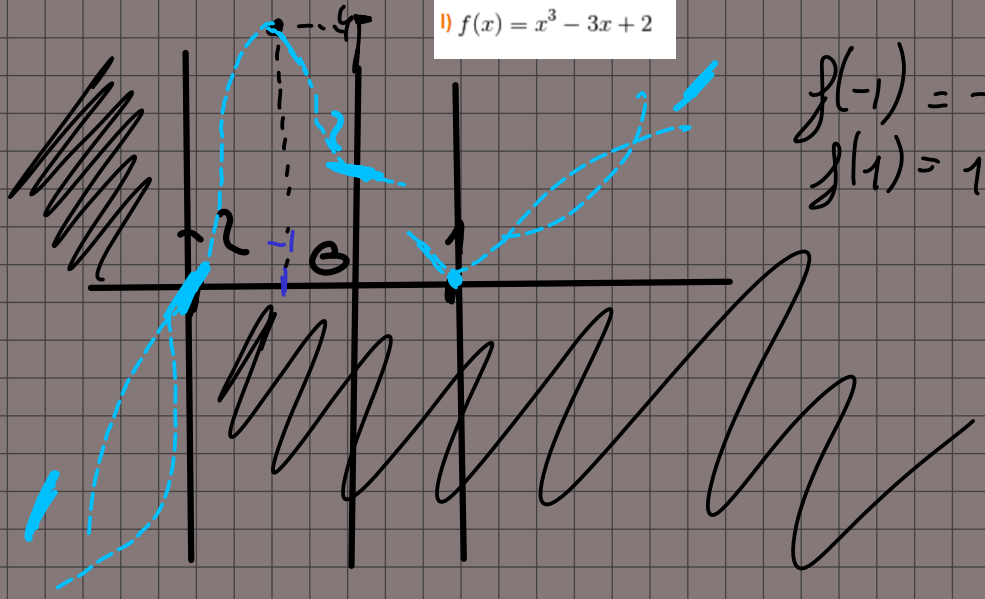
5) $f'(x) > 0$ $\quad f'(x) = 3x^2 - 3$

$3x^2 - 3 > 0$ $\quad x < -1 \wedge x > 1$

$3x^2 - 3 = 0$ $\quad 3x^2 = 3$



I) $f(x) = x^3 - 3x + 2$



$f(-1) = -1 + 3 + 2 = 4$

$f(1) = 1 - 3 + 2 = 0$

6) $f''(x) > 0$ $\quad f''(x) = 6x$ $\quad 6x > 0$

$3x^2 - 3$

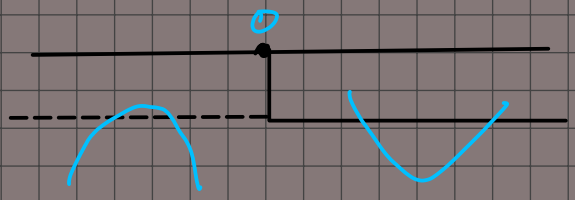
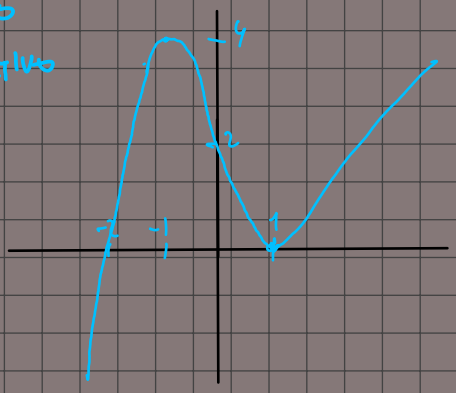


GRAFICO QUALITATIVO



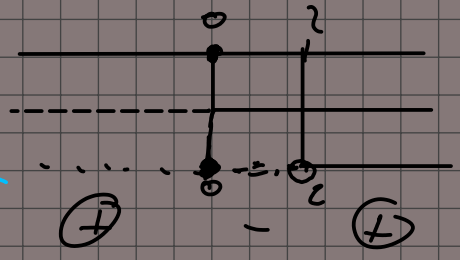
II) $f(x) = \dfrac{6x}{x-2}$

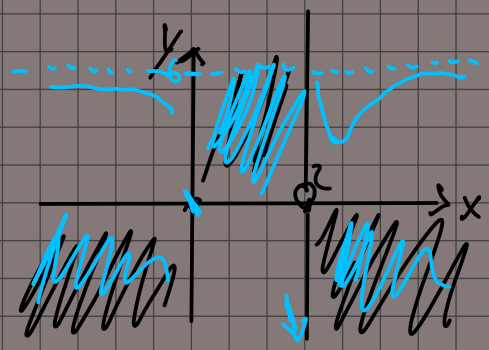
1) DOMINIO $\boxed{x \neq 2}$

2) $f(-x) \rightarrow \dfrac{6(-x)}{-x-2} = \dfrac{-6x}{-x-2} = \dfrac{6x}{x+2}$

$x-2>0$
$x>2$

3) SEGNO $\dfrac{6x}{x-2} \geq 0$





$x=2$ ASINTOTO VERT.
$y=6$ " " ORIZZ.

4) LIMITI

$\lim_{x \to +\infty} \dfrac{6x}{x-2} = \left[\dfrac{\infty}{\infty}\right] \dfrac{6x}{x\left(1-\frac{2}{x}\right)} = 6$ ($\frac{2}{x} \to 0$)

$\lim_{x \to -\infty} \dfrac{6x}{x-2} = \dfrac{6x}{x\left(1-\frac{2}{x}\right)} = 6$ ($\frac{2}{x} \to 0$)

$\lim_{x \to 2^+} \dfrac{6x}{x-2} = \dfrac{6x}{x\left(1-\frac{2}{x}\right)} = \dfrac{6}{0^+} = +\infty$

$\lim_{x \to 2^-} \dfrac{6x}{x-2} = -\infty$

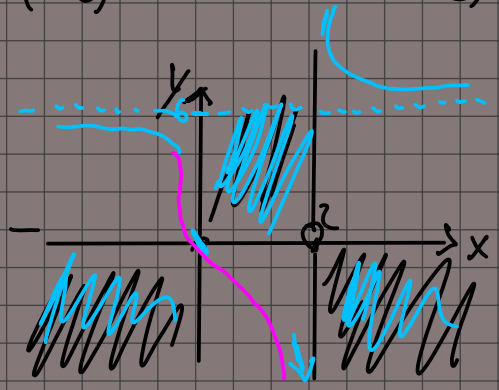
5) DERIVATE $f'(x) > 0$

$f(x) = \dfrac{6x}{x-2} \rightarrow f'(x) = \dfrac{6 \cdot (x-2) - (6x)}{(x-2)^2} = \dfrac{6x - 12 - 6x}{(x-2)^2} = \dfrac{-12}{(x-2)^2}$

(-12) ----------

$(x-2)^2$ +++ 2 +

TOT - - - 2 - - -



6) $f''(x) \rightarrow f'(x) = \frac{-12}{(x-2)^2}$

$(x-2)^2 \xrightarrow{\frac{d}{dx}} 2(x-2)\cdot 1$

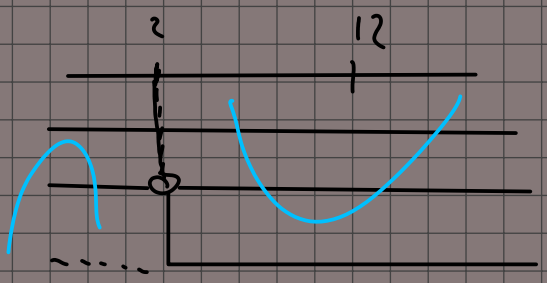
$$f''(x) = \frac{-(-12)\cdot 2(x-2)}{(x-2)^4}$$

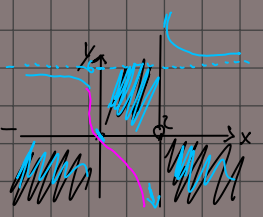
$$f''(x) = \frac{12\cdot(2x-4)}{(x-2)^4} > 0$$

