

# Masked Computation of the Floor Function and Its Application to the FALCON Signature

Pierre-Augustin Berthet<sup>1,3</sup>[0009–0005–5065–2730], Justine Paillet<sup>2,3</sup>[0009–0009–6056–7766], and Cédric Tavernier<sup>3</sup>[0009–0007–5224–492X]

<sup>1</sup> Télécom Paris, Palaiseau, France, [berthet@telecom-paris.fr](mailto:berthet@telecom-paris.fr)

<sup>2</sup> Université Jean-Monnet, Saint-Étienne, France,

[justine.paillet@univ-st-etienne.fr](mailto:justine.paillet@univ-st-etienne.fr)

<sup>3</sup> Hensoldt SAS FRANCE, Plaisir, France,

[<pierre-augustin.berthet,justine.paillet,cedric.tavernier@hensoldt.net](mailto:pierre-augustin.berthet,justine.paillet,cedric.tavernier@hensoldt.net)

**Abstract.** With the ongoing standardization of new Post Quantum Cryptography (PQC) primitives by the National Institute of Standards and Technology (NIST), it is important to investigate the robustness of new designs to Side Channel Analysis (SCA). Amongst those future standards is Falcon, a lattice-based signature which relies of rational numbers. It thus requires an implementation using floating point arithmetic, which is harder to design well and secure. While recent work proposed a solution to mask the addition and the multiplication, some roadblocks remains, most noticeably how to protect the floor function. In this work we propose several methods to protect the computation of the floor function. We provide mathematical proofs of our methods as well as formal security proof in the probing model using the Non-Interference concepts. We also discuss their application to the FALCON Signature.

**Keywords:** Floor Function · Floating-Point Arithmetic · Post-Quantum Cryptography · FALCON · Side-Channel Analysis · Masking

## 1 Introduction

With the rise of quantum computing, mathematical problems which were hard to solve with current technologies will be easier to breach. Amongst the concerned problem is the Discrete Logarithm Problem (DLP) which can be solved in polynomial times by the Shor quantum algorithm [29]. As much of the current asymmetric primitives rely on this problem and will be breach, new cryptographic primitives are studied. The National Institute of Standards and Technology (NIST) launched a post-quantum standardization process [9]. The finalists are CRYSTALS Kyber [7,23], CRYSTALS Dilithium [11,22], SPHINCS+ [4,24] and FALCON [26].

Another concern for the security of cryptographic primitives is their robustness to a Side-Channel opponent. Side-Channel Analysis (SCA) was first introduced by Paul Kocher [20] in the mid-1990. This new branch of cryptanalysis focuses on studying the impact of a cryptosystem on its surroundings. AS computations

take time and energy, an opponent able of accessing the variation of one or both could find correlations between its physical observations and the data manipulated, thus resulting in a leakage and a security breach. Thus, the study of weaknesses in implementations of new primitives and the ways to protect them is an active field of research.

While they have been many works focusing on CRYSTALS Dilithium and CRYSTALS Kyber, summed up by Ravi et al. [27], FALCON is noticeably harder to protect. Indeed, the algorithm relies on floating-point arithmetic, for which there is little literature on how to protect it.

**Related Work** Previous works have identified two main weaknesses within the signing process of Falcon : the pre-image computation and the Gaussian sampler. The pre-image computation was proved vulnerable by Karabulut and Aysu [19] using an ElectroMagnetic (EM) attack. Their work was later improved by Guereau et al. [15]. To counter those attacks, Chen and Chen [8] propose a masked implementation of the addition and multiplication of FALCON. However, they did not delved into the second weakness of Falcon, the Gaussian sampler. The Gaussian sampler is vulnerable to timing attacks, as shown by previous work [14,12,21,25]. A isochronous design was proposed by Howe et al. [16]. However, a successful single power analysis (SPA) was proposed by Guereau et al. [15] and further improved by Zhang et al. [30]. There is currently no masking countermeasure for FALCON’s Gaussian Sampler. Existing work [13] tends to re-write the Gaussian Sampler to remove the use of floating arithmetic, thus avoiding the challenge of masking the floor function.

**Our Contribution** In this work we further expand the countermeasure from Chen and Chen [8] and apply it to the Gaussian Sampler. We propose a masking method based on the mantissa truncation to compute the floor function. We also propose a generic method to arithmetize the computation of the floor function. We discuss the application of both methods to masking FALCON.

Relying on the previous work of Chen and Chen [8], we also verify the higher-order security of our method in the probing model. Our formal proofs rely on the Non-Interference (NI) security model first introduced by Barthe et al. [1].

Finally, we provide some performances of our methods and compare them with the reference unmasked implementation and the previous work of Chen and Chen [8]. The implementation is tested on a personal computer with an Intel-Core i7-11850H CPU.

## 2 Notation and Background

### 2.1 Notation

- We denote by  $\mathbb{R} \setminus \mathbb{Z}$  any real number which is not an integer. For  $x \in \mathbb{R}$ , we denote the floor function of  $x$  by  $\lfloor x \rfloor$ .

- We will use the dot . as the separator between the integer part  $i$  and the fractional part  $f$  of a real number  $x = i.f$ .
- We denote the floor function of  $x$  with  $n$  decimals of absolute precision by  $\lfloor x \rfloor_n$ . Absolute precision implies  $\lfloor x \rfloor_n = \lfloor x \rfloor + 0.\{0\}^n \parallel [0; 9]^*$ . This also implies that  $0 \leq \lfloor x \rfloor_n - \lfloor x \rfloor < 1 \times 10^{-n}$ .
- We denote the floor function of  $x$  with  $n$  decimals of relative precision  $\alpha \in [0.1; 1)$  by  $\lfloor x \rfloor_n^\alpha$ . Relative precision implies  $-\alpha \times 10^{-n} < \lfloor x \rfloor_n^\alpha - \lfloor x \rfloor < \alpha \times 10^{-n}$ .
- Let  $a, b \in \mathbb{R}^2$ . Let  $k \in \mathbb{N}$ . We say that  $a \equiv_k b$  if  $\lfloor a \rfloor = \lfloor b \rfloor$  and they have the same number of 0 at least for the first  $k$  decimals:  $\lfloor a \rfloor + 0.\{0\}^k \parallel [0; 9]^*$ . For instance,  $4.05 \equiv_k 4.09$  at precisions  $k = 0$  and  $k = 1$  but not at precision  $k = 2$ .
- If  $(b_i)$  is a 1-bit Boolean shares for value  $b$ , we denote  $(-b_i)$  as the 64-bit Boolean shares for  $2^{64} - b$ . It means that if  $b = 0$ ,  $(-b_i)$  is a 64-bit boolean shares for 0, and  $b = 1$ ,  $(-b_i)$  is a 64-bit boolean shares for 0xFFFFFFFFFFFFFFFF.

## 2.2 FALCON Sign

FALCON [26] is a Lattice-Based signature using the GPV framework over the NTRU problem. In this paper we will focus on the Gaussian Sampler used in the signature algorithm. For more details on the key generation or the verification, please refer to the reference paper [26].

**Signature** The signature follows the Hash-Then-Sign strategy. The message  $m$  is salted with a random value  $r$  and then hashed into a challenge  $c$ . The remainder of the signature aims at building an instance of the SIS problem upon  $c$  and a public key  $h$ , *id est* finding  $\mathbf{s} = (s_1, s_2)$  such as  $s_1 + s_2 h = c$ . To do so, the need to compute  $\mathbf{s} = (\mathbf{t} - \mathbf{z})\mathbf{B}$ , with  $\mathbf{t}$  a pre-image vector and  $\mathbf{z}$  provided by a Gaussian Sampler. Chen and Chen [8] focuses on masking the pre-image vector computation. In this work we intend to mask the Gaussian Sampler. The signature algorithm is detailed in Algorithm 1:

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**Algorithm 1:** FALCON Sign [26]

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### Gaussian Sampler

*ApproxExp.* This function return  $2^{63} \times ccs \times e^{-x}$ .

$$C = \begin{bmatrix} 0x00000004741183A3, & 0x00000036548CFC06, \\ 0x0000024FDCBF140A, & 0x0000171D939DE045, \\ 0x0000D00CF58F6F84, & 0x000680681CF796E3, \\ 0x002D82D8305B0FEA, & 0x011111110E066FD0, \\ 0x0555555555070F00, & 0x155555555581FF00, \\ 0x400000000002B400, & 0x7FFFFFFFFFFFF4800, \\ x8000000000000000 & \end{bmatrix}$$

**Algorithm 2:** ApproxExp(x,ccs) [26]

---

**Data:** Floating-point values  $x \in [0, \ln(2)]$  and  $ccs \in [0, 1]$   
**Result:** An integral approximation of  $2^{63} \cdot ccs \cdot \exp(-x)$

```

1  $y \leftarrow C[0];$  //  $y$  and  $z$  remain in  $\{0 \dots 2^{63} - 1\}$  the whole algorithm
2  $z \leftarrow \lfloor 2^{63} \cdot x \rfloor;$ 
3 for  $i$  from 1 to 12 do
4    $y \leftarrow C[i] - (z \cdot y) >> 63;$ 
5  $z \leftarrow \lfloor 2^{63} \cdot ccs \rfloor;$ 
6  $y \leftarrow (z \cdot y) >> 63;$ 
7 return  $y;$ 

```

---

*BerExp.* This function return 1 with proba  $ccs \times e^{-x}$ .

