Algoritmi di Parsing: Bottom-up Parsing

Slides based on material
by Ras Bodik available at
http://inst.eecs.berkeley.edu/~cs164/fa04

Welcome to the running example

- we'll build a parser for this grammar:

  $E \rightarrow E + T \mid E - T \mid T$

  $T \rightarrow T * int \mid int$

  (non ambiguous grammar with no useless variables)

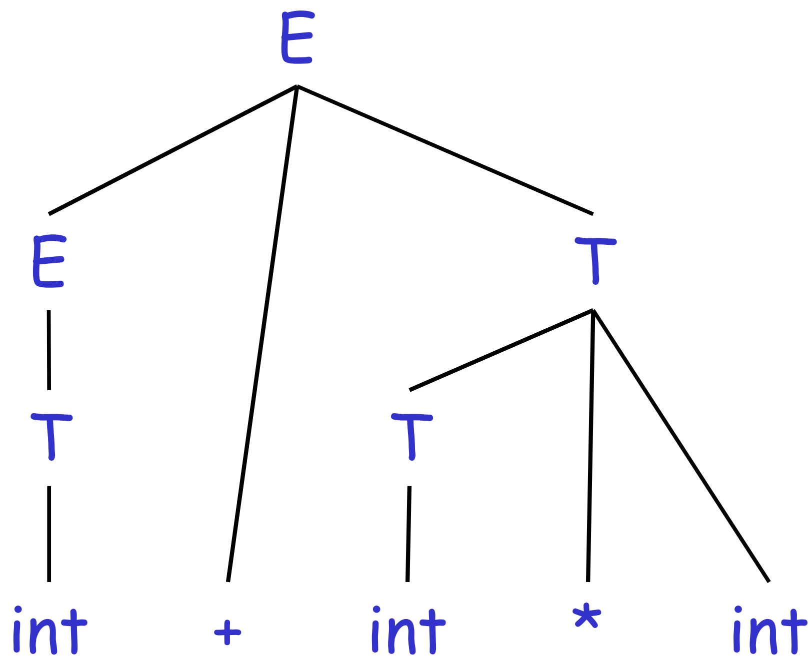
- see, the grammar is
  - left-recursive
  - not left-factored
- … and our parser won't mind!

Example input, parse tree

- input:

  int + int * int

- its parse tree:



Chaotic bottom-up parsing

Key idea: build the derivation in reverse

E

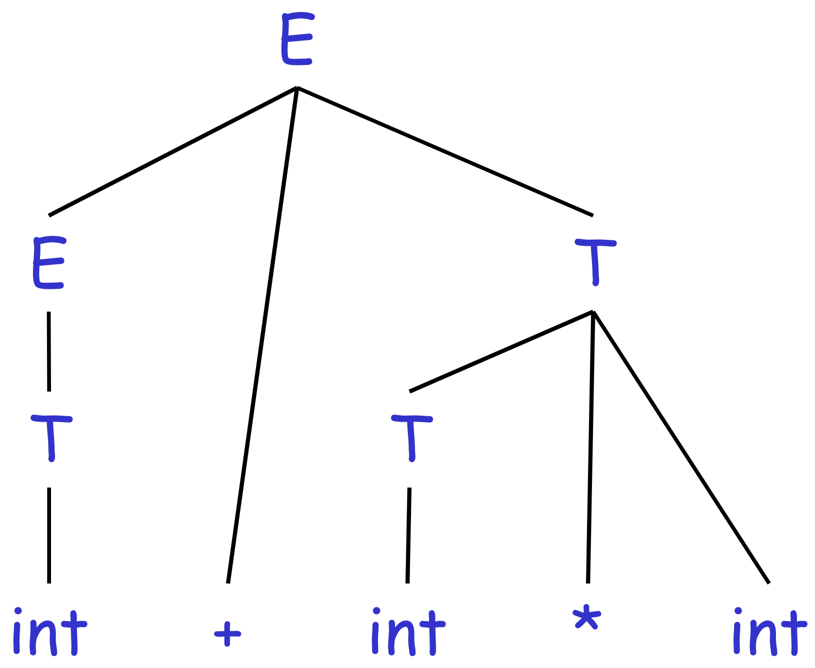
E + T

T + T

T + T * int

int + T * int

int + int * int



Chaotic bottom-up parsing

- The algorithm:
  1. stare at the input string *s*
     - feel free to look anywhere in the string
  2. find in *s* a right-hand side *r* of a production $N \rightarrow r$
     - ex.: found int for a production T→int
  3. **<u>reduce</u>** the found string *r* into its non-terminal *N*
     - ex.: replace int with T
  4. if all string reduced to start non-terminal
     - we're done, string is parsed, we got a parse tree
  5. otherwise continue in step 1

Don't celebrate yet!

- not guaranteed to parse a correct string
  - is this surprising?
- example:

and we are stuck

E + E * E

E + E * T

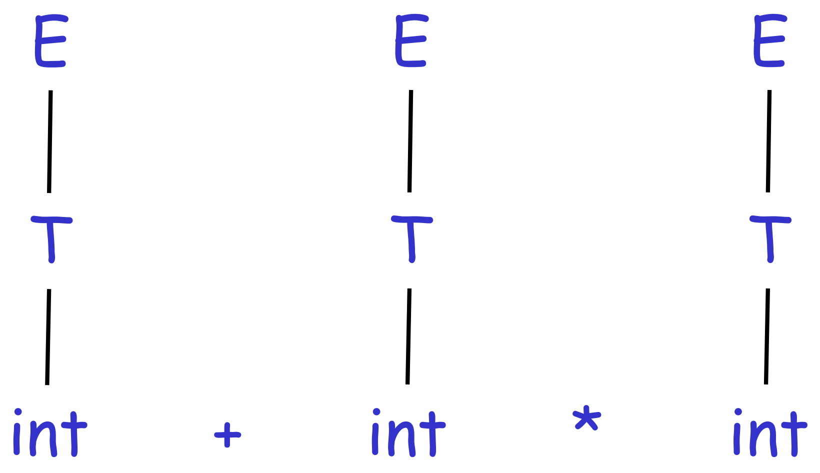
E + E * int

T + E * int

int + E * int

int + T * int

int + int * int



Lesson from chaotic parser

- Lesson:
  - if you're <u>lucky</u> in selecting the string to reduce next, then you will successfully parse the string
- How to "beat the odds"?
  - that is, how to find a lucky sequence of reductions that gives us a derivation of the input string?
  - use non-determinism!

Non-deterministic chaotic parser

The algorithm:

1. find in input all strings that can be reduced
   - assume there are k of them
2. create k copies of the (partially reduced) input
   - it's like spawning k identical <u>instances of the parser</u>
3. in each instance, perform one of k reductions
   - and then go to step 1, advancing and further spawning all parser instances
4. stop when at least one parser instance reduced the string to start non-terminal

Properties of the n.d. chaotic parser

Claim:

- the input will be parsed by (at least) one parser instance

But:

- exponential blowup: k*k*k*…*k parser copies

Also:

- Multiple (usually many) instances of the parser produce the correct parse tree (due to multiple corresponding derivations). This is wasteful.

Overview

- Chaotic bottom-up parser
  - it will give us the parse tree, but only if it's lucky
- Non-deterministic chaotic parser
  - creates many parser instances to make sure at least one builds the parse tree for the string
  - an instance either builds the parse tree or gets stuck
- Non-deterministic LR parser (next)
  - restrict where a reduction can be made
  - as a result, fewer instances necessary

Non-deterministic LR parser

- What we want:
  - create multiple parser instances
    - to find the lucky sequence of reductions
  - but the parse tree is found by at most **one** instance
    - zero if the input has syntax error

Two simple rules to restrict # of instances

1. split the input in two parts:
   - **right**: unexamined by parser
   - **left**: in the parser (we'll do the reductions here)

int ▸ + int * int after reduction: T ▸ + int * int

2. reductions allowed only on part adjacent to split

allowed: T + int ▸ * int after reduction: T + T ▸ * int

not allowed: int + int ▸ * int after reduction: T + int ▸ * int

☛ hence, left part of string can be kept on the stack

Wait a minute!

Aren't these restrictions fatally severe?

- **the doubt:** no instance succeeds to parse the input

No. Recall:

one parse tree corresponds to multiple derivations

- in n.d. chaotic parser, the instances that build the same parse tree each follow a different derivation

Wait a minute! (cont)

recall: two interesting derivations

- left-most derivation, right-most derivation

LR parser builds right-most derivation

- but does so <u>in reverse</u>: first step of derivation is the last reduction (the reduction to start nonterminal)
- example coming in two slides

hence the name:

- L: scan input left to right
- R: right-most derivation

so, if there is a parse tree, LR parser will build it!

- this is the key theorem

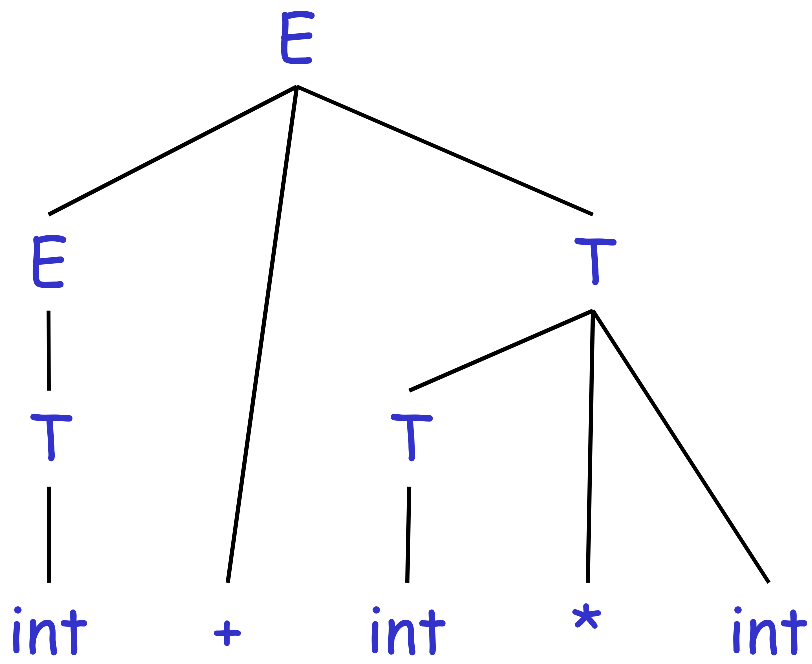
LR parser actions

- The left part of the string will be on the stack
  - the ▸ symbol is the top of stack
- Two simple actions
  - **reduce:**
    - like in chaotic parser,
    - but must replace a string on top of stack
  - **shift:**
    - shifts ▸ to the right,
    - which moves a new token from input onto stack, potentially enabling different reductions
- These actions will be chosen non-deterministically

Example of a correct LR parser sequence

A "lucky" sequence of shift/reduce actions (string parsed!):

E ▸
E + T ▸
E + T * int ▸
E + T * ▸ int
E + T ▸ * int
E + int ▸ * int
E + ▸ int * int
E ▸ + int * int
T ▸ + int * int
int ▸ + int * int
▸ int + int * int



Example of an incorrect LR parser sequence

will be stuck after reduction to T + E!

why can't we reduce to E + T, instead ?

