**AUTOMATIC CONTROL**

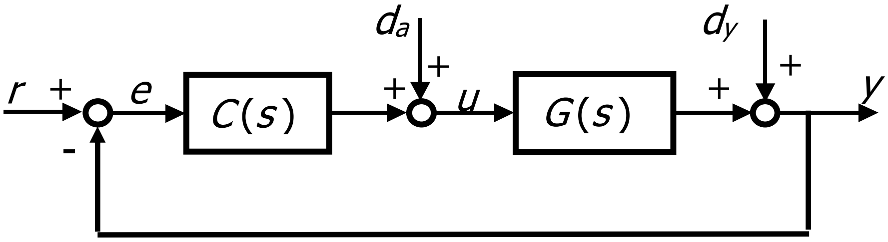
Computer Engineering and Electronic and Communications Engineering

# *Laboratory practice n. 6*

Objectives: Steady state analysis and design, loop shaping design.

**Problem 1** : loop shaping design of feedback control systems

Consider the feedback control system below




where:

$$G(s) = \frac{10}{s(s+5)(s+10)},\ d_a(t) = \delta_a \varepsilon(t),\ |\delta_a| \le 0.3,\ d_y(t) = \delta_y \sin(t),\ |\delta_a| \le 0.3$$

Design a cascade controller C(s) to meet the following requirements:

1. $|e_r^\infty| \le 1$ in the presence of a linear ramp reference signal with unitary slope;
2. $|y_{d_a}^\infty| \le 0.1$;
3. $\hat{S} \le 8.5\%$;
4. $t_{s,2\%} \le 0.75\ s$.

Evaluate through time domain simulation

- requirements satisfaction;
- the maximum magnitude of the input signal $u(t)$ in the presence of a step reference signal with amplitude 0.1;
- the maximum magnitude of the output signal $y(t)$ in the presence of both a step reference signal with amplitude 0.1 and the disturbance $d_a$

After the design evaluate

- the resonant peak $T_p$ (in dB) of the complementary sensitivity function as well as its bandwidth $\omega_B$;
- the resonant peak $S_p$ (in dB) of the sensitivity function as well as its bandwidth $\omega_{BS}$.

Write the expression of the final controller in the dc-gain form.

**Conceptual problem**

**Problem 2**: steady state analysis

$|e_r^\infty|$

| $h$ / type $g$ | 0 (step) $r(t)=\rho\varepsilon(t)$ | 1 (linear ramp) $r(t)=\rho t\varepsilon(t)$ | 2 (parabolic ramp) $r(t)=\rho(t^2/2)\varepsilon(t)$ |
|---|---|---|---|
| 0 | $\left\lvert \frac{\rho}{1+K_0} \right\rvert$ | $\infty$ | $\infty$ |
| 1 | 0 | $\left\lvert \frac{\rho}{K_1} \right\rvert$ | $\infty$ |
| 2 | 0 | 0 | $\left\lvert \frac{\rho}{K_2} \right\rvert$ |

linear ramp, unitary slope $\Rightarrow r(t) = t\varepsilon(t)$

$\hookrightarrow |e_r^\infty| \leq 1 \rightarrow$ finite

$\hookrightarrow |e_r^\infty| = \left| \frac{\rho}{K_1} \right|$

0 pole

$$G(s) = \frac{10}{s(s+5)(s+10)}$$

$\hookrightarrow$ type 1 system needed

$\hookrightarrow$ has 0 pole $\hookrightarrow \boxed{C_{ss} = k_c}$

$$K_1 = \lim_{s\to 0} sL(s) = \lim_{s\to 0} s\,G(s)\underbrace{C(s)}_{C_{ss}(s)\,C_T(s)} = \lim_{s\to 0} \cancel{s} \frac{10}{\cancel{s}(s+5)(s+10)}\, k_c \underbrace{C_T(s)}_{1} = \boxed{\frac{k_c}{5}}$$

$$|e_r^\infty| = \left| \frac{1}{k_c/5} \right| = \left| \frac{5}{k_c} \right| \leq 1 \Rightarrow \boxed{|5| \leq k_c}$$

$\boxed{k_c = 5}$

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$d_a = \delta_a \varepsilon(t)$

$|y_{d_a}^\infty| \leq 0.1 \rightarrow$ finite

$|y_{d_a}^\infty|$

| $h$ / $g_c$ | 0 (step) $d_a(t)=\delta_a\varepsilon(t)$ | 1 (linear ramp) $d_a(t)=\delta_a t\varepsilon(t)$ | 2 (parabolic ramp) $d_a(t)=\delta_a t^2/2\varepsilon(t)$ |
|---|---|---|---|
| 0 | $\left\lvert \frac{\delta_a}{K_0} \right\rvert$ | $\infty$ | $\infty$ |
| 1 | 0 | $\left\lvert \frac{\delta_a}{K_1} \right\rvert$ | $\infty$ |
| 2 | 0 | 0 | $\left\lvert \frac{\delta_a}{K_2} \right\rvert$ |

$$K_0 = \begin{cases} K_c & \text{if } G(s) \text{ has poles in } 0 \\ \frac{1+K_cK_G}{K_G} & \text{if } G(s) \text{ has not poles in } 0 \end{cases} \qquad K_{g_c} = \lim_{s\to 0} s^{g_c} C(s),\ g_c \geq 1$$

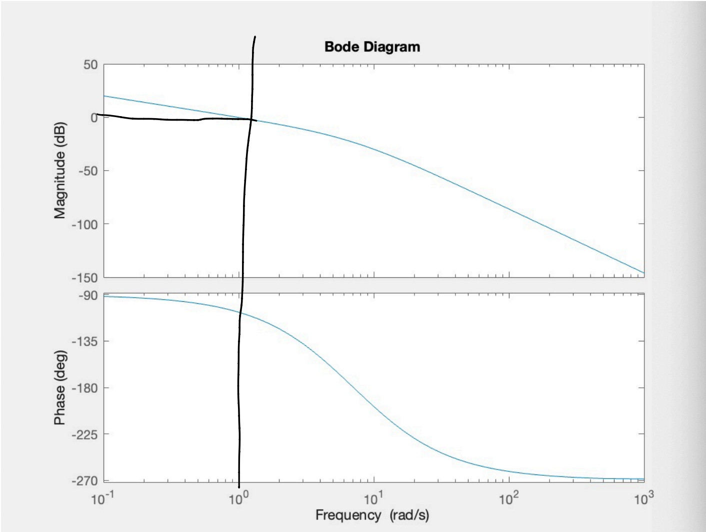
$\hookrightarrow$ since $G(s)$ has poles in 0; $k_0 = k_c \Rightarrow |y_{d_a}^\infty| = \left| \frac{\delta_a}{k_c} \right|$