**Algorithm 3:** BerExp(x,ccs) [26]

---

**Data:** Floating-point values  $x, ccs \geq 0$   
**Result:** A single bit, equal to 1 with probability  $\approx ccs \cdot \exp(-x)$

```

1  $s \leftarrow \lfloor x / \ln(2) \rfloor;$  // Compute the unique decomposition  $x = \ln(2^s) + r$  with
    $(r, s) \in [0, \ln(2)) \times \mathbb{Z}^+$ 
2  $r \leftarrow x - s \cdot \ln(2);$ 
3  $s \leftarrow \min(s, 63);$ 
4  $z \leftarrow (2 \cdot \text{APPROXEXP}(r, ccs) - 1) >> s;$ 
5  $i \leftarrow 64;$ 
6 do
7    $i \leftarrow i - 8;$ 
8    $w \leftarrow \text{UNIFORMBITS}(8) - ((z >> i) \& 0\text{xFF});$ 
9 while  $((w = 0) \text{ and } (i > 0));$ 
10 return  $\llbracket w < 0 \rrbracket;$ 

```

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*SamplerZ.* Gaussian Sampler

RCDT is define in Falcon Specification

### 2.3 Floor Function

The floor function is defined as follows:

**Definition 1.**  $\forall x \in \mathbb{R}$ , the floor function of  $x$ , denoted by  $\lfloor x \rfloor$ , returns the greatest integer  $z$  such as  $z \leq x$ .

There are several ways of computing the floor function. The first one relies on floating-point arithmetic. A floating-point is composed of a sign bit, exponent bits and a mantissa [18]. Computing the floor function on a floating-point is performed by truncating the mantissa according to the value of the exponent

**Algorithm 4:** SamplerZ( $\mu, \sigma'$ ) [26]

---

**Data:** Floating-point values  $\mu, \sigma' \in \mathcal{R}$  such that  $\sigma' \in [\sigma_{\min}, \sigma_{\max}]$   
**Result:** An integer  $z \in \mathbb{Z}$  sampled from a distribution very close to  $D_{\mathbb{Z}, \mu, \sigma'}$

```

1  $r \leftarrow \mu - \lfloor \mu \rfloor$ ;
2  $ccs \leftarrow \sigma_{\min} / \sigma'$ ;
3 while 1 do
4    $z_0 \leftarrow \text{BASESAMPLER}()$ ;
5    $b \leftarrow \text{UNIFORMBITS}(8) \& 0x1$ ;
6    $z \leftarrow b + (2 \cdot b - 1)z_0$ ;
7    $x \leftarrow \frac{(z-r)^2}{2\sigma'^2} - \frac{z_0^2}{2\sigma_{\max}^2}$ ;
8   if  $\text{BEREXP}(x, ccs) = 1$  then
9     return  $z + \lfloor \mu \rfloor$ ;
```

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**Algorithm 5:** BaseSampler() [26]

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**Data:** –  
**Result:** An integer  $z_0 \in \{0, \dots, 18\}$  such that  $z \sim \chi$

```

1  $u \leftarrow \text{UNIFORMBITS}(72)$ ;
2  $z_0 \leftarrow 0$ ;
3 for  $i$  from 0 to 17 do
4    $z_0 \leftarrow z_0 + \llbracket u < \text{RCDT}[i] \rrbracket$ ;
5 return  $z_0$ ;
```

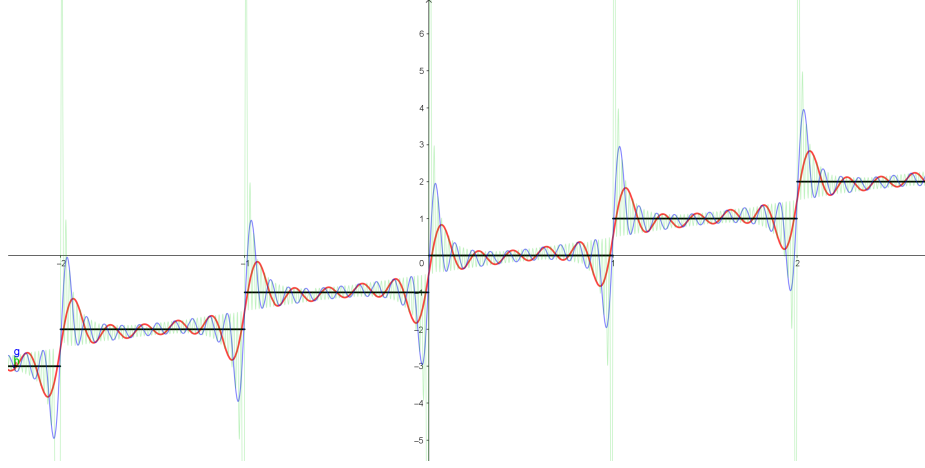
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and of the sign. However, while this method is fast and efficient, it requires the use of the exponent and the sign. In this work we will present a method to perform this truncation in a secure manner.

Another way of computing the floor function is to use its associated Fourier series:

$$\forall x \in \mathbb{R} \setminus \mathbb{Z}, \lfloor x \rfloor = x - \frac{1}{2} + \frac{1}{\pi} \sum_{k=1}^n \frac{\sin(2\pi kx)}{k}, \text{ with } n \rightarrow \infty \quad (1)$$

However, due to its discontinuities, Equation 1 suffers from the Gibbs phenomenon [6], as illustrated in Figure 2.3. It means this series cannot be used as it is to mask the floor function in reasonable time. Indeed, we extrapolate through empirical observations that we would require  $n = 2.5 \times 10^{13}$  to have at least one decimal of absolute precision on the floor computation of  $1 \times 10^{-16}$ . Nonetheless, we will see in this work how we can adapt this Fourier series to compute the floor function in a secure way and in reasonable time with the required precision.



**Fig. 1.** Gibbs phenomenon for Equation 1 with  $n = 5, 10, 50$  and  $\text{floor}(x)$  in black

## 2.4 Masking

# 3 Masking of the Floor Function

## 3.1 Arithmetization method

The floor function can be arithmetized using Equation 1. However, as stated in Section 2.3, this method cannot be performed in reasonable time. We thus elaborate a strategy to optimize this arithmetization.

**Absolute Precision** The first idea relies on constructing the floor of  $x \in \mathbb{R}$  rather than computing it. Indeed, if we can compute the floor of  $x$  with one decimal of precision, we can then multiply the result by 10 and then reapply our computation to the new value  $y = 10 \times \lfloor x \rfloor_1$ . Thus, we have the following proposition:

**Proposition 1.** *Let  $x \in \mathbb{R}$ . Let  $y = 10 \times \lfloor x \rfloor_1$ . Then  $\lfloor y \rfloor_1 / 10 \equiv_2 \lfloor x \rfloor_2$ .*

*Proof.* Let recall that  $\lfloor x \rfloor_1 = \lfloor x \rfloor + 0.0 \parallel [0; 9]^*$ . Thus,  $y = 10 \times \lfloor x \rfloor_1 = \lfloor x \rfloor \parallel 0 + 0. \parallel [0; 9]^*$  as multiplying by 10 can be seen as a left-shift in base 10. By computing the floor of  $y$  with absolute precision 1 we have  $y_1 = \lfloor y \rfloor_1 = \lfloor x \rfloor \parallel 0 + 0.0 \parallel [0; 9]^*$ . Finally,  $y_1 / 10 = \lfloor x \rfloor + 0.00 \parallel [0; 9]^* \equiv_2 \lfloor x \rfloor_2$  as dividing by 10 can be seen as a right-shift in base 10.

Thus, we can use Proposition 1 to construct the floor of  $x$  with the following Algorithm 6:

**Algorithm 6:** ConstructFloor( $x, prec$ )**Data:**  $x \in \mathbb{R}$  and the precision  $prec \in \mathbb{N}$ **Result:**  $\lfloor x \rfloor_{prec}$ 


---

```

1 for  $i$  from 0 to  $prec-1$  do
2    $y \leftarrow \lfloor x \rfloor_1$ ;
3    $x \leftarrow 10 \times y$ ;
4 return  $x \times 10^{-prec}$ 

```

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**Theorem 1.** Let  $x \in \mathbb{R}$ . Algorithm 6 returns  $\lfloor x \rfloor_{prec}$  for precision  $prec$ .

*Proof.* We use a recursive proof. The initial state is proven by Proposition 1. Let Theorem 1 be true for precision  $prec - 1$ . We have  $X = \lfloor x \rfloor_{prec-1}$  the output of Algorithm 6. We reapply Algorithm 6 to  $X \times 10^{prec-1}$  but with precision 1 and then divide by  $10^{prec-1}$ . This is strictly equal to applying Algorithm 6 with precision  $prec$ . Algorithm 6 with precision 1 is proven by Proposition 1. Thus, we have  $\lfloor X \rfloor_1 = \lfloor x \rfloor \parallel \{0\}^{prec-1} + 0.0 \parallel [0; 9]^*$ . By dividing by  $10^{prec-1}$ , we shift all the zeros stored in the integer part of  $X$  into the fractional part. Thus we have  $\lfloor X \rfloor_1 / 10^{prec-1} = \lfloor x \rfloor + 0.\{0\}^{prec} \parallel [0; 9]^* \equiv_{prec} \lfloor x \rfloor_{prec}$ . We thus proved Theorem 1.

**Relative Precision** We discuss how to achieve absolute precision from relative precision. In relative precision, we have the possibility that the error from the result can be negative. This results in  $\lfloor x \rfloor_n^\alpha = \lfloor x \rfloor - 0.\{0\}^n \parallel [0; 9]^* = \lfloor x \rfloor - 1 + 0.\{9\}^n \parallel [0; 9]^*$ . Thus, we cannot construct the floor function using directly  $\lfloor x \rfloor_n^\alpha$ , as in Algorithm 6. We introduce a second idea: controlled bias.

We have the following Proposition 2:

**Proposition 2.** Let  $x \in \mathbb{R}$ . Then  $\lfloor x \rfloor_1 \equiv_1 \lfloor x \rfloor_2^\alpha + \alpha \times 10^{-2}$ .