T + T ▸

T + T * int ▸

T + T * ▸ int

T + T ▸ * int

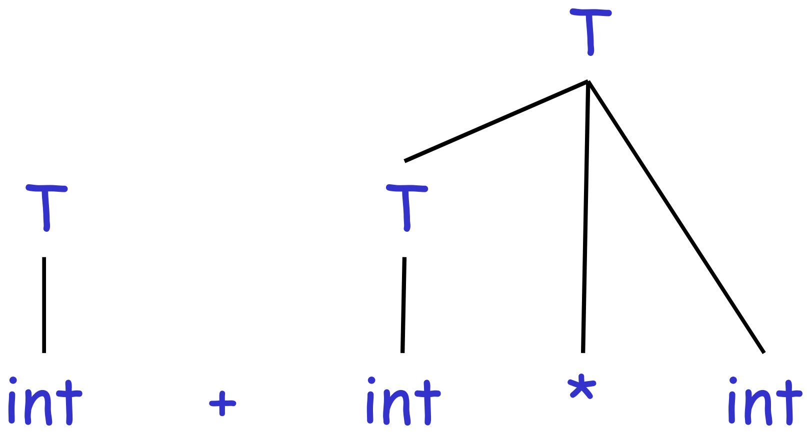
T + int ▸ * int

T + ▸ int * int

T ▸ + int * int

int ▸ + int * int

▸ int + int * int



Where did the parser instance make the mistake?

Non-deterministic LR parser

The algorithm: (compare with chaotic n.d. parser)

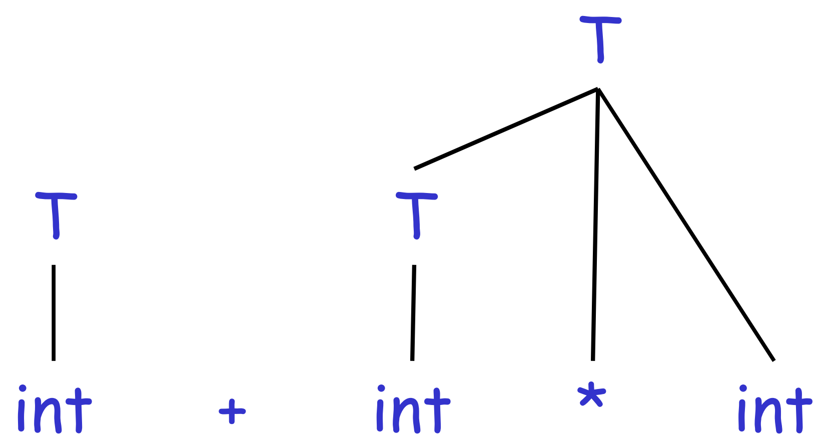
1. find all reductions allowed on top of stack
   - assume there are k of them
2. create k new identical instances of the parser
3. in each instance, perform one of the k reductions; in original instance, do no reduction, shift instead
   - and go to step 1
4. stop when a parser instance reduced the string to start non-terminal

Overview

- Chaotic bottom-up parser
  - tries one derivation (in reverse)
- Non-deterministic chaotic parser
  - tries all ways to build the parse tree
- Non-deterministic LR parser
  - restricts where a reduction can be made
  - as a result,
    - only one instance succeeds (on an unambiguous grammar)
    - all others get stuck
- Generalized LR parser (next)
  - **idea:** kill off instances that are going to get stuck ASAP

Revisit the <u>incorrect</u> LR parser sequence

T + T ▸
T + T * int ▸
T + T * ▸ int
T + T ▸ * int
T + int ▸ * int
T + ▸ int * int
T ▸ + int * int
int ▸ + int * int
▸ int + int * int



Key question:

What was the earliest stack configuration where we could tell this instance was doomed to get stuck?

Doomed stack configurations

The parser made a mistake to shift to

T + ▸ int * int

rather than reducing to

E ▸ + int * int

The first configuration is doomed

- because the T will never appear on top of stack so that it can be reduced to E
- hence this instance of the parser can be killed (it will never produce a parse tree)

How to find doomed parser instances?

- Look at their stack!
- How to tell if a stack is doomed:
  - list all legal (non yet doomed) stack configurations
  - if a stack is not legal, kill the instance
- Listing legal stack configurations
  - list **prefixes (stack content)** of **all right-most derivations**
  - describe them **as a DFA**
  - if the stack configuration is not from the DFA, it's doomed

Our example grammar

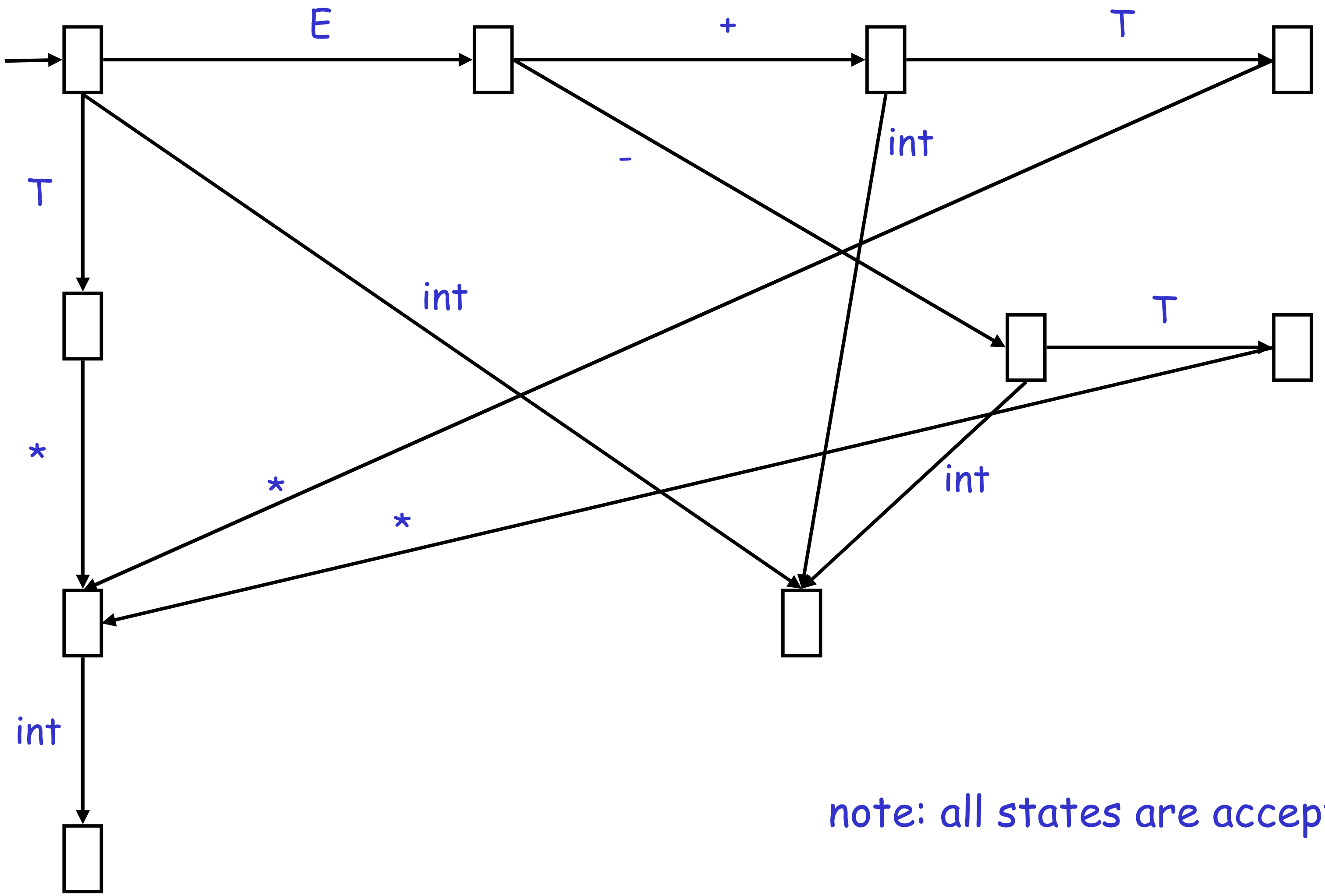
- we'll build a parser for this grammar:

  $E \rightarrow E + T \mid E - T \mid T$

  $T \rightarrow T * \text{int} \mid \text{int}$

- which are **prefixes (stack content)** of strings reached during **right-most derivations?**
  - E.g. those of length one.
    - They are "E", "T" and "int"
  - Then think about those of length two, etc…

