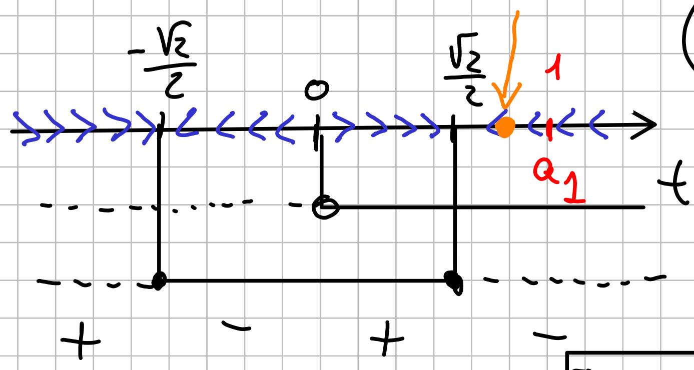
Esercizio 3. Calcolare il limite della successione

$$\begin{cases} a_1 = 1 \\ a_{n+1} = \dfrac{2a_n^2+1}{4a_n} \end{cases}$$

$$f(t) = \frac{2t^2+1}{4t} \qquad p(t) = \frac{2t^2+1}{4t} - t$$

$$\to \frac{2t^2+1-4t^2}{4t} = \boxed{\frac{-2t^2+1}{4t} > 0}$$

$$p(t) > 0 \quad \begin{cases} -2t^2+1 \geq 0 \\ 4t > 0 \end{cases} \quad \begin{cases} t = \pm\frac{1}{\sqrt{2}} \to \pm\frac{\sqrt{2}}{2} \\ t > 0 \end{cases}$$



$1^2 < \frac{\sqrt{2}}{4}$

$\frac{3}{4} > \frac{\sqrt{2}}{2}$

$\frac{9}{16} > \frac{2}{4}$, $36 > 32$

$]-\infty, -\frac{\sqrt{2}}{2}]$ $a_n \to -\frac{\sqrt{2}}{2}$	$[-\frac{\sqrt{2}}{2}, 0[$ $a_n \to -\frac{\sqrt{2}}{2}$	$]0, \frac{\sqrt{2}}{2}]$ $a_n \to \frac{\sqrt{2}}{2}$	$[\frac{\sqrt{2}}{2}, +\infty[$ $a_n \to \frac{\sqrt{2}}{2}$

2) LOCALIZZAZIONE

- $a_1 = 1$ $\quad \frac{\sqrt{2}}{2} \leq a_1 < +\infty$
- $a_1 = 1 \in [\frac{\sqrt{2}}{2}, +\infty[$ $\quad a_n \to \frac{\sqrt{2}}{2}$

$$\frac{2t^2+1}{4t} = \frac{3}{4}$$

$$\boxed{a_{n+1} = \frac{3}{4} \in \left[\frac{\sqrt{2}}{2}, +\infty\right[, \quad a_{n+1} \to \frac{\sqrt{2}}{2}}$$

Esercizio 3. Determinare il limite della successione

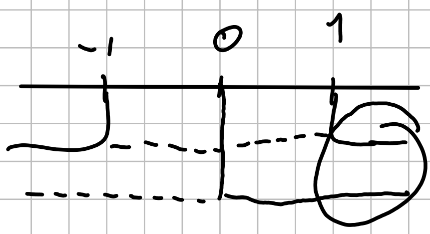
$$\begin{cases} a_1 = 2 \\ a_{n+1} = a_n^2 + |a_n| - 1 \quad \text{per ogni } n \in \mathbb{N}. \end{cases}$$

$g(t) = t^2 + |t| - 1$ $\qquad$ $f(t) = t^2 + |t| - 1 - t > 0$

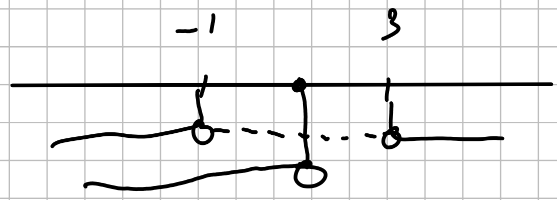
$\begin{cases} t \geq 0 \\ t^2 - 1 > 0 \end{cases}$ $\qquad$ $\begin{cases} t < 0 \\ t^2 - 2t - 1 > 0 \end{cases}$

$\Delta = 4 - 4(-1) = 16$

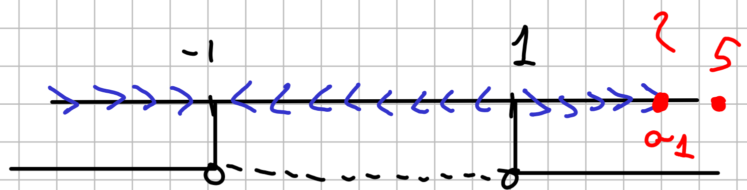
$t_{1,2} = \dfrac{2 \pm 4}{2} \left\langle \begin{array}{l} 3 \\ -1 \end{array} \right.$