*Proof.* We have two cases to cover:

1. The error is negative. Thus  $-\alpha \times 10^{-2} < \lfloor x \rfloor_2^\alpha - \lfloor x \rfloor \leq 0$  and  $0 < \lfloor x \rfloor_2^\alpha + \alpha \times 10^{-2} - \lfloor x \rfloor \leq \alpha \times 10^{-2} < 1 \times 10^{-2}$ . Thus, in this case, we have  $\lfloor x \rfloor_2^\alpha + \alpha \times 10^{-2} \equiv_2 \lfloor x \rfloor_2$  and by construction of  $\lfloor x \rfloor_n$ , we have  $\lfloor x \rfloor_2^\alpha + \alpha \times 10^{-2} \equiv_1 \lfloor x \rfloor_1$ .
2. The error is positive. Thus  $0 \leq \lfloor x \rfloor_2^\alpha - \lfloor x \rfloor < \alpha \times 10^{-2}$  and  $\alpha \times 10^{-2} \leq \lfloor x \rfloor_2^\alpha + \alpha \times 10^{-2} - \lfloor x \rfloor \leq 2\alpha \times 10^{-2}$ . This implies  $0 < \lfloor x \rfloor_2^\alpha + \alpha \times 10^{-2} - \lfloor x \rfloor < 1 \times 10^{-1}$ . Thus, in this case, we have  $\lfloor x \rfloor_2^\alpha + \alpha \times 10^{-2} \equiv_1 \lfloor x \rfloor_1$ .

We demonstrate that we can have an absolute precision of 1 from a relative precision of 2.

However, we can improve upon Proposition 2 by limiting the maximum value of  $\alpha$ . We have the following proposition:

**Proposition 3.** Let  $x \in \mathbb{R}$  and  $\alpha < 0.5$ . Then  $\lfloor x \rfloor_1 \equiv_1 \lfloor x \rfloor_1^\alpha + \alpha \times 10^{-1}$ .

*Proof.* We have already proven in Proposition 2 that for the negative bias,  $\lfloor x \rfloor_1^\alpha + \alpha \times 10^{-1} - \lfloor x \rfloor \leq \alpha \times 10^{-1}$ . As  $\alpha < 0.5$ , we have  $\lfloor x \rfloor_1 \equiv_1 \lfloor x \rfloor_1^\alpha$ . For the positive bias, according to Proposition 2 we have  $\alpha \times 10^{-1} \leq \lfloor x \rfloor_1^\alpha + \alpha \times 10^{-1} - \lfloor x \rfloor \leq 2\alpha \times 10^{-1}$ . As  $\alpha < 0.5$ ,  $2\alpha \times 10^{-1} < 1 \times 10^{-1}$ . We have  $\lfloor x \rfloor_1 \equiv_1 \lfloor x \rfloor_1^\alpha$ .

**Fourier Series** We now need a way of computing the floor function with a relative precision of at least 2 in reasonable time. Our final idea revolves around aborting Equation 1 early for a small value of  $n$ . Then, we reapply Equation 1 on the new value, with a small twist:

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**Algorithm 7:** ArFloorStep( $x, n, iter$ )

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**Data:**  $x \in \mathbb{R} \sim \mathbb{Z}$ , the series limiter  $n \in \mathbb{N}$  and the number of iterations  $iter \in \mathbb{N}$

**Result:**  $\lfloor x \rfloor_2^\alpha + \frac{1}{2}$

```

1 for  $i$  from 0 to  $iter-1$  do
2    $x \leftarrow x + \frac{1}{\pi} \sum_{k=1}^n \frac{\sin(2\pi kx)}{k};$ 
3 return  $x$ 
```

---

Instead of computing the floor function, Algorithm 7 gives the floor function with a positive bias of  $\frac{1}{2}$ . Indeed, applying Equation 1 and aborting without this bias does not ensure correctness. For instance, for a value close to 0, using Equation 1 with a small  $n$  results in  $-\frac{1}{2} + e$ , with  $e$  small. Reapplying Equation 1 will give  $-\frac{1}{2} + e - \frac{1}{2} + e_2 \approx -1$  as a result, which is not correct. Thus, it is important to remove  $-\frac{1}{2}$  and only apply it after the successive early aborts and reapplications. As an example, Algorithm 7 is proven by Theorem 2 for a specific set of parameters:

**Theorem 2.** *Let  $x \in (0; 1) \subset \mathbb{R} \sim \mathbb{Z}$  with up to 19 decimals, id est  $x = 0.[0; 9]^{19}||0^*$ . We set  $n = 3$  and  $\alpha = 0.4$ . Algorithm 7 returns  $\lfloor x \rfloor_1^\alpha + \frac{1}{2}$  in 24 iterations.*

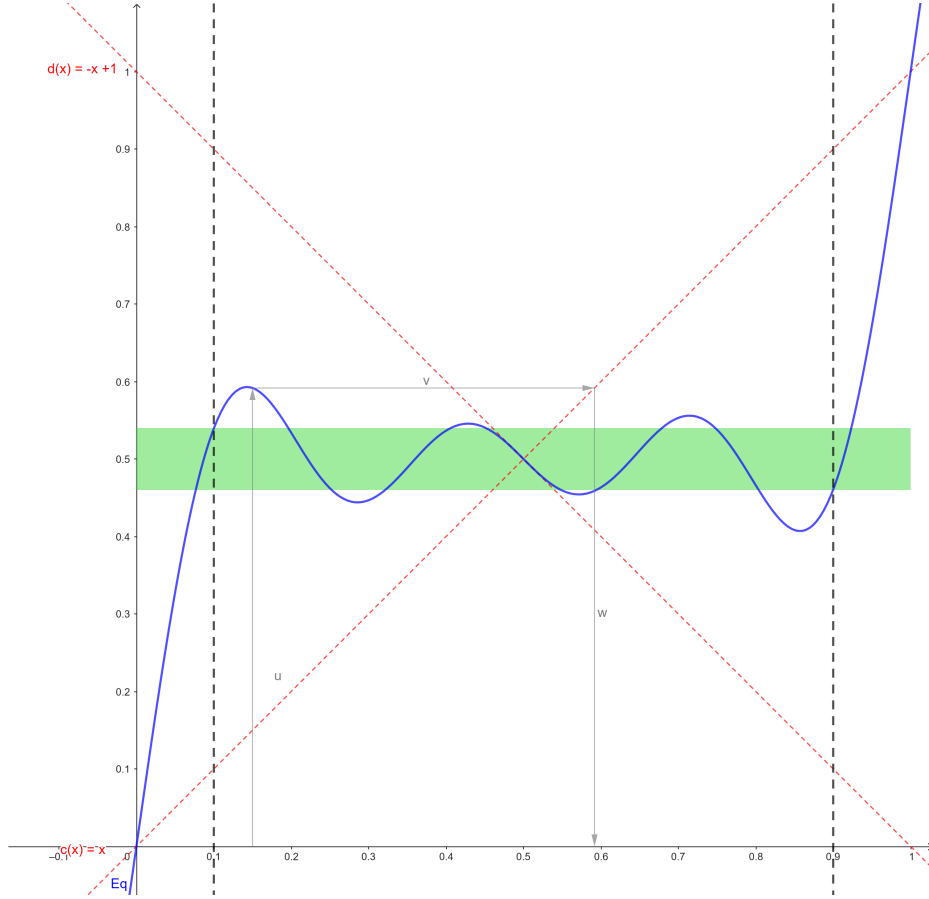
*Proof.* We denote  $x + \frac{1}{\pi} \sum_{k=1}^3 \frac{\sin(2\pi kx)}{k}$  by  $f(x)$ . Let set  $(U_n)_{n \in \mathbb{N}}$  such as  $U_{n+1} = f(U_n)$ . First, we will prove the convergence of  $(U_n)_{n \in \mathbb{N}}$ . Then, we use the Newton method and a visual proof to demonstrate Theorem 2.

### PREUVE DE LA CONVERGENCE A FAIRE!!!!!!

For the second part of the proof, we use the Newton method on a graphical representation of  $f(x)$  (labeled Eq). The Newton method, in grey arrows in Figure , allows to graphically compute the next term  $U_{n+1}$  from  $U_n$ . We start at the point  $(U_n, 0)$  (0.15 in this example) and find the point  $(U_n, f(U_n))$  (arrow labeled u). We then find the point of  $c(x) = x$  with ordinate  $f(U_n)$  (arrow labeled v) and look at its abscissa (arrow labeled w), which gives us  $U_{n+1}$ . Thanks to this method, we can visually compute the number of iterations of Algorithm 7 required to have  $0.46 \leq U_{n+1} \leq 0.54$  (green area). The amount of iterations is shown in Table 1 (due to the symmetric nature of  $\sin(x)$ , we restrict the study on  $(0; 0.5]$ ).

*Remark 1.* The choice of setting  $\alpha = 0.4$  in Theorem 2 is not the most optimal as taking  $\alpha = 0.5$  is more advantageous as shown in Proposition 3. However, by taking  $\alpha = 0.4$ , we have that Algorithm 7 returns  $\lfloor x \rfloor + err$  such as  $0.46 \leq err \leq 0.54$ . Thus, by retrieving to this result 0.45, we have  $0.01 \leq err - 0.45 \leq 0.09$ . By multiplying by 10, we have  $0.1 \leq 10 * (err - 0.45) \leq 0.9$ . As a result, when applying Algorithm 6 we avoid some extreme points and reduce the number of



**Fig. 2.** Equation 1 with  $n = 3$ **Table 1.** Subsets of  $(0; 0.5]$  and number of iterations to reach  $[0.46; 0.54]$ 

Number of iterations	Subsets
1	$U_1 = [0.0767; 0.1002] \vee [0.1962; 0.2522] \vee [0.3213; 0.4060] \vee [0.4514; 0.5]$
At most 2	$U_2 = U_1 \vee [0.0110; 0.0143] \vee [0.0287; 0.0373] \vee [0.0488; 0.0648] \vee [0.0746; 0.263] \vee [0.3095; 0.5]$
At most 3	$U_3 = U_2 \vee [0.0016; 0.002] \vee [0.0041; 0.0052] \vee [0.007; 0.0092] \vee [0.0107; 0.0391] \vee [0.0469; 0.5]$

iterations required by Algorithm 7. This strategy is not specific to the current set of parameters and will always benefit our method.

