Problem 1

Given the LTI system

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -0.2 & -1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \end{cases}, x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

- Suppose that the system state can be measured. Design, if possible, a state feedback controller of the form

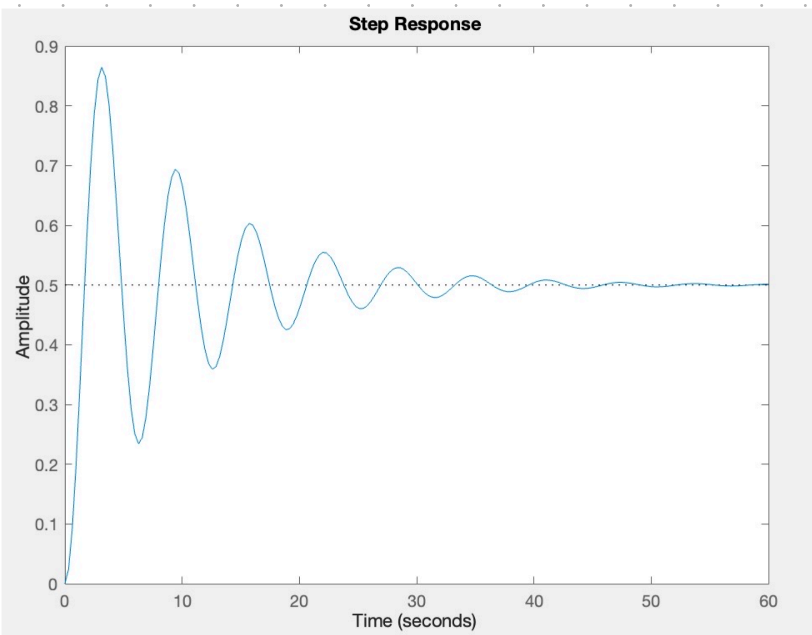
$$u(t) = -Kx(t) + Nr(t)$$

to meet the following requirements:

1. unitary dc-gain for the controlled system
2. $\hat{s} \leq 6\%$
3. $t_{s,2\%} \leq 2$ s

- Compute the analytical expression of the output response y(t) of the controlled system in the presence of a step reference input, i.e., r(t)=ε(t).

first lets examine the asymptotic behavior!



it is asymptotically stable

as $t \to \infty$ goes to 0!

Converges!

Secondly let's examine the reachability!

$M_R = \begin{bmatrix} 0.5 & -0.1 \\ 0 & 0.5 \end{bmatrix} \Rightarrow \rho(M_R) = 2$; since $R^{n,n}$; $n=2$

and $\rho(M_R) = n$

system is Reachable!

$$\hat{S} = e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}} \Rightarrow \zeta = \frac{|\log(\hat{S})|}{\sqrt{\pi^2 + \log^2(\hat{S})}} = 0.15$$

$$t_{s,\%2} = \frac{\log(0.02)^{-1}}{w_n \zeta} \Rightarrow w_n = \frac{3.9}{t_{s,\%2}\, \zeta} = 13.03$$

