## Report 2 : Forecasting and Analysis of the Polish Power System Time Series

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### Abstract

The Load of the Polish Power System Dataset has been analyzed with various methods and a forecast have been made out of this analysis. Two Fourier methods (time independent and time dependent) have been applied to extract information about the periodical behavior of the signal. The wavelet has been used to clean the signal from the noise, and the cleaned signal has been used to apply prediction with ARIMA and SARIMA processes and deconvolution methods. The best forecast gave a RMSE equal to the 9.7% of the maximum value.

## Chapter 1

## Introduction

An appealing kind of dataset that is worth studying in order to obtain a predictive ability about nature are the Time Series. The one studied in this report is about the load of the Polish Power System.

One of the most powerful tool of Data Analysis of a Time Series consists on using the Fourier transform on a periodic data. In fact, the Fourier Theorem states that a periodic function can be decomposed in sines and cosines terms. In this sense it is possible to perform a variable space from the original one (the x space) and analyze the behavior of the function with respect of the u frequency, thus going from f(x) to F(u). In particular, the absolute value of this function computed at a specific frequency  $u^*$  furnishes the amplitude that  $u^*$ has in the spectrum. By the distinction of the frequencies that could be considered as a form of "noise" and the frequencies of the "proper" signal, and by the accurate treatment of the non-stationary nature of the signal, it is possible to use the information of the frequency spectrum to filter the signal and gain a forecast ability.

Another important tool to clean the signal consists in the use of the Wavelet transform. As the Fourier Analysis, the wavelet trans-

form compute the projection of the original signal on an orthogonal basis of a function called wavelet. By using this wavelet at different scales, it is possible to detect the noise scale as it has frequencies that can be assumed to be above the typical band of frequency of the signal.

A predictive method that is more powerful of the Fourier Analysis consists in the use of ARIMA and SARIMA processes. By the use of this processes it is possible to construct an approximated "auto-regressive" and "moving average" model of the original signal. In particular, the main difference between this method and the other ones that has been used is the ability to use a different numbers of values to look at both in the auto-regressive and the moving-average processes to obtain a prediction. Moreover, in the SARIMA processes, the non stationary nature of the signal could be treated with an extra-attention as the method considers the seasonal elements of the data.

The last method that has been used to forecast and analyze the signal is the deconvolution. This approach considers the signal as the convolution of one single shape known as kernel with a series of pulses. A refinement of this method has been applied considering three different kernels.

## Chapter 2

## Data and Method Description

### 2.1 Data overview

The analyzed data is the load of Polish Power System time series. The dataset is extremely simple as it consists in a time report of all the loads in the Polish Power System. The columns of the dataset are:

- **Day**, Calendar Day in the following format ('YYYY/MM/DD')
- Hour, Hour of the end of the detection
- Minute, Minute of the end of the detection
- Load, Load (MW) during that time interval

Measurements are taken each 15 minutes and the Load column reports, in the (n)-th row, the load between the (n-1)th and the (n)th row (i.e. in 15 minutes of time). The site that has been used to extract the dataset uploads real time information about the Polish Load. Due to computational limits, our specific dataset is limited from 2008 (2008-01-01) to 2016 (2016-12-31).

### 2.2 Data preprocessing

Even if the initial format of the dataset is imediate to read and to understand, it is convenient to convert the temporal scale in a way that the temporal continuity is visible and appreciable. To do so, a 5-th column has been added to the dataset, consisting in a temporal line from 900 seconds (15 minutes) and increasing by the same factor (900 seconds) for each row: (900,1800,2700,...). Using this temporal scale, an effective visualization of the load can be obtained:

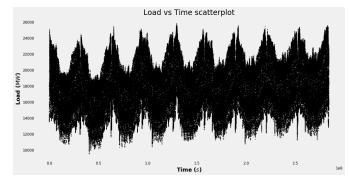


Figure 2.1: Load(MW)vs Time(s) plot.

The analysis we want to perform about the dataset is based on the Fourier Transform. As it has already been said, the goal of this analysis is to detect the frequencies that

characterize the phenomenon of interest. In this sense, the mean value and the global temporal trend of the signal are not interesting and tend to disturb the target analysis. As it can be appreciated from Figure 2.1 the mean value of the signal is not 0 and a global linear ascending trend can be easily identify. The first step is thus "detrending" the original signal, and the final result is expressed in Figure 2.2:

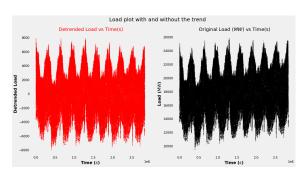


Figure 2.2: **De-trended Load vs Time**(s) **plot** to the left and Load (MW) vs Time (s plot to the right.

The mean value of the red signal is extremely low ( $\approx 10^{-11}$ ). To make it even lower, the mean value is subtracted from the red signal, thus obtaining a signal with mean value = 0 (numerically the mean value is the lowest possible:  $\approx 10^{-13}$ ). From now on, the "detrended" mean=0 Load will be simply intended as load. As it has already been said in chapter 1, the Fourier Analysis is made for periodical signal. The first important thing to notice is if the first point (i = 0) and the last point (i = N)of the dataset has the same Load quantity. As there is in fact a notable difference between  $Load_0$  and  $Load_N$  ( $|Load_0|$  $Load_N$  = 2267.67 that is almost the 30% of the maximum load), a window function needs to applied to smooth the signal and make the endpoint of the signal meet. The first standard windowing function that has been applied to do so is the Hanning function [6] and the result of the product between the signal and the hanning function is shown in Figure 2.3

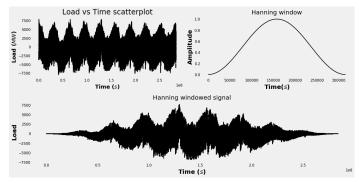


Figure 2.3: Detended Signal Load vs Time scatteplot (up-left), Hanning window (up-right) and their product windowed signal. As it is possible to see, the windowed signal has the same value on its border.

### 2.3 Fourier Transform

As it has already been said in the introduction, the signals that can be analyzed using Fourier Transform are by definition stationary. This condition appear to be a forced one when the signal is extracted by some real world data like the one that it has been considered. For example the behavior of the Load of a civilized country during the Christmas Holidays is not equal to the behavior during the rest of the year. For this reason, the blind application of the (Fast) Fourier Transform may seems a weak ap-

proach. Nonetheless some basic time period could still be verified from the Fourier transform of the Load. In fact by just looking at the plot Amplitude vs Period (h) it is possible to notice some natural order periods like:

- 12h (the length of a day divided by 2)
- 24h (the length of a day)
- $\approx 33h (1 day + 8h)$
- 168h (7 days, a week)

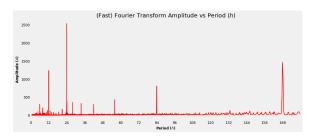


Figure 2.4: **Amplitude vs Period** (h) **plot.** As it is possible to notice some relevant peaks appear in some specific time ticks (12h,24h,33h,168h).

This specific periods seem to be high correlated with the working hours and the natural duration of a day. In fact, it is possible to assume that the 12h periodicity could be related to the day and the night hours of a day. 24h is the duration of a day, so it is still a natural period. 33h is the sum of 24h (the lenght of the day) + 8h (the mean working hours per day) and it could be related to some working periodicity. Even the week and the middle week periods are stressed by the 84h and 168h peaks. Thus, even if the signal is not stationary and some extra work needs to be

done in order to interpret the signal correctly and thus to be able to predict the following year, it is still important to analyze this periodicity and understand if it is possible to reconstruct the signal starting with few frequencies val**ues**. As the numpy algorithm that has been used is a robust one, transforming the original signal using Fourier transform and then inverse transforming the Fourier transform, the original signal is accurately reproduced. The mean absolute difference between the reconstructed signal and the original one is in fact almost 0 ( $\approx 10^{-13}$ ) and the signal are almost unrecognizable from each other as it is possible to see from Figure 2.5:

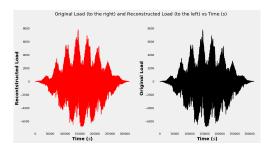


Figure 2.5: Reconstructed (to the left) and original signal (to the right) comparison. As it is possible to see, the signal are unrecognizable from each other, as the numpy FFT algorithm is robust.

After that the robustness of the algorithm has been tested, it is interesting to analyze which frequencies are really informative in terms of the reconstruction of the original signal. In particular, a threshold has been applied to the frequency spectrum, setting to 0 the values that are greater than that threshold. After this operation the inverse fourier transform has been applied, thus obtaining a reconstructed signal. The root

mean squared error (RMSE) has been computed between the original signal and the reconstructed one.

# 2.3.1 Uniform varying threshold

Varying the threshold (k) in an uniform manner  $(k \in \{0, 2475\})$  the RMSE becomes larger as one could expect. To deeply understand the meaning of this RMSE and how the information is filtered out by the threshold value, some k values filtered plot has been shown in Figure 2.6

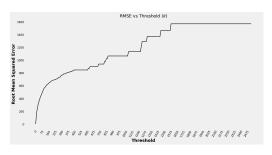


Figure 2.6: **RMSE** vs k plot. As the threshold  $(Th_k = k)$  increases the RMSE increases too, thus highlighting the loss of information that it is verified during the reconstruction. The highest growth rate is in the first zone of the graph, where  $k \in \{0, 300\}$ 

As Figure 2.7 suggests, as the k value increases, only the frequencies with highest amplitude are not filtered out. In fact, when k becomes bigger than a certain value, only the highest peak, that is the one related to the 24h period, "survives".

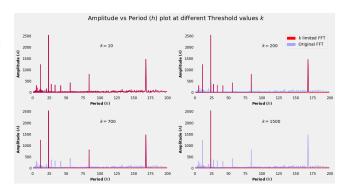


Figure 2.7: **FFT** of the original signal and k-filtered **FFT** vs Period (h) plot. As the threshold  $(Th_k = k)$  increases only the frequencies with the highest amplitude are not filtered out.

# 2.3.2 Maximum related threshold

The analysis suggests that to understand the relevance of the sinusoidal components, it could be more convenient to vary the threshold k scale with respect to the maximum amplitude value. As it is possible to see from Figure 2.8, if the frequencies with amplitude under the 30% of the maximum are set to 0, a big portion of the noise is removed, and the waving behavior of the signal is still visible and cleaned. As the threshold becomes higher, only the 24h period survives. Its frequency, as it is possible to see from Figure 2.4, is in fact so high to be indistinguishable from a full colored square, as the last subplot in Figure 2.8 highlights. A clearer version of Figure 2.6 has been reproduced, computing the RMSE at the varying of the k with respect to the maximum amplitude value (Figure 2.9). Even if, in general the RMSE furnishes an efficient comparison method be-

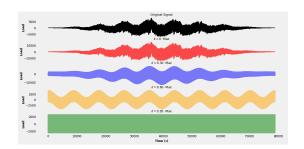


Figure 2.8: Reconstructed Load vs Time at various k threshold value. If k = 0.3·Max, the original signal can be reconstructed with a discrete level of accuracy.

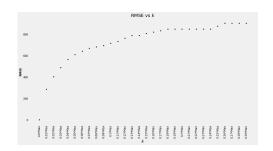


Figure 2.9: **RMSE** vs k plot. This plot is analog to the one reproduced in Figure 2.6, but it is simpler to read as it is related to the maximum amplitude value.

tween a real signal and a reconstructed one (low RMSE implies good correspondence between the two), it is important to understand if the RMSE value is dominated by the background noise of the original signal. In fact, while reconstructing the signal with the threshold limited fourier transform, the noisy behavior of the original Load is set to zero and a more clean signal is reconstructed (example in Figure 2.10). This process summarize the purpose of this part of the report, as it cleans the original data and set to zero the not relevant information,

but increases the RMSE.

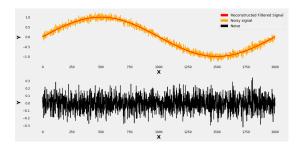


Figure 2.10: **Example of the noise phenomenon**. Filtering the fourier spectrum with a threshold, the noisy sinus wave looses its background noise. As it is possible to see, the noise is not related to the original signal.

To explore this phenomenon, it is important to understand how much the error is related or not to the original signal. The plot of the correlation value with respect to k predictably highlights that the correlation increases with the k threshold. The k value can thus be chosen with respect to this correlation coefficient. It is important to highlight the trade off that needs to be obtained here:

- An high threshold value assures that a rigid selection of the frequencies has been applied and the noise is deleted
- A low correlation coefficient assures that the residual signal is not correlated with the original signal and it can be properly assume as noise.

As it is possible to see from Figure 2.11, as the correlation decreases the threshold decreases too and vice versa. It is thus important, in order to delete only the signal, to keep the threshold as high as possible while the correlation doesn't increase over

a certain value. As it is possible to appreciate from Figure 2.11, the correlation between the error and the signal is not 0 even when the threshold is set to 0, thus highlighting that even if the error that is generated by the Fourier Transform is extremely low it is still slightly correlated with the signal (C = 4.3%) correlation). This

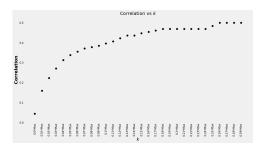


Figure 2.11: Correlation vs k scatter plot. As it is possible to see from the plot, as the k threshold value increases, the error becomes more correlated with the original signal.

consideration, together with the correlation instantly increasing in the first 3 thresholds  $(C_{k=0.05*Max} = 16\%, C_{k=0.02*Max} = 23\%, C_{k=0.03*Max} = 27\%)$ , furnishes a relevant warning to further decrease the k lower bound. As it is possible to see from Figure 2.12 this detailed observation permits to choose a less correlated signal reconstruction by setting the optimal threshold as the highest one with correspondent correlation lower than 10%  $(k_{opt} = 0.004 \cdot Max)$ .

The optimal signal reconstruction is thus obtained by inverse transforming the filtered power spectrum and the results are shown in Figure 2.13.

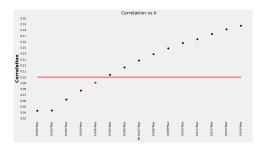


Figure 2.12: Correlation vs k scatter plot. As it is possible to see from the plot, as the k threshold value increases, the error becomes more correlated with the original signal. The optimal threshold

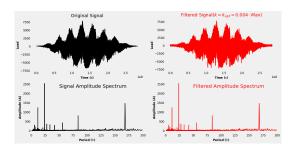


Figure 2.13: Original (Up-left) and Filtered (Up-right) Signal with correspondent amplitude spectra (Down-left and Down-right). As it is possible to see, the signals appear similar but does have some differences in the highest frequencies.

A double check of the analysis has been done to verify the distribution plot of the difference between the reconstructed signal and the original one. In fact in order to be considered white gaussian noise, it is expected to be fitted with low  $\chi^2$  value to a gaussian [5]. As it is possible to see from Figure 2.14 the error appears to have a gaussian shape. It is thus interesting

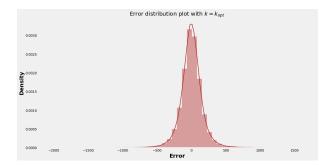


Figure 2.14: Error Distribution plot with  $k = k_{opt}$ . As it is possible to see, the distribution has a gaussian shape in its core.

to perform the  $\chi^2$  test on a gaussian distribution, as it has been shown in Figure 2.15. Unsurprisingly, the  $\chi^2$  value is low  $\chi^2 = 6.27 \times 10^{-2}$  while the p value [7] is extremely high  $p \approx 1.0$ , thus highlighting the excellent correlation between the gaussian fit and the original data. Even if this

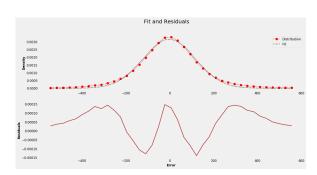


Figure 2.15: Error Distribution plot with  $k = k_{opt}$  and gaussian fit (Up), Residuals plot (Down). As it is possible to see, the gaussian fit with high accuracy the distribution plot. The residuals (R) are at a significant lower scale with respect to the distribution values (D):  $\frac{D_{max}}{R_{max}} = 22.360$ 

method shows promising statistical properties, the important test that the recon-

structed signal has to pass is the prediction of the new data. As it is important to highlight as a background clarification, the prediction that can be done is about the oscillatory behavior of the core of the signal. In fact, the linear trend (the growth of the signal) and the mean value are both subtracted from the original signal in order to process it correctly with the Fourier analysis. Moreover, two important hypothesis that has been made are the following:

- The nature of the signal is periodical: all the events can be accurately modeled as sines and cosines terms.
- The signal is stationary: the frequencies of the fourier spectrum are time invariant.

While the first assumption is a crucial one when the Fourier transform is applied, the second one will be relaxed with the application of the **spectrogram method**. In order to perform the prediction test, the dataset has been split in two sets. Borrowing in a not rigorous manner the Machine Learning notation they have been defined as following:

- The training set, where the algorithm that has been described in the previous page has been applied and the prediction has been performed.
- The test set, that is the set of data that has been used for testing the prediction.

In order to apply the algorithm in the most efficient way as possible, the training set has been obtained by extracting the first 8 years of the dataset, while the remaining year (2016) has been used as test set. The previous described algorithm has been applied in the training set, but as it is necessary to predict the real values no smoothing has been applied to the original signal. In order to keep both RMSE and Correlation values under control  $(RMSE \leq 300, C \leq 10\%)$ , the optimal threshold as been reduced:  $k_{opt} = 0.004 \cdot Max$ . The plot of both RMSE and Correlation values are reported on Figure 2.16 while the spectrum with the  $k_{opt}$ threshold has been reported in Figure 2.17.

The spectrum in Figure 2.17 furnishes a

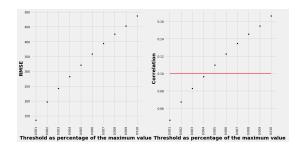


Figure 2.16: RMSE and Correlation vs k**plot**). In order to keep  $RMSE \leq 300$  and  $C \leq 10\%$ , the  $k_{opt}$  value has been chosen to be  $k_{opt} = 0.004 \cdot Max$ .

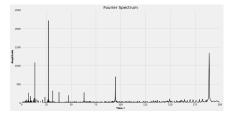


Figure 2.17: RMSE and Correlation vs k**plot**). In order to keep  $RMSE \leq 300$  and  $C \leq 10\%$ , the  $k_{opt}$  value has been chosen to be  $k_{opt} = 0.004 \cdot Max$ .

by their frequency  $\omega$  and their complex correspondent value  $(F(\omega) = F_R(\omega) + jF_I(\omega))$ . Given a specific frequency  $\omega$  and a specific time t the correspondent waving function is given by:

$$x(t,\omega) = \frac{2}{N} (F_R(\omega)\cos(2\pi\omega t) + F_I(\omega)\sin(2\pi\omega t))$$

In general, for all the frequencies that has not been filtered out  $(\omega \in \Omega)$  we have:

$$x(t) = \sum_{\omega \in \Omega} \frac{2}{N} (F_R(\omega) \cos(2\pi\omega t) + F_I(\omega) \sin(2\pi\omega t))$$

By applying this rule with  $t \in T_{train}$  with  $T_{train}$  that is defined by all the time steps (s) of the training data, the reconstructed signal is simply the inverse (filtered) fourier transform of the original one. Nonetheless, when  $t \notin T_{train}$  but  $t \in T_{test}$  the signal is extended over its inverse fourier definition by using the above rule. In this sense, as the algorithm "didn't see" the test set, a prediction has been made. As the reconstruction has been done on the training set frequencies, the difference between the original signal and the reconstructed one is at its minimum values in the training set and it increases on the test set. Indeed, it is possible to see that the increase of the difference between the predicted signal and the real one doubles in the test set.