According to Table 1, all values of  $x \in [0.1; 0.9]$  converge towards  $[0.46; 0.54]$  in less than 3 iterations. Thus, in accordance with Remark 1 we can limit the number of iterations in the following steps of our floor computation to only 3. We can use Algorithm 8 to compute the floor function of  $x \in \mathbb{R} \setminus \mathbb{Z}$  in reasonable times.

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**Algorithm 8:** ArFloor( $x, prec$ )

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**Data:**  $x \in \mathbb{R} \setminus \mathbb{Z}$  and the precision  $prec \in \mathbb{N}$

**Result:**  $\lfloor x \rfloor_{prec}$

```

1  $x \leftarrow \text{ArFloorStep}(x, 3, 24) - 0.45;$ 
2  $x \leftarrow 10 \times x;$ 
3 for  $i$  from 1 to  $prec-1$  do
4    $x \leftarrow \text{ArFloorStep}(x, 3, 3) - 0.45;$ 
5    $x \leftarrow 10 \times x;$ 
6 return  $x \times 10^{-prec}$ 
```

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## 4 Truncation method : Masked Floor Function

In this part we will consider  $f$  as one of these three functions : floor, truncature and round. There's some little differences between those functions and we will explain it in times.

### 4.1 Overview

Recall that we're working with floating point in a specific representation, the standardized<sup>4</sup> one, it's really important to understand the encoding method because all operation algorithms using floating point are based on. We can define this encoding as follow : let  $x \in \mathbb{R}$ ,  $x$  is represented by a tuple of three elements  $(s, e, m)$ , where  $s$ ,  $e$  and  $m$  are respectively 1-bit length, 11-bit length and 52-bit length, such as

$$x = (-1)^s \times 2^{e-1023} \times (1 + m \times 2^{-52})$$

We call  $s$  the sign bit,  $e$  the exponent and  $m$  the mantissa. The most important element we will use to construct  $f$  between those three is the exponent  $e$ . Just by analyzing it, we can distinguish three different cases :

1. If  $e - 1023 < 0$ , the result will be 0. However, it exists a subtlety if  $f = \text{round}$ . In this case we must take care of the rounding and test if  $e - 1022 < 0$ . It's only by not rejecting the case of  $e = 1022$  that we can compute the correct rounding;
2. If  $e - 1075 \geq 0$ , the input  $x$  is already an integer, so the output  $y$  will be equal to  $x$ ;
3. If  $0 \leq e - 1023 \leq 51$ , the result must be computed by removing the  $52 - (e - 1023)$  decimals, and adjusting the result if  $f = \text{round}$ .

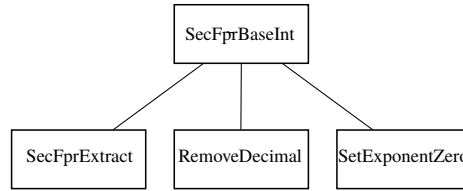
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<sup>4</sup> Binary64 – IEEE 754

These three distinguished cases provide a fixed structure shown in algorithm 9 for  $f$ . The main difficulty here is to compute securely all the checks on masked values, and to change in consequence our mantissa, exponent and eventually our sign bit.

## 4.2 Masked Floor Function

The function `SecFprBaseInt` is the main function to compute masked floor, masked truncature or masked round. Its result depends on the inputed function : gadgets and `Zerof` are different.



**Fig. 3.** `SecFprBaseInt` and its gadgets

In this subsection, suppose  $f = \text{floor}$ . We will first present the main structure and then present the gadgets. Gadgets from `trunc` and `round` function can be found in annexe (the little differences between gadgets will be explain too):

- `SetFprExtract`;
- `RemoveDecimaltrunc` and `SetExponentZerotrunc`;
- `RemoveDecimalround` and `SetExponentZeroround`.

The particularity of our shift method for calculating floor function is that it requires the gadgets proposed by Chen and Chen [8]. So by proposing this method, we are extending their work to mask the Gaussian sampler by using only the gadgets of their creation. By way of summary, we propose a table showing the functions used to create our gadgets and the sources from which they come.

**SetFprBaseInt<sub>floor</sub> – algorithm 9** As explained in overview, these three functions can be summarised in a global structure. What will differentiate the three functions will in fact be the gadgets and the `Zerof` parameter. To perform the floor function, we first extract the data from the encoding and place it in three variables  $s_y, e_y$  and  $m_y$ , which will be linked directly to the output of the algorithm. By extracting this data footnotePseudo-code in appendix: `SecFprExtract` – algorithm 13, we carry out a few operations on the mantissa and the exponent to match the conventions of Chen’s implementation.. Indeed, the input  $x$  is a 64-bit boolean share, consequently we’ll have to transform the exponent into a 16-bit arithmetic share by performing a boolean to arithmetic conversion. Another important thing to add is the bit implicit in the mantissa. This bit which

---

**Algorithm 9:** SecFprBaseInt( $x, f$ )

---

**Data:** 64-bit boolean shares  $(x_i)_{1 \leq i \leq n}$  for value  $x$

A function  $f$  : floor, round or trunc.

**Result:** 64-bit boolean shares  $(y_i)_{1 \leq i \leq n}$  for mantissa value  $y =$   
 $\text{floor}(x)/\text{round}(x)/\text{trunc}(x)$ .

```

1  $((my_i), (ey_i), (sy_i)) \leftarrow \text{SecFprExtract}((x_i));$ 
2  $(cx_i) \leftarrow (ey_i);$ 
3  $cx_1 \leftarrow ey_1 - \text{Zero}_f;$ 
4  $(c_i) \leftarrow \text{A2B}((cx_i^{(16)}));$ 
5  $(c_i) \leftarrow \text{SecNonZero}((c_i));$  // check if the exponent is negative
6  $(b_i) \leftarrow \neg(-c_i);$ 
7  $\text{Refresh}((cx_i));$ 
8  $(my_i) \leftarrow \text{SecAnd}((my_i), (b_i));$ 
9  $(my_i), (ey_i), (Rnd_i) \leftarrow \text{RemoveDecimal}_f((my_i), (ey_i), (sy_i), (cx_i));$ 
10  $(my_i), (ey_i) \leftarrow \text{SecFprNorm64}((my_i), (ey_i));$ 
11  $(my_i) \leftarrow (my_i \gg 11);$ 
12  $(my_i) \leftarrow (my_i^{[52:1]});$ 
13  $ey_1 \leftarrow ey_1 + 11;$ 
14  $(ey_i), (sy_i) \leftarrow \text{SetExponentZero}_f((ey_i), (b_i), (s_i), (Rnd_i));$ 
15  $(y_i^{(64)}) \leftarrow (sy_i);$ 
16  $(y_i^{[63:53]}) \leftarrow (ey_i);$ 
17  $(y_i^{[53:1]}) \leftarrow (my_i);$ 
18 return  $(y_i);$ 

```

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Algorithm	Description	Security	Reference
SecAnd	AND of Boolean shares	$t$ -SNI	[17], [2]
SecAdd	Addition of Boolean shares	$t$ -SNI	[10], [3]
A2B	Arithmetic to Boolean conversion	$t$ -SNI	[28]
B2A	Boolean to Arithmetic conversion	$t$ -SNI	[5]
RefreshMasks	$t$ -NI refresh of masks	$t$ -NI	[2], [5]
Refresh	$t$ -SNI refresh of masks	$t$ -SNI	[2]
SecOr	OR of Boolean shares	$t$ -SNI	[8]
SecNonZero	NonZero check of shares	$t$ -SNI	[8]
SecFprUrsh	Right-shift keeping sticky bit updated	$t$ -SNI	[8]
SecFprNorm64	Normalization to $[2^{63}, 2^{64})$	$t$ -NI	[8]

is not present in the  $x$  information, in order to save one bit and gain in precision, is nevertheless very important. In particular, it enables us to normalise our shares correctly. Once these changes have been made, we can start working on the exponent.

Without any surprise, the first step consists of checking whether the result is an obvious zero or not. To do so, we compare  $e_y$  and  $\text{Zero}_f$  – which is the corresponding exponent to  $1 = 2^0$  – by calculating  $c_x = e_y - \text{Zero}_f$ . If  $c_x$  is negative,  $|x| < 1$  and all decimals can be removed by putting  $my = 0$ . We just need to be careful at the end of the algorithm because of some particular cases<sup>5</sup>. This step without taking care of particular case corresponds in reality to the truncature. Otherwise, the  $m_y$  mantissa will remain unchanged to be adjusted during the next steps.

