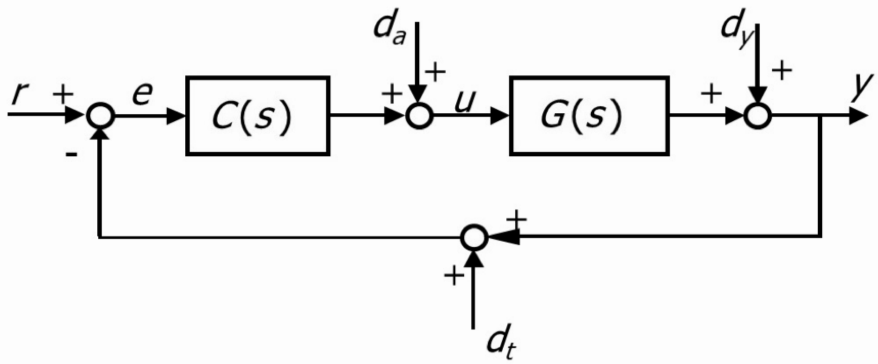
Problem 1

Objectives: computation of transfer functions and response of a feedback control system; evaluation of the stability; numerical simulation.

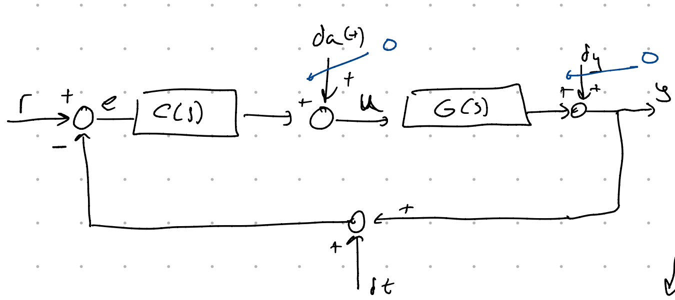
Let us consider the feedback control system:



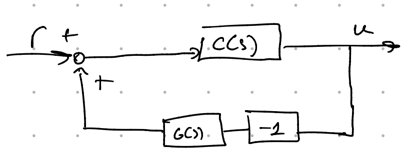
where $G(s) = \frac{1}{(1+s)^2}$ and $C(s) = \frac{(1+s)^2}{s\left(1+\frac{s}{4}\right)}$. We assume that $G(s)$ and $C(s)$ are minimal.

1. Compute analytically the time response of the control input $u(t)$, when $r(t)$ is a unit step function and the other inputs $d_a(t)$ and $d_y(t)$ are set to zero.
2. Is this feedback control system well-posed?
3. Check the stability of the feedback control system by using th definition.
4. Check the stability of the feedback control system by using the Nyquist criterion.
5. If possible, compute analytically the steady state response $e_{ss}(t)$ of the tracking error $e(t)$ when $d_y(t) = 0.5\sin(t)\epsilon(t)$, while the other inputs $r(t)$ and $d_a(t)$ are set to zero.
6. If possible, compute analytically the steady state response $y_{ss}(t)$ of the controlled output $y(t)$ when $r(t) = 3\epsilon(t)$, $d_y(t) = 2\epsilon(t)$ and $d_a(t) = 0$.
7. Perform a numerical simulation of the system with Simulink. Verify the analytical expressions obtained at 1., 5., 6. through the numerical results.