The stack-checking DFA



note: all states are accepting states

Simple LR parsing

Introducing a new grammar initial variable

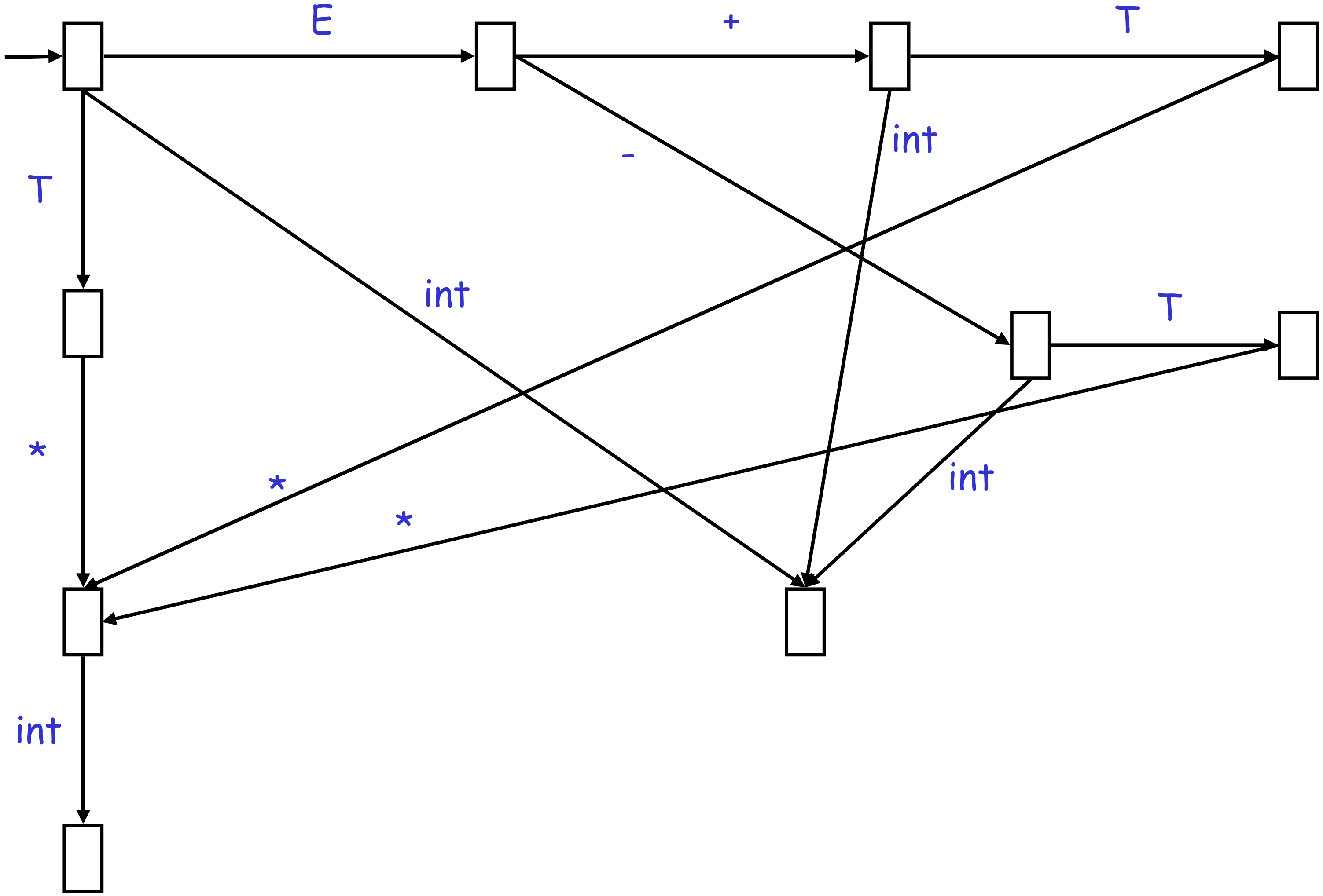
- In order to compute the stack checking DFA we preliminarily add to the grammar
  - a new **initial variable E'** and
  - a new production **E'** $\rightarrow$ **E** (with **E** being the old initial variable)
- Our example grammar thus becomes:

$$E' \rightarrow E$$

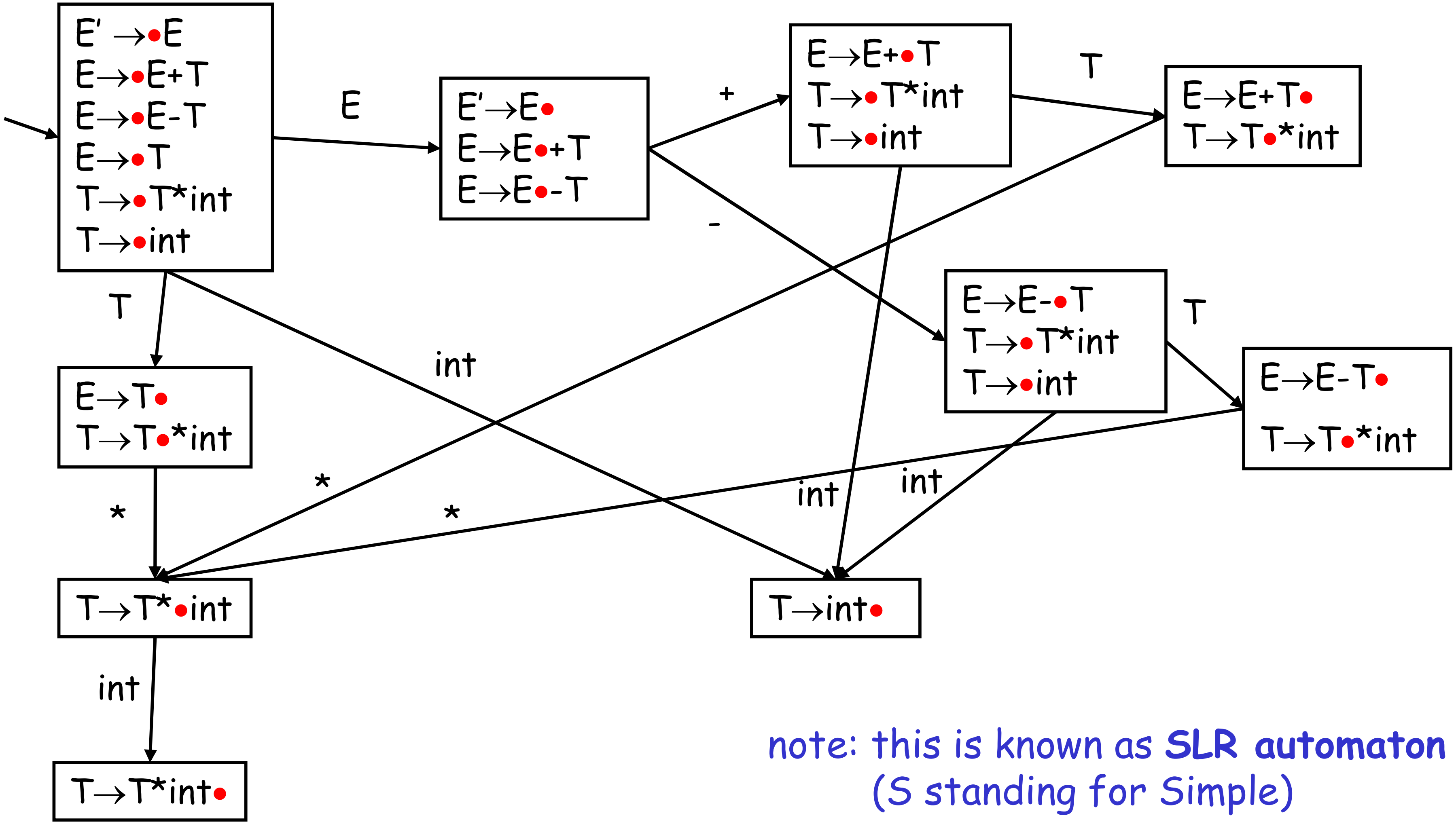
$$E \rightarrow E + T \mid E - T \mid T$$

$$T \rightarrow T * \text{int} \mid \text{int}$$

The stack-checking DFA



Constructing the stack-checking DFA



note: this is known as **SLR automaton**
(S standing for Simple)

SLR Items

- A ***SLR item*** has the form:

$$X \rightarrow \alpha \bullet \beta$$

  with $X \rightarrow \alpha\beta$ being a production

- $X \rightarrow \alpha \bullet \beta$ describes **context** information
  - We are **trying to find an X**, and
  - We have α already on **top of the stack**
  - Thus we need to see next a **string derived from** β

DFA state: a set of SLR items

- A DFA state describes a **parsing context**: a **set of SLR items** representing possible productions to apply
  - We are trying to find $E \rightarrow E + \bullet T$
    - on the top of the stack we have $E +$ already
  - **Thus** we are also trying to find $T \rightarrow \bullet\ T*\text{int}$ *or* $T \rightarrow \bullet\ \text{int}$
    - on the top of the stack we do not have anything yet