$2x-4 > 0$

$2x = 4$

$x = 2$





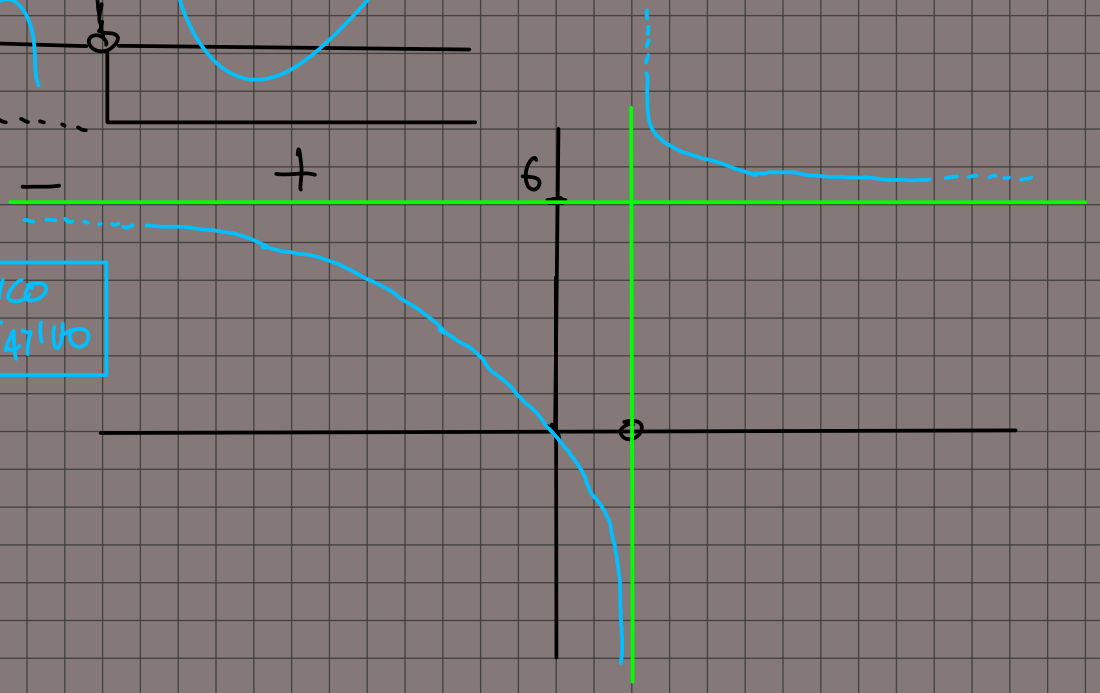


GRAFICO QUALITATIVO

$$f(x) = \frac{x^2-1}{x}$$

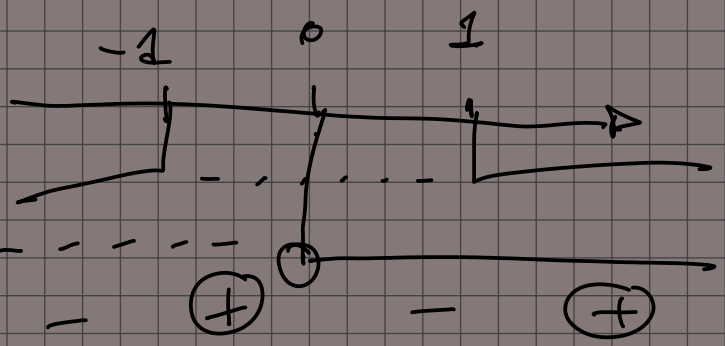
1) DOMINIO → $x \in \mathbb{R} \mid x \neq 0$

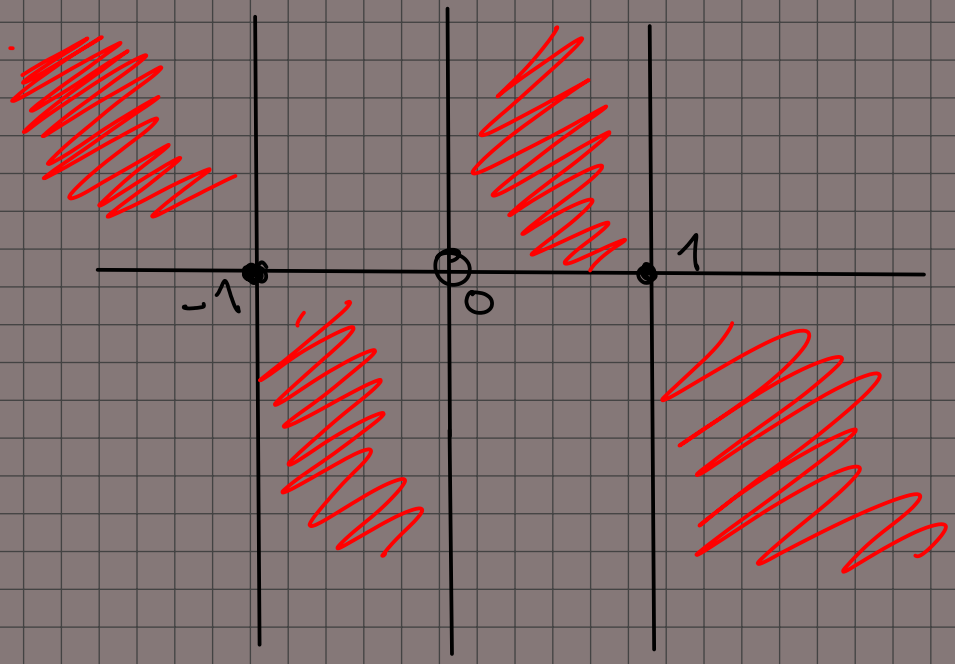
2) SIMMETRIA → $f(-x) = -\frac{x^2-1}{x}$

3) SEGNO

$$\frac{x^2-1}{x} \geq 0$$

$$\left.\begin{array}{l} x^2-1 \geq 0 \\ x > 0 \end{array}\right\} \quad \begin{array}{l} x = \pm 1 \text{ ext} \\ x > 0 \end{array}$$





0 (+/−)

4) limiti $\nearrow +\infty$

$\searrow -\infty$

$$\lim_{x \to +\infty} \frac{x^2-1}{x} = \frac{x^2\left(1-\frac{1}{x^2}\right)}{x} = x = +\infty$$

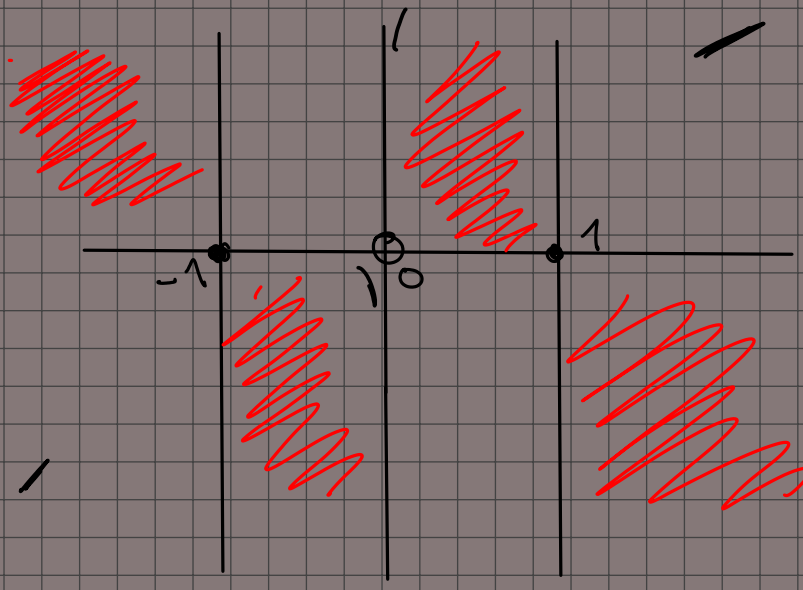
$-\infty$

$\lim_{x \to -\infty}$

$$\lim_{x \to 0^+} \frac{x^2-1}{x} = \frac{x\left(1-\frac{1}{x^2}\right)}{x} \to \infty \cdot (-\infty) = -\infty$$

$$\lim_{x \to 0^-} [\,///\,] = +\infty$$

$$\lim \begin{cases} +\infty \to +\infty \\ -\infty \to -\infty \\ 0^+ \to -\infty \\ 0^- \to +\infty \end{cases}$$