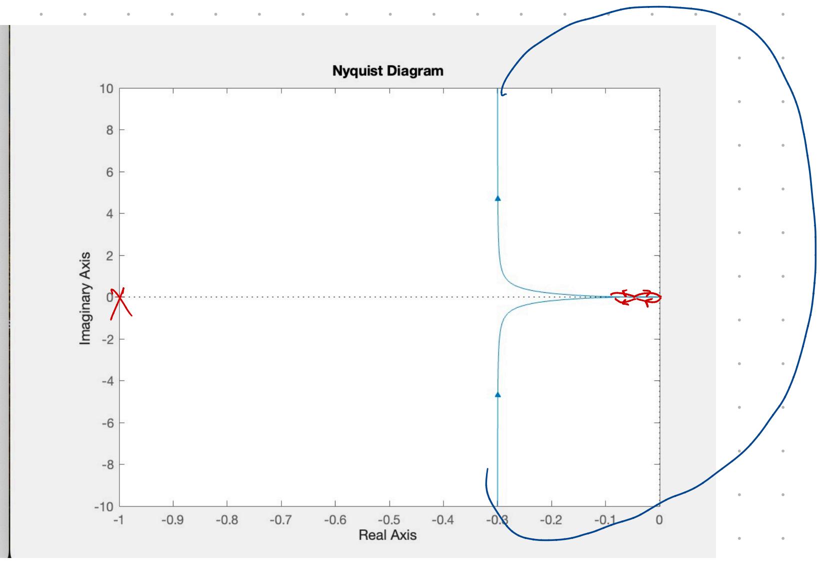
$\delta_a = 0.3$

$$|y_{d_a}^\infty| = \left| \frac{0.3}{k_c} \right| \leq 0.1$$

$\Rightarrow |3| \leq k_c$

$\Longrightarrow \boxed{C_{ss} = 5}$

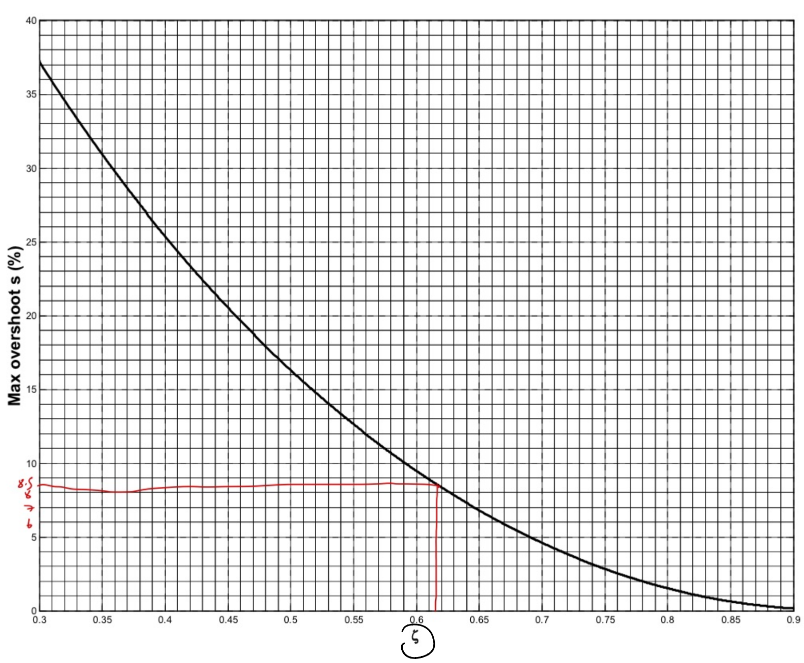


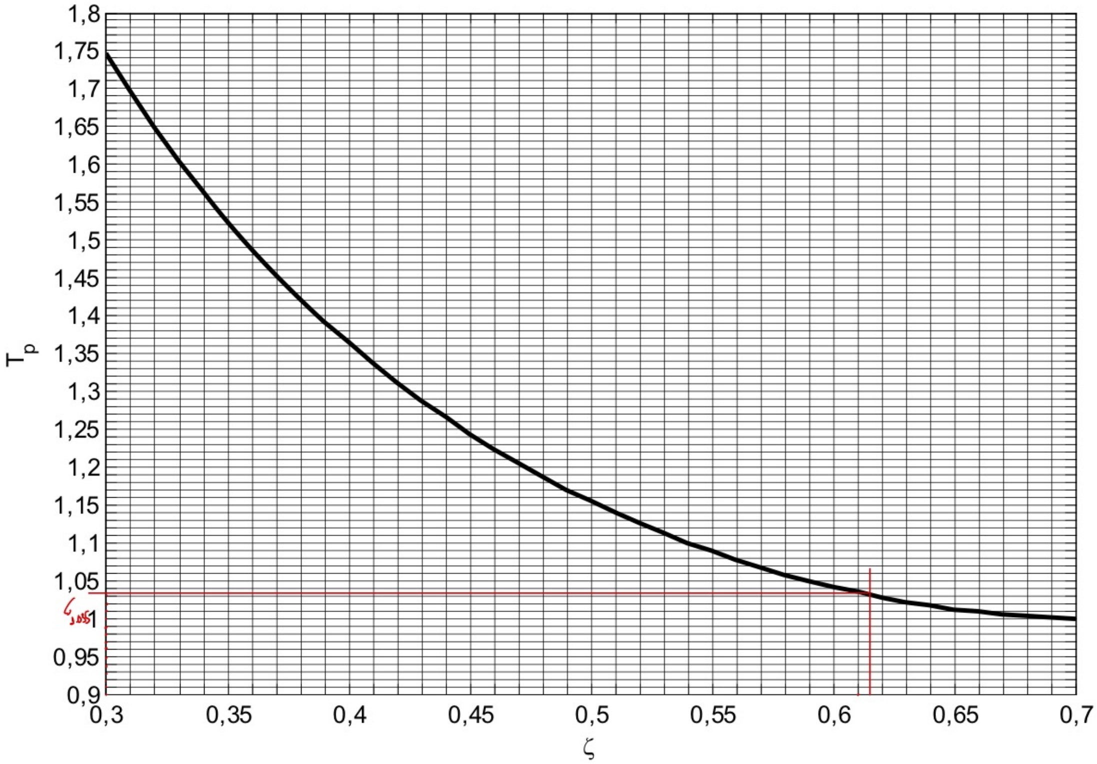

N = 0

p = 0 ( number of positive poles)

