

# A1 Portfolio Optimization

WEALTH MANAGEMENT: INVESTMENT RISK MANAGEMENT AND  
OPTIMIZATION IN R

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# A1 Portfolio Analysis

## Portfolio Presentation

Allocation Percentages of Assets in the Original Portfolio

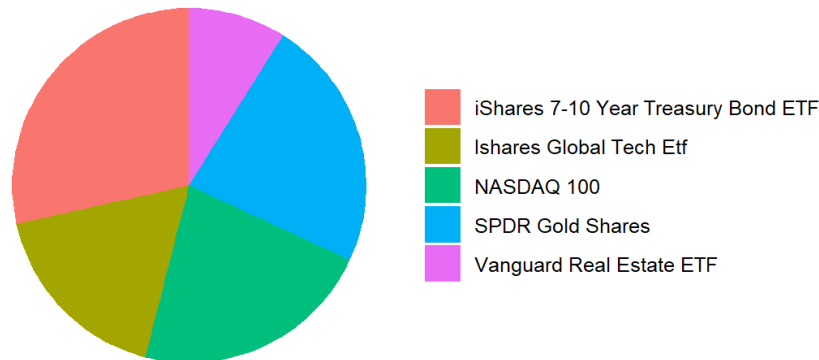


Figure 1 - Original Portfolio Allocation

The **allocation** percentage is the following: **Equities** (IXN + QQQ): 39.6%, **Fixed Income** (IEF): 28.5%, **REIT** (VNQ): 8.9% and **Commodities** (GLD): 23%. The assets are briefly described in the following lines:

- **iShares Global Tech ETF (IXN)**: An ETF focused on global technology stocks, representing strong exposure to innovation and growth in the tech sector.
- **NASDAQ 100 (QQQ)**: An ETF that tracks the performance of the 100 largest non-financial stocks listed on the NASDAQ, providing exposure to US large-cap technology and growth companies.
- **iShares 7-10 Year Treasury Bond ETF (IEF)**: An ETF that invests in U.S. Treasury bonds with maturities between 7 and 10 years.
- **Vanguard Real Estate ETF (VNQ)**: An ETF that invests in U.S. real estate securities, providing exposure to commercial and residential real estate.
- **SPDR Gold Shares (GLD)**: An ETF that aims to reflect the performance of the price of gold.

Considering the fact that Fixed Income Asset Class weight is only the 28.5% of the portfolio and considering that, according to Morningstar [2024]<sup>1</sup>, not only, the VNQ ETF is quite risky<sup>2</sup> but also that Gold in the last years has been more volatile than equities, it is acceptable to say that the portfolio can be considered “aggressive”. Clearly there are more aggressive portfolios but this is risky enough to be considered as such.

Assuming that the client goal is to maximize return and assuming that the weights remained constant in the last years, the analysis is presented following in the next pages.

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<sup>1</sup> Gold’s annualized volatility over the last 30 years is 15.44% – not dramatically higher than the S&P 500—which posted a 14.32% annualized volatility over the same period!

