FORMULE DI PASSAGGIO.

$P_0 \to P_\infty$

$\boxed{x = \frac{x'}{t'}}$ $\boxed{y = \frac{y'}{t'}}$

COND. ORTOGONALITA'

$\bar{V}_1 = (l_1, m_1, n_1)$
$\bar{V}_2 = (l_2, m_2, n_2)$ > $\boxed{\bar{V}_1 \cdot \bar{V}_2 = 0}$

$\left\{ \begin{array}{l} r_1 \perp \pi_1 \\ r_2 \perp \pi_2 \end{array} \right.$ DOPPIA COND.

COND. PARALLELISMO

$\bar{V}_1 = (l_1, m_1, n_1)$
$\bar{V}_2 = (l_2, m_2, n_2)$ > $\boxed{\bar{V}_1 = \lambda \bar{v}_2}$ -> $\bar{V}_1 = (\lambda l_2, \lambda m_2, \lambda n_2)$

RETTA -> PARAM. DIRETTORI

$r: \left\{ \begin{array}{l} ax + by + c = 0 \\ t' = 0 \end{array} \right. \left\{ \begin{array}{l} ax' + by' + ct' = 0 \\ t' = 0 \end{array} \right. \left\{ \begin{array}{l} \dots \\ \dots \end{array} \right. \Rightarrow P_\infty(x, y, z, \overset{0}{\overset{\shortparallel}{t}})$

$\bar{V}_r = (x, y, z)$

RETTA r

$$\boxed{\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}}$$

PASSA PER

$\boxed{P_0 = (x_0, y_0, z_0)}$

PIANO π $ax_0 + by_0 + cz_0 + d = 0$

$\bar{V}_r = (l, m, n)$ / $\bar{V}_\pi = (a, b, c)$

Angolo fra 2 rette =

$$\frac{\bar{V}_1 \cdot \bar{V}_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} = \cos\alpha$$

->SCALARE

dist. Punto-PIANO

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

PUNTO SIMMETRICO

Medio

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

x_M y_M z_M

Piano contenente 2 rette r e s

- FASCIO DI RETTE CON ASSE LA 1° retta

  $r: \left\{ \begin{array}{l} ax + by + cz + d = 0 \ (\pi_1) \\ a'x + b'y + c'z + d' = 0 \ (\pi_2) \end{array} \right.$

  $F = \pi_1 + K \cdot (\pi_2) = 0$

- Scelgo 1 PUNTO P_0 su s, $\boxed{P_0 \in s}$ e $\boxed{P_0 \notin r}$ SODDISFA "s" NON SODDISFA "r"

  ↳ risolvo sist. di s e trovo 1. Inc. lib

  ↳ DO 1 valore e verifico le condizioni

- Impongo il passaggio del fascio per P_0 e trovo K.
- Sost. K nell'eq. del fascio.

2 Rette sghembe det ≠ 0

$r: \left\{ \begin{array}{l} z = 0 \\ y + z - 1 = 0, \end{array} \right.$ e $s: \left\{ \begin{array}{l} x - y + z - 1 = 0 \\ x - 2y = 0 \end{array} \right.$

TUTTO AL I MEMBRO

$$\det_{4\times4} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -2 & 0 & 0 \end{pmatrix} \begin{array}{l} | r \\ \\ | s \end{array} = \dots$$

Coniche spezzate e punti Base

Servono almeno 2 coniche spezzate Γ_1, Γ_2

$\Gamma_1 = \det B = 0$ (TABELLA)

Γ_2 = Prendo il fascio, raccolgo K e i termini Con la K comune mi danno Γ_2.

Γ_2 = è spezzata? (RACCOGLIM., PROD. NOTEVOLE)

Punti Base $\left\{ \begin{array}{l} \Gamma_1 \\ \Gamma_2 \end{array} \right. \left\{ \begin{array}{l} \dots \\ \dots \end{array} \right.$ $B_1(,) \dots$ $B_2(,) \dots$

$$x_{1,2} = \frac{-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - ac}}{a}, \text{i}$$

$a_{11}x^2 + a_{22}y^2 + a_{33} + 2a_{12}xy + 2a_{13}x + 2a_{23}y = 0$ Conica

$$B = \begin{pmatrix} a_{11} & xy/2 & x/2 \\ xy/2 & a_{22} & y/2 \\ x/2 & y/2 & a_{33} \end{pmatrix} A = \begin{pmatrix} a_{11} & xy/2 \\ xy/2 & a_{22} \end{pmatrix}$$

$|B|$, $\rho(B)$, $|A|$, $TrA = a_{11} + a_{22}$

Iniziamo e calcoliamo il det B.
Se risulta det $B = 0$

- In tal caso la conica si spezza in due rette. A questo punto calcoliamo il rango $\rho(B) < 3$:
  a) se $\rho(B) = 2$ allora la conica si spezza in due rette distinte
  b) se $\rho(B) = 1$ allora la conica si spezza in due rette coincidenti.

Invece se risulta det $B \neq 0$

- In tal caso la conica si dice irriducibile e andremo a calcolare il det A:
  a) se det $A > 0$ allora la conica è: Ellisse reale se $TrA \cdot \det B < 0$;
  invece Ellisse immaginaria se $TrA \cdot \det B > 0$.
  Infine se $a_{11} = a_{22} \neq 0$, $a_{12} = 0$ avremo Circonferenza;
  b) se det $A = 0$ allora la conica è Parabola;
  c) se det $A < 0$ allora la conica è Iperbole. Se inoltre la $Tr(A) = 0$ allora si tratta di iperbole equilatera

- Vedo se si spezza se è prod. notevole, se posso raccogliere.

$(a + b) \cdot (a - b) = a^2 - b^2$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

↳ Altrimenti faccio la matrice dell'equaz. trovata e RIVEDO LA TABELLA

CONICHE |B| ≠ 0

IPERBOLE/ELLISSE

FORMA RIDOTTA 1.

$$\boxed{\alpha X^2 + \beta Y^2 = \gamma}$$

$$\gamma = \frac{-\det B}{\det A}$$

Se parliamo di ELLISSE O IPERBOLE abbiamo

$$I) : \alpha x^2 + \beta y^2 = \gamma$$

Determinare α, β, γ.

$$B = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & -\gamma \end{pmatrix}, A = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$$

det $B = -\alpha\beta\gamma$, det $A = \alpha\beta$ Da cui

$$\gamma = -\frac{\det B}{\det A}$$

ed α, β si ricavano dal P.C.(A) poichè sono gli autovalori della matrice A. Nell'iperbole e nell'ellisse posso scegliere io chi chiamare α e chi chiamare β basta solo poi essere coerente a questa scelta.

$$P_A(T) = \det\begin{vmatrix} a_{11} - T & a_{12} \\ a_{12} & a_{22} - T \end{vmatrix} = 0 \dashrightarrow = 0 \left\langle \begin{array}{l} \alpha \\ \beta \end{array} \right.$$

CENTRO

$$\begin{pmatrix} a_{11} & xy/2 & x/2 \\ xy/2 & a_{22} & y/2 \\ \cancel{x/2} & \cancel{y/2} & \cancel{a_{33}} \end{pmatrix} \cdot \begin{pmatrix} x_c \\ y_c \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

ASSI se $a_{12} \neq 0$ (term. xy)

$$m_1 = \frac{a_{11} - \alpha}{a_{12}}$$

ASSE 1: $y - y_c = m_1(x - x_c)$
ASSE 2: $y - y_c = -\frac{1}{m}(x - x_c)$

ASSI se $a_{12} = 0$ (term xy)

ASSE 1: $x = x_c$
ASSE 2: $y = y_c$

VERTICI ELLISSE (4)

$V_1, V_3 \left\{ \begin{array}{l} \text{ellisse} \\ \text{asse 1} \end{array} \right.$ / $V_2, V_4 \left\{ \begin{array}{l} \text{ellisse} \\ \text{asse 2} \end{array} \right.$

VERTICI IPERBOLE

$\left\{ \begin{array}{l} \text{ASSE 1} \\ \text{fascio} \end{array} \right.$ oppure $\left\{ \begin{array}{l} \text{ASSE 2} \\ \text{fascio} \end{array} \right.$

Uno dei 2 non ha soluzioni.

