# Problem 1

Let us consider the following transfer functions:

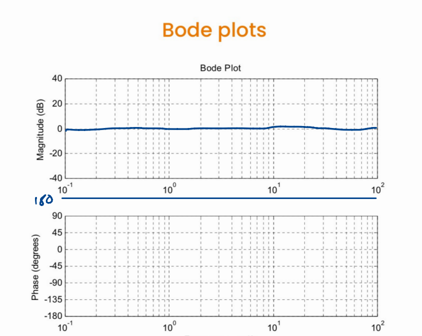
1. $H(s) = \frac{-1}{(1+s)(1-s)^2}$
2. $H(s) = \frac{10}{s^2\left(1-\frac{s}{2}\right)^2}$
3. $H(s) = \frac{9(s-1)}{s(s^2-9)}$

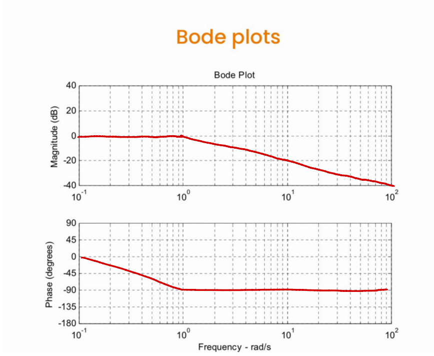
for later

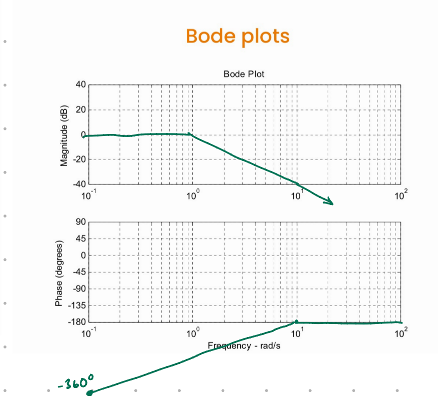
For each of them, draw the asymptotic Bode plot by hand and draw the Bode plot with MATLAB. Based on the Bode plot, sketch the polar and Nyquist plots by hand. Finally, draw the Nichols diagram with MATLAB.

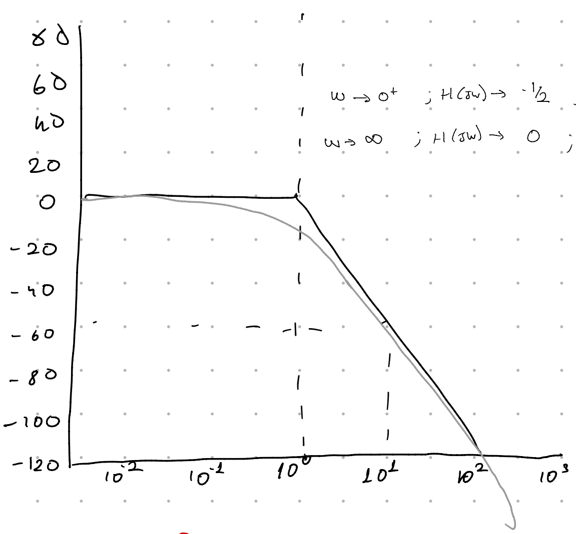
1) $H(s) = \frac{-1}{(1+s)(1-s)^2}$ ; $K=-1$ ; $P_1 = 1$ , $P_2 = -1$