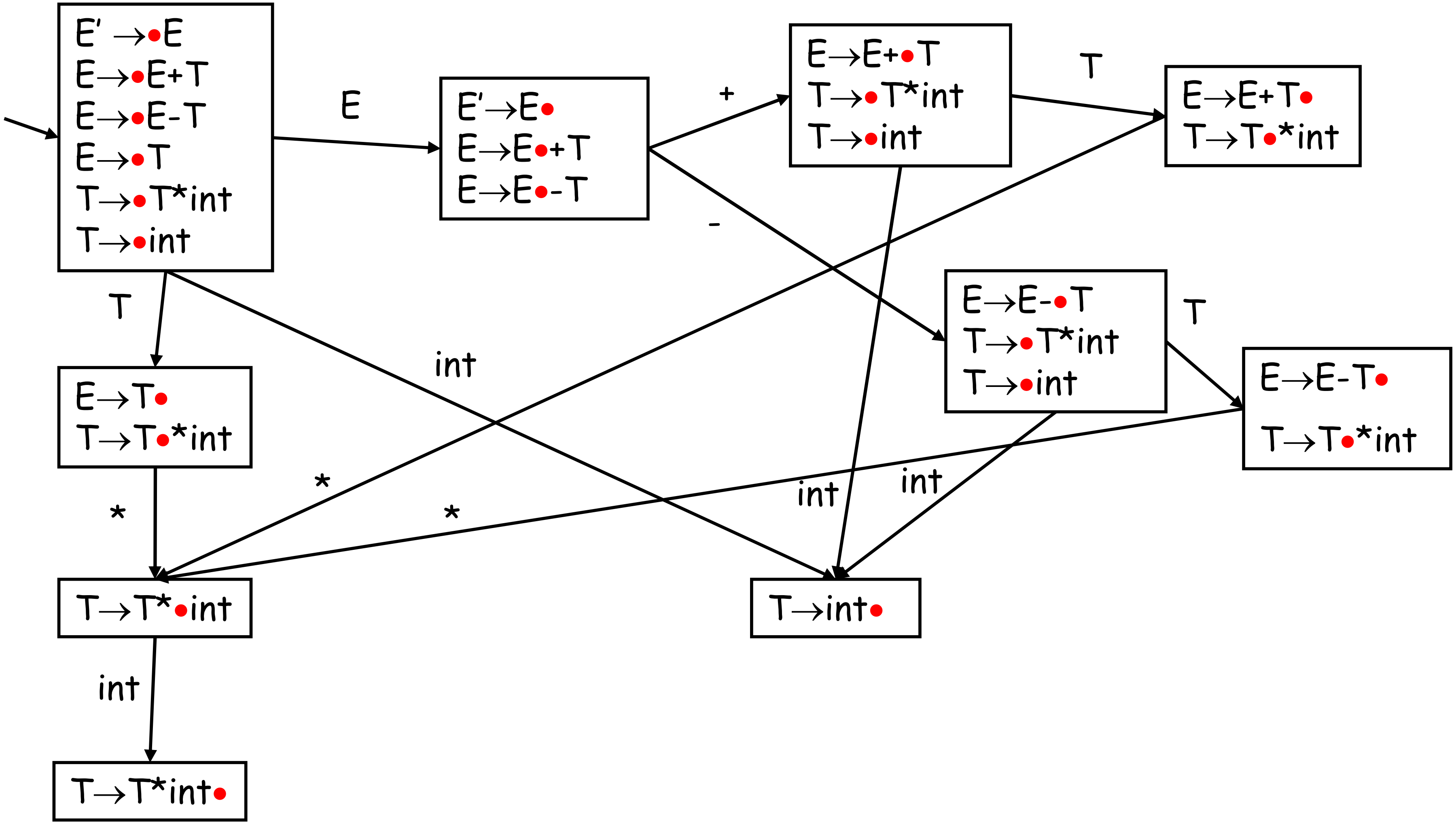
Note

- The symbol ▸ was used before to separate the stack from the rest of input
  - α ▸ γ, where α is the stack and γ is the remaining string of terminals
- In SLR items $\bullet$ is used to mark a prefix of a production rhs:

$$X \to \alpha \bullet \beta$$

  - Here β might contain non-terminals as well
- In both cases the stack is on the left

Constructing the stack-checking DFA



Constructing the stack-checking DFA

- A DFA state is a *closed* set of SLR(1) items
  - This means that we performed Closure

- The start state is Closure({E' $\rightarrow$ •E})

The Closure Operation

- The operation of extending the context with items is called the ***closure operation***

Closure(Items) =
  repeat
    for each $X \to \alpha \bullet Y\beta$ in Items
      for each production $Y \to \gamma$
        add $Y \to \bullet\gamma$ to Items
  until Items is unchanged

Constructing the stack-checking DFA

$E' \to \bullet E$
$E \to \bullet E+T$
$E \to \bullet E-T$
$E \to \bullet T$
$T \to \bullet T*int$
$T \to \bullet int$

The DFA Transitions

- A state State that contains at least an item $X \to \alpha \bullet y\beta$ has a transition labeled y to a state that contains the items: Transition(State, y)
  - y can be a terminal or a non-terminal

Transition(State, y) =

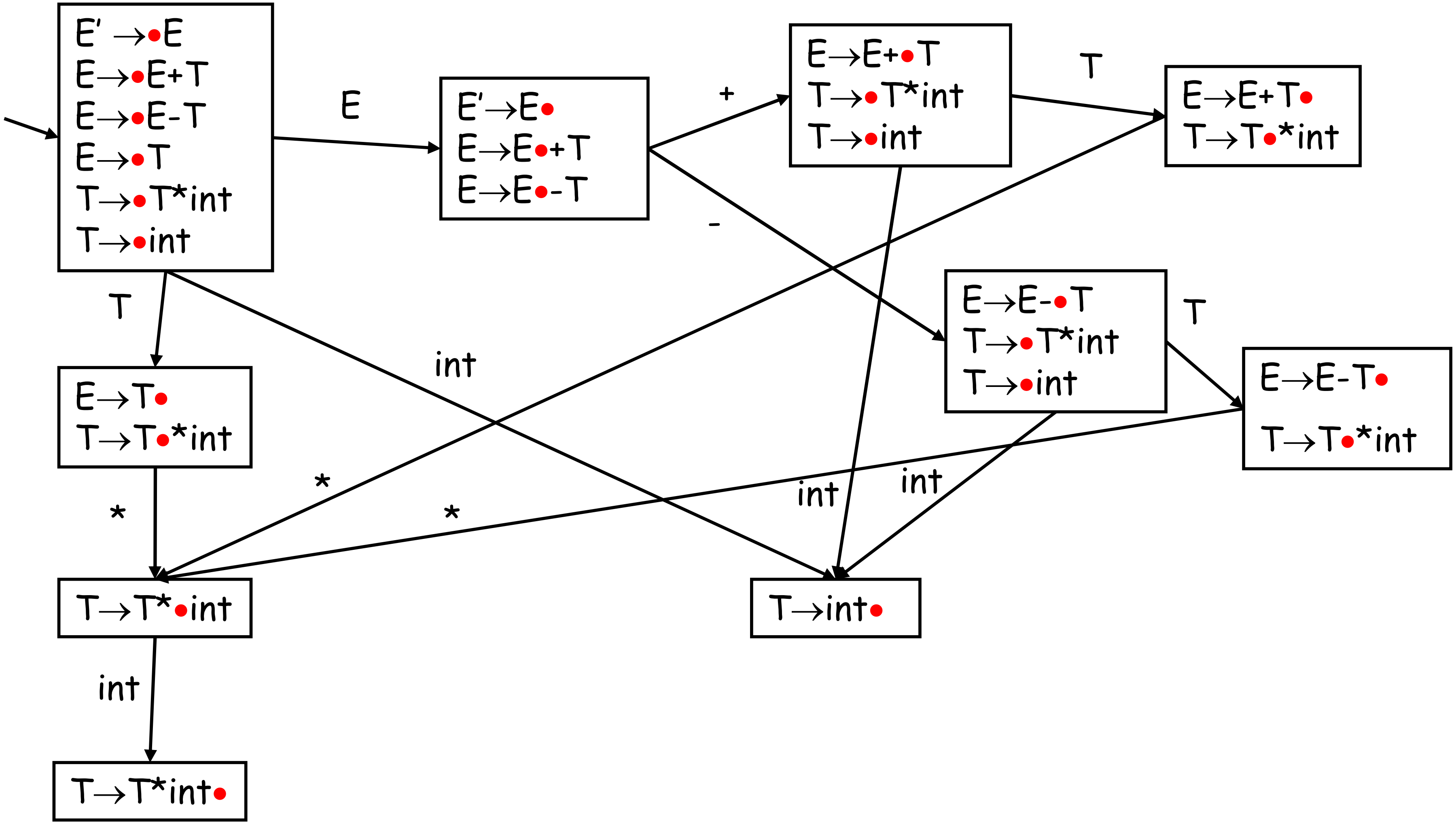
  Items $\leftarrow \varnothing$

  for each $X \to \alpha \bullet y\beta \in$ State

    add $X \to \alpha y \bullet \beta$ to Items

  return Closure(Items)