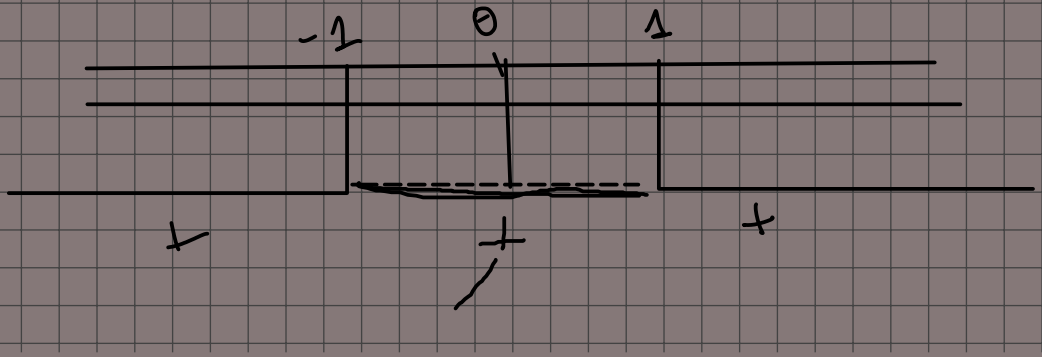


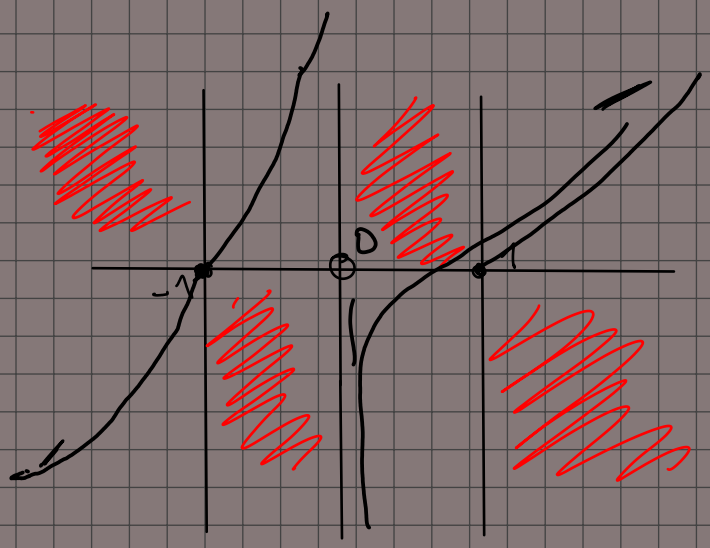
5) DERIVATA PRIMA

D. 1° → 2x
D. 2° → 1

$$f(x) \frac{x^2-1}{x} \rightsquigarrow f'(x) = \frac{2x^2 - x^2 + 1}{x^2} \rightsquigarrow f'(x) = \frac{x^2+1}{x^2}$$

- $x^2 > 0$ ✓ $\forall x$
- $x^2 + 1 > 0 \to x^2 + 1 = 0 \rightsquigarrow x = \pm 1 \rightsquigarrow$ ext.





6) DERIVATA SECONDA

$x^2+1 \rightarrow 2x$

$x^2 \rightarrow 2x$

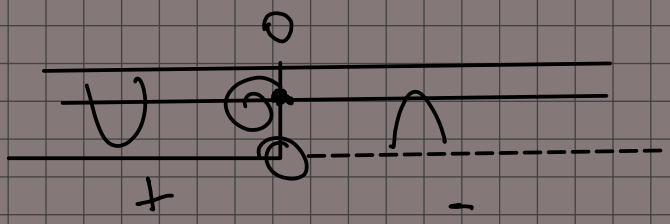
$$f'(x) = \frac{x^2+1}{x^2} = \frac{2x(x^2) - 2x(x^2+1)}{x^4}$$

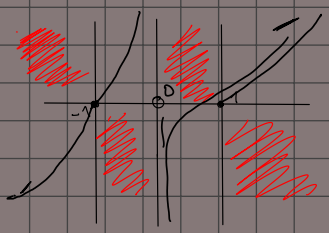
$$2x^3 - 2x^3 - 2x = f''(x)$$

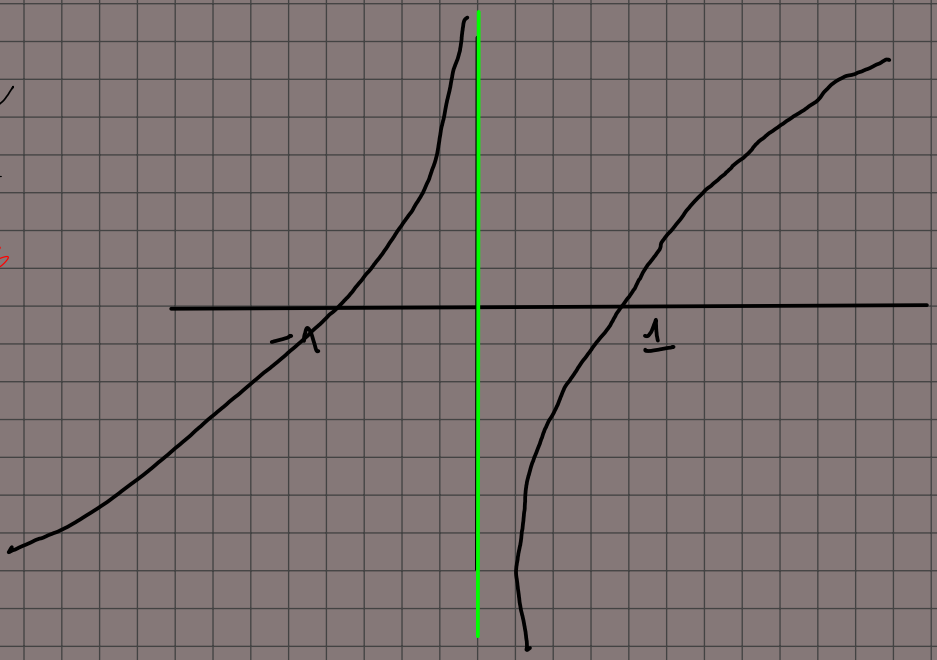
$x^4 > 0 \quad \forall x \mid x \neq 0$

$-2x > 0$

$x < 0$







VI) $f(x) = \frac{1}{\sqrt{x+3}}$

1) Dominio: $x + 3 > 0 \rightarrow x > -3$

$x \neq -3$

2) SIMMETRIA:

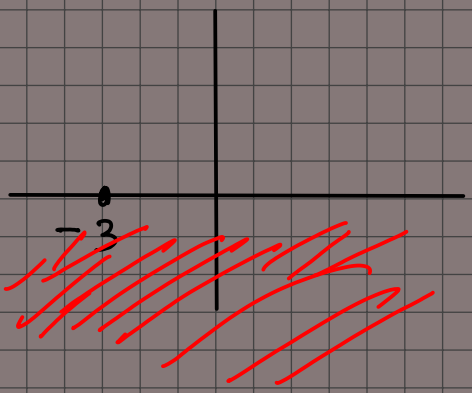
$f(-x) = \frac{1}{\sqrt{-x+3}}$ NB

3) $\frac{1}{\sqrt{x+3}} \geq 0$

$1 > 0$ SEMPRE

$\sqrt{\ } > 0$ SEMPRE

$\rightarrow$ MAI sotto X.