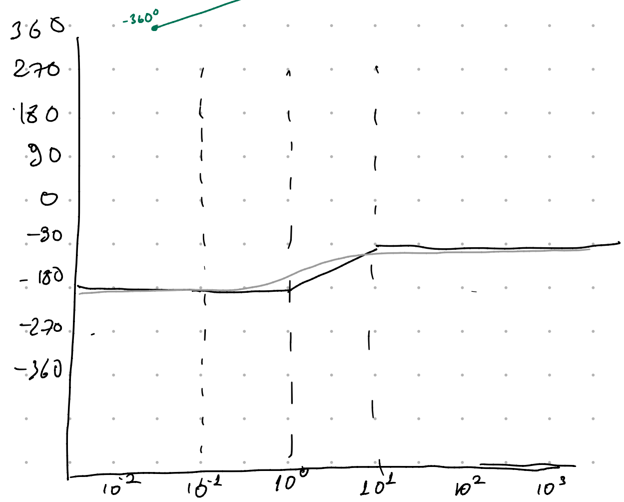
$$H = \underset{H_0(j\omega)}{(-1)} \cdot \underset{H_1(j\omega)}{\frac{1}{(1+j\omega)}} \cdot \underset{H_2(j\omega)}{\frac{1}{(1-j\omega)^2}}$$



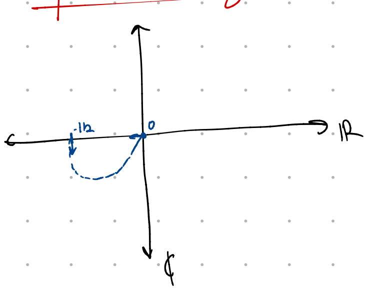



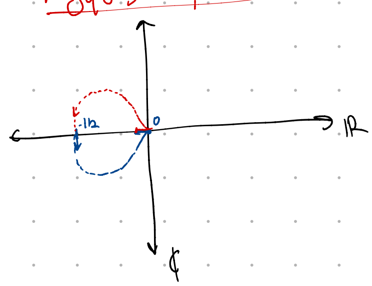



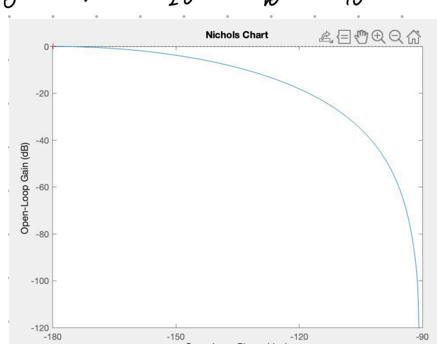




Polar Diagram:




Nyquist plot:



# Problem 2

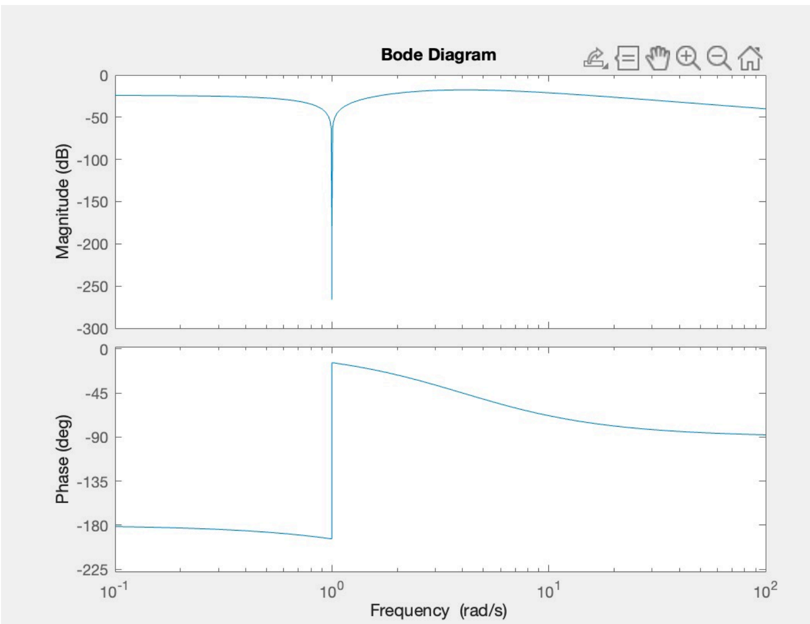
Given the transfer function

$$H(s) = \frac{1+s^2}{(s^2-4)(s+4)}$$

draw its Bode plot with MATLAB. Based on the Bode plot, sketch the polar and Nyquist plots by hand. Then, check their correctness with MATLAB. Finally, draw the Nichols diagram with MATLAB.