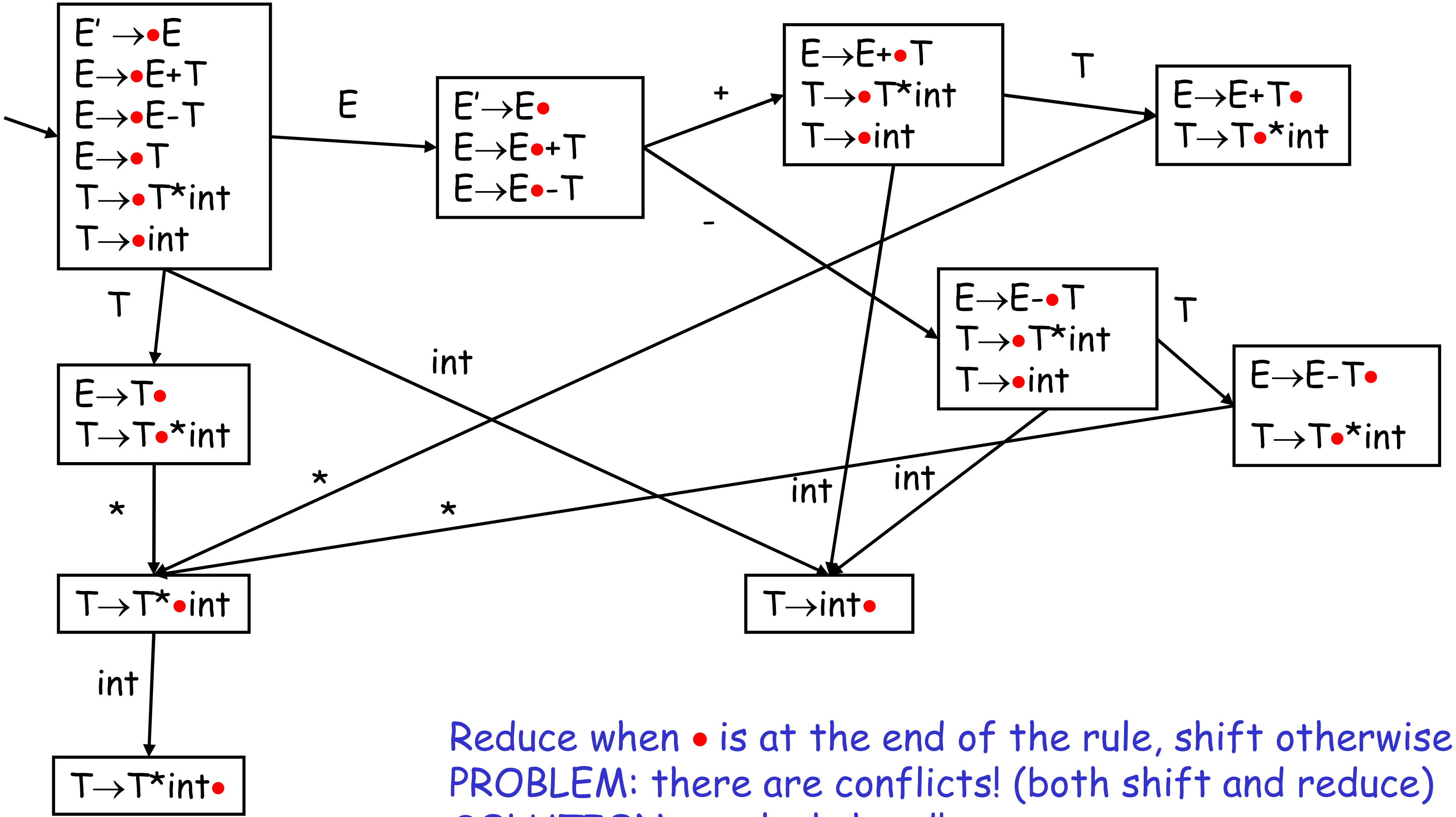
Constructing the stack-checking DFA



How to use the SLR automaton

- The automaton indicates the correct stack configurations…
- …but it also dictates when to do shift/reduce actions…

Shift/reduce based on stack-checking DFA



Reduce when • is at the end of the rule, shift otherwise
PROBLEM: there are conflicts! (both shift and reduce)
SOLUTION: use lookahead!

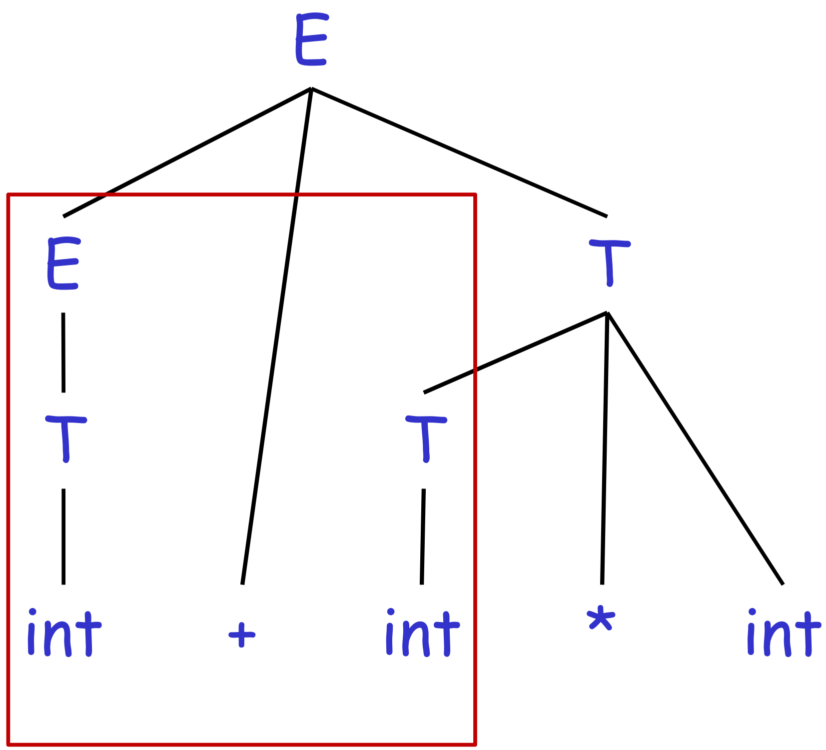
Example of a correct LR parser sequence

If we do not use lookahead ("*") we still have non-determinism!

The lookahead, instead, tells that here we cannot reduce E+T to E

- because it would cause E to be followed by "*" and we know $* \notin$ Follow(E)

E ▸
E + T ▸
E + T * int ▸
E + T * ▸ int
E + T ▸ * int
E + int ▸ * int
E + ▸ int * int
E ▸ + int * int
T ▸ + int * int
int ▸ + int * int
▸ int + int * int



How to use the SLR automaton

- Lookahead for "shift":
  - "Shift" only on the expected terminals
- Lookahead for "reduce":
  - "Reduce" only on terminals in the **Follow set** of the variable on the l.h.s. of the rule used to reduce

Computation of first and follow sets

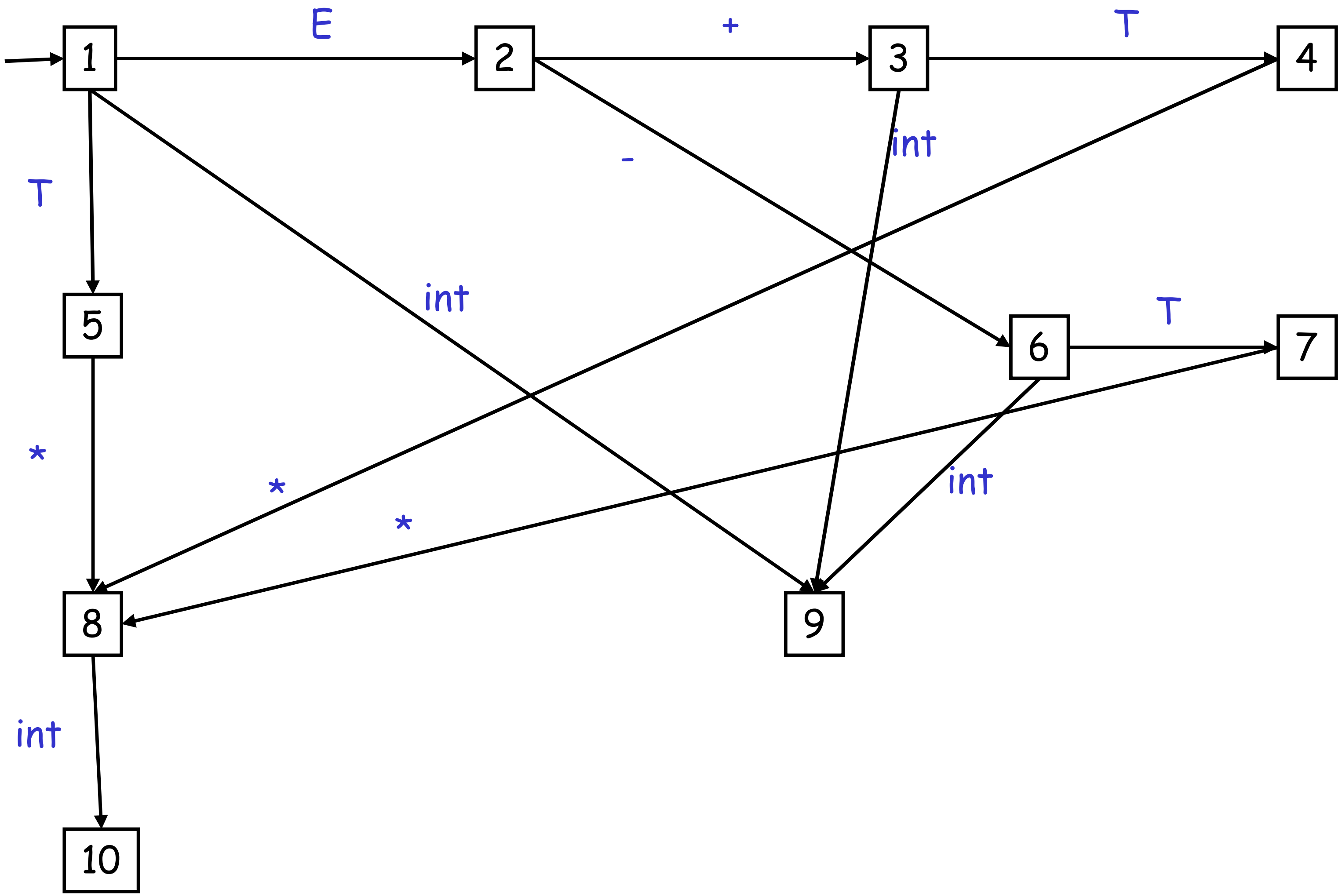
- Consider our example grammar

$$E' \to E$$
$$E \to E + T \mid E - T \mid T$$
$$T \to T * int \mid int$$

First(E')={int} (no need to calculate it)
First(E)={int}
First(T)={int}

Follow(E')={\$} (no need to calculate it, always {\$})
Follow(E)={\$,+,-}
Follow(T)={\$,+,-,*}