In particular the RMSE in the training set is:

$$RMSE_{Train} = 278.32$$

While the RMSE in the test set is:

$$RMSE_{Test} = 2624.89$$

This result comes with little surprise. In set of sines and cosines that can be defined fact no temporal information has been given

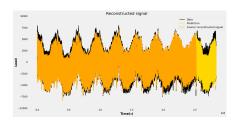


Figure 2.18: **Original Signal**, **Reconstructed Signal and Predicted Signal vs Time**(s)

about the frequencies and the signal is considered to be stationary. A close look of the original signal and the reconstructed one in the test set highlights that the errors of the prediction become significantly higher with respect to the time.

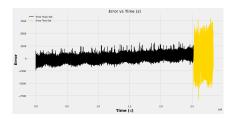


Figure 2.19: **Error vs Time**. The error becomes consistently higher when the test set is explored. In both training set and test set the signal appears to be slightly correlated to the error.

Moreover the error increases with respect to the time, thus highlighting the weaknesses of the stationarity assumption. As it has already been told, the RMSE could be a inappropriate metric. A deeper analysis has been made to see whether or not high correlation could be found between the original signal and the error. Unfortunately the signal and the error are highly anticorrelated ( $C_{data,error} \approx$ 

-0.6) and the *P*-value test highly rejects the gaussian hypothesis ( $P \approx 0$ ), thus confirming the failure of the method. The

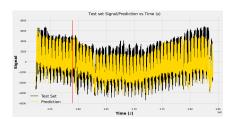


Figure 2.20: **Signal/Prediction vs Time**). It is possible to see that the prediction becomes worse at high time scale, while it seems to be acceptable for the first entries.

RMSE has been computed reducing the time range and it can be seen that the first prediction have the lowest error values. In particular the prediction for the first 30 days is both the most precise one in terms of RMSE and the less error-related. As the

		$T_fin$ (s)	RMSE	$\Delta T$ (s)	C <sub>data</sub> ,  error	$\Delta Days$
-	0	254188800	2569.99	1727100	0.64	20
	1	255052800	2439.43	2591100	0.61	30
:	2	257644800	2458.33	5183100	0.62	60
	3	260236800	2780.33	7775100	0.65	90
	4	262828800	2686.71	10367100	0.64	120

Figure 2.21: Summary of the RMSE at different time ranges). As it is possible to see, the prediction gets better for lower values of time step. In particular the best RMSE and correlation (the lowest) are verified for a 30 days time step.

(best) RMSE is the 33% of the maximum value of the signal, the method has to be considered as failed.

# 2.3.3 Time dependent Fourier Transform

The algorithm that has been presented in Chapter 2.3.2 had the important defect of being completely time independent. In fact the signal has been interpreted as a stationary one, thus obtaining a weak reconstruction and an even weaker prediction. The following approach [4] is used to consider the non stationary nature of the dataset, and it is based on the following division of the original data:

- The training set, that contains the signal from 2008 to 2015
- The validation set, that contains the signal data of the year 2015
- The test set that contains the signal data of the year 2016

The training set has been divided in 5 annual periods. For each one of those, the fourier transform has been applied. The mean of all these transform has been applied, thus obtaining a mean fourier transform of the first 5 years of the dataset.

The validation set has been used to apply the algorithm that has been described in chapter 2.3.2 to the mean fourier transform that has been obtained from the training set. The big difference between this approach and the one described in chapter 2.3.2 is that the RMSE and the correlation are computed with respect to a portion of the dataset that the algorithm "does not know": the algorithm is not been trained on this portion of the dataset. To consider

this difference, the RMSE threshold and the correlation one are less strict than the one consider in chapter 2.3.2 ( $RMSE_{max} < 2000, |C_{min}| < 0.82$ ).

The threshold value with the lowest  $C_{min}$  and within the  $RMSE_{max}$  value has been considered as the optimal threshold. The mean fourier transform that has been computed in the training set has thus been filtered with the optimal threshold, and the result has been compared with the test set signal. These has been summed together and divided by 8, thus obtaining a mean fourier transform that has been adopted to predict the values of the 9th year.

	RMSE	Days	$C_{data,error}$
0	1693.717665	10	-0.499780
1	1653.486625	20	-0.584994
2	1625.299205	30	-0.551923
3	1750.071345	40	-0.572060
4	1705.124815	50	-0.576772
5	1742.991733	60	-0.582522
6	1739.389256	70	-0.587244
7	1757.974530	80	-0.591003
8	1945.728359	90	-0.620888

Figure 2.22: Summary of the RMSE after a certain number of days of observation. Even if the RMSE is in general still remarkably high the values are lower than the ones that have been obtained with the previous method. Even the correlation values are lower than the ones of the previous method.

This method remarkably outclass the previous one in terms of RMSEs, obtaining lower RMSEs even for larger period of time. The method is better in terms of correlation between the original signal and the error too. In fact data are almost 10% less correlated

with the error with respect to the previous method.

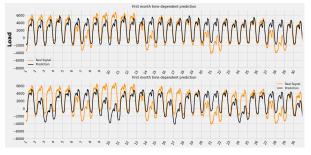


Figure 2.23: First month prediction using Time dependent Fourier Transform method).

The best RMSE assumes a value that is the 21% of the maximum value thus obtaining a more accurate method with respect of the (filtered) global fourier transform one.

### 2.4 Wavelet

A wide part of the algorithm in section 2.3.2 and 2.3.1 is based on cleaning the original signal from the (presumed) noise. In general, an important step in order to have an accurate prediction of a time-series is based on cleaning the original data. So far, this cleaning process has been done following the step indicated above:

- Analyzing the fourier spectrum
- Applying a threshold to it
- Selecting the upper bound of the threshold based on the RMSE
- Selecting the optimal threshold as the highest one with the correlation value as close as possible to 10%

This methodology has not been proved helpful even if it is based on the reasonable assumption that the residual signal has to be not correlated with the signal to be considered as noise. A different approach to clean the signal is based on using the so called wavelet transform. The wavelet transform is in some way similar to the Fourier one as it computes the product between the original signal and a basis function  $(\psi)$ . Nonetheless this basis function is stretched and contracted in relation with a parameter called **scale** (s). Thus, at a fixed scale s, the basis function will "slide" on the original signal time steps outputting the projection between the signal f at that specific time and the wavelet computed at that specific scale and that specific time. The sum of all this time contribute will give the wavelet coefficient at a specific time (u) and at a specific scale (s):

$$W_f(s,u) = \int_{-\infty}^{+\infty} f(t) \frac{1}{\sqrt{s}} \psi^*(\frac{t-u}{s}) dt$$

The power of this method relies on the ability of discern and decompose the original signal at different scale. Indeed, when the scale decreases, higher frequencies of the original signal are detected and vice versa. By using low value of scale, the wavelet method is able to discern the highest frequencies of the signal, that are the one that can be considered to belong to the white noise frequency range. In particular, the method that has been used to implement this idea is the discrete wavelet transform [2]. This method is based on the decomposition of the signal in detail and approximation coefficients on various level. The first level is obtained by using

the wavelet with the lowest possible scale: i.e. the highest frequency  $(\nu_{s_{min}})$  with respect to the highest frequency of the signal  $(\nu_{max})$  according to the Nyquist Shannon theorem  $\left(\frac{\nu_{max}}{2} = \nu_{s_{min}}\right)$  [1]. In particular, the detail coefficients of the first level are represented by  $W_f(s_{min}, u)$  and the approximation coefficients are represented by the residual between the original signal and the detail coefficients. Both the detail and the approximation coefficients, according to the Nyquist Shannon theorem can be downsampled by a factor  $2^{n_{level}}$  (2 for the first level, 4 for the second level, 8 for the third level). As it can be assumed that the denoised signal will have specific band limited frequency range, the highest frequencies of the studied signal can be considered as the white noise frequencies [3]. That means that it is possible to detect the noise signal in the lowest level of the discrete wavelet transform detail coefficients.

The first level presents a not negligible excess of kurtosis, thus discouraging from looking at other levels, as too much relevant information about the signal would be lost. Moreover, in order to select the part of the detail coefficient that can't be considered as "noise", a certain portion of the detail coefficient has been set to 0. This portion has been chosen with respect of a threshold value  $\theta_k$  that is a function of k, that is a real number between 0 and 13.25  $(k \in [0, 13.25])$  and the fitted sigma value from the gaussian fit:

$$\theta_k = k\sigma$$

This value has to be intended as a threshold in the following sense: all the values of

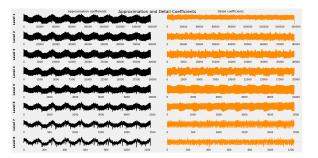


Figure 2.24: **Approximation and Detail Coefficients**). As it is possible to see, the periodical behavior of the signal does not present itself in the first detail coefficients, while it starts appear when the  $n_{level}$  increases. At the same time the approximation coefficients are really similar to the original signal for the first level, and they loose accuracy while the level increases.

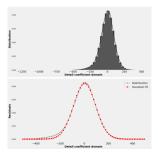


Figure 2.25: **Detail Coefficient distribution**). As it is possible to see, the distribution is almost gaussian, but presents an excess of kurtosis on its tail.

the details coefficient that are found between  $-\theta_k$  and  $+\theta_k$  are set to 0. The k upper bound is due to the fact that if the threshold is chosen to be that high, the entire first detail coefficient is set to 0. While k, (and  $\theta_k$ ) increases, so does the RMSE. Indeed if  $\theta_0 = 0$  the approximation and detail coefficient are naively summed, as no value of the detail coefficient is set to 0. Nevertheless, if the RMSE increments are due to the noise they are harmless as they don't imply a loss of information about the original signal. In order to detect whether or not the filtering is actually cutting only the noise out, the correlation coefficient between the original signal and the difference between the latter and the reconstructed one (intended as the sum of the approximation coefficient and the filtered detail coefficient both of the first level) has been computed. The best threshold has been chosen to be the one with the lowest correlation. In particular it has been proven to be 1:

$$\theta_{opt} = \sigma$$

. This optimal threshold permitted to have the following error-signal correlation

$$C = 0.2\%$$

The effect of the threshold on the original detail coefficient domain has been shown in Figure 2.27. This optimal threshold has been used to construct a reconstructed signal, that is as less as possible influenced by the noise. As it is possible to see from Figure 2.28, it is really difficult to spot the differences between the original signal and the reconstructed one in general terms. On the other hand the smoothing effect can be appreciated at lower scales, where the original signal and the reconstructed one are still similar, but the reconstructed signal follows a smoother line as it is not 'disturbed' by the noise.

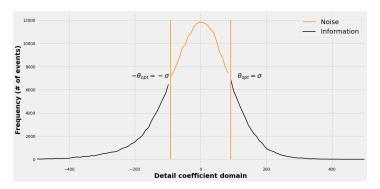


Figure 2.26: **Detail Coefficient** distribution filter). In orange it is possible to see the range of points that has been filtered out from the original detail coefficient signal. In black, it is possible to see the remaining part, that has been summed to the approximation coefficient signal to obtain the reconstructed filtered signal.

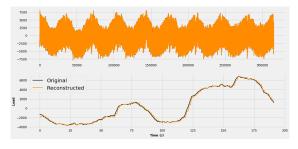


Figure 2.27: Reconstructed Signal (Orange) and Original Signal (Black) at different time scales. While the signals appear to be indistinguishable in the larger time scales, the effect of the de-noising wavelet process appears to be visible at lower scales.

### 2.5 Arima and Sarima Processes

In this report we wanted to test the ability of a class of models widely used in forecasting, the Autoregressive–moving-average (ARMA) models. The basic idea is to use these models to describe the dataset and extrapolate a forecast.

The main goal in time series analysis is identifying an appropriate stochastic process that has trajectories that adapt to the data, in order to then be able to formulate forecasts. An important class of stochastic processes that allows us to uniquely identify the process and obtain a consistent estimate is represented by stationary processes. In particular, if what we observe is interpreted as a finite realization of a stochastic process that enjoys particular properties, then it is possible to find a single model suitable to represent the temporal evolution of the phenomenon under study. Intuitively, a stochastic process is stationary if its probabilistic structure (average value, variance, etc.) is invariant over time. Generally a stochastic process  $(X_t)_{t\in\mathbb{Z}}$  is said to be stationary or weakly stationary if it satisfies the following conditions:

- 1.  $E[|X_t|^2] < \infty$
- 2.  $E[X_t] = m \ \forall t \in \mathbb{Z}$
- $3.\gamma_X(r,s) = \gamma_X(r+t,s+t) \forall r,s,t \in \mathbb{Z}$

Considering a white noise with zero mean and variance  $\sigma^2$  identified as WN(0,  $\sigma^2$ ) and introducing the delay operator B defined as

$$BX_t = X_{t-1} \tag{2.1}$$

we can begin to define some of the fundamental characteristics of these processes. An **Autoregressive process** of order p, identified as AR (p) can be written as:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \dots + \phi_n x_{t-n} + w_t$$
 (2.2)

formally

$$W_t = \phi_0 X_t - \sum_{i=1}^p \phi_i X_{t-i} = \phi(B) X_t \quad (2.3)$$

In particular,  $\phi(B)$  is a polynomial of degree p, given by

$$\phi(z) = \phi_0 - \phi_1 z - \dots - \phi_p z^p$$

Where  $x_t$  is stationary and  $\phi_1, \phi_2, ..., \phi_p$  are constant  $(\phi_p \neq 0)$ . It is generally assumed that  $w_t$  is a Gaussian white noise with zero mean and variance  $\sigma_{w_t}^2$ .

In a very similar way it is possible to define a **Moving Avarage** process of order q, with  $q \in \mathbb{N}$ , briefly MA(q), if it is of the form:

$$X_{t} = \theta_{0} + \theta_{1}W_{t} + \theta_{2}W_{t-1} + \dots + \theta_{q}W_{t-q} =$$

$$= \sum_{i=1}^{q} \theta_{i}W_{t-i} = \theta(B)W_{t}$$

where  $\theta(B)$  is the polynomial of degree q, given by

$$\theta(z) = \theta_0 + \theta_1 z + \dots + \theta q z^q$$

A stationary process  $(X_t)_{t\in\mathbb{Z}}$  is called Au- $toRegressive\ Moving\ Average$ , more briefly ARMA(p, q), if there are coefficients  $\phi_1, ..., \phi_p, \theta_1, ..., \theta_q$  such that

$$x_{t} - \phi_{1} x_{t-1} - \dots - \phi_{p} x_{t-p} = w_{t} + \theta_{1} w_{t-1} + \dots + \theta_{p} w_{t-q}$$

$$(2.4)$$

Using the polynomials AR and MA defined above and the delay operator B, we can write the equation in compact form:

$$\phi(B)X_t = \theta(B)W_t$$

These models are widely used to study and model time series in particular they are very much related to stationary processes due to their nature. Furthermore, they are very simple models which, however, manage to capture complex data behaviors. However, the specific dataset we have analyzed is very extensive and we have decided to test the power of these models on a monthly average of the original dataset, for two reasons: first of all this would have considerably reduced the computational complexity and would have allowed us to compare different configurations, but also because in order to be able to describe all the facets of the original dataset, the model would have required an enormous amount of parameters, and this would have been in contradiction with the nature of these models.

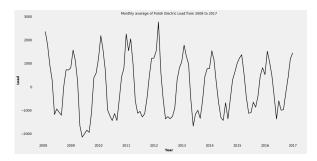


Figure 2.28: Monthly averege of original dataset

### Arima modeling

The first step in analyzing the time series is to check its stationarity, and it was done using Augmented Dickey–Fuller test which tests that the time series can be represented by a unit root(has some time-dependent structure). The alternate hypothesis (rejecting the null hypothesis) is that the time

series is stationary. The values of the parameters relating to this test are shown in the following table under the "monthly average" column.

Differencing	ADF Statistic	P value
monthly average	-2.64	0.085
1st order	-2.27	0.18
2nd order	-10.99	$6.80 \times 10^{-20}$

This shows us that according to the criteria of the hypothesis test used, it is necessary to reject the null hypothesis, i.e. that the series is stationary. Table 2.5 also shows the values of the test parameters for the first and second differencing order; In order to make the series stationary, the series has been differencied up to 2 times.

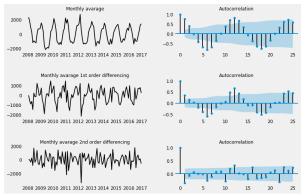


Figure 2.29: Lineplot and autocorrelation plot for the time series, for the 1st. and 2nd order differencing of the Monthly average of Polish Electric Load from 2008 to 2017.

The test is actually successful only for the second order differencing, moreover Figure 2.29 shows how differentiation manages to regularize the series by making it stationary, in particular, the autocorrelation plot for the 2nd differencing is very similar to the autocorrelation of a white noise; These facts would suggest that the differentiation did the trick! However, a closer look at the autocorrelation plot for the 2nd differencing the lag goes into the far negative zone fairly quick, which indicates, the series might have been over differenced. Considering what has been said above, we decided to build two ARIMA models and compare them, one with a differentiation parameter d equal to 0 and the other equal to 2. The parameters relating to the auto-regression p and to the moving average q were selected by comparing the Akaike Information Criterion (AIC) of different models and have been fitted to the data The main information relating to the two models has been reported in the following table:

Coeff	d=0	d=2
ar.L1	0.0819	0.5279
ar.L2	0.1028	-0.2791
ar.L3	0.2692	-0.0252
ar.L4	0.3284	0.3296
ar.L5	-0.3148	-0.5174
ar.L6	-0.7677	-0.3888
ma.L1	0.3635	-1.9547
ma.L2	0.1579	1.3792
ma.L3	0.4566	-0.5751
ma.L4	-0.2928	-0.3030
ma.L5	0.4209	1.1751
ma.L6	0.8783	-0.7160
ma.L7	0.2458	Na
ma.L8	0.3423	Na
AIC	1385.880	1382.601
BIC	1423.707	1415.099

In Table 3 are shown values of autoregressive and moving avarage parameters of the models; in particular the comparison of the

AIC value for the two models suggested that the best configuration for the integrated one was ARIMA(p=6, d=2, q=6) and ARMA(p=6,q=8). Furthermore, it is interesting to note that the two models differ very little in terms of AIC and BIC score. The models were built using a statsmodel function called SARIMAX. One of the built-in methods of this function allows to get a diagnostic plot of the model which is very useful to get an idea of the agreement between models and data.