(Hints: $H(s)$ has two purely imaginary zeros, $\zeta = 0$.

$H(j\omega) = 0$ for $\omega = 1$, therefore the polar plot crosses the origin.)

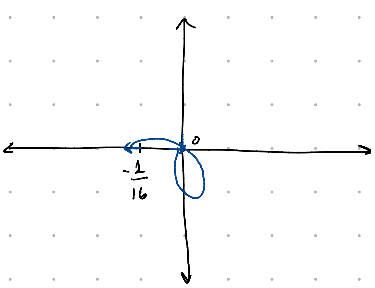



$\omega \to 0^+$ ; $H(j\omega) \to -\frac{1}{16}$ ; $\angle H(j\omega) \to -180°$

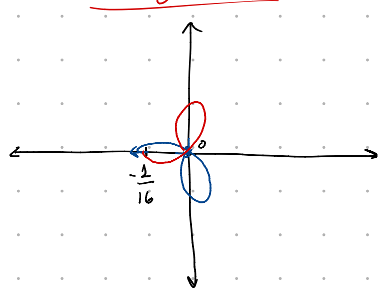
$\omega \to \infty$ ; $H(j\omega) \to 0$ ; $\angle H(j\omega) \to -90°$

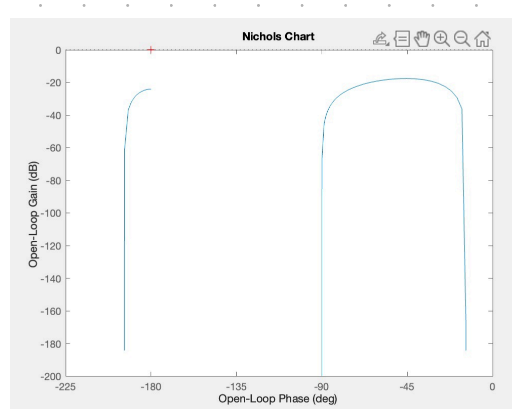
$\omega \to 1$ ; $H(j\omega) \to 0$ ; $\angle H(j\omega) \to 0°$

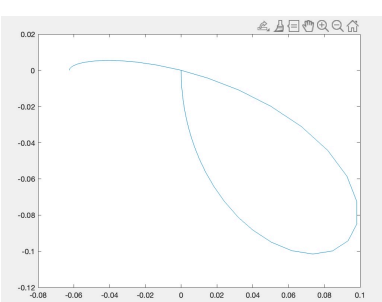
Polar :

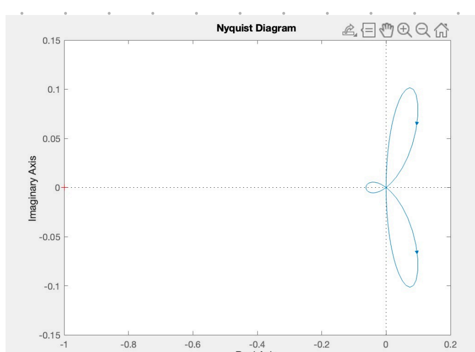



Nyquist :



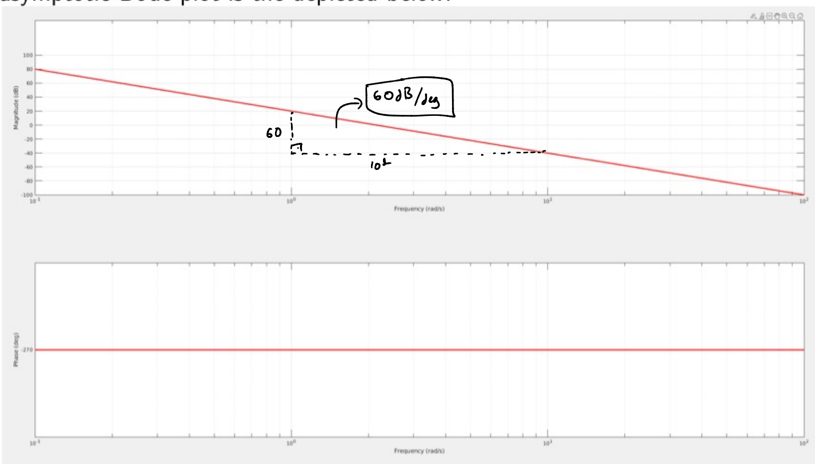





## Problem 3

Let us consider a minimal LTI system, with real singularities (zero and poles). Its asymptotic Bode plot is the depicted below.