<sup>2</sup> More will be added in Risk section of this report

## Returns Analysis (12, 18, 24 M Returns) – Question 1

### Returns by Asset since 2008

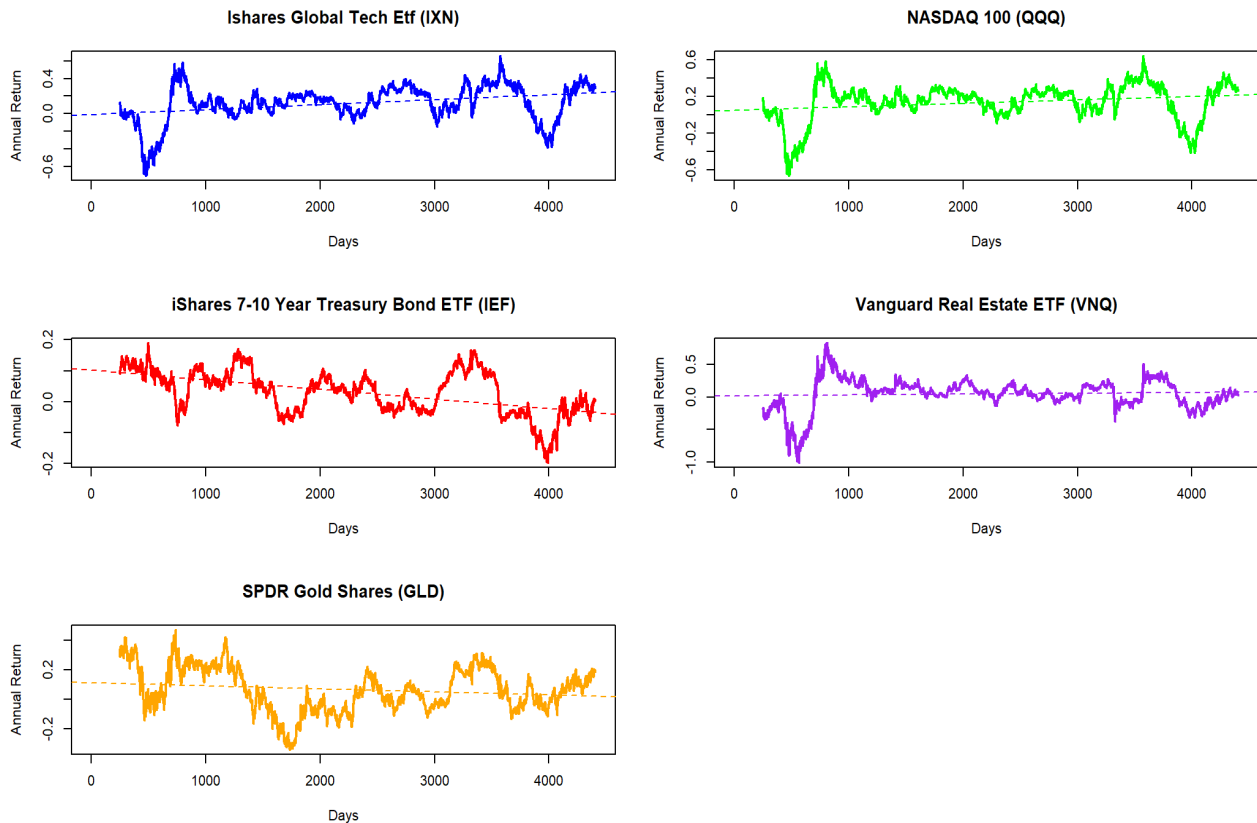


Figure 2 - 12 months returns by asset in the last decade

As it is possible to see from this time series the positive return in the last 17 years has been provided by the Equities Assets that focus on tech. This is coherent with the decennial trend that saw the tech companies driving the markets positive performance. The fixed income performance are, instead, negative. This is not surprising as well, since the long and only recently (post covid) broken low interests policy, that got us used to negative yield bonds for some countries in the past decades. Slightly positive and slightly negative performance trends are identified respectively for the VNQ REIT and for GOLD.

### Most recent 12 months returns for the assets

Coming to more recent and consequently more important data, the returns for the last year for all the 5 assets contained in the portfolio are shown in the following lines:

	IXN_12M_ret	QQQ_12M_ret	IEF_12M_ret	VNQ_12M_ret	GLD_12M_ret
01/07/2024	31,0%	27,0%	0,3%	2,4%	19,2%

As it is possible to notice the positive performance are mainly driven by equities. As confirmed by Blackrock [2024], buying tech ETFs has been very profitable in the last year, since the high performance of the magnificent 7 (AAPL, AMZN, GOOGL, META, MSFT, NVDA and TSLA) has driven the market growth. Interesting the performance for GLD, that this year has reached its highest historical peak. According to Investopedia [2024] gold has performed so well because its historical function of safe asset, targeted by investors in a moment of economic uncertainty as the one we're living. Low are the performance for real estate VNQ and the IEF bond ETF. The real estate is definitely not living its golden era due to the rise of interest rates and the risk of failure for big companies as Evergrande. About the bonds market, the inversion of the yield curve makes long term treasury bonds less attractive (also signaling economic risk) (US bank, 2024), and the possible cut of the interest rate mentioned by the FED (Reuters, 2024) could have a little decreasing effect on the performance as well.

### Most recent 18 months returns for the assets

Here are presented the results for the last 18 months:

	IXN_18M_ret	QQQ_18M_ret	IEF_18M_ret	VNQ_18M_ret	GLD_18M_ret
01/07/2024	63,9%	61,0%	0,6%	6,9%	23,1%

Even if the performance are the different, the overall relative performance are similar to the 12 months ones, standing on similar explanations as for the 12 months ones.

### Most recent 24 months returns for the assets

Here are presented the results for the last 24 months:

	IXN_24M_ret	QQQ_24M_ret	IEF_24M_ret	VNQ_24M_ret	GLD_24M_ret
01/07/2024	60,2%	52,6%	-4,5%	-1,8%	28,5%

Considering the two years the performance of the bonds and the REIT are negative. **These two are probably the most likely assets that will be impacted by the rebalancing.**

### Most recent 12, 18 & 24 months returns for the Portfolio

	portfolio12M	portfolio18M	portfolio24M
01/07/2024	16,1%	30,8%	27,3%

The overall performance of the portfolio are positive indicating anyway a good starting base for improving.

## Risk Analysis (Sigma, Tracking Error, Correlation, Beta) – Question 2 & 3

### Assets' Sigma



Figure 3 - Assets Sigma

As it is showed in the scatter plot, the internal (or total risk) for the VNQ REIT is definitely the highest among the assets. Especially when compared to the expected returns, it is clear that **VNQ asset is suboptimal** and **it has to be sold or substituted with another more performing and less risky asset**. For the remaining asset it could be said that they are positioned in expectable location according to their nature. Indeed, IEF bond is the less risky and the less performant in term of return, followed by GLD that, as said in the introduction, has been quite risky but definitely not as equities, providing anyway discrete returns especially in relation to the risk taken. Lastly QQQ and IXN are the riskier but providing the highest returns as it is expected from tech equities. Going to the specific, the fact that IXN is riskier than QQQ should not be a surprise, considering its major exposure to chipset production companies' stocks and what happened at geopolitical and supply chain-operational level in the last years (Citigroup, 2024).

These data were mainly related their intrinsic volatility, but it could be useful to check the risk in comparison to a benchmark. So the tracking error is analyzed in the next page.