PARABOLA

FORMA RIDOTTA 2.

$$\boxed{\beta Y^2 = 2\gamma X}$$

Se parliamo di PARABOLA la sua forma canonica è

$$\beta y^2 = 2\gamma \cdot x$$

$\beta = ?$ $\gamma = ?$

Determinare β, γ.

$$B = \begin{pmatrix} 0 & 0 & -\gamma \\ 0 & \beta & 0 \\ -\gamma & 0 & 0 \end{pmatrix}, A = \begin{pmatrix} 0 & 0 \\ 0 & \beta \end{pmatrix}$$

det $B = -\beta\gamma^2$, det $A = 0$ Da cui

$$\boxed{\beta = TrA, \gamma = +\sqrt{-\frac{|B|}{TrA}}}$$

Abbiamo preso il segno positivo per γ poichè per convenzione scegliamo il verso positivo dell'asse X.

CENTRO PARABOLA ∄

ASSE PARABOLA, è UNICO
NON SI CALCOLA IN GENERALE

ASSE PARABOLA SOLO SE E' PARALLELO ALL'ASSE Y

$y = ax^2 + bx + c$

ASSE $x = x_{\text{Ⓥ}}$ vertice

$V = \left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$

ASSE PARABOLA SOLO SE E' PARALLELO ALL'ASSE X

$y = ax^2 + bx + c$

ASSE $y = y_{\text{Ⓥ}}$ vertice

$V = \left(-\frac{\Delta}{4a}, -\frac{b}{2a}\right)$

- Dato centro e raggio ($r > 0$), la circonferenza ha equazione

$$(x - \alpha)^2 + (y - \beta)^2 = r^2$$

Forma n.2 dell'equazione della circonferenza del piano $z = 0$:

$$1x^2 + 1y^2 + ax + by + c = 0$$

con centro $C = (-\frac{a}{2}, -\frac{b}{2})$ e raggio $r = \sqrt{(-\frac{a}{2})^2 + (-\frac{b}{2})^2 - c}$

1) Data una retta nello spazio, determinare i parametri direttori e scriverla come intersezione tra due piani:

$a)\frac{x-2}{2}=\frac{y+1}{0}=z-2$

$b)\frac{3x-1}{2}=\frac{y-1}{2}=z$

$c)\frac{x-2}{0}=\frac{y+1}{0}=z-2$

$d)\frac{x-2}{2}=\frac{y+1}{4}=\frac{4z}{4}$

a) $\frac{x-2}{2}=\frac{y+1}{0}=z-2$ $\qquad \frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$

$$r:\begin{cases} y+1=0 \\ \frac{x-2}{2}=z-2 \end{cases} \quad \begin{cases} y+1=0 \\ \frac{x-2}{\cancel{2}}=2z-4 \end{cases} \quad \begin{cases} \boxed{y+1=0} \; \pi_1 \\ x-2-2z+4 \end{cases} \quad \begin{cases} \\ \boxed{x-2z+2=0} \; \pi_2 \end{cases} \quad \pi_1\cap\pi_2$$

$$\bar{V}_r=(\underset{l}{2},\underset{m}{0},\underset{n}{1})$$

$b)\frac{3x-1}{2}=\frac{y-1}{2}=z$ $\qquad \bar{V}_r=(2,2,1)$

$$r:\begin{cases} \frac{3x-1}{2}=\frac{y-1}{2} \\ \frac{y-1}{2}=z \end{cases} \quad \begin{cases} 3x-1=y-1 \\ y-1=2z \end{cases} \quad \begin{cases} 3x-y=0 \; (\pi_1) \\ y-2z-1=0 \; (\pi_2) \end{cases} \quad \boxed{\pi_1\cap\pi_2}$$

$c)\frac{x-2}{0}=\frac{y+1}{0}=z-2$ $\qquad \bar{V}_r=(0,0,1)$ $\qquad$ 1°,3° $\quad$ 2°,3

$$\begin{cases} \frac{x-2}{0}=\frac{y+1}{0} \\ \frac{y+1}{0}=z-2 \end{cases} \begin{cases} 0=0 \\ y+1=0 \end{cases} \begin{cases} 0=0 \\ y=-1 \end{cases} \Bigg\| \begin{cases} \frac{x-2}{0}=z-2 \\ \frac{x-2}{0}=\frac{y+1}{0} \end{cases} \begin{cases} x=2 \\ \cancel{0=0} \end{cases} \Bigg\| \begin{cases} \frac{x-2}{0}=z-2 \\ \frac{y+1}{0}=z-2 \end{cases} \begin{cases} x=2 \\ y=-1 \end{cases}$$

$d)\frac{x-2}{2}=\frac{y+1}{4}=\frac{4z}{4}$ $\qquad \boxed{\hat{V}_r=(2,4,4)}$

$$\begin{cases} \frac{x-2}{2}=\frac{y+1}{4} \\ \frac{y+1}{4}=\frac{4z}{4} \end{cases} \quad \begin{cases} 4x-8=2y+2 \\ y+1=4z \end{cases} \quad \begin{cases} \underset{2}{\cancel{4}}x-\cancel{2}y-\underset{5}{\cancel{8+2}}=0 \to \boxed{2x-y-5=0}\,(\pi_1) \\ \boxed{y+1-4z=0}\,(\pi_2) \end{cases} \quad \pi_1\cap\pi_2$$

2) Data una retta e un piano, stabilire se la retta r è contenuta nel piano π:

a) $r: \begin{cases} 2y - z &= -1 \\ x - y &= -3 \end{cases}, \pi: x + y - z - 2 = 0$

b) $r: \begin{cases} x - y - z &= 0 \\ x + y + z &= -2 \end{cases}, \pi: 2x + y + z - 4 = 0$

P. GENERICO DI r OPP. SISTEMA.

P. GENERICO

$\begin{cases} 2y - z = -1 \\ x - y = -3 \end{cases} \begin{cases} z = 2y + 1 \\ x = y - 3 \end{cases} \quad \forall y$

$\forall y$

P. generico sulla retta $(\underset{x}{y-3}, \underset{y}{y}, \underset{z}{2y+1}) \rightarrow (-2, 1, 3)$

Sost. nel piano $\pi: x + y - z - 2 = 0$

$-2 + 1 - 3 - 2 = 0$

$-1 - 3 - 2 = 0$

$-6 = 0$ ASSURDO $r \not\in \pi$

La retta non è contenuta nel piano!

---

b) $r: \begin{cases} x - y - z &= 0 \\ x + y + z &= -2 \end{cases}, \pi: 2x + y + z - 4 = 0$

$\begin{cases} x - y - z = 0 \\ x + y + z = -2 \end{cases} \begin{cases} x = z + y \\ z + y + y + z = -2 \end{cases} \begin{cases} x = \dots \\ \cancel{2}y + \cancel{2}z = -\cancel{2} \end{cases} \{ y + z = -1 \begin{cases} x = z + y \\ z = -1 - y \end{cases}$

$\forall y$

P. generico $= (z + y, y, -1 - y) \ \forall z$

$\hookrightarrow (-1, 1, -2)$

Sost. nel piano

$2(-1) + 1 - 2 - 4 = 0$

$-2 + 1 - 2 - 4 = 0$

$-7 = 0$ ASSURDO $r \not\in \pi$

---

3) Dato un punto fissato $P_0 = (1, 1, -1)$, determinare la retta passante per P_0
avente parametri direttori (5,0,5).