1. Is this system stable?
2. Compute its transfer function $H(s)$
3. Based on the Bode plot, sketch the polar and Nyquist plots by hand. Then, check their correctness with MATLAB. Finally, draw the Nichols diagram with MATLAB.

**Solution**
Unstable, $H(s) = \frac{10}{s^3}$.

1) $\lim_{\omega \to \infty} |H(s)|_{dB} = -\infty$ hence unstable

2) magnitude decreases with a slope of $60\ dB/rad$ and $\omega \to 0 \Rightarrow H(s) \Rightarrow \infty$

hence $H(s) = k \frac{1}{s^3}$ ; at $\omega = 1$; $20 \log H(s) = 20$

$\log H(s) = 1$

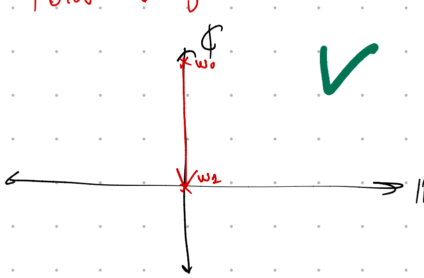
$H(s) = 10^1 = 10 \Rightarrow \boxed{k = 10}$

$$H(s) = \frac{10}{s^3}$$

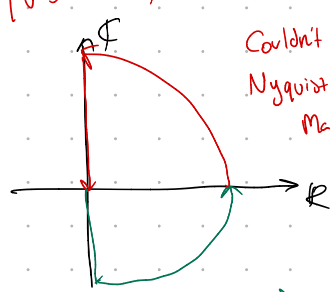
3) $\omega \to 0^+$ ; $H(j\omega) \to +\infty$ ; $\angle H(j\omega) \to -270° = 90°$ $\quad$ $\cos(90°) = 0$; real part 0

$\omega \to \infty^+$ ; $H(j\omega) \to 0$ ; $\angle H(j\omega) \to -270° = 90°$ $\quad$ $\sin(90°) = 1$; positive imaginary

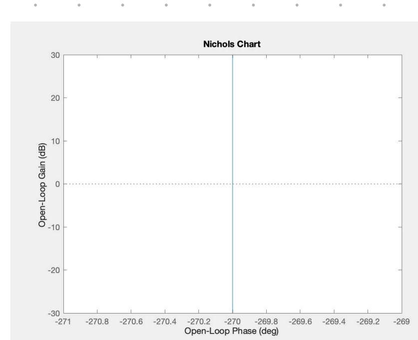
Polar Diagram




Nyquist Plot

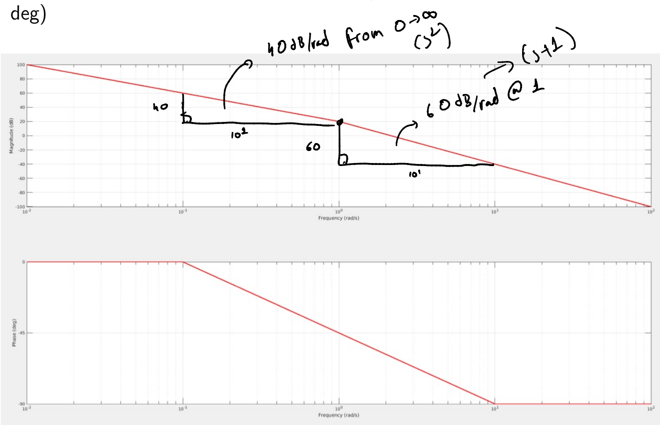



Couldn't plot Nyquist plot in Matlab?