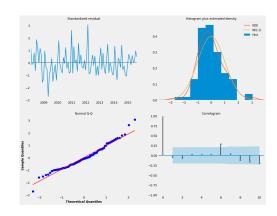


Figure 2.30: Diagnostic plot of the ARIMA(6,2,6) model. The plots show a good match between the model and the data as the residuals are very similar to Gaussian white noise

Figure 2.30 shows the agreement that exists between the ARIMA(6,2,6) model and the data and suggests that the chosen model could be an acceptable representation of the process represented by the data (Diagnostic for the ARMA(6,8) is very similar). Once the information about the two models is gathered, it's time to see how they perform in the forecast; For both models, a training set equal to about 80% of the volume of the series was used to calculate the parameters

and was asked to predict the last recorded fact, we know that the studied dataset cycle (still monthly average).

has recurring patterns with an annual pe-

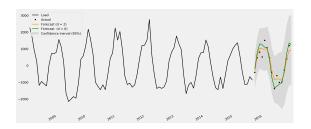


Figure 2.31: Forcasting of the monthly avarage of Polish Electric Load for **2016.** Two different models were used: Arima(6,2,6)[Orange] and Arma(6,8)[Green].

The two models behave similarly on unexplored data, in particular the integrated model performs slightly better as it is given an RMSE of 270.22 which is approximately 9% of the global maximum of the data; The ARMA model, on the other hand, has an RMSE equal to 345.64 (12% of the maximum).

To conclude, it is interesting to note how the non-integrated model seems to be able to better capture the characteristic frequency of the data at this level, this prompted us to investigate this class of models better to verify if there was a way to better represent the time series under exam.

#### Sarima modeling

The results obtained by building the ARIMA model just described have high-lighted some very interesting aspects of the dataset. First of all, the fact that the autoregressive terms necessary to describe the data are a fairly high number suggests that there is a correlation between the data at least 6 lag apart. In

fact, we know that the studied dataset has recurring patterns with an annual period and it might be more appropriate to build a model that takes into account the **seasonal component** to obtain accurate forecasts. For this purpose, a model called SARIMA was taken into consideration which represents a more sophisticated version of the ARMA models and introduces parameters to manage the seasonality of the dataset, generally indicated with the acronym SARIMA(p,d,q)x(P,D,Q,s) where

- **p** and seasonal **P**: indicate number of autoregressive terms (lags of the stationarized series)
- d and seasonal **D**: indicate differencing that must be done to stationarize series
- q and seasonal Q: indicate number of moving average terms (lags of the forecast errors)
- s: indicates seasonal length in the data

Retracing the steps described above, two approaches were taken to determine the ideal SARIMA parameters: ACF and PACF plots, and a grid search. Let's proceed step by step, first we used a function of statsmodel called *seasonal decompose* which allows to obtain a decomposition of the time series in an additive sense.

$$y(t) = T(t) + S(t) + N(t)$$

where:

- y(t) is the time series at the time step t
- T(t) is the **trend** component at the time step t

- S(t) is the **seasonal** component at the time step t
- N(t) is the **noise** component at the time step t

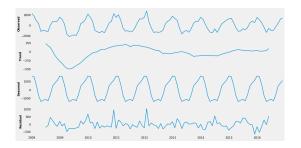


Figure 2.32: Series Additive decompositition of monthly avarage dataset.

Figure 2.32 actually shows that there is a strong seasonal component of length equal to 7 time steps. The idea now, as already mentioned, is to identify the regular (d) and seasonal (D) integration parameters and the seasonal one (s), then extrapolate the others through grid search. We already have information on the parameter d thanks to the previous analysis, to try to identify D we proceed by carrying out a hypothesis test AD Fuller on the seasonally differentiated series to defend seasonal lag values.

The P values are shown in figure 2.33, which shows how according to this test the best value for parameter s should be 6. It is important to underline that this hypothesis was subsequently confirmed by carrying out a bit of grid search also on this parameter, in fact the configurations with the lowest AIC are those that have the parameter s equal to 6. Going over again the process applied for the ARMA model, the series seems

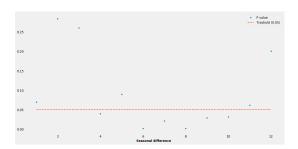


Figure 2.33: Scatter plot for P Values releted to AD-Fuller test for stationarity of differentiated series. On X axis the differencing order taken for the time series.

to assume a stationary behavior by applying a second order differentiation to the seasonally differentiated series at the first order, in short, the best configuration for the integration parameters appears to be d=2 and D=1.

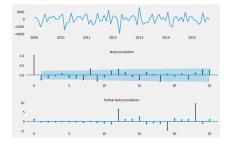


Figure 2.34: Autocorrelation and Partial Autocorrelation plots for the differentiated series of order d = 2 and D = 1.

Recalling what has been said for the ARIMA model, let's proceed by testing the behavior of both cases. The grid search on the parameters p, q, P, Q showed that the models which are in best agreement with the time series are SARIMA(3, 2, 1)x(1, 1, 2, 6) and SARIMA(3, 0, 1)x(1, 1, 2, 6). The following table shows the values of the

parameters relating to both models. Once again the integrated model appears to be better according to the information criteria and diagnostic confirms the agreement of the model to the time series.

Coeff	d=0	d=2
ar.L1	-0.7437	-0.7343
ar.L2	0.6337	-0.3098
ar.L3	. 0.3774	0.0616
ma.L1	1	0.9992
ar.S.L1	-1	-0.9992
ma.S.L1	-0.1438	-0.2194
ma.S.L2	-0.8533	-0.7738
AIC	1293.954	1269.353
BIC	1313.589	1288.799

Once again the integrated model appears to be better according to the information criteria and diagnostic confirms the agreement of the model to the time series.

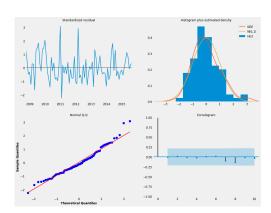


Figure 2.35: Diagnostic plot for SARIMA(3, 2, 1)x(1, 1, 2, 6) model.

At this point it remains only to check the behavior on the test set of these models. Finally, both models seem to learn better how to model the frequency that characterizes the time series, despite the fact that

the error is higher than that estimated by not considering seasonality; in fact RMSE for the integrated model is equal to 498.38 which is approximately 18% of the maximum, while the other one is 332.94, approximately 12% of the maximum.

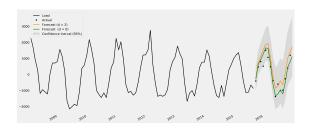


Figure 2.36: Forcasting of the monthly avarage of Polish Electric Load for 2016. Two different models were used: SARIMA(3, 2, 1)x(1, 1, 2, 6)[Orange] and SARIMA(3, 0, 1)x(1, 1, 2, 6)[Green].

### 2.6 Deconvolution

The last method that was used in the time series analysis was a deconvolution. More in detail, the idea was to hypothesize that the signal was the result of a convolution between a series of pulses and a kernel function that modeled its shape.

$$TS = d \circledast K$$

where TS represents the time series, d the pulse signal, which for convenience will be called Delta signal and K the kernel function

The main purpose of this approach is to obtain a model capable of making predictions, to achieve this, it is necessary to know the delta signal and have an estimate of the Kernel function form.

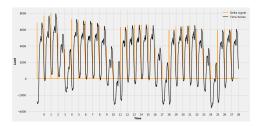


Figure 2.37: Lineplot of the first 30 days of of Polish Electric Load[Black], Representation of the pulse signal through a series of deltas centered at the starting point of each day and with an amplitude equal to the maximum relative to that day[Orange].

For simplicity, it has been assumed that the pulse signal was composed of a series of delta functions linked together with an amplitude equal to the maximum value of the time series during the day to which they refer; as regards the kernel function, it was obtained by considering an average of the time series at a daily level, in order to capture the behavior of the "average day". In fact, the figure shows how generally weekdays have a very similar trend, while holidays seem to have their own. This way we get 3 different kernels, one for weekdays, one for Saturday and one for Sunday.

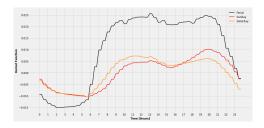


Figure 2.38: Kernel functions used for convolution.

stake, the model was built in the following way: The Delta signal, which represents the average year, was obtained by taking the average over all years of the maximums relating to each specific day, i.e. the amplitude of the delta relating to the first Monday of January is the average of the maximums of the first Monday of January from 2008 to 2015, the amplitude of the first Tuesday is the average of the highs of all the first Tuesdays, etc. The convolution of this delta signal with a kernel function that discriminates the weekdays from Saturday and Sunday, allows to obtain a representation of a "typical" year of the time series, thus obtaining the realization of an "average" full year. the convolution was obtained by exploiting the fourer space in which, as we know, the covolution becomes a product between the delta function and the kernel function:

$$F[TS] = F[d \circledast K] = F[d] \cdot F[K]$$

In particular we compared the model with the year 2016, which is used as a test set to see if it could actually be compared with the average behavior of the previous 8 years.

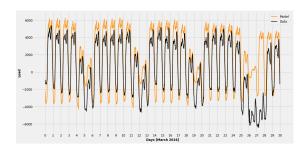


Figure 2.39: Comparison between model and time series for March 2016

Obviously this model, for how it was built Once defined who are the elements at is an approximation of the average behavior of the Polish electric load, figure 2.39 shows its limits; It can be seen clearly in the figure that the model manages to capture the behavior of the data, but it remains quite inaccurate. Quantitatively, The correlation between the model residuals and the time series is 0.55 and the RMSE for the data showed in 2.39 is approximately 2150 which is the 31.39% of the maximum.

## Chapter 3

## Results and Conclusion

A wide analysis has been made about the load of the Polish Power Systems using different techniques. The information that has been extracted by this analysis has been used to gain some forecasting abilities about the dataset that has been considered.

As it was predictable, the time series presents a recurrent periodical behavior. This periodicity of the data permitted to use the **Fourier Transform method**. In order to make the Fourier Analysis more robust and clean, the signal has been preprocessed in the following way:

- **De-Trend**: A linear fit has been subtracted from the signal in order to make it "more stationary".
- **De-Mean**: The mean of the signal has been subtracted from the original one in order not to disturb the frequency spectrum (F(w=0) = Mean = 0)
- Smoothing: In order to make the endpoints of the signal meet and make the periodic assumption more valid, the hanning window has been applied to the signal

After this preprocessing part, the signal has been analyzed in the Fourier spectrum,

thus retrieving life ticked frequencies like the 12h, 24h and 168h ones. The Fourier spectrum has been filtered by setting the frequencies with amplitude under a certain threshold to 0. This threshold has been chosen to vary as a certain fraction of the maximum amplitude of the original spectrum. In particular, a reconstruction algorithm that considered the correlation between the original signal and the reconstructed one permitted to obtain an optimal reconstruction by using the threshold  $k_{out} = 0.004 \cdot Max$ . The same reconstruction algorithm has been applied to the first 7 years of the dataset (training set) in order to obtain the optimal frequency spectrum. This spectrum has been used to predict the last year (test set) signal obtaining an RMSE up to the 33% of the maximum value.

A refinement of this algorithm [4] has been used to consider the non-stationary nature of the signal. This new approach considered a tripartite division (training/validation/test) and computed annual Fourier transforms, thus obtaining an RMSE up to the 21% of the original maximum.

A more robust cleaning method on the signal that has been applied is the **Wavelet** 

one. Under the safe assumption that the "real" signal is band-limited and the highest frequencies of the signal could be related to the noise, an algorithm has been developed in order to reconstruct a signal that could be as much as possible entirely and exclusively not influenced by the noise [3]. This algorithm permitted to have a signal that has an error almost uncorrelated with the original signal: C = 0.2%.

Furthermore, the dataset has been studied using a class of models widely used to model time series, the ARMA models. This analysis showed how these models are able to adapt to the low frequencies of the dataset and manage to capture their behavior in a convincing way. In particular, different models were built and compared, starting from simpler models up to more sophisticated ones. It is interesting to note that in terms of RMSE the one that seems to perform better is the ARIMA(6,2,6) which does not take into account seasonality directly, but the information of the correlation between the data is mainly entrusted to the autoregres-However, the two SARIMA sive terms. models considered seem to be able to better capture the frequency-level behavior of the dataset by not forcibly chasing those data that could be associated with fluctuations. Probably a more precise tuning of the parameters could highlight other aspects, however this analysis has shown how also relatively simple models can be extremely powerful in this field. The last analysis performed on the time series was deconvolution, assuming that the series could be represented by the convolution of a Kernel function with a pulse function on a daily scale. The aim was to be able to create an

impulse signal that represented the average annual behavior of the series, in order to use it to make predictions. Unfortunately this method did not prove to be particularly efficient, although it is possible to obtain some information; the match between the data and the model is not satisfactory for several reasons, However, we must think that there have been different degrees of approximation and the impulse signal has been constructed on the basis of a priori assumptions that do not necessarily reflect the nature of the process. Anyway looking closely at the comparison between the data and the model, it would seem that one of the problems is related to the kernel function, in fact it is possible that the average behavior of weekdays and holidays is not a good approximation of the signal behavior at the daily level because this changes consistently for each specific day. The following table shows the RMSE values related to the various methods used for forcasting the series

Method	RMSE
Fourier	
Wavelet	
ARMA(6,8)	. 345.64
ARIMA(6,2,6)	270.22
SARIMA(3, 2, 1)x(1, 1, 2, 6)	498.38
SARIMA(3, 0, 1)x(1, 1, 2, 6)	332.94
Deconvolution	2150

## Chapter 4

## Appendix

In this section, the codes that permitted to obtain the ouput shown in the report have been reported. The codes are collected in their relative GitHub page. The description of the notebook is the following:

- 1. "datapreprocessing.ipynb" In this notebook data has been preprocessed: the mean value of the data and a linear fit has been subtracted by the original data. Moreover an hanning window has been applied.
- 2. "fouriermethod.ipynb" Time independent Fourier Analysis has been performed. By the usage of an opportune filter, a forecast has been 5 made.
- 3. "fouriermethod2.ipynb" been performed. By the usage of an opportune filter, a forecast has been import astropy made.
- 4. "waveletfiltering.ipynb" A wavelet filter has been adopted to clean the signal.
- 5. "SARIMA.ipynb" Sarima and Arima processes have been curve\_fit

made in order to perform a forecast on the dataset that has been cleaned by the wavelet filtering.

6. "Deconvolution.ipynb" A deconvolution algorithm has been applied and a forecast has been made on the last year available (2016)