## Assets' Tracking Error

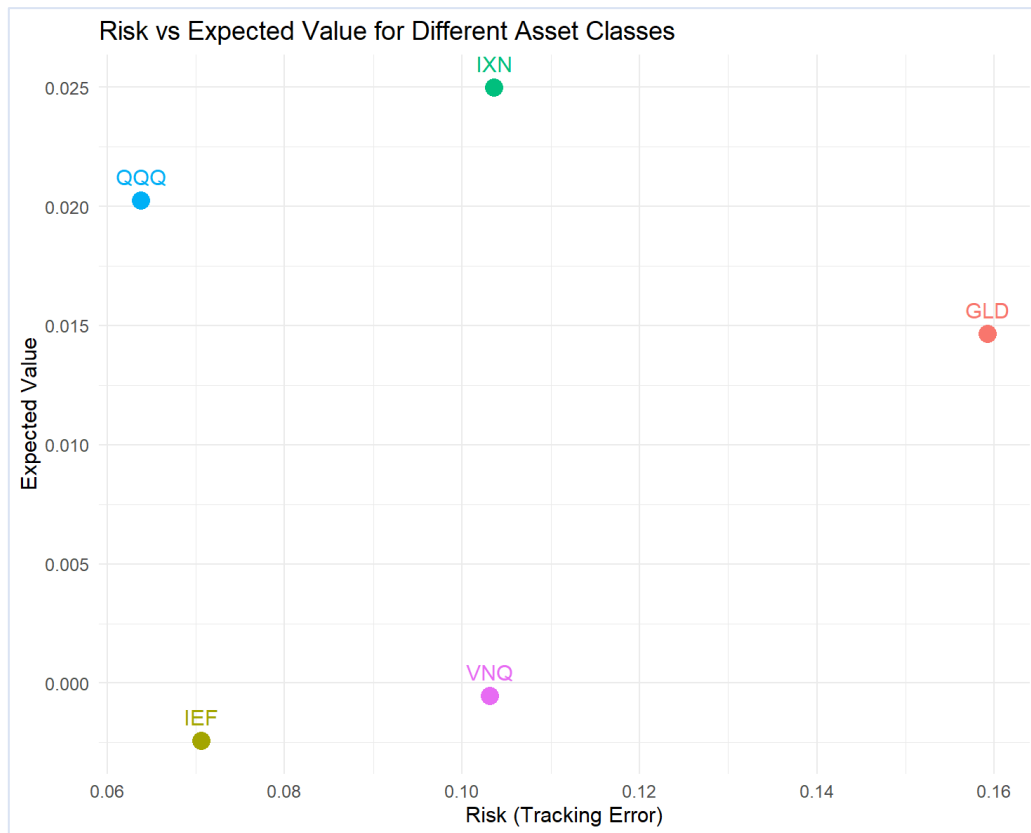


Figure 4 - Assets' Tracking Error

Considering the tracking error the situation is, indeed, different. To calculate the TE the **AOA (iShares Core Aggressive Allocation ETF)** std. deviation has been used. AOA is an aggressive ETF leaning towards large growth stocks, focusing on SP500 and other US market and having less than 20% bond holdings. **It has been considered an appropriate benchmark for this quite aggressive portfolio** (Morningstar, 2024). In this scenario the tracking error is of course lower for the equities ETF since clearly their performance are consistently close to the one of a similar equities market based ETF. Also in this case the IXN is confirmed as riskier than QQQ, probably for the same reasons stated in the previous paragraph. IEF keep its status of secure assets as in its nature, while VNQ performance deviation with respect to the benchmark is in line with the IXN one. Surprisingly, **GLD shows the highest deviation (16%)! This shows as GLD will likely be a likely candidate for diversify the portfolio, as its returns don't move consistently with the AOA (Equities) ones.** This will be confirmed by running a correlation in the next section.