A mathematical explanation of  $\text{Zero}_f$ 's choice can be summarized in a few simple calculus. In case  $f = \text{floor}$  or  $\text{trunc}$ ,  $\text{Zero}_f = 1023$ . Indeed if  $e_y - 1023 < 0$ ,

$$2^{e_x - 1023} \times (1 + m_x \times 2^{-52}) \leq 2^{-1} \times (1 + m_x \times 2^{-52}) < 1$$

We remark a difference for  $f = \text{round}$ . The main reason of this difference is that we want to avoid to reject the case where the binary decomposition of  $x$  contains a 1 at indices  $2^{-1} = 0.5$ , since we will no longer be able to round off the final result. This justifies the choice of  $\text{Zero}_f = 1022$  for rounding function. This time, if  $e_y - 1022 < 0$ :

$$2^{e_x - 1023} \times (1 + m_x \times 2^{-52}) \leq 2^{-2} \times (1 + m_x \times 2^{-52}) < 0.5$$

To sum up, after the first step, only  $my$  can change. The two values it can take are  $my$  itself (no changes) or 0. The second step The second step removes the decimals by shifting them to the right, using `RemoveDecimal` – algorithm 10. As this algorithm does not normalise the mantissa, we then apply the `SecFprNorm64` function, followed by a refresh (`SecFprNorm64` is a  $t$ -NI gadget) and a computation of shifted  $my$  and  $ey$  to set the mantissa back

<sup>5</sup> For example, if  $-1 < x < 0$ , we need to return encoding -1, not zero.

to bits  $[52 : 1]$  and update  $ey$ . Finally, the last step in the algorithm, before reformatting the initial encoding, is to apply the specific encoding of "0" if this is the result to be returned. To do this, we apply the  $\text{SetExponentZero}_f$  function – algorithm 12.

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**Algorithm 10:**  $\text{RemoveDecimal}_{\text{floor}}((my_i), (ey_i), (sy_i), (cx_i))$ 


---

**Data:** 64-bit boolean shares  $(my_i)_{1 \leq i \leq n}$  for mantissa value  $my$ ;  
 16-bit arithmetic shares  $(ey_i)_{1 \leq i \leq n}$  for exponent value  $ey$ ;  
 1-bit boolean shares  $(sy_i)_{1 \leq i \leq n}$  for sign value  $sy$   
 16-bit arithmetic shares  $(cx_i)_{1 \leq i \leq n}$  for value  $cx = \text{ex-2013}$ .  
**Result:** 64-bit boolean shares  $(my_i)_{1 \leq i \leq n}$  for mantissa value  
 $my \gg (52 - cx)$ ;  
 16-bit arithmetic shares  $(ey_i)_{1 \leq i \leq n}$  for exponent value  $ey + (52 - cx)$ ;  
 1  $cx_1 \leftarrow cx_1 - 52$ ;  
 2  $(c_i) \leftarrow \text{A2B}((cx_i))$ ;  
 3  $(cp_i) \leftarrow (c_i^{(16)})$ ;  
 4  $(cp_i) \leftarrow \text{SecNonZero}(cp_i)$ ;  
 5  $(c'_i) \leftarrow (-cp_i)$ ;  
 6  $(c_i) \leftarrow \text{SecAnd}((c_i), (c'_i))$ ;  
 7  $(cx_i) \leftarrow \text{B2A}((c_i))$ ;  
 8  $(cd_i) \leftarrow (-cx_i)$ ;  
 9  $(my_i), (rot_i) \leftarrow \text{SecFprUrsh}_f((my_i), (cd_i))$ ;  
 10  $(b_i) \leftarrow \text{SecNonZero}(rot_i)$ ;  
 11  $(cp_i) \leftarrow \text{SecAnd}((cp_i), (sy_i))$ ;  
 12  $(cp_i) \leftarrow \text{SecAnd}((cp_i), (b_i))$ ;  
 13  $(my_i) \leftarrow \text{SecAdd}((my_i), (cp_i))$ ;  
 14  $(ey_i) \leftarrow (ey_i + cd_i)$ ;  
 15 **return**  $(\text{Refresh}(my_i), \text{Refresh}(ey_i))$ ;

---

**RemoveDecimal<sub>floor</sub> – algorithm 10** As we explained at the beginning of this section, we need to remove the decimals from the input number. After checking a first result, the mantissa is equal to 0, if  $e - \text{Zero}_f < 0$ , or has remained unchanged. This leaves us with the case where a shift is needed. To do so, we write  $cx = e - 1023$  as the shift to be performed. The case where  $cx < 0$  has already been treated by modifying (or not) the mantissa, whatever shift is made, we would have  $0 \gg cx = 0$ . If  $cx \geq 52$ ,  $x$  is already in the correct form. In fact, we have :

$$| (-1)^s \times 2^{cx} \times (1 + m \times 2^{-52}) | \geq 2^{52} \times (1 + m \times 2^{-52}) = 2^{52} + m \in \mathbb{N}$$

To avoid removing information if it isn't needed, we replace  $cx$  by 0. Be careful if we want to round a number : it's important to compare  $cx$  to 53 instead of 52. The reason is that we first subtract to  $ey$  1022 instead of 1023 when we were checking is the result was 0 or not. So here we need to add 1. If  $0 \leq e_y - 1023 \leq 51$ ,

we need to modify the mantissa to remove – if necessary – the decimals. Remarks that even in this case, the mantissa can be in the good format. For example if  $x = 5$ , we have  $0 \leq e_y - 1023 = 1025 - 1023 = 2 \leq 51$  and  $x$  is an integer.

After verifying if  $x$  is or isn't a "big" number, i.e. without decimal in its representation, we can shift<sup>6</sup>  $my$  by  $cd = 52 - cx$  by applying a modified<sup>7</sup> SecFprUrsh<sub>*f*</sub> – algorithm 11: we don't want to keep the sticky bit, so we just removed this part. We had an extra output, returning the part we removed. Shift a value is a good thing, but that's not exactly the floor function. After this step we just have the truncature. When  $x$  positive there's no problem at all, floor function is equal to truncature. But when  $x$  is negative there is one. It comes from floor properties. Indeed if  $x < 0$  :

$$\text{floor}(-x) = \begin{cases} \text{trunc}(-x) & \text{if } x \in \mathbb{N} \\ \text{trunc}(-x) - 1 & \text{if } x \in \mathbb{R}/\mathbb{N} \end{cases} \quad (2)$$

On one hand, checking if  $x$  is negative is quite easy : we just have to take a look at if  $sy$  is equal to 1. On the other hand, checking that  $x$  is an integer is trickier. This is where the second modification to SecFprUrsh<sub>*f*</sub> comes in handy. As this function performs a rotation, the bit information is retained until a mask is applied to calculate the shift. We therefore decided to invert the mask in order to retain the suppressed information. If this information has zero Hamming weight then  $x$  is an integer. We can then use the SecNonZero gadget on this information, which will return 0 if  $x$  is an integer, and 1 if not. We denote this result  $b = \text{SecNonZero}((my_i^{[52-cx:1]}))$ . In the following truth table (Figure 4) if the result  $cp = s \text{ AND } b$  is 1, the mantissa must be changed. Another question

$sy$	$b$	$cp = sy \text{ AND } b$	Interpretation
0	$b$	0	Positive number
1	0	0	$x$ is an integer
1	1	1	Negative real

**Fig. 4.** Do we need to subtract  $s$ : Truth table

is now : how to subtract  $s$  from the mantissa if the mantissa is always positive? The sign of  $x$  is only define by  $sy$ , so  $|x| = 2^{e_y-1023} \times (1 + my \times 2^{-52})$ . We already know that if  $x$  is positive or an integer,  $my = my \pm 0 = my \pm cp$ . Now let's focus on  $x$  a non integer negative number. Let  $x$  be a positive real, from

<sup>6</sup> In pseudo-code the subtraction is implicate due to the format of a masked value.

<sup>7</sup> If we want to use Chen's algorithm, it's possible. We just need to compute one shift less then described in our algorithm and then compute an extra shift manually. The disadvantages of using it, is that you need to check if  $cx$  is different from 0 – to not compute a shift if it isn't needed – and the cost of computing the sticky bit.

Equation 2, we can write :

$$my = \lfloor \text{floor}(-x) \rfloor = \lfloor \text{trunc}(-x) - 1 \rfloor \quad (3)$$

By using  $\lfloor \text{trunc}(-x) \rfloor = \lfloor -\text{trunc}(x) \rfloor$ , we can transform Equation 3 :

$$my = \lfloor \text{floor}(-x) \rfloor = \lfloor -\text{trunc}(x) - 1 \rfloor = \lfloor \text{trunc}(x) + 1 \rfloor = my + cp \quad (4)$$

From this, we only have to compute a secure addition between  $cp$  and  $my$ . The

---

**Algorithm 11:** SecFprUrsh<sub>floor</sub>(( $my_i$ ), ( $cx_i$ ))

---

**Data:** 6-bit arithmetic shares ( $cx_i$ )<sub>1 ≤ i ≤ n</sub> for value  $cx$ ;  
 64-bit boolean shares ( $my_i$ )<sub>1 ≤ i ≤ n</sub> for sign value  $my$ .  
**Result:** 64-bit boolean shares ( $my'_i$ )<sub>1 ≤ i ≤ n</sub> for value  $my \gg cx$   
 64-bit boolean shares ( $rot_i$ )<sub>1 ≤ i ≤ n</sub> for value  $my^{[cx:1]}$ .

```

1 ( $m_i$ )1 ≤ i ≤ n ← ((1 << 63), 0, ..., 0);
2 for  $i$  from 1 to  $n$  do
3   Right-Rotate ( $my_i$ ) by  $cx_i$ ;
4   ( $my_i$ ) ← RefreshMasks(( $my_i$ ));
5   Right-Rotate ( $m_i$ ) by  $cx_i$ ;
6   ( $m_i$ ) ← RefreshMasks(( $m_i$ ));
7  $len$  ← 1;
8 while  $len \leq 32$  do
9   ( $m_i$ ) ← ( $m_i \oplus (m_i \gg len)$ );
10   $len$  ←  $len \ll 1$ ;
11 ( $my'_i$ ) ← SecAnd(( $my_i$ ), ( $m_i$ ));
12 ( $m_i$ ) ← ( $\neg(m_i)$ );
13 ( $rot_i$ ) ← SecAnd(( $my_i$ ), ( $m_i$ ));
14 return (( $my'_i$ ), ( $rot_i$ ));
```

---

last step of this algorithm is to add the shift  $cd$  to  $ey$  to keep this data update.