#### 4.1 datapreprocessing.py

```
#!/usr/bin/env python
                               2 # coding: utf-8
                                # # Visualization Notebook: Pre-
                                    processing
                               6 # In[4]:
Time dependent Fourier Analysis has #Importing the libraries to watch
                                   the 'fits' image and get the
                                    data array
                              import plotly.graph_objects as go
                              12 from astropy.io import fits
                              13 #Importing a library that is useful
                                     to read the original file
                               15 import pylab as plb
                              16 import matplotlib.pyplot as plt
                              17 from scipy.optimize import
```

```
cmap_big(np.linspace(0.7, 0.95,
18 from scipy import asarray as ar, exp
19 #Importing a visual library with
                                            256)))
     some illustrative set up
                                      49 bbox_props = dict(boxstyle="round,
                                           pad=0.3", fc=colors[0], alpha
20 import matplotlib.pyplot as plt
21 import matplotlib.colors as mcolors
22 from matplotlib import cm
23 import numpy as np
                                      51
                                      52 # In[8]:
24 import math
25 import seaborn as sns
plt.style.use('fivethirtyeight')
                                      54
plt.rcParams['font.family'] = 'sans55 data=pd.read_csv('data.csv',sep=';'
     -serif'
28 plt.rcParams['font.serif'] = '
     Ubuntu'
29 plt.rcParams['font.monospace'] = '58 # In[10]:
     Ubuntu Mono'
30 plt.rcParams['font.size'] = 14
                                     60
g1 plt.rcParams['axes.labelsize'] = 12g1 data=data.rename(columns={'Data':'
32 plt.rcParams['axes.labelweight'] =
                                           Day', 'Godzina': 'hour', 'Minuty':
     'bold'
                                            'minutes', 'Wolumen': 'Load'})
33 plt.rcParams['axes.titlesize'] = 1262
34 plt.rcParams['xtick.labelsize'] = 63
                                      64 # In[11]:
plt.rcParams['ytick.labelsize'] =
                                      65
     12
  plt.rcParams['legend.fontsize'] =
                                     67 #Building a continous time array
                                      data['seconds']=np.arange(0,len(
37 plt.rcParams['figure.titlesize'] =
                                           data) *900,900)
     12
38 plt.rcParams['image.cmap'] = 'jet' 70
39 plt.rcParams['image.interpolation']71 # In[15]:
      = 'none'
                                      72
40 plt.rcParams['figure.figsize'] =
                                      73
     (16, 8)
                                      74 plt.plot(data.seconds, data.Load,','
41 plt.rcParams['lines.linewidth'] = 2 ,color='k')
42 plt.rcParams['lines.markersize'] = 75 plt.grid(True)
                                      76 plt.xlabel('Time (s)',fontsize=20)
43 plt.rcParams["axes.grid"] = False 77 plt.ylabel('Load (MW)',fontsize=20)
                                      78 plt.title('Load vs Time scatterplot
45 colors = ['xkcd:pale orange', 'xkcd
                                            ',fontsize=20)
     :sea blue', 'xkcd:pale red', '
     xkcd:sage green', 'xkcd:terra
     cotta', 'xkcd:dull purple', '
                                      81 # In[16]:
     xkcd:teal', 'xkcd: goldenrod',
     'xkcd:cadet blue',
46 'xkcd:scarlet']
                                      84 from scipy import signal
cmap_big = cm.get_cmap('Spectral', 85
     512)
48 cmap = mcolors.ListedColormap(
                                   87 # In[17]:
```

```
1 #!/usr/bin/env python
88
                                       2 # coding: utf-8
89
  #Detrend the signal of its constant 3
       and linear trend
                                       4 # # Time Independent Fourier
                                            Transform
91
92
93 # In[18]:
                                       6 # In[2]:
94
95
  detrended_sig=signal.detrend(data.
                                       9 #Importing the libraries to watch
                                            the 'fits' image and get the
     Load)
                                            data array
97
                                       10 import astropy
98
99 # In [29]:
                                       import plotly.graph_objects as go
                                       12 from astropy.io import fits
                                       13 #Importing a library that is useful
102 plt.subplot(1,2,1)
                                             to read the original file
103 plt.title('Detrended Signal',color=14 import pandas as pd
      'red', fontsize=20)
                                      15 import pylab as plb
plt.plot(data.seconds,detrended_sig16 import matplotlib.pyplot as plt
      ,',',color='red',label='
                                      17 from scipy.stats import chisquare
      Detrended')
plt.xlabel('Time (s)')
                                      19 from scipy.optimize import
plt.ylabel('Load (MW)')
                                            curve_fit
107 plt.subplot(1,2,2)
                                      20 from scipy import asarray as ar, exp
108 plt.title('Original Signal',color='21 #Importing a visual library with
     k', fontsize=20)
                                            some illustrative set up
plt.plot(data.seconds,data.Load,','22 import matplotlib.pyplot as plt
      ,color='black',label='Original' 23 import matplotlib.colors as mcolors
                                      24 from matplotlib import cm
plt.xlabel('Time (s)')
                                      25 import numpy as np
plt.ylabel('Load (MW)')
                                      26 import math
                                      27 from sklearn.metrics import
112
                                            mean_squared_error
114 # In [32]:
                                       28 import seaborn as sns
                                       29 from scipy import signal
                                       30 plt.style.use('fivethirtyeight')
117 #Multiplying the signal by an
                                       31 plt.rcParams['font.family'] = 'sans
     hanning window
                                            -serif'
                                       plt.rcParams['font.serif'] = '
plt.plot(detrended_sig*np.hanning(
                                           Ubuntu'
      len(data)),',',color='k')
                                       plt.rcParams['font.monospace'] = '
120 plt.grid(True)
                                           Ubuntu Mono'
plt.xlabel('Time($s$)')
                                       34 plt.rcParams['font.size'] = 14
plt.ylabel('Load(MW)')
                                       plt.rcParams['axes.labelsize'] = 12
                                       36 plt.rcParams['axes.labelweight'] =
                                            'bold'
         fouriermtehod.ipynb<sub>37 plt.rcParams['axes.titlesize'] = 12</sub>
                                       38 plt.rcParams['xtick.labelsize'] =
```

```
39 plt.rcParams['ytick.labelsize'] =
                                      66 # In [4]:
                                       67
40 plt.rcParams['legend.fontsize'] =
                                      68
     12
                                      69 data=data.rename(columns={'Data':'
                                            Day','Godzina':'hour','Minuty':
41 plt.rcParams['figure.titlesize'] =
                                            'minutes','Wolumen':'Load'})
     12
42 plt.rcParams['image.cmap'] = 'jet' 70
43 plt.rcParams['image.interpolation']71
      = 'none'
                                      72 # In [5]:
plt.rcParams['figure.figsize']
                                      73
     (16, 8)
                                      74
45 plt.rcParams['lines.linewidth'] = 275 #Building a continous time array
46 plt.rcParams['lines.markersize'] = 76 data['seconds']=np.arange(0,len(
                                            data) *900,900)
47 plt.rcParams["axes.grid"] = False
49 colors = ['xkcd:pale orange', 'xkcd79 # In[7]:
     :sea blue', 'xkcd:pale red', '
     xkcd:sage green', 'xkcd:terra
                                      81
     cotta', 'xkcd:dull purple', '
                                      82 detrended_sig=signal.detrend(data.
     xkcd:teal', 'xkcd: goldenrod',
                                            Load)
     'xkcd:cadet blue',
                                       83 sig=detrended_sig*np.hanning(len()
'xkcd:scarlet']
                                            detrended_sig))
51 cmap_big = cm.get_cmap('Spectral', 84
     512)
52 cmap = mcolors.ListedColormap(
                                       86 # ## Uniform varying threshold
     cmap_big(np.linspace(0.7, 0.95, 87
      256)))
                                       88 # In[8]:
bbox_props = dict(boxstyle="round, 89
     pad=0.3", fc=colors[0], alpha
     =.5)
                                       91 #Computing the FFT
                                      92 FFT=np.fft.fft(sig)
54
                                       93
56 # In[3]:
                                      94
                                      95 # Tn[9]:
57
58
  data=pd.read_csv('data.csv',sep=';'97
                                       98 #Constructing the period x axis
60 data=data.rename(columns={'Data':'
                                            with the hours
     Day','Godzina':'hour','Minuty': 99 new_N=int(len(FFT)/2)
     'minutes','Wolumen':'Load'})
                                      100 f_nat=1/900
61 data['seconds']=np.arange(0,len(
                                      new_X = np.linspace(10**-12, f_nat
     data) *900,900)
                                            /2, new_N, endpoint=True)
62 detrended_sig=signal.detrend(data.102 new_Xph=1.0/(new_X*60*60)
63 sig=detrended_sig*np.hanning(len( 104
                                      105 # In[10]:
     detrended_sig))
```

```
148 #Computing the RMSE for each
108 FFT_abs=np.abs(FFT)
                                              reconstruction
plt.plot(new_Xph,2*FFT_abs[0:int( 149 K=np.arange(0,2475+75,75)
                                        150 RMSE = []
      len(FFT)/2.)]/len(new_Xph),
      color='red')
                                        151 for k in K:
plt.xlabel('Period ($h$)',fontsize152
                                               rec_four=fft_filter(k)
      =20)
                                               rec=np.fft.ifft(rec_four)
                                        153
plt.ylabel('Amplitude', fontsize=20)54
                                               RMSE.append(np.sqrt(
plt.title('(Fast) Fourier Transform
                                              mean_squared_error(rec.real, sig
       Method Algorithm', fontsize=20)
                                              )))
plt.grid(True)
114
  plt.xlim(0,200)
                                        157 # In [14]:
115
                                        158
117 # In [11]:
                                        160 plt.plot(K,RMSE,color='k')
118
                                        plt.xlabel('Threshold')
119
120 #Minimal difference has been shown 162 plt.ylabel('RMSE')
      in the reconstruction
                                        163 plt.grid(True)
121 plt.subplot(1,2,1)
                                        164 plt.xticks(K,rotation=45)
plt.plot(data.seconds,np.fft.ifft(165
      FFT),',',color='red')
123 plt.grid(True)
                                        167 # In [15]:
plt.xlabel('Time ($s$)')
                                        168
plt.ylabel('Reconstructed Signal') 169
126 plt.subplot(1,2,2)
                                        170 #Showing the plots at different
plt.plot(data.seconds, sig,',', color
                                              thresholds values
      = 'k')
                                        171 #Defining the amplitude filtering
128 plt.grid(True)
                                              function
plt.xlabel('Time ($s$)')
                                        172 def fft_filter_amp(th):
plt.ylabel('Original Signal')
                                               fft_tof=FFT.copy()
                                        173
                                               fft_tof_abs=np.abs(fft_tof)
131
                                        174
                                               fft_tof_abs=2*fft_tof_abs/len(
132
                                        175
133 # In [12]:
                                              new_Xph)
                                               fft_tof_abs[fft_tof_abs<=th]=0
134
                                               return fft_tof_abs[0:int(len())]
                                        177
136 #Defining the filtering function
                                              fft_tof_abs)/2.)]
  def fft_filter(th):
       fft_tof=FFT.copy()
                                        179
138
                                        180 # In[16]:
       fft_tof_abs=np.abs(fft_tof)
139
       fft_tof_abs=2*fft_tof_abs/len(181
140
      new_Xph)
       fft_tof[fft_tof_abs <= th] = 0
                                        183 K_plot = [10,200,700,1500]
141
       return fft_tof
                                        184 j=0
142
                                        185 for k in K_plot:
143
                                        186
                                               j = j + 1
144
145 # In [13]:
                                               plt.subplot(2,2,j)
                                        187
                                               plt.title('k=%i',%(k))
146
                                        188
                                               plt.xlim(0,200)
```

```
plt.plot(new_Xph,2*FFT_abs[0: 229
190
      int(len(FFT)/2.)]/len(new_Xph),230 # In[19]:
      color='navy',alpha=0.5,label=' 231
      Original')
       plt.grid(True)
                                        233 #Performing the same RMSE process
191
       plt.plot(new_Xph,fft_filter_amp
                                               as before, but relating to the
      (k), 'red', label = 'Filtered')
                                               maximum value
       plt.xlabel('Time($h$)')
                                        234 #Computing the RMSE for each
193
                                               reconstruction
       plt.ylabel('Amplitude')
194
       plt.legend()
                                        235 K=np.arange(0.0,0.31,0.01)
  plt.subplots_adjust(hspace=0.5)
                                        236 RMSE = []
                                        237 for k in K:
197
                                                rec_four=fft_filter(k)
                                        238
198
                                                rec=np.fft.ifft(rec_four)
  # ## Maximum related threshold
199
                                                RMSE.append(np.sqrt(
201 # In [17]:
                                               mean_squared_error(rec.real, sig
                                               )))
202
                                        241
204 #Maximum relate filter function
205 def fft_filter(perc):
                                        243 # In[20]:
       th=perc*(2*FFT_abs[0:int(len(
206
      FFT)/2.)]/len(new_Xph)).max()
       fft_tof=FFT.copy()
                                        246 plt.plot(K,RMSE,'.',color='k')
207
       fft_tof_abs=np.abs(fft_tof)
                                        247 plt.grid(True)
208
       fft_tof_abs=2*fft_tof_abs/len(248 plt.xlabel('Threshold as percentage
209
                                                of the maximum value ',
      new_Xph)
       fft_tof[fft_tof_abs<=th]=0
                                               fontsize=20)
       return fft_tof
                                        249 plt.xticks(K,rotation=90)
211
                                        plt.ylabel('RMSE', fontsize=20,
212
                                               rotation=90)
213
214 # In[18]:
215
                                        252
                                        253 # In [21]:
216
217 #Showing some plots at different
      threshold values
                                        255
218 K_plot_values = [0.0,0.30,0.60,0.95] 256 #Computing the correlation between
219 j = 0
                                               the original signal and its
220 for k in K_plot_values:
                                               error
                                        257 CORR = []
       j += 1
221
                                        258 for k in K:
       plt.subplot(4,1,j)
222
                                                rec_four=fft_filter(k)
       plt.plot(data.seconds, np.fft.
                                        259
      ifft(fft_filter(k)),color=
                                                rec=np.fft.ifft(rec_four)
                                        260
      colors[j])
                                        261
                                                error=rec.real-sig
       plt.title('k=\%.2f of the
                                                CORR.append(np.abs(np.corrcoef(
224
                                        262
      maximum' %(k))
                                               sig, error) [0] [1]))
       plt.xlabel('Time ($s$)')
                                        263
225
       plt.ylabel('Load')
                                        264
                                        265 # In[22]:
plt.subplots_adjust(hspace=0.8)
```

```
267
                                       304 plt.ylabel('Reconstructed Signal')
268 #Plotting the correlation between
                                       305 plt.subplot(2,2,2)
      the signal and its error
                                       plt.plot(data.seconds, sig,',',color
plt.plot(K,CORR,'.',color='k')
                                             = 'k')
270 plt.grid(True)
                                       307 plt.grid(True)
271 plt.xlabel('Threshold as percentageos plt.xlabel('Time($s$)')
       of the maximum value ',
                                       309 plt.ylabel('Original Signal')
                                       310 plt.subplot (2,2,4)
      fontsize=20)
272 plt.xticks(K,rotation=90)
                                       311 plt.xlim(0,200)
  plt.ylabel('Correlation Coefficients2 plt.plot(new_Xph,2*FFT_abs[0:int(
      ',fontsize=20,rotation=90)
                                             len(FFT)/2.)]/len(new_Xph),
                                             color='k',alpha=1.0,label='
274
                                             Original')
275
276 # In [23]:
                                       313 plt.legend()
                                       315 plt.grid(True)
278
279 #Reducing the range and selecting
                                       316 plt.xlabel('Time($h$)')
      the best K value
                                       317 plt.ylabel('Original Fourier
280 K=np.arange(0.001,0.015,0.001)
                                             Transform amplitude')
                                       318 plt.subplot(2,2,3)
281 CORR = []
282 for k in K:
                                       319 plt.xlim(0,200)
       rec_four=fft_filter(k)
                                       320 plt.plot(new_Xph,2*np.abs(
283
       rec=np.fft.ifft(rec_four)
                                             fft_filter(opt_perc))[0:int(len
       error=rec.real-sig
                                             (FFT)/2.)]/len(new_Xph),color='
285
       CORR.append(np.abs(np.corrcoef(
                                             red', alpha=1.0, label='
286
      sig, error) [0] [1]))
                                             Reconstructed')
plt.plot(K,CORR,'.',color='k')
                                       321 plt.grid(True)
288 plt.plot(K, np.zeros(len(K))+0.10,
                                       plt.xlabel('Time($h$)')
                                       323 plt.ylabel('Reconstructed Fourier
      color='red')
289 plt.grid(True)
                                             Transform amplitude ')
plt.xlabel('Threshold as percentage24 plt.legend()
       of the maximum value ',
                                       325
      fontsize=20)
plt.xticks(K,rotation=90)
                                       327 # In [25]:
plt.ylabel('Correlation Coefficient28
      ',fontsize=20,rotation=90)
                                       330 #Distribution of the error
293
                                       331 sns.distplot(sig-np.fft.ifft(
295 # In [24]:
                                             fft_filter(opt_perc)).real,
                                             color='firebrick')
296
                                       332 plt.grid(True)
298 #Displaying the optimum values
                                       333 plt.xlabel('Error distribution
299 opt_perc=0.004
                                             values',fontsize=20)
300 plt.subplot(2,2,1)
                                       334 plt.ylabel('Distribution', fontsize
plt.plot(data.seconds,np.fft.ifft(
                                             =20)
      fft_filter(opt_perc).real),',',335
      color='red')
302 plt.grid(True)
                                       337 # In [26]:
plt.xlabel('Time($s$)')
```

```
Distribution')
340 y_1=np.histogram(sig-np.fft.ifft( 374 ax_11.grid(True)
      fft_filter(opt_perc)).real,500)375 ax_11.set_xlabel('Distribution)
                                             Values')
341 x_1=np.histogram(sig-np.fft.ifft( 376 ax_11.set_ylabel('Distribution')
      fft_filter(opt_perc)).real,500)377
      [1][0:len(y_1)]
                                      378 #plt.y_1label('Residuals')
                                      379 ax_11.plot(x_1,gaus(x_1,*popt),'red
342
                                             ',label='fit')
343
  # In[27]:
                                      380 plt.grid(True)
344
                                      381 ax_11.legend()
                                      _{382} res = y_1 - gaus(x_1,*popt)
346
                                      383 ax_12.plot(x_1,res,color=')
347
348 def gaus(x,a,x0,sigma):
                                             darkorange',label='Residuals')
           return a*np.exp(-(x-x0))
                                      384 ax_11.set_xlabel('Distribution
      **2/(2*sigma**2))
                                             Values')
                                      ax_11.set_ylabel('Residuals')
350
                                      386 ax_12.legend()
352 # In [28]:
                                      387 plt.show()
                                      388
353
354
                                      389
<500))]
                                      391
y_1=y_1[np.where((x_1>-500) & (x_1)^2 # # Train Test Split)
      <500))]
  x_1=np.linspace(-500,500,len(y_1))_{394} # In[30]:
                                      395
358
359
                                      396
360 # In [29]:
                                      397 #Selecting the first 80% of the
                                             signal
                                      398 sig=data.Load
363 #Plot of the values
                                      399 sig=signal.detrend(sig)
364 val_medio=0
                                      400 sig=sig[0:365*96*8].copy()
_{365} n = len(x_1)
                                      401 FFT=np.fft.fft(sig)
      #the number of data
                                      402 FFT_abs=np.abs(FFT)
mean = sum(x_1*y_1)/n
                                      403
           #note this correction
                                      404
sigma = sum(y_1*(x_1-val_medio)**2)_{05} # In[31]:
     /n
                #note this correction406
p0 = [max(y_1), val_medio, 10]
  popt,pcov = curve_fit(gaus,x_1,y_1408 #Maximum relate filter function
                                      409 def fft_filter(perc):
      p0=p0)
370 \text{ fig, } (ax_11, ax_12) = plt.subplots_{410}
                                             sig=data.Load
      (2, 1)
                                              sig=signal.detrend(sig)
                                      411
plt.suptitle('Fit and rediduals',
                                              sig=sig[0:365*96*8].copy()
      fontsize=20)
                                              FFT=np.fft.fft(sig)
                                      413
#ax_11.set_y_1_label('Intensity_1 414
                                              FFT_abs=np.abs(FFT)
                                             th=perc*(2*FFT_abs[0:int(len(
      ADU]')
                                      415
ax_11.plot(x_1,y_1,'navy',label='
                                             FFT)/2.]/len(new_Xph)).max()
```

```
fft_tof=FFT.copy()
                                        plt.plot(K, np.zeros(len(K)) + 0.10,
416
       fft_tof_abs=np.abs(fft_tof)
                                              color='red')
417
       fft_tof_abs=2*fft_tof_abs/len(454 plt.xticks(K,rotation=90)
      new_Xph)
                                        455 plt.ylabel('Correlation', fontsize
       fft_tof[fft_tof_abs <= th] = 0
                                              =20, rotation =90)
419
       return fft_tof
                                        456
420
421
                                        457
                                        458 # In[34]:
422
423 # In[32]:
                                        459
424
                                        461 opt_perc=0.004
426 #Performing the same RMSE process
                                        462 fft_filter(opt_perc)
      as before, but relating to the 463 plt.xlim(0,200)
      maximum value
                                        464 plt.ylim(0,2500)
427 #Computing the RMSE for each
                                        465 plt.plot(new_Xph,np.abs(fft_filter(
      reconstruction
                                               opt_perc))[0:len(new_Xph)]/len(
428 K=np.arange(0.001,0.011,0.001)
                                              new_Xph),color='k',alpha=1.,
429 RMSE = []
                                              label='Original')
430 CORR = []
                                        466 plt.xlabel('Time $h$')
  for k in K:
                                        467 plt.ylabel('Amplitude')
431
       rec_four=fft_filter(k)
                                        468 plt.grid(True)
432
       rec=np.fft.ifft(rec_four)
                                        469 plt.title('Fourier Spectrum',
433
       RMSE.append(np.sqrt(
                                              fontsize=20)
434
      mean_squared_error(rec.real, sig470
      )))
       rec=np.fft.ifft(rec_four)
                                        472 # In [203]:
       error=rec.real-sig
436
       CORR.append(np.abs(np.corrcoef(74
437
                                        475 #Prediction:
      sig, error) [0] [1]))
                                        476 #Using the Fourier formula
438
                                        477 fourier=fft_filter(opt_perc)
440 # In[33]:
                                        478 space=np.array(data.seconds.tolist
                                               ())
441
                                        479 f_nat=1/(space[1]-space[0])
443 plt.subplot(1,2,1)
                                        480 P=np.pi*2
plt.plot(K,RMSE,'.',color='k')
                                        481 #N=int(len(space)/2)
445 plt.grid(True)
                                        482 freq=np.linspace(0,f_nat/2,len(
446 plt.xlabel('Threshold as percentage
                                              fourier))*2*np.pi
       of the maximum value ',
                                        483 #fourier=f_try[1]
      fontsize=20)
                                        484 J_LIST=np.where(fourier!=0)[0]
447 plt.xticks(K,rotation=90)
                                        485 real=fourier.real
  plt.ylabel('RMSE', fontsize=20,
                                        486 imag=fourier.imag
      rotation=90)
                                        T=np.linspace(0,f_nat/2,len(freq),
449 plt.subplot(1,2,2)
                                              endpoint=True)
plt.plot(K,CORR,'.',color='k')
                                        488 I=np.arange(1,101,1)
                                        489 x_t=np.zeros(len(space))
451 plt.grid(True)
plt.xlabel('Threshold as percentage90 q=0
                                        491 SEEN = []
       of the maximum value ',
      fontsize=20)
                                        492 for j in J_LIST:
```

```
493
       q = q + 1
       x_t=x_t+2/(len(space))*(np.abs(26))
494
      fourier[j].real)*np.cos(freq[j 527 # In[307]:
      ]*2*space)-fourier[j].imag*np. 528
      sin(2*freq[j]*space))
495 #print(i,len(space))
                                       530 RMSE=mean_squared_error(sig
       if int(100*q/len(J_LIST)) in I
                                             [365*8*96:len(data)],np.array(
      and int(100*q/len(J_LIST)) not
                                             x_t[365*8*96:len(data)]).real)
      in SEEN:
           print('%i'%(int(100*q/len(532
497
      J_LIST)))+ ' % of the
                                       533
                                         # In[309]:
      frequencies reconstructed')
           SEEN.append(int(100*q/len(535
498
      J LIST)))
                                         print('RMSE is '+ str(RMSE))
       #X_T.append(x_t)
499
500
                                       539 # In [314]:
501
502 # In[200]:
                                       540
503
                                       plt.plot(data.seconds[365*8*96:len(
504
                                             data)],sig[365*8*96:len(data)],
x_t=np.array(x_t)
x_t[np.where(x_t>sig.max())]=sig.
                                             '.', color='gold', label='
      max()
                                             Prediction', markersize=5)
                                       plt.plot(data.seconds[365*8*96:len(
507
                                             data)],x_t[365*8*96:len(data)],
508
                                             '.', color='k', label='Real',
509
  # In[312]:
                                             markersize=5)
                                       544 plt.legend()
#plt.plot(np.fft.ifft(fourier))
                                       545 plt.grid(True)
513 sig=data.Load
                                       546 plt.xlabel('Time($s$)')
514 sig=signal.detrend(sig)
                                       547 plt.ylabel('Signal')
plt.plot(space,sig,color='k')
plt.plot(space[0:len(fourier)],x_t549
                                       550 # In [321]:
      [0:len(fourier)],color='
      darkorange')
                                       551
plt.plot(space[len(fourier):len(
                                       552
      data)],x_t[len(fourier):len(
                                       553 #Plotting the errors
      data)],color='gold')
                                       554 plt.plot(np.array(data.seconds)
                                             [0:365*8*96],(sig-x_t)
518 plt.grid(True)
plt.xlabel('Time($s$)')
                                             [0:365*8*96], color='k')
520 plt.ylabel('Load (MW)')
                                       plt.plot(np.array(data.seconds)
  #plt.plot(space[len(fourier)+10:len
                                             [365*8*96:len(data)],(sig-x_t)
                                             [365*8*96:len(data)],color='
      (data)],exp,color='black')
#plt.plot(data.seconds[0:365*8*96],
                                             gold')
      np.fft.ifft(fourier))
                                       556 plt.xlabel('Time($s$)')
##plt.plot(data.seconds
                                       557 plt.ylabel('Load($MW$)')
      [365*8*96::],sig[365*8*96::])
                                       558 plt.grid(True)
#plt.plot(space,x_t*max(sig)/max(
                                      559
      x_t))
                                       560
```