4) Limiti

$+\infty$

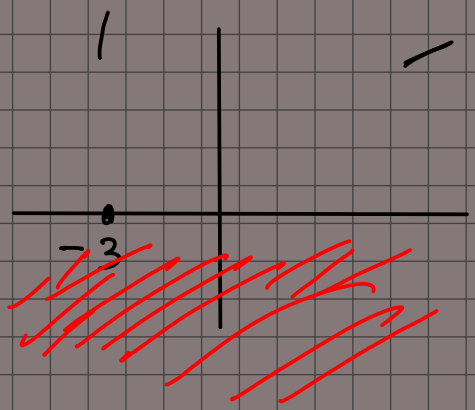
$-\infty$

-3^+

$\lim_{x \to \infty} \frac{1}{\sqrt{x+3}} = \frac{1}{\sqrt{x\left(1+\frac{3}{x}\right)}} = \frac{1}{\sqrt{\infty}} = \frac{1}{\infty} = 0.$

$\lim_{x \to -\infty}$ ∄

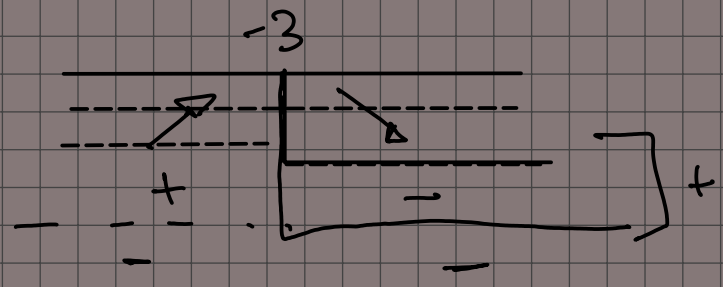
$\lim_{x \to -3^+} \frac{1}{\sqrt{x+3}} = \frac{1}{\sqrt{0}} = \frac{1}{0} = +\infty$



5) DERIVATA PRIMA

$$f(x) = \frac{1}{\sqrt{x+3}} = \frac{0 - \frac{1}{2}(x+3)\cdot 1}{(x+3)}$$

$$-\frac{1}{2}(x+3) > 0$$



$$-\frac{1}{2} > 0 \quad \nexists$$

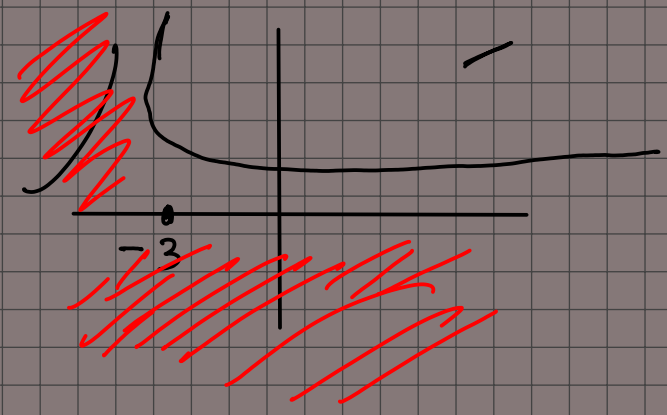
$$= \begin{cases} x+3 > 0 \rightarrow x > -3 \\ x+3 \geq 0 \rightarrow x \geq -3 \end{cases}$$

$$(x+3)^{\frac{1}{2}-1}$$

$$\frac{1}{2}(x+3)^{-\frac{1}{2}} \cdot 1$$

$$\frac{\frac{1}{2}\cdot\frac{1}{x+3^{\frac{1}{2}}}}{x+3} \qquad \frac{1}{2(x+3)^{\frac{1}{2}}}\cdot\frac{1}{x+3}$$

$$\frac{1}{2(x+3)^{\frac{3}{2}}} = -\frac{1}{2\sqrt{(x+3)^3}}$$



6) DERIVATA SECONDA

$$\frac{-\frac{1}{2}(x+3)}{x+3}$$

$$f(x) = -\frac{1}{2}(x+3) \rightarrow f'(x) = 0(x+3) + \left(-\frac{1}{2}\right)\cdot 1 = -\frac{1}{2} \qquad \frac{3}{2} - 1 \; \frac{3-2}{2} = \frac{-1}{2}$$

$\frac{1}{2(x+3)^{\frac{3}{2}}} = f''(x) = \frac{0 \cdot (\dots) - 1 \cdot 3(x+3)^{-\frac{1}{2}}}{2(x+3)^3} = \quad \frac{1}{2}+3 = \frac{1+6}{2}$

$2(x+3)^{\frac{3}{2}} \rightsquigarrow 3(x+3)^{-\frac{1}{2}} \quad = -1 \cdot \frac{3}{(x+3)^{\frac{1}{2}}} \cdot \frac{1}{2(x+3)^3} = -\frac{3}{2(x+3)^{\frac{7}{2}}}$

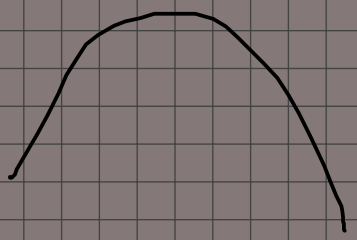
$= -\frac{3}{2(x+3)^{\frac{7}{2}}} \rightarrow \ominus \quad \frac{3}{2\sqrt{x+3}^7}$ $\cup \rightarrow \cap$

$(x+3)^{(4)}(x+3)^3 = (x+3)^7$

$\hookrightarrow 2(x+3)^2\sqrt{(x+3)^3}$

$-\frac{3}{4(x+3)^{5/2}}$ $\quad -\frac{3}{4}$

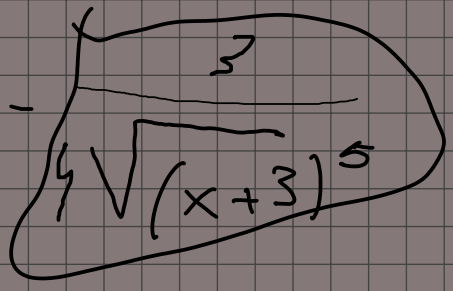
$-\frac{3}{4 \cdot (x+3)^2\sqrt{x+3}}$ $\quad \sqrt{(x+3)^5}$

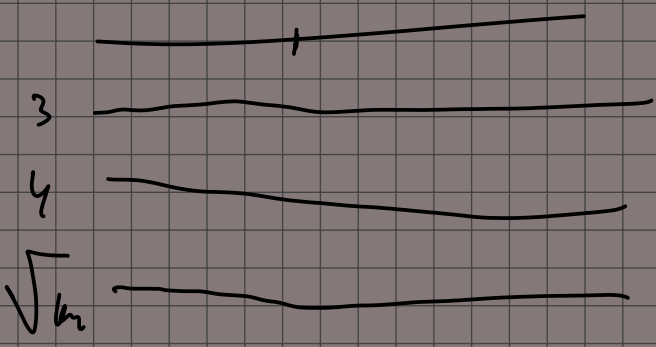


$\sqrt{(x+3)^5} = \sqrt{(x+3)^4 \cdot (x+3)} =$

$\sqrt[2]{(x+3)^{(4)}} \cdot \sqrt{(x+3)} = (x+3)^2\sqrt{(x+3)}$

$-\frac{3}{4\sqrt{(x+3)^5}}$





$D(N) : -\frac{1}{2}$

$D(D) : 1$

$$\frac{0 \cdot (x+3) - \left(-\frac{1}{2}(x+3) \cdot 1\right)}{(x+3)^2} = -\left(-\frac{1}{2} \cdot -\frac{3}{2}\right)$$

$$\frac{\frac{1}{2} \cdot + \frac{3}{2}}{(x+3)^2} = f''(x)$$

$$\frac{-\frac{1}{2}(x+3) - 1\left(-\frac{1}{2}(x+3)\right)}{(x+3)^2}$$

$$\frac{-\frac{1}{2} + \frac{1}{2}(x+3)}{x+3} = 0$$

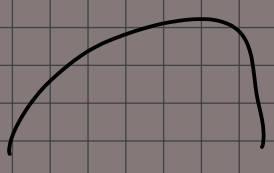
$$f(x) = \frac{1}{\sqrt{x+3}}$$

$$-\frac{1}{2}(x+3) \rightsquigarrow -\frac{1}{2}$$

$$f'(x) = \frac{-\frac{1}{2}(x+3)}{x+3}$$

$$f''(x) = \frac{-\frac{1}{2}(x+3) - \left(-\frac{1}{2}(x+3)\right)}{(x+3)^2} = \frac{-\frac{1}{2}(x+3) + \frac{1}{2}(x+3)}{(x+3)^2}$$

$$= \frac{(x+3)\overbrace{\left(-\frac{1}{2} + \frac{1}{2}\right)}^{0}}{(x+3)^2} = 0$$