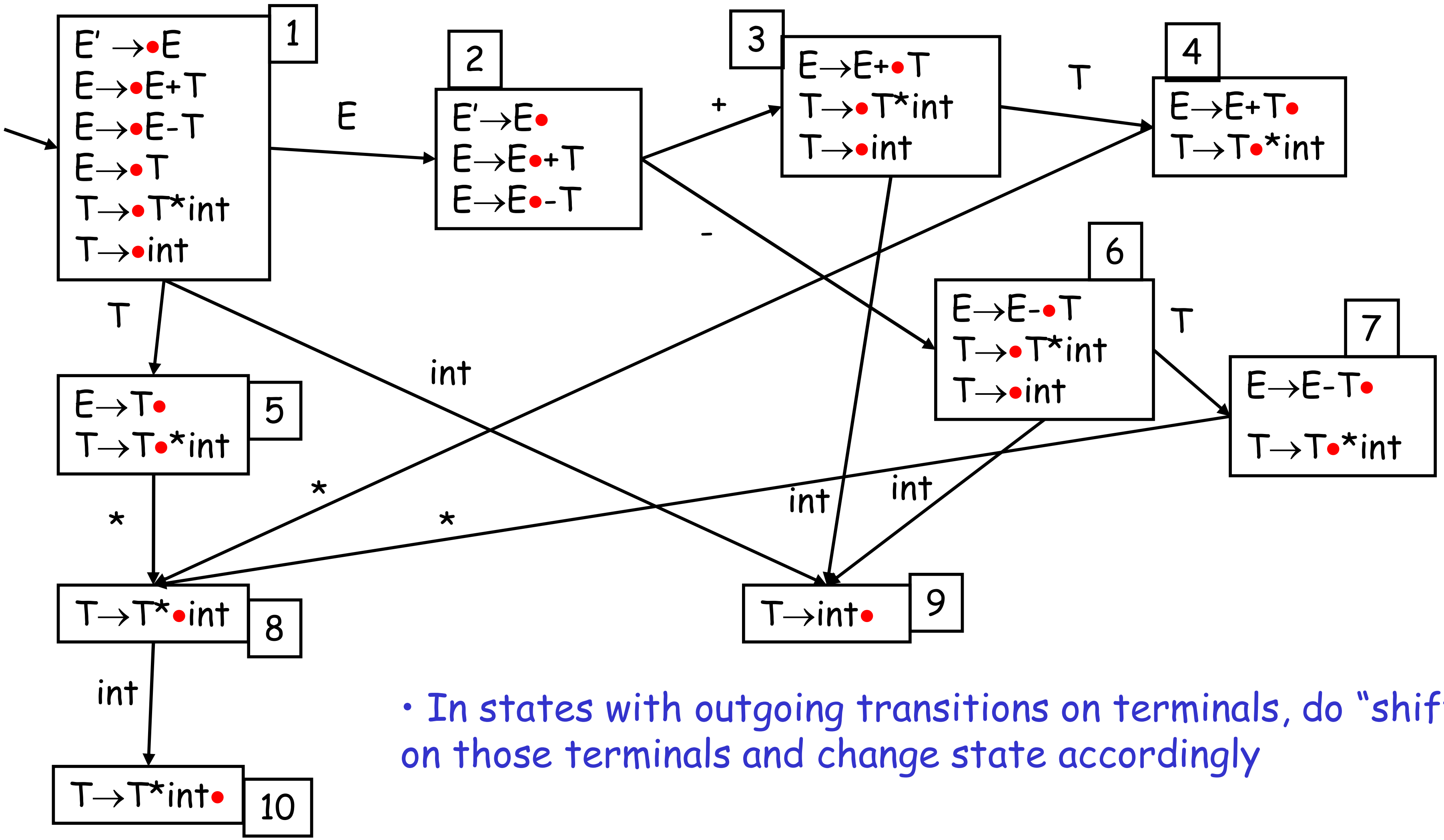
How to use the SLR automaton



How to use the SLR automaton (tabular representation)

	int	+	-	*	E	T
1	9				2	5
2		3	6			
3	9					4
4				8		
5				8		
6	9					7
7				8		
8	10					
9						
10						

How to use the SLR automaton

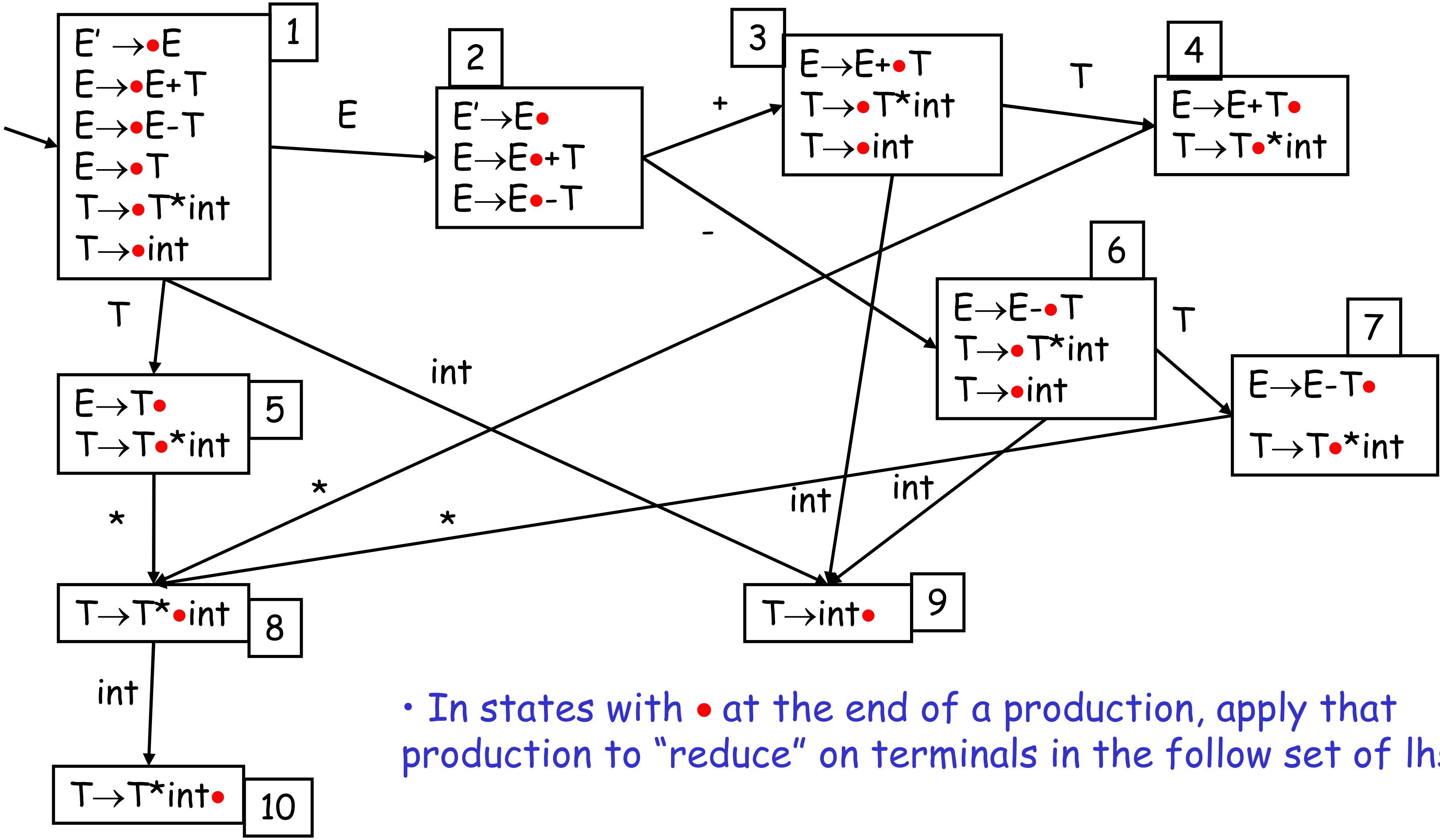


• In states with outgoing transitions on terminals, do "shift" on those terminals and change state accordingly

How to use the SLR automaton (tabular representation)

	int	+	-	*	E	T
1	shift,9				2	5
2		shift,3	shift,6			
3	shift,9					4
4				shift,8		
5				shift,8		
6	shift,9					7
7				shift,8		
8	shift,10					
9						
10						

How to use the SLR automaton

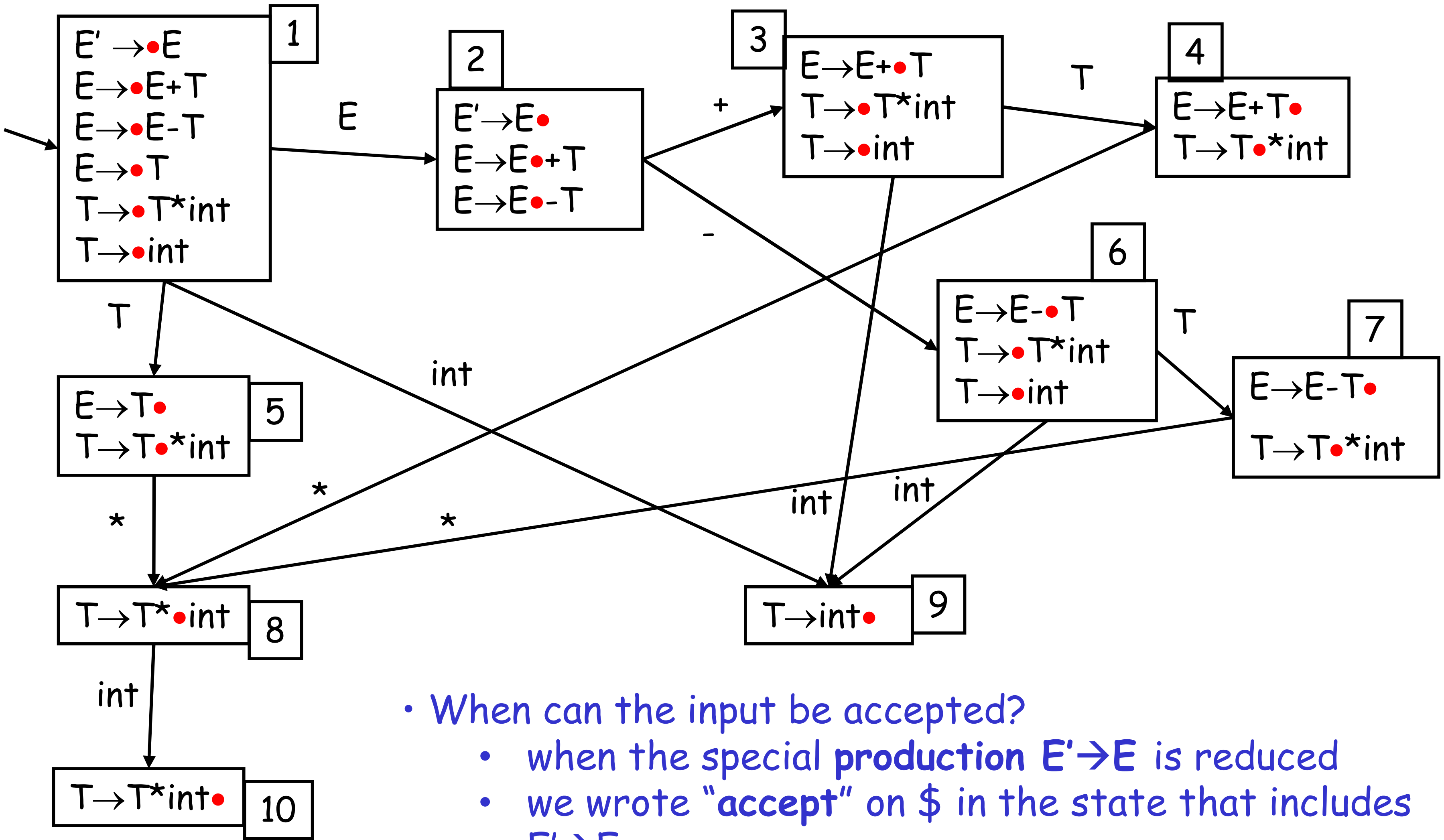


- In states with • at the end of a production, apply that production to "reduce" on terminals in the follow set of lhs