### 561 # In [352]: 562 564 #RMSE for each interval D = [0, 20, 30, 60, 90, 120]567 D=np.array(D)\*900 568 start=365\*900\*8 569 T\_in=(D+start)[:-1] 570 T\_fin=(D+start)[1::] D = [0, 20, 30, 60, 90, 120]572 new\_D=np.array(D) 573 new\_D=new\_D\*96 574 new\_start = 365 \* 96 \* 8 575 RMSE = [] 576 C=[] 577 for d in range(len(D)-1): x\_t\_=np.array(x\_t[(new\_start+ new\_D[d]):(new\_start+new\_D[d +1])]) 579 new\_start+new\_D[d+1])] error=x\_t\_.real-sig\_ 580 RMSE.append(np.sqrt( 581 mean\_squared\_error(sig\_,x\_t\_. real))) 582 )[0][1]) 583 585 # In [357]: 586 588 RMSE\_data=pd.DataFrame() 589 RMSE\_data['\$T\_fin\$(s)']=T\_fin 590 RMSE\_data['RMSE']=RMSE 591 RMSE\_data['\$\Delta T\$(s)']=T\_fin- $T_{in}$ 592 RMSE\_data['\$C\_{data,error}\$']=C 593 RMSE\_data['\$\Delta Days\$']=D[1::] 596 # In [360]: 597

599 RMSE\_data

#### fouriermethod2.ipynb 4.3

```
1 #!/usr/bin/env python
                                 2 # coding: utf-8
                                 4 # # Time Dependent method
                                 6 # In[2]:
                                 9 #Importing the libraries to watch
                                     the 'fits' image and get the
                                     data array
                                10 import astropy
                                import plotly.graph_objects as go
                                12 from astropy.io import fits
                                13 #Importing a library that is useful
                                      to read the original file
                                14 import pandas as pd
                                15 import pylab as plb
sig_=sig[(new_start+new_D[d]):(16 import matplotlib.pyplot as plt
                                17 from scipy.stats import chisquare
                                19 from scipy.optimize import
                                     curve_fit
                                20 from scipy import asarray as ar, exp
C.append(np.corrcoef(error, sig_{-21} #Importing a visual library with
                                     some illustrative set up
                                22 import matplotlib.pyplot as plt
                                23 import matplotlib.colors as mcolors
                                24 from matplotlib import cm
                                25 import numpy as np
                                26 import math
                                27 from sklearn.metrics import
                                     mean_squared_error
                                28 import seaborn as sns
                                29 from scipy import signal
                                plt.style.use('fivethirtyeight')
                                31 plt.rcParams['font.family'] = 'sans
                                     -serif'
                                32 plt.rcParams['font.serif'] = '
                                     Ubuntu'
                                plt.rcParams['font.monospace'] = '
                                     Ubuntu Mono'
                                34 plt.rcParams['font.size'] = 14
                                plt.rcParams['axes.labelsize'] = 12
                                36 plt.rcParams['axes.labelweight'] =
                                      'bold'
                                37 plt.rcParams['axes.titlesize'] = 12
```

```
38 plt.rcParams['xtick.labelsize'] = 64
     12
39 plt.rcParams['ytick.labelsize'] =
                                      66 # In[4]:
                                      67
40 plt.rcParams['legend.fontsize'] =
                                      68
     12
                                       69 load_no_line=signal.detrend(data.
41 plt.rcParams['figure.titlesize'] =
                                            Load,type='linear')
                                       70 clean_load=np.array(load_no_line)-
42 plt.rcParams['image.cmap'] = 'jet'
                                            np.array(load_no_line).mean()
43 plt.rcParams['image.interpolation']71 data['clean_load']=clean_load
      = 'none'
                                       72 data=data.drop(columns=['Load']).
44 plt.rcParams['figure.figsize'] =
                                            rename(columns={'clean_load':'
     (16, 8)
                                            Load'})
45 plt.rcParams['lines.linewidth'] = 273
46 plt.rcParams['lines.markersize'] = 74
                                      75 # In [5]:
47 plt.rcParams["axes.grid"] = True
                                      76
48 #plt.rcParams['']
49 colors = ['xkcd:pale orange', 'xkcd78 #Mean FFT on Training set
     :sea blue', 'xkcd:pale red', '
                                      79
     xkcd:sage green', 'xkcd:terra
                                      80 data.Day=pd.to_datetime(data.Day)
     cotta', 'xkcd:dull purple', '
                                      81 data['Year']=data.Day.dt.year #
     xkcd:teal', 'xkcd: goldenrod',
                                            Years
     'xkcd:cadet blue',
                                       82 data['Month'] = data.Day.dt.month #
'xkcd:scarlet']
                                            Months
51 cmap_big = cm.get_cmap('Spectral', 83 YEARS=np.sort(list(set(data.Year.
     512)
                                            tolist()))).tolist() #Year list
52 cmap = mcolors.ListedColormap(
                                      84 load_yf=0
     cmap_big(np.linspace(0.7, 0.95, 85 for y in range(len(YEARS)-2): #len
      256)))
                                            of the training set
bbox_props = dict(boxstyle="round, 86
                                            df_year=data[data['Year']==
     pad=0.3", fc=colors[0], alpha
                                            YEARS [y]]
                                             if y==0 or y==4: #leap years :)
     =.5)
                                                 df_year=df_year.drop(
                                            df_year[df_year.Day == (str(YEARS))
55
56 # In [3]:
                                            [y]) + '-02-29')].index)
                                             load_yf = load_yf + np.fft.fft(
57
                                            df_year.Load)
59 data=pd.read_csv('data.csv',sep=';'90 four_year=load_yf/(len(YEARS)-2)
60 data=data.rename(columns={'Data':' 92
     Day','Godzina':'hour','Minuty': 93 # In[6]:
     'minutes','Wolumen':'Load'})
                                      94
61 data['seconds']=np.arange(0,len(
     data) *900,900)
                                       96 #Fourier transform is defined on
62 detrended_sig=signal.detrend(data.
                                            the frequency axis.
                                      97 #In order to make it more visible,
sig=detrended_sig*np.hanning(len(
                                            the conversion on the period
     detrended_sig))
                                            has been applied
```

```
98 new_N=int(len(four_year)/2)
                                             .copy()
99 f_nat=1/900
                                                      t_new_abs_year[
                                      128
new_X = np.linspace(10**-12, f_nat
                                             t_new_abs_year < (t_new *
      /2, new_N, endpoint=True)
                                             t_new_abs_year.max())]=0
new_Xph=1.0/(new_X*60*60)
                                                      t_new_four_year=
102 plt.xlim(0,40)
                                             four_year.copy()
plt.plot(new_Xph,np.abs(2*four_year30
                                                      t_new_four_year[
                                             t_new_abs_year == 0] = 0
      [:int(len(four_year)/2.)]/len(
      four_year)),color='red')
                                                      E=np.fft.ifft(
plt.ylabel('Amplitude')
                                             t_new_four_year).real-df_year.
plt.xlabel('Period ($h$)')
                                             Load
106 plt.ylim(0,3000)
                                                      corr=pd.DataFrame({'A':
plt.title('Mean Fourier Spectrum',
                                             df_year.Load,'B':E}).corr().
      fontsize=20)
                                             values [0] [1]
                                                      if abs(corr) <= 0.54: #</pre>
                                             Second requirement: LOW
110 # In[7]:
                                             CORRELATION
                                                          opt_t.append(t_new)
                                      134
111
                                                          opt_corr.append(
113 #Validation
                                             corr)
114 TH=np.arange(0.01,0.5,0.01) #
                                      136 opt_corr=np.abs(opt_corr)
      threshold Values
                                      137
opt_t=[] #Threshold values that aress
       acceptable in terms of RMSE
                                      139 # In [8]:
      and Correlation values
                                      140
opt_corr=[] #Acceptable Correlatiom41
       values
                                      142 #Selecting the best reconstruction
abs_year=np.array(abs(four_year))
                                      143 best_th=opt_t[opt_corr.argmin()] #
                                             Best threshold value
  for t in TH:
      t_abs_year=abs_year.copy()
                                      abs_year=np.array(abs(four_year))
      t_abs_year[t_abs_year<(t*
                                      145 t_abs_year=abs_year.copy()
      t_abs_year.max())]=0 #Apply the146 t_abs_year[t_abs_year<(best_th*
       threshold on the absolute
                                             t_abs_year.max())]=0
                                       147 med_four=four_year.copy()
       t_four_year=four_year.copy()
                                      148 med_four[t_abs_year==0]=0 #Best
      t_four_year[t_abs_year==0]=0 #
                                             fourier transform
      Apply the threshold on Fourier 149
      transform
      RMSE=np.sqrt(mean_squared_error51 # In[9]:
123
      (np.fft.ifft(t_four_year).real, 152
      data[data.Year == 2015].Load)) # 153
                                      154 RMSEs = []
      inverse transform and compute
                                      test=data[data['Year']==2016] #Test
      RMSE
      if RMSE < 2000: #First</pre>
                                              year
124
      requirement: LOW RMSE
                                      156 test=test.drop(test[test.Day==(')
           new_TH=np.arange(t*0.1,t,t
                                            2016-02-29')].index) #Again,
      *0.1) #More specific threshold
                                             leap year
           for t_new in new_TH:
                                      157 C=[] #Correlation list
126
               t_new_abs_year=abs_years8 TIME=np.arange(0,100,10) #Timestep
```

## we want to check rmse\_four=np.fft.ifft(med\_four) # Take our prediction test=np.array(test.Load) 161 162 for t in range(len(TIME)-1): Rmse\_test=test[96\*(TIME[0]) :96\*(TIME[t+1])] #Take the load between two timestep Rmse\_four=rmse\_four[96\*(TIME [0]):(96\*(TIME[t+1]))] #Take the prediction between the same timestep RMSEs.append(np.sqrt( 165 mean\_squared\_error(Rmse\_four. real, Rmse\_test))) #Compute the C.append(np.corrcoef(Rmse\_four. $_{13}$ import pandas as pd real-Rmse\_test, Rmse\_test) [0][1]) #Compute the correlation res\_data=pd.DataFrame({'RMSE':RMSEs , 'Days':TIME[1:], '\$C\_{data, error}\$': C}) #Store them in a dataframe res\_data #Et voil 169 171 # In [11]: 174 print('Percentage of the maximum: + str(100\*np.array(RMSEs).min() import matplotlib.patches as /data.Load.max())+ '%') 175 176 177 # In [17]: 178 plt.plot(rmse\_four[0:30\*96],color= plt.plot(test[0:30\*96],color=' darkorange') plt.xlabel('Time(Day)') plt.ylabel('Load') 184 plt.xticks(np.arange(0,30\*96,96),np .arange(0,30))