1) $u(t) = ?$; $r(t) = \varepsilon(t)$; $d_a(t) = d_y(t) = 0$



$$H(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)}$$



$$\frac{C(s)}{1+G(s)C(s)} = Q(s) \; ; \; u(s) = Q(s)\, d_a(s) = \frac{C(s)}{1+G(s)C(s)} \times \frac{1}{s}$$

$$U(s) = \frac{4(s+1)^2}{s(s+2)^2} \rightarrow \text{res } U = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \qquad \text{pol } U = \begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix}$$

$$\boxed{u(t) = \left(3e^{-2t} - 2te^{-2t} + 1\right)\varepsilon(t)}$$

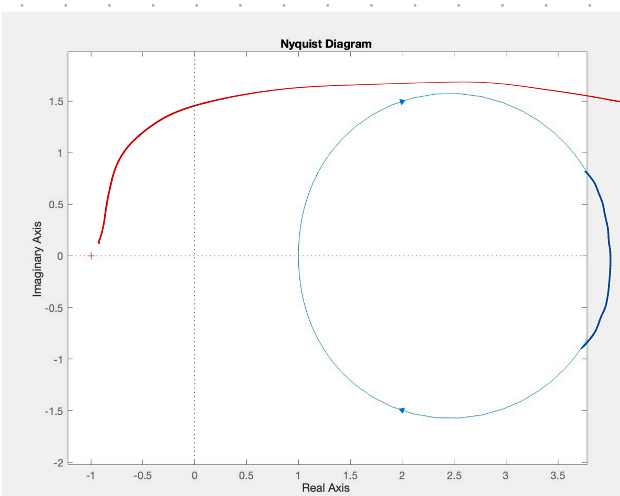
2) $Q(s) \Rightarrow \frac{4(s+1)^2}{(s+2)^2} \rightarrow$ degree in numarator does not pass degree in denumarator (response does not become unbounded as frequency increases)

$\hookrightarrow$ proper

$\hookrightarrow$ Well posed

3) signals are bounded and the transfer function is proper $\Rightarrow$ Stable by definition

4)



$\rightarrow$ criticle point is out of the loop hence $N=0$

$Z =$ number of poles of $E(s)$ with real part >0

$P =$ number of poles of $Q(s)$ with real part >0

$Q(s) = \frac{4(s+1)^2}{(s+2)^2} \rightarrow P = 0$ since $\boxed{P_{1,2} = -2}$

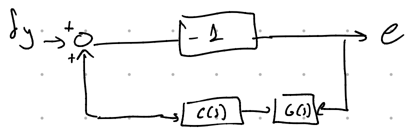
$E(s) = \frac{4(s+1)^2}{s(s+2)^2} \rightarrow Z = 0$ since $\boxed{P_1 = 0 ;\ P_{2,3} = -2}$

$\rightarrow 0 = 0 - 0$ ✓

if $\boxed{N = Z - P} \rightarrow$ stabile ✓

5) $\mathcal{L}\{\frac{1}{2}\sin(t)\varepsilon(t)\} = \frac{1}{2} \times \frac{1}{s^2+1} = d_y(s)$

$\varepsilon(t) = ?$; $d_y(t) = \sin(t)\varepsilon(t)$; $r(t) = d_a(t) = 0$



$H(s) = \frac{H_1(s)}{1 - H_1(s) H_2(s)}$

$\boxed{\begin{aligned} H_1(s) &= -1 \\ H_2(s) &= C(s)G(s) \end{aligned}}$

$\Rightarrow H(s) = \frac{-1}{1 + C(s)G(s)}$

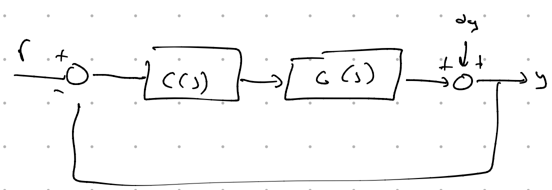
$E(s) = H(s) \times d_y(s)$

$\text{res } E = \begin{bmatrix} 0.32 \\ 0.4 \\ -0.16 + 0.13j \\ -0.16 - 0.13j \end{bmatrix}$ $\quad$ $\text{pole } E = \begin{bmatrix} -2 \\ -2 \\ j \\ -j \end{bmatrix}$

$e_{ss}(t) = \lim_{t \to \infty} e(t) = 0.4123 \sin(t - 2.4593)$

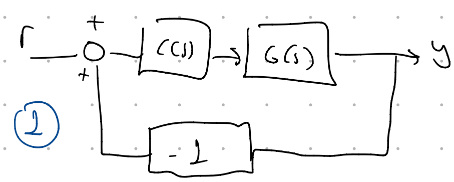
↳ not the answer?