$f(x) = |x^3 - x|$

1) $D = \mathbb{R}$

$$\begin{cases} x^3 - x \geq 0 \\ x^3 - x \end{cases}$$

$$\begin{cases} x(x^2-1) \geq 0 \\ x^3 - x \end{cases}$$

$$\begin{cases} -1 \leq x \leq 0 \ \wedge \ x \geq 1 \\ x^3 - x \end{cases}$$

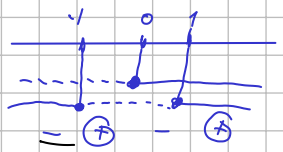
$$\begin{cases} x^3 - x < 0 \\ -x^3 + x \end{cases}$$

$$\begin{cases} x(x^2-1) < 0 \\ -x^3 + x \end{cases}$$

$$\begin{cases} x < -1 \quad \vee \quad 0 < x < 1 \\ -x^3 + x \end{cases}$$

$x \geq 0 \ \vee \ x^2 - 1 \geq 0$

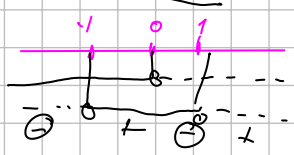
$x \geq 0 \ \vee \ (x \leq -1 \ \wedge \ x \geq 1)$



$-1 \leq x \leq 0 \ \wedge \ x \geq 1$

$x < 0 \ \vee \ x^2 - 1 < 0$

$x < 0 \ \vee \ (-1 < x < 1)$



$x < -1 \quad 0 < x < 1$

$$\begin{cases} x^3 - x & \text{SE} \quad -1 \leq x \leq 0 \ \wedge \ x \geq 1 \\ -x^3 + x & \text{SE} \quad x < -1 \quad \vee \quad 0 < x < 1 \end{cases}$$

2) DISPARI

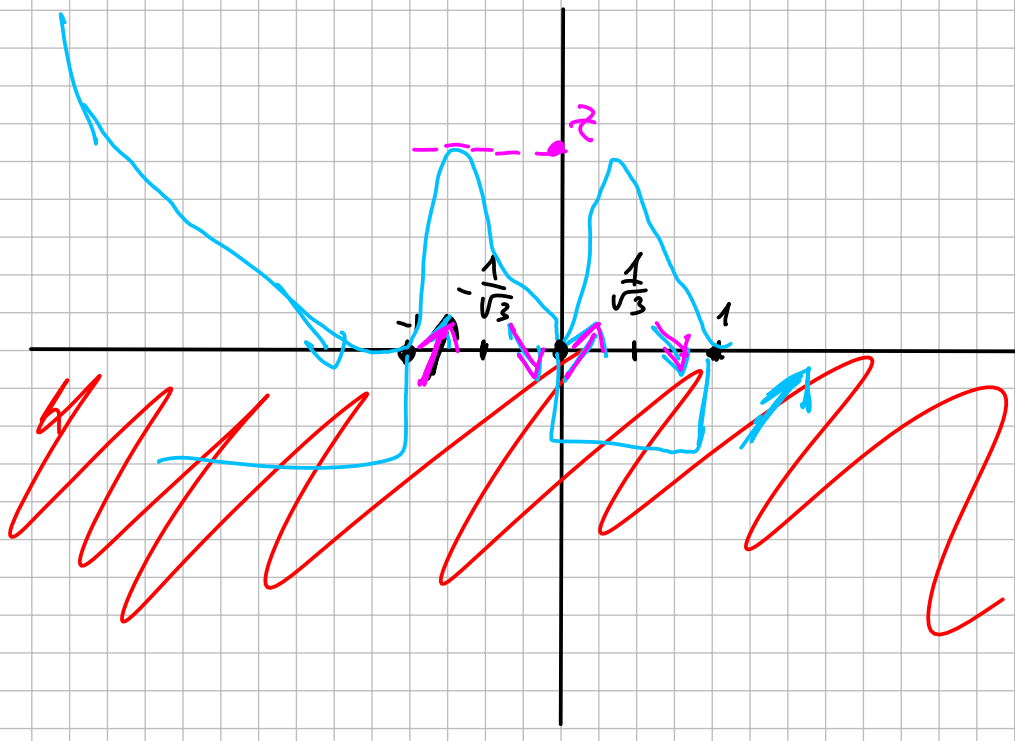
3) $f(x) \geq 0$

4) LIMITI

$$\lim_{x \to +\infty} x^3 - x = +\infty$$

$$\lim_{x \to -\infty} -x^3 + x = +\infty$$

5) DERIVATA.



$$\begin{cases} x^3 - x & \text{se } -1 \le x \le 0 \ \wedge \ x \ge 1 \\ -x^3 + x & \text{se } x < -1 \ \vee \ 0 < x < 1 \end{cases}$$

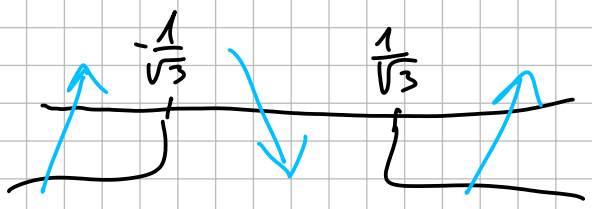
$$\begin{cases} 3x^2 - 1 & \text{se } -1 \le x \le 0 \ \wedge \ x \ge 1 \\ -3x^2 + 1 & \text{se } x < -1 \ \vee \ 0 < x < 1 \end{cases}$$

$$\begin{cases} 3x^2 - 1 > 0 \\ -3x^2 + 1 > 0 \end{cases}$$

$3x^2 - 1 = 0$

$3x^2 = 1$

$x = \pm \frac{1}{\sqrt{3}}$



$-3x^2 + 1 = 0$

$-3x^2 = -1$

$3x^2 = 1$

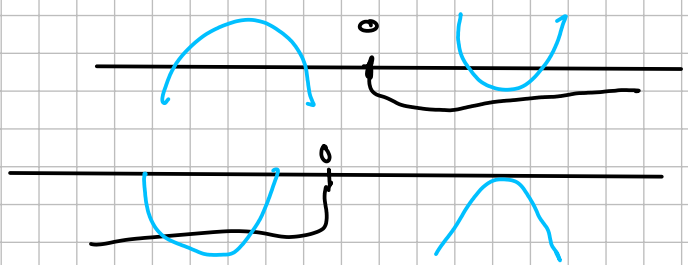
6) DER'

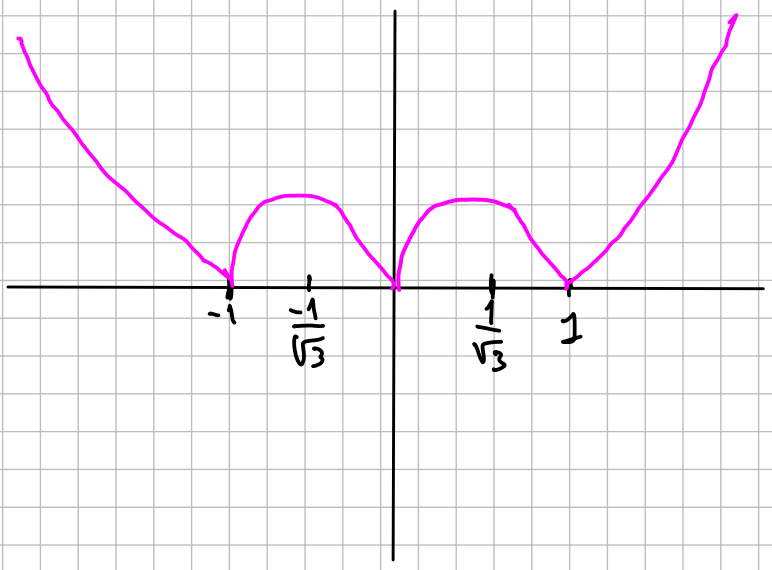
$$\begin{cases} 3x^2 - 1 & \text{se } -1 \le x \le 0 \ \wedge \ x \ge 1 \\ -3x^2 + 1 & \text{se } x < -1 \ \vee \ 0 < x < 1 \end{cases}$$

$$\begin{cases} 6x & \text{se } -1 \le x \le 0 \ \wedge \ x \ge 1 \\ -6x & \text{se } x < -1 \ \vee \ 0 < x < 1 \end{cases}$$