**SetExponentZero<sub>floor</sub>** – **algorithm 12** This last function, which is useful in the algorithm, uses the data collected throughout the calculations of the whole algorithm to modify  $ey$  and  $sy$  if the expected result is 0. The encoding of 0 is special because it is encoded by itself. It must therefore be possible to update  $ey$  and  $sy$  if necessary. For the floor function, we need the  $sy$  sign bit and the  $b$  mantissa zero condition. The desired result is zero only if  $|x| < 1$  and  $sy = 0$ . Recall that if  $sy = 1$  and  $|x| < 1$ ,  $\text{floor}(x) = -1$ . Minus 1's encoding is  $sy = 1$ ,  $ey = 1023$  and  $my = 0$ .

## 5 Security Proof

### 5.1 SetExponentZero<sub>floor</sub>

FAIRE LA PREUVE



**Algorithm 12:** SetExponentZero<sub>floor</sub>(( $ey_i$ ), ( $sy_i$ ), ( $b_i$ ))

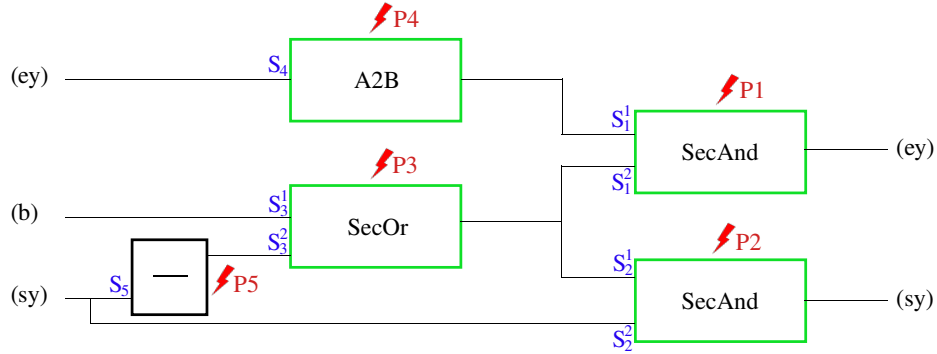
**Data:** 16-bit arithmetic shares ( $ey_i$ ) $_{1 \leq i \leq n}$  for exponent value  $ey$ ;  
 1-bit boolean shares ( $sy_i$ ) $_{1 \leq i \leq n}$  for sign value  $sy$   
 64-bit boolean shares ( $b_i$ ) $_{1 \leq i \leq n}$ .  
**Result:** 16-bit boolean shares ( $ey_i$ ) $_{1 \leq i \leq n}$  for exponent value  $ey + (52 - cx)$ ;  
 1-bit boolean shares ( $sy_i$ ) $_{1 \leq i \leq n}$  for sign value.

```

1 ( $ey_i$ )  $\leftarrow$  A2B( $ey_i$ );
2 ( $b'_i$ )  $\leftarrow$  ( $-sy_i$ );
3 ( $b'_i$ )  $\leftarrow$  SecOr( $b'_i$ , ( $b_i$ ));
4 ( $ey_i$ )  $\leftarrow$  SecAnd( $ey_i$ ,  $b'_i$ );
5 ( $sy_i$ )  $\leftarrow$  SecAnd( $sy_i$ ,  $b'_i$ );
6 return ( $ey_i$ ), ( $sy_i$ );

```

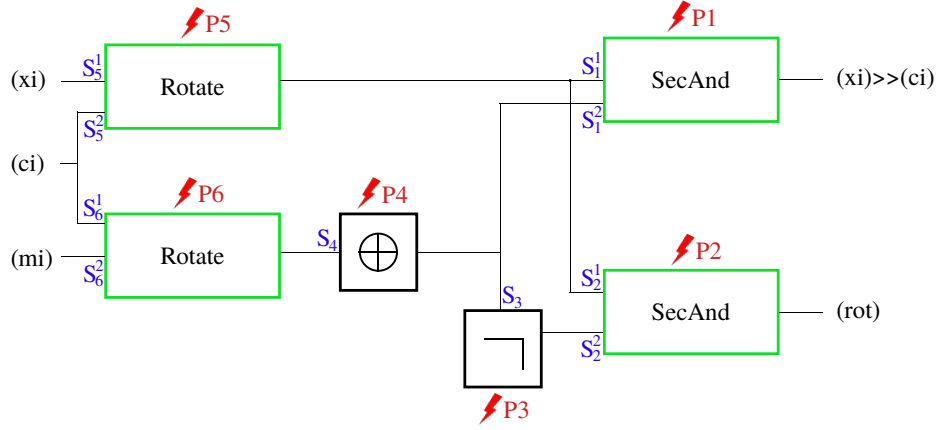
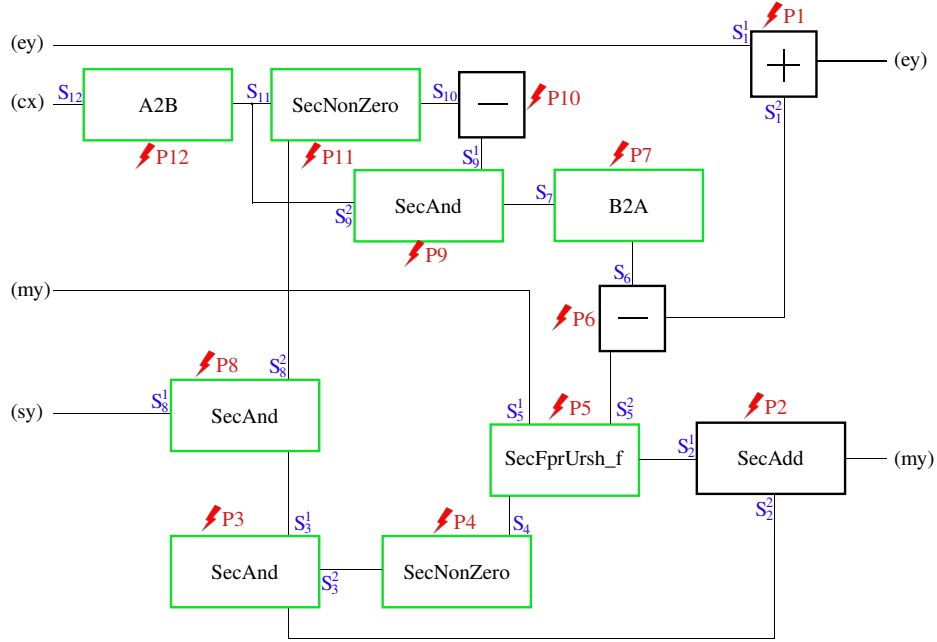
$-sy$	$b$	$-sy$ OR $b$	Interpretation
$0 \dots 0$	$0 \dots 0$	$0 \dots 0$	"Small" positive number : $ey = 0$ and $sy = 0$
$1 \dots 1$	$0 \dots 0$	$1 \dots 1$	"Small" negative number : $ey = 1023$ and $sy = 1$
$-sy$	$1 \dots 1$	$01 \dots 1$	Non zero number : $ey = ey$ and $sy = sy$