6) $r(t) = 3\varepsilon(t)$; $d_y(t) = 2\varepsilon(t)$ $d_a(t) = 0$



$\Rightarrow$

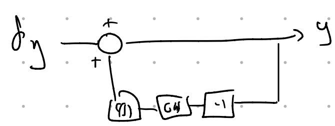
$d_y = 0$



(1)

$r = 0$

(2)



(1) $\qquad\qquad$ (2)

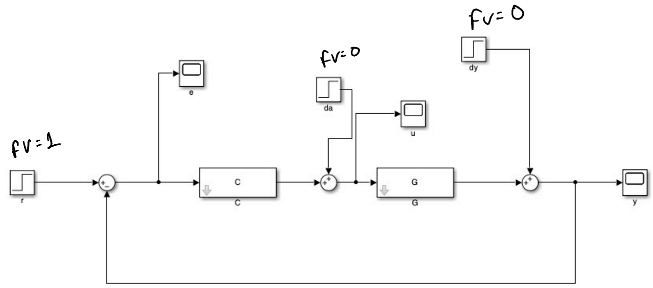
$H_1(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$; $H_2(s) = \frac{1}{1 + C(s)G(s)}$

$y(s) = r(s) H_1(s) + d_y(s) H_2(s) = \frac{2(s^2 + 4s + 6)}{s(s+2)^2}$

$\text{res } y = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$ $\quad$ $\text{pol } y = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix}$

$\boxed{y_{ss}(t) = 3\varepsilon(t)}$

7)



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fv=1
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u
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C
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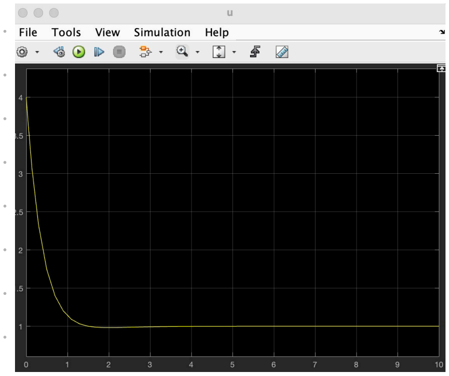
r
y



u
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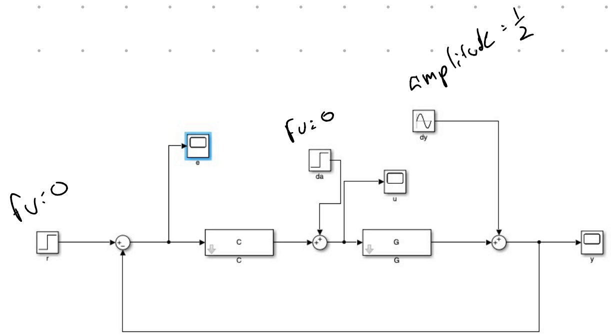

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fv=0
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da
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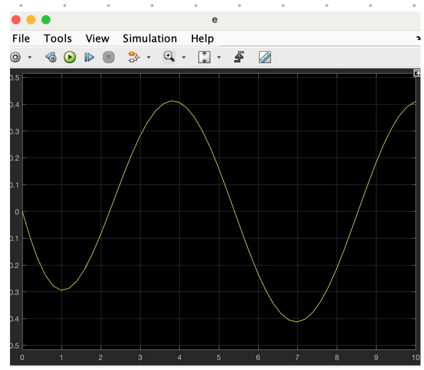