RETTA = $\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$

$$\frac{x-1}{5} = \frac{y-1}{0} = \frac{z+1}{5}$$

$$r_1 \begin{cases} y-1=0 \\ \frac{x-1}{5} = \frac{z+1}{5} \end{cases} \quad \begin{cases} y=1 \\ x-1=z+1 \end{cases} \quad \begin{cases} \boxed{y=1 \ (\pi_1)} \\ \boxed{x-z-2=0 \ (\pi_2)} \end{cases} \quad \pi_1 \cap \pi_2$$

---

4) Date le seguente rette e i seguenti piani, trovare i parametri direttori di ciascuno:

a) $r_1 : \begin{cases} x+y & = 2 \\ x+y+z & = 2 \end{cases}; r_2 : \begin{cases} 2x-3z+1 & = 0 \\ x-y+2z-1 & = 0 \end{cases}; r_3 \begin{cases} x & = 0 \\ y & = z-3 \end{cases}$

b) $\pi_1 : 2x+y-3z+1 = 0; \pi_2 : x-z = 0; \pi_3 : x+y+4 = 0$

$$r_1 : \begin{cases} x+y = 2 \\ x+y+z = 2 \end{cases}$$

omogenizzo

$$\begin{cases} x'+y' = \cancel{2t'} \\ x'+y'+z' = \cancel{2t'} \\ t'=0 \end{cases} \quad \begin{cases} x' = -y' \\ \cancel{-y'}\cancel{+y'}+z'=0 \\ t'=0 \end{cases} \quad \begin{cases} x'=-y' \\ z'=0 \\ t'=0 \end{cases} \quad \forall y'$$

Vettore generico $= (\underbrace{-y', y', 0}_{P.d.}, 0)$

$\bar{v}_{r_1} = (-1, 1, 0)$

$$r_2 : \begin{cases} 2x-3z+1=0 \\ x-y+2z-1=0 \end{cases} \xrightarrow{OMOG.} \begin{cases} 2x'-3z'\cancel{+t'}=0 \\ x'-y'+2z'\cancel{-t'}=0 \\ t'=0 \end{cases}$$

$$\begin{cases} 2x' - 3z' = 0 \\ x' - y' + 2z' = 0 \\ t' = 0 \end{cases} \quad \begin{cases} \cancel{2}x' = \frac{3}{2}z' \\ -\frac{3}{2}z' - y' + 2z' = 0 \end{cases} \quad \begin{cases} x' = \dots \\ \frac{1}{2}z' = y \end{cases} \quad \begin{cases} x = \frac{3}{2}z' \\ y = \frac{1}{2}z' \end{cases} \quad \forall z$$

$\bar{V}$ generico $= \left(\frac{3}{2}z', \frac{1}{2}z', z'\right)$

$$\boxed{\bar{V}_{r_2} = \left(\frac{3}{2}, \frac{1}{2}, 1\right)}$$

$$\begin{cases} x = 0 \\ y = z - 3 \end{cases} \xrightarrow{\text{OMOG}} \begin{cases} x' = 0 \\ y' = z' - \cancel{3t'} \\ t' = 0 \end{cases} \quad \begin{cases} x' = 0 \\ y' = z' \\ t' = 0 \end{cases}$$

Vett. generico $= (0, z, z, 0)$ $\forall z$

$\bar{V}_{r_3} = (0, 1, 1)$

b) $\pi_1 : 2x + y - 3z + 1 = 0; \pi_2 : x - z = 0; \pi_3 : x + y + 4 = 0$

PIANO. $ax + by + cz + d = 0$ $\quad V_\pi = (a, b, c)$

$\bar{V}_{\pi_1} = (2, 1, -3)$ $\quad \bar{V}_{\pi_2} = (1, 0, -1)$ $\quad \bar{V}_{\pi_3} = (1, 1, 0)$

5) Dato un punto P_0 e una retta r, determinare la retta s passante per P_0 e parallela alla retta r e una retta t passante per P_0 e ortogonale alla retta s:

a) dato $P_0 = (2, -2, 0), r : \begin{cases} x + y & = 2 \\ x - z & = 3 \end{cases}$

b) dato $P_0 = (1, 0, -3), r : \begin{cases} 2x + y & = -3 \\ 2x + y + z & = 0 \end{cases}$

$s: \frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$ $\quad P_0 = (2, -2, 0)$

SOST. $\frac{x - 2}{l} = \frac{y + 2}{m} = \frac{z - 0}{n}$ $\quad V = (l, m, n)$??

$s // r$ quindi $\bar{V}_s = V_r$. TROVO $\bar{V}_r$. OMOGENIZZO.

$$r: \begin{cases} x + y = 2 \\ x - z = 3 \end{cases} \quad \begin{cases} x + y = 2t \\ x - z = 3t \\ t = 0 \end{cases} \quad \begin{cases} x + y = 0 \\ x - z = 0 \\ t = 0 \end{cases} \quad \begin{cases} y = -x \\ z = x \\ t = 0 \end{cases} \quad \forall x$$

Vett. generico = $(x, -x, x, 0)$

$\bar{V}_r = (1,-1,1)$ → diventa $\boxed{\bar{V}_s = (1,-1,1)}$

$\frac{x-2}{1} = \frac{y+2}{-1} = \frac{z-0}{1} \rightarrow x-2 = -y-2 = z$

$$\boxed{s : \begin{cases} x-\cancel{2} = -y\cancel{-2} \\ -y-2 = z \end{cases} \begin{cases} x+y=0 \quad (\pi_1) \\ z+y+2=0 \quad (\pi_2) \end{cases}}$$

$s : \pi \cap \pi_2$

$t \perp \boxed{s}$ passante per P_0

$$\boxed{t = \frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}}$$

$$\boxed{P_0 = (2,-2,0)}$$

$\bar{V}_s = (1,-1,1)$

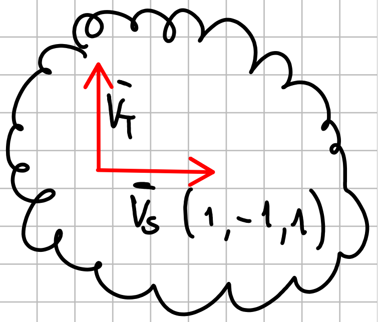
$\bar{V}_s \cdot \bar{V}_t = 0$

$\bar{V}_s = (l, m, n)$

$V_t = (l', m', n')$

$$\boxed{(l, m, n) \cdot (l', m', n') = 0}$$

$(1, -1, 1) \cdot (l', m', n') = 0$

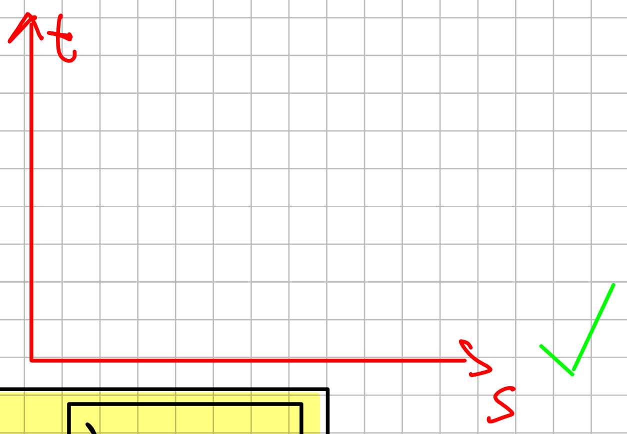


$t = \frac{x-2}{l'} = \frac{y+2}{m'} = \frac{z-0}{n'}$

$\bar{V}_t \perp \bar{V}_s$

$$\boxed{l' - m' + n' = 0}$$

$\overset{l}{\overset{\shortparallel}{1}} l' - \overset{m}{\overset{\shortparallel}{1}} m' + \overset{n}{\overset{\shortparallel}{1}} n' = 0$



$l' = m' - n'$

$$\boxed{t = \frac{x-2}{m'-n'} = \frac{y+2}{m'} = \frac{z-0}{n'} \quad \boxed{\forall m', n'}}$$

Infinite rette ortogonali nello spazio!

Un $\bar{V}_t = (1, 1, 0)$, $\bar{V}_t$ generico $= (m'-n', m', n')$

b) dato $P_0 = (1,0,-3), r : \begin{cases} 2x+y & = -3 \\ 2x+y+z & = 0 \end{cases}$

s: per P_0 $// r$?

