

**Esercizio 1.** Risolvere l'equazione

$$\frac{z^2}{|z|} = 2$$

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$$z = a + ib$$

$$z^2 = (a + ib)^2 = a^2 - b^2 + 2iab$$

$$|z| = \sqrt{a^2 + b^2}$$

$$\frac{a^2 - b^2 + 2iab}{\sqrt{a^2 + b^2}} = 2$$

$$\frac{a^2 - b^2 + 2iab}{\sqrt{a^2 + b^2}} - 2 = 0$$

$$\frac{a^2 - b^2 + 2iab - 2(\sqrt{a^2 + b^2})}{\sqrt{a^2 + b^2}} = 0$$

$$a^2 - b^2 + 2iab - 2\sqrt{a^2 + b^2} = 0$$

$$a^2 - b^2 - 2\sqrt{a^2 + b^2} + 2iab = 0$$

$$\begin{cases} a^2 - b^2 - 2\sqrt{a^2 + b^2} = 0 \\ 2ab = 0 \end{cases}$$

$$\begin{cases} a^2 - 2\sqrt{a^2} = 0 \\ b = 0 \end{cases} \quad (1)$$

$$\begin{cases} -b^2 - 2b = 0 \\ a = 0 \end{cases} \quad (2)$$

$$(1) \begin{cases} a^2 - 2a = 0 \\ b = 0 \end{cases} \quad \begin{cases} a(a - 2) = 0 \\ b = 0 \end{cases}$$

$$\begin{cases} a = 0 \\ b = 0 \end{cases}$$

$$\begin{cases} a = 2 \\ b = 0 \end{cases}$$

$$\begin{cases} b = 0 \\ a = 0 \end{cases}$$

$$\begin{cases} b = -2 \\ a = 0 \end{cases}$$

$$(2) \begin{cases} b^2 + 2b = 0 \\ a = 0 \end{cases} \quad \begin{cases} b(b + 2) = 0 \\ a = 0 \end{cases}$$

$$\begin{cases} a=0 \\ b=0 \end{cases}$$

$$\begin{cases} a=2 \\ b=0 \end{cases}$$

$$\begin{cases} b=0 \\ a=0 \end{cases}$$

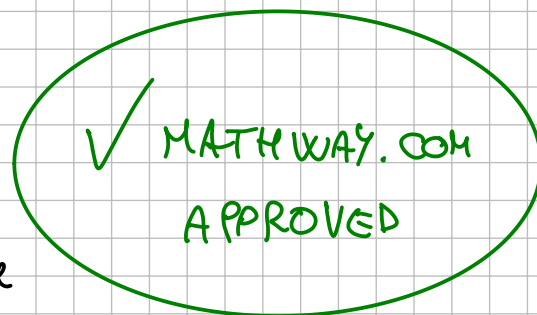
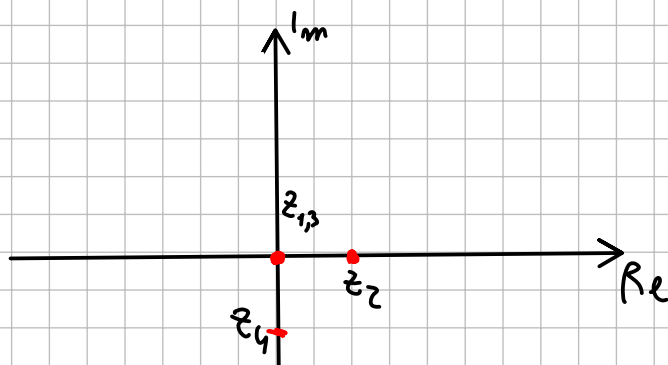
$$\begin{cases} b=-2 \\ a=0 \end{cases}$$

$$z_1 = 0 + i0$$

$$z_2 = 2 + i0$$

$$z_3 = 0 + i0$$

$$z_4 = 0 - 2i$$



**Esercizio 1.** Risolvere l'equazione

$$|z|^2 = 3z$$

nel campo complesso e rappresentarne graficamente le soluzioni.