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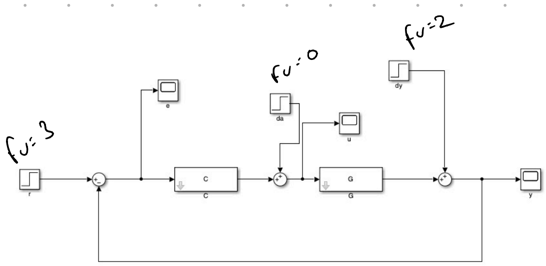

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fv=0
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e
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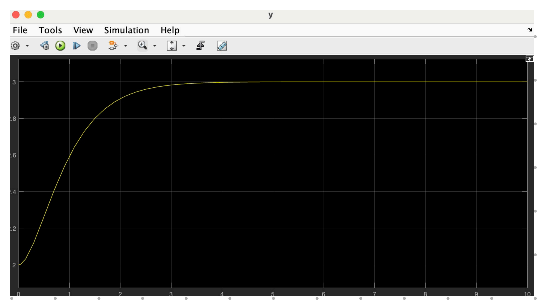

y
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Problem 2

Objectives: feedback control systems simulation.

Packet information flow in a router working under TCP/IP can be modelled, in a suitable working condition, through the following transfer function (see C. V. Hollot et al., "Analysis and Design of Controllers for AQM Routers Supporting TCP Flows", IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 47, NO. 6, pp. 945-959, 2002)

$$G(s) = \frac{q(s)}{p(s)}, \quad q(s) = \frac{c^2}{2N} e^{-sR}, \quad p(s) = \left(s + \frac{2N}{R^2 c}\right)\left(s + \frac{1}{R}\right)$$

where

- q = queue length (packets)
- p = probability of packet mark/drop
- c = link capacity (packets/sec)
- N = load factor (number of TCP sessions) item R = round trip time (sec)

The objective of an active queue management (AQM) algorithm is to choose automatically the packet mark/drop probability p, so that the sender can tune the window size to keep the queue length at a constant level. This system can be represented by the standard feedback structure reported in the previous figure.

Several AQM algorithms are available, but the one that has received special attention is the random early detection (RED) algorithm. The RED algorithm dynamics can be approximated through the linear controller

$$C(s) = \frac{\ell}{1 + \frac{s}{k}}.$$

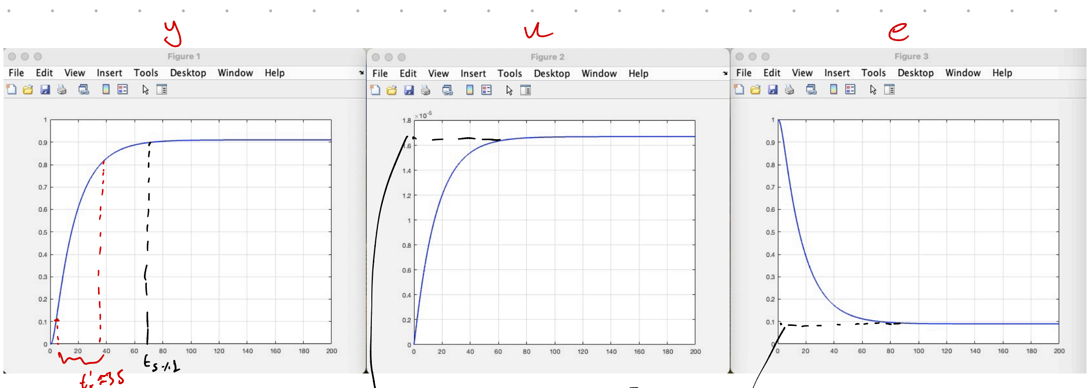
1. Given the following controller and plant parameter values

   $$\ell = 1.86 \cdot 10^{-4}, \;\; k = 0.005, c = 3750, \;\; N = 60, R = 0.246$$

   build a suitable Simulink scheme to, simulate the given AQM. In particular, evaluate the transient performance in terms of maximum overshoot, $10-90\%$ rise time and settling time 1%, the steady state tracking error and the maximum amplitude of the input magnitude $\max|u(t)|$ when the reference signal is a step function with amplitude 1 and the other inputs are set to 0.