$\boxed{\text{stability } \checkmark}$

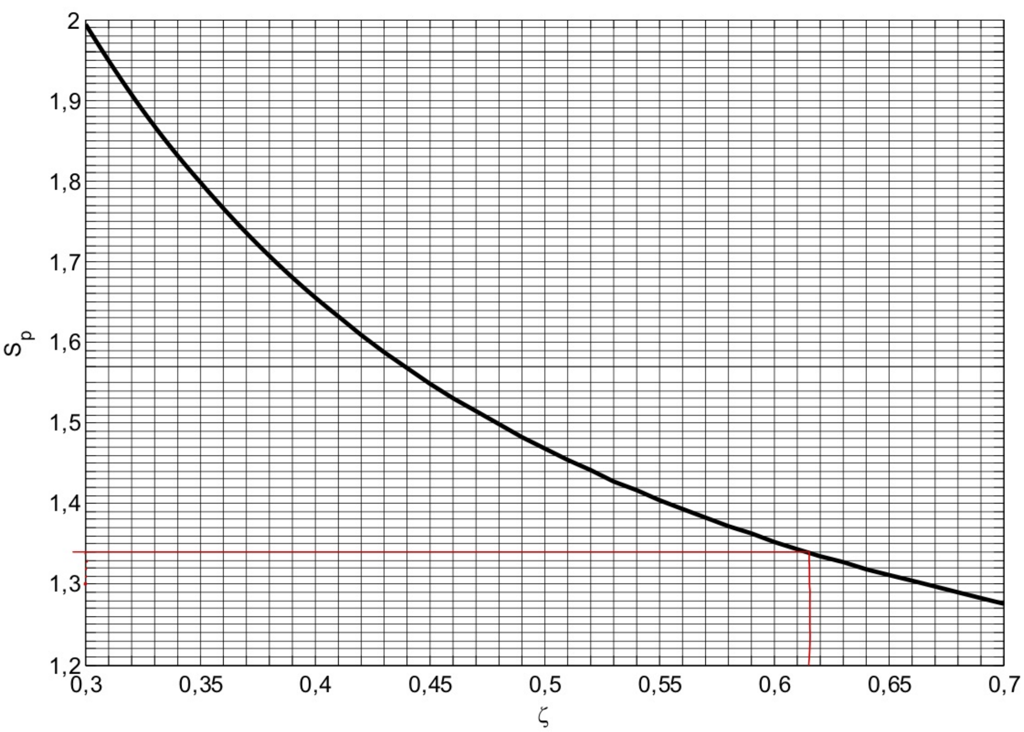



$\zeta = 0.615$



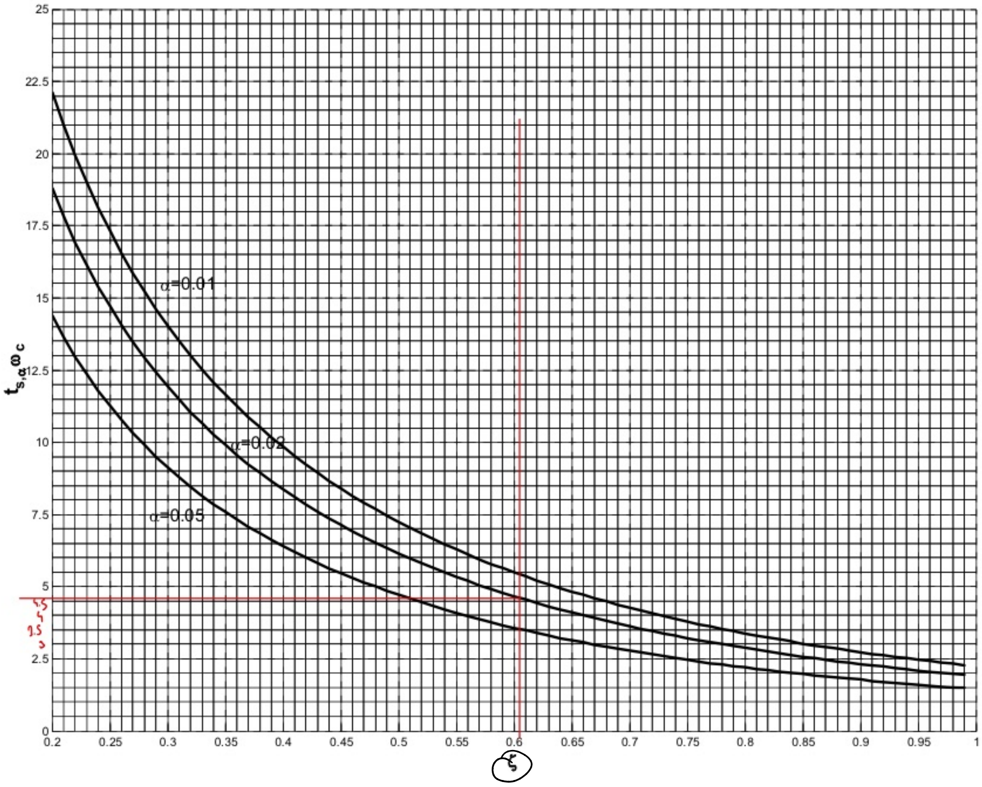

$T_p = 0.34$

dB




$S_p = 2.93$

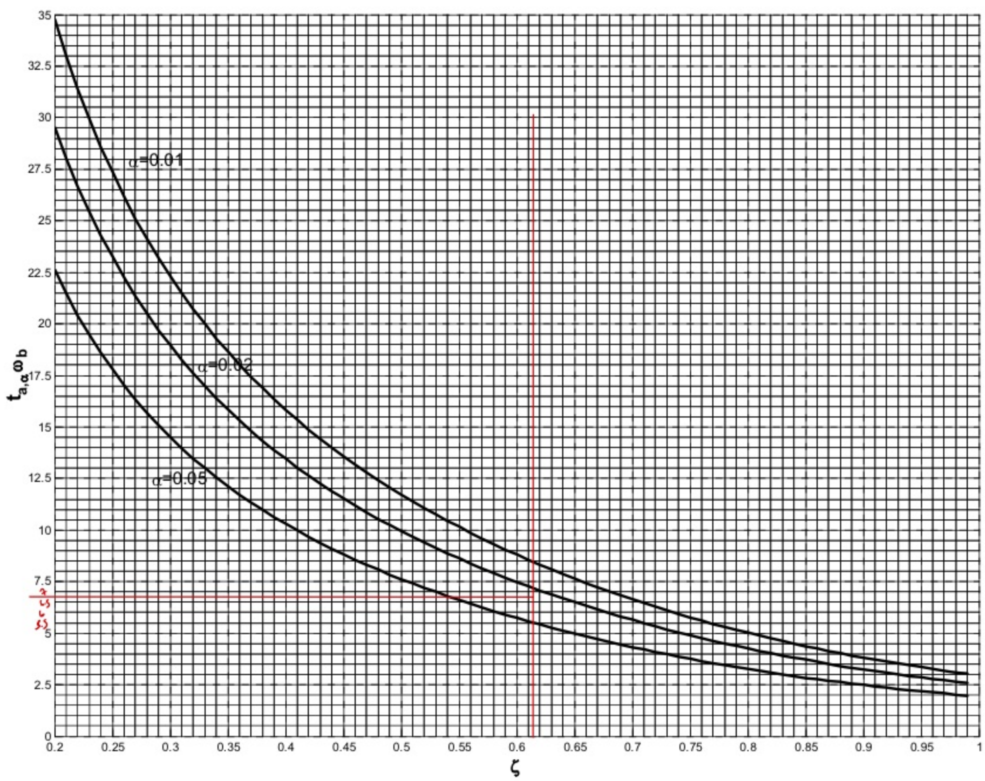
dB




$w_c = 6$

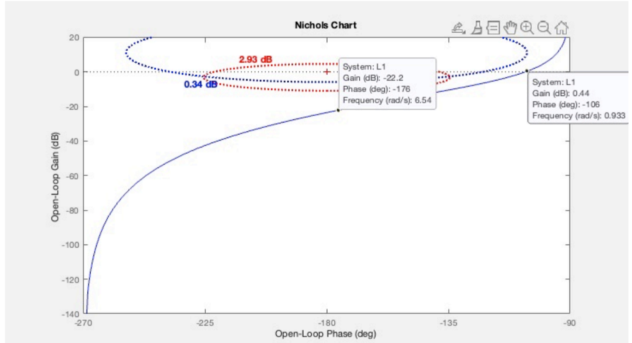