## Portfolio Sigma and TE

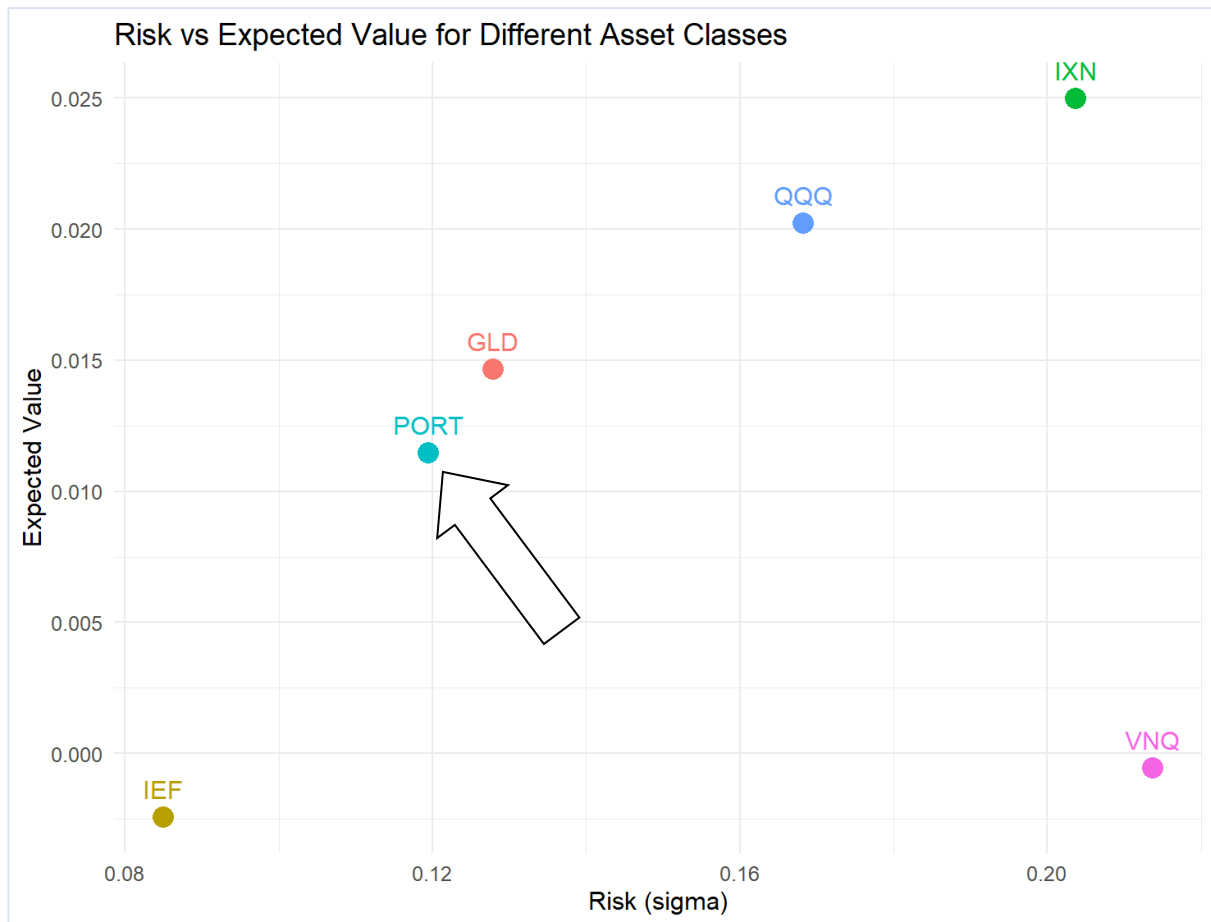


Figure 5 - Portfolio Sigma

Adding the portfolio to the scatter plot ( $E(x)$  vs  $\sigma$ ) it can be said that, as it is supposed to be, the portfolio is less risky than the other assets (except IEF, that is the safest being a fixed income asset class) but it still produce a decent amount of returns, almost reaching the GLD. **The portfolio Tracking error is instead 0.04**, way lower than any other single asset tracking error. This is of course due to the weighted average of the different assets returns that clearly lead to a bigger deviation with respect to the benchmark. Following two recap tables with Sigma and TE values (please note that these are rounded values).

IXN_Sigma	QQQ_Sigma	IEF_Sigma	VNQ_Sigma	GLD_Sigma	Pf_Sigma
0,20	0,17	0,08	0,21	0,13	0,12

IXN_TE	QQQ_TE	IEF_TE	VNQ_TE	GLD_TE	Pf_TE
0,10	0,06	0,07	0,10	0,16	0,04



## Correlation among Portfolio's Assets

	IXN	QQQ	IEF	VNQ	GLD	Portfolio	SPY (Benchmark)
IXN		97,9%	82,0%	86,5%	16,3%	94,7%	94,8%
QQQ	97,9%		84,1%	90,7%	8,1%	93,9%	97,4%
IEF	82,0%	84,1%		88,3%	15,2%	88,8%	82,3%
VNQ	86,5%	90,7%	88,3%		23,6%	93,7%	91,6%
GLD	16,3%	8,1%	15,2%	23,6%		38,9%	19,2%
Portfolio	94,7%	93,9%	88,8%	93,7%	38,9%		94,6%
SPY (Benchmark)	94,8%	97,4%	82,3%	91,6%	19,2%	94,6%	

Table 1 - Correlations

Correlation among assets is overall quite high (Corr > 0.8). Clearly, the equities EFT are the most correlated among themselves. Indeed, IXN and QQ have 97,9% of positive correlation, and both are highly correlated to the SPY. Also the entire portfolio, being structured as aggressive, shows high correlation with the equities stocks and with the equities markets. **VNQ is confirmed to be a not efficient asset in the portfolio, producing low returns, being risky and, as shown in the table, being highly correlated with IXN and QQQ (and also with IEF fixed income asset).** Also IEF bonds are correlated with equities, signaling a dangerous economic situation, being a symptom of an overvalued bubble equities market [Financial Times, 2023]. Concluding with **GLD**, this shows an expected pleasant low correlation with all the other portfolio's assets, acting as a source of diversification and reconfirming itself as a good asset to hold in the portfolio.