2. Repeat the previous points with

   $$C(s) = K_c \frac{1 + \frac{s}{z}}{s}, \quad K_c = 9.64 \cdot 10^{-6}, \quad z = 0.53$$



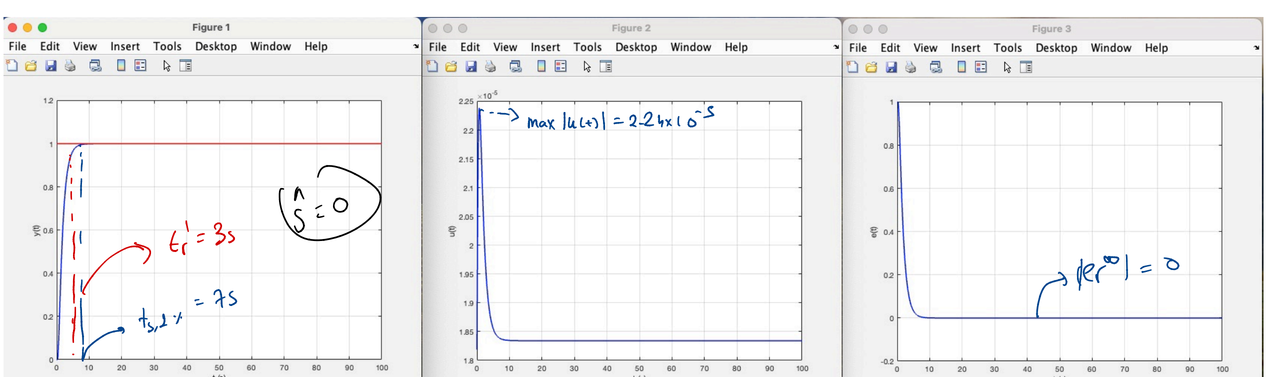
$\max|u(t)| = 2.67 \times 10^{-5}$

$|e_r^{\infty}| = 0.09$

$\hat{S} = 0$

$t_r' \approx 35s$

$t_{s,1\%} \approx 74$



Problem 3

Let us consider a feedback control system with loop transfer function

$$L(s) = \frac{1}{s\left(1+\frac{s}{2}\right)^3}$$

1. Verify that the feedback control system is stable by using the Nyquist criterion

2. Verify that the feedback control system is stable by using the Nichols diagram of $L(s)$

3. Compute the gain margin and the phase margin (hints: $\angle L(j\omega_0) = -180°$ for $\omega_0 = 1.15$ rad/s; $L(j\omega_c) = -1$ for $\omega_c = 0.8$)

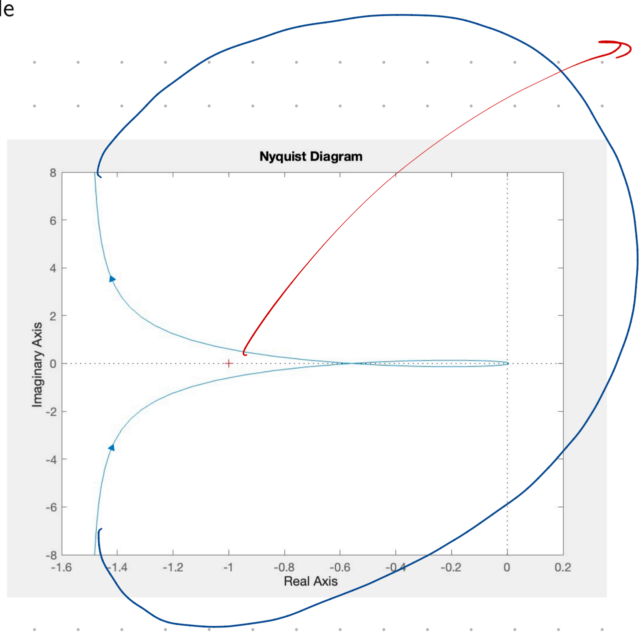
4. Verify that the feedback control system with loop transfer function

$$L_\Delta(s) = 3L(s)$$

is unstable

1)