↳ $\underbrace{t_{s,\%2}}_{0.75}\ w_c = 4.5$

$0.75 \rightarrow 6$



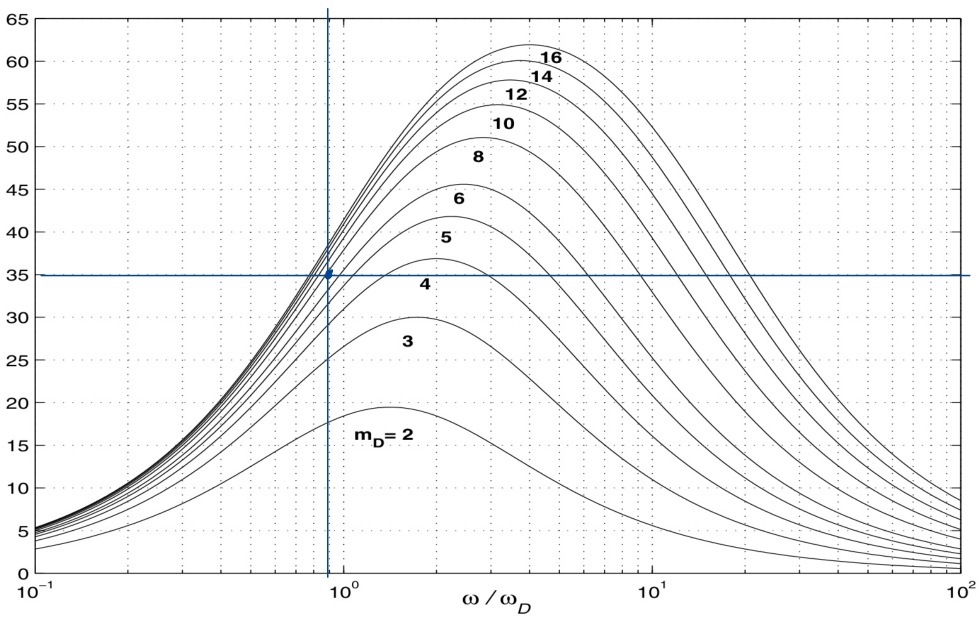

$w_b = 9$

$L' = G \times C_{ss} \Rightarrow$




↳ $\omega_c = 6$

↳ $\omega_{c,des} = 6.5$ $\Rightarrow$ We need to apply phase lead for $70°$ degrees