#### waveletfiltering.ipynb 4.4

```
#!/usr/bin/env python
 2 # coding: utf-8
 4 # # Wavelet Filtering
 6 # In [4]:
 9 #Importing the libraries to watch
     the 'fits' image and get the
      data array
10 import astropy
11 from astropy.io import fits
12 #Importing a library that is useful
       to read the original file
14 import pylab as plb
15 import matplotlib.pyplot as plt
16 from scipy.optimize import
      curve_fit
17 import pywt
18 from scipy.stats import chisquare
19 from scipy import asarray as ar, exp
20 #Importing a visual library with
21 from sklearn.metrics import
      mean_squared_error
22 import matplotlib.pyplot as plt
23 import matplotlib.colors as mcolors
,<sup>24</sup> import random
25 from matplotlib import cm
      mpatches
28 import numpy as np
29 import math
30 import seaborn as sns
31 import datetime
plt.style.use('fivethirtyeight')
plt.rcParams['font.family'] = 'sans
     -serif'
34 plt.rcParams['font.serif'] = '
      Ubuntu'
35 plt.rcParams['font.monospace'] = '
      Ubuntu Mono'
36 plt.rcParams['font.size'] = 14
37 plt.rcParams['axes.labelsize'] = 12
38 plt.rcParams['axes.labelweight'] =
```

```
69 data=data.rename(columns={'Data':'
39 plt.rcParams['axes.titlesize'] = 12
                                            Day','Godzina':'hour','Minuty':
40 plt.rcParams['xtick.labelsize'] =
                                            'minute','Wolumen':'Load'})
41 plt.rcParams['ytick.labelsize'] =
                                       71
                                       72 # In[7]:
     12
42 plt.rcParams['legend.fontsize'] =
     12
                                       74
43 plt.rcParams['figure.titlesize'] = 75 SECONDS=np.arange(900,900*len(data)
                                            +900,900)
44 plt.rcParams['image.cmap'] = 'jet' 76
45 plt.rcParams['image.interpolation']77
      = 'none'
                                      78 # In [8]:
46 plt.rcParams['figure.figsize'] =
     (16, 8)
47 plt.rcParams['lines.linewidth'] = 2s1 data['seconds']=SECONDS
48 plt.rcParams['lines.markersize'] = 82
     8
49 plt.rcParams["axes.grid"] = False
                                       84 # In [9]:
50
                                       85
colors = ['xkcd:pale orange', 'xkcd86
     :sea blue', 'xkcd:pale red', ' 87 LOAD=data.Load
     xkcd:sage green', 'xkcd:terra 88
     cotta', 'xkcd:dull purple', '
                                      89
     xkcd:teal', 'xkcd: goldenrod', 90 # In[10]:
     'xkcd:cadet blue',
'xkcd:scarlet']
53 cmap_big = cm.get_cmap('Spectral', 93 from scipy import signal
     512)
                                       94 load_no_line=signal.detrend(LOAD,
                                            type='linear')
54 cmap = mcolors.ListedColormap(
     cmap_big(np.linspace(0.7, 0.95, 95 #load_no_constant=signal.detrend(
      256)))
                                            load_no_line,type='constant')
55 bbox_props = dict(boxstyle="round, 96
     pad=0.3", fc=colors[0], alpha
     =.5)
                                       98 # In[11]:
56
                                       99
                                      100
57
# # Pre-processing steps
                                      clean_load=np.array(load_no_line)-
                                            np.array(load_no_line).mean()
60 # In [5]:
                                      102
61
                                      103
                                      104 # In[12]:
  data=pd.read_csv('data.csv',sep='; 105
63
                                      106
                                      107 data['clean_load']=clean_load
64
                                      108
66 # In [6]:
                                      110 # In[13]:
67
```

```
([])
data=data.drop(columns=['Load']).
                                               #print(len(coeff_d))
      rename(columns={'clean_load':'
                                               COEFF_D.append(np.repeat(
                                        145
      Load'})
                                               coeff_d,2**k))
                                               DATASET.append(np.repeat(
114
                                        146
                                              dataset,2**k))
                                               k=k+1
116 # # Wavelet Transform
                                        147
                                        148 plt.tight_layout()
117
118 # In [14]:
                                        149 plt.show()
119
                                        151
121 #Performing the Wavelet transform
                                        152
      using a sym2
                                        153
122 week=data
                                        154
123 time=week.seconds.max()
                                        155 #Preparing the gaussian fit
124 sample_rate=1/900.
                                        def gaus(x,a,x0,sigma):
125 size= int(sample_rate*time)
                                                    return a*np.exp(-(x-x0)
126 t = np.linspace(0, time, num=size)
                                              **2/(2*sigma**2))
127 dataset = np.array(week.Load.tolists
      ())
128 waveletname = 'sym2'
                                        160
129 levels=8
                                        161
130 fig, axarr = plt.subplots(nrows=
                                        162 # In[16]:
      levels, ncols=2, figsize
                                        163
      =(20,10))
                                        164
131 COEFF_D = []
                                        165 #Histogram of the first coefficient
132 DATASET = []
                                        x=np.histogram(COEFF_D[0],500)
133 k = 1
                                               [1][0:500]
                                        y=np.histogram(COEFF_D[0],500)[0]
134 for ii in range(levels):
       (dataset, coeff_d) = pywt.dwt(168
      dataset, waveletname, mode = 'per'169
                                        170 # In[17]:
       axarr[ii, 0].plot(dataset, '
136
                                        171
      black')
       axarr[ii, 1].plot(coeff_d, '
                                        173 #To restrict the number of values
137
      darkorange')
                                              and fit the gaussian correctly,
       axarr[ii, 0].set_ylabel("Level
                                               this function has been used
138
      {}".format(ii + 1), fontsize
                                        174 def takeClosest(num, collection):
      =14, rotation = 90)
                                               collection=collection.tolist()
                                        175
       axarr[ii, 0].set_yticklabels
                                               collection=np.array(collection)
139
      ([])
                                               if num >= 0:
                                        177
       if ii == 0:
                                                    collection = np.abs(
140
                                        178
           axarr[ii, 0].set_title("
                                               collection[np.where(collection
141
      Approximation coefficients",
                                              >0)])
      fontsize=14)
                                                    a= min(collection, key=
           axarr[ii, 1].set_title("
                                               lambda x:abs(x-abs(num)))
142
      Detail coefficients", fontsize
                                       180
      =14)
                                                    collection=np.abs(
                                        181
       axarr[ii, 1].set_yticklabels
                                               collection[np.where(collection
```

```
<0)])
                                        220 x=sns.distplot(COEFF_D[0],color='k'
                                               ).get_lines()[0].get_data()[0][
182
           a= -min(collection, key=
                                               a:b]
      lambda x:abs(x-abs(num)))
                                        y=sns.distplot(COEFF_D[0],color='k'
       return a
                                               ).get_lines()[0].get_data()[1][
184
                                               a:b]
185
                                        222 plt.ylabel('Distribution')
186
  # In[18]:
                                        223 plt.xlabel('Distribution Values of
187
                                              the First Level')
188
                                           plt.grid(True)
190 #Gaussian Fit
                                        225
191 x_try=x
192 b=x.tolist().index(takeClosest(500327 # In[20]:
      x))
                                        228
193 a=x.tolist().index(takeClosest
      (-500,x))
                                        230 Sigma=popt[2]
194 x = x [a:b]
                                        231
195 y=y[a:b]
                                        232
                                        233 # In [36]:
                                        234
197
val_medio=x[int(len(x)/2)]
                                        235
_{199} n = len(x)
                                       #236 #Defining the reconstruction with
      the number of data
                                               the threshold
200 mean = sum(x*y)/n
                                        237 def recons_from_th_zero(threshold):
                                                lim_1=-threshold*Sigma
       #note this correction
                                        238
sigma = sum(y*(x-val_medio)**2)/n
                                                lim_2=-lim_1
                                       239
                                                coeff_0=np.array(COEFF_D[0].
            #note this correction
p0 = [max(y), val_medio, 10]
                                               copy())
popt,pcov = curve_fit(gaus,x,y,p0=241
                                                #mask = np.where((coeff_0<lim_2</pre>
                                               ) & (coeff_0>lim_1))
      p0)
204 fig, (ax1, ax2) = plt.subplots(2, 242
                                               coeff_0[np.where((coeff_0<lim_2</pre>
                                               ) & (coeff_0>lim_1))]=0
      1)
205 ax1.set_ylabel('Distribution ')
                                               level_0=coeff_0+DATASET[0]
                                        243
206 ax1.plot(x,y,'navy',label='First
                                                MAX=level_0.max()
                                        244
      Coefficient Distribution')
                                               return level_0*(data.Load.max()
                                        245
207 ax1.grid(True)
                                               /MAX)
plt.ylabel('Residuals')
                                        246
209 ax1.plot(x,gaus(x,*popt),'ro:',
                                        247
      label='Gaussian Fit')
                                        248 # In[24]:
210 ax1.legend()
                                        249
res = y - gaus(x,*popt)
                                        250
212 ax2.plot(res,color='darkorange')
                                        251 #Data test
213 ax2.grid(True)
                                        252 test=np.array(data.Load)
214 plt.show()
                                        253
215
                                        254
                                        255 # In [25]:
217 # In [19]:
                                        256
                                        257
218
                                        258 #Threshold, large range
```

```
259 TH=np.arange(0.0,13.25,0.5)
                                        302
260 C_TH=[]
                                        303 # In [101]:
  for t in TH:
                                        304
       rec=recons_from_th_zero(t)
                                        305
       C_TH.append(np.abs(np.corrcoef(06 x_hist=np.histogram(coeff_0,199)[1]
263
      rec-test, test) [0] [1]))
                                        y_hist=np.histogram(coeff_0,200)[0]
                                        308 filt_coeff_0=coeff_0[np.where((
                                               coeff_0<lim_2) & (coeff_0>lim_1
265
266 # In [29]:
267
                                        309
                                        310
269 #Threshold, narrow range
                                        311 # In [104]:
270 TH=np.arange(0.0,1.01,0.01)
                                        312
271 C_TH=[]
                                        313
272 for t in TH:
                                        314 x_left=x_hist[np.where((x_hist
273
       rec=recons_from_th_zero(t)
                                               lim_neg) & (x_hist>-500))]
       C_TH.append(np.abs(np.corrcoef( x_right=x_hist[np.where(x_hist>
274
      rec-test, test) [0] [1]))
                                               lim_pos)]
                                        316 y_left=y_hist[np.where((x_hist
275
                                               lim_neg) & (x_hist>-500))]
276
277 # In [37]:
                                        317 y_right=y_hist[np.where(x_hist>
                                               lim_pos)]
278
                                        318 x_mid=x_hist[np.where((x_hist>
279
280 #Best threshold
                                               lim_neg) & (x_hist<lim_pos))]</pre>
281 print('The Correlation has the
                                        319 y_mid=y_hist[np.where((x_hist>
      following percentage: '+str
                                               lim_neg) & (x_hist<lim_pos))]</pre>
      (100*C_TH[-1]) + '%')
                                        320
282
                                        321
283
                                        322
                                        323 # In[111]:
284 # In [53]:
285
                                        324
                                        325
287 #Mean and extreme value
                                        plt.plot(x_left,y_left,color='k')
288 mean=popt[1]
                                        plt.plot(x_right, y_right, color='k',
                                               label='Information')
289 lim_neg=mean-1*Sigma
290 lim_pos=mean+1*Sigma
                                        328 plt.plot(x_mid,y_mid,color=')
291
                                               darkorange',label='Noise')
                                        plt.plot(np.zeros(12000)+lim_neg,np
293 # In [78]:
                                               .arange(0,12000),color='red')
                                        plt.plot(np.zeros(12000)+lim_pos,np
294
                                               .arange(0,12000),color='red')
lim_1 = -1 * Sigma
                                        332 plt.grid(True)
297 lim_2=-lim_1
298 coeff_0=np.array(COEFF_D[0].copy())33 plt.xlabel('Coefficient values')
299 #mask = np.where((coeff_0<lim_2) &334 plt.ylabel('Coefficient
      (coeff_0>lim_1))
                                               Distribution')
300 #coeff_0[np.where((coeff_0<lim_2) &s plt.legend(fontsize=20)
       (coeff_0 > lim_1))] = 0
```

