CALCOLARE LA DERIVATA.

$$f(x)=\sqrt{x+2} = (x+2)^{\frac{1}{2}} = \frac{1}{2}(x+2)^{-\frac{1}{2}} = \frac{1}{2}\frac{1}{\sqrt{x+2}} = \frac{1}{2\sqrt{x+2}}$$

STUDIARE LA FUNZIONE $f(x)=x^3-4x$

- DOMINIO $\mathbb{R}$
- SIMMETRIE $f(-x)=?$

$$f(-x)=(-x)^3-4(-x)=-x^3+4x \quad (\text{DISPARI})$$

- SEGNO E INTERSEZ. ASSI

$x^3-4x\geq 0 \qquad x(x^2-4)\geq 0$

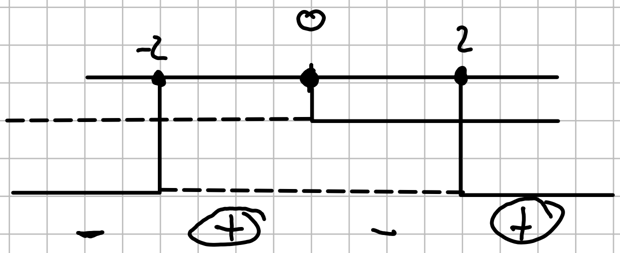
$$\begin{cases} x\geq 0 \\ x^2-4\geq 0 \end{cases}$$

$x^2-4=0$

$x^2=4 \rightarrow x=\pm 2$

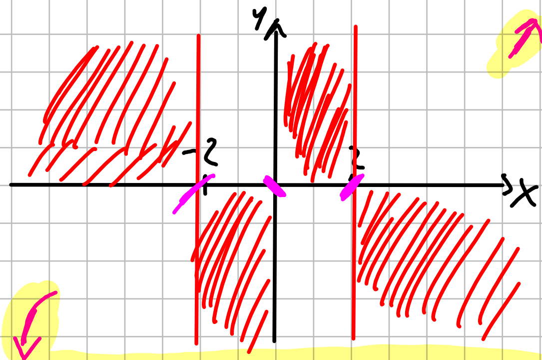
$\rightarrow x\leq 2 \wedge x\geq 2$

$$\begin{cases} -2\leq x\leq 0 \\ x\geq 2 \end{cases}$$



INTERSEZ. ASSI. $x=-2$
$x=0$
$x=2$
$(0,0)$

BOZZA



4) LIMITI $\pm\infty$

$$\lim_{x\to+\infty} x^3-4x = \underbrace{x^3}_{+\infty}\left(1-\underbrace{\frac{4x}{x^3}}_{\to 0}\right) = +\infty$$

$$\lim_{x\to-\infty} x^3-4x = \underbrace{x^3}_{-\infty}\left(1-\underbrace{\frac{4x}{x^3}}_{\to 0}\right) = -\infty$$

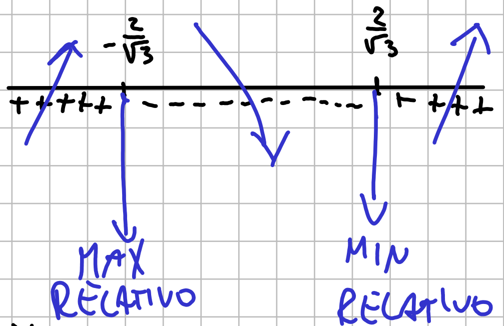
5) $f'(x) > 0$

$f(x) = x^3 - 4x \longrightarrow f'(x) = 3x^2 - 4$

$3x^2 - 4 > 0$ — $3x^2 - 4 = 0$

$\cancel{3}x^2 = \pm\sqrt{\frac{4}{3}}$ $\quad \pm \frac{2}{\sqrt{3}}$

$x > \frac{2}{\sqrt{3}} \wedge x < -\frac{2}{\sqrt{3}}$



BOZZA

TROVO PUNTO MAX/MIN REL.