$$\lambda_{1,2} = \sigma_0 \pm J w_0 = -\zeta w_n \pm J w_n \sqrt{1-\zeta^2}$$

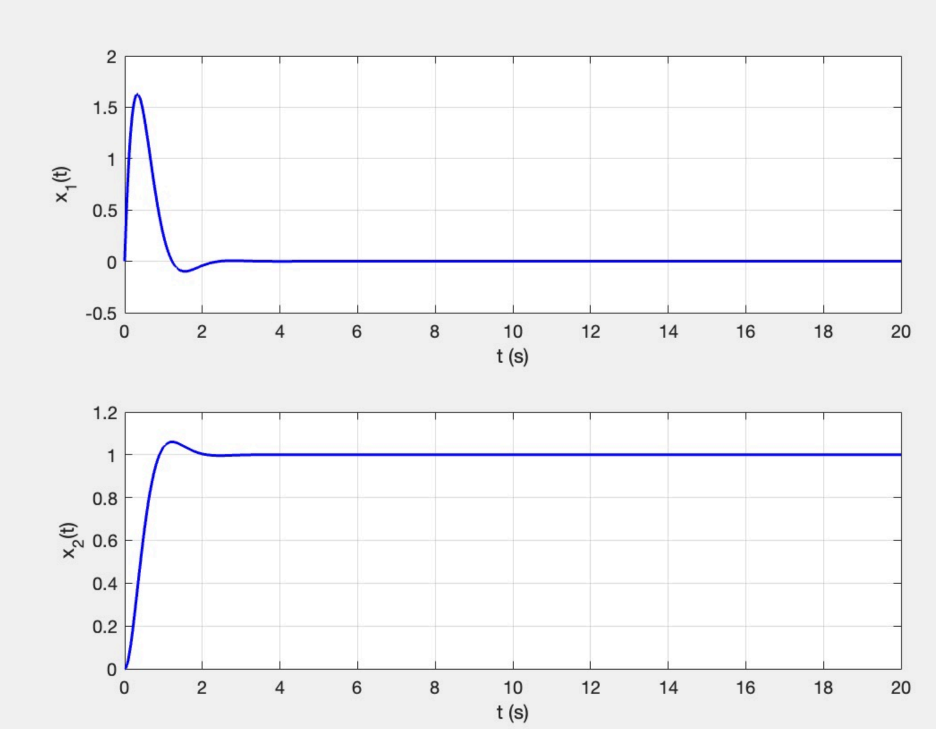
$$\lambda_1 = -1.96 + 12.03 J$$

$$\lambda_2 = -1.96 - 12.03 J$$

$$K = [7.4240 \quad 295.07]$$

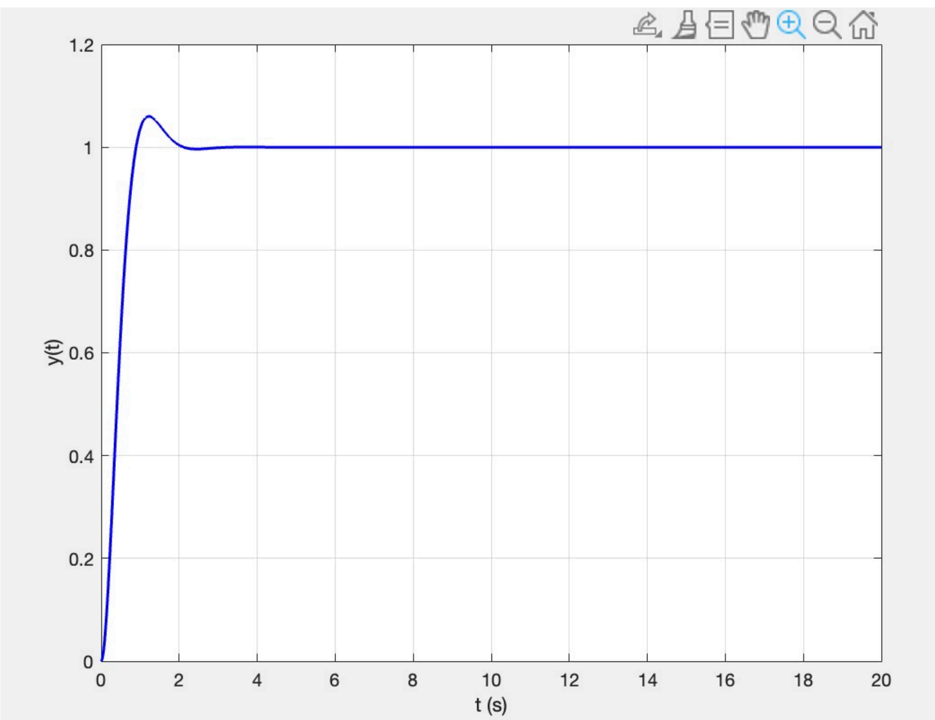
$$u = -K x(t) + N u(t)$$

$$\boxed{N = 297.07}$$



$t_{s,\%2} \leq 2s$

$\hat{S} \leq \%6$



Problem 2

Given the LTI system

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -0.2 & -1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \end{cases}, x(0) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

- Suppose that the system state cannot be measured. Design, if possible, a state feedback controller of the form

$$u(t) = -K\hat{x}(t) + Nr(t)$$

to account for the following requirements:

1. unitary dc-gain for the controlled system
2. controlled system eigenvalues characterized by $\zeta = 0.66$ and $\omega_n = 2.93$ rad/s