$\bar{V}_s = \boxed{\bar{V}_r}$

Omogenizzo

$$\begin{cases} 2x+y=0 \\ 2x+y+z=0 \\ t=0 \end{cases} \quad \begin{cases} y=-2x \\ 2x-2x+z=0 \end{cases} \quad \begin{cases} y=-2x \\ z=0 \\ t=0 \end{cases}$$

$\bar{V}_r = (x, -2x, 0)$

$\bar{V}_r = (1, -2, 0)$

$\|$

$\boxed{\bar{V}_s = (1,-2,0)}$

$$\frac{x-\overset{1}{\overset{\|}{x_0}}}{\underset{\underset{1}{\|}}{l}} = \frac{y-\overset{0}{\overset{\|}{y_0}}}{\underset{\underset{-2}{\|}}{m}} = \frac{z-\overset{-3}{\overset{\|}{z_0}}}{\underset{\underset{0}{\|}}{n}} \Longrightarrow x-1 = \frac{y-0}{-2} = \frac{z+3}{0}$$

$$s: \begin{cases} x-1 = \dfrac{y}{-2} \\ x-1 = \dfrac{z+3}{0} \end{cases} \quad \begin{cases} -2x+2 = y \\ z=-3 \end{cases} \quad \begin{cases} 2x+y-2=0 & \pi_1 \\ z+3=0 & \pi_2 \end{cases} \; s:$$

6) Dato un punto P_0e un piano π, determinare il piano $\pi_{\|}$ passante per P_0 e parallela al piano π

a) dato $P_0 = (0,2,0), \pi : x+y=0$

$\bar{V}_r^{(\pi)} = (1,1,0)$ $\quad$ $\bar{V}_{r\,//} = (1,1,0)$

$\pi_{//} = ax+by+cz+d=0$

$$\underset{\underset{0}{//}}{x} + \underset{\underset{2}{//}}{y} + 0 + \underset{\underset{?}{\|}}{d} = 0 \qquad P_0 = (0,2,0)$$

$0+2+0+d=0$

$2+d=0 \quad d=-2$

$\boxed{x+y-2=0}$ $\quad$ $\boxed{\pi_{//}^{(\pi_1)} \text{ che passa per } P_0.}$

b) dato$P_0 = (1, 1, -3), \pi : 2x + 2y + 2z$

$\bar{V}_\pi = (2,2,2) \to (1,1,1)$

$\parallel$

$\bar{V}_{\pi_\parallel} = (1,1,1)$

EQ. PIANO.

$ax+by+cz+d=0$

$x+y+z+d=0 \to$ Impongo il passaggio per P_0.

$1+1-3+d=0$

$d=3-2=1$ $\boxed{x+y+z+1=0}$ $\pi_\parallel$

---

7) Data una retta r un punto P_0e un piano π, determinare la retta t_1 passante per P_0, ortogonale al piano π e il piano $\pi_\perp$ passante per P_0, ortogonale alla retta r

7a) Data una retta $r : \begin{cases} x - y + z & = 0 \\ x + y + z - 2 & = 0 \end{cases}, P_0 = (0, 0, 1), \pi : x + 3y - 1 = 0$

7b) Data una retta $r : \begin{cases} y + z & = 0 \\ 3x - z - 1 & = 0 \end{cases}, P_0 = (1, -1, 1), \pi : x + y - z = 0$

t_1 per P_0 e $\perp \pi$ retta. $\dfrac{x-x_0}{l} = \dfrac{y-y_0}{m} = \dfrac{z-z_0}{n}$

$\bar{V}_\pi = (1,3,0)$ è $\parallel$ a $\bar{V}_t = (1,3,0)$

t_1: $\boxed{\dfrac{x-0}{1} = \dfrac{y-0}{3} = \dfrac{z-1}{0}}$

$\begin{cases} x - \frac{1}{3}y = 0 & (\pi_1) \\ z-1=0 & (\pi_2) \end{cases}$

$\pi_\perp$ per P_0 $\perp$ r $\bar{V}_{\pi_\perp} = \bar{V}_r$.

$\begin{cases} x-y+z=0 \\ x+y+z=\cancel{+z}^{t}_{0} \\ t=0 \end{cases}$ $\begin{cases} x=y-z \\ y-\cancel{z}+y\cancel{+z} \quad =0 \\ t=0 \end{cases}$ $\begin{cases} x=y-z \\ \cancel{2}y=0 \\ t=0 \end{cases}$ $\begin{cases} x=-z \\ y=0 \\ t=0 \end{cases}$

Vettore direttivo di r generico $:= (-z, 0, z)$ $\forall z$

$\bar{V}_r = (-1,0,1)$

P_0 era $(0,0,1)$ e $\pi_\perp$ è

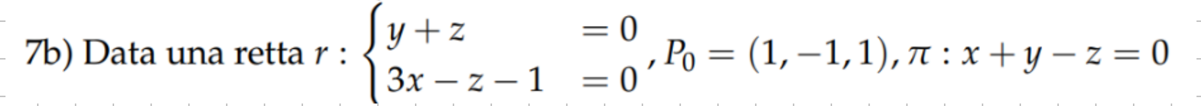
$\pi_\perp \quad ax+by+cz+d=0$

$-x+z+d=0 \rightsquigarrow$ Passa per $P_0 \rightarrow -0+1+d=0$

$\hookrightarrow d=-1$

Eq. $\pi_\perp = -x+z-1=0$

$\hookrightarrow \boxed{x-z+1=0}$



7b) Data una retta $r: \begin{cases} y+z &= 0 \\ 3x-z-1 &= 0 \end{cases}$, $P_0=(1,-1,1)$, $\pi: x+y-z=0$

t_1 per P_0 e $\perp \pi$ retta. $\dfrac{x-x_0}{l} = \dfrac{y-y_0}{m} = \dfrac{z-z_0}{n}$

$\bar{V}_\pi = (1,1,-1) \Longleftrightarrow \bar{V}_{t_1} = (1,1,-1)$

$t: \dfrac{x-1}{1} = \dfrac{y+1}{1} = \dfrac{z-1}{-1}$

$\begin{cases} x-1 = y+1 \\ y+1 = \dfrac{z-1}{-1} \end{cases} \quad \begin{cases} x-y-2=0 \\ -y\cancel{-1}-z\cancel{+1}=0 \end{cases} \quad \boxed{t: \begin{cases} x-y-2=0 \\ y+z=0 \end{cases}}$

$\pi_\perp \perp r$ passa per P_0.

$\bar{V}_{\pi_\perp} = \bar{V}_r$

$\begin{cases} y+z=0 \\ 3x-z-\cancel{1}t=0 \\ t=0 \end{cases} \quad \begin{cases} y=-z \\ z=3x \\ t=0 \end{cases} \forall x \quad P_\infty = (x, -z, 3x, 0)$

$\bar{V}_{\pi_\perp} = (\underset{a}{1}, \underset{b}{-3}, \underset{c}{3})$

PIANO. $ax+by+cz+d=0$

$x-3y+3z+d=0$ Passa per $P_0(1,-1,1)$

$1+3+3+d=0$

$d=-7$

$\boxed{\text{PIANO } \pi_\perp = x-3y+3z-7=0}$

8) Date due rette r_1, r_2 e un punto P_0, determinare la retta t che passa per P_0 ed è ortogonale ad entrambe le rette

8a) $r_1 : \begin{cases} x+y-z &= 0 \\ 2x-y &= 1 \end{cases}, r_2 : \begin{cases} y+z &= 0 \\ x &= 0 \end{cases}, P_0 = (1, 0, -3)$

8b) $r_1 : \begin{cases} x-z &= 0 \\ x-y &= 1 \end{cases}, r_2 : \begin{cases} x+z &= 0 \\ z &= 0 \end{cases}, P_0 = (0, 0, 3)$

t per P_0 $\perp$ r_1 e r_2 $\quad \frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$

$$\frac{x-1}{l} = \frac{y-0}{m} = \frac{z+3}{n}$$

$t \perp r_1 \qquad \overline{V}_{r_1} = ?$

$$\begin{cases} x+y-z=0 \\ 2x-y=1 \\ t=0 \end{cases} \begin{cases} z=x+y \\ y=2x \\ t=0 \end{cases} \begin{cases} z=x+2x \\ y=2x \\ t=0 \end{cases} \begin{cases} z=3x \\ y=2x \\ t=0 \end{cases} \forall x \qquad P_\infty = (x, 2x, 3x, 0)$$

$\overline{V}_{r_1} = (1, 2, 3)$

$$r_2: \begin{cases} y+z=0 \\ x=0 \\ t=0 \end{cases} \begin{cases} z=-y \\ x=0 \\ t=0 \end{cases} \qquad P_\infty = (0, y, -y, 0)$$