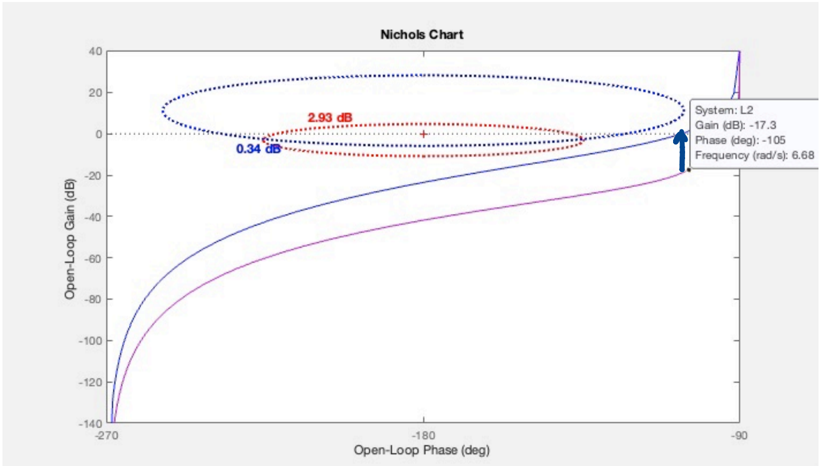

Since the wanted phase lead is $>60°$ we will use double phase lead network.

$m_{D_1} = 8$ ; $\omega_{D_1} = 0.9$

$m_{D_2} = 8$ ; $\omega_{D_2} = 0.9$

$> \omega_{d1} = \omega_{D2} = \omega_{c,des} / \omega_{d1}$

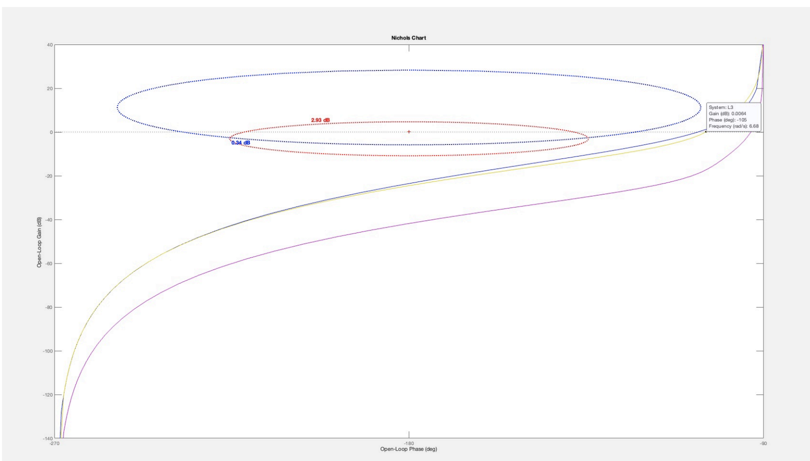
$$C_D = \left( \frac{1 + \frac{s}{\omega_{D_1}}}{1 + \frac{s}{m_{D_1} \omega_{D_1}}} \right) \left( \frac{1 + \frac{s}{\omega_{D_2}}}{1 + \frac{s}{m_{D_2} \omega_{D_2}}} \right) \Rightarrow L'' = L' \, C_D$$




We need to employ magnitude increase of (17.3) to take the $\omega_{c,des}$ to 0 gain.

↳ $K = 10^{\frac{17.3}{20}}$

$L''' = L'' \times K$

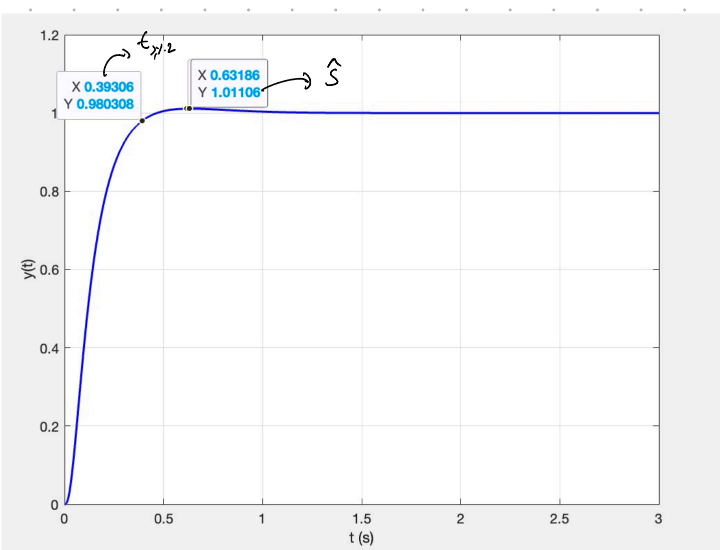



even though it's not perfect tangent we took the $\omega_{c,des}$ to 0 gain.

Now we can look at the simulation results.

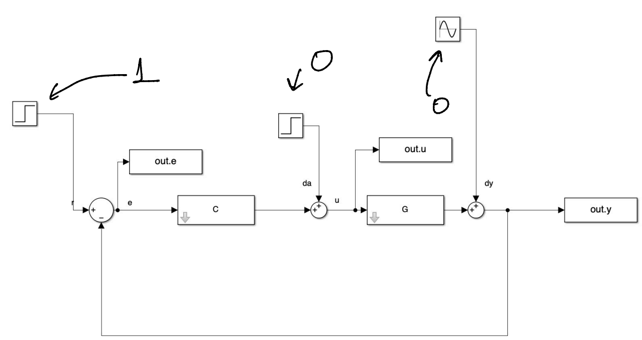
Note:

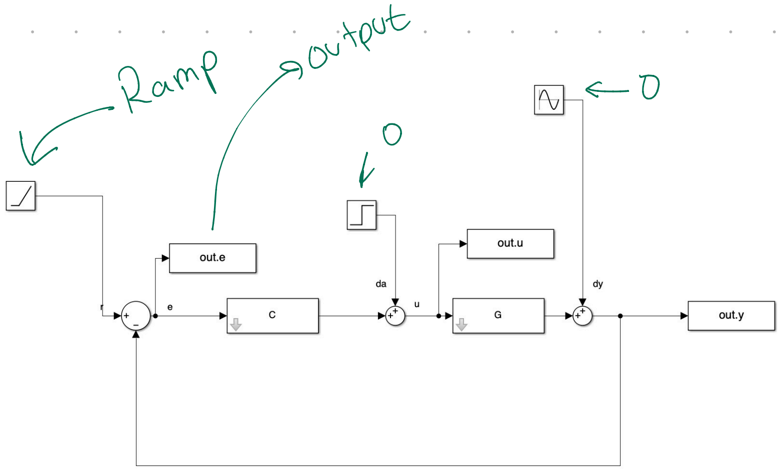
When we have have to check satisfaction of transient requirements we only have to activate a step as reference and deactivate all the other.