- Evaluate the maximum overshoot of the output unitary step response in the presence of the given initial condition.

```
M_r =

    0.5000   -0.1000
         0    0.5000

rho_M =

     2
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```
Mo =

     0     1
     1     0

rho_Mo =

     2
```

the system is reachable

$\lambda_1 = -1.33 + 2.25$

$\lambda_2 = -1.33 - 2.25$

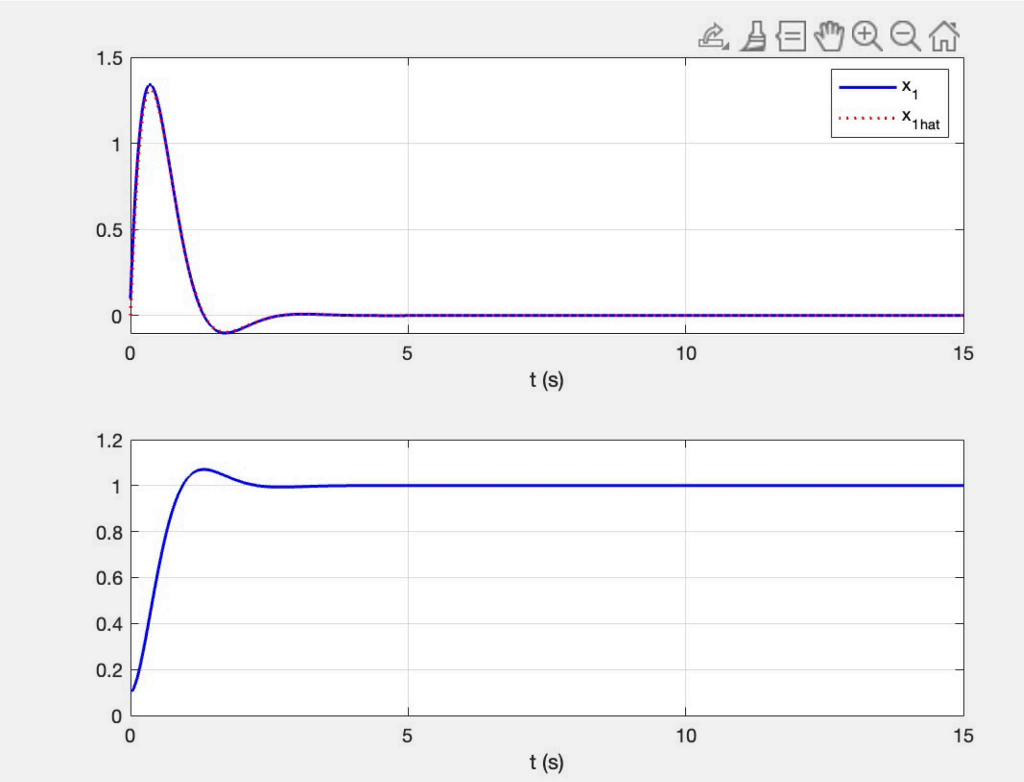
$K = [7.34 \;, \; 15.17]$

$N = 17.17$

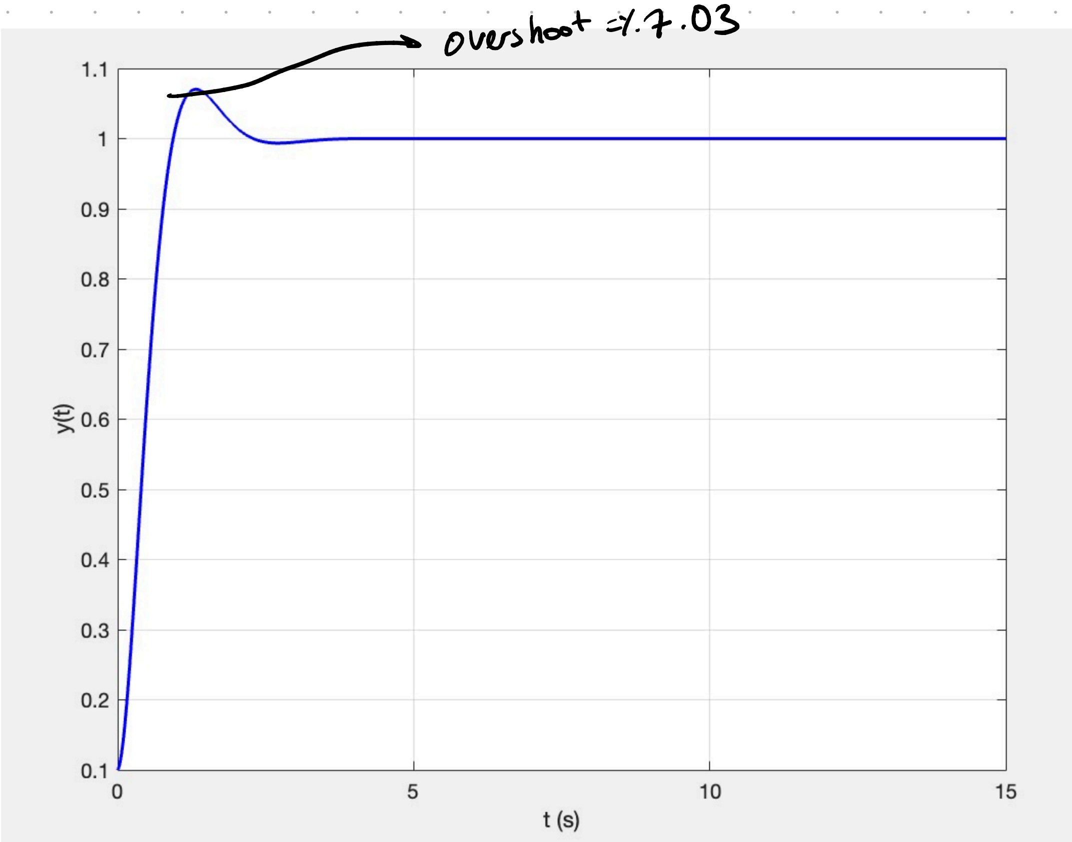
state feedback design

$L = \begin{bmatrix} 2.01 \\ 3.67 \end{bmatrix}$

Observer design



overshoot =%7.03



Problem 3
Consider the LTI system

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 0 & 10 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \end{cases}$$