$$|z|^2 = \left(\sqrt{a^2 + b^2}\right)^2 \rightarrow |z|^2 = a^2 + b^2$$

$$z = a + ib \rightarrow 3z = 3a + 3ib$$

$$a^2 + b^2 = 3a + 3ib$$

$$a^2 + b^2 - 3a - 3ib = 0$$

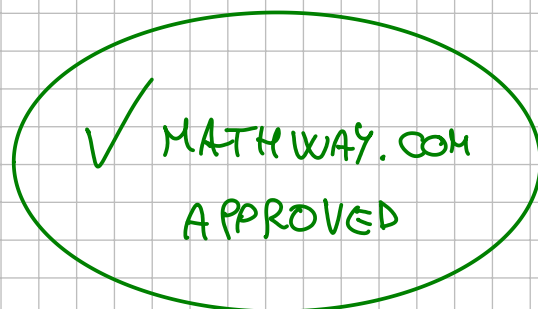
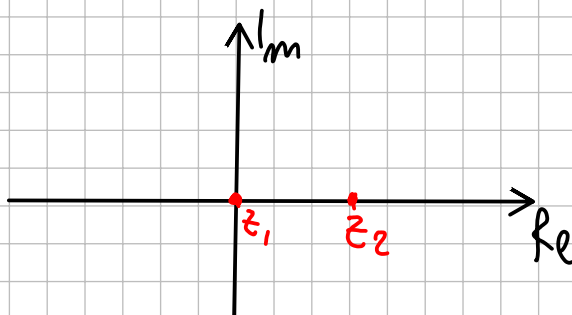
$$\begin{cases} a^2 + b^2 - 3a = 0 \\ -3b = 0 \end{cases} \rightarrow \begin{cases} a^2 - 3a = 0 \\ b = 0 \end{cases} \rightarrow \begin{cases} a(a-3) = 0 \\ b = 0 \end{cases}$$

$$\begin{cases} a=0 \\ b=0 \end{cases}$$

$$\begin{cases} a=3 \\ b=0 \end{cases}$$

$$z_1 = 0 + i0$$

$$z_2 = 3 + i0$$



# Esercizio 1. Risolvere l'equazione

$$z^3 + |z|^3 = 0$$

nel campo dei numeri complessi e disegnare l'insieme delle soluzioni.

$$z^3 = (a + ib)^3 \quad |z|^3 = (\sqrt{a^2 + b^2})^3$$

$$z^3 = a^3 + 3ia^2b + 3ia^2b^2 + i^3b^3$$

$$\hookrightarrow a^3 + 3ia^2b - 3ab^2 - ib^3 \quad (z^3)$$

$$\hookrightarrow a^3 - 3ab^2 + 3ia^2b - ib^3 \quad \leftarrow$$

$$|z|^3 = (\sqrt{a^2 + b^2})^3 = (a^2 + b^2)^{\frac{3}{2}} = (a^2 + b^2)^{\frac{1}{2}} \cdot (a^2 + b^2)^{\frac{2}{2}}$$

$$\frac{3}{2} = \frac{1}{2} + \frac{2}{2}$$

$$\frac{\sqrt{a^2 + b^2} \cdot (a^2 + b^2)}{a^2 \sqrt{a^2 + b^2} + b^2 \sqrt{a^2 + b^2}}$$

$$|z|^3 = a^2 \sqrt{a^2 + b^2} + b^2 \sqrt{a^2 + b^2}$$

$$a^3 - 3ab^2 + \underline{3ia^2b} - \underline{ib^3} + a^2 \sqrt{a^2 + b^2} + b^2 \sqrt{a^2 + b^2} = 0$$

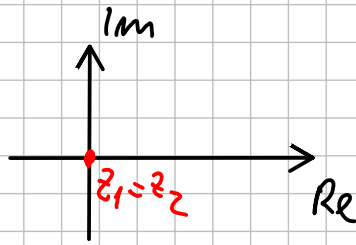
$$\begin{cases} a^3 - 3ab^2 + a^2 \sqrt{a^2 + b^2} + b^2 \sqrt{a^2 + b^2} = 0 \\ 3a^2b - b^3 = 0 \end{cases}$$

$$\begin{cases} \text{---} \\ b(3a^2 - b^2) = 0 \end{cases} \begin{cases} \begin{cases} a^3 + a^3 = 0 \\ b = 0 \end{cases} \begin{cases} 2a^3 = 0 \\ b = 0 \end{cases} \begin{cases} a = 0 \\ b = 0 \end{cases} \\ \begin{cases} \text{---} \\ 3a^2 - b^2 = 0 \end{cases} \begin{cases} a^3 - 9a^3 + a^2 \sqrt{a^2 + 3a^2} + 3a^2 \sqrt{a^2 + 3a^2} = 0 \\ b^2 = 3a^2 \end{cases} \end{cases}$$

$$\begin{cases} -8a^3 + a^2 \sqrt{4a^2} + 3a^2 \sqrt{4a^2} = 0 \\ b^2 = 3a^2 \end{cases} \begin{cases} -8a^3 + 2a^3 + 12a^3 = 0 \\ b^2 = 3a^2 \end{cases}$$

$$\begin{cases} 6a^3 = 0 \\ b^2 = 3a^2 \end{cases} \quad \begin{cases} a = 0 \\ b = 0 \end{cases}$$

$$z_1 = z_2 = 0 + i0$$



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