## Problem 4

Let us consider a minimal LTI system, with real singularities (zero and poles). Its asymptotic Bode plot is the depicted below (remark: in the phase plot, $0 = -360$ deg)




1. Is this system stable?
2. Compute its transfer function $H(s)$
3. Based on the Bode plot, sketch the polar and Nyquist plots by hand. Then, check their correctness with MATLAB. Finally, draw the Nichols diagram with MATLAB.

1) as $\omega \to \infty$ $H(j\omega) \to -\infty$

Not stable

2)

$$H(s) = K \frac{1}{(s+1)\, s^2}$$

: $w = 1$; $20 \log(H(s)) = 20$

$H(s) = 10$

but with degree 0

if $k = 10$ it would start with $-180°$

Here $k = -10$

$$\boxed{H(s) = \frac{-10}{(s+1)\, s^2}}$$

3. $w \to 0$; $H(jw) \to \infty$; $\angle H(jw) = 0°$ $\quad \cos(0°) = 1 \quad \sin(0°) = 0$

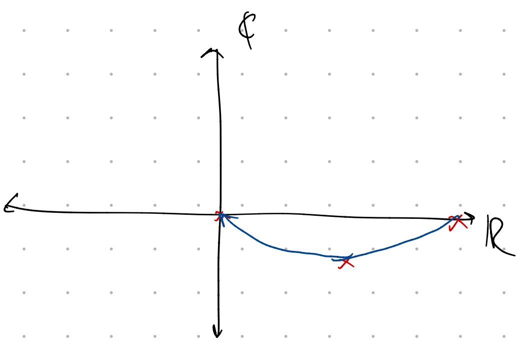
positive Real

$w \to \infty$; $H(jw) \to 0$ ; $\angle H(jw) = -90°$

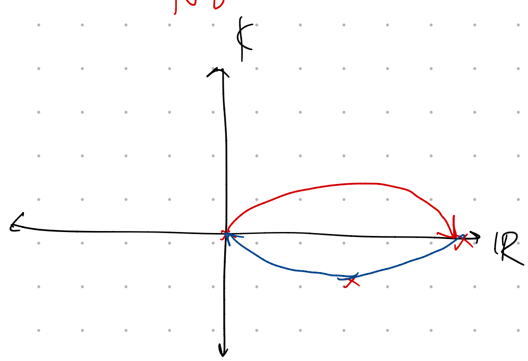
$w = 1$ ; $H(jw) \to 20$ ; $\angle H(jw) = -45°$ $\to \sin(45) = -\cos(45°)$