1. Study the stability properties of the given system.
2. Supposing that the system state can be measured, is it possible to compute a state feedback controller of the form $u(t) = -Kx(t) + Nr(t)$ to stabilize the given system? Motivate your answer.
3. Supposing that the system state cannot be measured, is it possible to compute a state feedback controller of the form $u(t) = -K\hat{x}(t) + Nr(t)$ to stabilize the given system? Motivate your answer.

1. $\mathrm{eig}(A) = \begin{bmatrix} -1 \\ 10 \end{bmatrix}$

$\lambda_1 = -1$; $\lambda_2 = 10$

↳ convergent ↳ Divergent

$H(s) = \dfrac{1}{(s+1)}$

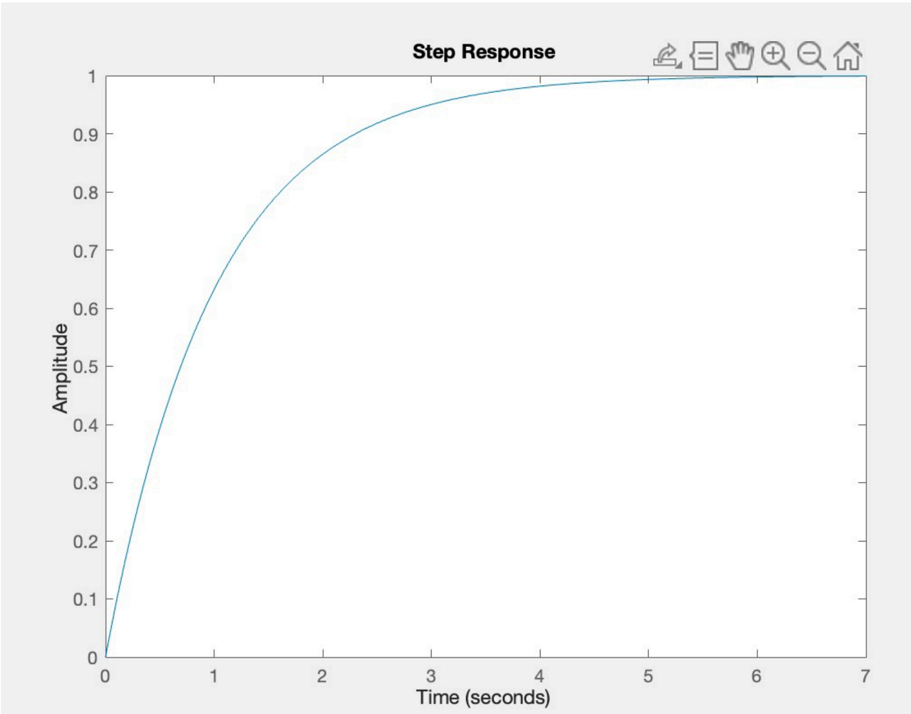
Since $y(t) = \underbrace{e^{\lambda_1 t}}_{\text{conv}} + \underbrace{e^{\lambda_2 t}}_{\text{Divergent}}$