$6x > 0$

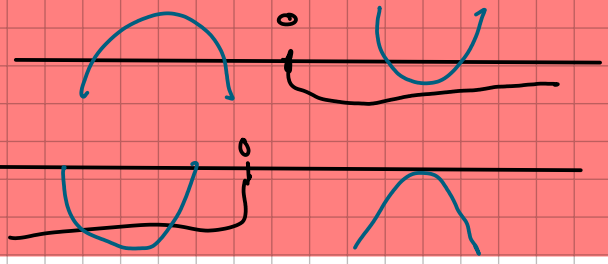
$-6x > 0 \quad x < 0$





$-1 \leq x \leq 0 \ \wedge \ x \geq 1$

$x < -1 \ \text{or} \ 0 < x < 1$



Studiare e tracciare il grafico della funzione $f(x) = \dfrac{x}{\sqrt{x-2}}$

1) DOMINIO

$x > 2$

$\begin{cases} x-2 \geq 0 \\ \sqrt{x-2} \neq 0 \end{cases} \quad \begin{cases} x-2 > 0 \\ x \neq 2 \end{cases} \quad \begin{cases} x > 2 \\ x \neq 2 \end{cases}$

2) Simmetria

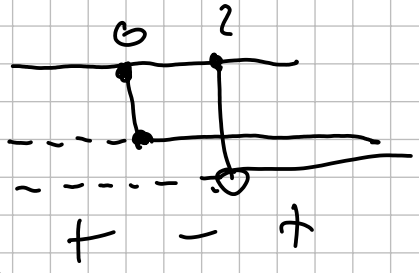
$f(-x) = \dfrac{-x}{\sqrt{-x-2}} \neq -f(x) \vee f(x)$ (NE PARI, NE DISPARI)

3) SEGNO.

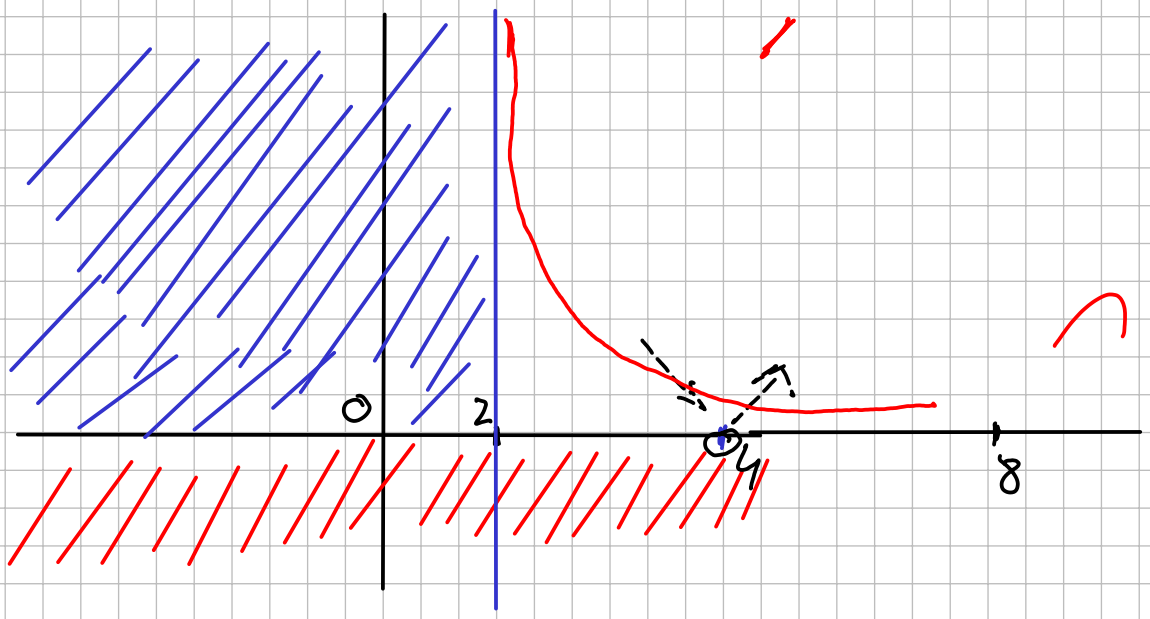
$\dfrac{x}{\sqrt{x-2}} \geq 0$

$\dfrac{x}{\sqrt{x-2}} = 0$

$\begin{cases} x \geq 0 \\ x > 2 \end{cases}$



$\begin{cases} x \leq 0 \\ x > 2 \end{cases} \Rightarrow D = (x > 2)$



4) LIMITI

$\lim_{x \to 2^+} \dfrac{x}{\sqrt{x-2}} = \dfrac{2}{0^+} = +\infty$

$\lim_{x \to +\infty} \dfrac{x}{\sqrt{x-2}} = \left[\dfrac{\infty}{\infty}\right] = +\infty$

$\dfrac{x}{\sqrt{x-2}} \cdot \dfrac{\sqrt{x-2}}{\sqrt{x-2}} = \dfrac{x\sqrt{x-2}}{x-2} =$

$= \dfrac{x(x-2)}{x(1-\frac{2}{x})} \to \infty$

5) DERIVATA 1

$\frac{x}{\sqrt{x-2}}$

$(x-2)^{\frac{1}{2}}$ $\quad \frac{1}{2}(x-2)^{-\frac{1}{2}} \cdot 1$

$\frac{1}{2} \cdot \frac{1}{\sqrt{x-2}} = \frac{1}{2\sqrt{x-2}}$

$$f'(x) = \frac{1 \cdot (\sqrt{x-2}) - x \cdot \left(\frac{1}{2\sqrt{x-2}}\right)}{x-2} = \frac{\left(\sqrt{x-2} - x \cdot \frac{1}{2\sqrt{x-2}}\right)}{x-2}$$

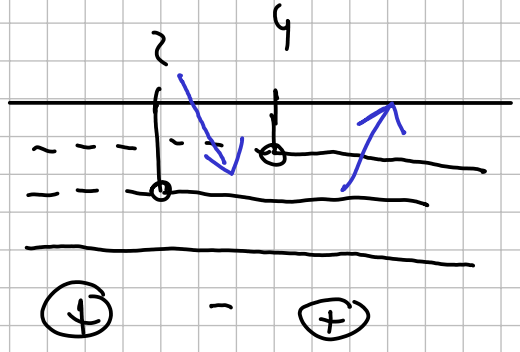
$$= \frac{\left(\sqrt{x-2} - \frac{x}{2\sqrt{x-2}}\right)}{x-2} = \frac{\frac{2(x-2)-x}{2\sqrt{x-2}}}{x-2} =$$

$$\frac{2(x-2)-x}{2\sqrt{x-2}} \cdot \frac{1}{x-2} = \frac{2x-4-x}{2\sqrt{x-2}} \cdot \frac{1}{x-2} = \frac{x-4}{2\sqrt{x-2}} \cdot \frac{1}{x-2} =$$

$$\frac{x-4}{2(x-2)^{\frac{1}{2}} \cdot (x-2)} = \boxed{\frac{x-4}{2(x-2)^{\frac{3}{2}}}} \rightarrow f'(x)$$

$$\frac{x-4}{2(x-2)^{\frac{3}{2}}} > 0$$

$$\begin{cases} x-4>0 \\ 2>0 \\ (x-2)^{\frac{3}{2}} > 0 \end{cases} \quad \begin{cases} x>4 \\ \\ \left(\sqrt{x-2}\right)^3 > 0 \end{cases} \quad \begin{cases} x>4 \\ \\ \sqrt{x-2} > 0 \end{cases} \quad \begin{cases} \\ \\ x>2 \end{cases}$$



$$\begin{cases} x \geq 2 \\ x > 4 \end{cases} \rightarrow D = (x>2)$$

6) DER. 2 $\quad f'(x) \to$

$$\frac{x-4}{2(x-2)^{\frac{3}{2}}}$$

$(x-4) = (1)$

$\frac{3}{2} - 1 = \frac{3-2}{2} = +\frac{1}{2}$

$$f''(x) = \frac{1 \cdot 2(x-2)^{\frac{3}{2}} - (x-4)\cdot 3(x-2)^{\frac{1}{2}}}{\left(2(x-2)^{\frac{3}{2}}\right)^2} =$$