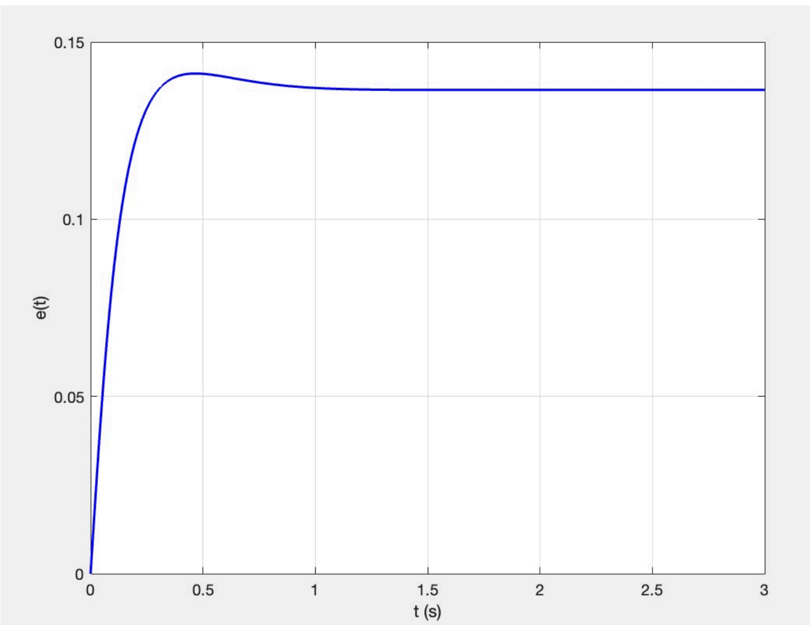



$\hat{S} = \%1 \leq \%8.5$

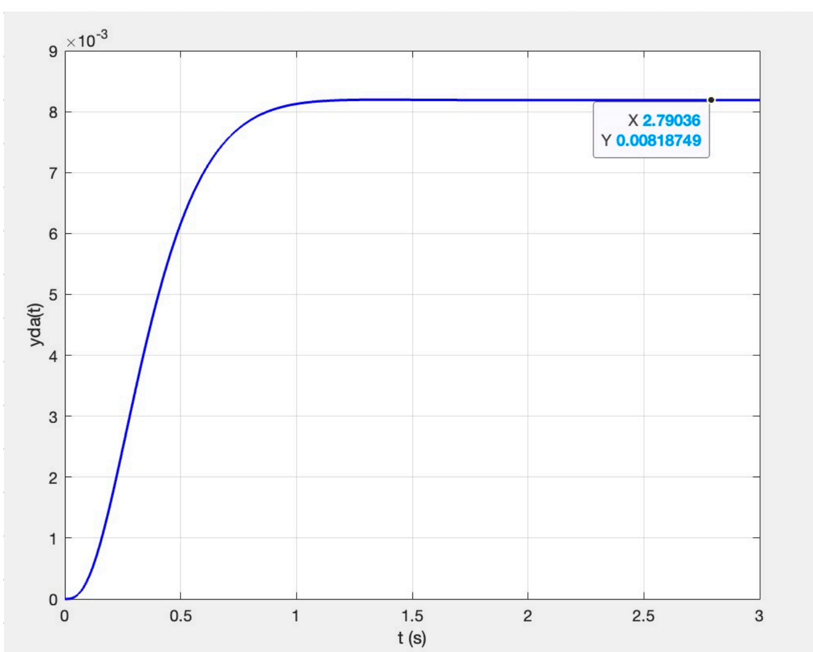
$t_{s,\%2} = 0.39\,s \leq 0.75\,s$



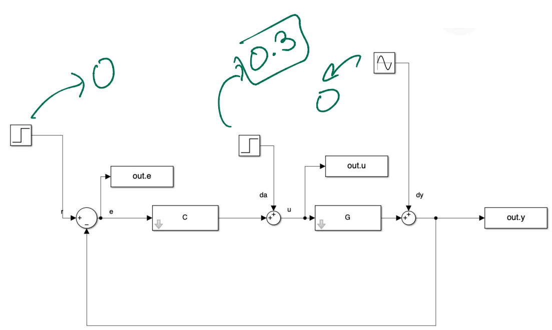


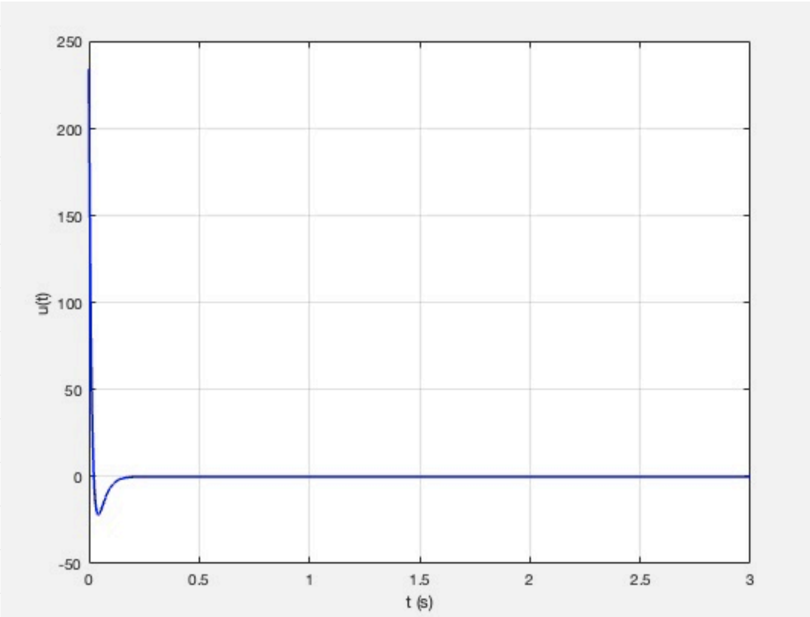




$$\rightarrow \quad |e_r^\infty| = 0.14 \le 1$$

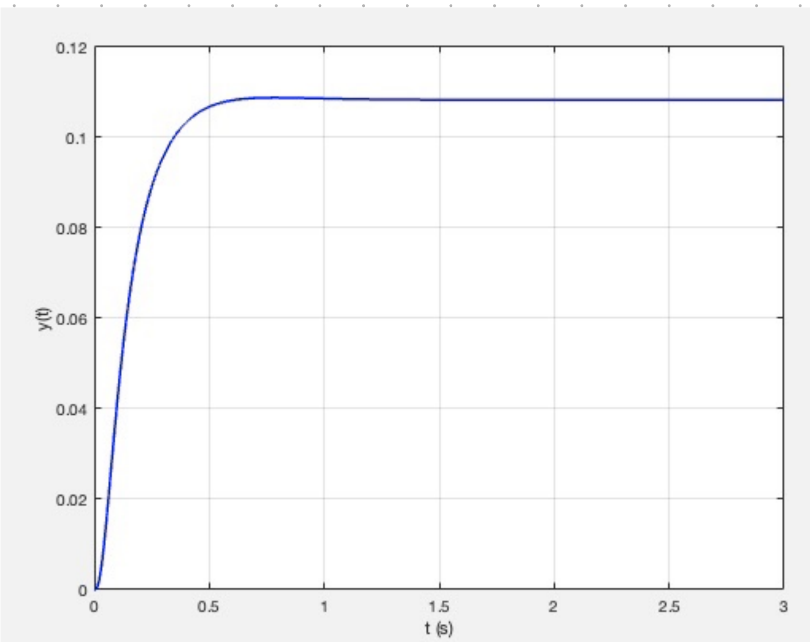



$$|y_{da}^\infty| = 0.082 \le 0.1$$






$\max(u(t)) \approx 220$ ?




$\max\ y(t) = 0.11$

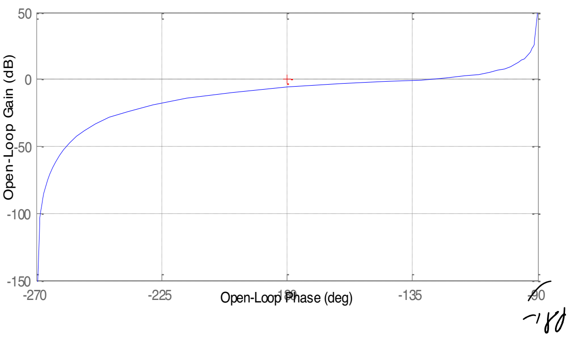
$\omega_B = 9$

$C = C_{ss} \times C_D \times K$

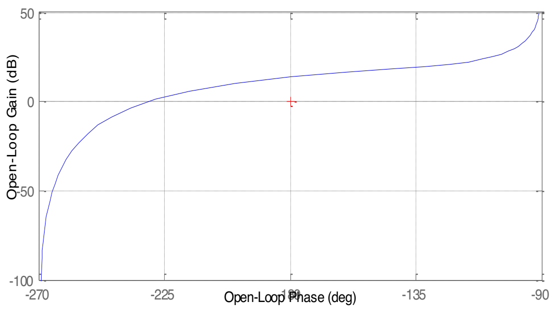
$$\boxed{C = 5 \times \left(\frac{1 + \frac{s}{7.2}}{1 + \frac{s}{8 \times 7.2}}\right)^2 \times 10^{\frac{17.3}{20}}}$$

Consider the following Nichols plots of four different loop functions L(s) of a unitary negative feedback, cascade compensation control system architecture