#### SARIMA.ipynb 4.5

```
#!/usr/bin/env python
2 # coding: utf-8
4 # # SARIMA MODELING
6 # Import dataset
8 # In[2]:
#Importing the libraries to watch
     the 'fits' image and get the
     data array
12 import astropy
import plotly.graph_objects as go
14 from astropy.io import fits
15 #Importing a library that is useful
      to read the original file
16 import pandas as pd
17 import pylab as plb
18 import matplotlib.pyplot as plt
19 from scipy.optimize import
     curve_fit
20 from scipy import asarray as ar, exp
21 #Importing a visual library with
     some illustrative set up
22 import matplotlib.pyplot as plt
23 import matplotlib.colors as mcolors
24 from matplotlib import cm
25 import numpy as np
26 import math
27 import seaborn as sns
                                      56
plt.style.use('fivethirtyeight')
plt.rcParams['font.family'] = 'sans<sup>57</sup> from statsmodels.graphics.tsaplots
     -serif'
plt.rcParams['font.serif'] = '
     Ubuntu'
plt.rcParams['font.monospace'] = , 59 from statsmodels.tsa.arima_process
     Ubuntu Mono'
plt.rcParams['font.size'] = 14
plt.rcParams['axes.labelsize'] = 12
plt.rcParams['axes.labelweight'] = ^{61} from statsmodels.tsa.statespace.
plt.rcParams['axes.titlesize'] = 12^{62} from statsmodels.tsa.stattools
36 plt.rcParams['xtick.labelsize'] =
37 plt.rcParams['ytick.labelsize'] =
```

```
38 plt.rcParams['legend.fontsize'] =
     12
39 plt.rcParams['figure.titlesize'] =
     12
40 plt.rcParams['image.cmap'] = 'jet'
41 plt.rcParams['image.interpolation']
      = 'none'
42 plt.rcParams['figure.figsize'] =
     (16, 8)
43 plt.rcParams['lines.linewidth'] = 2
44 plt.rcParams['lines.markersize'] =
45 plt.rcParams["axes.grid"] = False
47 colors = ['xkcd:pale orange', 'xkcd
     :sea blue', 'xkcd:pale red', '
     xkcd:sage green', 'xkcd:terra
     cotta', 'xkcd:dull purple', '
     xkcd:teal', 'xkcd: goldenrod',
     'xkcd:cadet blue',
48 'xkcd:scarlet']
49 cmap_big = cm.get_cmap('Spectral',
     512)
50 cmap = mcolors.ListedColormap(
     cmap_big(np.linspace(0.7, 0.95,
      256)))
bbox_props = dict(boxstyle="round,
     pad=0.3", fc=colors[0], alpha
54 # In[3]:
     import plot_pacf
58 from statsmodels.graphics.tsaplots
     import plot_acf
     import ArmaProcess
60 from statsmodels.stats.diagnostic
     import acorr_ljungbox
     sarimax import SARIMAX
     import adfuller
63 from statsmodels.tsa.stattools
     import pacf
```

```
64 from statsmodels.tsa.stattools
                                       102
      import acf
                                       103 data = pd.read_csv(dir+file,
65 from tqdm import tqdm_notebook
                                             skiprows=0, sep=';')
66 import matplotlib.pyplot as plt
                                       104 data.head()
67 import numpy as np
                                       105
68 import pandas as pd
                                       106
                                       data.columns= ['Day','hour','minute
70 from itertools import product
                                             ','Load']
                                       #plt.figure(figsize=(10, 10))
                                       #plt.plot(F606,F814,',k')
73 # In[4]:
                                       110 #plt.ylim(32,12)
74
                                       111 #plt.xlim(32,12)
                                       112 #plt.show()
76 dataset = pd.read_csv('/Users/
                                       113 #plt.close
      Simone/Desktop/University/DDA/
      Lab/Relazione_2_TimesSeries/
                                       115
      reconstruction.csv')
                                       116 # In[6]:
                                       117
  dataset = dataset.drop(columns=['
      Unnamed: 0'])
                                       119 N= 10000# data.shape[0]
                                       120 print("Using {} entries on {}
                                             available".format(N, data.shape
80 full_load = pd.Series(dataset.R)
                                             [0])
81
                                       121 load=np.zeros(N) #vogliamo mettere
82
83 # In[5]:
                                             i vettori tempo in formato np,
                                             per fare successive analisi, in
                                              formato datetime pu essere
85
86 import csv
                                             scomodo
87 import datetime as dat
                                       122 time=np.zeros(N)
88 import pandas as pd
                                       123
89 import numpy as np
                                       txt0 = "{} {:02d}:{:02d}:00".format
90 import matplotlib.pyplot as plt
                                             (data['Day'][0],data['hour'
92 dir='/Users/Simone/Desktop/
                                             [0],0)
      University/DDA/Lab/
                                       126 TO = dat.datetime.strptime(txt0, '%Y
      Relazione_2_TimesSeries/'
                                             -\%m - \%d \%H : \%M : \%S')
93 file='daneOkresoweKSE.csv'
                                       127
                                       128 dday=0
95 # Read data from file 'filename.csw29 for i in range(N):
                                              load[i]=data['Load'][i]
96 # (in the same directory that your 131
     python process is based)
                                              if data['hour'][i] == 24:
97 # Control delimiters, rows, column 133
                                                  data['hour'][i]=0
     names with read_csv (see later)134
                                                  dday=1
98 data = pd.read_csv(dir+file)
                                              txt = "{} {:02d}:{:02d}:00".
99 # Preview the first 5 lines of the 136
     loaded data
                                             format(data['Day'][i],data['
                                             hour'][i],data['minute'][i])
100 data.head()
                                              date= dat.datetime.strptime(txt
```

```
python process is based)
      ,'%Y-%m-%d %H:%M:%S')
       #Serve questo loop perch i
                                       # Control delimiters, rows, column
138
      polacchi segnano la mezzanotte
                                          names with read_csv (see later)
      in maniera differente
                                       178 data = pd.read_csv(dir+file, sep ='
       if dday==1:
                                             ; ')
           date=date+dat.timedelta(
                                       179 data=data.rename(columns={'Data':'
140
      days=1)
                                             Day','Godzina':'hour','Minuty':
       Secs=(date-T0).total_seconds()
                                             'minute','Wolumen':'Load'})
141
       time[i]=Secs
                                       SECONDS=np.arange(900,900*len(data)
142
       dday=0
                                             +900,900)
143
                                       data['seconds']=SECONDS
plt.figure(figsize=(10,10))
                                       182 LOAD=data.Load
plt.plot(time,load,'.k')
                                       183 load_no_line=signal.detrend(LOAD,
                                             type='linear')
147
plt.xlabel('Time [s]')
                                       184 clean_load=np.array(load_no_line)-
149 plt.ylabel('Load [MW]')
                                             np.array(load_no_line).mean()
plt.title('Polish Electric Load
                                       185 data['clean_load']=clean_load
      since {} {:02d}:{:02d}:00'.
                                       186 data=data.drop(columns=['Load']).
      format(data['Day'][0], data['
                                             rename(columns={'clean_load':'
      hour'][0],0))
                                             Load'})
plt.show()
                                       187
152 #
                                       188
                                       189 # In[11]:
153
                                       190
154
155 # In [8]:
                                       191
                                       _{192} s = '2008-01-01 00:15:00'
                                          end = '2017-01-01 00:00:00'
157
s = data.Day.loc[0]
                                       194
159 data.iloc [96]
                                       195
                                       196 # In[12]:
161
                                       197
162 # In [9]:
                                       198
                                       index = pd.date_range(start = s
163
                                             end = end , freq='15T')
                                       200 series = pd.Series(data.Load.tolist
165 data.tail()
                                             (), index=index)
166
                                       201 series.resample('1W').mean()
167
168 # In [10]:
                                       202
169
                                       203
                                       204 # In[13]:
171 from scipy import signal
                                       205
172 dir='/Users/Simone/Desktop/
      University/DDA/Lab/
                                       W=pd.DataFrame({'Time':series.
      Relazione_2_TimesSeries/'
                                             resample('1W').mean().index,'
173 file='daneOkresoweKSE.csv'
                                             Load':series.resample('1W').
                                             mean()})
174
175 # Read data from file 'filename.cswo8 M=pd.DataFrame({'Time':series.
                                             resample('1M').mean().index,'
# (in the same directory that your
                                             Load':series.resample('1M').
```

```
mean()})
D=pd.DataFrame({'Time':series.
                                        249 # In[20]:
      resample('1D').mean().index,'
                                        250
      Load':series.resample('1D').
                                        251
                                        252 plt.plot(M.L_diff_2,color = 'k')
      mean()})
                                        253 plt.xlabel('Year')
210
                                        254 plt.ylabel('Load')
211
                                        255 plt.title('Monthly avarage of
212 # In [14]:
                                              Polish Electric Load from 2008
213
                                              to 2017')
M = M.drop(M.index[-1])
                                        256
216
                                        258 # In [64]:
217
218 # In[15]:
                                        259
                                        _{261} M = M.fillna(0)
plt.plot(M.Load,color = 'k')
                                        262
plt.xlabel('Year')
                                        263
223 plt.ylabel('Load')
                                        264 # In [65]:
224 plt.title('Monthly avarage of
      Polish Electric Load from 2008 266
      to 2017')
                                        267 import statsmodels.api as sm
                                        268 from statsmodels.tsa.arima_model
                                              import ARIMA
227 # # Stationarity
                                        269
                                        fig = plt.figure(figsize=(12,8))
229 # Let's check stationary properties 1 ax1 = fig.add_subplot(211)
      , let's plot the series, the
                                        fig = sm.graphics.tsa.plot_acf(M.
      series differencied 1 and 2
                                              Load, ax=ax1)
      times, and test stationarity
                                        273 ax2 = fig.add_subplot(212)
      with ADF test
                                        274 fig = sm.graphics.tsa.plot_pacf(M.
                                              Load, ax=ax2)
230
231 # In [56]:
                                        275
                                        276
                                        277 # In [66]:
233
234 M['L_diff'] = M.Load.diff()
                                        278
                                        279
235
                                        280 fig = plt.figure(figsize=(12,8))
237 # In [18]:
                                        281 ax1 = fig.add_subplot(211)
                                        182 fig = sm.graphics.tsa.plot_acf(M.
238
                                              L_diff, ax=ax1)
240 plt.plot(M.L_diff)
                                        283 ax2 = fig.add_subplot(212)
241
                                        284 fig = sm.graphics.tsa.plot_pacf(M.
                                              L_diff, ax=ax2)
242
243 # In[19]:
                                        285
244
                                        287 # In [70]:
246 M['L_diff_2'] = M.L_diff.diff()
                                        288
```

```
330 for key, value in result[4].items()
290 fig = plt.figure(figsize=(12,8))
291 ax1 = fig.add_subplot(211)
fig = sm.graphics.tsa.plot_acf(M.
                                              print('\t{}: {}'.format(key,
      L_diff_2, ax=ax1, lags =25)
                                             value))
293 ax2 = fig.add_subplot(212)
fig = sm.graphics.tsa.plot_pacf(M.333
      L_diff_2, ax=ax2, lags = 25)
                                       335
295
                                       336
296
                                       337 result = adfuller(M.L_diff)
  # In [75]:
297
                                       338 print('ADF Statistic: {}'.format(
                                             result[0]))
fig = plt.figure(figsize=(12,8))
                                       339 print('p-value: {}'.format(result
301 ax11 = fig.add_subplot(321)
                                             [1]))
302 plt.plot(M.Load,color = 'k')
                                       340 print('Critical Values:')
                                       341 for key, value in result[4].items()
plt.title('Monthly avarage')
                                              print('\t{}: {}'.format(key,
ax12 = fig.add_subplot(322)
                                             value))
307 fig = sm.graphics.tsa.plot_acf(M.
      Load, ax=ax12, lags =25)
                                       344
ax21 = fig.add_subplot(323)
                                       345 # In[77]:
  plt.plot(M.L_diff,color = 'k')
                                       347
310
  plt.title('Monthly avarage 1st
                                       348 result = adfuller(M.L_diff_2)
                                       349 print('ADF Statistic: {}'.format(
      order differencing')
ax22 = fig.add_subplot(324)
                                             result[0]))
fig = sm.graphics.tsa.plot_acf(M.
                                       350 print('p-value: {}'.format(result
      L_diff, ax=ax22, lags =25)
                                             [1]))
ax21 = fig.add_subplot(325)
                                       351 print('Critical Values:')
plt.plot(M.L_diff_2,color = 'k')
                                       352 for key, value in result[4].items()
317 plt.title('Monthly avarage 2nd
                                              print('\t{}: {}'.format(key,
                                       353
      order differencing')
                                             value))
ax22 = fig.add_subplot(326)
                                       354
fig = sm.graphics.tsa.plot_acf(M.
                                       355
      L_diff_2, ax=ax22, lags =25)
                                       356 # # ARMA
320 fig.tight_layout(pad=3.0)
                                       357
                                       358 # First of all we try to build An
321
                                             ARMA model on the data
323 # In [69]:
                                       359
                                         # In[24]:
324
                                       361
326 result = adfuller(M.Load)
                                       362
327 print('ADF Statistic: {}'.format(
                                      _{363} N = len(M.Load)
      result[0]))
                                       364 \text{ split} = 0.85
328 print('p-value: {}'.format(result 365 training_size = round(split*N)
      [1]))
                                       366 test_size = round((1-split)*N)
329 print('Critical Values:')
```

```
368 series = M.Load[:training_size]
                                         _{404} ps = range(0, 10, 1)
                                         405 d = 0
369
                                         qs = range(0, 10, 1)
371 # In [25]:
                                         408 # Create a list with all possible
372
                                                combination of parameters
373
  def optimize_ARIMA(order_list, exogno parameters = product(ps, qs)
      ):
                                         410 parameters_list = list(parameters)
       0.00
375
           Return dataframe with
                                         412 order_list = []
376
      parameters and corresponding
                                         413
      AIC
                                         414 for each in parameters_list:
                                                 each = list(each)
                                         415
377
                                                 each.insert(1, d)
            order_list - list with (p,416
378
      d, q) tuples
                                                 each = tuple(each)
                                         417
            exog - the exogenous
                                                 order_list.append(each)
                                         418
379
      variable
                                         419
       0.00
                                         420 result_df = optimize_ARIMA(
380
                                                order_list, exog = series)
381
       results = []
                                         421
382
                                         422 result_df
383
       for order in tqdm_notebook(
                                         423
      order_list):
                                         424
                                         425 # Try to fit the best model
           try:
385
                                               according to AIC
                model = SARIMAX(exog,
386
      order=order).fit(disp=-1)
                                         426
            except:
                                         427 # In [76]:
387
                continue
                                         428
388
389
                                         429
            aic = model.aic
                                         430 best_model = SARIMAX(series, order
            results.append([order,
                                                =(6,2,6)).fit()
391
      model.aic])
                                         431 print (best_model.summary())
                                         432 s_best_model = SARIMAX(series,
392
       result_df = pd.DataFrame(
                                                order=(6,0,8)).fit()
                                         433 print(s_best_model.summary())
      results)
       result_df.columns = ['(p, d, q)34
394
      ', 'AIC']
       #Sort in ascending order, lowers6 # In[77]:
395
       AIC is better
       result_df = result_df.
396
                                         438
      \verb|sort_values(by='AIC', ascending|_{439}| best_model.plot_diagnostics(figsize)| \\
      =True).reset_index(drop=True)
                                               =(15,12)
                                         440 s_best_model.plot_diagnostics(
397
       return result_df
                                                figsize=(15,12))
398
                                         441
                                         442
401 # In [26]:
                                         443 # In[29]:
                                         444
402
403
```

```
446 original_1 = M.L_diff_2.cumsum() + = ['f'], index = x1)
      M.L_diff.max()-M.L_diff_2.
                                       478 forec.f.plot(ax=ax,color = '
                                             Darkorange',label = 'Forecast (
      cumsum().max()
447 original_1[0] = M.L_diff[0]
                                             d = 2),
448 original_2 = original_1.cumsum() +479 ax.fill_between(x1, ci['lower Load'
                                             ], ci['upper Load'],alpha=0.2,
      M.Load.max() - original_1.cumsum
      ().max()
                                             label = 'Confidence inerval
449 original_2[0] = M.Load[0]
                                             (95%)', color='grey')
450 plt.plot(M.Load)
                                       480
451 plt.plot(original_2)
                                       s_forec = pd.DataFrame(s_forec,
                                             columns=['f'], index = x1)
                                       482 s_forec.f.plot(ax=ax,color = 'green
454 # Let's see how forecast performs
                                             ', label = 'Forecast (d = 0)')
                                       483 #ax.fill_between(x1, s_ci['lower
                                             Load'], s_ci['upper Load'],
456 # In [86]:
                                             alpha=0.2, label = 'Confidence
457
                                             inerval (95%)',color='grey')
458
                                       484
460 fore_l= test_size -1
                                       485
461 forecast = best_model.
                                       486 plt.legend(loc = 'upper left')
      get_prediction(start=
                                       487 plt.show()
      training_size, end=
                                       488
      training_size+fore_1)
462 forec = forecast.predicted_mean
                                       490 # In[89]:
463 ci = forecast.conf_int(alpha=0.05)491
465 s_forecast = s_best_model.
                                       493 from sklearn.metrics import
      get_prediction(start=
                                             mean_squared_error
      training_size, end=
                                       494 RMSE_Arima = np.sqrt(
      training_size+fore_1)
                                             mean_squared_error(M.Load[
466 s_forec = s_forecast.predicted_mean
                                             training_size:training_size+
467 s_ci = forecast.conf_int(alpha
                                             fore_1],forec.f[0:len(forec)
      =0.05)
                                             -1]))
                                       495 print('RMSE : {:.02f}'.format(
469 fig, ax = plt.subplots(figsize
                                             RMSE_Arima))
      =(16,8), dpi=300)
470 x0 = M.Load.index[0:training_size] 497
x1=M.Load.index[training_size:
                                       498 # In [90]:
      training_size+fore_l+1]
                                       499
#ax.fill_between(forec, ci['lower 500
      Load'], ci['upper Load'])
                                       501 RMSE_Arima/M.Load.max()
  plt.plot(x0, M.Load[0:training_size02
      ],'k', label = 'Load')
                                       504 # In[91]:
474
plt.plot(M.Load[training_size:
                                       505
      training_size+fore_l], '.k',
                                       506
      label = 'Actual')
                                       507 from sklearn.metrics import
                                             mean_squared_error
476
477 forec = pd.DataFrame(forec, columnsom RMSE_Arima = np.sqrt(
```

```
mean_squared_error(M.Load[
                                              Standard Deviation')
      training_size:training_size+
                                               plt.show(block=False)
      fore_1],s_forec.f[0:len(forec)
                                       538
      -1]))
                                               # DickeyFuller test:
509 print('RMSE : {:.02f}'.format(
                                               result = adfuller(timeseries)
                                        540
                                               print('ADF Statistic: {}'.
      RMSE_Arima))
                                        541
                                              format(result[0]))
                                               print('p-value: {}'.format(
511
                                        542
512 # In [92]:
                                              result[1]))
                                               print('Critical Values:')
                                        543
                                               for key, value in result[4].
515 RMSE_Arima/M.Load.max()
                                              items():
                                                   print('\t{}: {}'.format(key
516
                                        545
                                              , value))
517
518 # # SARIMA
                                        546
519
                                        547
520 # Let's see if taking care of
                                        548 # In [95]:
      seasonality could help building549
       a better model, tre process is550
       very similar to the one used
                                        551 from statsmodels.tsa.seasonal
      for ARIMA modeling, however a
                                              import seasonal_decompose
      decomposition tool helped us
                                        552
      checking the stationarity
                                        result = seasonal_decompose(M.Load,
      components.
                                               model='additive')
                                        554 result.plot()
522 # In [100]:
                                          plt.show()
523
                                        557
524
525 def get_stationarity(timeseries,
                                        558 # In [101]:
      window):
                                        560
       # rolling statistics
                                        561 len(S_series),len(RM_Load)
527
       rolling_mean = timeseries.
                                        562
      rolling(window=window).mean()
       rolling_std = timeseries.
                                        564 # In[102]:
      rolling(window=window).std()
                                        565
                                        566
530
       # rolling statistics plot
                                        567 N = len(M.Load)
       original = plt.plot(timeseries 568 split = 0.85
       color='blue', label='Original'569 training_size = round(split*N)
                                        570 test_size = round((1-split)*N)
       mean = plt.plot(rolling_mean,
                                        571
      color='red', label='Rolling
                                        572 S_series = M.Load[:training_size]
      Mean')
       std = plt.plot(rolling_std,
                                        574
      color='black', label='Rolling
                                        575 # In [103]:
      Std')
                                        576
       plt.legend(loc='best')
                                        577
       plt.title('Rolling Mean &
                                        578 len(RM_Load)
```

```
579
                                              RM_Load, ax=ax1, lags =25)
                                        623 ax2 = fig.add_subplot(212)
580
                                        fig = sm.graphics.tsa.plot_pacf(
581 # In [105]:
                                              RM_Load, ax=ax2, lags = 25)
                                        625
583
584 \text{ window} = 6
                                        626
585 rolling_mean = S_series.rolling(
                                        627 # In [645]:
      window=window).mean()
                                        628
586 RM_Load = S_series.diff(window)
                                        629
RM_Load.dropna(inplace=True)
                                        fig = plt.figure(figsize=(12,8))
                                        ax3 = fig.add_subplot(311)
588 get_stationarity(RM_Load, window)
589 plt.plot(RM_Load)
                                        ax3.plot(RM_Load.diff().dropna().
590 plt.plot(S_series)
                                              diff().dropna())
                                        633 ax1 = fig.add_subplot(312)
                                        634 fig = sm.graphics.tsa.plot_acf(
593 # In [106]:
                                              RM_Load.diff().dropna().diff().
                                              dropna(), ax=ax1,lags =25)
594
                                        ax2 = fig.add_subplot(313)
validates = range(1,13,1)
                                        636 fig = sm.graphics.tsa.plot_pacf(
597 p_0 = []
                                              RM_Load.diff().dropna().diff().
598 for v in validates:
                                              dropna(), ax=ax2,lags=25)
       rolling_mean = S_series.rolling; fig.tight_layout(pad=3.0)
      (window=v).mean()
       RM = S_series.diff(v)
                                        639
600
       RM.dropna(inplace=True)
                                        640 # In[121]:
601
       result = adfuller(RM)
                                        641
       p_0.append(result[1])
                                        642
603
                                        643 def optimize_SARIMA(parameters_list
604
                                              , d, D, s, exog):
606 # In [111]:
                                        644
                                                   Return dataframe with
607
                                        645
                                              parameters, corresponding AIC
608
p_value = np.full(len(validates),
                                              and SSE
      fill_value= 0.05)
plt.plot(validates,p_0,'.',label
                                                   parameters_list - list with
                                      = 647
      'P value')
                                               (p, q, P, Q) tuples
plt.plot(validates,p_value,'--',
                                                   d - integration order
                                        648
      label = 'Trashold (0.05)')
                                                   D - seasonal integration
                                        649
612 plt.xlabel('Seasonal difference')
                                              order
613 plt.legend()
                                                   s - length of season
                                        650
                                                   exog - the exogenous
614 #plt.ylim(0,0.01)
                                        651
                                              variable
                                               0.00
616
                                        652
617 # In [644]:
                                        653
                                               results = []
618
                                        654
619
                                        655
fig = plt.figure(figsize=(12,8))
                                               for param in tqdm_notebook(
                                        656
ax1 = fig.add_subplot(211)
                                              parameters_list):
fig = sm.graphics.tsa.plot_acf(
                                                   try:
```

```
658
               model = SARIMAX (exog,
                                              comparison
      order=(param[0],d, param[1]),
                                       696
      seasonal_order=(param[2], D,
                                        697 # In [113]:
      param [3], s)).fit(disp=-1)
           except:
659
                                       699
                continue
                                        700 best_model = SARIMAX(S_series,
660
                                              order=(3, 2, 1), seasonal_order
           aic = model.aic
                                              =(1, 1, 2, 6)).fit(disp=-1)
662
           results.append([param, aic 701 print(best_model.summary())
663
      ])
                                        702 s_best_model = SARIMAX(S_series,
                                              order=(3,0, 1), seasonal_order
664
       result_df = pd.DataFrame(
                                              =(1, 1, 2, 6)).fit(disp=-1)
665
      results)
                                       703 print(s_best_model.summary())
       result_df.columns = ['(p,q)x(P_{704})]
666
      Q)', 'AIC']
       #Sort in ascending order, lowerof # In[120]:
667
       AIC is better
                                        707
       result_df = result_df.
      sort_values(by='AIC', ascending709 best_model.plot_diagnostics(figsize
      =True).reset_index(drop=True)
                                              =(15,12);
669
                                        710
       return result_df
                                        711
670
                                        712 # In [115]:
671
                                       713
672
673 # In [122]:
                                       714
674
                                       716 fore_l= test_size -1
675
p = range(0, 4, 1)
                                       717 forecast = best_model.
677 d = 0
                                              get_prediction(start=
q = range(0, 4, 1)
                                              training_size, end=
P = range(0, 4, 1)
                                              training_size+fore_1)
680 D = 1
                                        718 forec = forecast.predicted_mean
Q = range(0, 4, 1)
                                        719 ci = forecast.conf_int(alpha=0.05)
682 s = 6
                                       721 s_forecast = s_best_model.
parameters = product(p, q, P, Q)
684 parameters_list = list(parameters)
                                              get_prediction(start=
685 print(len(parameters_list))
                                              training_size, end=
                                              training_size+fore_1)
                                        722 s_forec = s_forecast.predicted_mean
687
688 # In [123]:
                                        723 s_ci = forecast.conf_int(alpha
                                              =0.05)
689
691 result_df = optimize_SARIMA(
                                        fig, ax = plt.subplots(figsize
      parameters_list, d, D, s ,
                                              =(16,8), dpi=300)
                                        x0 = M.Load.index[0:training_size]
      S_series)
692 result_df
                                        727 x1=M.Load.index[training_size:
                                              training_size+fore_l+1]
693
                                        728 #ax.fill_between(forec, ci['lower
694
                                              Load'], ci['upper Load'])
695 # Again building two models for
```

```
729 plt.plot(x0, M.Load[0:training_size59
      ],'k', label = 'Load')
                                       761
731 plt.plot(M.Load[training_size:
                                       762
      training_size+fore_l], '.k',
      label = 'Actual')
733 forec = pd.DataFrame(forec, columns
      =['f'], index = x1)
forec.f.plot(ax=ax,color = '
      Darkorange',label = 'Forecast (
      d = 2),
ax.fill_between(x1, ci['lower Load?66
      ], ci['upper Load'],alpha=0.2, 767 # In[125]:
      label = 'Confidence inerval
      (95%)', color='grey')
737 s_forec = pd.DataFrame(s_forec,
      columns=['f'], index = x1)
738 s_forec.f.plot(ax=ax,color = 'green
      ',label = 'Forecast (d = 0)')
739 #ax.fill_between(x1, s_ci['lower
      Load'], s_ci['upper Load'],
      alpha=0.2, label = 'Confidence
      inerval (95%)',color='grey')
742 plt.legend(loc = 'upper left')
743 plt.show()
745
746
747 # In[116]:
749
750 RMSE_Sarima = np.sqrt(
      mean_squared_error(M.Load[
      training_size:training_size+
      fore_1],forec.f[0:len(forec)
      -1]))
751 print('RMSE : {:.02f}'.format(
      RMSE_Sarima))
752
753
754 # In [117]:
755
757 RMSE_Sarima/M.Load.max()
```