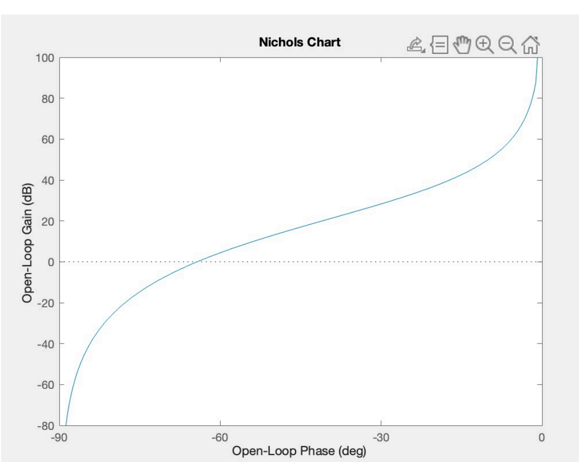
pos Real ; neg Img

Polar




Nyquist



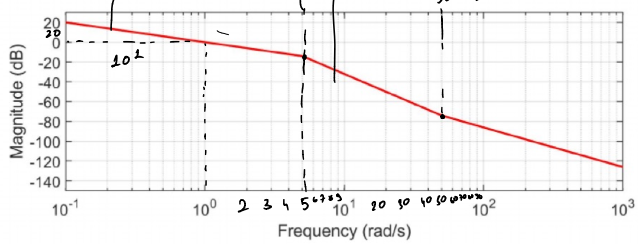



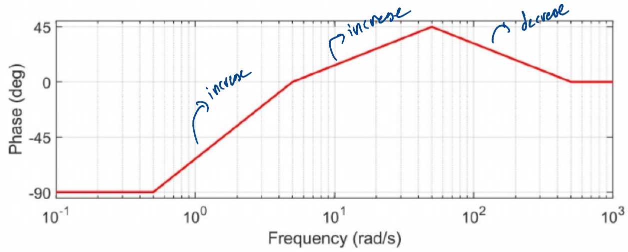
## Problem 5

Let us consider a minimal LTI system, with real singularities (zero and poles). Its asymptotic Bode plot is the depicted below.

20dB/rad $\left(\frac{1}{s}\right)$   5   ≈ 60 dB/rad $\left(\frac{1}{(1-\frac{s}{5})^2}\right)$ decrease in magnitude hence a pole ; increase in degree hence positive real

50 ⟶ ≈ 40dB/rad $(1-\frac{s}{50})$ increase in mag hence zero ; decrease in degree hence pos real




1. Is this system stable?
2. Given the input $u(t) = 2\epsilon(t)$ compute the analytical expression of the output response $y(t)$.
3. If possible, evaluate the steady state $y_{ss}(t)$ in the presence of the input $u(t) = 2\epsilon(t)$.

1) Unstable

2) $20\log(|H(j\omega)|) = 0 \quad @\ \omega = 1$

$H(j\omega) = 1 \quad @\ \omega = 1$

$\hookrightarrow k = 1$

$$\boxed{H(s) = \frac{1 - \frac{s}{50}}{s(s-\frac{1}{5})^2}}$$

$u(t) = 2\epsilon(t) \Longleftrightarrow \frac{2}{s}$

$Y(s) = H(s)U(s)$

$$\boxed{Y(s) = 2\,\frac{1 - \frac{s}{50}}{s^2(1-\frac{s}{5})^2}}$$

$$\boxed{\begin{matrix} \lambda_1 = 0 & \lambda_2 = 5 & z_1 = 50 \\ \mu_1 = 2 & \mu_2 = 2 & \end{matrix}}$$

$$\text{res } y = \begin{bmatrix} -0.76 \\ 1.8 \\ 0.76 \\ 2 \end{bmatrix} \qquad \text{poly} = \begin{bmatrix} 5 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

$$y(t) = \left(-0.76e^{5t} + 1.8te^{5t} + 0.76 + 2t\right)\epsilon(t)$$

the system is not stable

3) Dcgain is inf ; no steady state

## Problem 6 (INF only)

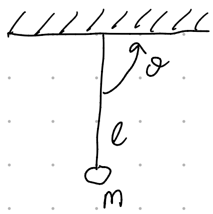
For the pendulum system with state equation

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -\frac{g}{l}\sin(x_1(t)) - \frac{\beta}{ml^2}x_2(t) + \frac{1}{ml^2}u(t)$$

with $m = 0.1$ kg, $l = 0.5$ m, $\beta = 0.1$ N s rad$^{-1}$, $g = 9.81$ m s$^{-1}$.

Compute the equilibrium input $\bar{u}$ that corresponds to the equilibrium state $\bar{x} = \begin{pmatrix} \frac{\pi}{2} \\ 0 \end{pmatrix}$ and study its stability properties through the linearized model.