$t > 1$



$t < -1$



$]-\infty, -1[$ $\qquad$ $]-1, 1[$ $\qquad$ $]1, +\infty[$

$a_n \to -1$ $\qquad$ $a_n \to -1$ $\qquad$ $a_n \to +\infty$

$t^2 + |t| - 1 \to \quad 2^2 + 2 - 1 = 5$

$a_{n+1} \to +\infty$

Esercizio 3. Determinare il limite della successione

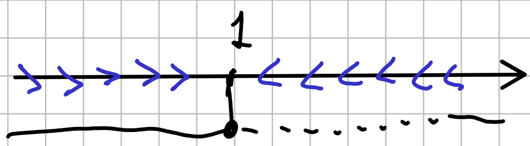
$$\begin{cases} a_1 = \lambda \\ a_{n+1} = \dfrac{1+a_n}{1+a_n^2} \quad \text{per ogni } n \in \mathbb{N} \end{cases}$$

$$f(t) = \frac{1+t}{1+t^2} \qquad \varphi(t) = \frac{1+t}{1+t^2} - t$$

$$\frac{1+\cancel{t}-\cancel{t}-t^3}{1+t^2} = \frac{-t^3+1}{1+t^2} > 0$$

$$-t^3+1 \geq 0, \quad -t^3 \geq -1, \quad t^3 \leq 1, \quad t \leq 1$$

$$1+t^2>0, \quad t^2+1>0, \quad t^2>-1 \quad \underline{\forall t}$$



$]-\infty, 1]$ $\qquad$ $[1, +\infty[$

$a_n \to 1$ $\qquad$ $a_n \to 1$

$$a_{n+1} \to 1 \quad \forall n$$

Esercizio 3. Calcolare il limite della successione

$$\begin{cases} a_1 = \lambda \\ a_{n+1} = a_n|a_n| \end{cases}$$

$f(t) = t|t|$

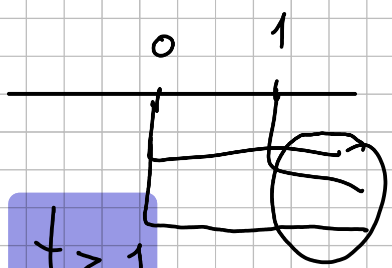
$f(t) = \underline{t|t| - t}$

$\hookrightarrow t(|t|-1) > 0$

$$\begin{cases} t \geq 0 \\ t^2 - t > 0 \end{cases}$$

$t(t-1) > 0$

$t > 0, \ t > 1$

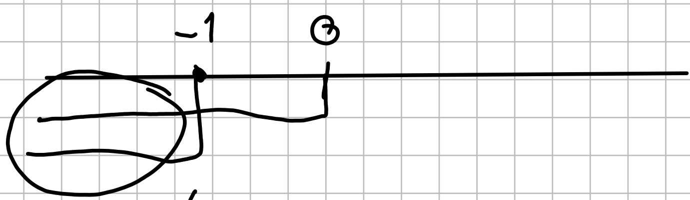


$\boxed{t > 1}$

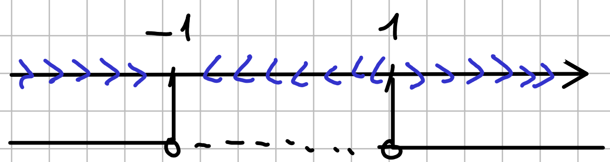
$$\begin{cases} t < 0 \\ -t^2 - t > 0 \end{cases}$$

$t^2 + t < 0 ; \ t(t+1) < 0$

$t < 0, \ t < -1$



$t < -1$



$]-\infty, -1[$ $\quad]-1, 1[$ $\quad]1, +\infty[$

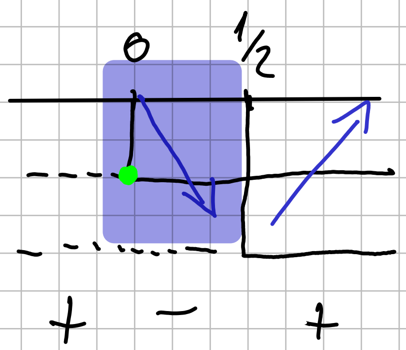
$a_n \to -1$ $\quad a_n \to -1$ $\quad a_n \to +\infty$