## 4.6 Deconvolution.ipynb

```
#!/usr/bin/env python
2 # coding: utf-8
4 # # Deconvolution
6 # In[79]:
9 #Importing the libraries to watch
     the 'fits' image and get the
     data array
10 import astropy
#import plotly.graph_objects as go
12 from astropy.io import fits
13 #Importing a library that is useful
      to read the original file
14 import pandas as pd
15 import pylab as plb
16 import matplotlib.pyplot as plt
17 from scipy.optimize import
     curve_fit
18 from scipy import asarray as ar, exp
19 #Importing a visual library with
     some illustrative set up
20 import matplotlib.pyplot as plt
21 import matplotlib.colors as mcolors
22 from matplotlib import cm
23 import numpy as np
```

```
52 # First step we import filtered
24 import math
25 import seaborn as sns
                                            dataset. (wavelet)
plt.style.use('fivethirtyeight')
27 plt.rcParams['font.family'] = 'sans54 # In[80]:
     -serif'
plt.rcParams['font.serif'] = '
     Ubuntu'
                                      57 dataset = pd.read_csv('
29 plt.rcParams['font.monospace'] = '
                                           reconstruction.csv')
     Ubuntu Mono'
                                      58 dataset = dataset.drop(columns=['
plt.rcParams['font.size'] = 14
                                           Unnamed: 0'])
garante plt.rcParams['axes.labelsize'] = 1259
plt.rcParams['axes.labelweight'] = 60
     'bold'
                                      61 # In [81]:
33 plt.rcParams['axes.titlesize'] = 1262
34 plt.rcParams['xtick.labelsize'] =
                                      data=pd.read_csv('deconv.csv',sep='
                                            ; ')
35 plt.rcParams['ytick.labelsize'] =
36 plt.rcParams['legend.fontsize'] =
     12
                                      67 # In[82]:
37 plt.rcParams['figure.titlesize'] = 68
     12
38 plt.rcParams['image.cmap'] = 'jet' 70 data=data.rename(columns={'Data':'
39 plt.rcParams['image.interpolation']
                                          Day','Godzina':'hour','Minuty':
      = 'none'
                                           'minute','Wolumen':'Load'})
40 plt.rcParams['figure.figsize'] =
     (16, 8)
                                      72
41 plt.rcParams['lines.linewidth'] = 273 # In[83]:
42 plt.rcParams['lines.markersize'] = 74
43 plt.rcParams["axes.grid"] = False 76 data['Load'] = dataset.R
45 colors = ['xkcd:pale orange', 'xkcd78
     :sea blue', 'xkcd:pale red', ' 79 # In[84]:
     xkcd:sage green', 'xkcd:terra
     cotta', 'xkcd:dull purple', '
                                     81
     xkcd:teal', 'xkcd: goldenrod', 82 data.head()
     'xkcd:cadet blue',
46 'xkcd:scarlet']
                                      84
47 cmap_big = cm.get_cmap('Spectral', 85 # Coverting data into DateTime
     512)
                                           objects.
48 cmap = mcolors.ListedColormap(
                                      86
     cmap_big(np.linspace(0.7, 0.95, 87 # In[85]:
      256)))
49 bbox_props = dict(boxstyle="round, 89
     pad=0.3", fc=colors[0], alpha
                                     90 data.Day = pd.to_datetime(data.Day)
     =.5)
                                      91
                                      92
                                      93 # In[86]:
```

```
141 Years = data.year.tolist()
94
                                          _{142} \text{ hd} = []
95
96 data.Day = pd.to_datetime(data.Day)43 for i in range(len(data)):
97 data['year'] = data.Day.dt.year
                                                 if date(Years[i], Months[i],
                                          144
98 data['month'] = data.Day.dt.month
                                                Days[i]) in pl_holidays :
99 data['day'] = data.Day.dt.day
                                                          hd.append(1)
                                          145
                                                  else :
100
                                          146
                                                      hd.append(0)
                                          147
102 # In [87]:
                                          148 data['holidays'] = hd
103
                                          149
data['month'] = data.Day.dt.month
                                         151
                                             # In [94]:
                                          152
106
107
                                          153
108 # In [88]:
                                          154 data.head()
109
                                          155
                                          156
data['day'] = data.Day.dt.day
                                          157 # In [95]:
112
                                          158
                                          159
114 # In[89]:
                                          160 weekdays = data.weekday.
115
                                                 drop_duplicates().tolist()
                                             weekdays = ['Monday', 'Tuesday', '
117 data['weekday']=data.Day.dt.
                                                 Wednesday', 'Thursday', 'Friday
                                                 ', 'Saturday', 'Sunday']
      day_name()
                                          162
                                          163
119
120 # In [91]:
                                          # Building kernel functions for
                                                weekdays and weekend without
                                                holidays.
123 data.head()
                                          165
                                          166 # In [97]:
124
125
                                          167
126 # Distinguish Holidays from
                                          168
      weekdays
                                          169 \text{ Mean} = []
                                             for w in weekdays:
127
                                          170
                                                 W = data[data['weekday'] == w]
128 # In [92]:
                                          171
                                                 W = W[W['holidays'] == 0]
129
                                          172
                                                 W_Day = W.Day.drop_duplicates()
                                          173
130
131 import holidays
                                                 .tolist()
  pl_holidays = holidays.Poland()
                                                 mean = pd.DataFrame()
                                          174
                                                 mean = np.zeros(96)
133
134
                                                 k = 0
135 # In [93]:
                                                 for d in W_Day:
                                          177
                                                      value = W[W['Day'] == d].
136
                                          178
                                                Load
137
138 from datetime import date
                                                      try:
139 Days = data.day.tolist()
                                                          mean =
                                                                   mean + np.
                                          180
                                                 asarray(value)
140 Months = data.month.tolist()
```

```
k = k+1
                                          227
181
            except :
182
                                          228
                                          229 # In[103]:
                 continue
184
       mean = mean/k
                                         231
185
       Mean.append(mean)
                                          we= np.array(Mean[5])+np.array(Mean
186
                                                [6])
187
                                          _{233} \text{ we=we/2.}
188
189 # In [98]:
                                          234 we=we/we.sum()
190
                                          235
                                          236
192 plt.figure(1)
                                            # In[104]:
plt.subplot(711)
                                         238
194 plt.plot(Mean[0])
                                         239
195 plt.subplot(712)
                                         240 day=data.Day.drop_duplicates()
plt.plot(Mean[1])
                                         241
plt.subplot(713)
                                         242
198 plt.plot(Mean[2])
                                          243 # ## Building the delta signal for
199 plt.subplot (714)
                                                convolution
200 plt.plot(Mean[3])
                                         244
plt.subplot(715)
                                          245 # Some days are longer than others,
plt.plot(Mean[4])
                                                 while other days were snaller.
203 plt.subplot (716)
                                          246 # We fix this in the following way.
204 plt.plot(Mean[5])
                                         247
205 plt.subplot(717)
                                         248 # In[105]:
  plt.plot(Mean[6])
                                         249
207
                                         _{251} MAX = []
208
209 # In[99]:
                                         252 PROB_D=[]
210
                                          253 DAY=np.array(day)
                                          254 for t in range(len(DAY)-1):
                                                 startdata=data[(data.Day==DAY[t
212 d=data.Day.drop_duplicates()
                                          255
                                                ])].index.tolist()[0]
213
                                                 enddata=data[(data.Day==DAY[t
215 # In [101]:
                                                +1])].index.tolist()[0]
                                                 if enddata-startdata!=96:
216
                                          257
217
                                                      PROB_D.append(DAY[t])
                                          258
218 Ferial = np.array(Mean[0]) + np.
      array(Mean[1]) + np.array(Mean
                                                 arr_= np.array(data.Load.loc[
                                         260
      [2]) + np.array(Mean[3]) + np.
                                                startdata:enddata])
      array (Mean [4])
                                          261
219 Ferial=Ferial/5.
                                                 #arr_= arr_-arr_.mean()
  Ferial=Ferial/Ferial.sum()
220
                                          263
                                                 MAX.append(arr_.max())
                                                 #print(t,len(DAY)-1)
221
                                          264
                                          265
222
223 # In [102]:
                                          267 # In [106]:
224
225
                                          268
plt.plot(Ferial), Ferial.mean()
```

```
OLD_PROB_D=PROB_D.copy()
                                         303 PROB_D=[]
                                         304 #data=data.reset_index()
271
                                         RANGE=np.arange(0,len(fix_data),96)
273 # In [107]:
                                            for t in RANGE:
                                                 MAX.append(fix_data.loc[t:t
274
                                         307
275
                                                +96-1]. Load. max())
276 fix_data=data.copy()
1 = np.arange (0, len (OLD_PROB_D) + 1, 1) 309
for i in range(len(I)-1):
                                         310 # In [110]:
       if i%2==0:
279
                                         311
            to_add=fix_data.loc[
280
                                         312
      fix_data[(fix_data.Day==
                                         313 def build_day(k):
      OLD_PROB_D[I[i+1]]) & (fix_data314
                                                 day=np.zeros (96)
      .hour==2)].index.tolist()
                                                 day[0]=k
                                         315
      [0:4]].Load
                                                 return day
            line = fix_data.loc[to_add317
281
      index[0]:to_add.index[0]+3]
            start=fix_data[(fix_data. 319 # In[111]:
      Day == OLD_PROB_D[I[i]]) & (
      fix_data.hour==1)].index.max()
                                         322 DELTA = []
            try_data=fix_data.copy()
283
                                         323 for m in MAX:
            try_data=try_data.drop(np.324
                                                 DELTA.append(build_day(m))
284
      arange(start,len(data)))
                                         325 #DELTA=np.abs(DELTA)
            fix_data=try_data.append( 326
285
      line,ignore_index=True)
            fix_data.loc[len(fix_data)328 # In[112]:
286
      -4:len(fix_data)].Day=np.repeat329
      (OLD_PROB_D[I[i]],4)
                                         330
            fix_data=fix_data.append( 331 DELTA=np.array(DELTA).ravel()
287
      data.loc[fix_data.index.max()
      -3:<u>len</u>(data)])
                                         333
            fix_data=fix_data.drop(line34 # In[118]:
288
       .index)
289
                                         337 data['r']=DELTA
290
291 # Fix data is used to build Delta 338
      Signal
                                         340 # In[119]:
292
  # In[108]:
293
                                         341
294
                                         342
                                         343 fix_data['r']=DELTA
296 data=fix_data
                                         344
297
                                         345
                                         346 # In [143]:
299 # In [109]:
                                         347
300
                                         348
                                         349 plt.plot(np.array(fix_data.r
301
302 MAX = []
                                                [96:96*30]), 'Darkorange', label
```

```
for y in range(len(
       = 'Delta signal')
                                          390
plt.plot(np.array(fix_data.Load
                                                 YEARS) -1):
       [96:96*30]), 'k', label = 'Time 391
                                                                try:
      Series')
                                                                    add=np.array(
                                          392
                                                 mon[(mon['month']==m)&(mon.year
351 plt.grid(True)
352 plt.ylabel('Load')
                                                 ==YEARS[y])].reset_index().loc[
353 plt.xlabel('Time')
                                                 J[j]:J[j]+95].r
                                                                    j_rel=j_rel+add
354 plt.xticks(np.arange(96,96*30,96),393
      np.arange(0,30))
                                                                    k = k + 1
  plt.legend()
                                          395
                                                                except:
356
  plt.show()
                                          396
                                                                    continue
357
                                                           K.append(k)
                                          397
                                                           W.append(j_rel/k)
358
                                          398
  # In[121]:
                                                      DAYS.append(W)
359
                                          399
                                          400
                                                  TOT.append(DAYS)
                                          401
361
362 data=fix_data
                                          402
363
                                          403
                                             # The calendar of 2016 has been
                                          404
364
365 # In [122]:
                                                 buit
366
                                          405
                                          406 # In[144]:
367
  YEARS=data.year.drop_duplicates().407
      tolist()
369
                                          409
                                          cal=pd.DataFrame({'Day':data['Day'
  # Each specific day of the months
                                                 ], 'month':data['month'], 'year':
      for each year has been stored
                                                 data['year'],'Weekday':data['
                                                 weekday']})
372 # e.g.
                                          411 cal['hour'] = data.hour
373 # The first Monday of the third
      week of May
                                          412 cal['minute'] = data.minute
                                          413 cal=cal[cal['year']==2016]
374 # the second Friday of the first
      week of April
                                          414
                                          415
376 # In [123]:
                                          416 # In [145]:
                                          417
377
                                          418
378
379 TOT = []
                                          419
   for m in range(1,13):
                                          420 \text{ NEW}_Z = []
380
       DAYS = []
                                             for k in range(1,13):
381
                                          421
       for d in weekdays:
                                                  mese=cal[cal['month']==k]
                                          422
382
            mon=data[data['weekday']==d23
                                                  mese=mese.reset_index().drop(
383
                                                 columns = ['index'])
            W = []
                                                  z = []
384
                                          424
            J=np.arange(0,96*7,96)
                                          425
                                                  for j in range (1,6):
            K = []
                                                      sett=mese.loc[(j-1)
                                          426
386
            for j in range(len(J)):
                                                 *7*96:96*(j)*7]
387
                 j_rel=np.zeros(96)
                                                      #print(j-1)
388
                                          427
                 k=0
                                                      ZERO=np.zeros(len(sett)-1)+
                                          428
```

```
if day == 'Tuesday':
                                            474
            z.append(ZERO)
                                                        i=1
429
                                            475
                                                    if day == 'Wednesday':
        new_z = []
430
                                            476
        for i in z:
431
                                            477
                                                    if day == 'Thursday':
            for j in i:
                                           478
432
                 new_z.append(j)
                                                        i=3
433
                                           479
        new_z = new_z [0:len(mese)] + [new_z]
                                                    if day == 'Friday':
       [len(new_z)-1]]
                                                        i=4
                                            481
       NEW_Z.append(new_z)
                                                    if day == 'Saturday':
435
                                            482
                                                        i=5
436
                                            483
                                                   if day == 'Sunday':
437
                                            484
                                            485
                                                        i=6
438
439 # In [146]:
                                                    return i
                                            486
440
                                            487
                                            489 # In[151]:
442
443 ZERO_FIN=[]
                                            490
444 for i in NEW_Z:
                                            491
        for j in i:
                                            492 d_n=[]
            ZERO_FIN.append(j)
                                            493
                                               for day in tot_cal.Weekday.tolist()
446
447
                                                    d_n.append(week_to_number(day))
448
                                            494
449 # In [147]:
                                            495
450
                                            496
                                              # In[152]:
                                            497
   cal['n']=ZERO_FIN
                                            498
453
                                            499
                                            500 tot_cal['d_n']=d_n
454
   # In[148]:
455
                                            501
456
                                           502
                                            503 # The model of each day has been
tot_cal=cal.drop(columns=['hour','
                                                  used to create a synthetic 2016
      minute'])
                                            504
tot_cal=tot_cal.drop_duplicates(
                                            505 # In [153]:
       subset = 'Day')
                                            506
460
                                            507
                                            508 SIGNAL=np.array([])
461
462 # In [149]:
                                               for m in range (1,13):
                                                   mon=tot_cal[tot_cal['month']==m
463
                                            510
   tot_cal=tot_cal.reset_index()
                                                   mon=mon.reset_index()
465
                                            511
466
                                                   for i in range(len(mon)):
467
                                           513
   # In[150]:
                                                             dn=int(mon.loc[i]['d_n'
468
                                           514
                                                  ])
469
                                                             N=int(mon.loc[i]['n'])
470
                                           515
471 def week_to_number(day):
                                                             SIGNAL=np.append(SIGNAL
       if day == 'Monday':
                                                   ,TOT[m-1][dn][N])
472
            i=0
```

```
560 # In[160]:
519 # Debugging the split years
                                         561
                                         562
521 # In [154]:
                                         563 Ferial = np.array(Mean[0]) + np.
                                                array(Mean[1]) + np.array(Mean
                                                [2]) + np.array(Mean[3]) + np.
523
524 bis_1=data[data.year==2008]
                                                array(Mean[4])
525 bis_2=data[data.year==2012]
                                         564 Ferial=Ferial/5.
526 first=np.array(bis_1[bis_1['Day']=#65 Ferial=Ferial/Ferial.sum()
      '2008-02-29'].r)
                                         566 Ferial=Ferial+Ferial.mean()-Ferial.
second=np.array(bis_2[bis_2['Day'
                                                min()-Ferial.max()
      ] == '2012-02-29'].r)
  bis=(first+second)/2.
528
                                         568
                                         569 # In [161]:
529
531 # In [155]:
                                         571
                                         572 Ferial=Ferial/5.
532
                                         573
a=np.argwhere(np.isnan(SIGNAL)).
                                         574
      ravel().min()
                                         575 # In [162]:
b=np.argwhere(np.isnan(SIGNAL)).
      ravel().max()+1
                                         577
  SIGNAL[a:b]=bis
                                            Ferial=Ferial/Ferial.sum()
                                         578
                                         579
537
538
                                         580
539 # In [157]:
                                         581 # In [163]:
540
                                         582
541
                                         583
                                         584 Ferial=Ferial+Ferial.mean()-Ferial.
542 OLD_SIGNAL=SIGNAL.copy()
                                                min()-Ferial.max()
544
                                         585
545 # In [158]:
                                         586
                                         587 # Performing the convolution with
546
                                                Ferial days
548 SIGNAL=np.abs(OLD_SIGNAL)
                                         588
                                         589 # In [166]:
549
550
                                         590
551 # In [159]:
                                         592 X=signal.convolve(SIGNAL, Ferial)
552
                                         593 X=X*(data.Load.max())/X.max()
553
Ferial = np.array(Mean[0]) + np.
                                         594
      array(Mean[1]) + np.array(Mean
                                         595
      [2]) + np.array(Mean[3]) + np.
                                         596
                                            # In[167]:
      array (Mean [4])
                                         597
555 Ferial=Ferial/5.
                                         598
556 Ferial=Ferial/Ferial.sum()
                                         599 test=np.array(data[data.year
                                                ==2016].Load)
557 Old_Ferial=Ferial.copy()
558
                                         600
559
                                         601
```

```
602 # In [168]:
                                        646 sat=sat/sat.sum()
                                        647 sat=sat-sat.mean()
603
                                        648 plt.plot(Ferial, 'k', label = 'Ferial
_{605} X=X[0:len(X)-95]
                                        plt.plot(sun,'red',label = 'Sunday'
606
607
                                        plt.plot(sat,'Darkorange',label = '
608 # In [169]:
                                               Saturday')
609
                                        651 plt.legend()
610
  plt.plot(X[60*96:90*96],color='blue52 plt.grid(True)
                                        653 plt.ylabel('Kernel Function')
612 plt.plot(np.array(test)
                                        654 plt.xlabel('Time [Hours]')
      [60*96:90*96], color='red')
                                        655 plt.xticks(np.arange(0,96,4),np.
#error=np.array(test)[12*96:50*96]-
                                              arange(0,24))
      original
                                        656 plt.show()
#plt.plot(np.abs(error),color='
                                        657
      black')
                                        658
                                        # Prediction using weekend days
615
616
                                               kernels for entire calendar.
# Building convolution for weekend 660
                                        661 # In [175]:
      days
                                        662
618
619 # In [170]:
                                        664 SUN=signal.convolve(sun, SIGNAL)
620
                                        SUN=SUN*data[data.year!=2016].Load.
                                               max()/SUN.max()
622 start=20*96
end = (20 + 7 * 40) * 96
                                        666 SAT=signal.convolve(sat, SIGNAL)
624 prev=X[start:end]
                                        667 SAT=SAT*data[data.year!=2016].Load.
                                               max()/SAT.max()
t=np.arange(0,len(prev),1)
                                        668
627
                                        669
628 # In [173]:
                                        670 # In [176]:
629
                                        671
sat = np.array(Mean[6])
                                        673 plt.plot(SAT[60*96:90*96])
632 sat=sat-sat.min()
                                        674 plt.plot(test[60*96:90*96])
633 sat=sat/sat.sum()
                                        675
634 sat=sat-sat.mean()
                                        676
                                        677 # Dataset of the 3 Kernels
635
                                              reconstruction
636
637 # In [204]:
                                        678
                                        679 # In[]:
638
                                        680
639
640 sun= np.array(Mean[6])
                                        681
                                        682 recons_data=data[data['year'
641 sun=sun-sun.min()
642 sun=sun/sun.sum()
                                              ] = 2016].copy()
643 sun=sun-sun.mean()
                                        683 recons_data['PSFFerial']=X[0:len(
644 sat = np.array(Mean[5])
                                               recons_data)]
645 sat=sat-sat.min()
                                        recons_data['PSFSunday']=SUN[0:len(
```

```
recons_data)]
                                                color='Darkorange',label='Model
recons_data['PSFSaturday']=SAT[0:
                                                <sup>,</sup> )
                                          724 plt.plot(test[60*96:90*96],color='k
      len(recons_data)]
                                                ',label = 'Data')
686
                                          725 plt.grid(True)
687
688 # In [178]:
                                          726 plt.legend()
                                          727 plt.xticks(np.arange(0*96,31*96,96)
                                                ,np.arange(0,31))
690
                                          728 plt.xlabel('Days [March 2016]')
691 recons_data
                                          729 plt.ylabel('Load')
692
                                          730
694
  # In[180]:
                                          732 # In [214]:
695
                                          733
697 psf_data=pd.DataFrame()
                                          734
psf_data['weekday']=recons_data['
                                         735 from sklearn.metrics import
      weekday']
                                                mean_squared_error
psf_data['Ferial']=recons_data.
                                          736
      PSFFerial
                                          737
700 psf_data['Sunday']=recons_data.
                                          738 # In [216]:
      PSFSunday
                                          739
701 psf_data['Saturday']=recons_data.
                                         740
      PSFSaturday
                                          741 finaldata=data[data['year']==2016].
  psf_data=psf_data.reset_index()
                                                copy()
                                          742 #finaldata['Prediction']=
703
704
                                                final_recons
705
  # In[182]:
706
                                          744
                                          745 # In [217]:
708 final_recons=np.array(psf_data.
                                          746
      Ferial)
709 for i in range (35136):
                                          748 finaldata.to_csv('pred.csv')
       day_data=str(psf_data.loc[i].
710
                                         749
      weekday)
       if day_data == 'Sunday':
                                          751 # In [218]:
711
            final_recons[i]=psf_data.
712
                                         752
      loc[i].Sunday
                                          753
       if day_data == 'Saturday':
                                         754 finaldata=pd.read_csv('pred.csv')
            final_recons[i]=psf_data.
                                         755
714
      loc[i].Saturday
                                          756
                                          757 # In[1158]:
715
                                          758
                                          759
717
718 # Final Prediction
                                         760 test=np.array(finaldata.Load)
                                          761 pred=np.array(final_recons)
720 # In [226]:
                                         762
721
                                          763
                                          764 # In[29]:
722
723 plt.plot(final_recons[60*96:90*96] 765
```

```
766
767 from sklearn.metrics import
      mean_squared_error as mse
768
769
770 # In[41]:
771
772
PERF = [np.corrcoef(test-pred,test)
      [0][1], np.sqrt(mse(pred
      [0*96:30*96], test [0*96:30*96]))
      ,np.sqrt(mse(pred[30*96::],test
      [30*96::]))/test[30*96::].max()
      ]
774
776 # In[45]:
777
779 print ('The correlation between the
       error and the original signal
      is '+str(PERF[0]))
780 print ('The RMSE for the first
      month of data is ' + str(PERF
      [1]))
781 print ('This RMSE is the '+str(PERF
  [2]*100) + '% of the maximum')
```

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