$f(x) = x^3 - 4x$

$\pm\frac{2}{\sqrt{3}}$

$f\left(\frac{2}{\sqrt{3}}\right) = \left(\frac{2}{\sqrt{3}}\right)^3 - 4\left(\frac{2}{\sqrt{3}}\right) =$

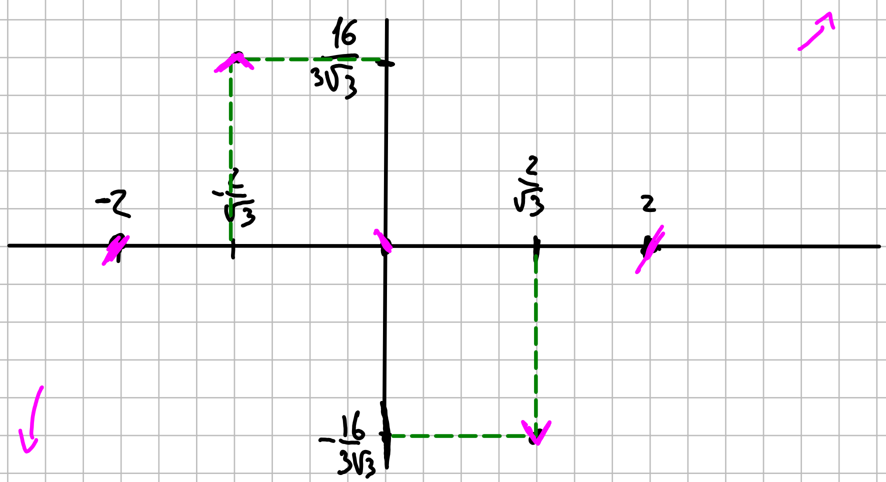
$\left(\frac{2}{\sqrt{3}}\right)^3 = \frac{8}{\sqrt{27}} = \frac{8}{\sqrt{9 \cdot 3}} = \frac{8}{3\sqrt{3}}$

$= \frac{8}{3\sqrt{3}} - \frac{8}{\sqrt{3}} = \frac{8 - 24}{3\sqrt{3}} = -\frac{16}{3\sqrt{3}}$ (y)

MAX REL. $\left(\frac{2}{\sqrt{3}}, -\frac{16}{3\sqrt{3}}\right)$

MIN REL. $\left(-\frac{2}{\sqrt{3}}, \frac{16}{3\sqrt{3}}\right)$

non calcolato perché la $f(x)$ è DISPARI.



CONCAVITA'

$f(x) = x^3 - 4x \qquad f'(x) = 3x^2 - 4 \qquad f''(x) = 6x$

$6x > 0 \qquad x > 0$

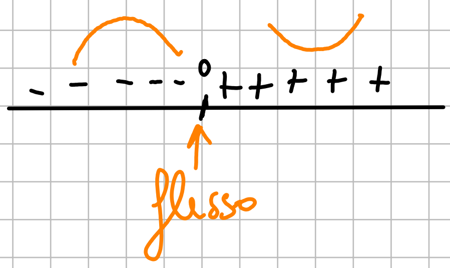
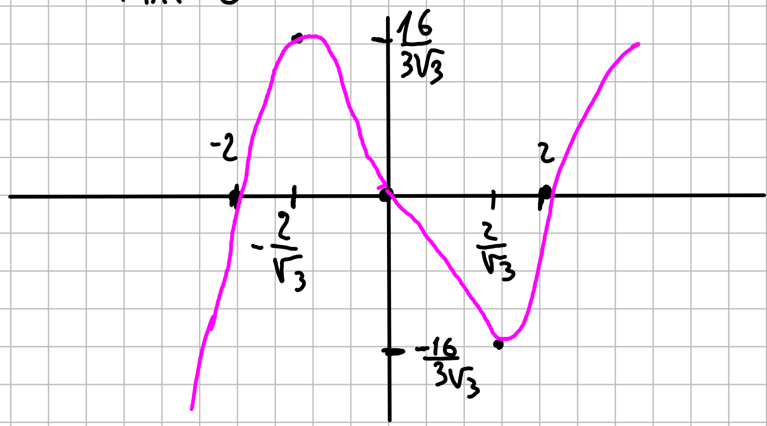