$\overline{V}_{r_2} = (0, 1, -1)$

$$\begin{cases} V_t \perp V_{r_1} \to \text{PROD SCALARE} = 0 \\ V_t \perp V_{r_2} \to \text{PROD SCALARE} = 0 \end{cases}$$

$(l, m, n) \cdot (1, 2, 3) = 0 \to l + 2m + 3n = 0$

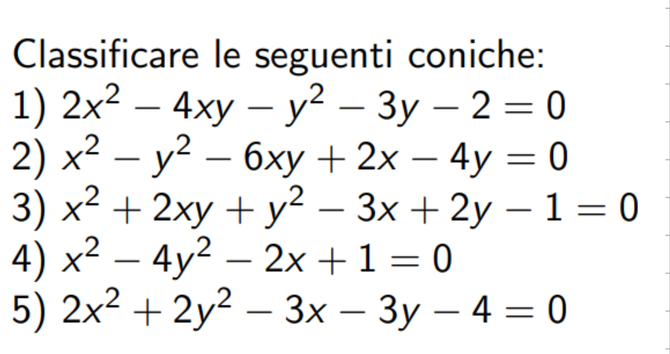
$(l, m, n) \cdot (0, 1, -1) = 0 \to 0l + m - n = 0$

$$\begin{cases} l+2m+3n=0 \\ m-n=0 \end{cases} \begin{cases} l+2n+3n=0 \\ m=n \end{cases} \begin{cases} l=-5n \\ m=n \end{cases} \forall n$$

$$\overline{V}_t = (-5n, n, n) \to (-5, 1, 1)$$

$$t: \frac{x-1}{-5} = \frac{y-0}{1} = \frac{z+3}{1}$$

$$t: \begin{cases} \frac{x-1}{-5} = \frac{y-0}{1} \\ \frac{y-0}{1} = \frac{z+3}{1} \end{cases} \begin{cases} x-1=-5y \\ y-z-3=0 \end{cases} \begin{cases} x+5y-1=0 & (\pi_1) \\ y-z-3=0 & (\pi_2) \end{cases}$$



Classificare le seguenti coniche:

1) $2x^2 - 4xy - y^2 - 3y - 2 = 0$

2) $x^2 - y^2 - 6xy + 2x - 4y = 0$

3) $x^2 + 2xy + y^2 - 3x + 2y - 1 = 0$

4) $x^2 - 4y^2 - 2x + 1 = 0$

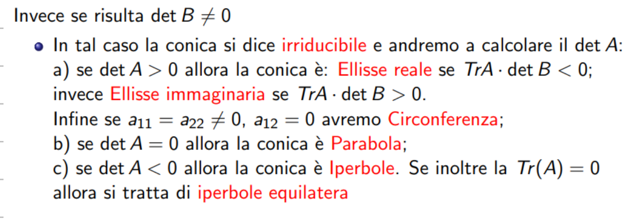
5) $2x^2 + 2y^2 - 3x - 3y - 4 = 0$

1) $2x^2 - 4xy - y^2 - 3y - 2 = 0$

$$B = \begin{pmatrix} 2 & -2 & 0 \\ -2 & -1 & -\frac{3}{2} \\ 0 & -\frac{3}{2} & -2 \end{pmatrix} \quad A = \begin{pmatrix} 2 & -2 \\ -2 & -1 \end{pmatrix}$$

$$|B| = 4 - \left[-8 + \frac{9}{2}\right] = 4 + 8 - \frac{9}{2} = \frac{8+16-9}{2} = \frac{15}{2}$$

$|B| \neq 0$

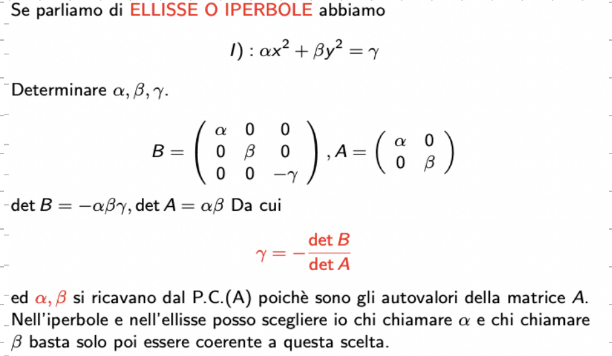


Invece se risulta det $B \neq 0$

- In tal caso la conica si dice irriducibile e andremo a calcolare il det A:
  a) se det $A > 0$ allora la conica è: Ellisse reale se $TrA \cdot \det B < 0$; invece Ellisse immaginaria se $TrA \cdot \det B > 0$.
  Infine se $a_{11} = a_{22} \neq 0$, $a_{12} = 0$ avremo Circonferenza;
  b) se det $A = 0$ allora la conica è Parabola;
  c) se det $A < 0$ allora la conica è Iperbole. Se inoltre la $Tr(A) = 0$ allora si tratta di iperbole equilatera

$|A| = -2 - [4] = -2 - 4 = -6$ $\quad |A| < 0$, $\boxed{\text{IPERBOLE}}$

$TrA = 2 - 1 = 1$, $\boxed{\text{non equilatera.}}$



Se parliamo di ELLISSE O IPERBOLE abbiamo

$$I) : \alpha x^2 + \beta y^2 = \gamma$$

Determinare α, β, γ.

$$B = \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & -\gamma \end{pmatrix}, A = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$$

det $B = -\alpha\beta\gamma$, det $A = \alpha\beta$ Da cui

$$\gamma = -\frac{\det B}{\det A}$$

ed α, β si ricavano dal P.C.(A) poichè sono gli autovalori della matrice A. Nell'iperbole e nell'ellisse posso scegliere io chi chiamare α e chi chiamare β basta solo poi essere coerente a questa scelta.

$$P_A(T) = \det\begin{pmatrix} a_{11}-T & a_{12} \\ a_{12} & a_{22}-T \end{pmatrix} = 0 \rightarrow = 0 < \begin{matrix} \alpha \\ \beta \end{matrix}$$

FORMA CANONICA

$$\alpha X^2 + \beta Y^2 = \gamma$$

$$\gamma = -\frac{|B|}{|A|} = \frac{\frac{15}{2}}{-6} = +\frac{15}{2} \cdot \frac{1}{6} = \frac{\cancel{15}^{5}}{\cancel{12}_{4}}$$

$\gamma = \frac{5}{4}$

$$\alpha X^2 + \beta Y^2 = \frac{5}{4}$$

Cerco α, β.

$$A = \begin{pmatrix} 2-T & -2 \\ -2 & -1-T \end{pmatrix} \rightarrow \begin{pmatrix} 2-T & -2 \\ -2 & -1-T \end{pmatrix}$$

$$|A_T| = 0 \rightarrow (2-T)(-1-T) - 4 = 0 \rightarrow -2 - 2T + T + T^2 - 4 = 0$$

$-6 - T + T^2 = 0 \qquad T^2 - T - 6 = 0$

$b^2 - 4ac \rightarrow 1 - 4(-6) = 25$

$$T_{12} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{1 \pm 5}{2} \begin{cases} \frac{1+5}{2} = 3 & T_1 = \alpha \\ \frac{1-5}{2} = -2 & T_1 = \beta \end{cases}$$

$$\boxed{3X^2 - 2Y^2 = \frac{5}{4}}$$

CENTRO

$$B = \begin{pmatrix} 2 & -2 & 0 \\ -2 & -1 & -\frac{3}{2} \\ \cancel{0} & \cancel{-\frac{3}{2}} & \cancel{-2} \end{pmatrix} \cdot \begin{pmatrix} x_c \\ y_c \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 2x_c - 2y_c = 0 \\ -2x_c - y_c - \frac{3}{2} = 0 \end{cases} \quad \begin{cases} \cancel{2}x_c = \cancel{2}y_c \\ -2yc - yc - \frac{3}{2} = 0 \end{cases} \quad \begin{cases} x_c = y_c \\ -3y_c = \frac{3}{2} \end{cases} \quad \begin{cases} x_c = y_c \\ \cancel{3}y_c = -\frac{\cancel{3}^1}{\cancel{6}_2} \end{cases}$$

$$C = (y_c, y_c) \rightarrow C = \left(-\frac{1}{2}, -\frac{1}{2}\right)$$

ASSI

$$B = \begin{pmatrix} 2 & -2 & 0 \\ -2 & -1 & -\frac{3}{2} \\ 0 & -\frac{3}{2} & -2 \end{pmatrix}$$