$\to N = 0$

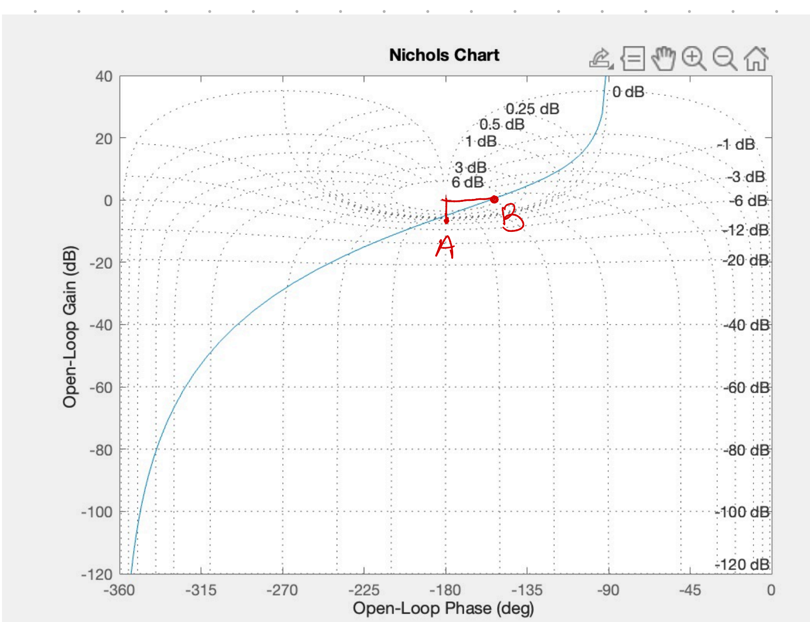


$T = \dfrac{---}{s^2(s+2)^6} \to -2<0 = \quad Z = 0$

$L = \dfrac{---}{s\left(1+\frac{s}{2}\right)^3} \to -2<0 \quad P = 0$

$N = Z - P$ ✓ $0 = 0 - 0$ ✓

2)

