Automatic Control
Laboratory practice 2

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Objectives: evaluation of the steady state; step response of prototype models.

Problem 1

Consider the LTI system described by the transfer function

$$H(s) = \frac{1}{s^3 + 2s^2 + 5.25s + 4.25}.$$

(a) Analyse the BIBO stability
(b)**If possible**, compute the steady state response $y_{ss}(t)$ in the presence of the input

$$u(t) = (3\sin(0.1t + 1))\varepsilon(t)$$

(c) **If possible**, compute the maximum amplitude $\bar{u}$ of a sinusoidal input of the form

$$u(t) = \bar{u}\sin(3t)\varepsilon(t)$$

such that the steady state response $y_{ss}(t)$ satisfies $|y_{ss}(t)| \leq 1$.

Solution
(a) The system is BIBO stable. Thus, the steady state response exists.
(b) $y_{ss}(t) = (0.7038\sin(0.1t - 0.1232) + 0.4706)\varepsilon(t)$
(c) $\bar{u} \leq 17.7658$

Problem 2

Consider the 2nd order system described by the transfer function

$$H(s) = \frac{10}{s^2 + 1.6s + 4}.$$

(a) evaluate the natural frequency ω_n, the damping coefficient ζ and the time constant τ of the poles;
(b) In MATLAB, plot the step response by using the function step and get the following data from the graph:

1. steady state value y_∞
2. maximum overshoot $\hat{s}$
3. peak time $\hat{t}$
4. rise time t_r
5. 5% settling time $t_{s,5\%}$

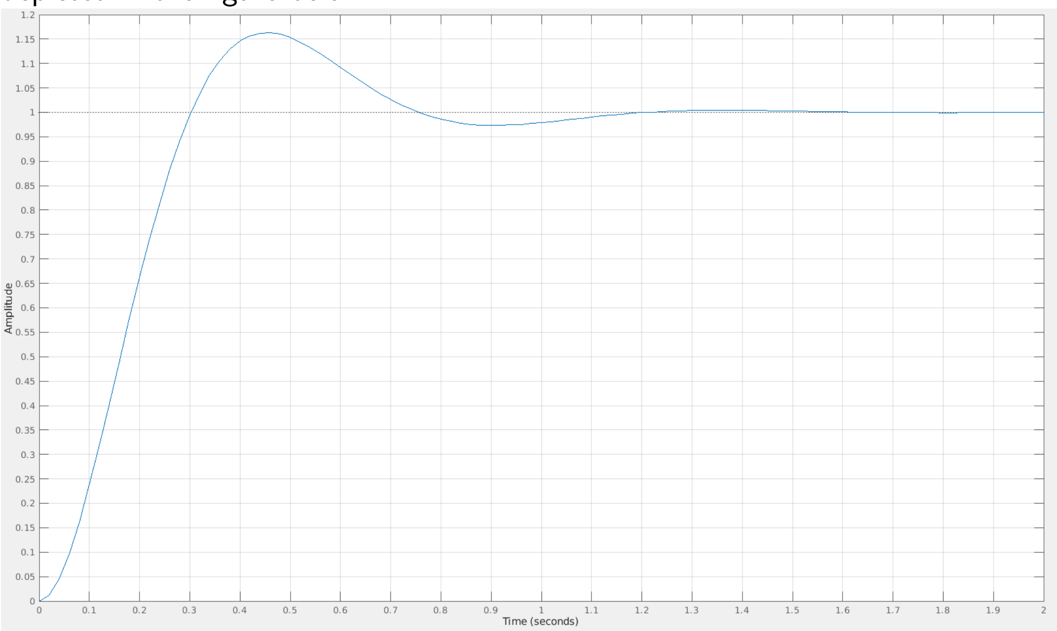
Solution
(a) $\omega_n = 2$, $\zeta = 0.4$, $\tau = 1.25$ s
(b) $y_\infty = 2.5$, $\hat{s} = 25.37$,$\hat{t} = 1.7$ s, $t_r = 1.08$ s, $t_{s,5\%} = 3.8$ s

Problem 3

Consider a 2nd order LTI system described by

$$H(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

Its (zero state) output response in the presence of a step input of amplitude 5 is depicted in the figure below.



Compute the values of the parameters K, ω_n and ζ.

Solution

$K = 0.2$, $\omega_n = 8$, $\zeta = 0.5 \Rightarrow H(s) = \frac{12.8}{s^2+8s+64}$