How to use the SLR automaton (tabular representation, $ column added)

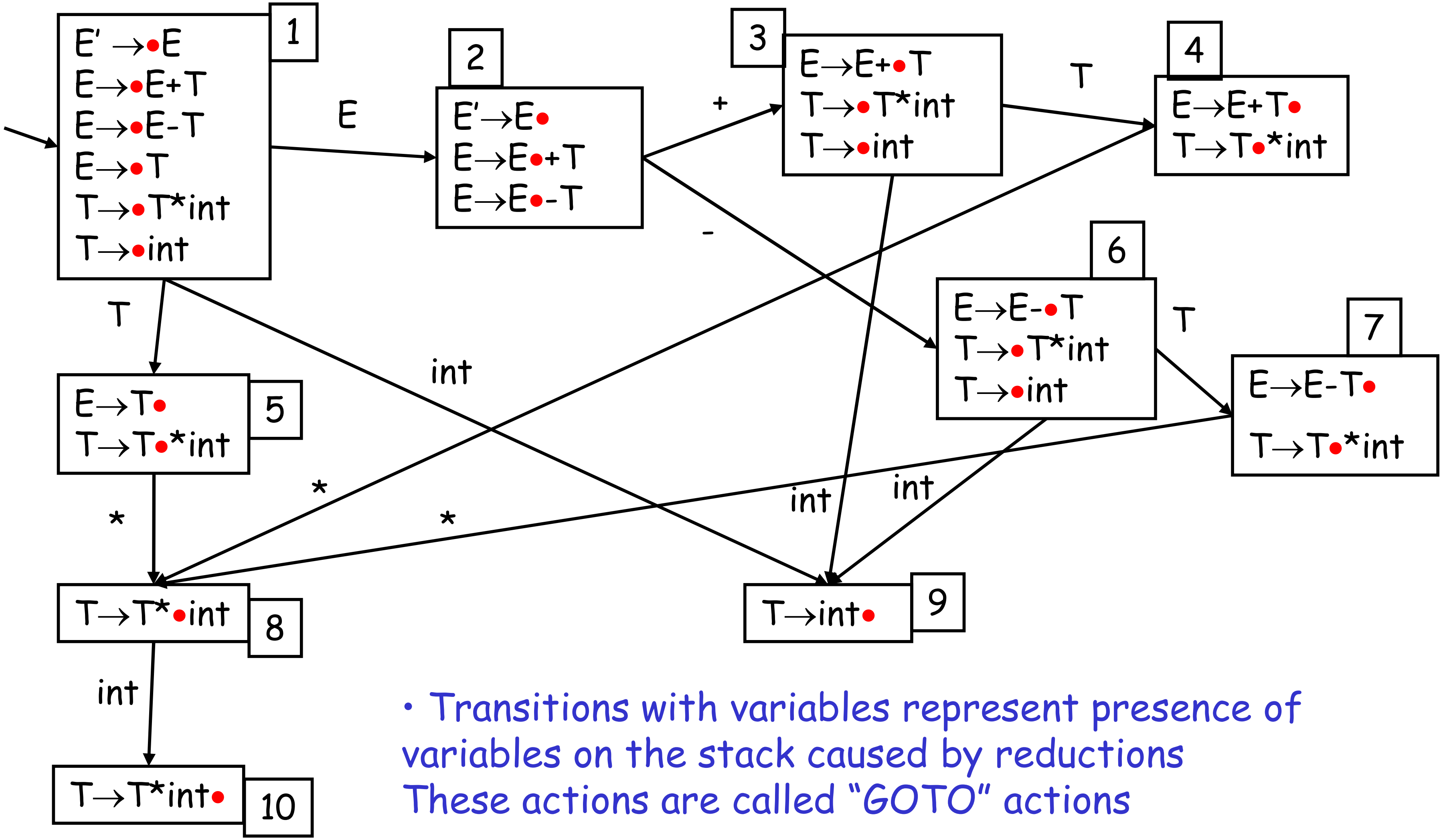
	int	+	-	*	$	E	T
1	shift,9					2	5
2		shift,3	shift,6		accept		
3	shift,9						4
4		red,E→E+T	red,E→E+T	shift,8	red,E→E+T		
5		red, E→T	red, E→T	shift,8	red, E→T		
6	shift,9						7
7		red,E→E-T	red,E→E-T	shift,8	red,E→E-T		
8	shift,10						
9		red,T→int	red,T→int	red,T→int	red,T→int		
10		red, T→T*int	red, T→T*int	red, T→T*int	red, T→T*int		

How to use the SLR automaton



- When can the input be accepted?
  - when the special **production E'→E** is reduced
  - we wrote "**accept**" on $ in the state that includes E'→E•

How to use the SLR automaton



• Transitions with variables represent presence of variables on the stack caused by reductions These actions are called "GOTO" actions

How to use the SLR automaton (complete tabular representation)

	int	+	-	*	$	E	T
1	shift,9					goto,2	goto,5
2		shift,3	shift,6		accept		
3	shift,9						goto,4
4		red,E→E+T	red,E→E+T	shift,8	red,E→E+T		
5		red, E→T	red, E→T	shift,8	red, E→T		
6	shift,9						goto,7
7		red,E→E-T	red,E→E-T	shift,8	red,E→E-T		
8	shift,10						
9		red,T→int	red,T→int	red,T→int	red,T→int		
10		red, T→T*int	red, T→T*int	red, T→T*int	red, T→T*int		

How to use the SLR automaton

- The LR predictive algorithm governed by this table is called SLR(1) parsing algorithm
  - Note that it works only if each cell of the table contains at most one action! (i.e. no conflict)
  - In this case, the grammar is a SLR(1) grammar

Bottom-up parsing algorithm

Remember the idea of LR parsing

- LR parsing *reduces* a string to the start symbol by inverting productions:

str such that str $\rightarrow^*$ input string of terminals

while str $\neq$ S:

  - Identify β in str such that $A \rightarrow \beta$ is a production and S $\rightarrow^* \alpha\ A\ \gamma\ \rightarrow \alpha\ \beta\ \gamma\ =$ str
  - Replace β by A in str (so $\alpha\ A\ \gamma$ becomes new str)

- Stronger than top-down parsing!
  - make decisions *after* seeing all symbols β rather than before (LL(1) make decisions knowing at most one of those symbols, the lookahead)

Bottom-up parsing algorithm

- Add $ at the end of the input
- Execute the bottom-up parsing algorithm using an automaton (e.g. the SLR automaton) to decide whether to shift or reduce
  - Every time a decision should be taken, use the stack as input for the automaton
  - The automaton will communicate whether to shift or reduce (based on the lookahead)