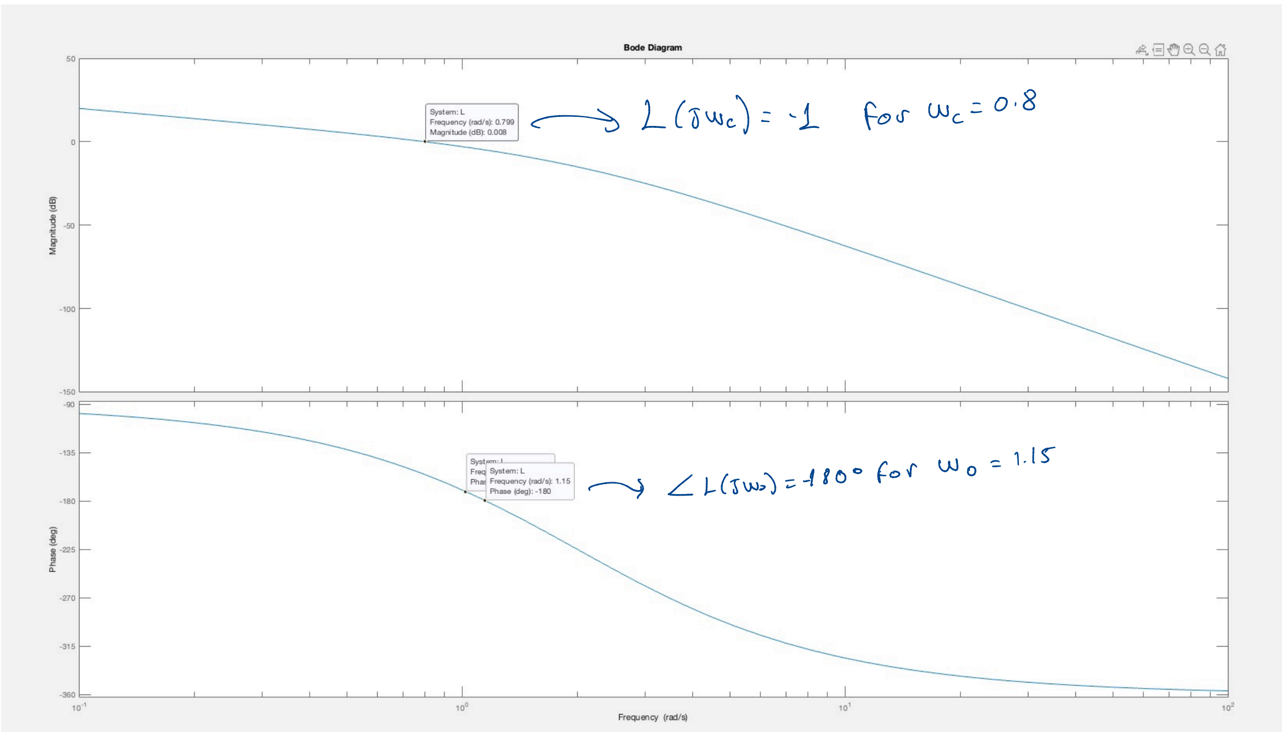


Point A is below, point B is on the right of critical point hence stable

3) Gain Margin $= \dfrac{1}{|L(j\omega_c)|}$

Phase Margin $= -180^\circ - \angle L(j\omega_o)$

if the hint is not given; we have to draw the bode plot



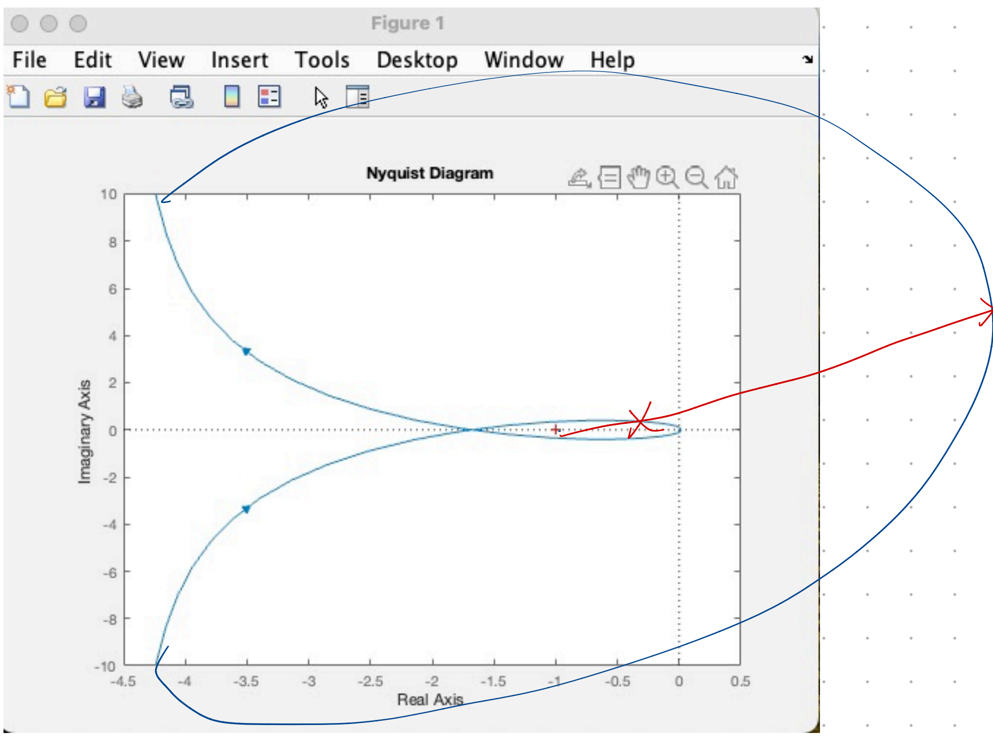
$\omega_c = 0.8$ $\qquad \omega_o = 1.15$ (can also be find by margin(L))