$\boxed{\text{Divergent}}$ => internally unstable

Nodal Analysis $\boxed{y(t) = e^{-t} + e^{10t}}$

$\dfrac{1}{s+1}$ => Converges to 0 ; $\boxed{\text{BIBO stable}}$

2. System state can be measured!



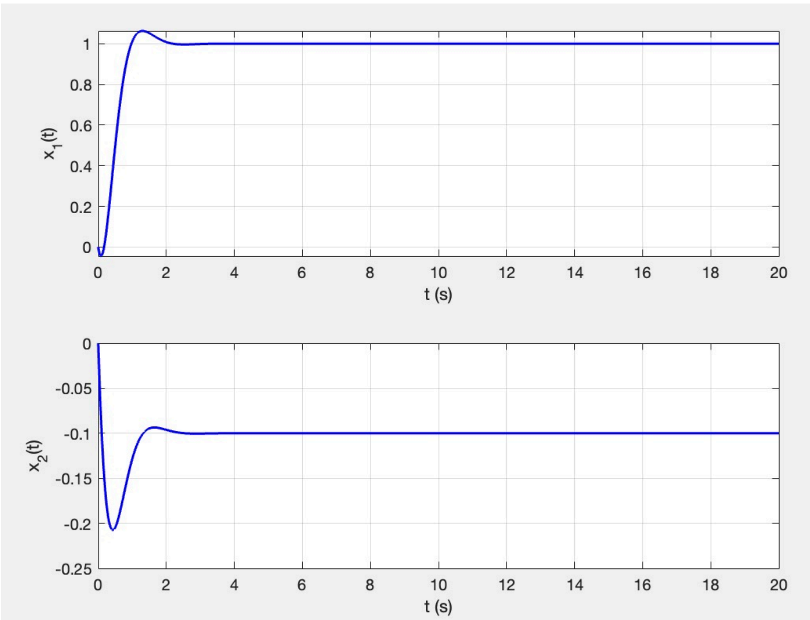
the step response is converging to 1

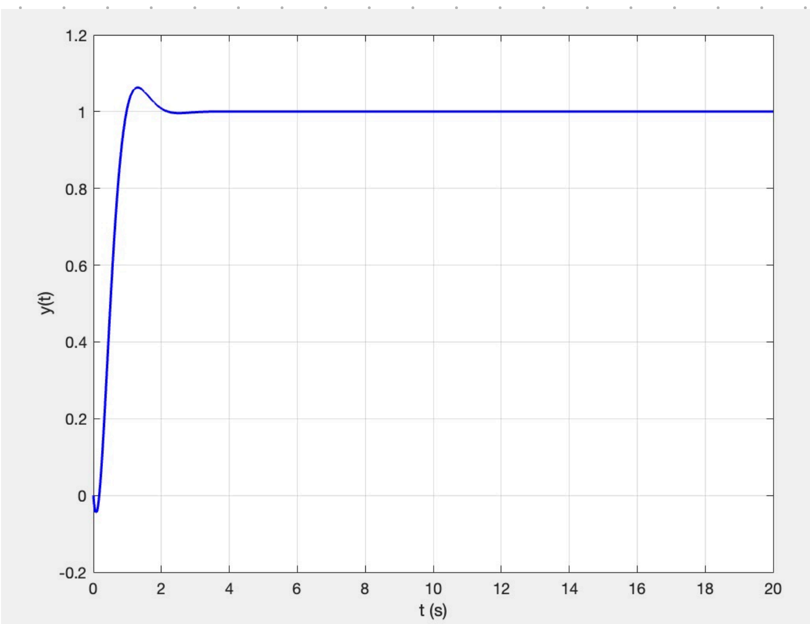
$$m_R = \begin{pmatrix} 1 & -1 \\ 1 & 10 \end{pmatrix} \quad \text{and} \quad \boxed{\rho(m_R) = 2}$$

hence the system is reachable

Since the system is reachable hence it can be stabilized.

Example for the conditions in question 1:





3. if cannot be measured ; we have to use an observer. And for that the system needs to be observable

$$M_O = \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix} \quad ; \quad \rho(M_O) = 1$$

Hence not observable hence we cannot stabilize it