## Betas of the Assets and Beta of the Portfolio

Using the CAPM model as a predictive tool is not anymore possible due to the contingencies of the times we are living, but it is still possible to use it as a risk analysis tool, so a brief summary of the models is presented and then the beta is extracted to evaluate the risk. In this case the SPY has been used for the regression since the model works properly considering the market beta, using AOA it would have been incorrect.

	p-value	R - Squared	Market Beta
IXN CAPM	<2e-16	0.84	1.13
QQQ CAPM	<2e-16	0.84	1.09
IEF CAPM	0.238	0.06	Garbage Model
VNQ CAPM	<2e-16	0.57	1.09
GLD CAPM	0.181	0.008	Garbage Model

Table 2 - CAPM Regressions

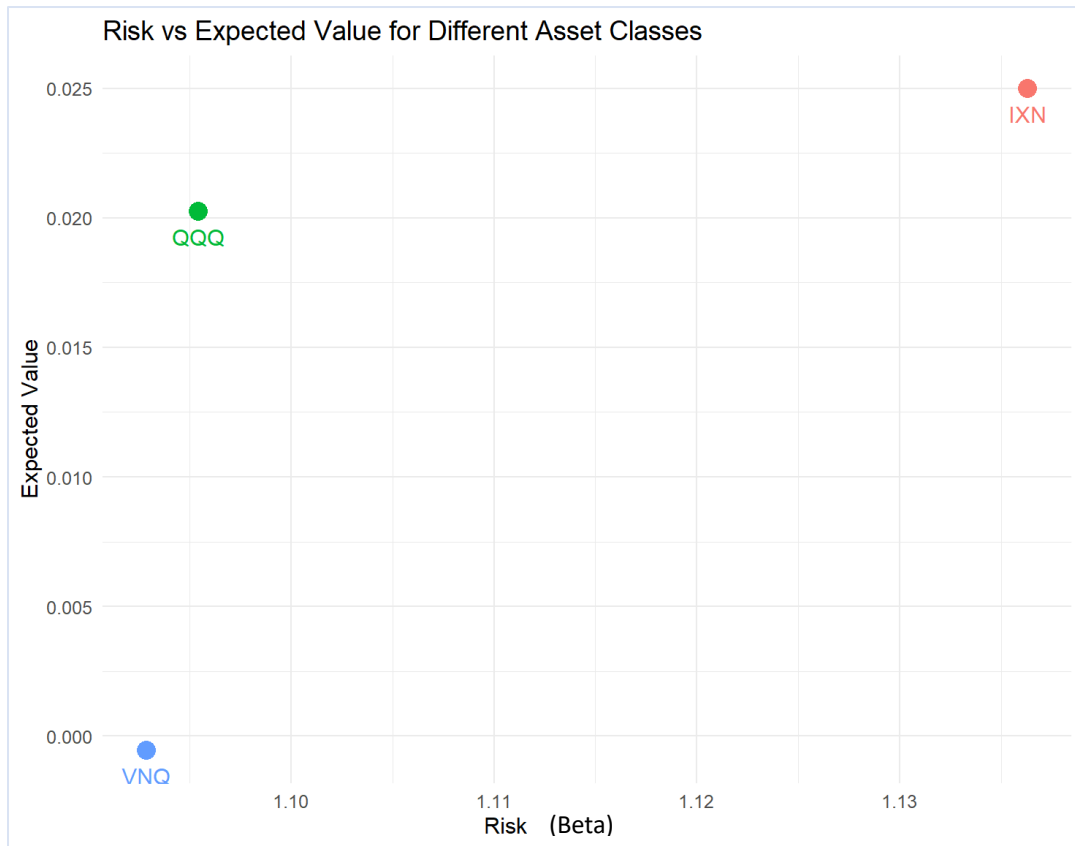


Figure 6 - Asset  $E(x)$  vs Betas

In the graph the only assets shown are the ones sustained by a reliable model. Before providing for a comment it must be signaled the “distortion” due to the way this graph is built<sup>3</sup>. Indeed IXN is located in the right up corner being the most dangerous among the three assets in relation to SPY. Now, as is true that IXN is riskier than the S&P 500, it must be clear that the graph highlight the relative differences between Betas values, showing IXN far from the other assets, while the absolute difference is around 0.04-0.03. That’s why it is possible to say that all the assets are just slightly riskier than SPY. Of course this result for the equities assets is normal, while it is still an anomaly for **VNQ that again it’s confirmed as an inefficient assets**.

<sup>3</sup> Actually this graph is misleading and should not be used in an official report according to the official data visualization best practices, but since the preference that the professor expressed for scatter plots rather than bar charts or other methods, and since this is “just” an academical exercise, it is still proposed.

## Performance Analysis (Sharpe, Sortino, Treynor)

Asset	Sharpe_Ratio	Sortino_Ratio	Treynor_Ratio
IXN	1,69	2,36	0,30
QQQ	1,61	1,86	0,25
IEF	-0,34	-0,20	-
VNQ	-0,03	-0,05	0,19
GLD	1,49	1,31	-
Portfolio	1,23	1,76	-

Table 3 - Ratios

Looking at the table it is immediately clear that **IEF and VNQ are underperforming!** The negative Sharpe/Sortino ratio suggests that the investment are not providing adequate returns for the level of risk taken for both the assets (taking for granted that the impact of the risk free<sup>4</sup> used is irrelevant). The **Highest Sharpe/Sortino ratio is for the equities assets IXN and QQQ**, as expectable from the empirical analysis of the graph (Figure 3 and Figure 5) **indicating that the highest economic util is derived from them.** Nice performance also for **GLD that present** lower (with respect to IXN and QQQ) but **satisfying ratios values** given the asset class it belongs to. Anyway it must be said that **Gold is the only asset class for which the Sortino ratio is lower than the Sharpe**, making clear that the effect of negative standard deviation is higher than the positive, in other words the asset has a notable downside risk compared to its overall risk, and this should be monitored. **Two relevant downsides are the quite high Treynor ratios for VNQ**, expected according what has been said in the previous page, **and the fact that the portfolio ratios are lower than the majority of the other assets with positive returns (especially Sharpe Ratio is lower than all the others)!** **The portfolio need to be optimized.**