$$2(x-2)^{\frac{3}{2}} \xrightarrow{D} 0 \cdot (x-2)^{\frac{3}{2}} + 2\cdot\left(\frac{3}{2}(x-2)^{\frac{1}{2}}\cdot 1\right)$$

$$= 2\left(\frac{3}{2}(x-2)^{\frac{1}{2}}\right) = 3\sqrt{x-2}$$

$$\frac{2(x-2)^{\frac{3}{2}} - (x-4)\cdot 3(x-2)^{\frac{1}{2}}}{4(x-2)^3} = \frac{(x-2)^{\frac{1}{2}}\left[2(x-2) - 3x + 12\right]}{4(x-2)^{3-\frac{1}{2}}} =$$

$$\frac{1}{2} - 3 = \frac{1-6}{2} = -\frac{5}{2}$$

$$= \frac{2x-4-3x+12}{4(x-2)^{-5/2}} = \frac{8-x}{4(x-2)^{-5/2}}$$

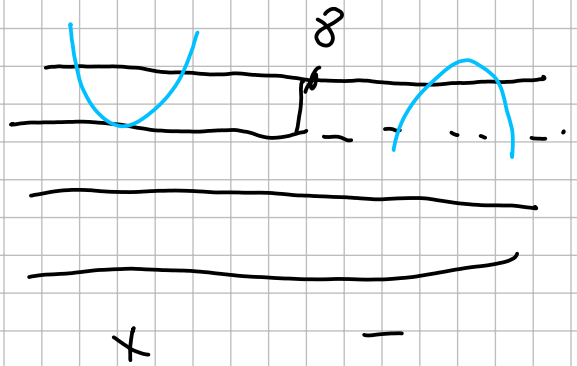
$f''(x) > 0 \quad \begin{cases} 8-x>0 \\ 4(x-2)^{-5/2} > 0 \end{cases}$

$x < 8 \quad \checkmark$

$4 > 0 \quad \checkmark$

$(x-2)^{-\frac{5}{2}} > 0$

$\frac{1}{(x-2)^{\frac{5}{2}}} > 0 \Rightarrow 1 > 0 \quad \checkmark$



Esercizio 2. Studiare la funzione definita dalla legge

5 settembre 2022

$$-2x + \sqrt{|x-1|}$$

e tracciarne un grafico qualitativo.

1) $D = \mathbb{R}$

① $\begin{cases} x-1 \geq 0 \\ -2x+\sqrt{x-1} \end{cases}$ $\begin{cases} x \geq 1 \\ -2x+\sqrt{x-1} \end{cases}$

② $\begin{cases} x-1 < 0 \\ -2x+\sqrt{-x+1} \end{cases}$ $\begin{cases} x < 1 \\ -2x+\sqrt{-x+1} \end{cases}$

FORMA A TRATTI

$$\begin{cases} -2x+\sqrt{x-1} & \text{se } x \geq 1 \\ -2x+\sqrt{-x+1} & \text{se } x < 1 \end{cases}$$

2) SIMMETRIA $-2x + \sqrt{|x-1|}$

$f(-x) = -2(-x)+\sqrt{|-x-1|} = 2x+\sqrt{|-x-1|}$ NO PARI/DISPARI

3) SEGNO $f(x) \geq 0$

$-2x+\sqrt{|x-1|} \geq 0$

$\left(-2x+\sqrt{|x-1|}\right)^2 = 0$

$= 4x^2 + |x-1| - 2(-2x)\left(\sqrt{|x-1|}\right) = 0$

$-2x+\sqrt{|x-1|} \geq 0$

$\sqrt{|x-1|} \geq 2x$

$|x-1| \geq 4x^2$

$$\begin{cases} x \geq 1 \\ x - 1 \geq 4x^2 \end{cases}$$

$$\begin{cases} x \geq 1 \\ 4x^2 - x + 1 \leq 0 \end{cases}$$

$$\downarrow$$

$$b^2 - 4ac = 1 - 4(4) = -15$$

DISCORDI $\emptyset$

$$\begin{cases} x < 1 \\ -x + 1 \geq 4x^2 \end{cases}$$

$$\begin{cases} x < 1 \\ 4x^2 + x - 1 \leq 0 \end{cases}$$

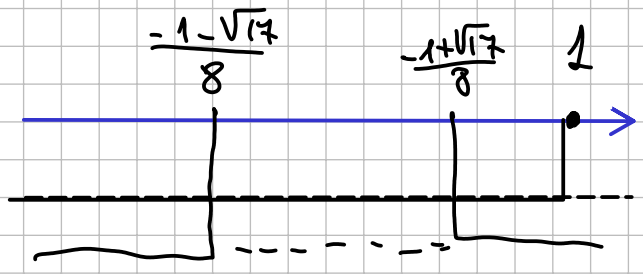
$$b^2 - 4ac =$$

$$1 - (4)(4)(-1) =$$

$$= 1 - (-16) = 17$$

$$\begin{cases} x < 1 \\ \frac{-1-\sqrt{17}}{8} < x < \frac{-1+\sqrt{17}}{8} \end{cases}$$

$-0{,}64$ $\qquad$ $0{,}39$



Studiare la seguente funzione:

$$f(x) = \sqrt{1 + |x|}$$

$f(0) = \sqrt{1+|0|} = \pm 1$

1) DOMINIO $\mathbb{R}$

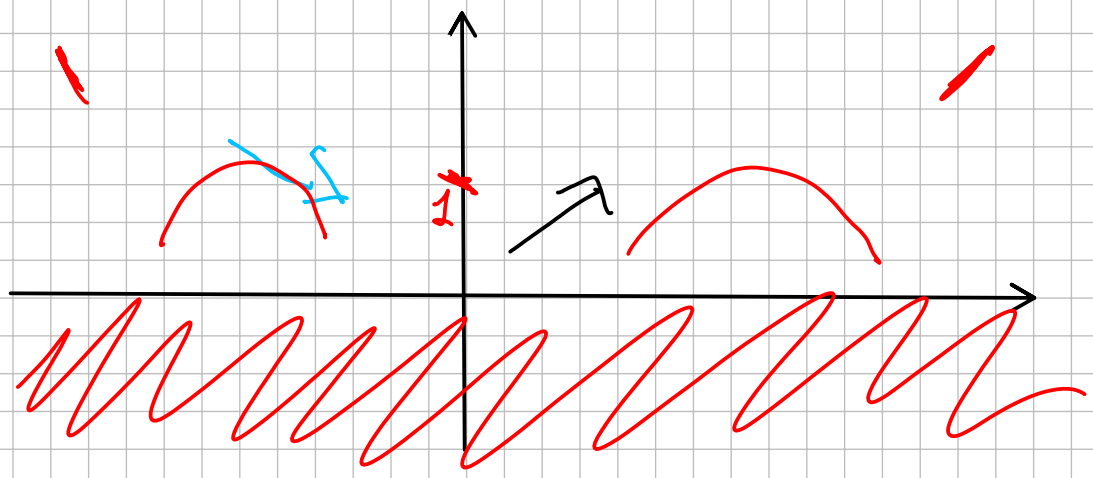
2) $f(-x) = \sqrt{1+|-x|} = f(x)$ PARI

3) SEGNO

$\sqrt{1+|x|} > 0$ SEMPRE

$\rightsquigarrow$ intersez. ASSE $\vec{y}$

$f(0) = \sqrt{1+0} = 1$ (y)



4) LIMITI e forma a tratti.