Graphical Representation:




at the equilibrium state:

$\dot{x}_1 = 0 \quad ; \dot{x}_2 = 0 \qquad \Rightarrow \bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} \pi/2 \\ 0 \end{bmatrix}$

$\hookrightarrow 0 = -\frac{g}{l}\sin(x_1) - \frac{\beta}{ml^2}x_2 + \frac{1}{ml^2}\bar{u}$

$0 = -\frac{g}{l} + \frac{1}{ml^2}\bar{u} \Rightarrow \frac{g}{\cancel{l}} = \frac{\bar{u}}{ml^{\cancel{2}}} \Rightarrow \boxed{mlg = \bar{u}}$

$= \boxed{0.49}$

$\dot{x}_1 = x_2 = f_1$

$\dot{x}_2 = a\sin(x_1) + b\,x_2 + c\,u = f_2$

$a \hookrightarrow -g/l \qquad b \hookrightarrow \beta/ml^2 \qquad c \hookrightarrow 1/ml^2$

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & b \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{\partial f_1}{\partial u} & \frac{\partial f_2}{\partial u} \end{pmatrix}^T = \begin{pmatrix} 0 & c \end{pmatrix}^T$$

$$\mathrm{eig}(A) = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

$0 \rightarrow$ can not say

$-4 \rightarrow$ Stable

A conclusion can not be drawn

Consider the nonlinear dynamical systems described by

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -19.62\sin(x_1(t)) - 4x_1(t) - 4x_2(t) + 40u(t)$$

Compute the equilibrium states $\bar{x}$ corresponding to the equilibrium input $\bar{u} = 0$ and study their stability properties through the linearized model.

(Hint: the solving equation for equilibrium computation can be solved graphically through a MATLAB plot.)

$\dot{x}_1 = 0 \quad ; \dot{x}_2 = 0$

since $\bar{x}_2 = \dot{\bar{x}}_1 \Rightarrow \bar{x}_2 = 0$

$\Rightarrow 0 = -19.62 \sin(\bar{x}_1) - 4\bar{x}_1 \Rightarrow 19.62 \sin(\bar{x}_1) = -4\bar{x}_1$

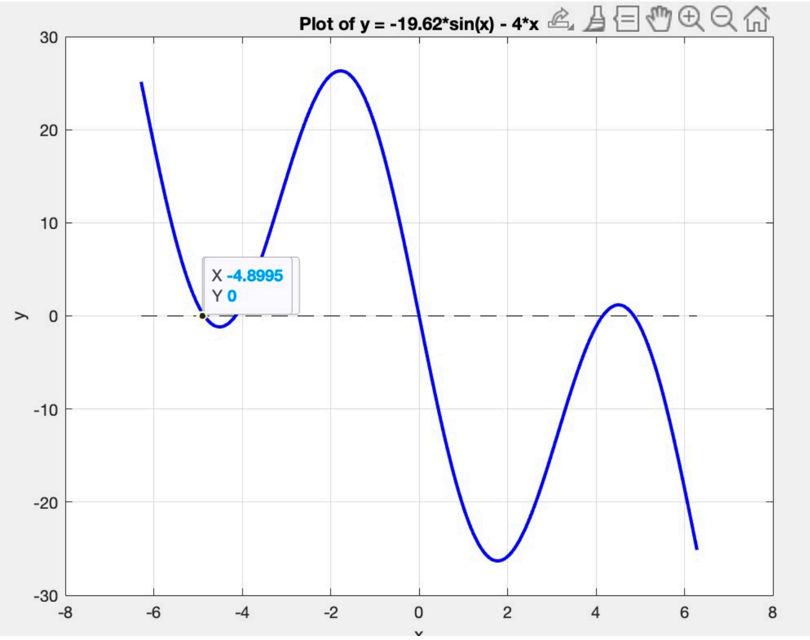
roots $= (-4.85, -4.15, 0, 4.15, 4.85)$

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -19.62\cos(x_1) - 4 & -4 \end{pmatrix}$$

$\begin{bmatrix} -4.85 \\ 0 \end{bmatrix} \rightarrow$ stable $\qquad \begin{bmatrix} -4.15 \\ 0 \end{bmatrix} \rightarrow$ unstable

$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow$ stable $\qquad \begin{bmatrix} 4.15 \\ 0 \end{bmatrix} \rightarrow$ unstable

$\begin{bmatrix} 4.85 \\ 0 \end{bmatrix} \rightarrow$ stable




Alternatively it is possible to find the roots by looking at the graph