**Fig. 5.** Encoding 0, minus 1 or others: Truth table**Fig. 6.** SetExponentZero**5.2** SecFprUrsh<sub>floor</sub>

FAIRE LA PREUVE

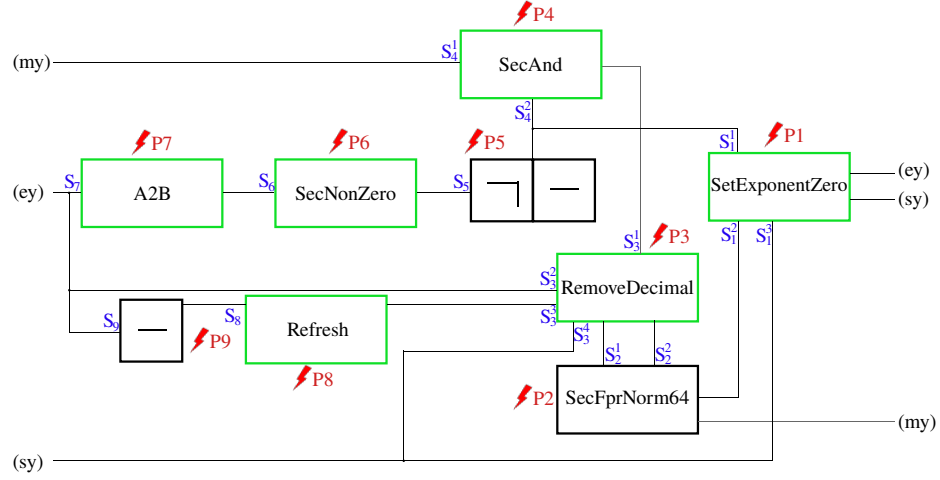
**5.3** RemoveDecimal<sub>floor</sub>

FAIRE LA PREUVE

Fig. 7. SecFprUrsh<sub>f</sub>Fig. 8. SecFprUrsh<sub>f</sub>

#### 5.4 SecBaseInt<sub>floor</sub>

FAIRE LA PREUVE

Fig. 9. SecFprUrsh<sub>f</sub>

## 6 Application to FALCON

### 6.1 Masked Floor Function For FALCON

### 6.2 Masking the Gaussian Sampler

## 7 Performances

## 8 Conclusion

## Acknowledgments

## References

1. Barthe, G., Belaïd, S., Dupressoir, F., Fouque, P.A., Grégoire, B., Strub, P.Y., Zucchini, R.: Strong non-interference and type-directed higher-order masking. In: Proceedings of the 2016 ACM SIGSAC Conference on Computer and Communications Security. p. 116–129. CCS '16, Association for Computing Machinery, New York, NY, USA (2016). <https://doi.org/10.1145/2976749.2978427>, <https://doi.org/10.1145/2976749.2978427>
2. Barthe, G., Belaïd, S., Dupressoir, F., Fouque, P.A., Grégoire, B., Strub, P.Y., Zucchini, R.: Strong non-interference and type-directed higher-order masking. In: Proceedings of the 2016 ACM SIGSAC Conference on Computer and Communications Security. pp. 116–129 (2016)
3. Barthe, G., Belaïd, S., Espitau, T., Fouque, P.A., Grégoire, B., Rossi, M., Tibouchi, M.: Masking the glp lattice-based signature scheme at any order. In: Advances in Cryptology–EUROCRYPT 2018: 37th Annual International Conference on the Theory and Applications of Cryptographic Techniques, Tel Aviv, Israel, April 29–May 3, 2018 Proceedings, Part II 37. pp. 354–384. Springer (2018)
4. Bernstein, D.J., Hülsing, A., Kölbl, S., Niederhagen, R., Rijneveld, J., Schwabe, P.: The sphincs+ signature framework. In: Proceedings of the 2019 ACM SIGSAC Conference on Computer and Communications Security. p. 2129–2146. CCS '19, Association for Computing Machinery, New York, NY, USA (2019). <https://doi.org/10.1145/3319535.3363229>, <https://doi.org/10.1145/3319535.3363229>
5. Bettale, L., Coron, J.S., Zeitoun, R.: Improved high-order conversion from boolean to arithmetic masking. IACR Transactions on Cryptographic Hardware and Embedded Systems pp. 22–45 (2018)
6. Bocher, M.: Introduction to the theory of fourier’s series. *Annals of Mathematics* **7**(3), 81–152 (1906), <http://www.jstor.org/stable/1967238>
7. Bos, J., Ducas, L., Kiltz, E., Lepoint, T., Lyubashevsky, V., Schanck, J.M., Schwabe, P., Seiler, G., Stehle, D.: Crystals - kyber: A cca-secure module-lattice-based kem. In: 2018 IEEE European Symposium on Security and Privacy (EuroS&P). pp. 353–367 (April 2018). <https://doi.org/10.1109/EuroSP.2018.00032>
8. Chen, K.Y., Chen, J.P.: Masking floating-point number multiplication and addition of falcon: First- and higher-order implementations and evaluations. *IACR Transactions on Cryptographic Hardware and Embedded Systems* **2024**(2), 276–303 (Mar 2024). <https://doi.org/10.46586/tches.v2024.i2.276-303>, <https://tches.iacr.org/index.php/TCHES/article/view/11428>
9. Chen, L., Chen, L., Jordan, S., Liu, Y.K., Moody, D., Peralta, R., Perlner, R.A., Smith-Tone, D.: Report on post-quantum cryptography, vol. 12. US Department of Commerce, National Institute of Standards and Technology ... (2016)
10. Coron, J.S., Großschädl, J., Tibouchi, M., Vadnala, P.K.: Conversion from arithmetic to boolean masking with logarithmic complexity. In: Fast Software Encryption: 22nd International Workshop, FSE 2015, Istanbul, Turkey, March 8–11, 2015, Revised Selected Papers 22. pp. 130–149. Springer (2015)
11. Ducas, L., Kiltz, E., Lepoint, T., Lyubashevsky, V., Schwabe, P., Seiler, G., Stehlé, D.: Crystals-dilithium: A lattice-based digital signature scheme. *IACR Transactions on Cryptographic Hardware and Embedded Systems* **2018**(1), 238–268 (Feb 2018). <https://doi.org/10.13154/tches.v2018.i1.238-268>, <https://tches.iacr.org/index.php/TCHES/article/view/839>
12. Espitau, T., Fouque, P.A., Gérard, B., Tibouchi, M.: Side-channel attacks on bliss lattice-based signatures: Exploiting branch tracing against strongswan and electromagnetic emanations in microcontrollers. In: Proceedings of the 2017 ACM

- SIGSAC Conference on Computer and Communications Security. p. 1857–1874. CCS '17, Association for Computing Machinery, New York, NY, USA (2017). <https://doi.org/10.1145/3133956.3134028>, <https://doi.org/10.1145/3133956.3134028>
13. Espitau, T., Fouque, P.A., Gérard, F., Rossi, M., Takahashi, A., Tibouchi, M., Wallet, A., Yu, Y.: Mitaka: A simpler, parallelizable, maskable variant of falcon. In: Dunkelman, O., Dziembowski, S. (eds.) *Advances in Cryptology – EUROCRYPT 2022*. pp. 222–253. Springer International Publishing, Cham (2022)
14. Groot Bruinderink, L., Hülsing, A., Lange, T., Yarom, Y.: Flush, gauss, and reload – a cache attack on the bliss lattice-based signature scheme. In: Gierlichs, B., Poschmann, A.Y. (eds.) *Cryptographic Hardware and Embedded Systems – CHES 2016*. pp. 323–345. Springer Berlin Heidelberg, Berlin, Heidelberg (2016)
15. Guerreau, M., Martinelli, A., Ricosset, T., Rossi, M.: The hidden parallel piped is back again: Power analysis attacks on falcon. *IACR Transactions on Cryptographic Hardware and Embedded Systems* **2022**(3), 141–164 (Jun 2022). <https://doi.org/10.46586/tches.v2022.i3.141-164>, <https://tches.iacr.org/index.php/TCHES/article/view/9697>
16. Howe, J., Prest, T., Ricosset, T., Rossi, M.: Isochronous gaussian sampling: From inception to implementation. In: Ding, J., Tillich, J.P. (eds.) *Post-Quantum Cryptography*. pp. 53–71. Springer International Publishing, Cham (2020)
17. Ishai, Y., Sahai, A., Wagner, D.: Private circuits: Securing hardware against probing attacks. In: *Advances in Cryptology-CRYPTO 2003: 23rd Annual International Cryptology Conference*, Santa Barbara, California, USA, August 17-21, 2003. Proceedings 23. pp. 463–481. Springer (2003)
18. Kahan, W.: Ieee standard 754 for binary floating-point arithmetic. *Lecture Notes on the Status of IEEE* **754**(94720-1776), 11 (1996)
19. Karabulut, E., Aysu, A.: Falcon down: Breaking falcon post-quantum signature scheme through side-channel attacks. In: *2021 58th ACM/IEEE Design Automation Conference (DAC)*. pp. 691–696 (Dec 2021). <https://doi.org/10.1109/DAC18074.2021.9586131>
20. Kocher, P.C.: Timing attacks on implementations of diffie-hellman, rsa, dss, and other systems. In: Koblitz, N. (ed.) *Advances in Cryptology — CRYPTO '96*. pp. 104–113. Springer Berlin Heidelberg, Berlin, Heidelberg (1996)
21. McCarthy, S., Howe, J., Smyth, N., Brannigan, S., O'Neill, M.: Bearz attack falcon: Implementation attacks with countermeasures on the falcon signature scheme. *Cryptology ePrint Archive*, Paper 2019/478 (2019), <https://eprint.iacr.org/2019/478>, <https://eprint.iacr.org/2019/478>
22. NIST: Module-lattice-based digital signature standard. NIST FIPS (2024). <https://doi.org/10.6028/NIST.FIPS.204.ipd>
23. NIST: Module-lattice-based key-encapsulation mechanism standard. NIST FIPS (2024). <https://doi.org/10.6028/NIST.FIPS.203.ipd>
24. NIST: Stateless hash-based digital signature standard. NIST FIPS (2024). <https://doi.org/10.6028/NIST.FIPS.205.ipd>
25. Pessl, P., Bruinderink, L.G., Yarom, Y.: To bliss-b or not to be: Attacking strongswan's implementation of post-quantum signatures. In: *Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security*. p. 1843–1855. CCS '17, Association for Computing Machinery, New York, NY, USA (2017). <https://doi.org/10.1145/3133956.3134023>, <https://doi.org/10.1145/3133956.3134023>
26. Prest, T., Fouque, P.A., Hoffstein, J., Kirchner, P., Lyubashevsky, V., Pornin, T., Ricosset, T., Seiler, G., Whyte, W., Zhang, Z.: Falcon. Post-Quantum Cryptography Project of NIST (2020)

27. Ravi, P., Chattopadhyay, A., D’Anvers, J.P., Baksı, A.: Side-channel and fault-injection attacks over lattice-based post-quantum schemes (kyber, dilithium): Survey and new results. *ACM Trans. Embed. Comput. Syst.* **23**(2) (mar 2024). <https://doi.org/10.1145/3603170>, <https://doi.org/10.1145/3603170>
28. Schneider, T., Paglialonga, C., Oder, T., Güneysu, T.: Efficiently masking binomial sampling at arbitrary orders for lattice-based crypto. In: *Public-Key Cryptography–PKC 2019: 22nd IACR International Conference on Practice and Theory of Public-Key Cryptography*, Beijing, China, April 14–17, 2019, *Proceedings, Part II* 22. pp. 534–564. Springer (2019)
29. Shor, P.W.: Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. *SIAM Review* **41**(2), 303–332 (1999). <https://doi.org/10.1137/S0036144598347011>, <https://doi.org/10.1137/S0036144598347011>
30. Zhang, S., Lin, X., Yu, Y., Wang, W.: Improved power analysis attacks on falcon. In: Hazay, C., Stam, M. (eds.) *Advances in Cryptology – EUROCRYPT 2023*. pp. 565–595. Springer Nature Switzerland, Cham (2023)

## Appendix

### Extract

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**Algorithm 13:** SecFprExtract( $x$ )

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**Data:** 64-bit boolean shares  $(x_i)_{1 \leq i \leq n}$  for value  $x$   
**Result:** 64-bit boolean shares  $(mx_i)_{1 \leq i \leq n}$  for mantissa value  $mx$ ;  
 16-bit arithmetic shares  $(ex_i)_{1 \leq i \leq n}$  for exponent value  $ex$ ;  
 1-bit boolean shares  $(sx_i)_{1 \leq i \leq n}$  for sign value  $s$ .