GRAFICO QUALITATIVO



STUDIARE LA FUNZIONE $f(x) = \dfrac{2x^3}{x^2-1}$

1) DOMINIO $x^2-1 \neq 0$ $\boxed{x \neq 1 \quad \wedge \quad x \neq -1}$

2) Simmetrie

$$f(-x) = \frac{2(-x)^3}{x^2-1} = -\frac{2x^3}{x^2-1} = \text{DISPARI}$$

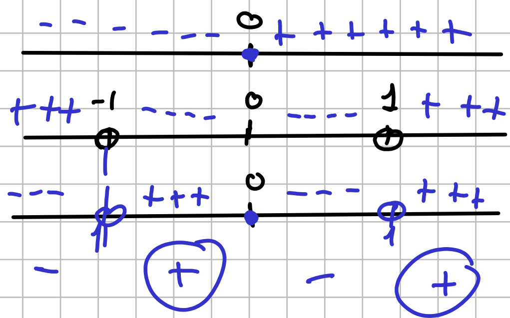
3) Segno e intercett. assi.

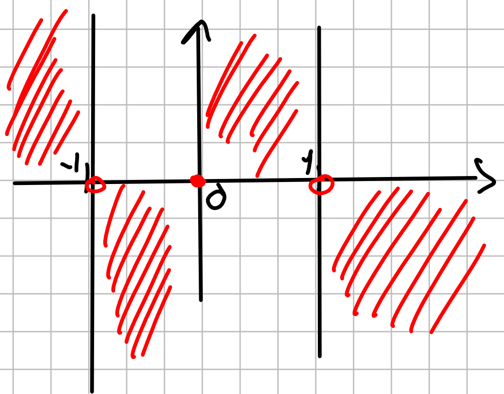
$$\frac{2x^3}{x^2-1} \geq 0$$

$2x^3 \geq 0$

$x^2-1>0$

$\dfrac{2x^3}{x^2-1}$





4) LIMITI

$$\lim_{x\to+\infty} \frac{2x^3}{x^2-1} = \frac{\overbrace{2x^{\cancel{3}}}^{\to+\infty}}{\cancel{x^2}\left(1-\underbrace{\frac{1}{x^2}}_{\to 0}\right)} = +\infty$$

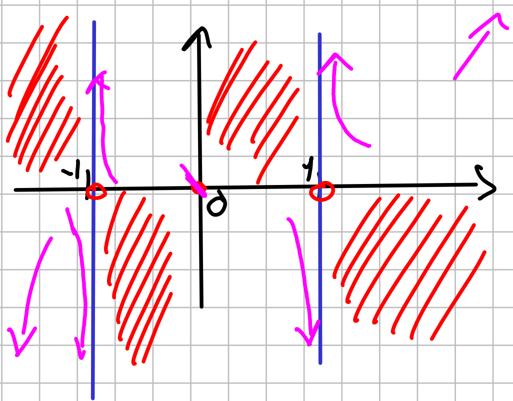
$$\lim_{x\to-\infty} \frac{2x^3}{x^2-1} = \frac{\overbrace{2x^{\cancel{3}}}^{\to-\infty}}{\cancel{x^2}\left(1-\underbrace{\frac{1}{x^2}}_{\to 0}\right)} = -\infty$$