## Educated guess on what to sell and what to buy – Question 4

Considering the ratios listed in Table 3 and what said in the previous analysis, the analyst suggestions are:

- Sell the entire VNQ holdings, since it is risky and provide low/negative<sup>5</sup> returns
- Sell or drastically reduce IEF. This is actually not so risky but too correlated with the other assets while providing very low/negative<sup>6</sup> returns.
- Choose among IXN and QQQ<sup>7</sup>, since they are very similar in their behavior and performance (considering the overall performance IXN is better especially looking at the Sortino Ratio).
- Keep GLD as a valid source of diversification and return.

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<sup>4</sup> Risk Free = 0,0001

<sup>5</sup> In base on the period considered

<sup>6</sup> idem

<sup>7</sup> This depends on the max. weight the analyst in going to set but in any case IXN should prevail QQQ

## FAMA & FRENCH model application

	IXN FF3		
	residuals	actuals	model_pred
1	-0,026	-0,026	-0,001
2	0,008	0,010	0,002
3	0,020	0,016	-0,004
4	0,007	0,005	-0,002
5	0,000	-0,001	-0,001
6	0,014	0,015	0,001
7	0,004	0,003	-0,001
8	-0,022	-0,022	0,000
avg (abs val)	0,013		

	QQQ FF3		
	residuals	actuals	model_pred
1	0,001	0,000	-0,001
2	-0,005	-0,004	0,001
3	0,008	0,009	0,001
4	0,006	0,004	-0,002
5	-0,009	-0,007	0,002
6	-0,013	-0,001	0,002
7	-0,004	-0,002	0,002
8	0,008	0,005	-0,003
avg (abs val)	0,007		

	VNQ FF3		
	residuals	actuals	model_pred
1	-0,008	-0,009	-0,001
2	-0,022	-0,022	0,000
3	0,000	0,001	0,001
4	-0,005	-0,007	-0,001
5	-0,011	-0,010	0,001
6	0,014	0,015	0,001
7	0,017	0,019	0,002
8	0,000	-0,002	-0,002
avg (abs val)	0,010		

	IXN FF5		
	residuals	actuals	model_pred
1	-0,001	0,001	0,002
2	-0,003	-0,002	0,002
3	0,015	0,014	-0,001
4	0,005	0,004	0,000
5	0,000	-0,001	-0,001
6	-0,010	-0,012	-0,002
7	-0,003	-0,005	-0,002
8	0,004	0,002	-0,002
avg (abs val)	0,005		

	QQQ FF5		
	residuals	actuals	model_pred
1	-0,002	0,000	0,002
2	-0,004	-0,003	0,001
3	0,007	0,005	-0,001
4	0,005	0,006	0,000
5	-0,002	0,000	-0,001
6	-0,003	-0,005	-0,002
7	-0,003	-0,004	-0,001
8	-0,003	-0,004	-0,002
avg (abs val)	0,003		

	VNQ FF5		
	residuals	actuals	model_pred
1	0,003	0,003	0,000
2	0,005	0,005	0,000
3	0,003	0,002	-0,001
4	0,001	0,000	0,000
5	-0,006	-0,008	-0,001
6	0,000	-0,001	-0,001
7	-0,004	-0,004	-0,001
8	0,006	0,006	0,000
avg (abs val)	0,003		

Given the date limitation from the free Fama-French API the data from two weeks ago are shown in table (1 week time range). In general all the models do a good job in predicting the value for the day considered as the average of residuals absolute value shows. As it was possible to imagine **the Fama-French 5**, that include the profitability factor (RMW)<sup>8</sup> and the investment factor (CMA)<sup>9</sup> is even **more accurate than the three factor model, making this model preferable**.

<sup>8</sup> is the difference between the returns of firms with robust (high) and weak (low) operating profitability

<sup>9</sup> is the difference between the returns of firms that invest conservatively and firms that invest aggressively