ASSI se $a_{12} \neq 0$ (term. xy)

$m_1 = -\frac{a_{11} - \alpha}{a_{12}}$

ASSE 1 = $y - y_c = m_1(x - x_c)$

SSE 2 = $y - y_c = -\frac{1}{m}(x - x_c)$

ASSI se $a_{12} = 0$ (term xy)

ASSE 1 = $x = x_c$

ASSE 2 = $y = y_c$

$a_{12} \neq 0 \qquad m_1 = -\frac{2-3}{-2} = -\frac{-1}{-2} = -\frac{1}{2}$

ASSE 1 = $y - y_c = m_1(x - x_c)$

ASSE 2 = $y - y_c = -\frac{1}{m_1}(x - x_c)$

ASSE 1 = $y + \frac{1}{2} = -\frac{1}{2}\left(x + \frac{1}{2}\right)$

$y + \frac{1}{2} = -\frac{1}{2}x - \frac{1}{4} \Rightarrow$

$y + \frac{1}{2} + \frac{1}{4} + \frac{1}{2}x = 0 \qquad \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4}$

$$\boxed{y + \frac{1}{2}x + \frac{3}{4} = 0}$$

ASSE 2 = $y + \frac{1}{2} = 2\left(x + \frac{1}{2}\right)$

$y + \frac{1}{2} = 2x + 1 \rightarrow y + \frac{1}{2} - 1 - 2x = 0$

$$\boxed{y - 2x - \frac{1}{2} = 0}$$

VERTICI IPERBOLE

$\begin{cases} \text{ASSE 1} \\ \text{fascio} \end{cases}$ oppure $\begin{cases} \text{ASSE 2} \\ \text{fascio} \end{cases}$

Uno dei 2 non ha soluzioni.

$$\begin{cases} 2x^2-4xy-y^2-3y-2=0 \\ y+\frac{1}{2}x+\frac{3}{4}=0 \end{cases}$$

$$\begin{cases} 2x^2-4\left(-\frac{1}{2}x-\frac{3}{4}\right)x-\boxed{\left(-\frac{1}{2}x-\frac{3}{4}\right)^2}-3\left(-\frac{1}{2}x-\frac{3}{4}\right)-2=0 \\ y=-\frac{1}{2}x-\frac{3}{4} \end{cases}$$

1° $2x^2+(2x+3)x-\left(\frac{1}{4}x^2+\frac{9}{16}+\frac{3}{4}x\right)+\frac{3}{2}x+\frac{9}{4}-2=0$

$$2x^2+2x^2+3x-\frac{1}{4}x^2-\frac{9}{16}-\frac{3}{4}x+\frac{3}{2}x+\frac{9}{4}-2=0$$

$$\frac{15}{\cancel{4}}x^2+\frac{15}{\cancel{4}}x-\frac{5}{\cancel{16}_4}=0$$

$$15x^2+15x-\frac{5}{4}=0 \rightarrow 5x^2+5x-\frac{\cancel{5}^1}{\cancel{20}_4}$$

$$\boxed{5x^2+5x-\frac{1}{4}=0}$$

$$x_{12}=\frac{-5\pm\sqrt{20}}{10}$$

$$25-\cancel{4}\left(5\cdot\frac{1}{\cancel{4}}\right)$$

$$25-5=20$$

$$x_1=\frac{-5+\sqrt{20}}{10}=\frac{-\cancel{5}^1}{\cancel{10}_2}+\frac{\sqrt{20}}{10}$$

$$-\frac{1}{2}+\sqrt{4\cdot5}$$

$$-\frac{1}{2}+\frac{\cancel{2}\sqrt{5}}{\cancel{10}_5}=-\frac{1}{2}+\frac{\sqrt{5}}{5}$$

$$x_2=\frac{-5-\sqrt{20}}{10}=\frac{-5}{10}-\frac{\cancel{2}\sqrt{5}}{\cancel{10}_5}$$

$$\boxed{-\frac{1}{2}-\frac{\sqrt{5}}{5}=\frac{-5-2\sqrt{5}}{10}\quad x_2}$$

$$\boxed{x_1=\frac{-5+2\sqrt{5}}{10}}$$

2 SISTEMI

$$\begin{cases} x = \dfrac{-5-2\sqrt{5}}{10} \\ y = -\dfrac{1}{2}x - \dfrac{3}{4} \end{cases}$$

$$\begin{cases} x = \dots \\ y = -\dfrac{1}{2}\left(\dfrac{-5-2\sqrt{5}}{10}\right) - \dfrac{3}{4} \end{cases}$$

$$\begin{cases} x = \dots \\ y = -\dfrac{-5-2\sqrt{5}}{20} - \dfrac{3}{4} \end{cases}$$

$$\begin{cases} x = \dots \\ y = -\dfrac{\cancel{5}^{1}}{\cancel{20}_{4}} - \dfrac{\cancel{2}^{1}\sqrt{5}}{\cancel{20}_{10}} - \dfrac{3}{4} \end{cases}$$

$$\begin{cases} x = \dots \\ y = -\dfrac{1}{4} - \dfrac{\sqrt{5}}{10} - \dfrac{3}{4} \end{cases} \quad \begin{cases} x = \dots \\ y = -\dfrac{4}{4} - \dfrac{\sqrt{5}}{10} \end{cases}$$

$$\begin{cases} x = \dfrac{-5-2\sqrt{5}}{10} \\ y = \dfrac{\sqrt{5}}{10} \end{cases}$$ Vertice$_1$

$$\begin{cases} x = \dfrac{-5+2\sqrt{5}}{10} \\ y = -\dfrac{1}{2}x - \dfrac{3}{4} \end{cases}$$

$$\begin{cases} x = \dots \\ y = -\dfrac{1}{2}\left(\dfrac{-5+2\sqrt{5}}{10}\right) - \dfrac{3}{4} \end{cases}$$

$$\begin{cases} x = \dots \\ y = -\dfrac{-5+2\sqrt{5}}{20} - \dfrac{3}{4} \end{cases}$$

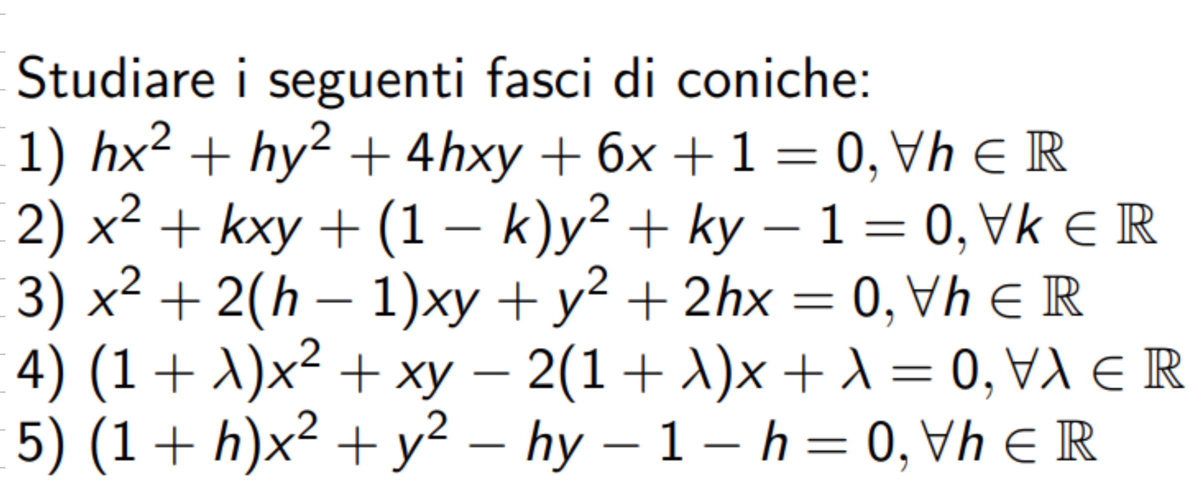
$$\begin{cases} x = \dots \\ y = -\dfrac{\cancel{5}^{1}}{\cancel{20}_{4}} + \dfrac{\cancel{2}^{1}\sqrt{5}}{\cancel{20}_{10}} - \dfrac{3}{4} \end{cases}$$

$$\begin{cases} x = \dots \\ y = -\dfrac{1}{4} + \dfrac{\sqrt{5}}{10} - \dfrac{3}{4} \end{cases} \quad \begin{cases} x = \dots \\ y = -\dfrac{4}{4} + \dfrac{\sqrt{5}}{10} \end{cases}$$

$$\begin{cases} x = \dfrac{-5-2\sqrt{5}}{10} \\ y = -\dfrac{\sqrt{5}}{10} \end{cases}$$ Vertice$_2$

$$V_1 = \left(\dfrac{-5-2\sqrt{5}}{10}, \dfrac{\sqrt{5}}{10}\right)$$

$$V_2 = \left(\dfrac{-5-2\sqrt{5}}{10}, -\dfrac{\sqrt{5}}{10}\right)$$



Studiare i seguenti fasci di coniche:

1) $hx^2 + hy^2 + 4hxy + 6x + 1 = 0, \forall h \in \mathbb{R}$

2) $x^2 + kxy + (1 - k)y^2 + ky - 1 = 0, \forall k \in \mathbb{R}$

3) $x^2 + 2(h - 1)xy + y^2 + 2hx = 0, \forall h \in \mathbb{R}$

4) $(1 + \lambda)x^2 + xy - 2(1 + \lambda)x + \lambda = 0, \forall \lambda \in \mathbb{R}$

5) $(1 + h)x^2 + y^2 - hy - 1 - h = 0, \forall h \in \mathbb{R}$

$\mathbb{R}^3 \to \mathbb{R}^3$, $f(x,y,z) = (-x-2y, -x, -x+(h+3)z)$ $h \in \mathbb{R}$

Im f. Ker f.

$$M(f) = \begin{pmatrix} -1 & -2 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & h+3 \end{pmatrix}$$

$$|M(f)| = -(2(h+3)) = -(2h+6)$$

$-2h-6 \neq 0$ per $\rho = 3$

$+\cancel{2}h \neq \cancel{\frac{6}{2}}^{-3}$ $\boxed{h \neq -3}$

$\dim \operatorname{Im} f = 3$, $\dim \operatorname{Ker} f = n - \rho = 0$

non è suriettive e iniettive, endomorfismo

Eq. cart. $\operatorname{Im} f = \{(x,y,z) \in \mathbb{R}^3 \mid x,y,z \in \mathbb{R}\}$ se $h \neq -3$

Eq. cart. $\operatorname{Ker} f = \{(x,y,z) \in \mathbb{R}^3 \mid x=y=z=0\}$ $\boxed{\text{se } h \neq -3}$

$\boxed{h = -3}$

$$M(f) = \begin{pmatrix} -1 & -2 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \qquad \rho < 2$$

$\rho = 2$ $\det = -2$ (2×2)

$\boxed{\dim \operatorname{Im} f = ②}$ $\dim \operatorname{Ker} = 3 - 2 = ①$

Eq. cart. Im f. $\operatorname{Im} f = \{C_1, C_2\}$

$$\begin{matrix} \to \\ \to \\ \to \end{matrix} \begin{vmatrix} -1 & -1 & -1 \\ -2 & 0 & 0 \\ x & y & z \end{vmatrix} = 0$$ per mantenere $\rho = 2$

$$\det = 2y - (-2z) = 0$$

$$\cancel{2}y + \cancel{2}z = 0$$

$$\boxed{y} = -z \quad \forall x, z$$

$$\boxed{h = -3}$$

$$\operatorname{Im} f = \{(x,y,z) \in \mathbb{R}^3 \mid \underbrace{y = -z}\} \quad \forall x, z.$$

$$\text{Base } \operatorname{Im} f = \{(x, -z, z)\} \to \{(1,0,0), (0,-1,1)\}$$

Eq. Ker f. $M(f) = \begin{pmatrix} -1 & -2 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{cases} -x - 2y = 0 \\ -x = 0 \\ -x = 0 \end{cases} \quad \begin{cases} 0 - 2y = 0 \\ x = 0 \\ \cancel{x = 0} \end{cases} \quad \begin{cases} -\cancel{2}y = 0 \\ x = 0 \end{cases} \quad \begin{cases} y = 0 \\ x = 0 \\ \forall z \end{cases}$$

$$\boxed{\operatorname{Ker} f = \{(x,y,z) \in \mathbb{R}^3 \mid x = y = 0\} \ \forall z \ \text{ se } h = -3}$$

$$\boxed{\text{Base } \operatorname{Ker} f = \{(0,0,z)\} \to (0,0,1)}$$

DIAGONALIZZABILE per quale h?

$$M(f) = \begin{pmatrix} -1 & -2 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & h+3 \end{pmatrix} = \text{POL. CAR.}$$

$$\det = 0$$

$$\begin{pmatrix} -1-T & -2 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & h+3-T \end{pmatrix} \Rightarrow -[2\underbrace{(h+3-T)}]$$

$$= -[2h + 6 - 2T]$$

$$= -\cancel{2}h - \underset{3}{\cancel{6}} + \cancel{2}T = 0$$

$$= \overline{-h-3} + T = 0 \to T = h+3$$

$T = h+3$ $\quad m_{h+3} = 1 \rightsquigarrow g_{h+3} = 1$ f e' semplice

posso solo controllare per $h=-3$

$$M(f) = \begin{pmatrix} -1 & -2 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -1-T & -2 & 0 \\ -1 & -T & 0 \\ -1 & 0 & -T \end{pmatrix}$$

$$T^2(-1-T) - [-2T]$$

$$T^2(-1-T) + 2T = 0$$

$$T[T(-1-T)+2] = 0$$

$$T = 0$$

$$-T - T^2 + 2 = 0$$

$\hookrightarrow T^2 + T - 2 = 0 \qquad 1 - 4(-2) \qquad 1 + 8 = 9 \qquad \frac{-1 \pm 3}{2} \begin{cases} \frac{2}{2} = 1 \\ -\frac{4}{2} = -2 \end{cases}$

$T=0,\ T=1,\ T=-2$

$$\left.\begin{matrix} m_0 = 1 & g_0 = 1 \\ m_1 = 1 & g_1 = 1 \\ m_{-2} = 1 & g_{-2} = 1 \end{matrix}\right\}$$ f e' semplice e diagonalizzabile $\forall h$

STUDIARE FASCIO DI coniche.

$$x^2 + hy^2 + 2hxy + hx - hy = 0$$

$$|B| = \begin{vmatrix} 1 & h & \frac{1}{2}h \\ h & h & -\frac{1}{2}h \\ \frac{1}{2}h & -\frac{1}{2}h & 0 \end{vmatrix} \begin{matrix} 1 & h \\ h & h \\ \frac{1}{2}h & -\frac{1}{2}h \end{matrix}$$

$$h\left(-\frac{1}{2}h\right)\left(\frac{1}{2}h\right)+h\left(\frac{1}{2}h\right)\left(-\frac{1}{2}h\right)-\left[\left(\frac{1}{2}h\right)^2+h\left(\frac{1}{2}h\right)^2\right]=$$

$$h\left(-\frac{1}{2}h\right)\left(\frac{1}{2}h\right)+h\left(\frac{1}{2}h\right)\left(-\frac{1}{2}h\right)-\left(\frac{1}{2}h\right)^2-h\left(\frac{1}{2}h\right)^2$$

$$h\left(-\frac{1}{2}h\right)\left(\frac{1}{2}h\right)+h\left(\frac{1}{2}h\right)\left(-\frac{1}{2}h\right)-\frac{1}{4}h^2-\frac{1}{4}h^3=$$

$$h\left(-\frac{1}{4}h^2\right)+\left(\frac{1}{2}h^2\right)\left(-\frac{1}{2}h\right)-\frac{1}{4}h^2-\frac{1}{4}h^3$$