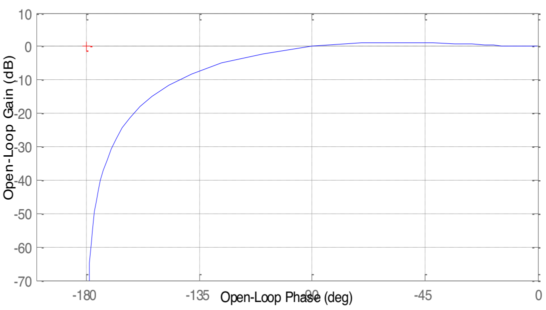
**1**



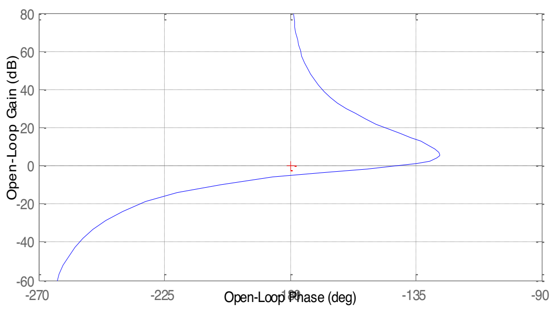

**2**




**3**




**4**




Suppose that, for each L(s), the generalized dc-gain is such that $K_g=\lim_{s\to 0} s^g L(s) > 0$, then, based on the Nichols plot only, determine which of the four

1. corresponds to a closed loop stable system
2. guarantees a finite value of $|e_r^\infty|$ in the presence of a constant reference signal
3. guarantees $|e_r^\infty| = 0$ in the presence of a constant reference signal
4. guarantees a finite value of $|e_r^\infty|$ in the presence of a linear ramp reference signal
5. guarantees $|e_r^\infty| = 0$ in the presence of a linear ramp reference signal
6. surely guarantees $|y_{d_a}^\infty| = 0$ in the presence of a constant actuator disturbance signal $d_a(t)$