$$\lim_{x\to1^+} \frac{2x^3}{x^2-1} = \frac{+2}{0^+} = +\infty$$

$$\lim_{x\to1^-} \frac{2x^3}{x^2-1} = \frac{2}{0^-} = -\infty$$

$$\lim_{x\to-1^+} \frac{2x^3}{x^2-1} = \frac{-2}{0^-} = +\infty$$

$$\lim_{x\to-1^-} \frac{2x^3}{x^2-1} = \frac{-2}{0^+} = -\infty$$



5) $f'(x)$ di $\frac{2x^3}{x^2-1}$

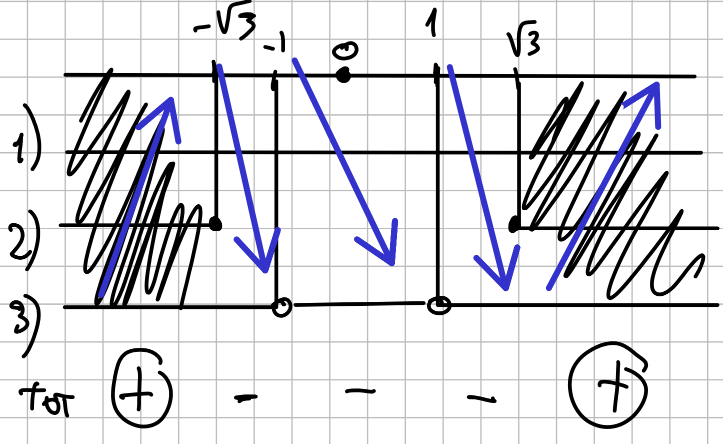
$$\frac{6x^2(x^2-1)-2x^3\cdot 2x}{(x^2-1)^2} = \frac{6x^4-6x^2-4x^4}{(x^2-1)^2} = \frac{2x^4-6x^2}{(x^2-1)^2}$$

$$\boxed{f'(x) = \frac{2x^2(x^2-3)}{(x^2-1)^2}} > 0 \qquad \begin{cases} 1)\ 2x^2 > 0 \\ 2)\ x^2-3 > 0 \\ 3)\ (x^2-1)^2 > 0 \end{cases}$$

$x^2 - 3 > 0$

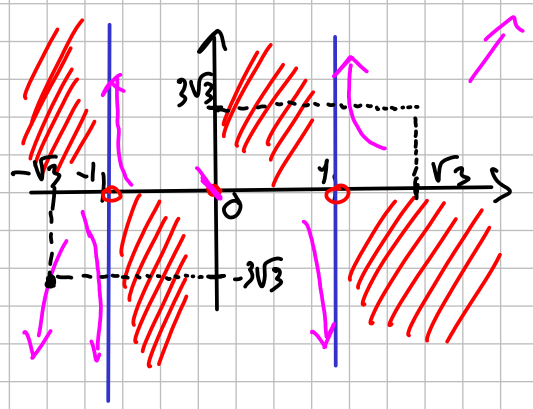
$\cancel{x^2} \pm\sqrt{3}$

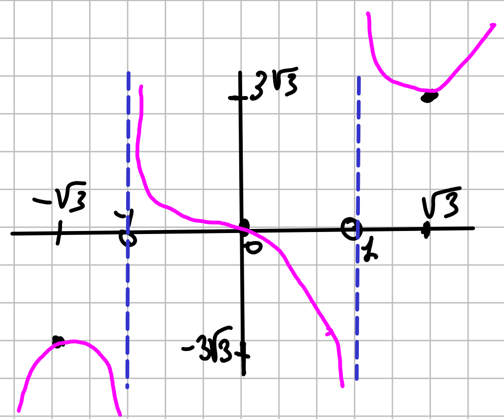
$(x^2-1)^2$



$x \leq -\sqrt{3}$, $x \geq \sqrt{3}$, $-1 < x < 1$

MAX. REL. $-\sqrt{3}$ / MIN REL. $\sqrt{3}$





MAX. REL. $f(-\sqrt{3})$

$\to (-\sqrt{3}, -3\sqrt{3})$

$$\frac{2x^3}{x^2-1} \to \frac{2(-\sqrt{3})^3}{\cancel{\sqrt{3}^2}-1} = \cancel{2}\left(\frac{-\sqrt{27}}{\cancel{2}}\right) -3\sqrt{3}$$

MIN REL $(\sqrt{3}, 3\sqrt{3})$

6) CONCAVITÀ

$$f''(x) \Rightarrow f'(x) = \frac{2x^2(x^2-3)}{(x^2-1)^2} = \boxed{\frac{2x^4-6x^2}{(x^2-1)^2}}$$