## Efficiency Frontier – Question 6

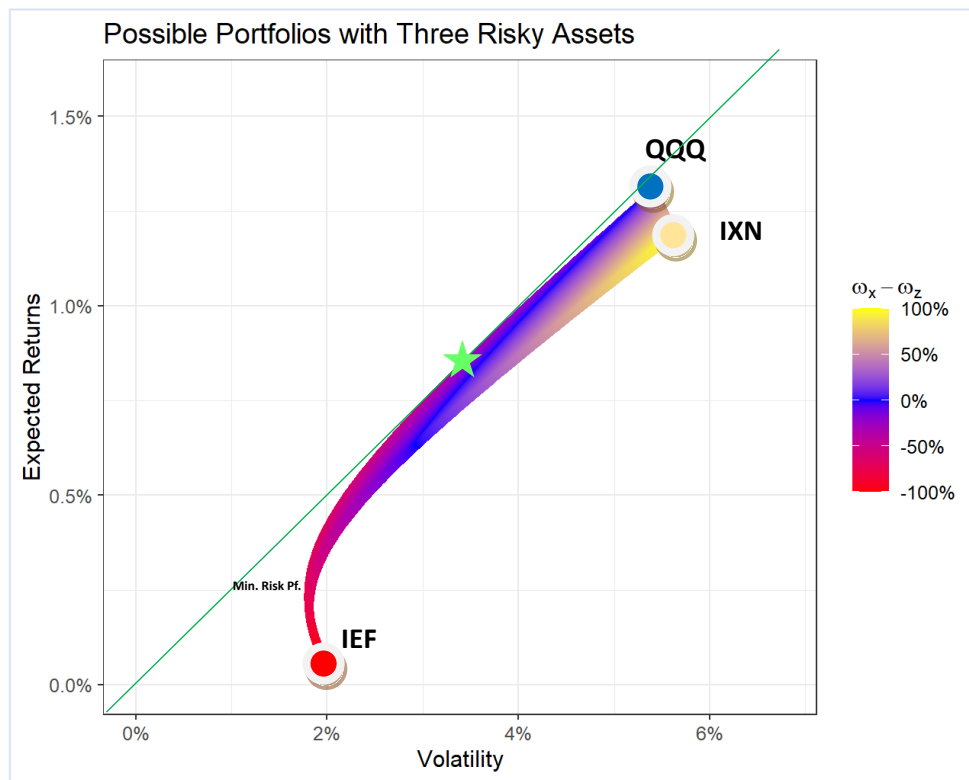


Figure 7 - Efficiency Frontier IXN, QQQ, IEF

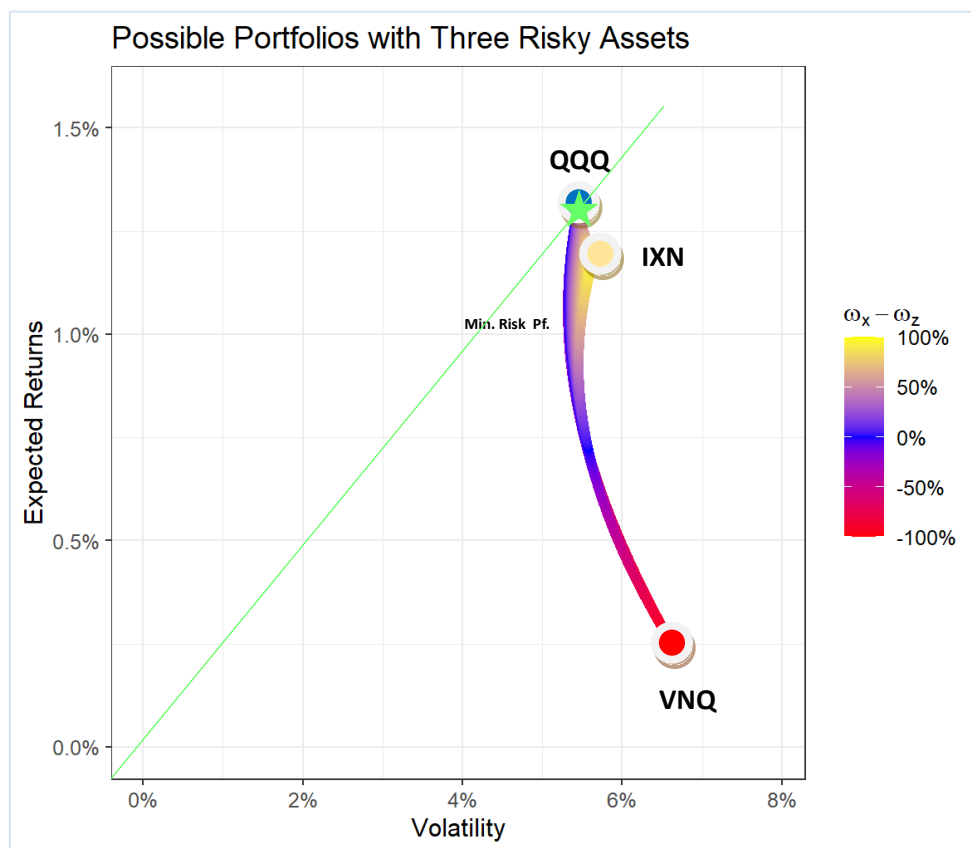


Figure 8 - Efficiency Frontier IXN, QQQ, VNQ

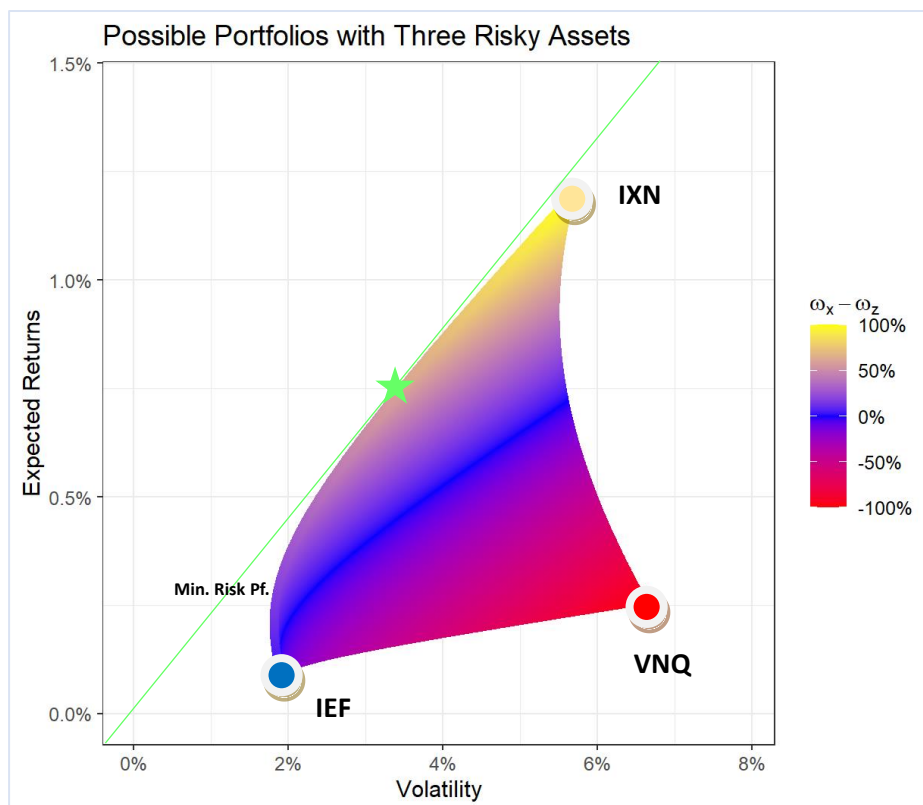


Figure 9 - Efficiency Frontier IXN, IEF, VNQ

★ = Optimal Pf. Assuming risk free = 0%

Using the code provided in class, these efficiency curves have been produced. In the Figure 7 and 8 the most performing equities asset is QQQ<sup>10</sup>, that is less risky than IXN, while being able to provide slightly higher returns. This probably depends on the time window taken in consideration and on the data and calculations differences. Assuming that the rebalancing code behaves in the same way, this little change could affect the rebalancing that will see QQQ prevailing on IXN, on the contrary of what stated in the previous educated guess<sup>11</sup>. **In Figure 8 and 9 come the final confirm (not necessary, but still a confirm) that VNQ will be reduce to 0, selling the 100% of its shares held in the portfolio.** Especially looking at Figure 8 it is interesting noticing how the optimization algorithm suggest to keep only QQQ in the portfolio, being the only efficient asset among the three. For Figure 7 and Figure 9 the optimal portfolio is instead composed of both IEF and the most efficient equity ETF, QQQ for Figure 7 and IXN for figure 9, this to reduce the overall risk but keeping at the same time a satisfying return percentage. Arrived to this point, the last task to perform is to create the new optimized portfolio, presented in the next page.

<sup>10</sup> This was determined checking the  $er_x$ ,  $er_y$  and  $sd_x$  and  $sd_y$  in the code

<sup>11</sup> After having performed rebalancing it has been proved that it works differently, but I leave this part to show the logical flow I had during the assignment.

## Portfolio Optimization – Question 5 pt 1

Pf. Simulation 1		Pf. Simulation 2		Pf. Simulation 3	
max. weight	40%	max. weight	80%	max. weight	30%
IXN	40%	IXN	74%	IXN	30%