$20\log_{10}(Gm) = 5\,dB$

Phase margin $= 24.57$

4) $L_\Delta(s) = \dfrac{24}{s(s+2)^3}$ $\qquad$ $T_\Delta(s) = \dfrac{24 \cdots\cdots}{s^2(s+2)^6}$

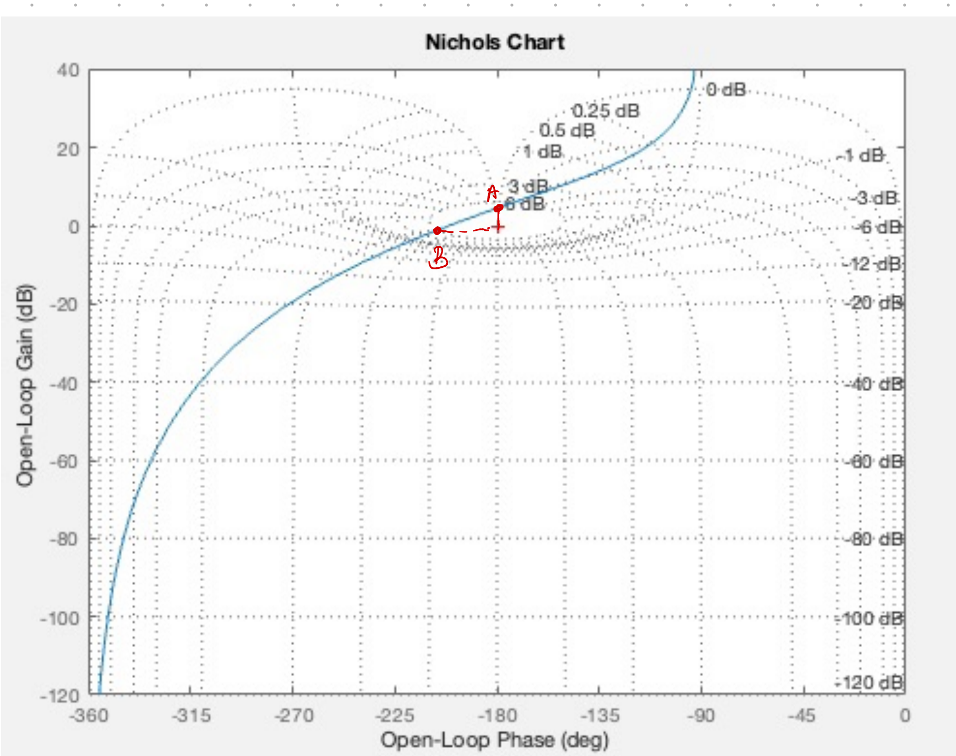
↳ $p = 0$ $\qquad$ ↳ $z = 0$



$N = 2$

$2 = 0 - 0$ ✗

↳ $\boxed{\text{unstable}}$



point A is on top, point B is on left of the critical point hence non-stable

2. Verify that the feedback control system is stable by using the Nichols diagram of $L(s)$

3. Compute the gain margin and the phase margin
(hints: $\angle L(j\omega_0) = -180°$ for $\omega_0 = 1.15$ rad/s; $L(j\omega_c) = -1$ for $\omega_c = 0.8$)

4. Verify that the feedback control system with loop transfer function

$$L_{\Delta}(s) = 3L(s)$$

is unstable

Solution

Gain margin: 5 dB
Phase margin: 24.6 deg