$(x^2-1)^2 =$

x^4+1-2x^2

$4x^3-4x$

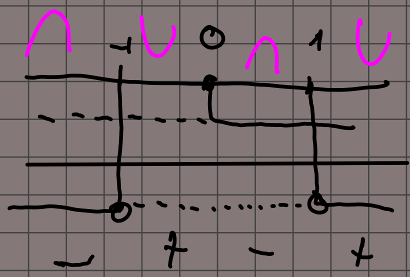
$$f''(x) = \frac{(8x^3-12x)\cdot(x^2-1)^2 - (2x^4-6x^2)\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4} =$$

$$= \frac{\cancel{(x^2-1)}\cdot\left[(8x^3-12x)(x^2-1) - \underbrace{2(2x^4-6x^2)(2x)}_{(-4x)}\right]}{(x^2-1)^{\cancel{4}\,3}} =$$

$$= \frac{\cancel{8x^5} - 8x^3 - 12x^3 + 12x - \cancel{8x^5} + 24x^3}{(x^2-1)^3} = \frac{4x^3+12x}{(x^2-1)^3} = \boxed{\frac{4x(x^2+3)}{(x^2-1)^3}}$$

$f''(x) > 0$

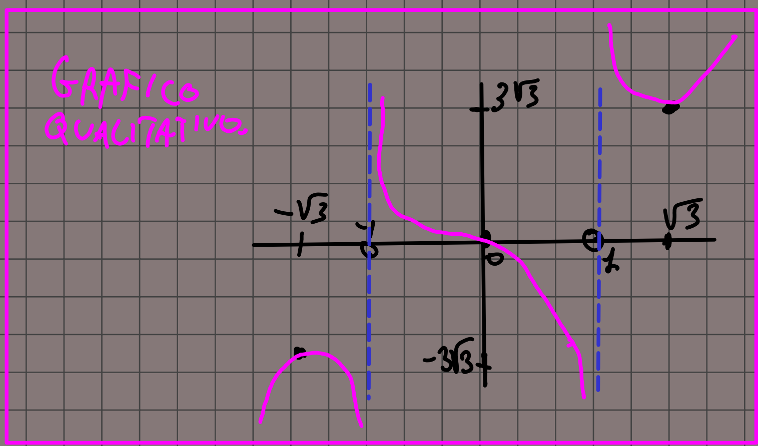
$$\begin{cases} 1)\ 4x > 0 \\ 2)\ x^2+3 > 0 \\ 3)\ (x^2-1)^3 > 0 \end{cases}$$



$(x^2-1)^3 = 0$

$x^2 - 1 = 0$

$x = \pm 1$



1) $f(x) = x^3 - 3x + 2$ STUDIARE LA FUNZIONE

1) DOMINIO $= \mathbb{R}$

2) $f(-x) = (-x)^3 - 3(-x) + 2$

$\hookrightarrow -x^3 + 3x + 2$

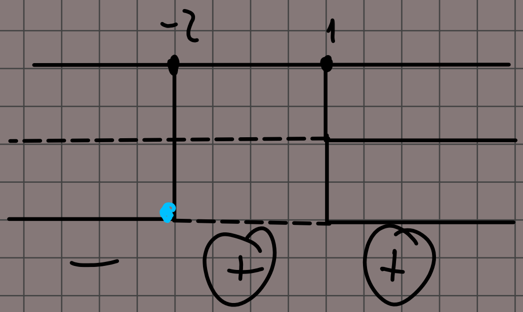
3) SEGNO $f(x) \geq 0 \rightarrow x^3 - 3x + 2 \geq 0$