- 1  $(mx_i) \leftarrow (x_i^{[52:1]})$ ;
- 2  $(mx_i) \leftarrow \text{SecAdd}((mx_i), (2^{52}, 0, \dots, 0))$ ;      // add implicit bit in the mantissa
- 3  $(ex_i) \leftarrow (x_i^{[63:53]})$ ;
- 4  $(ex_i) \leftarrow \text{B2A}((ex_i))$ ;
- 5  $(sx_i) \leftarrow (x_i^{(64)})$ ;
- 6 **return**  $((mx_i), (ex_i), (sx_i))$ ;

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### Truncature Function: Gadgets

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**Algorithm 14:** RemoveDecimal<sub>trunc</sub>(( $my_i$ ), ( $ey_i$ ), ( $sy_i$ ), ( $cx_i$ ))

---

**Data:** 64-bit boolean shares ( $my_i$ ) $_{1 \leq i \leq n}$  for mantissa value  $my$ ;  
 16-bit arithmetic shares ( $ey_i$ ) $_{1 \leq i \leq n}$  for exponent value  $ey$ ;  
 1-bit boolean shares ( $sy_i$ ) $_{1 \leq i \leq n}$  for sign value  $sy$   
 16-bit arithmetic shares ( $cx_i$ ) $_{1 \leq i \leq n}$  for value  $cx = \text{ex-2013}$ .  
**Result:** 64-bit boolean shares ( $my_i$ ) $_{1 \leq i \leq n}$  for mantissa value  
 $my >> (52 - cx)$ ;  
 16-bit arithmetic shares ( $ey_i$ ) $_{1 \leq i \leq n}$  for exponent value  $ey + (52 - cx)$ ;  
 1  $cx_1 \leftarrow cx_1 - 52$ ; // check if  $0 \leq c < 51$   
 2 ( $c_i$ )  $\leftarrow \text{A2B}((cx_i))$ ;  
 3 ( $cp_i$ )  $\leftarrow (c_i^{(16)})$ ;  
 4 ( $cp_i$ )  $\leftarrow \text{SecNonZero}(cp_i)$ ;  
 5 ( $c'_i$ )  $\leftarrow (-cp_i)$ ; // if  $cp = 0$   $cx = 0$ . if not  $cx = cx$   
 6 ( $c_i$ )  $\leftarrow \text{SecAnd}((c_i), (cp_i))$ ;  
 7 ( $cd_i$ )  $\leftarrow \text{B2A}((c_i))$ ;  
 8 ( $cd_i$ )  $\leftarrow (-cd_i)$ ;  
 9 ( $my_i$ )  $\leftarrow \text{SecFprUrsh}_f((my_i), (cd_i))$ ; //  $my >> 52 - cx$   
 10 ( $ey_i$ )  $\leftarrow (ey_i + cd_i)$ ;  
 11 **return** (( $my_i$ ), ( $ey_i$ ));

---



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**Algorithm 15:** SetExponentZero<sub>trunc</sub>(( $ey_i$ ), ( $sy_i$ ), ( $b_i$ ))

---

**Data:** 16-bit arithmetic shares ( $ey_i$ ) $_{1 \leq i \leq n}$  for exponent value  $ey$ ;  
 1-bit boolean shares ( $sy_i$ ) $_{1 \leq i \leq n}$  for sign value  $sy$   
 64-bit boolean shares ( $b_i$ ) $_{1 \leq i \leq n}$ .  
**Result:** 16-bit boolean shares ( $ey_i$ ) $_{1 \leq i \leq n}$  for exponent value  $ey + (52 - cx)$ ;  
 1-bit boolean shares ( $sy_i$ ) $_{1 \leq i \leq n}$  for sign value.  
 1 ( $ey_i$ )  $\leftarrow \text{A2B}((ey_i))$ ;  
 2 ( $ey_i$ )  $\leftarrow \text{SecAnd}((ey_i), (b_i))$ ;  
 3 ( $sy_i$ )  $\leftarrow \text{SecAnd}((sy_i), (b_i))$ ;  
 4 **return** (( $ey_i$ ), ( $sy_i$ ));

---



## Round Function: Gadgets

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**Algorithm 16:** RemoveDecimal<sub>round</sub>(( $my_i$ ), ( $ey_i$ ), ( $sy_i$ ), ( $cx_i$ ))

---

**Data:** 64-bit boolean shares ( $my_i$ ) $_{1 \leq i \leq n}$  for mantissa value  $my$ ;  
 16-bit arithmetic shares ( $ey_i$ ) $_{1 \leq i \leq n}$  for exponent value  $ey$ ;  
 1-bit boolean shares ( $sy_i$ ) $_{1 \leq i \leq n}$  for sign value  $sy$   
 16-bit arithmetic shares ( $cx_i$ ) $_{1 \leq i \leq n}$  for value  $cx = ex-2013$ .  
**Result:** 64-bit boolean shares ( $my_i$ ) $_{1 \leq i \leq n}$  for mantissa value  
 $my \gg (52 - cx)$ ;  
 16-bit arithmetic shares ( $ey_i$ ) $_{1 \leq i \leq n}$  for exponent value  $ey + (52 - cx)$ ;  
 1-bit boolean shares ( $Rnd_i$ ) for value  $Rnd$  – the bit at position -1.

```

1  $cx_1 \leftarrow cx_1 - 53$ ; // check if  $0 \leq c < 51$ 
2  $(c_i) \leftarrow A2B((cx_i))$ ;
3  $(cp_i) \leftarrow (c_i^{(16)})$ ;
4  $(cp_i) \leftarrow SecNonZero(cp_i)$ ;
5  $(rshORnot_i) \leftarrow (-cp[i])$ ;
6  $(c'_i) \leftarrow (-cp_i)$ ; // if  $cp = 0$   $cx = 0$ . if not  $cx = cx$ 
7  $cx_1 \leftarrow cx_1 + 1$ ;
8  $(c_i) \leftarrow A2B((cx_i))$ ;
9  $(c_i) \leftarrow SecAnd((c_i), (cp_i))$ ;
10  $(cx_i) \leftarrow B2A((c_i))$ ;
11  $(cd_i) \leftarrow (-cx_i)$ ;
12  $(my_i) \leftarrow SecFprUrsh((my_i), (cd_i))$ ; //  $my \gg 53 - cx$ 
13  $(Rnd_i) \leftarrow (my_i^{(1)})$ ;
14  $(Rnd_i) \leftarrow SecNonZero(Rnd_i)$ ;
15  $(Rnd_i) \leftarrow SecAnd((Rnd_i), (rshORnot_i))$ ;
16  $(my1_i) \leftarrow (my_i) \gg 1$ ,  $(e1_i) \leftarrow (1, 0, \dots, 0)$ ;
17  $(my2_i) \leftarrow (my_i)$ ,  $(e2_i) \leftarrow (0, \dots, 0)$ ;
18  $(my1_i) \leftarrow SecAnd((my1_i), (rshORnot_i))$ ,  $(e1_i) \leftarrow$   

       $SecAnd((e1_i), (rshORnot_i))$ ;
19  $(rshORnot_i) \leftarrow (-rshORnot_i)$ ;
20  $(my2_i) \leftarrow SecAnd((my2_i), (rshORnot_i))$ ,  $(e2_i) \leftarrow$   

       $SecAnd((e2_i), (rshORnot_i))$ ;
21  $(my_i) \leftarrow SecOr((my1_i), (my2_i))$ ;
22  $(my_i) \leftarrow SecAdd((my_i), (Rnd_i))$ ;
23  $(e1_i) \leftarrow SecOr((e1_i), (e2_i))$ ;
24  $(e1_i) \leftarrow B2A((e1_i))$ ;
25  $(ey_i) \leftarrow (ey_i + e1_i)$ ;
26  $(ey_i) \leftarrow (ey_i + cd_i)$ ;
27 return ( $(my_i)$ , ( $ey_i$ ), ( $Rnd_i$ ));
```

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**Algorithm 17:** SetExponentZero<sub>round</sub> $((ey_i), (sy_i), (b_i), (Rnd_i))$ 


---

**Data:** 16-bit arithmetic shares  $(ey_i)_{1 \leq i \leq n}$  for exponent value  $ey$ ;

1-bit boolean shares  $(sy_i)_{1 \leq i \leq n}$  for sign value  $sy$

64-bit boolean shares  $(b_i)_{1 \leq i \leq n}$ .

**Result:** 16-bit boolean shares  $(ey_i)_{1 \leq i \leq n}$  for exponent value  $ey + (52 - cx)$ ;

1-bit boolean shares  $(sy_i)_{1 \leq i \leq n}$  for sign value.

```

1  $(ey_i) \leftarrow \text{A2B}((ey_i));$ 
2  $(b'_i) \leftarrow (Rnd_i);$ 
3  $(b'_i) \leftarrow \text{SecOr}((b'_i), (b_i));$ 
4  $(ey_i) \leftarrow \text{SecAnd}((ey_i), (b'_i));$ 
5  $(sy_i) \leftarrow \text{SecAnd}((sy_i), (b'_i));$ 
6 return  $((ey_i), (sy_i));$ 

```

---