QQQ	34,20%	QQQ	0%	QQQ	30%
IEF	0%	IEF	0%	IEF	14,7%
VNQ	0%	VNQ	0%	VNQ	0%
GLD	23,80%	GLD	23,40%	GLD	24,30%

Table 4 - Portfolio proposals

After several simulations<sup>12</sup> changing the timeframe window, assets and max weight it has been ensured that at the current moment there is no ETF in bonds, cryptocurrencies or commodities that can compete in term of Sharpe Ratio (Expected Return, Risk) and correlation, with the Equities ETF, in the short term (1 Year) and in the long term (8-10 years). Several attempt has been made using IBIT, GBTC, CWB, EMB, USHY, GCC and more but the model always suggest to keep in only IXN, QQQ and GLD. Since this is a mathematical optimization of the portfolio, meaning that all qualitative aspects and personal preference are being ignored, the result must be reported as the model command. Before to go straight to the conclusion, it is interesting to notice how **the second portfolio simulation**, where the minimum weight has been set to 80%, **perfectly match with the (semi-empirical) recommendation given** in *Educated Guess on what to sell and what to buy* - Question 4 chapter. Indeed, the only asset kept in the portfolio are IXN and GLD as suggested. Instead, as expected in the simulation number 3, where the model has to choose between IEF and VNQ due to the weight boundary imposed, it selected the least inefficient i.e. IEF.

## Conclusions (New Expected Return, New Risk) – Question 5 pt 2

Given that simulation number three come with IEF that produce very low / negative return only the first two are taken in consideration. Among the two the highest Sharpe Ratio is obtained by the simulation number 1 with IXN, QQQ and GLD. **The new portfolio will have the following properties:**

Optimized Pf		Original Pf	
Expected Value	0.022	Expected Value	0.011
Sigma	0.145	Sigma	0.110
Ann. Exp. Val	0.308	Ann. Exp. Val	0.146
Sharpe Ratio	2.58	Sharpe Ratio	1.22

Table 5 - Final Portfolio

<sup>12</sup> Here are reported only the most relevant to keep the document clean

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