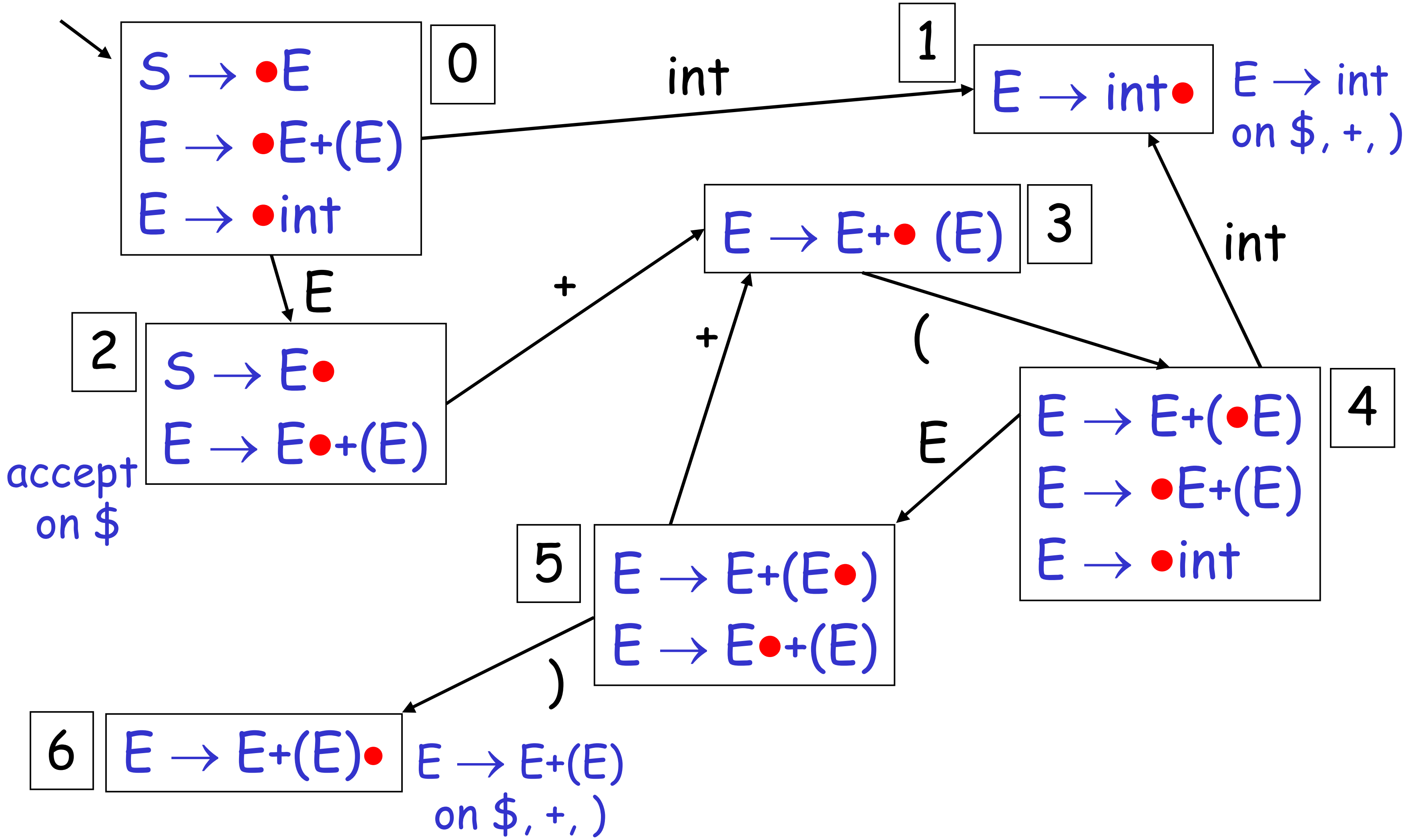
A Running Example

- We will study the bottom-up parsing algorithm considering
  - The grammar:

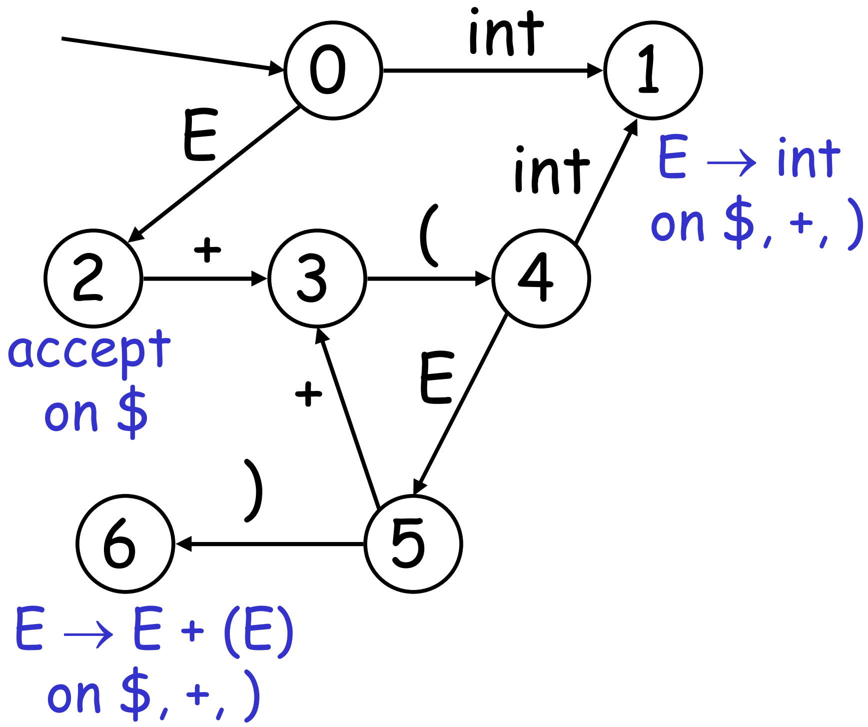
$$E \rightarrow E + (E) \mid int$$

  - An automaton/parsing table for such a grammar, i.e. its **SLR(1) automaton**
  - First(E) = {int}
  - Follow(E) = {$,+,)}

Constructing the Parsing DFA. Example.



Bottom-up parsing algorithm example



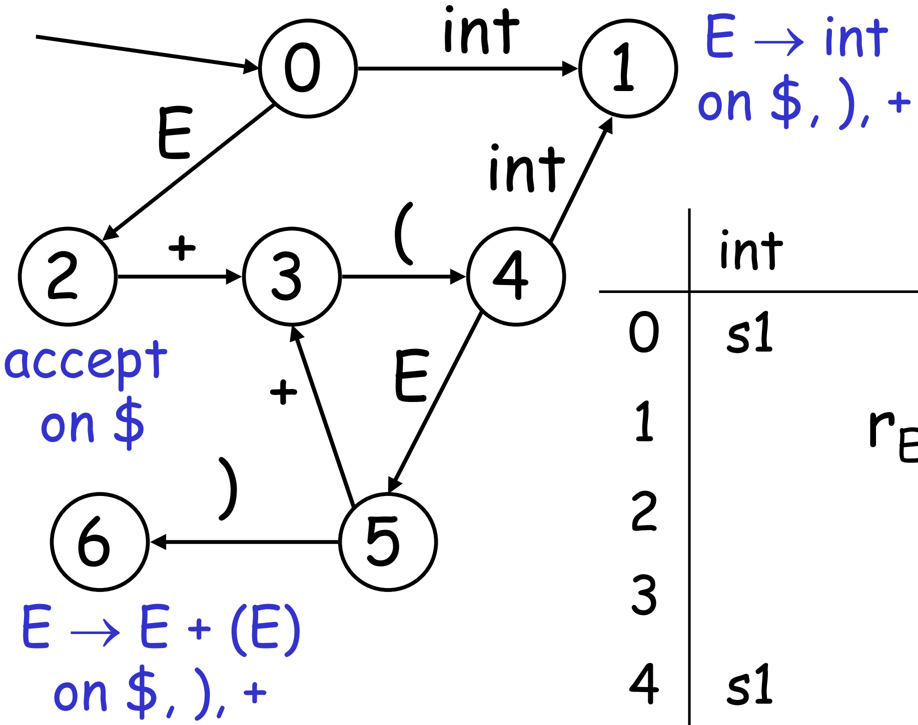
▸ int + (int) + (int)$	shift
int ▸ + (int) + (int)$	E → int
E ▸ + (int) + (int)$	shift(x3)
E + (int ▸) + (int)$	E → int
E + (E ▸) + (int)$	shift
E + (E) ▸ + (int)$	E → E+(E)
E ▸ + (int)$	shift (x3)
E + (int ▸)$	E → int
E + (E ▸)$	shift
E + (E) ▸ $	E → E+(E)
E ▸ $	accept

DFA table representation is actually used

- Parsers represent the DFA as a 2D table
  - As for table-driven lexical analysis
- Lines correspond to DFA states
- Columns correspond to terminals and non-terminals
- In classical treatments, columns are split into:
  - Those for terminals: action table
  - Those for non-terminals: goto table

Representing the DFA. Example

- The table for our DFA:



	int	+	(	)	$	E
0	s1					g2
1		$r_{E \rightarrow int}$		$r_{E \rightarrow int}$	$r_{E \rightarrow int}$	
2		s3			accept	
3			s4			
4	s1					g5
5		s3		s6		
6		$r_{E \rightarrow E+(E)}$		$r_{E \rightarrow E+(E)}$	$r_{E \rightarrow E+(E)}$	

The Bottom-up Parsing Algorithm

- After a shift or reduce action we rerun the DFA on the entire stack
  - This is wasteful, since most of the work is repeated

- So record, for each stack element, state of the DFA after that element

- Parser maintains a stack

$$\langle\, sym_1, state_1 \,\rangle \ldots \langle\, sym_n, state_n \,\rangle$$

$state_k$ is the final state of the DFA on $sym_1 \ldots sym_k$

The Bottom-up Parsing Algorithm

Let I = $w_1 w_2 \ldots w_n\$$ be initial input

Let j = 1

Let DFA state 0 be the start state

Let stack = $\langle$ dummy, 0 $\rangle$

repeat

  case **action[top_state(stack), I[j]]** of

    shift k: push $\langle$ I[j], k $\rangle$; j += 1

    reduce X $\rightarrow \alpha$:

      pop $|\alpha|$ pairs,

      push $\langle$X, **goto[top_state(stack), X]**$\rangle$

    accept: halt normally

    error: halt and report error

bison: a parser generator

- **bison** generates parsers building the LALR(1) automaton/table
  - slightly more complex variant of SLR(1) such that a conflict-free table is generated for more grammars
- bison input:
  - file with token description, priority and associativity rules, grammar rules, and some auxiliary ***C*** intructions
- Bison generates two files in output:
  - a ***C*** program containing the code for the parser
  - the LALR(1) table

bison: structure of bison input

```
%{
  C declarations
%}
  Bison declarations
%%
  Grammar rules
%%
  Programs
```

bison: example of input

```
%{
#include <stdio.h>
int yylex() ;
void yyerror(char *s){ printf("parser error") ; }
%}
%token ID WHILE BEGIN END DO IF THEN ELSE SEMI ASSIGN
%start prog
%%
prog        : stmlist;
stmlist     : stm
            | stmlist SEMI stm;
stm         : ID ASSIGN ID
            | WHILE ID DO stm
            | BEGIN stmlist END
            | IF ID THEN stm
            | IF ID THEN stm ELSE stm;
```

bison: example of output

- In the generated table we read:

  state 18 contains 1 shift/reduce conflict.

  . . .

  state 18

  variabili -> minuscolo
  terminali -> maiuscolo

  stm -> IF ID THEN stm . (rule 7) . = pallino rosso

  stm -> IF ID THEN stm . ELSE stm (rule 8)

  ELSE shift, and go to state 19

  ELSE [reduce using rule 7 (stm)]

- If there are no explicit precedence declarations, BISON follows the following:
  - In shift/reduce conflicts the shift is executed
  - In reduce/reduce conflicts the first rule is applied