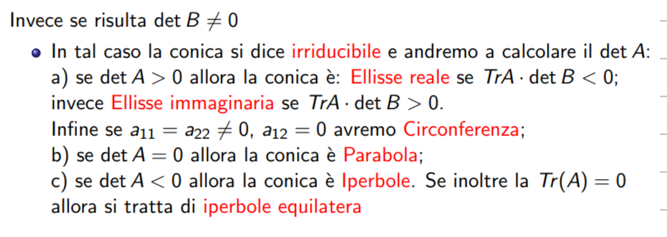
$$\cancel{-\frac{1}{4}h^3}+\cancel{\frac{1}{4}h^3}-\frac{1}{4}h^2-\frac{3}{4}h^3=$$ (PHOTO MATH)

$$h^2\left(-\frac{1}{4}-\frac{3}{4}h\right)\neq 0$$

$$h^2\neq 0\rightarrow\boxed{h\neq 0}$$

$$-\frac{1}{4}-\frac{3}{4}h\neq 0\rightarrow-\frac{3}{\cancel{4}}h\neq\frac{1}{\cancel{4}}\Rightarrow -3h\neq 1=\boxed{h\neq-\frac{1}{3}}$$

$$\begin{pmatrix}1&h&\frac{1}{2}h\\h&h&-\frac{1}{2}h\\\frac{1}{2}h&-\frac{1}{2}h&0\end{pmatrix}=B\qquad |A|=\begin{vmatrix}1&h\\h&h\end{vmatrix}$$



Invece se risulta det $B\neq 0$

- In tal caso la conica si dice irriducibile e andremo a calcolare il det A:
  a) se det $A>0$ allora la conica è: Ellisse reale se $TrA\cdot\det B<0$; invece Ellisse immaginaria se $TrA\cdot\det B>0$.
  Infine se $a_{11}=a_{22}\neq 0$, $a_{12}=0$ avremo Circonferenza;
  b) se det $A=0$ allora la conica è Parabola;
  c) se det $A<0$ allora la conica è Iperbole. Se inoltre la $Tr(A)=0$ allora si tratta di iperbole equilatera

$$h-h^2<0\qquad h^2-h=0\qquad h(h-1)=0$$

$$h=0\qquad h=1$$

$$\begin{cases}h<0\\h\neq 0\end{cases}\qquad\begin{matrix}h>1\\h\neq\frac{1}{3}\end{matrix}$$

$$T_rA=0$$

$$1+h=0$$

$$\boxed{h=-1}$$ ok ✓

$$\alpha X^2+\beta Y^2=\gamma$$

$$\gamma=-\frac{|B|}{|A|}=\frac{h^2\left(-\frac{1}{4}-\frac{3}{4}h\right)}{1+h}=\frac{-\frac{1}{4}h^2-\frac{3}{4}h^3}{1+h}$$

$$=-\frac{1}{4}h^2-\frac{3}{4}h^3\cdot\frac{1}{1+h}=1+h\cdot\left(-\frac{1}{4}h^2-\frac{3}{4}h^3\right)=$$

NON SO

$A=(0,0,1)$ PIANO π_1 ; $2x-y+3=0$ π_2 : $x+y-z-3=0$

$A'=?$ simm. di A, rispetto a π_1. $\bar{v}_{\pi_1}=(2,-1,3)$

retta t che passa per A $\perp$ π_2

$\bar{v}_{\pi_2}=(1,1,-1)$

DIST. PUNTO-PIANO

π_1 $2x-y+3=0$ $A=(0,0,1)$

$$\overline{P_0H}=\frac{|ax_0+by_0+cz_0|}{\sqrt{a^2+b^2+c^2}}=\overline{P_0H}=\frac{|0+0+3|}{\sqrt{4+1+9}}=\frac{3}{\sqrt{14}}$$

$$\overline{P_0H}=\frac{3}{\sqrt{14}}\rightarrow\frac{3\sqrt{14}}{14}$$

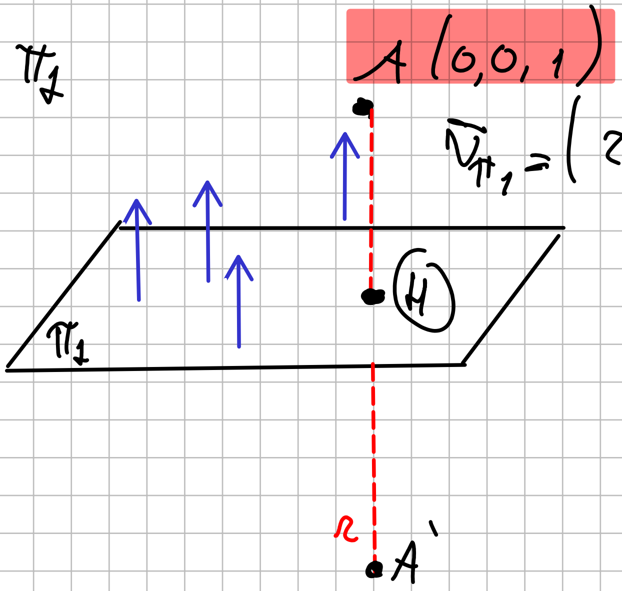
retta che passa per A, $\perp$ π_1

$$r:\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$$

$$:\frac{x-0}{l}=\frac{y-0}{m}=\frac{z-1}{n}$$

$$\frac{x}{2}=\frac{y}{-1}=\frac{z-1}{0}$$

$$\begin{cases}\frac{1}{2}x=-y\\-y=\frac{z-1}{0}\end{cases}\quad\begin{cases}\cancel{\frac{1}{2}}x=-2y\\z-1=0\end{cases}\quad z=1$$



$\bar{v}_{\pi_1}=(2,-1,0)$

Punto Intersezione r e π_1

$$\begin{cases}\frac{1}{2}x+y=0\\-3y-z+1=0\\2x-y+3=0\end{cases}\quad\begin{cases}\ldots\\-6x-9-z+1=0\\y=2x+3\end{cases}\rightarrow\begin{cases}\frac{1}{2}x+2x+3=0\\-6x-8-z=0\rightarrow z=-6x-8\\\ldots\end{cases}$$

$$\begin{cases} \frac{5}{2}x+3=0 \\ \dots \\ \dots \end{cases} \qquad \begin{cases} \frac{5}{2}x=-3 \quad 5x=-6 \quad x=-\frac{6}{5} \\ z=-6\left(-\frac{6}{5}\right)-8 \Rightarrow z=\frac{36}{5}-8 \quad \boxed{z}=\frac{36-40}{5}=\boxed{-\frac{4}{5}} \\ y=2\left(-\frac{6}{5}\right)+3 \quad y=-\frac{12}{5}+3 \quad \frac{-12+15}{5}=\frac{3}{5} \end{cases}$$

$$H=\left(-\frac{6}{5}, \frac{3}{5}, -\frac{4}{5}\right)$$

punto medio $M=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

$$\frac{A+A'}{2}=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

$$A=(\underset{\substack{\parallel\\0}}{x_A}, \underset{\substack{\parallel\\0}}{y_A}, \underset{\substack{\parallel\\1}}{z_A}) \qquad A'=(x_{A'}, y_{A'}, z_{A'})$$

PUNTO MEDIO $M=\left(-\frac{6}{5}, \frac{3}{5}, -\frac{4}{5}\right)$

$x_M=\frac{x_A+x_{A'}}{2}$ $\qquad 2x_M=x_A\boxed{+x_{A'}}$

$x_{A'}=2x_M-x_A$ $\qquad x_{A'}=2\left(-\frac{6}{5}\right)-0$

$x_{A'}=-\frac{12}{5}$

$y_M=\frac{y_A+y_{A'}}{2} \longrightarrow y_{A'}=2y_M-y_A$ $\qquad y_{A'}=\frac{6}{5}$

$z_M=\frac{z_A+z_{A'}}{2} \longrightarrow z_{A'}=2z_M-z_A$ $\qquad z_{A'}=-\frac{8}{5}-1=\frac{-8-5}{5}=-\frac{13}{5}$

A' simmetrico $=\left(-\frac{12}{5}, \frac{6}{5}, -\frac{13}{5}\right)$