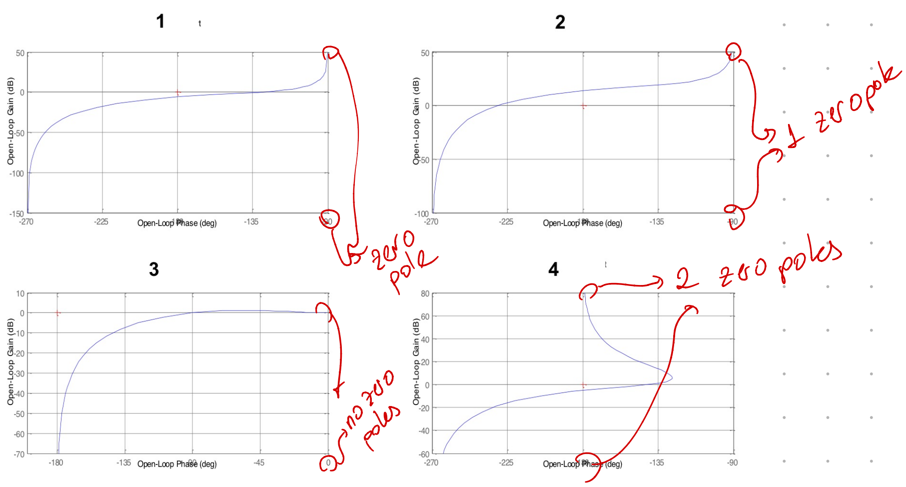
(Answer:
1. → 1,3,4   2. → 1,3,4   3. → 1,4   4. → 1,4   5. → 4   6. → none )

1) Nichols criterion: point A, on bottom point B on the right of critical point

(1,3,4)

Note:

having poles at the origin means having poles at frequency $\omega = 0$ which basically is the upper limit of the L graph so you trace the L graph up to the top. A single pole in the origin also means that a negative phase of $-90$ is introduced and a negative lead also so if the is at least a pole in the origin the graph should start up and come down from infinity due to the negative lead action.




Note:

We can analyse the steady state properties if the system is stabile

2)

constant reference signal

$|e_r^\infty|$

| $h$ / type $g$ | 0 (step) $r(t)=\rho\varepsilon(t)$ | 1 (linear ramp) $r(t)=\rho t\varepsilon(t)$ | 2 (parabolic ramp) $r(t)=\rho(t^2/2)\varepsilon(t)$ |
|---|---|---|---|
| 0 | $\left\vert\frac{\rho}{1+K_0}\right\vert$ | $\infty$ | $\infty$ |
| 1 | 0 | $\left\vert\frac{\rho}{K_1}\right\vert$ | $\infty$ |
| 2 | 0 | 0 | $\left\vert\frac{\rho}{K_2}\right\vert$ |

Any type

Any type g would be applicable, since 2 is not stabile

$\boxed{1, 3, 4}$

3)

$|e_r^\infty|$

| h / type g | 0 (step) $r(t)=\rho\varepsilon(t)$ | 1 (linear ramp) $r(t)=\rho t\varepsilon(t)$ | 2 (parabolic ramp) $r(t)=\rho(t^2/2)\varepsilon(t)$ |
|---|---|---|---|
| 0 | $\left\|\frac{\rho}{1+K_0}\right\|$ | $\infty$ | $\infty$ |
| 1 | 0 | $\left\|\frac{\rho}{K_1}\right\|$ | $\infty$ |
| 2 | 0 | 0 | $\left\|\frac{\rho}{K_2}\right\|$ |

It has to contain at least 1 zero pole to be have type $g \geq 1$

$\boxed{1,4}$

3 is not stable

4)

$|e_r^\infty|$

| h / type g | 0 (step) $r(t)=\rho\varepsilon(t)$ | 1 (linear ramp) $r(t)=\rho t\varepsilon(t)$ | 2 (parabolic ramp) $r(t)=\rho(t^2/2)\varepsilon(t)$ |
|---|---|---|---|
| 0 | $\left\|\frac{\rho}{1+K_0}\right\|$ | $\infty$ | $\infty$ |
| 1 | 0 | $\left\|\frac{\rho}{K_1}\right\|$ | $\infty$ |
| 2 | 0 | 0 | $\left\|\frac{\rho}{K_2}\right\|$ |

It has to contain at least 1 zero pole to be have type $g \geq 1$

$\boxed{1,4}$

3 is not stable

5)

$|e_r^\infty|$

| h / type g | 0 (step) $r(t)=\rho\varepsilon(t)$ | 1 (linear ramp) $r(t)=\rho t\varepsilon(t)$ | 2 (parabolic ramp) $r(t)=\rho(t^2/2)\varepsilon(t)$ |
|---|---|---|---|
| 0 | $\left\|\frac{\rho}{1+K_0}\right\|$ | $\infty$ | $\infty$ |
| 1 | 0 | $\left\|\frac{\rho}{K_1}\right\|$ | $\infty$ |
| 2 | 0 | 0 | $\left\|\frac{\rho}{K_2}\right\|$ |

it must have at least 2 zeros to have type $g = 2$

$\boxed{4}$

6)

$|y_{d_a}^\infty|$

| h / $g_c$ | 0 (step) $d_a(t)=\delta_a\varepsilon(t)$ | 1 (linear ramp) $d_a(t)=\delta_a t\varepsilon(t)$ | 2 (parabolic ramp) $d_a(t)=\delta_a t^2/2\varepsilon(t)$ |
|---|---|---|---|
| 0 | $\left\|\frac{\delta_a}{K_0}\right\|$ | $\infty$ | $\infty$ |
| 1 | 0 | $\left\|\frac{\delta_a}{K_1}\right\|$ | $\infty$ |
| 2 | 0 | 0 | $\left\|\frac{\delta_a}{K_2}\right\|$ |

$$K_0 = \begin{cases} K_c & \text{if } G(s) \text{ has poles in } 0 \\ \frac{1+K_cK_G}{K_G} & \text{if } G(s) \text{ has not poles in } 0 \end{cases} \qquad K_{g_c} = \lim_{s\to 0} s^{g_c} C(s),\ g_c \geq 1$$

if $g_c > h$, $|y_{d_a}^\infty| = 0$

↳ we must know the controller!!

we cannot surely guarantee!