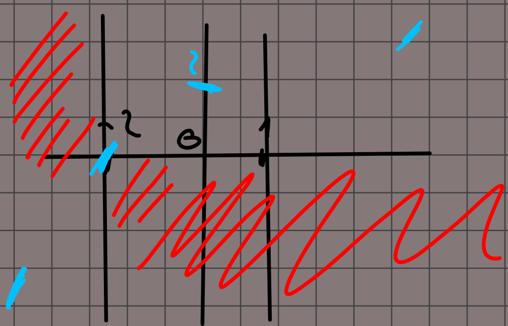
	1	0	-3	2
1		1	1	-2
	1	1	-2	0

$(x-1)(\underline{x^2+x-2}) \geq 0$ — $x \geq 1$

$\frac{-1 \pm 3}{2}$ — 1, -2 $x \leq -2 \wedge x \geq 1$

$1 - 4(-2) = 9$





4) LIMITI 1) $f(x) = x^3 - 3x + 2$

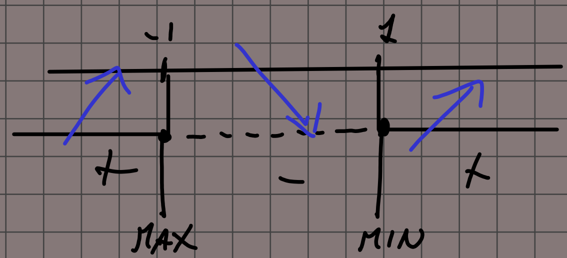
$$\lim_{x\to+\infty} x^3 - 3x + 2 = \underset{+\infty}{x^3}\left(1 - \underset{0}{\frac{3x}{x^2}} + \underset{0}{\frac{2}{x^3}}\right) = +\infty$$

$$\lim_{x\to-\infty} x^3 - 3x + 2 = \underset{-\infty}{x^3}\left(1 - \underset{0}{\frac{3x}{x^2}} + \underset{0}{\frac{2}{x^3}}\right) = -\infty$$

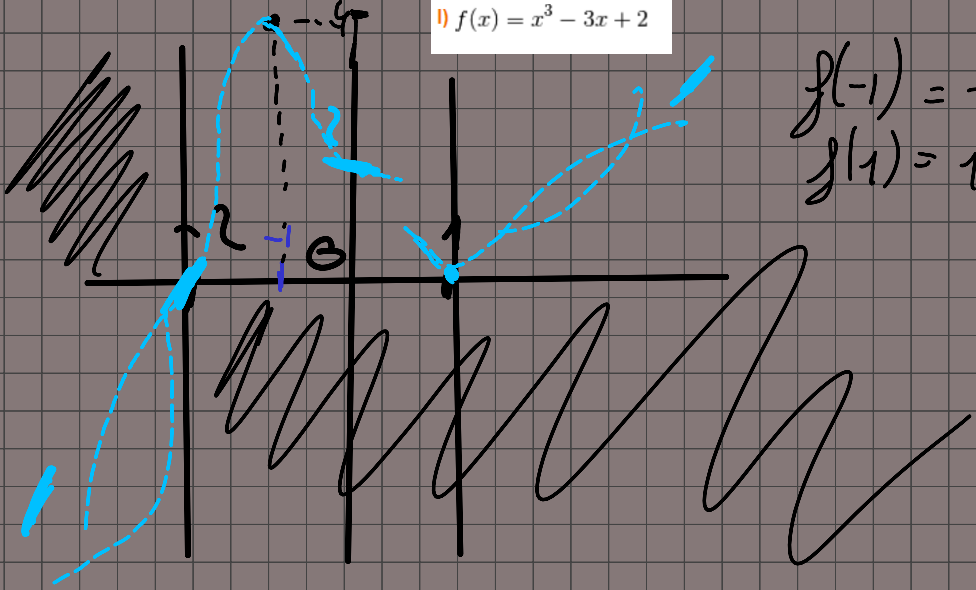
5) $f'(x) > 0$ $f'(x) = 3x^2 - 3$

$3x^2 - 3 > 0$ $\boxed{x < -1 \wedge x > 1}$

$3x^2 - 3 = 0$ $\cancel{3}x^{\cancel{2}} = \pm\cancel{3}1$



I) $f(x) = x^3 - 3x + 2$



$f(-1) = -1 + 3 + 2 = 4$

$f(1) = 1 - 3 + 2 = 0$

6) $f''(x) > 0$ $\quad f''(x) = 6x$

$6x > 0$

$3x^2 - 3$