$\begin{cases} x \geq 0 \\ \sqrt{1+x} \end{cases}$ $\begin{cases} x < 0 \\ \sqrt{1-x} \end{cases}$

$\lim_{x \to +\infty} \sqrt{1+x} = +\infty$

$\lim_{x \to -\infty} \sqrt{1-x} = +\infty$

5) DERIVATA

$f'(x)$

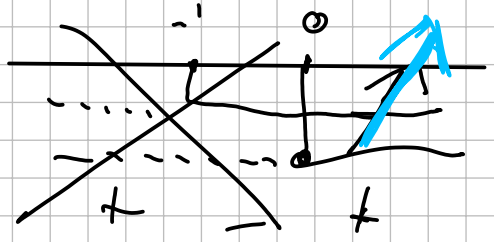
$\begin{cases} x \geq 0 \\ \frac{1}{2\sqrt{x+1}} \end{cases}$ $\begin{cases} x < 0 \\ -\frac{1}{2\sqrt{1-x}} \end{cases}$

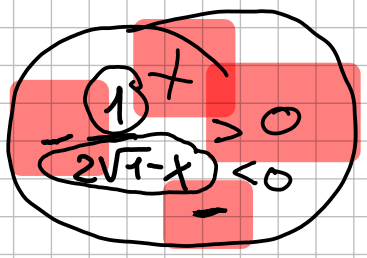
$x \geq 0$

$\frac{1}{2\sqrt{x+1}} > 0$ $\quad \frac{2\sqrt{x+1}}{4(x+1)} = \frac{2\sqrt{x+1}}{4x+4} > 0$

$2\sqrt{x+1}$: $+$ $+$

$\begin{cases} 2\sqrt{x+1} > 0 \\ 4x+4 > 0 \end{cases}$ $\quad \begin{cases} x+1 > 0 \\ 4x > -4 \end{cases}$ $\quad \begin{cases} x > -1 \\ x > -1 \end{cases}$



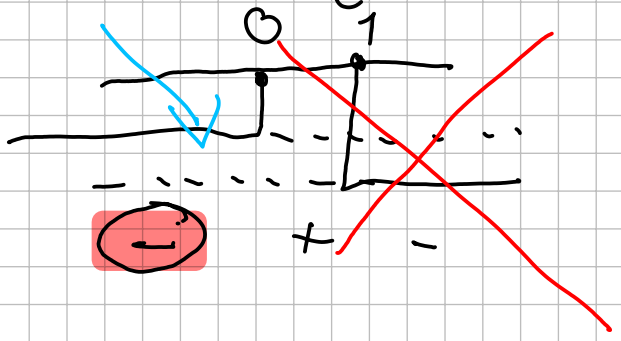


$2\sqrt{1-x} < 0$

$1-x < 0$

$-x < -1$

$\begin{cases} x > 1 \\ x < 0 \end{cases}$



6) DERIVATA 2

$2\sqrt{x+1} = 2(x+1)^{\frac{1}{2}} =$

$(x+1)^{-\frac{1}{2}} = \frac{1}{\sqrt{x+1}}$

$\begin{cases} x \geq 0 \\ -\frac{1}{\sqrt{x+1}} \cdot \frac{1}{4(x+1)} \end{cases}$

$4(x+1)^{\frac{1}{2}} \cdot (x+1)$

$\begin{cases} x \geq 0 \\ \frac{1}{2\sqrt{x+1}} \end{cases}$ $\quad f'(x)$ $\quad \begin{cases} x < 0 \\ -\frac{1}{2\sqrt{1-x}} \end{cases}$

$f''(x) = \frac{0 \cdot (2\sqrt{x+1}) - 1 \cdot \frac{1}{\sqrt{x+1}}}{4(x+1)} =$

$$\begin{cases} x \geq 0 \\ -\dfrac{1}{4(x+1)^{3/2}} \end{cases}$$

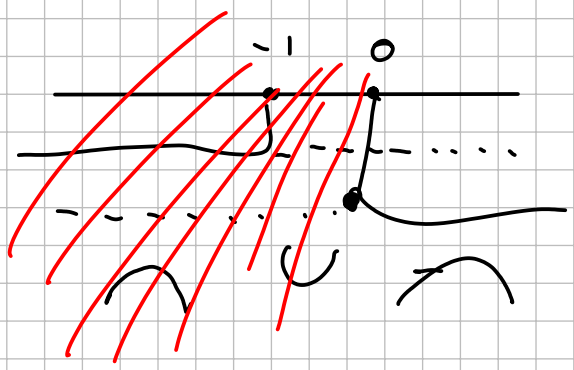
$$\left(\sqrt{x+1}\right)^3$$

$$\hookrightarrow -\frac{1}{4(x+1)^{3/2}} > 0 \rightarrow 4(x+1)^{3/2} < 0$$

$4 < 0$ Mal

$$(x+1)^{3/2} < 0$$

$\hookrightarrow x < -1$



$$2\sqrt{1-x} = 2(1-x)^{\frac{1}{2}}$$

$$(1-x)$$

$$\begin{cases} x < 0 \\ -\dfrac{1}{2\sqrt{1-x}} \end{cases} \qquad \begin{cases} x < 0 \\ -\dfrac{[0 \cdot (2\sqrt{1-x}) - 1 \cdot (1-x)^{-\frac{1}{2}} \cdot (-1)]}{4(1-x)} \end{cases}$$

$$\begin{cases} x < 0 \\ -\dfrac{\left[-\frac{1}{\sqrt{1-x}} \cdot (-1)\right]}{4(1-x)} \end{cases} \qquad \begin{cases} x < 0 \\ -\left[-\frac{1}{\sqrt{1-x}} \cdot \frac{1}{4(1-x)} \cdot (-1)\right] \end{cases} \qquad \begin{cases} x < 0 \\ -\dfrac{1}{4(1-x)^{3/2}} \end{cases}$$

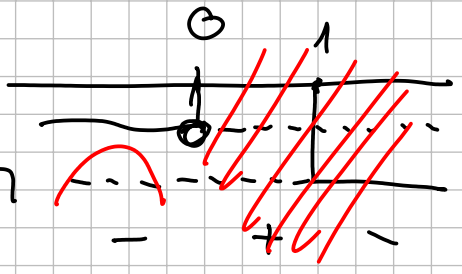
$$-\frac{1}{4(1-x)^{3/2}} > 0$$

$$(1-x)^{3/2} < 0$$

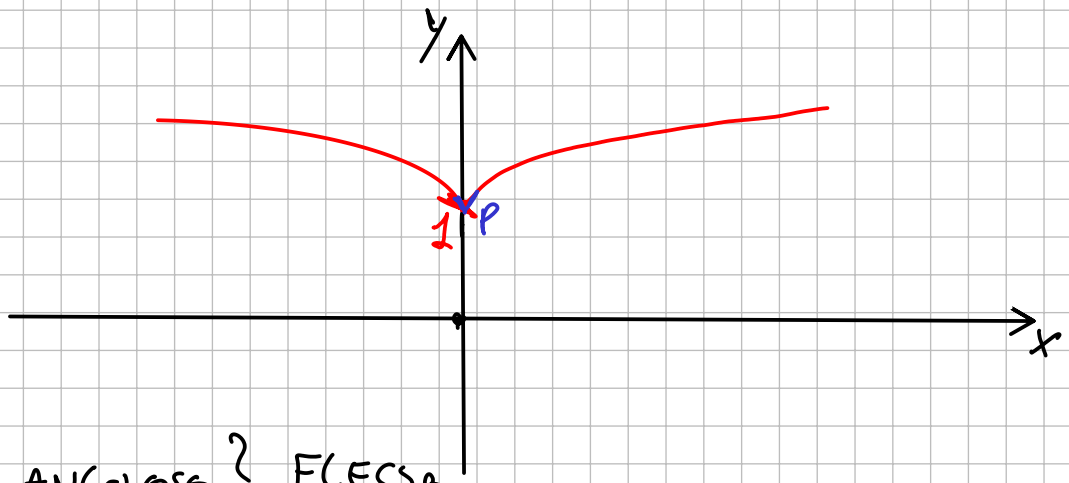
$$1 - x < 0$$

$$-x < -1$$

$$x > 1$$



7) GRAFICO QUALITATIVO



$P(0,1)$ È ANGOLOSO? FLESSO

$$\lim_{x\to 0^-} \frac{1}{2\sqrt{x+1}} = \frac{1}{2}$$

$$\lim_{x\to 0^+} -\frac{1}{2\sqrt{1-x}} = \frac{1}{2}$$

} DIVERSI

Il punto $(0,1)$ è un punto angoloso

$$\begin{cases} x \geq 0 \\ \dfrac{1}{2\sqrt{x+1}} \end{cases} \qquad f'(x) \qquad \begin{cases} x < 0 \\ -\dfrac{1}{2\sqrt{1-x}} \end{cases}$$