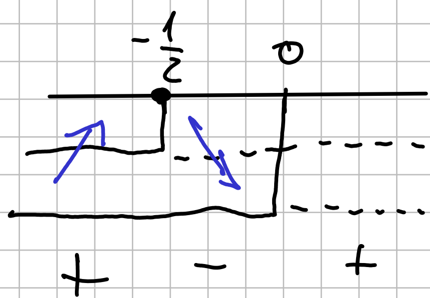
$f'(t) > 0 \quad t|t| \to$

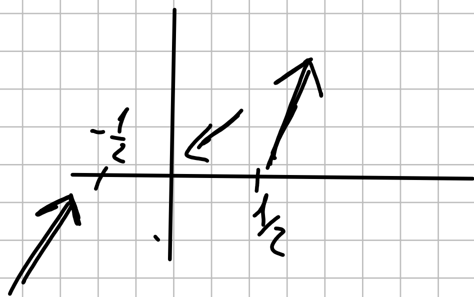
$$\begin{cases} t \geq 0 \\ t^2 - t > 0 \end{cases} \qquad \begin{cases} t < 0 \\ -t^2 - t > 0 \end{cases}$$

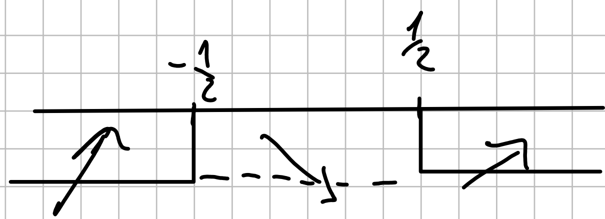
$$\begin{cases} t \geq 0 \\ 2t - 1 > 0 \end{cases} \qquad \begin{cases} t < 0 \\ 2t + 1 < 0 \end{cases}$$

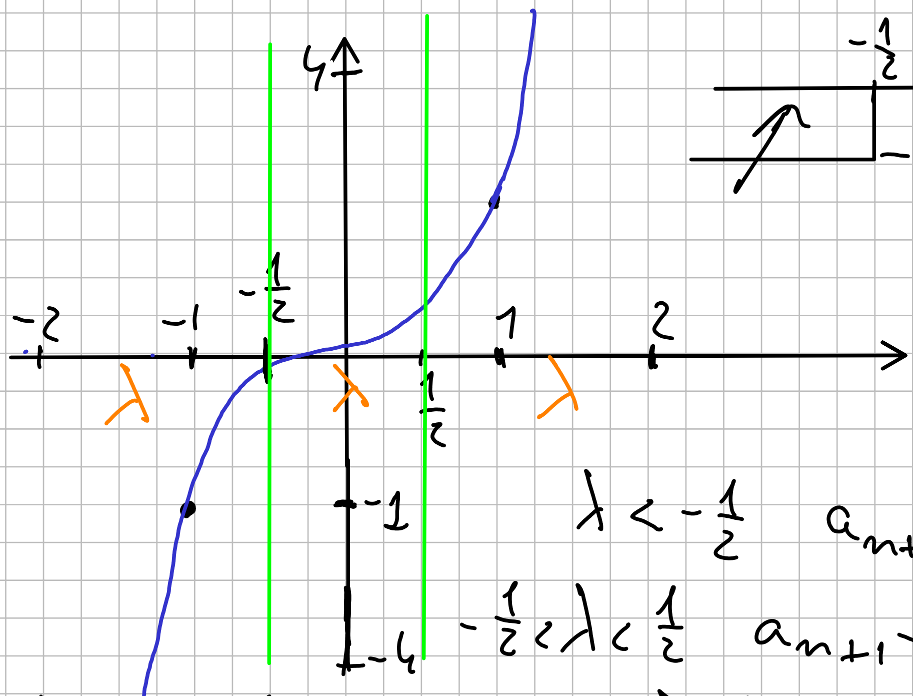
$t > \frac{1}{2}$ $\qquad$ $t < -\frac{1}{2}$











$\lambda < -\frac{1}{2}$ $\quad a_{n+1} \to -\infty$

$-\frac{1}{2} < \lambda < \frac{1}{2}$ $\quad a_{n+1} \to 0$

$\lambda > \frac{1}{2}$ $\quad a_{n+1} \to +\infty$

$t|t|$

$t = 1$	$f(t)$	$= 1$
$t = -1$	"	$= -1$
" $= 2$	"	$= 4$
$= -2$	"	$= -4$
$= 4$	"	$= 16$
$= -4$	"	$= -16$

$|z|^2 = 3z$

$x^2+y^2 = 3(x+iy)$

$x^2+y^2 = 3x+3iy$

$x^2+y^2-3x-3iy=0$

$\begin{cases} -3y=0 \\ x^2+y^2-3x=0 \end{cases}$ $\begin{cases} y=0 \\ x^2-3x=0 \end{cases}$ $\begin{cases} y=0 \\ x(x-3)=0 \end{cases}$ $\begin{cases} y=0 \\ x=0 \lor x=3 \end{cases}$

$\boxed{z_1=(0,0)}$ $\boxed{z_2=(3,0)}$

$|z| = 3z$

$z-\bar{z}=?$ → BOMBARDELLI?